Bayesian Time Series

Bayesian Time Series

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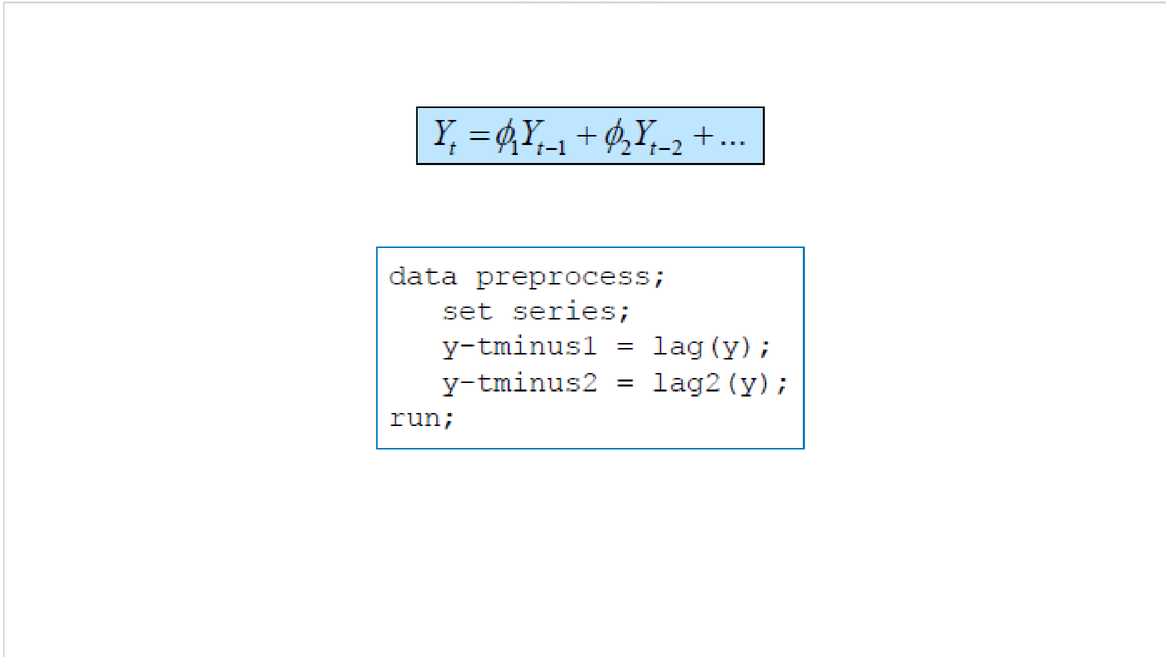
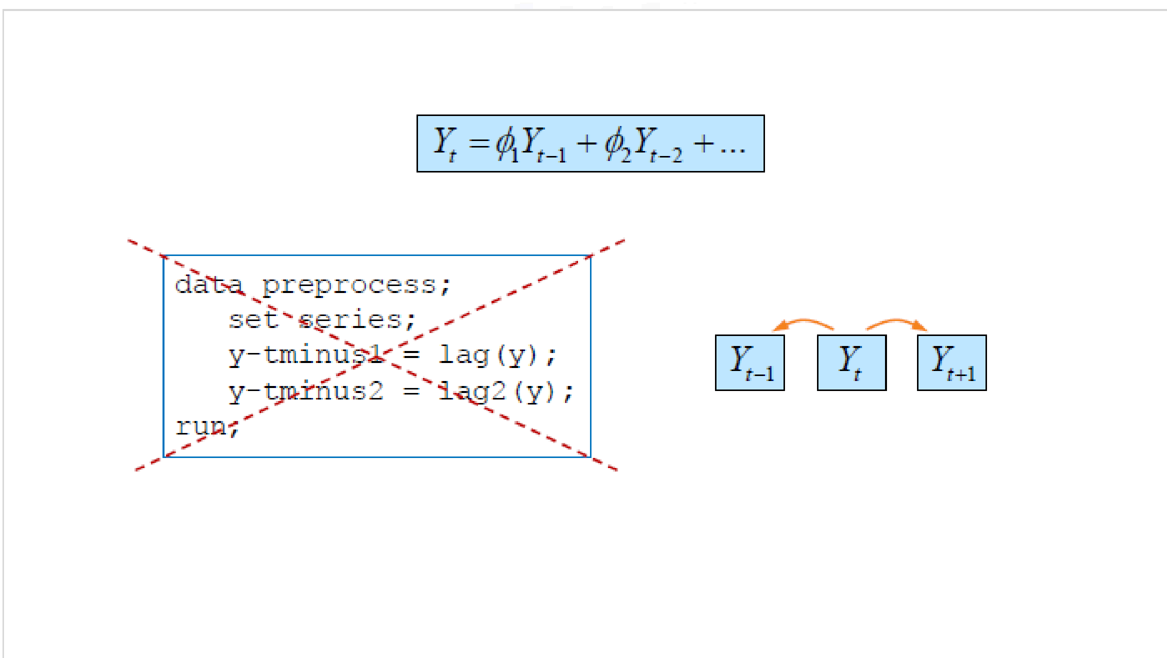
Objectives:

- Referencing (Lag) and (Next) values in Time Series analysis
- Adding Autoregressive components
- Adding Seasonal components dynamically
- Adding Exogenous components
- Forecasting Using PREDDIST



Let's begin our discussion of Bayesian time series structure with autoregressive elements. Prior to SAS/STAT 14.1, coding these elements was more time consuming than it is now. To fit autoregressive time series models in the past, you had to preprocess the data. This put the work on you to create variables within the data set that contained the lagged values of the response series.



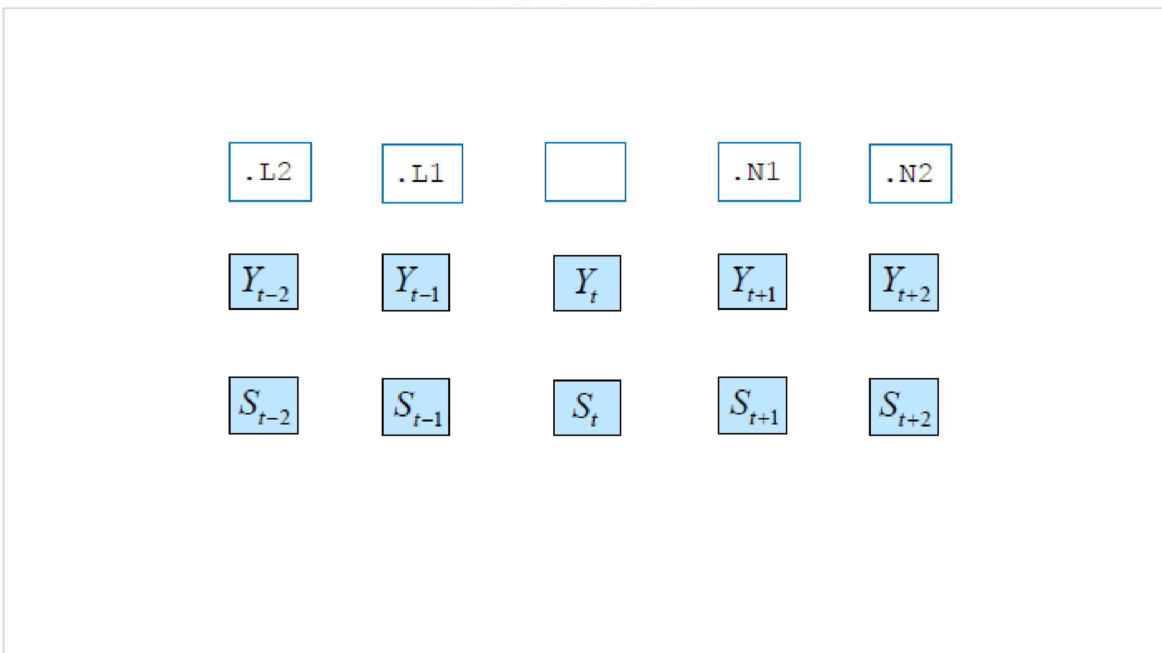
Now, we have access to lead and lagged values for random variables that are indexed. What do I mean by indexed? Two types of random variables are indexed in the MCMC procedure.

```
model y~normal(mu,var);
```

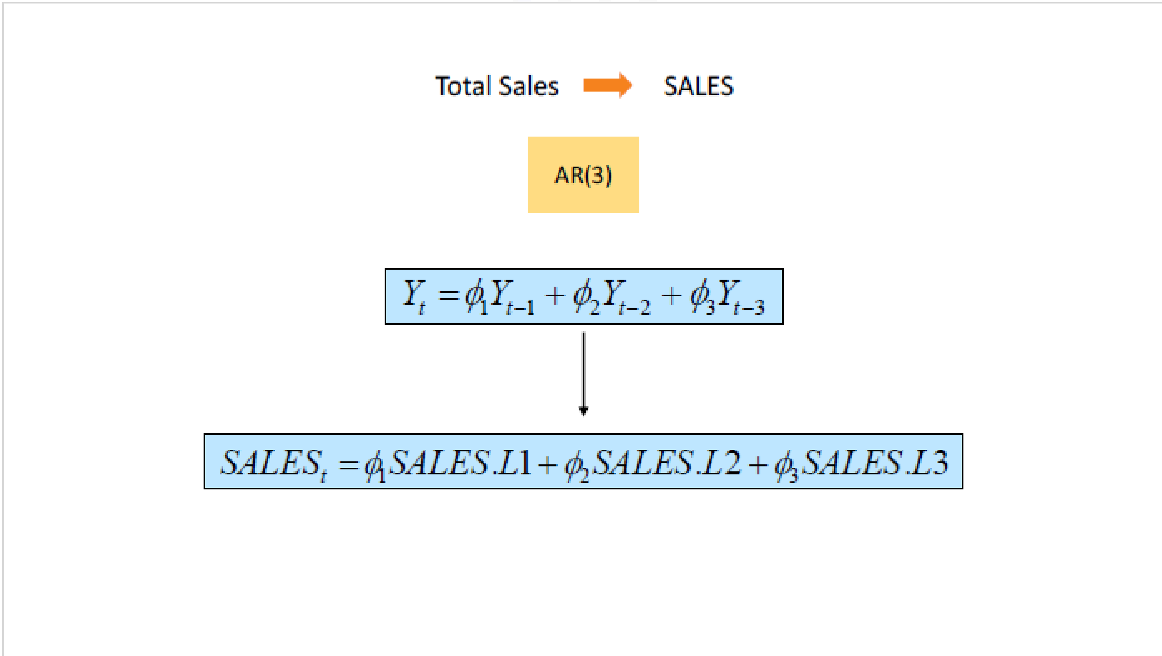
```
random s~normal(0,var)/
                subject=qtr;
```

obs	y
1	24
2	32
3	30
4	26
5	22

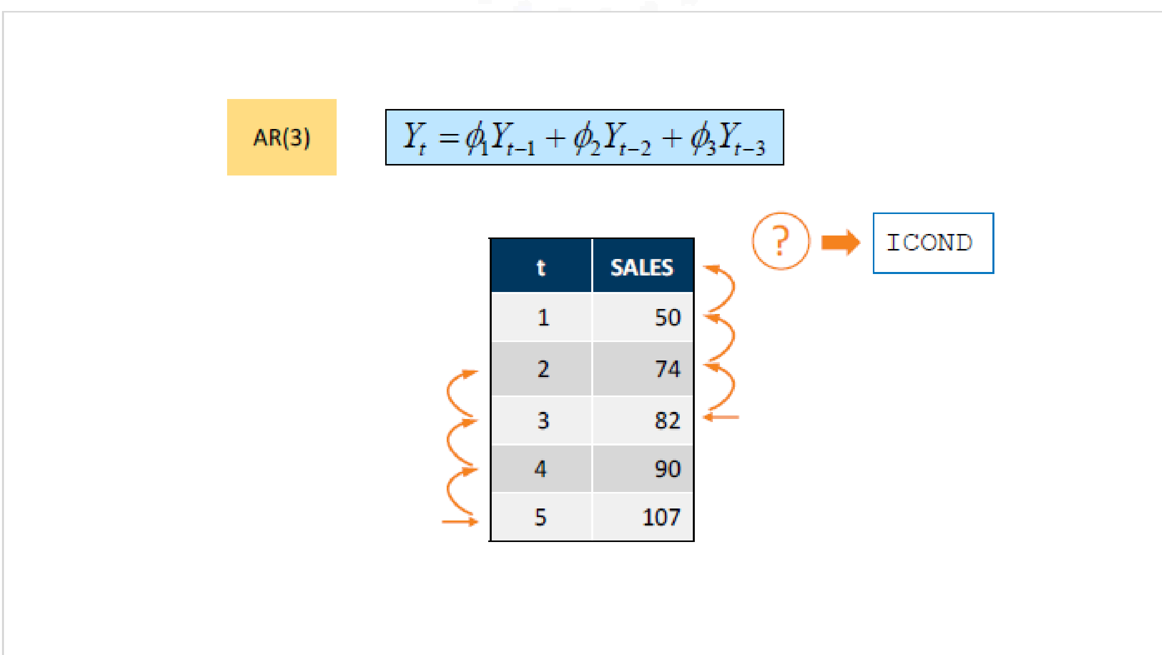
The first is the response variable, our time series. In the MODEL statement, this variable is indexed by observations. The second is a random variable placed in the RANDOM statement. This variable is indexed by the SUBJECT= option.



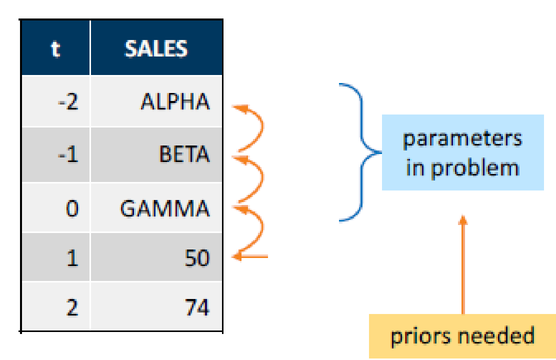
To access both the lead and lagged values of these indexed variables, we state the variable name followed by either .L or .N to access lagged or next values respectively. Let's look at an example.



Suppose you have a time series of total sales data accumulated monthly. During your exploration, you determine that a thirdorder autoregressive model (AR(3)) would be appropriate for your series. So, three lagged values of the response series would be included in your time series model. When adding these elements to our model, we would code SALES.L1 for the first lagged value, SALES.L2 for the second, and SALES.L3 for the third. We do not use it here, but if we wanted to look forward in time, we would use SALES.N1 to access the value one time unit ahead in the series. Creating these lagged values using a data set was not too complex, but with .L and .N, it is much easier. However, there was no built-in way to account for the initial states of these lagged variables. What do I mean by initial states?



To forecast the total sales at time position 5 in the series, we would include the values from time positions 4, 3, and 2. This is not a problem because those values are found within our response series. What happens if we wanted to forecast position 3 in the series? We have the total sales of time positions 2 and 1. You might now see the problem that we have. As we approach the start of the time series, we run out of information for our lagged time values. This was a problem before the ICOND= option. In the MODEL statement or the RANDOM statement, we can now account for these initial states (or initial conditions).



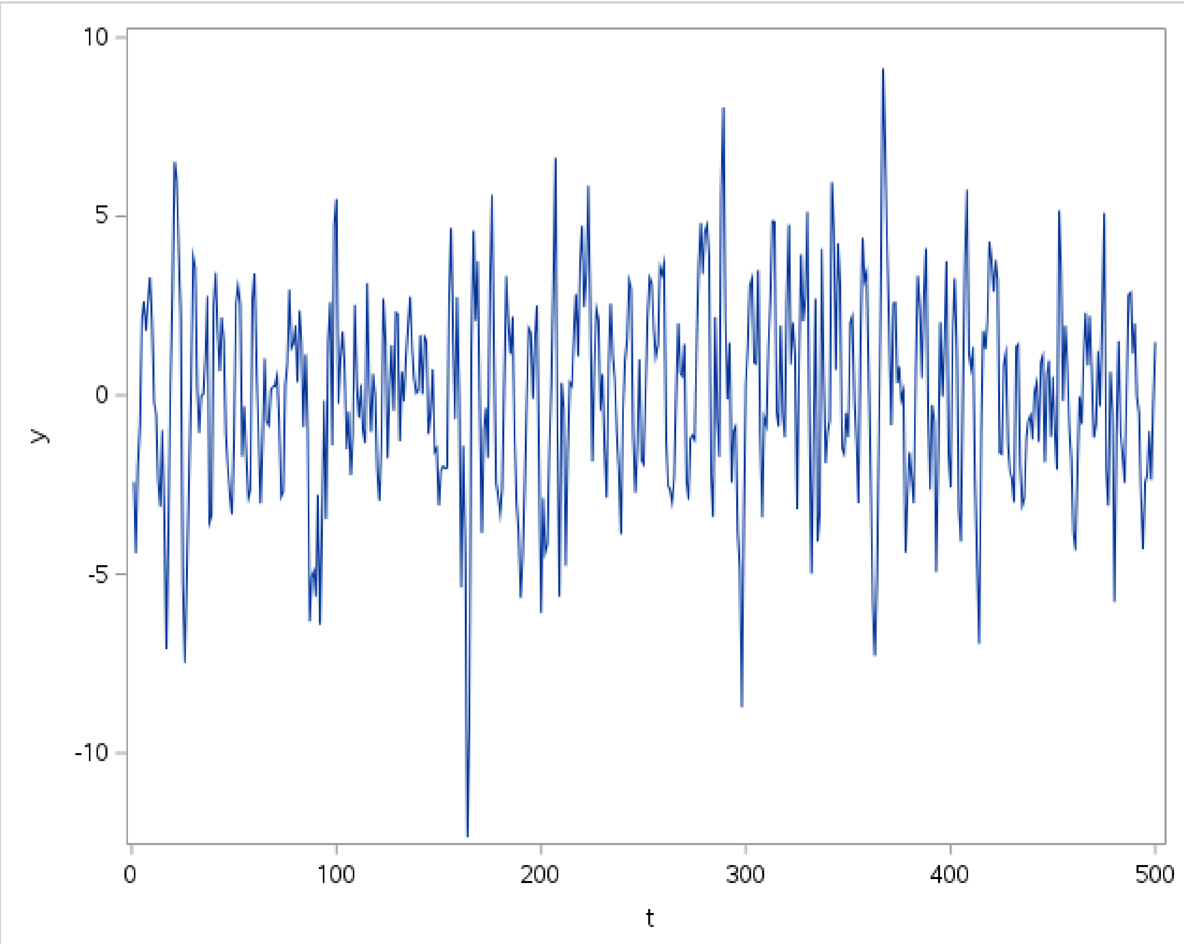
In our example, we can include ICOND= (alpha beta gamma) in the MODEL statement. These initial states are treated as parameters in the problem, and we place priors against them just like any other parameter in our model. Three items are listed due to the maximum number of initial states needed being three when we are at the very beginning of the series. Using this technique, we do not lose data at the front from missing values.

In [1]:

```
*Creating AR(4) data set;

%let N = 500;
%let p = 4;
%let sigma=2;
%let constant = 0;
data ar4(keep= t y);
  call streaminit(12345);
  array phi phi1 - phi&p (.8, -.64, .512, -.4096);
  array yLag   yLag1 - yLag&p;
  do j = 1 to dim(yLag);
    yLag[j] = 0;
  end;
  constant=&constant;
  do t = -100 to &N;
    e = rand('normal',0,&sigma);
    y = e;
    do j = 1 to dim(phi);
      y = y + phi[j] * yLag[j];
    end;
```

```
    y=y+constant;
    if t > 0 then output;
    do j = dim(yLag) to 2 by -1;
      yLag[j] = yLag[j-1];
    end;
    yLag[1] = y;
  end;
run;


ods graphics on;
proc sgplot data=ar4;
  series x=t y=y;
run;
```

SAS server started using Context SAS Studio compute context with SESSION_ID=93dd6aec-4b16-49c0-9d4d-cb4d015701c2-ses0000



In [2]:

```
*AR(4) Analysis;

proc mcmc data=ar4 nmc=100000 seed=100 nthreads=8 propcov=quanew
       outpost=ar4example;
  parms phi_1 phi_2 phi_3 phi_4;
  parms sigma2 1;
  parms Y_0 Y_1 Y_2 Y_3;
  prior phi_:~normal(0,var=1000);
  prior sigma2 ~ igamma(shape = 3/10, scale = 10/3);
```

```
  prior Y_: ~ n(0, var=1000 );
  mu=phi_1*y.l1 + phi_2*y.l2 + phi_3*y.l3 + phi_4*y.l4;
  model y~normal(mu, var=sigma2)
        icond=(Y_3 Y_2 Y_1 Y_0);
run;

proc contents data=ar4example;
run;
```

The MCMC Procedure

Number of Observations Read	500
Number of Observations Used	500

Parameters					
Block	**Parameter**	**Sampling Method**	**Threads**	**Initial Value**	**Prior Distribution**
1	phi_1	N-Metropolis	8	0	normal(0,var=1000)
	phi_2			0	normal(0,var=1000)
	phi_3			0	normal(0,var=1000)
	phi_4			0	normal(0,var=1000)
2	sigma2	N-Metropolis	8	1.0000	igamma(shape = 3/10, scale = 10/3)
3	Y_0	N-Metropolis	8	0	normal(0, var=1000)
	Y_1			0	normal(0, var=1000)
	Y_2			0	normal(0, var=1000)
	Y_3			0	normal(0, var=1000)

The MCMC Procedure

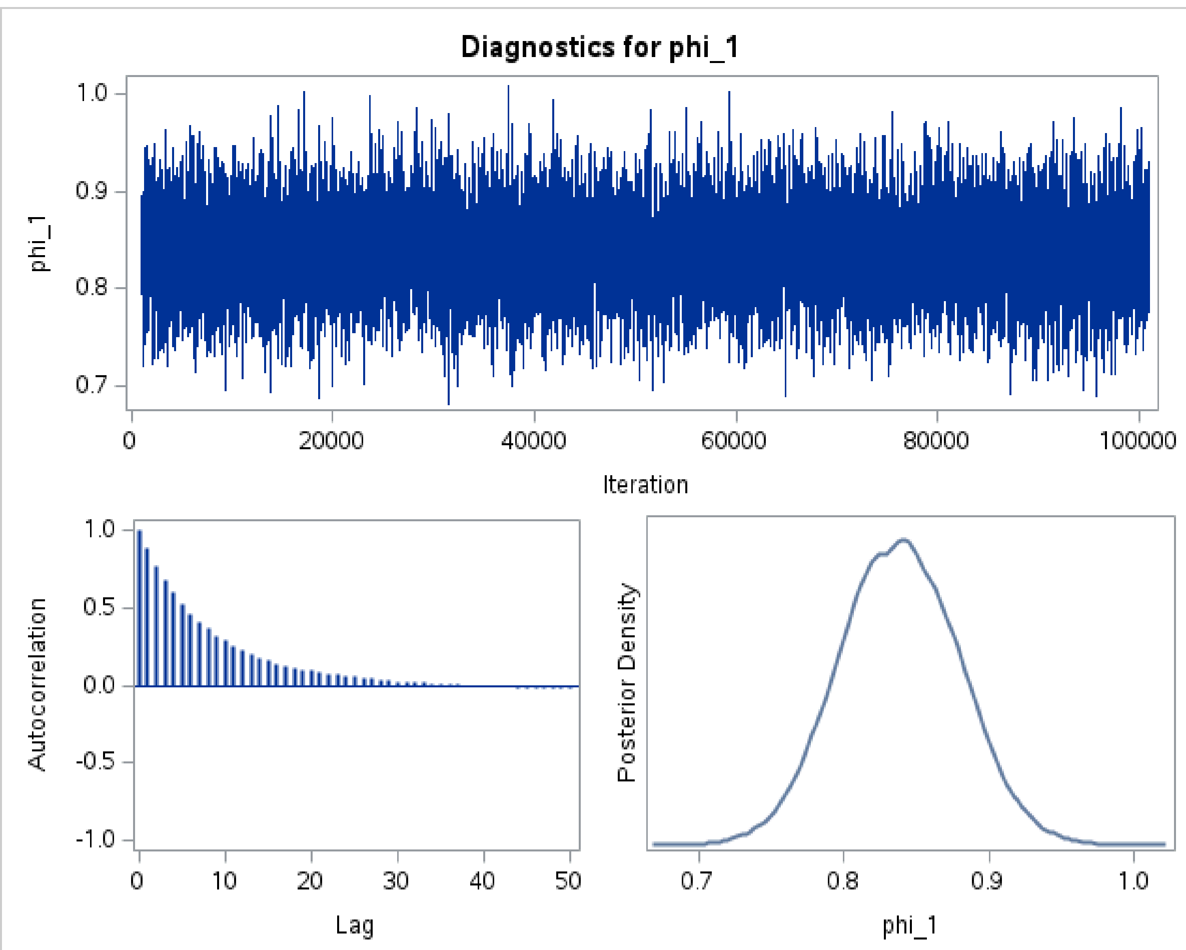
Posterior Summaries and Intervals					
Parameter	**N**	**Mean**	**Standard Deviation**	**95% HPD Interval**	
phi_1	100000	0.8382	0.0411	0.7584	0.9184
phi_2	100000	-0.5762	0.0518	-0.6749	-0.4729
phi_3	100000	0.4835	0.0519	0.3815	0.5847
phi_4	100000	-0.3951	0.0414	-0.4773	-0.3153
sigma2	100000	4.1109	0.2641	3.6000	4.6336
Y_0	100000	1.2669	5.0179	-8.7476	11.0021
Y_1	100000	0.6614	7.9088	-15.0633	16.2767
Y_2	100000	4.4767	7.9868	-11.0639	20.6039
Y_3	100000	13.0851	8.4739	-3.3648	29.8601

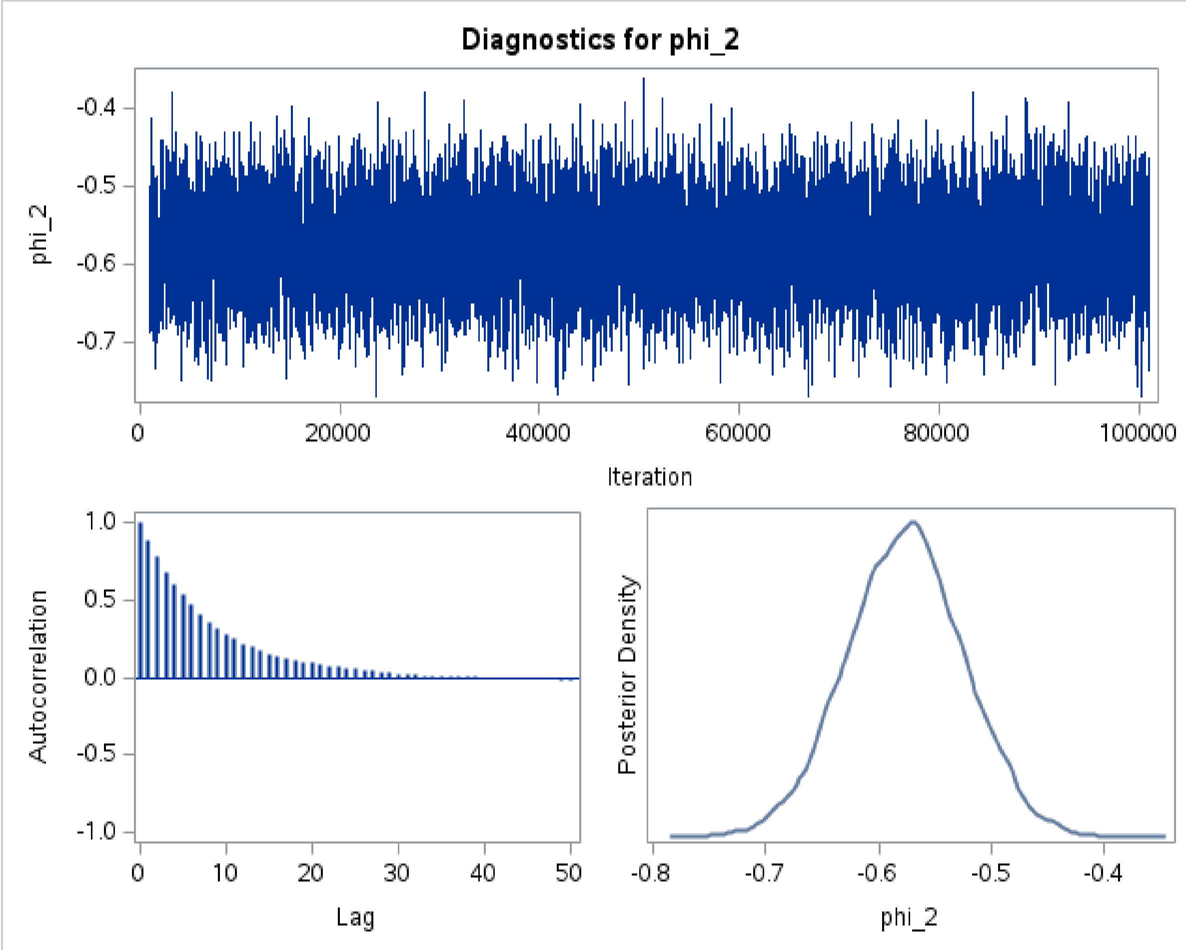
The MCMC Procedure

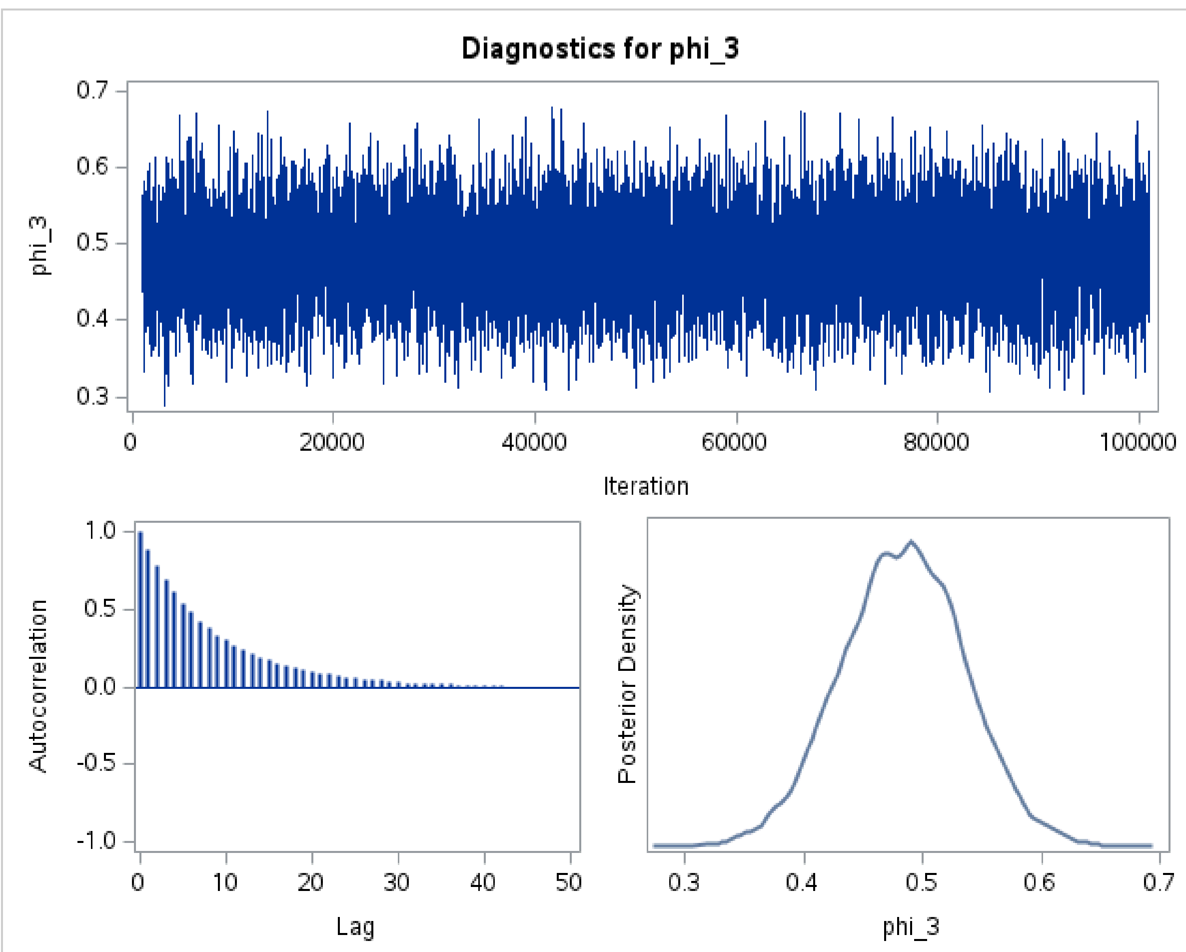
Effective Sample Sizes			
Parameter	ESS	Autocorrelation Time	Efficiency
phi_1	6244.6	16.0140	0.0624
phi_2	6271.0	15.9465	0.0627
phi_3	6002.0	16.6612	0.0600
phi_4	5626.9	17.7716	0.0563
sigma2	17163.8	5.8262	0.1716
Y_0	5997.8	16.6728	0.0600
Y_1	5272.0	18.9682	0.0527
Y_2	6358.4	15.7273	0.0636
Y_3	6677.8	14.9749	0.0668

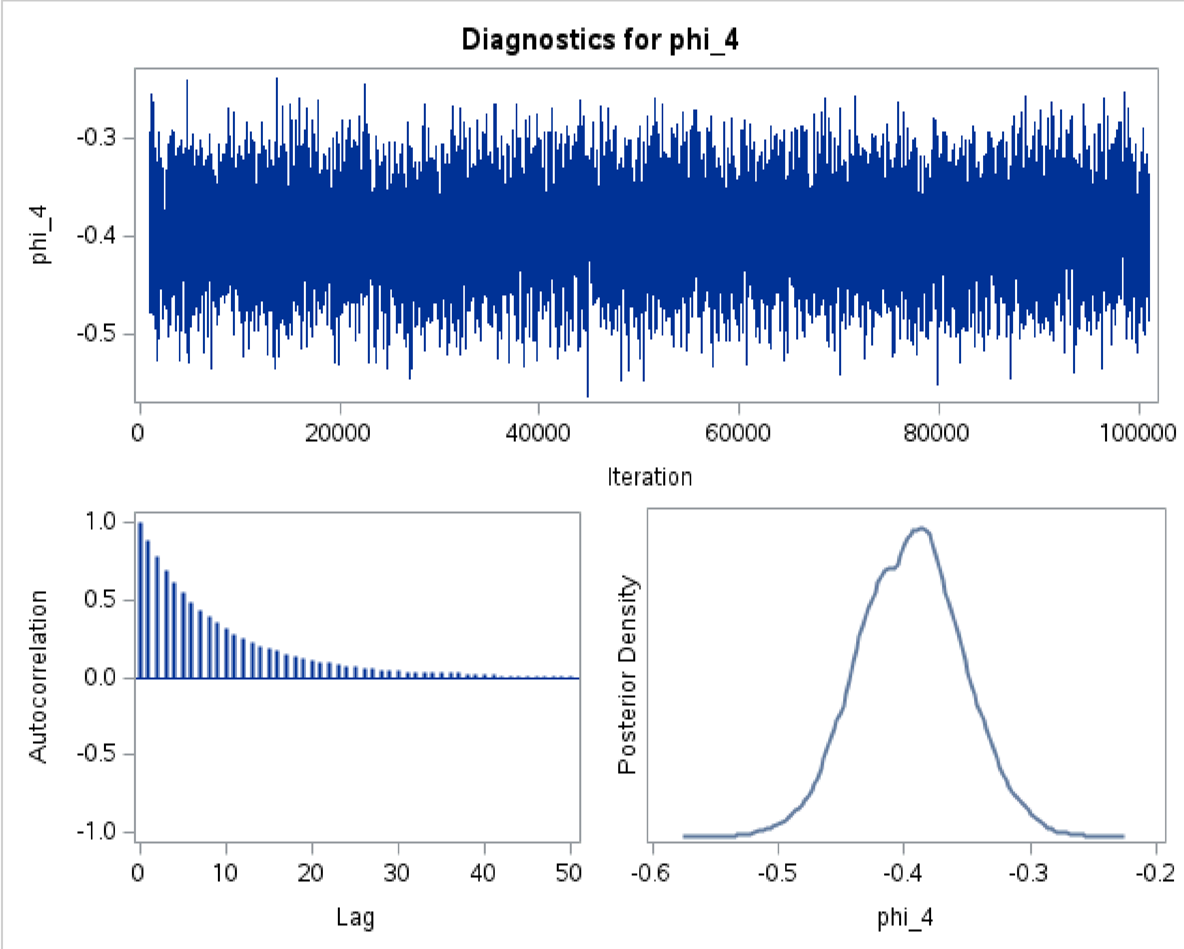
The SAS System

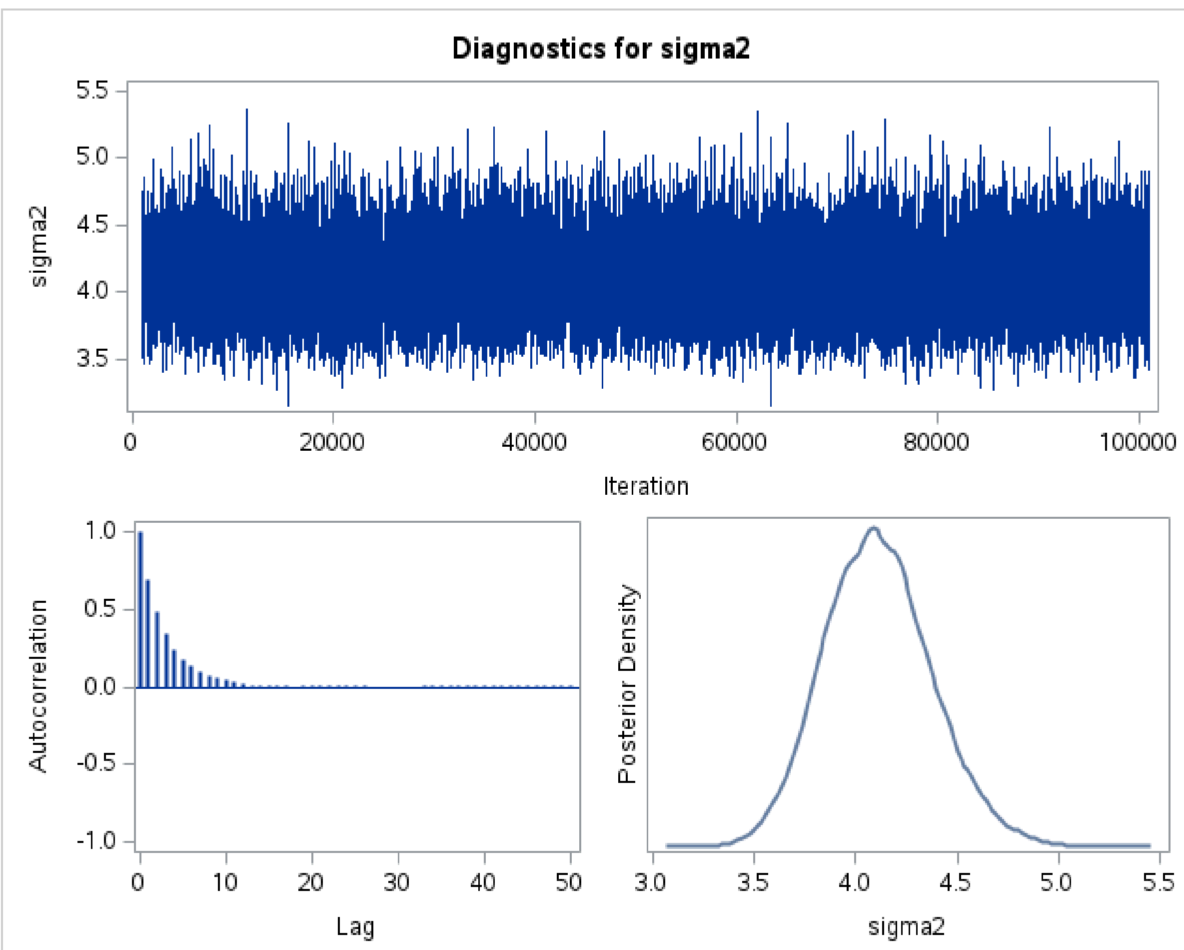
The MCMC Procedure

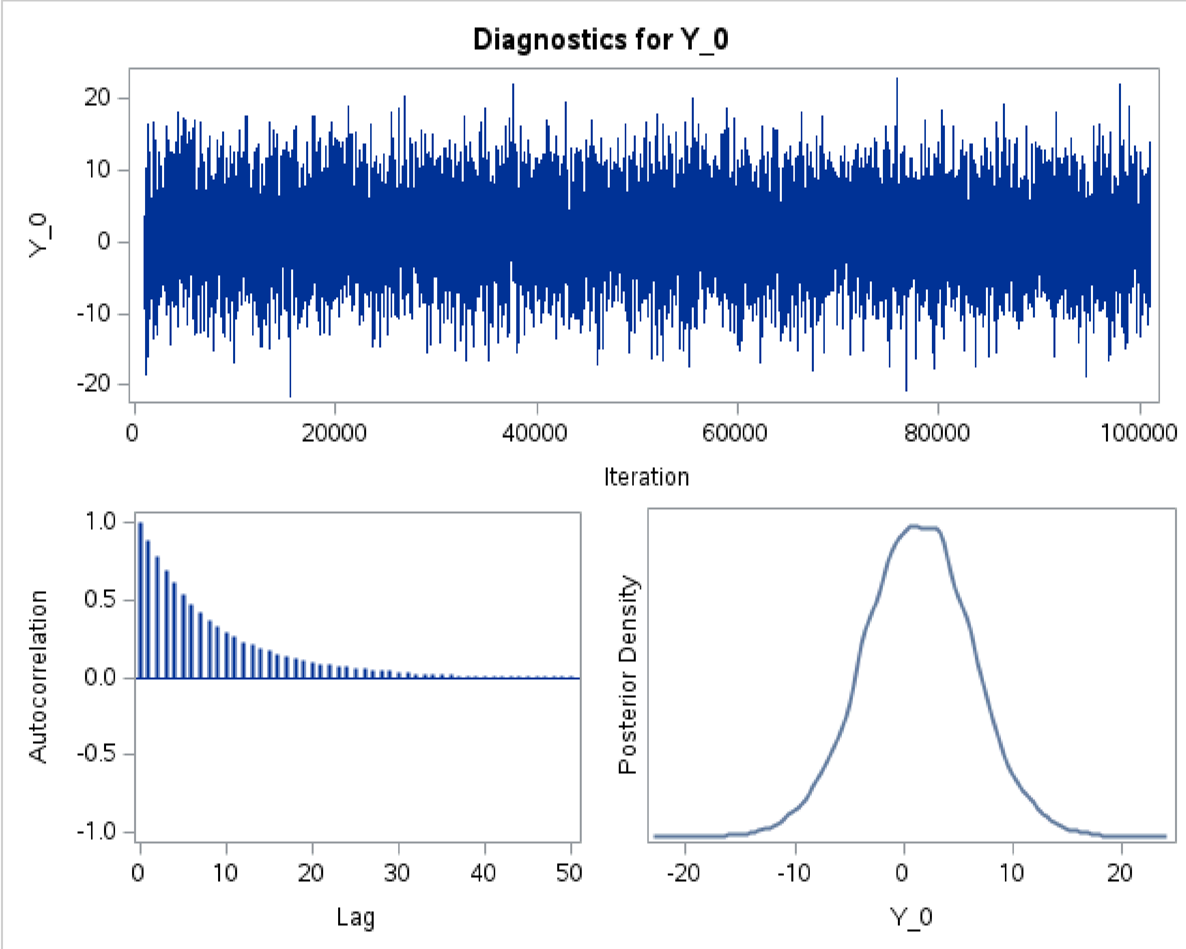


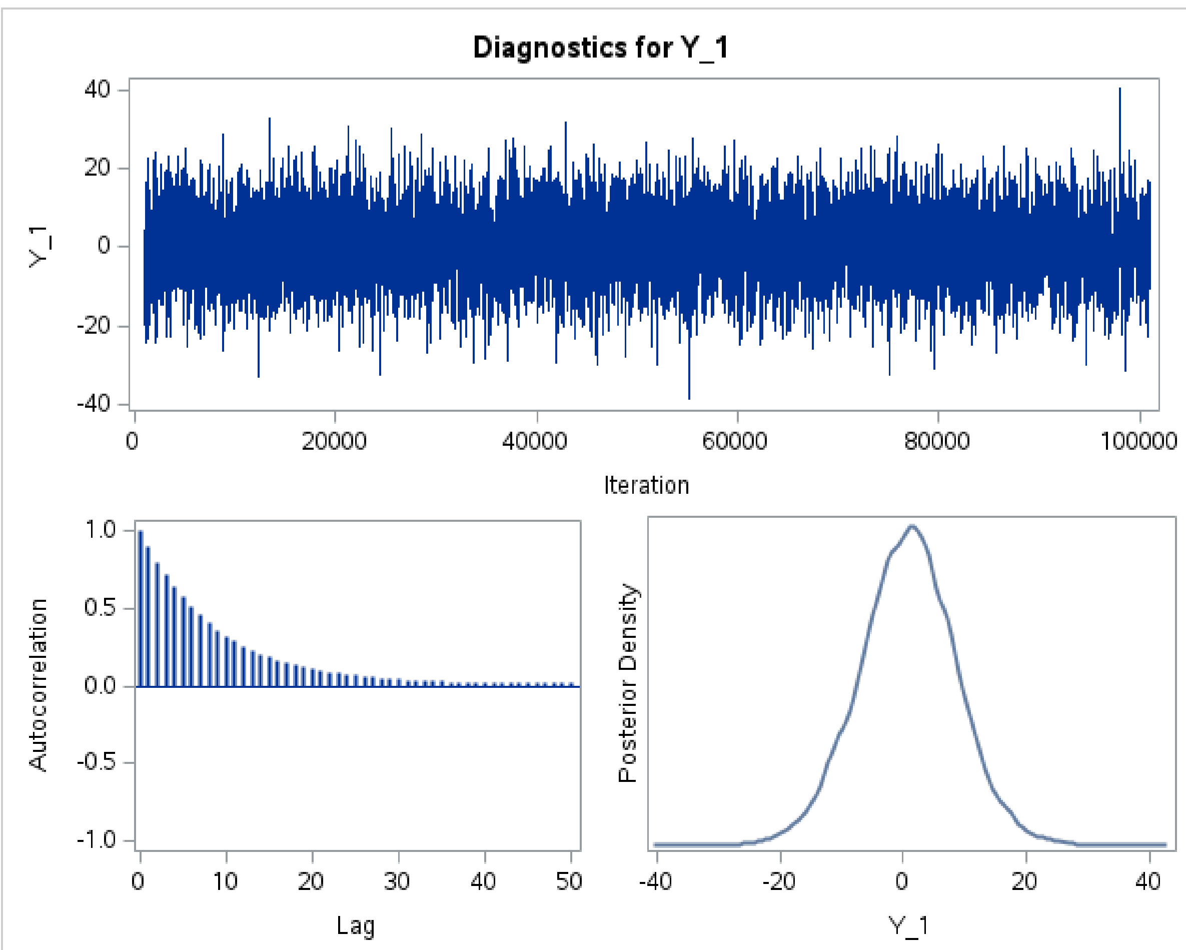

Diagnostics for phi_2
phi_2
Iteration
Autocorrelation
Lag
Posterior Density
phi_2

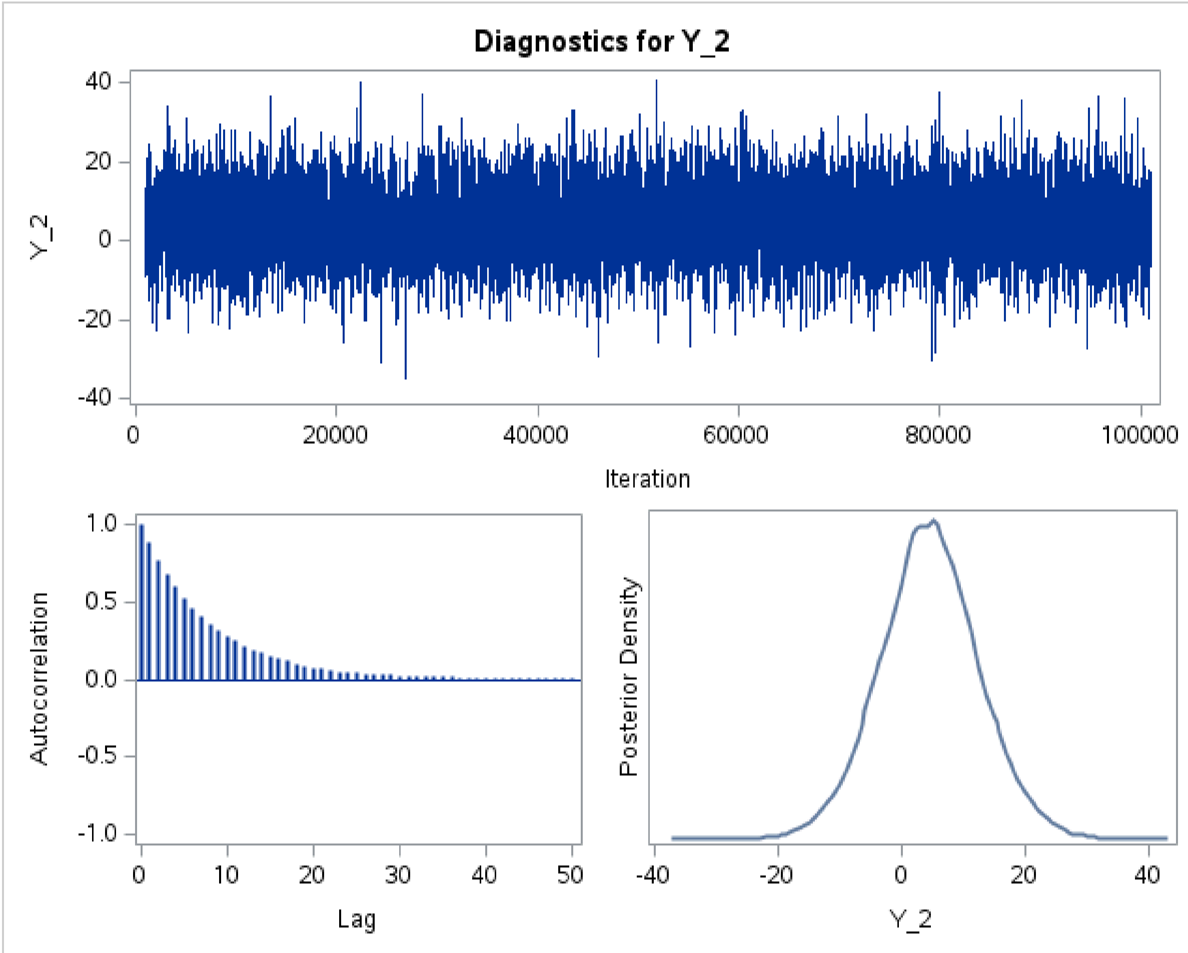

Diagnostics for phi_3
phi_3
Iteration
Autocorrelation
Lag
Posterior Density
phi_3

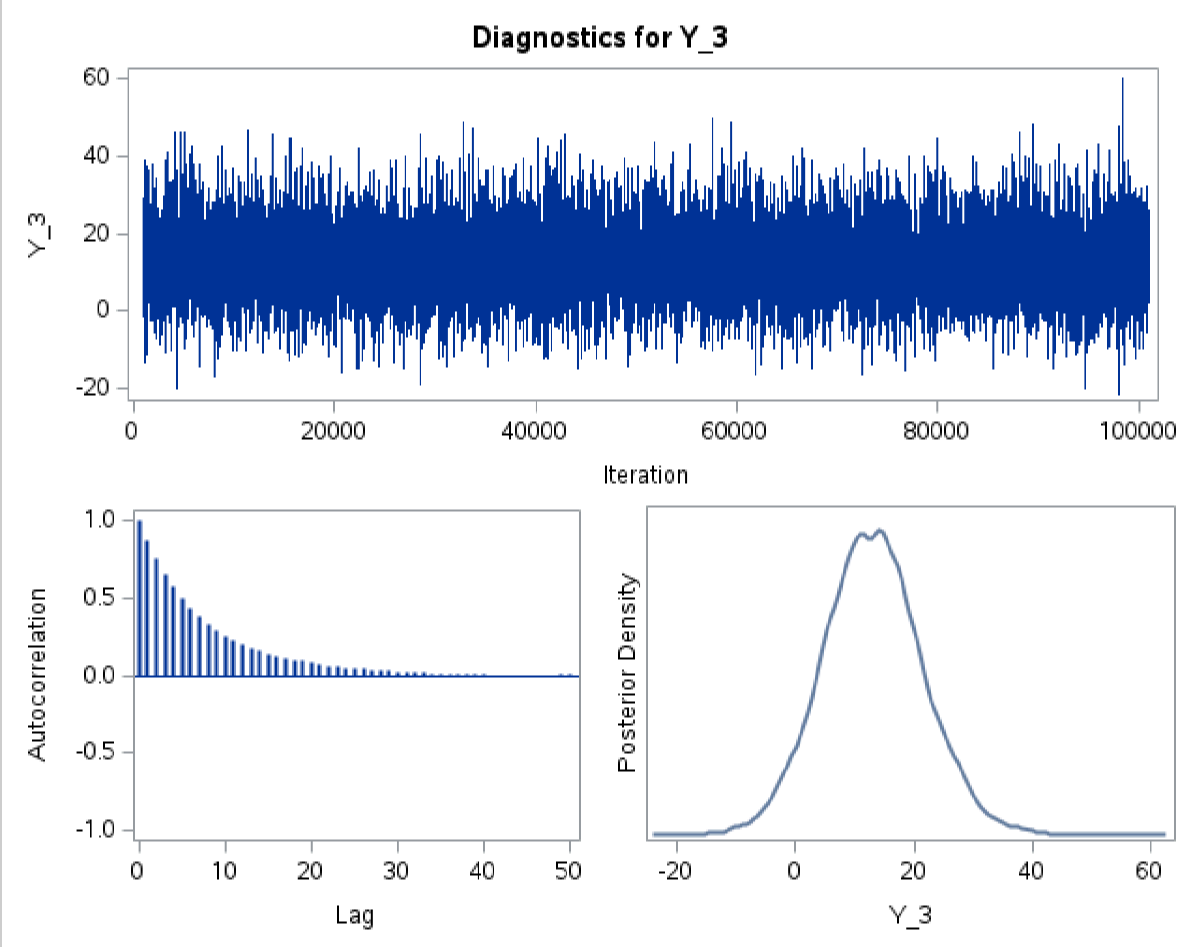

Diagnostics for phi_4
phi_4
-0.3
-0.4
-0.5
0
20000
40000
60000
80000
100000
Iteration
Autocorrelation
1.0
0.5
0.0
-0.5
-1.0
0
10
20
30
40
50
Lag
Posterior Density
-0.6
-0.5
-0.4
-0.3
-0.2
phi_4


Diagnostics for sigma2
sigma2
5.5
5.0
4.5
4.0
3.5
0
20000
40000
60000
80000
100000
Iteration
Autocorrelation
1.0
0.5
0.0
-0.5
-1.0
0
10
20
30
40
50
Lag
Posterior Density
3.0
3.5
4.0
4.5
5.0
5.5
sigma2


Diagnostics for Y_0
Y_0
Iteration
Autocorrelation
Lag
Posterior Density
Y_0


Diagnostics for Y_1
Y_1
Iteration
Autocorrelation
Lag
Posterior Density
Y_1


Diagnostics for Y_2
Y_2
40
20
0
-20
-40
0
20000
40000
60000
80000
100000
Iteration
Autocorrelation
1.0
0.5
0.0
-0.5
-1.0
0
10
20
30
40
50
Lag
Posterior Density
-40
-20
0
20
40
Y_2



The SAS System

The CONTENTS Procedure

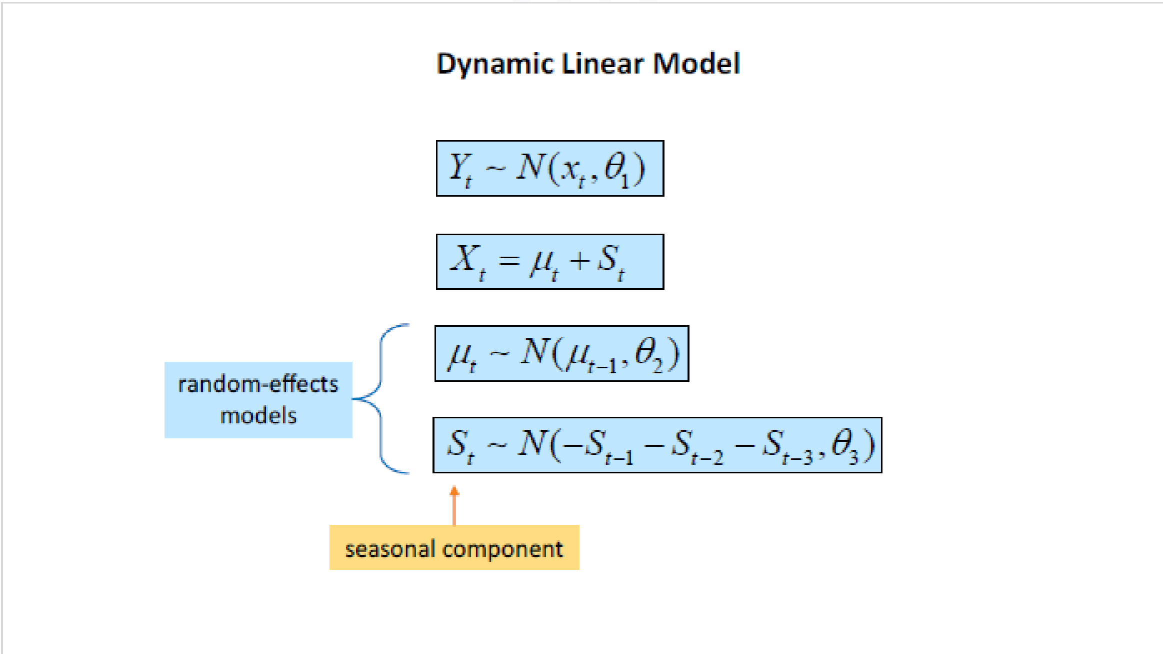
Data Set Name	WORK.AR4EXAMPLE	**Observations**	100000
Member Type	DATA	**Variables**	13
Engine	V9	**Indexes**	0
Created	04/15/2025 14:22:03	**Observation Length**	104
Last Modified	04/15/2025 14:22:03	**Deleted Observations**	0
Protection		**Compressed**	NO
Data Set Type		**Sorted**	NO
Label			
Data Representation	SOLARIS_X86_64, LINUX_X86_64, ALPHA_TRU64, LINUX_IA64, LINUX_POWER_64		
Encoding	utf-8 Unicode (UTF-8)		

Engine/Host Dependent Information

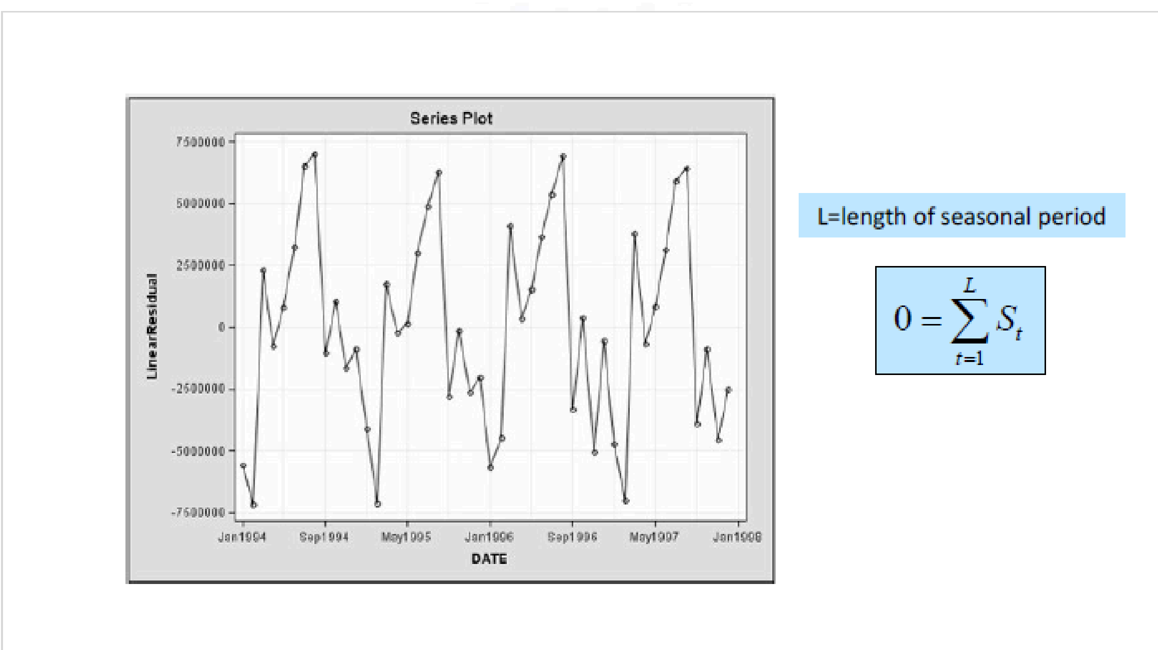
Engine/Host Dependent Information	
Data Set Page Size	65536
Number of Data Set Pages	160
First Data Page	1
Max Obs per Page	629
Obs in First Data Page	595
Number of Data Set Repairs	0
Filename	/opt/sas/viya/config/var/tmp/compsrv/default/93dd6aec-4b16-49c0-9d4d-cb4d015701c2/SAS_workEEB80000020B_sas-compute-server-a5564084-adbf-4dd6-816f-56d632996a06-16126/ar4example.sas7bdat
Release Created	V.0400M0
Host Created	Linux
Inode Number	35007694
Access Permission	rw-r--r--
Owner Name	UNKNOWN
File Size	10MB
File Size (bytes)	10551296

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
1	Iteration	Num	8	8.	
12	LogLike	Num	8	D8.	Log-Likelihood Value
13	LogPost	Num	8	D8.	Log Posterior Density
11	LogPrior	Num	8	D8.	Log Prior Density
7	Y_0	Num	8	D8.	
8	Y_1	Num	8	D8.	
9	Y_2	Num	8	D8.	
10	Y_3	Num	8	D8.	
2	phi_1	Num	8	D8.	
3	phi_2	Num	8	D8.	
4	phi_3	Num	8	D8.	
5	phi_4	Num	8	D8.	

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
6	sigma2	Num	8	D8.	



Performing a Bayesian time series analysis also enables you to use a dynamic linear model setup. This setup is a very general type of nonstationary time series model. With this, you can create models with time-varying coefficients where you can explore stochastic shifts in regression parameters. To do this, we use random-effects models that specify time dependence between successive parameter values in the form of smoothness priors. The best application of this structure is for seasonality components.



As you recall, seasonality components are deviations from the trend. These seasonality values sum to zero across the length of the seasonal period. For example, let's look at sales data that have been accumulated to quarterly averages. Upon inspection, we determine that there is a seasonal pattern existing across the quarterly values.

Quarterly Time Period

$$0 = \sum_{t=1}^{L} S_t = S_1 + S_2 + S_3 + S_4$$

$$\sum_{t=1}^{L} S_t \sim N(0, \theta)$$

From a deterministic approach, these quarterly seasonal component values will sum to zero across four consecutive time points. This is due to the period of quarterly data being 4 in length. Taking a more dynamic approach, this sum is zero in the mean of the distribution with an additional variability.

Dynamic Approach

lag and next

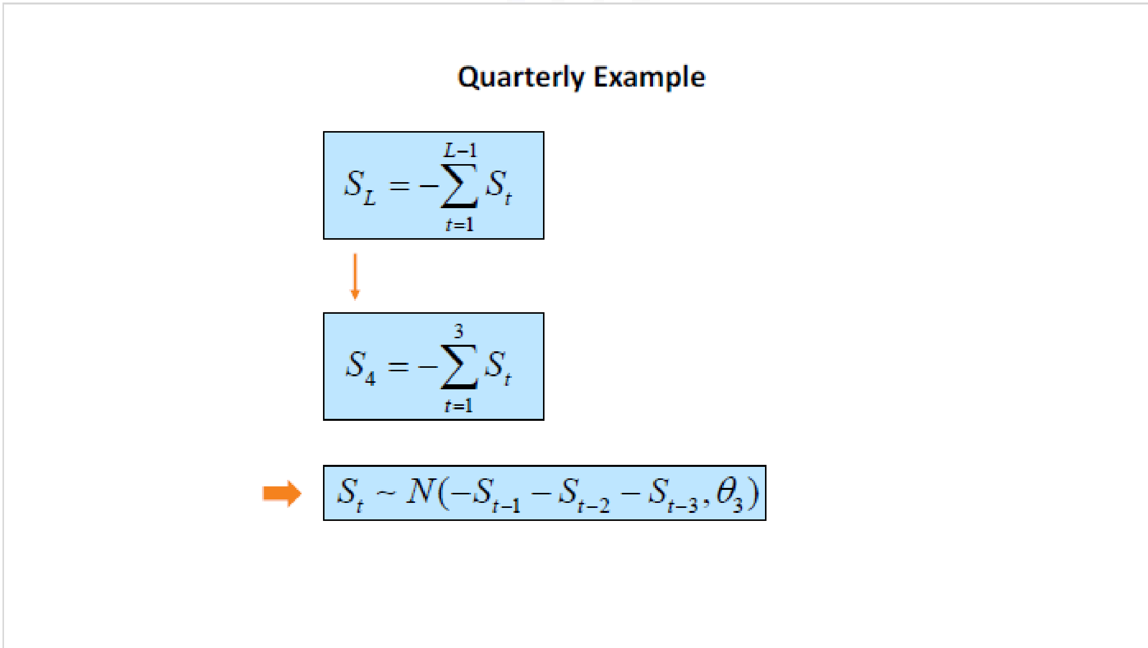
initial conditions

$$\sum_{t=1}^{L} S_t \sim N(0, \theta)$$

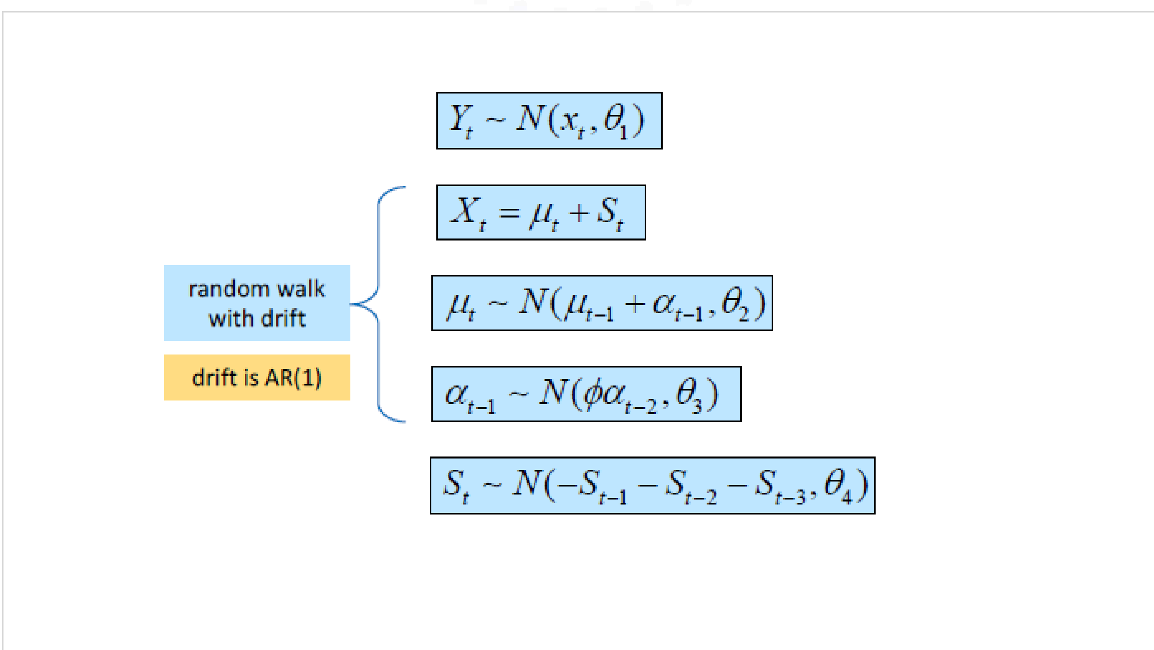
$$S_1 + \ldots + S_L$$

$$S_L = -\sum_{t=1}^{L-1} S_t$$

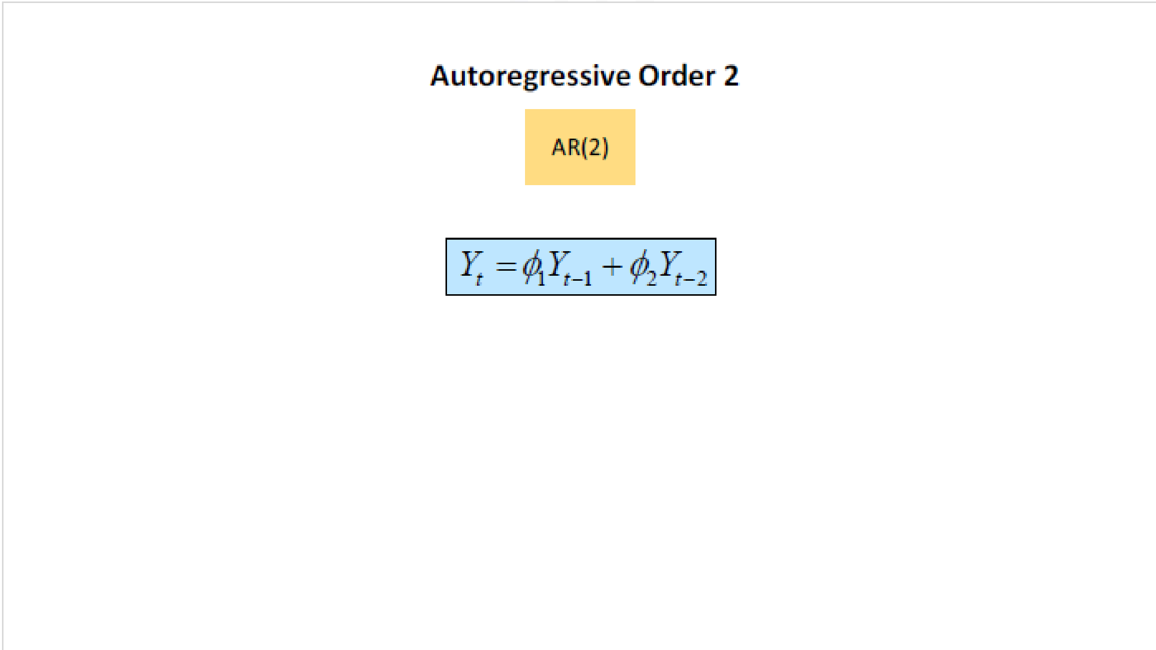
The additional benefit of using the dynamic approach to this seasonal component is that we can now use the lag and next elements as well as initial conditions during the modeling process. Because we know that the sum of all the seasonal components should add to zero in the mean, we can model the seasonal component at the current time point as the sum of the negative previous seasonal components.



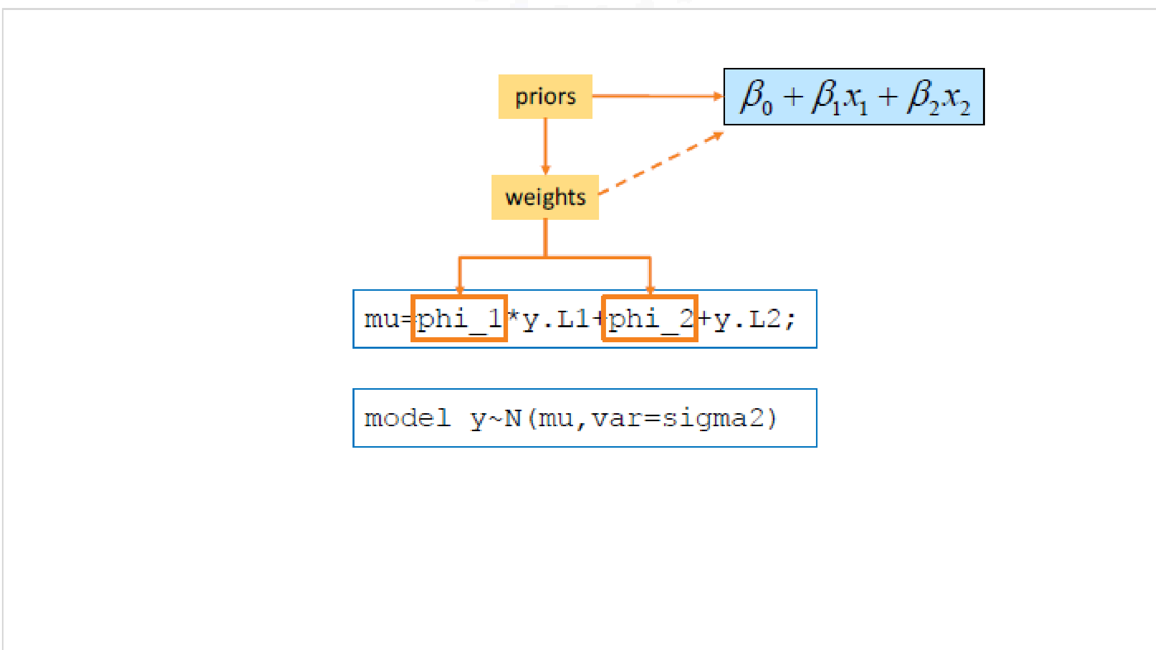
In our quarterly example, this would make the value of our current time point equal to the negative of the sum of the previous three seasonal components. We place this calculation in the mean of our distribution with a smoothness variance component.



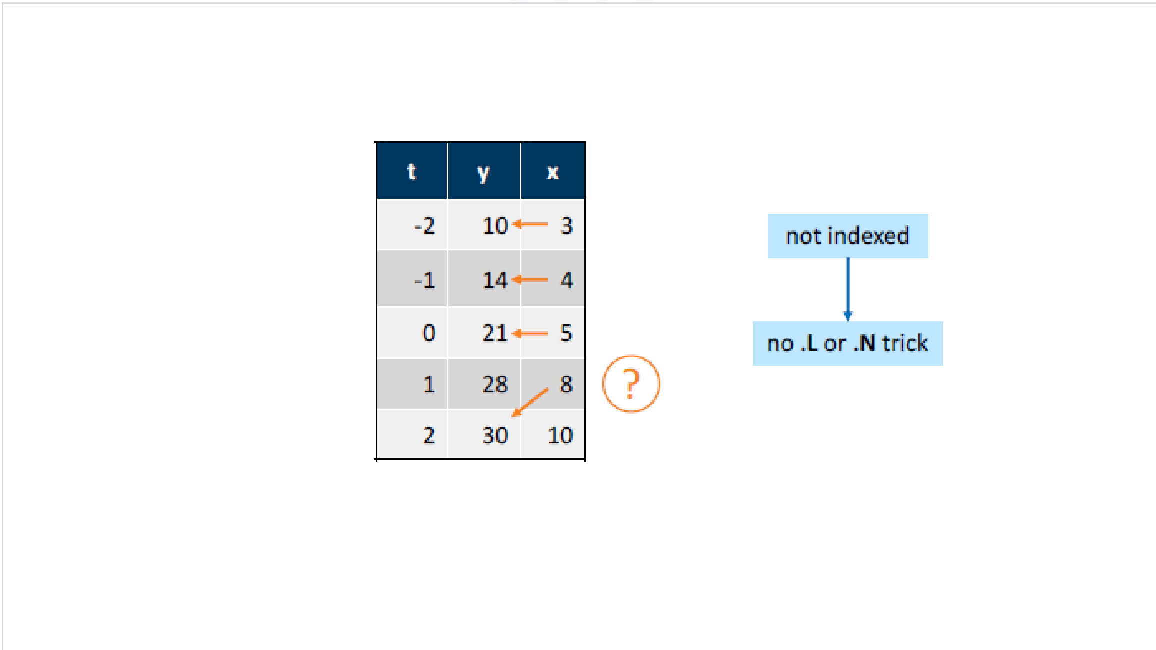
In addition to seasonality components, we could entertain trends that follow a random walk with drift. This drift could follow a first-order autoregressive process. The application of dynamic linear model setups within our time series models greatly expands the ability of our Bayesian approach to modeling.



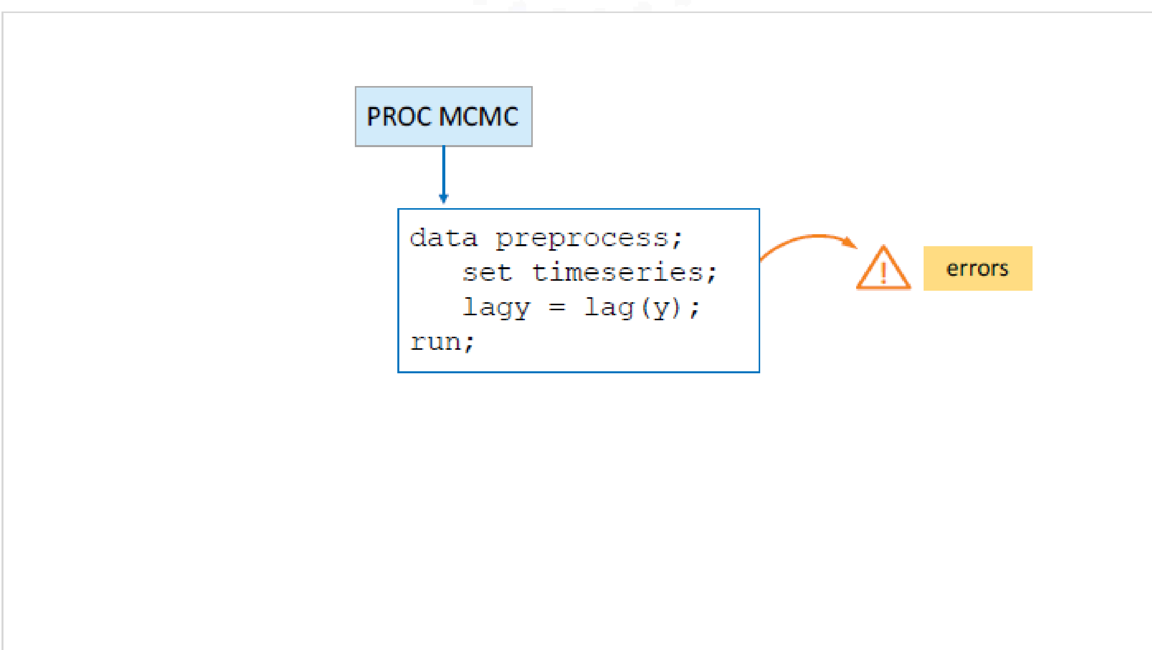
Previously, we discussed and practiced adding an autoregressive component to our model. The example was an AR(2).



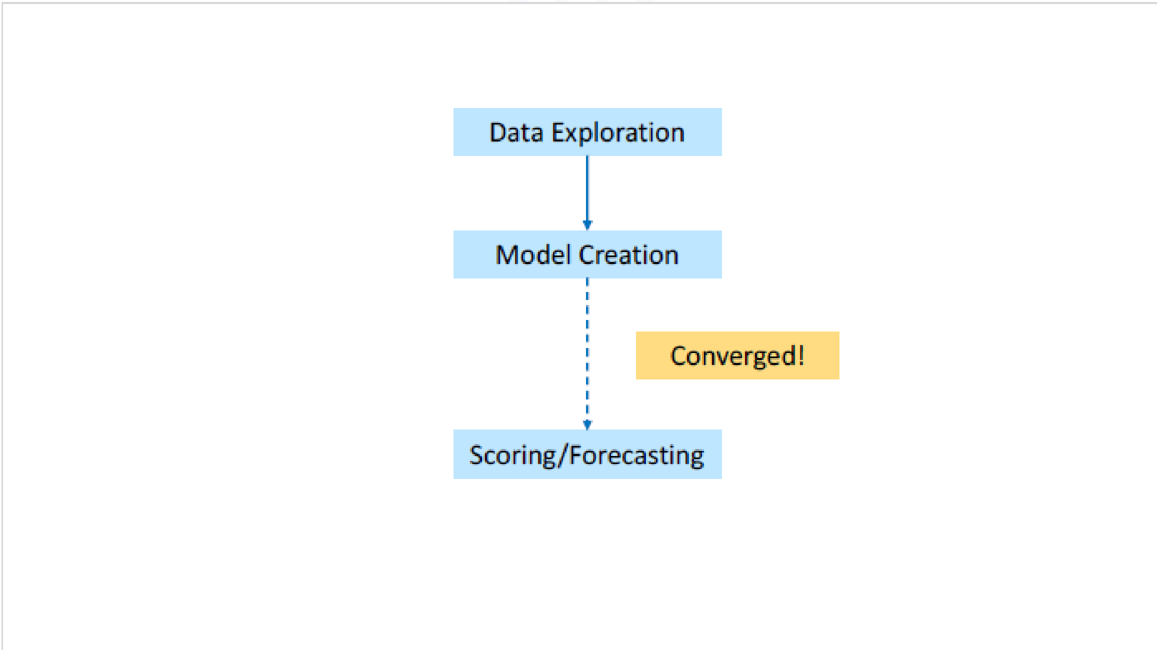
As you recall, prior to the model line in our code, we composed the mean from a weighted combination of the previous two values of the time series. The phi coefficients in the model were the weights. Think back to traditional regression equations from your past. You might see a resemblance of this autoregressive setup to that of a regression equation. In the regression version, our weights/coefficients are our beta estimates. Much like in the autoregressive viewpoint, we place prior distributions against the coefficients, as they are the parameters of the problem.



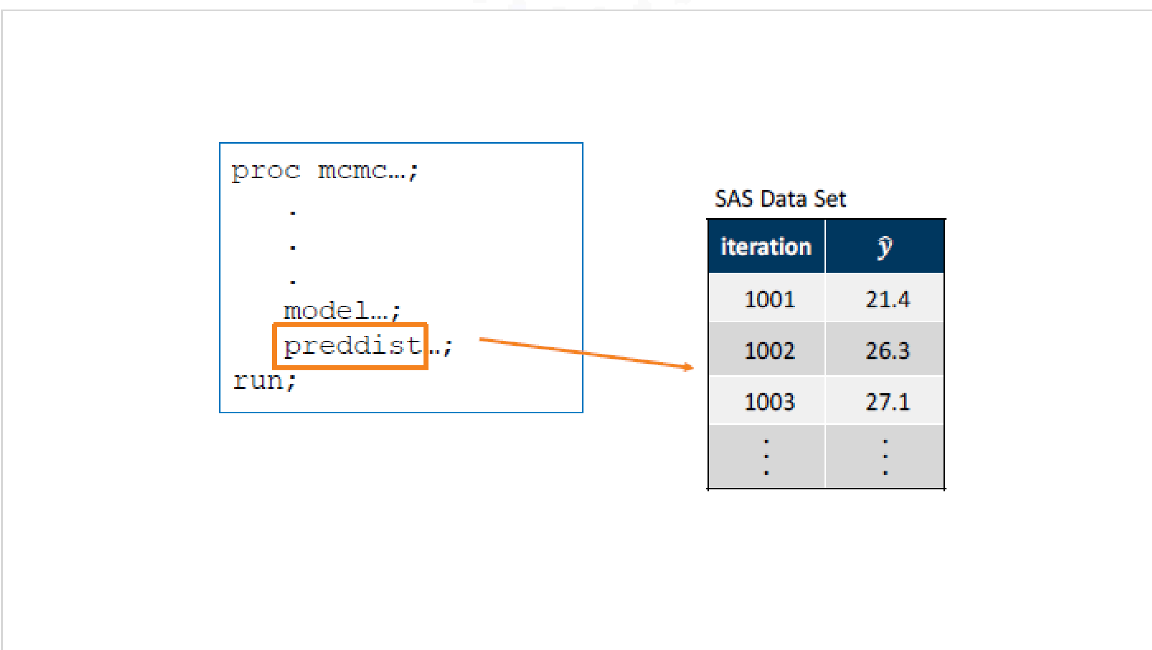
Adding a contemporaneous exogenous effect, where x at time t affects y at time t, is rather simple. However, what happens if there is a lag of the exogenous variable that influences the value of the series at time t? Unlike the series y and our seasonal components from before, our exogenous variable does not appear on a code line that makes it indexed. Therefore, we do not have the luxury of the .L and .N elements. So how do we bring in these lagged values on an exogenous variable?



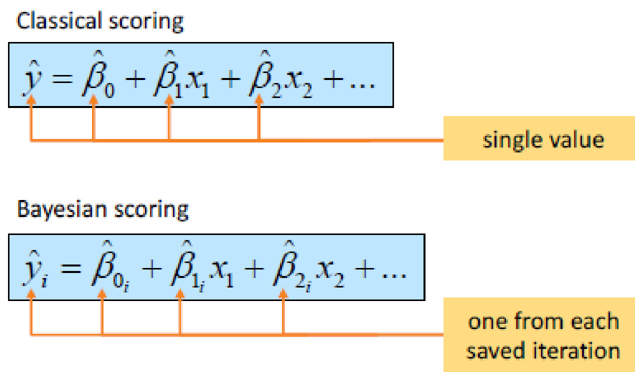
Prior to the execution of the Bayesian code, we will use a DATA step to produce the needed lag terms for the model. PROC MCMC does allow for the use of DATA step procedures within open code to construct the mean element of the model. However, using the lag DATA step function will generate missing values at the start of the series, and these will cause errors in the execution. There are several options of working with lagged values of exogenous variables. Using the external DATA step approach is the most direct. We will talk about this in the demonstration.



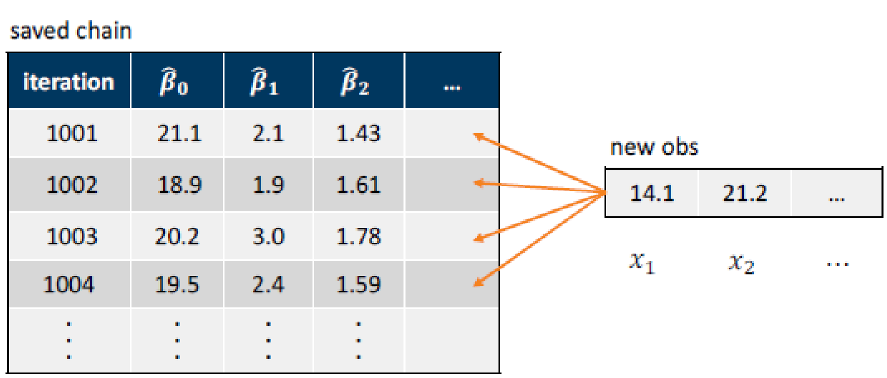
Once you have confirmed that you have a converged solution to your Bayesian analysis, you can then proceed with using this model to create forecasts (scoring). Trying to create forecasts before the solution has converged will waste your time.



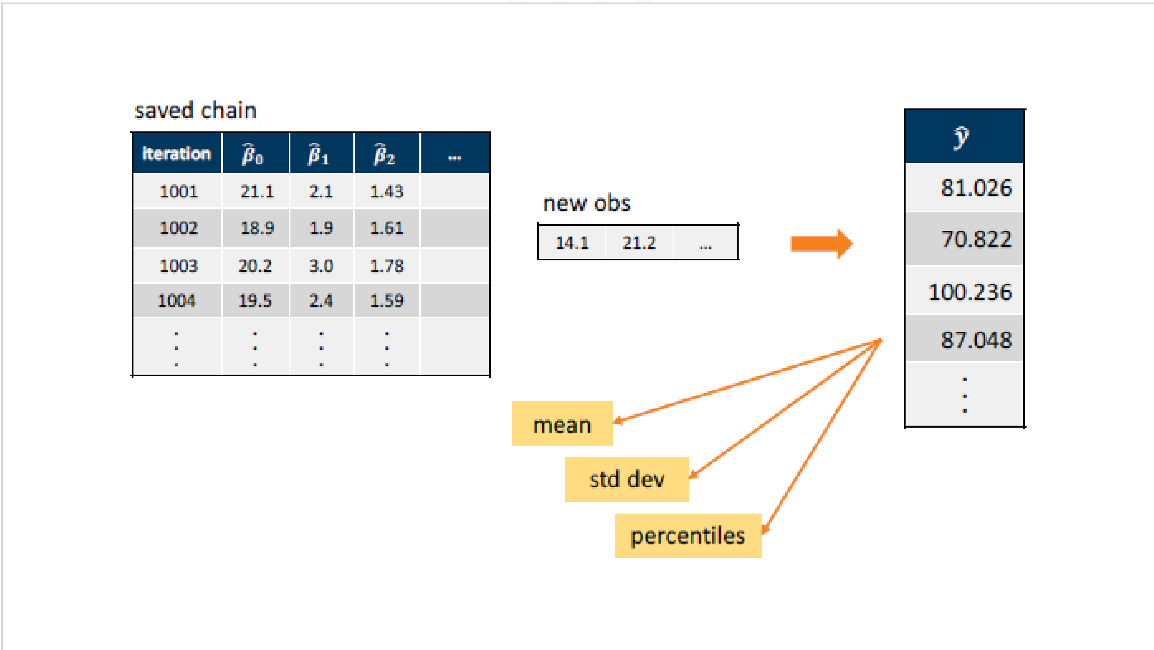
The PREDDIST statement in PROC MCMC is the tool for creating a new SAS data set that contains random samples from the posterior predictive distribution of the response variable, our time series.



There is a big difference between a classical approach to scoring and a Bayesian approach to scoring. In the classical approach, a single value is estimated for each of the parameters in the model. With the provided observation information and these parameter estimates, we arrive at a single prediction value for the current time point. In the Bayesian approach, this differs because there is no longer a single estimate of the parameter. Recall that we are treating the parameters as random variables and ultimately ending on a posterior distribution for the parameters given the data. It is for this reason that scoring changes in the Bayesian approach.



PROC MCMC uses an iterative approach to sample from the posterior distribution of the parameters. At each individual iteration, the current value of the parameter is what, at this moment, we perceive the value of that parameter to be. In the end, we have a chain of iterations saved to represent the sample from the posterior distribution of the parameter. When PREDDIST is active, the values of the observations are combined with each iteration value of the parameters, yielding a predicted response value for each iteration. This chain of predictions creates the sample of the posterior distribution of the prediction at a time point of the series.

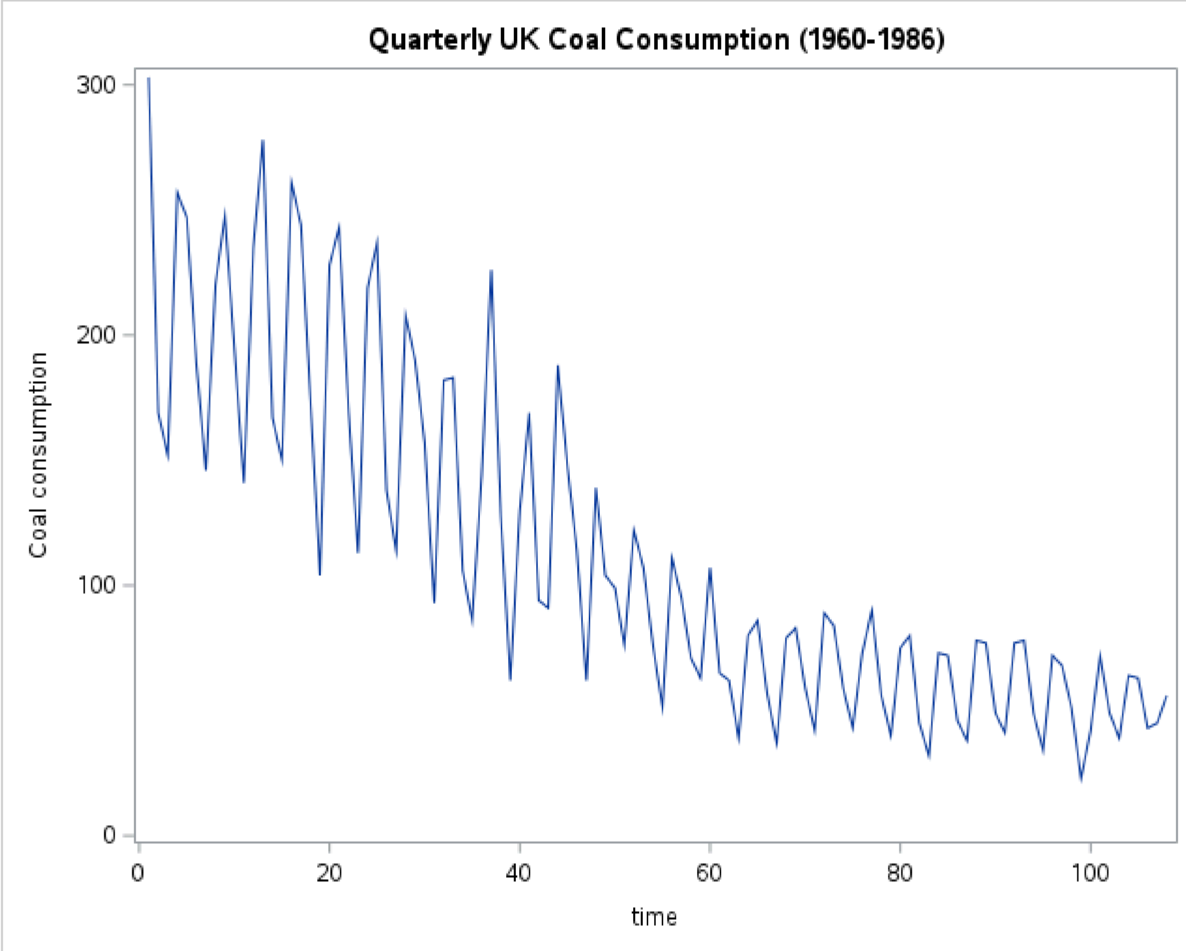


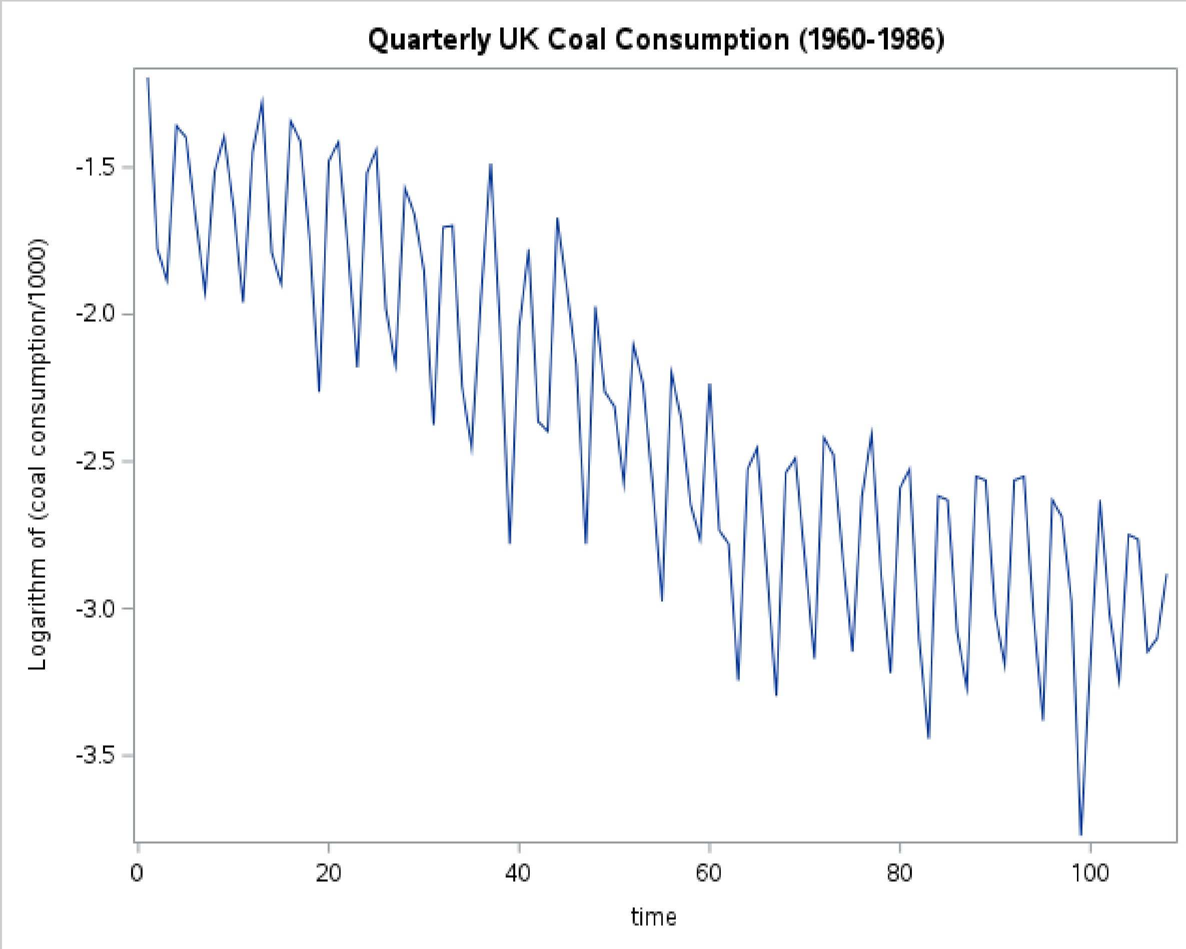
From this posterior sample, we can calculate the mean, standard deviation, and percentiles of that distribution to aid in discussion and presentation.

In [3]:

```
*Creating Coal Data Example;

data UKcoal;
  input coal year quarter @@;
  t=_N_;
  C=log(coal/1000);
  datalines;
303 1960 1 169 1960 2 152 1960 3 257 1960 4
247 1961 1 189 1961 2 146 1961 3 220 1961 4
248 1962 1 195 1962 2 141 1962 3 235 1962 4
278 1963 1 167 1963 2 150 1963 3 261 1963 4
244 1964 1 174 1964 2 104 1964 3 228 1964 4
243 1965 1 170 1965 2 113 1965 3 219 1965 4
237 1966 1 138 1966 2 114 1966 3 208 1966 4
190 1967 1 157 1967 2 93  1967 3 182 1967 4
183 1968 1 106 1968 2 86  1968 3 144 1968 4
226 1969 1 128 1969 2 62  1969 3 130 1969 4
169 1970 1 94  1970 2 91  1970 3 188 1970 4
148 1971 1 114 1971 2 62  1971 3 139 1971 4
104 1972 1 99  1972 2 76  1972 3 122 1972 4
107 1973 1 76  1973 2 51  1973 3 111 1973 4
95  1974 1 71  1974 2 63  1974 3 107 1974 4
65  1975 1 62  1975 2 39  1975 3 80  1975 4
86  1976 1 57  1976 2 37  1976 3 79  1976 4
83  1977 1 59  1977 2 42  1977 3 89  1977 4
```

```
84  1978 1 59  1978 2 43  1978 3 73  1978 4
90  1979 1 56  1979 2 40  1979 3 75  1979 4
80  1980 1 45  1980 2 32  1980 3 73  1980 4
72  1981 1 46  1981 2 38  1981 3 78  1981 4
77  1982 1 49  1982 2 41  1982 3 77  1982 4
78  1983 1 49  1983 2 34  1983 3 72  1983 4
68  1984 1 51  1984 2 23  1984 3 42  1984 4
72  1985 1 49  1985 2 39  1985 3 64  1985 4
63  1986 1 43  1986 2 45  1986 3 56  1986 4
;


proc sgplot data=UKcoal;
  title "Quarterly UK Coal Consumption (1960-1986)";
  series y=coal x=t;
  yaxis label="Coal consumption";
  xaxis label="time";
run;
title;

proc sgplot data=UKcoal;
  title "Quarterly UK Coal Consumption (1960-1986)";
  series y=c x=t;
  yaxis label="Logarithm of (coal consumption/1000)";
  xaxis label="time";
run;
title;

data UKcoal;
  set UKcoal;
  z=c;
  if year>1984 then c=.;
run;
```


Quarterly UK Coal Consumption (1960-1986)
Coal consumption
time
300
200
100
0
0
20
40
60
80
100



In [4]:

```
*Dynamic Linear Model;

proc mcmc data=UKcoal nmc=100000 seed=123456 outpost=posterior propcov=quanew;
  parms alpha0;
  parms mu0;
  parms s0 s1 s2;
  parms theta1;
  parms theta2;
  parms theta3;
  parms theta4;
  parms theta_phi;
  parms phi;
  prior phi~normal(0,var=exp(theta_phi));
  prior alpha0~normal(0,var=theta2);
  prior mu0~normal(0,var=100);
  prior s:~normal(0,var=theta3);
  prior theta:~igamma(shape = 3/10, scale = 10/3);
  random alpha~normal(phi*alpha.l1,var=exp(theta2)) subject=t icond=(alpha0);
  random s~normal(-s.l1-s.l2-s.l3,var=exp(theta3)) subject=quarter icond=(s2 s1 s0);
  random mu~normal(mu.l1 + alpha.l1,var=exp(theta1)) subject=t icond=(mu0);
  x=mu + s;
  model c~normal(x,var=exp(theta4));
  preddist outpred=TVCoutpred statistics=brief;
  ods output PredSumInt=TVCPredSumInt;
run;
```

```
data forecast;
  merge UKcoal TVCPredSumInt;
run;

proc sgplot data=forecast;
  series x=t y=c / lineattrs=(color=red);
  series x=t y=z / lineattrs=(color=red pattern=dot);
  series x=t y=mean / lineattrs=(color=blue pattern=dash);
  band x=t upper=hpdupper lower=hpdlower / transparency=.5;
run;

proc contents data=posterior;
run;
```

The MCMC Procedure

Number of Observations Read	108
Number of Observations Used	108

Missing Data Information Table			
Variable	**Number of Missing Obs**	**Observation Indices**	**Sampling Method**
C	8	**101 102 103 104 105 106 107 108**	Direct

Parameters				
Block	**Parameter**	**Sampling Method**	**Initial Value**	**Prior Distribution**
1	**alpha0**	N-Metropolis	0	normal(0,var=theta2)
2	**mu0**	N-Metropolis	0	normal(0,var=100)
3	**s0**	N-Metropolis	0	normal(0,var=theta3)
	s1		0	normal(0,var=theta3)
	s2		0	normal(0,var=theta3)
4	**theta1**	N-Metropolis	2.5641	igamma(shape = 3/10, scale = 10/3)
5	**theta2**	N-Metropolis	2.5641	igamma(shape = 3/10, scale = 10/3)
6	**theta3**	N-Metropolis	2.5641	igamma(shape = 3/10, scale = 10/3)
7	**theta4**	N-Metropolis	2.5641	igamma(shape = 3/10, scale = 10/3)
8	**theta_phi**	N-Metropolis	2.5641	igamma(shape = 3/10, scale = 10/3)
9	**phi**	N-Metropolis	0	normal(0,var=exp(theta_phi))

Random Effect Parameters					
Parameter	**Sampling Method**	**Subject**	**Number of Subjects**	**Subject Values**	**Prior Distribution**
alpha	N-Metropolis	**t**	**108**	**1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 ...**	**normal(phi*alpha.l1,var=exp(theta2))**
s	N-Metropolis	**quarter**	**4**	**1 2 3 4**	**normal(-s.l1-s.l2-s.l3,var=exp(theta3))**
mu	N-Metropolis	**t**	**108**	**1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 ...**	**normal(mu.l1 + alpha.l1,var=exp(theta1))**

The MCMC Procedure

Posterior Summaries and Intervals					
Parameter	N	Mean	Standard Deviation	95% HPD Interval	
alpha0	100000	-0.00581	0.6915	-1.3978	1.2828
mu0	100000	-1.4880	1.8117	-5.0158	2.0603
s0	100000	-0.0120	1.0895	-2.2098	2.1533
s1	100000	-0.1060	1.0863	-2.2653	2.1015
s2	100000	-0.0408	1.1643	-2.4439	2.1714
theta1	100000	0.4718	0.1162	0.2675	0.7067
theta2	100000	0.4914	0.1230	0.2606	0.7265
theta3	100000	1.6167	0.7693	0.4636	3.1733
theta4	100000	0.4243	0.1009	0.2440	0.6235
theta_phi	100000	2.7639	1.7233	0.4815	6.1892
phi	100000	-0.3213	0.1956	-0.6879	0.0703

The MCMC Procedure

Effective Sample Sizes			
Parameter	ESS	Autocorrelation Time	Efficiency
alpha0	8936.5	11.1901	0.0894
mu0	2378.3	42.0466	0.0238
s0	1405.9	71.1282	0.0141
s1	6154.3	16.2488	0.0615
s2	6266.2	15.9585	0.0627
theta1	9554.2	10.4666	0.0955
theta2	5249.1	19.0510	0.0525
theta3	992.0	100.8	0.0099
theta4	10646.5	9.3928	0.1065
theta_phi	10814.3	9.2470	0.1081
phi	2109.5	47.4056	0.0211

Posterior Summaries and Intervals for Prediction					
Parameter	N	Mean	Standard Deviation	95% HPD Interval	
C_1	100000	-1.2291	1.6215	-4.4699	1.8526
C_2	100000	-1.6941	1.5824	-4.7703	1.4512

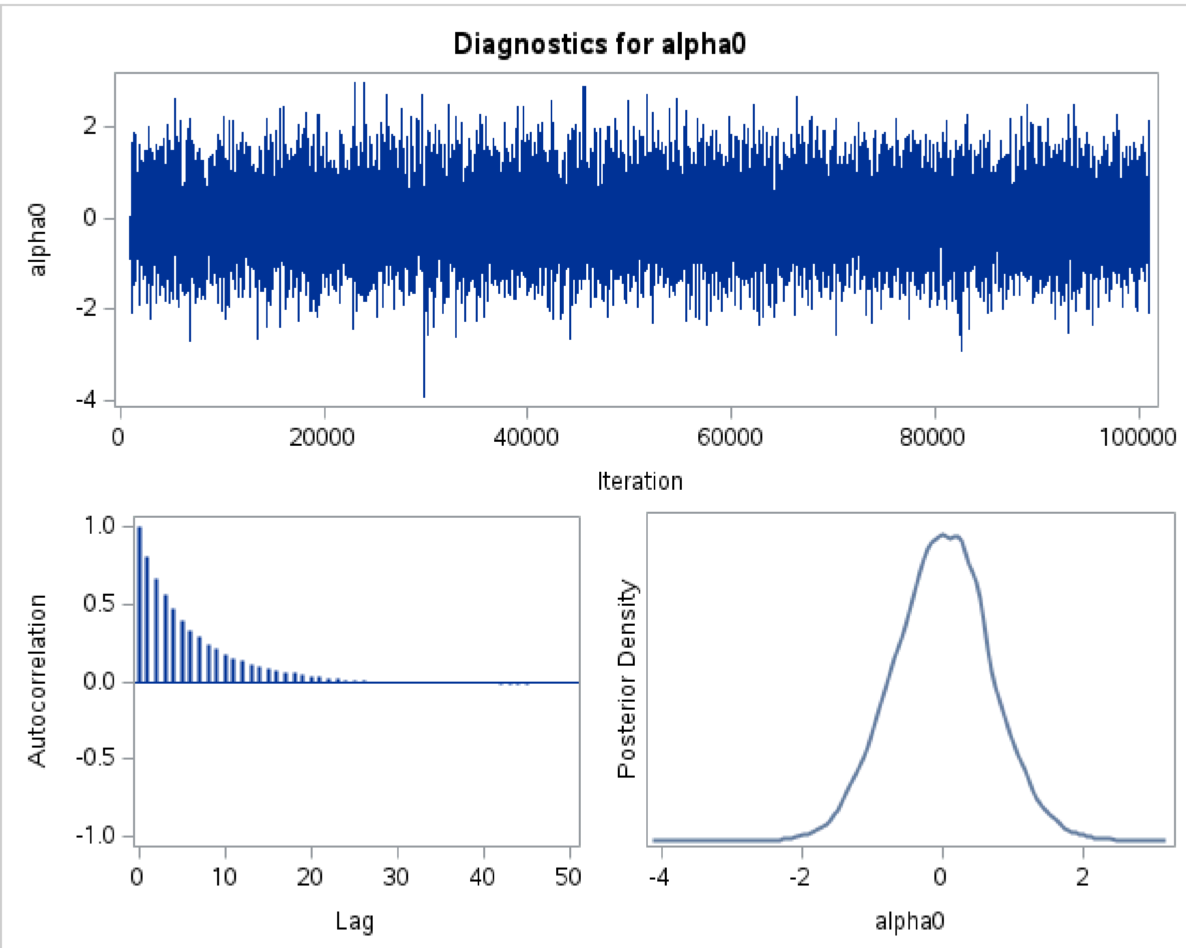
Posterior Summaries and Intervals for Prediction					
Parameter	N	Mean	Standard Deviation	95% HPD Interval	
C_3	100000	-1.9489	1.5804	-5.0755	1.1287
C_4	100000	-1.3558	1.5785	-4.4367	1.7588
C_5	100000	-1.3477	1.5772	-4.4933	1.6969
C_6	100000	-1.6973	1.5720	-4.7693	1.3871
C_7	100000	-1.9843	1.5736	-5.0672	1.0910
C_8	100000	-1.4619	1.5816	-4.5189	1.6841
C_9	100000	-1.3842	1.5843	-4.5719	1.6457
C_10	100000	-1.6820	1.5862	-4.7899	1.4204
C_11	100000	-1.9705	1.5765	-5.0579	1.1144
C_12	100000	-1.3868	1.5760	-4.4231	1.7621
C_13	100000	-1.3079	1.5693	-4.4105	1.7442
C_14	100000	-1.7230	1.5888	-4.8025	1.4231
C_15	100000	-1.9569	1.5792	-5.0779	1.1129
C_16	100000	-1.3415	1.5881	-4.4013	1.8403
C_17	100000	-1.3967	1.5817	-4.4688	1.7268
C_18	100000	-1.7622	1.5866	-4.8004	1.4121
C_19	100000	-2.2067	1.5834	-5.3124	0.9061
C_20	100000	-1.4955	1.5826	-4.6214	1.5820
C_21	100000	-1.4469	1.5838	-4.5370	1.6781
C_22	100000	-1.8322	1.5751	-4.9068	1.2786
C_23	100000	-2.1733	1.5713	-5.2492	0.8708
C_24	100000	-1.4979	1.5810	-4.5888	1.6017
C_25	100000	-1.4763	1.5805	-4.5893	1.5827
C_26	100000	-1.9303	1.5738	-5.0284	1.1540
C_27	100000	-2.2149	1.5737	-5.3541	0.8181
C_28	100000	-1.6029	1.5854	-4.7141	1.4829
C_29	100000	-1.6219	1.5771	-4.7461	1.4397
C_30	100000	-1.9092	1.5784	-4.9622	1.2238
C_31	100000	-2.3390	1.5810	-5.4626	0.7160
C_32	100000	-1.6981	1.5757	-4.7738	1.3949
C_33	100000	-1.7273	1.5785	-4.8368	1.3607
C_34	100000	-2.1692	1.5784	-5.2698	0.8973
C_35	100000	-2.4542	1.5792	-5.5386	0.6426

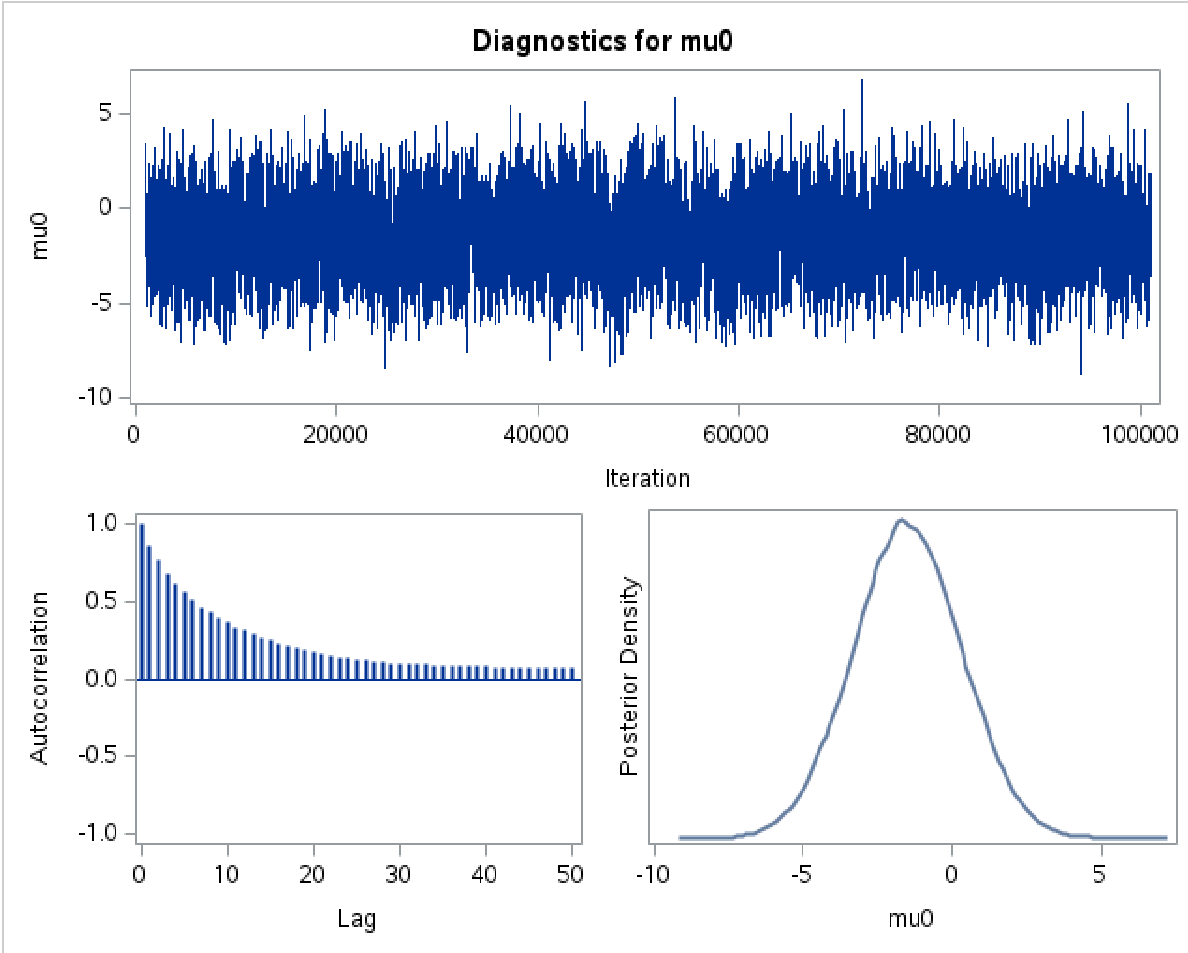
Posterior Summaries and Intervals for Prediction					
Parameter	N	Mean	Standard Deviation	95% HPD Interval	
C_36	100000	-1.8458	1.5791	-4.9387	1.2446
C_37	100000	-1.6205	1.5773	-4.7668	1.4220
C_38	100000	-2.1247	1.5770	-5.1560	1.0158
C_39	100000	-2.6603	1.5745	-5.7751	0.3939
C_40	100000	-1.9888	1.5802	-5.0751	1.1031
C_41	100000	-1.8396	1.5804	-4.9311	1.2624
C_42	100000	-2.2770	1.5802	-5.3337	0.8429
C_43	100000	-2.4642	1.5727	-5.5415	0.6386
C_44	100000	-1.7385	1.5777	-4.8424	1.3252
C_45	100000	-1.8669	1.5797	-5.0309	1.1859
C_46	100000	-2.2171	1.5833	-5.2722	0.9328
C_47	100000	-2.7211	1.5788	-5.8394	0.3412
C_48	100000	-2.0129	1.5707	-5.1179	1.0558
C_49	100000	-2.1409	1.5774	-5.1994	1.0057
C_50	100000	-2.3557	1.5767	-5.4323	0.7481
C_51	100000	-2.6482	1.5836	-5.7010	0.4966
C_52	100000	-2.1027	1.5870	-5.2273	0.9966
C_53	100000	-2.1771	1.5817	-5.2589	0.9444
C_54	100000	-2.5715	1.5794	-5.6757	0.5244
C_55	100000	-2.9678	1.5777	-6.0788	0.1257
C_56	100000	-2.2165	1.5751	-5.3627	0.8329
C_57	100000	-2.3000	1.5732	-5.3743	0.7921
C_58	100000	-2.5962	1.5800	-5.7031	0.4803
C_59	100000	-2.8476	1.5810	-5.9667	0.2438
C_60	100000	-2.2991	1.5843	-5.3775	0.8544
C_61	100000	-2.5519	1.5767	-5.6182	0.5862
C_62	100000	-2.8140	1.5801	-5.8923	0.2991
C_63	100000	-3.1964	1.5662	-6.3075	-0.1753
C_64	100000	-2.5229	1.5826	-5.6307	0.5853
C_65	100000	-2.4744	1.5692	-5.5495	0.5983
C_66	100000	-2.8636	1.5688	-5.9654	0.1785
C_67	100000	-3.2446	1.5737	-6.3420	-0.1572
C_68	100000	-2.5369	1.5694	-5.5766	0.5837

Posterior Summaries and Intervals for Prediction					
Parameter	N	Mean	Standard Deviation	95% HPD Interval	
C_69	100000	-2.4880	1.5699	-5.6394	0.5290
C_70	100000	-2.8160	1.5775	-5.8449	0.3456
C_71	100000	-3.1492	1.5770	-6.2578	-0.0644
C_72	100000	-2.4600	1.5783	-5.5478	0.6514
C_73	100000	-2.4789	1.5828	-5.5331	0.6650
C_74	100000	-2.8418	1.5880	-6.0362	0.1929
C_75	100000	-3.1452	1.5699	-6.2660	-0.0901
C_76	100000	-2.5620	1.5944	-5.6573	0.6148
C_77	100000	-2.4734	1.5786	-5.5971	0.5844
C_78	100000	-2.8746	1.5732	-5.9304	0.2417
C_79	100000	-3.2231	1.5761	-6.2723	-0.0921
C_80	100000	-2.6151	1.5736	-5.6970	0.4759
C_81	100000	-2.5746	1.5785	-5.6810	0.5147
C_82	100000	-3.0702	1.5736	-6.1050	0.0459
C_83	100000	-3.4023	1.5827	-6.5187	-0.3108
C_84	100000	-2.6768	1.5748	-5.6862	0.4860
C_85	100000	-2.6499	1.5727	-5.7397	0.4215
C_86	100000	-3.0224	1.5835	-6.1387	0.0733
C_87	100000	-3.2810	1.5772	-6.3675	-0.1925
C_88	100000	-2.5801	1.5774	-5.6556	0.5214
C_89	100000	-2.5879	1.5805	-5.7417	0.4600
C_90	100000	-3.0037	1.5747	-6.1079	0.0643
C_91	100000	-3.2282	1.5729	-6.3123	-0.1296
C_92	100000	-2.5867	1.5729	-5.6998	0.4762
C_93	100000	-2.5815	1.5809	-5.6893	0.4957
C_94	100000	-3.0002	1.5794	-6.1064	0.0915
C_95	100000	-3.3576	1.5784	-6.3625	-0.1840
C_96	100000	-2.6683	1.5802	-5.7388	0.4413
C_97	100000	-2.7068	1.5780	-5.8197	0.3686
C_98	100000	-3.0760	1.5737	-6.2158	-0.0390
C_99	100000	-3.6861	1.5930	-6.8302	-0.5873
C_100	100000	-3.1157	1.6403	-6.2934	0.1245
C_101	100000	-3.1130	2.4696	-7.9950	1.6276

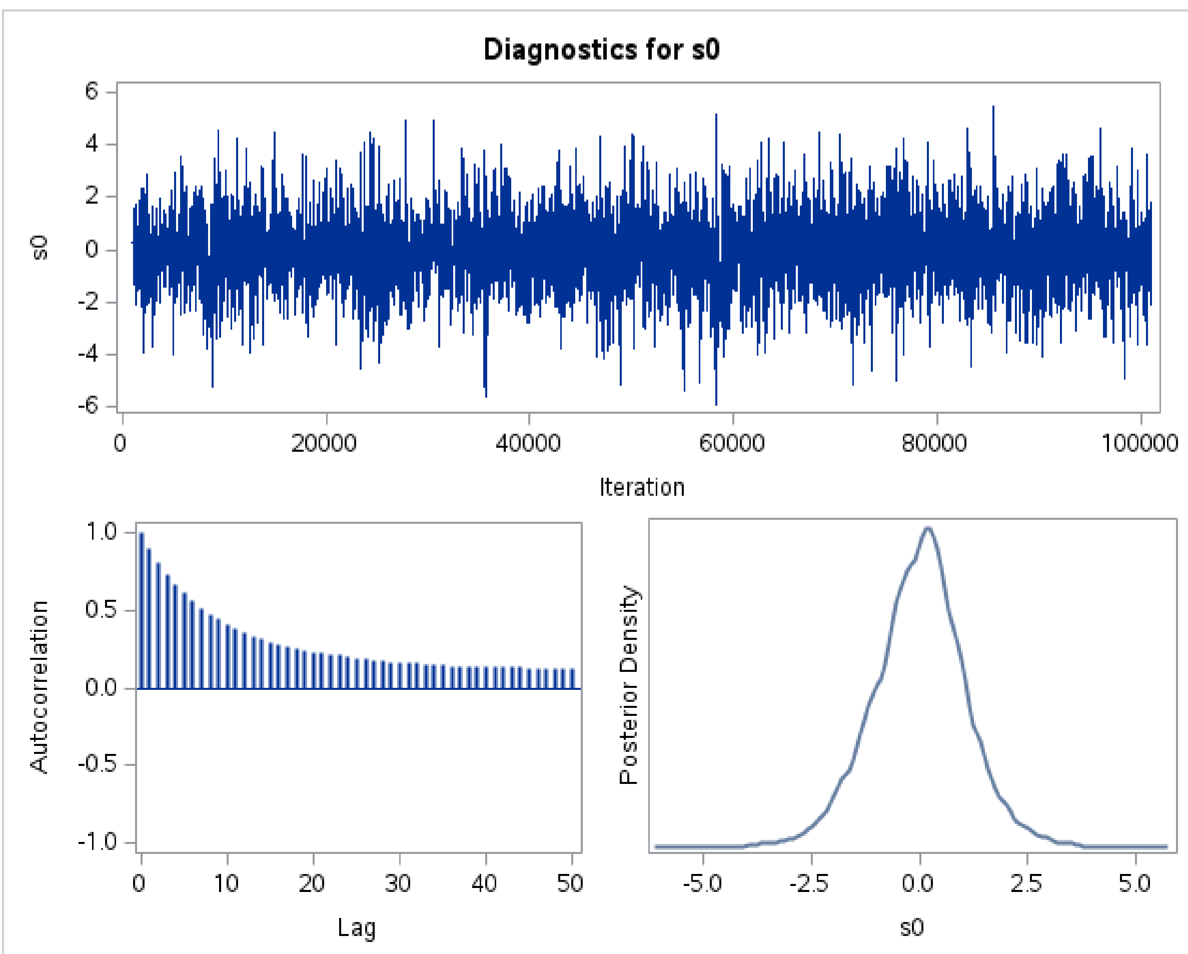
Posterior Summaries and Intervals for Prediction					
Parameter	N	Mean	Standard Deviation	95% HPD Interval	
C_102	100000	-3.4260	2.8991	-9.2436	2.0972
C_103	100000	-3.7443	3.2551	-10.2415	2.4024
C_104	100000	-3.1529	3.5388	-10.1918	3.6677
C_105	100000	-3.0946	3.9177	-10.9139	4.5433
C_106	100000	-3.4304	4.2191	-11.8424	4.6655
C_107	100000	-3.7470	4.5164	-12.6463	5.0729
C_108	100000	-3.0906	4.7834	-12.4284	6.3380

The MCMC Procedure

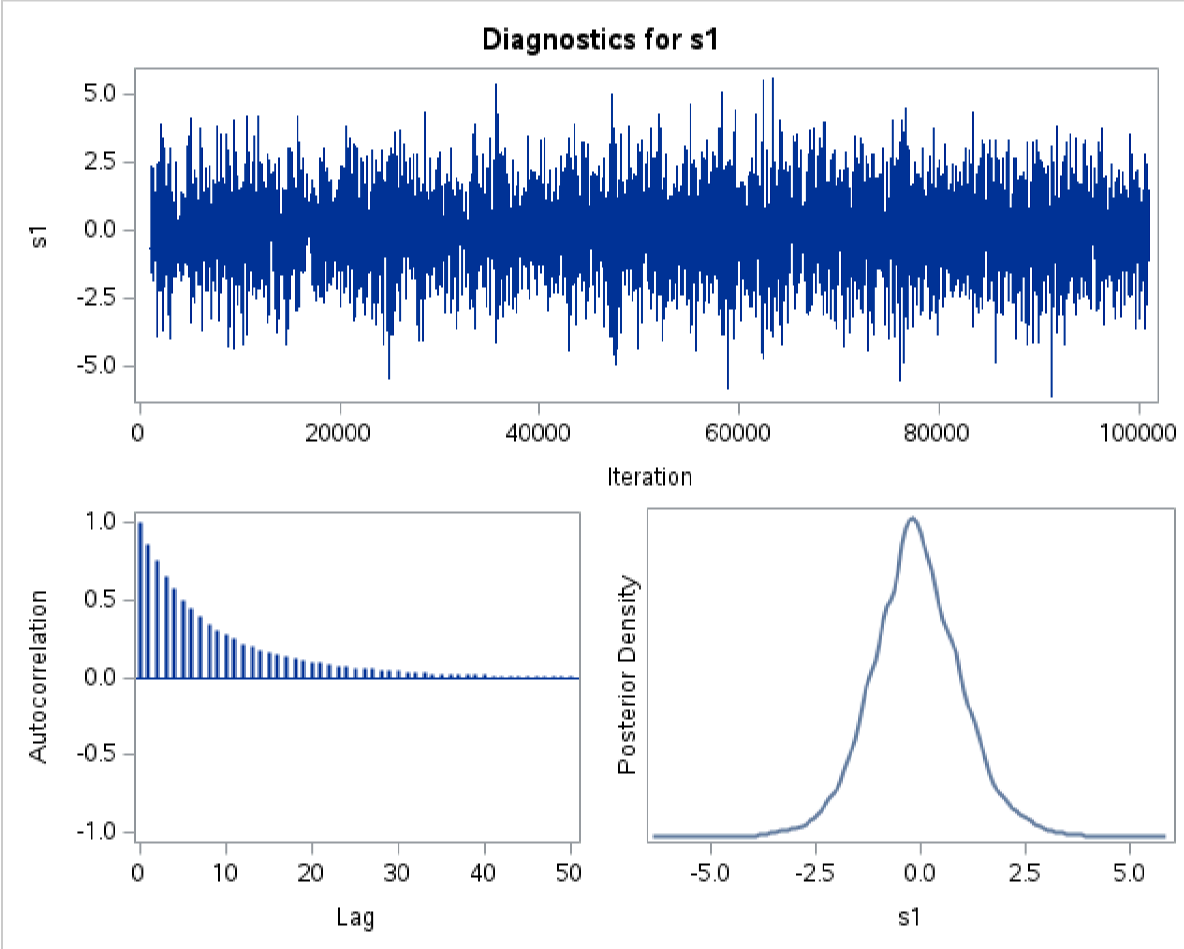




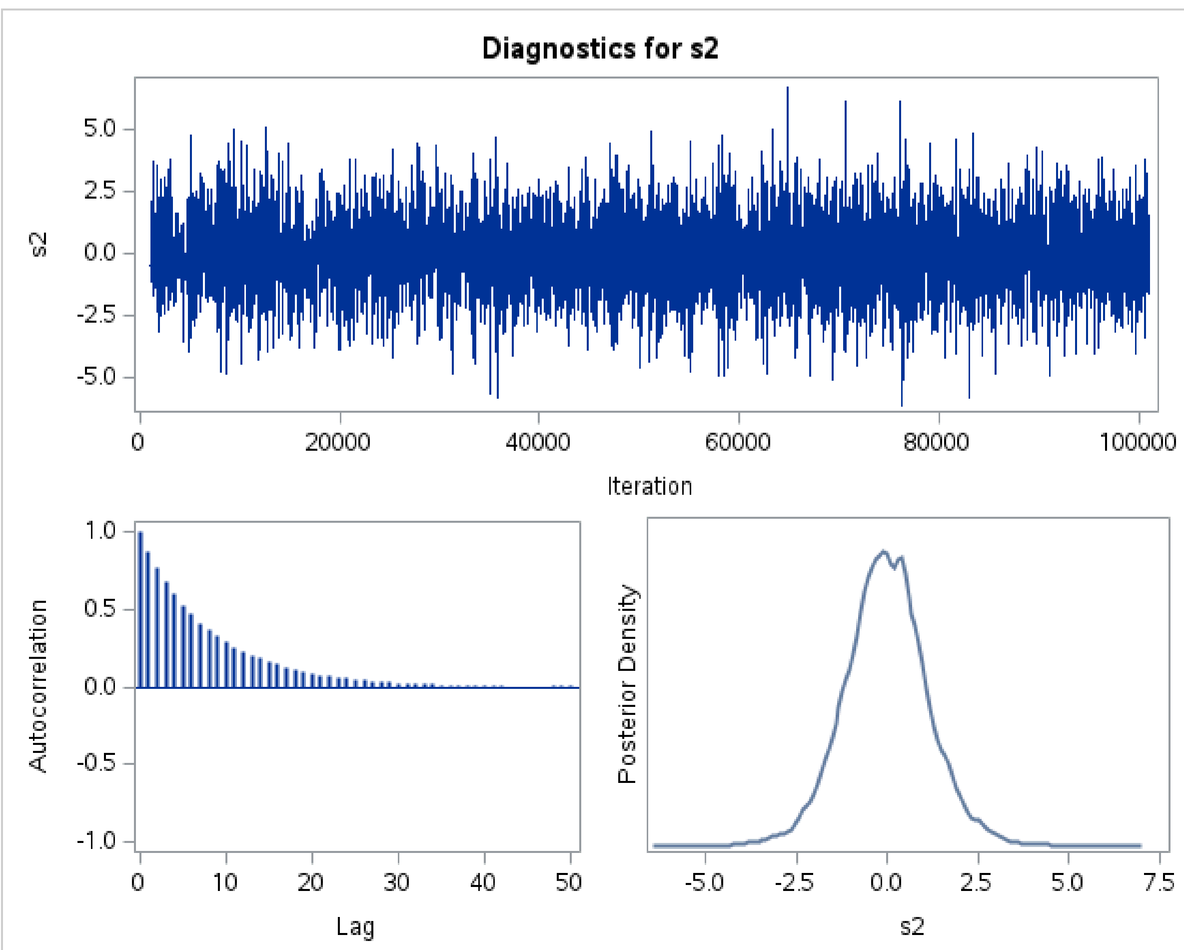
Diagnostics for mu0
mu0
Iteration
Autocorrelation
Lag
Posterior Density
mu0



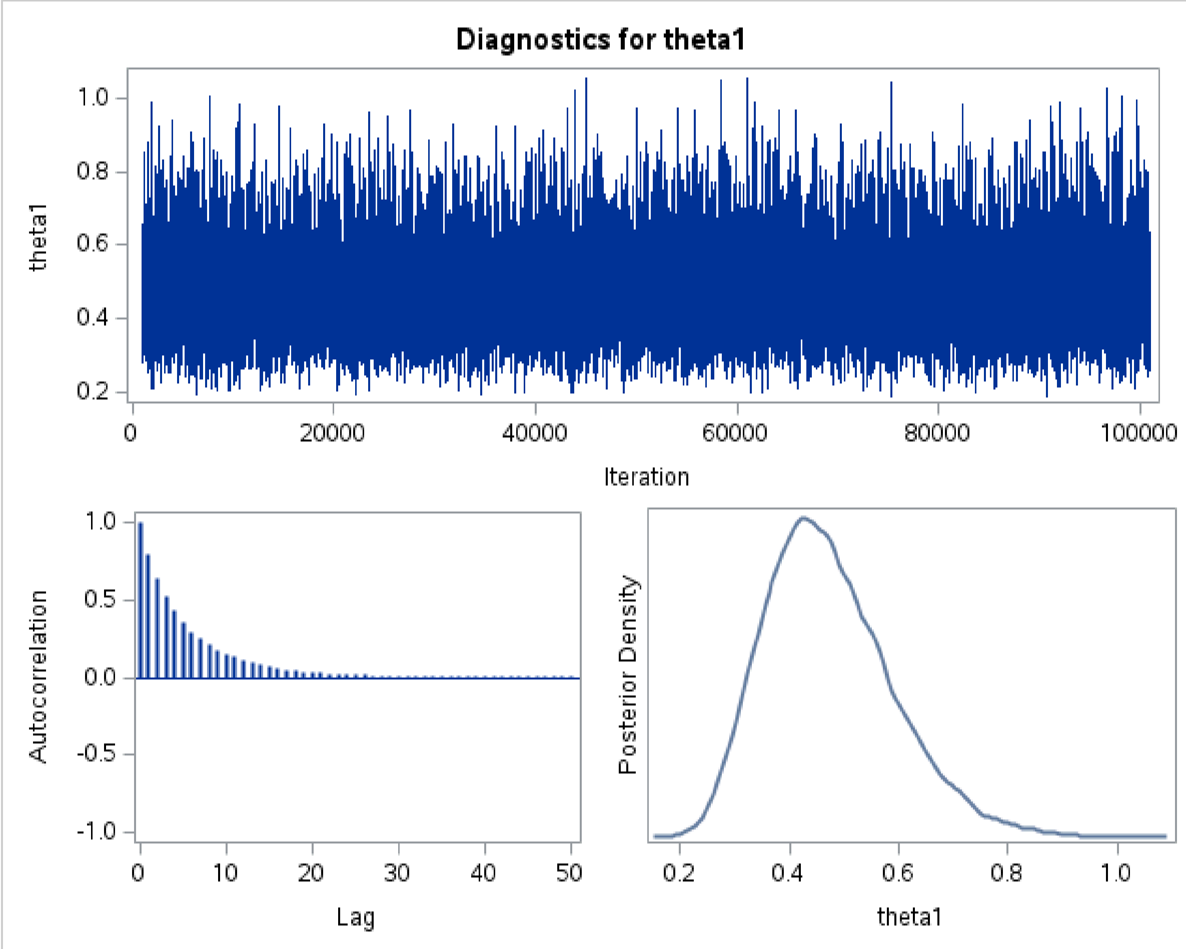
Diagnostics for s0
s0
Iteration
Autocorrelation
Lag
Posterior Density
s0

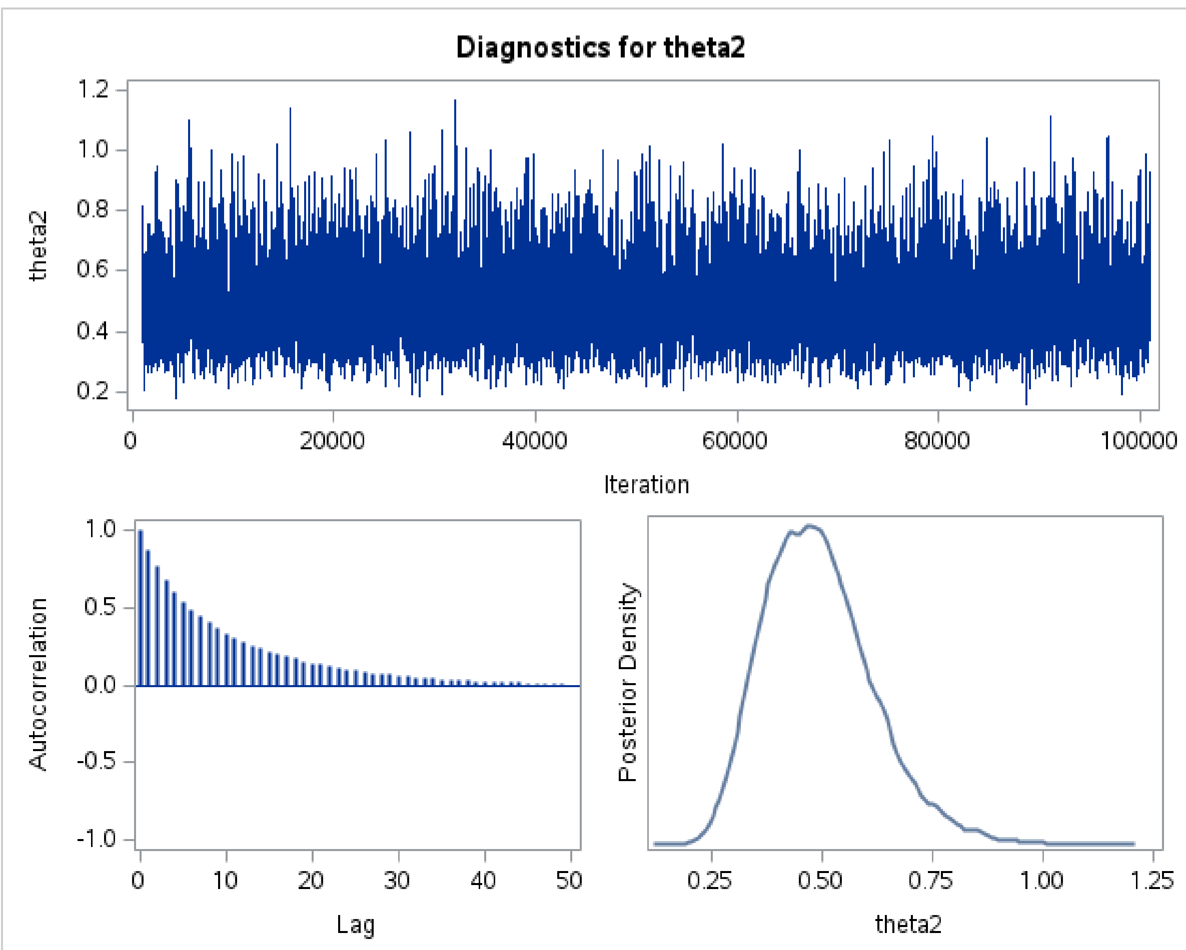


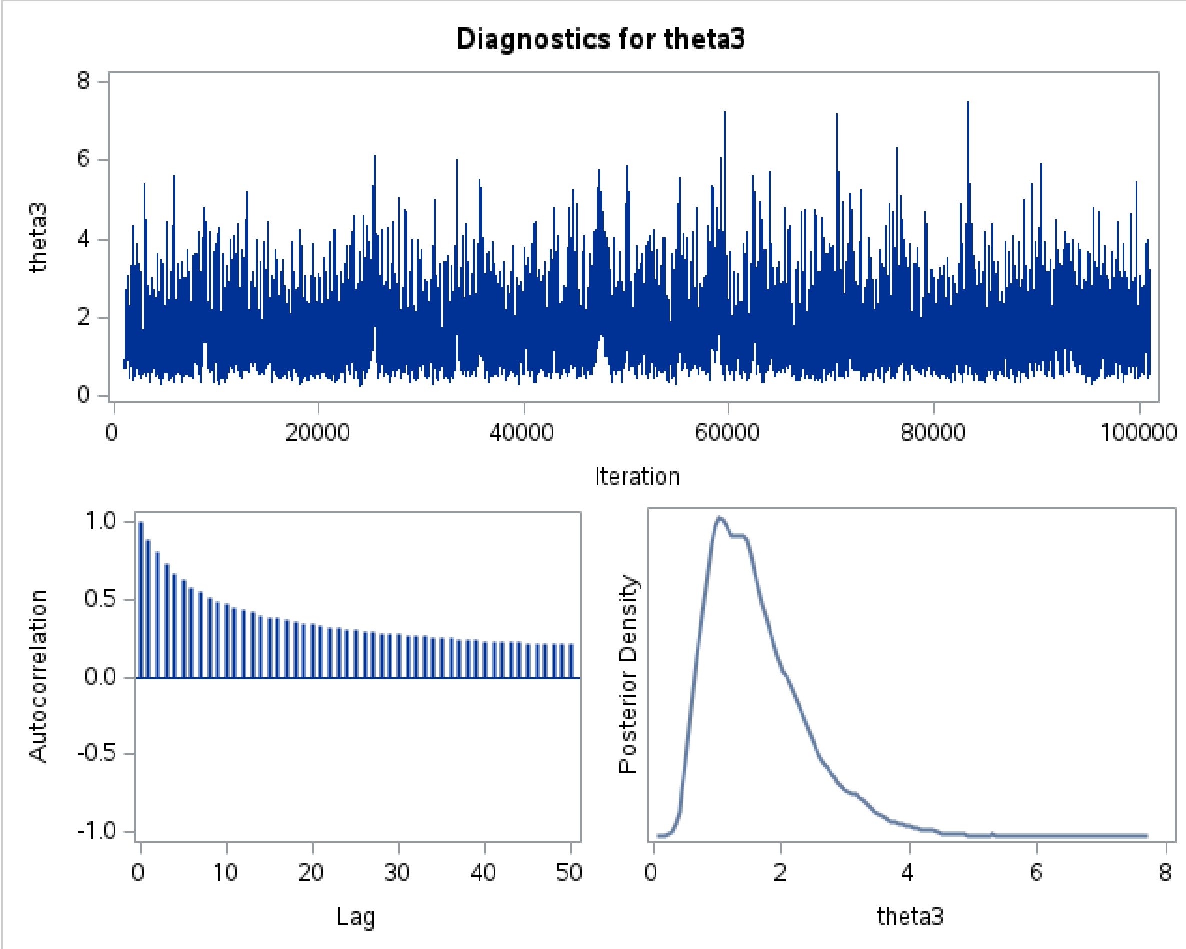
Diagnostics for s1
s1
Iteration
Autocorrelation
Lag
Posterior Density

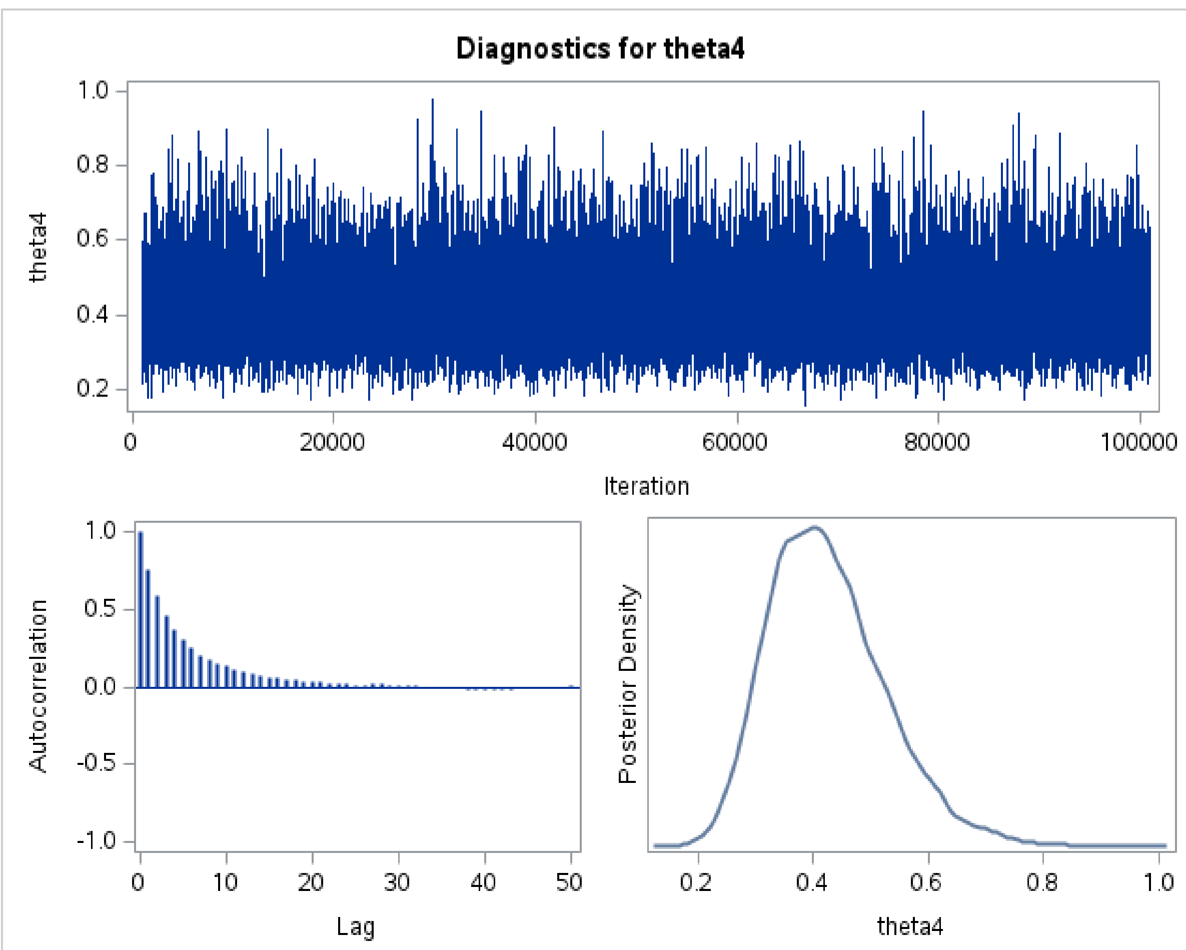


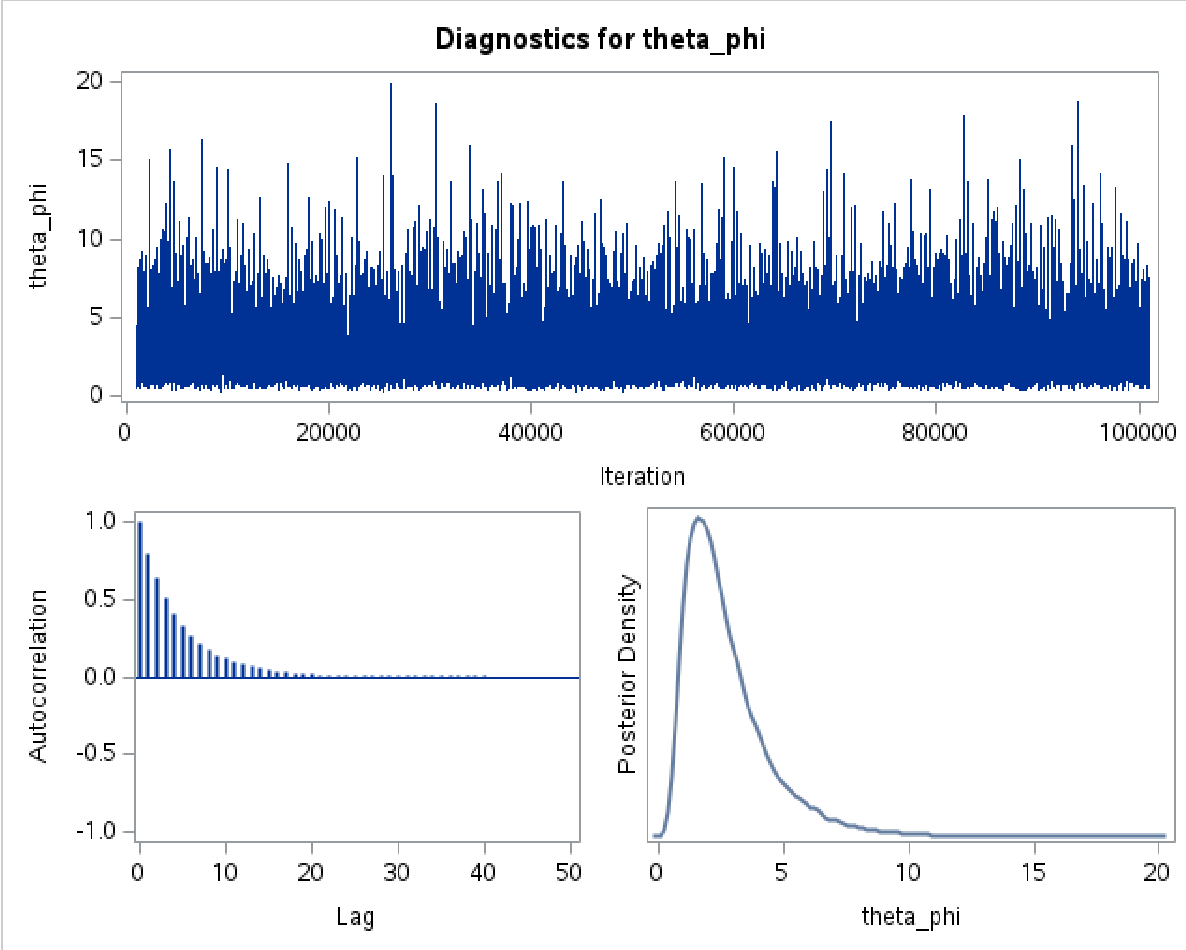
Diagnostics for s2
s2
Iteration
Autocorrelation
Lag
Posterior Density


Diagnostics for theta1
theta1
Iteration
Autocorrelation
Lag
Posterior Density
theta1

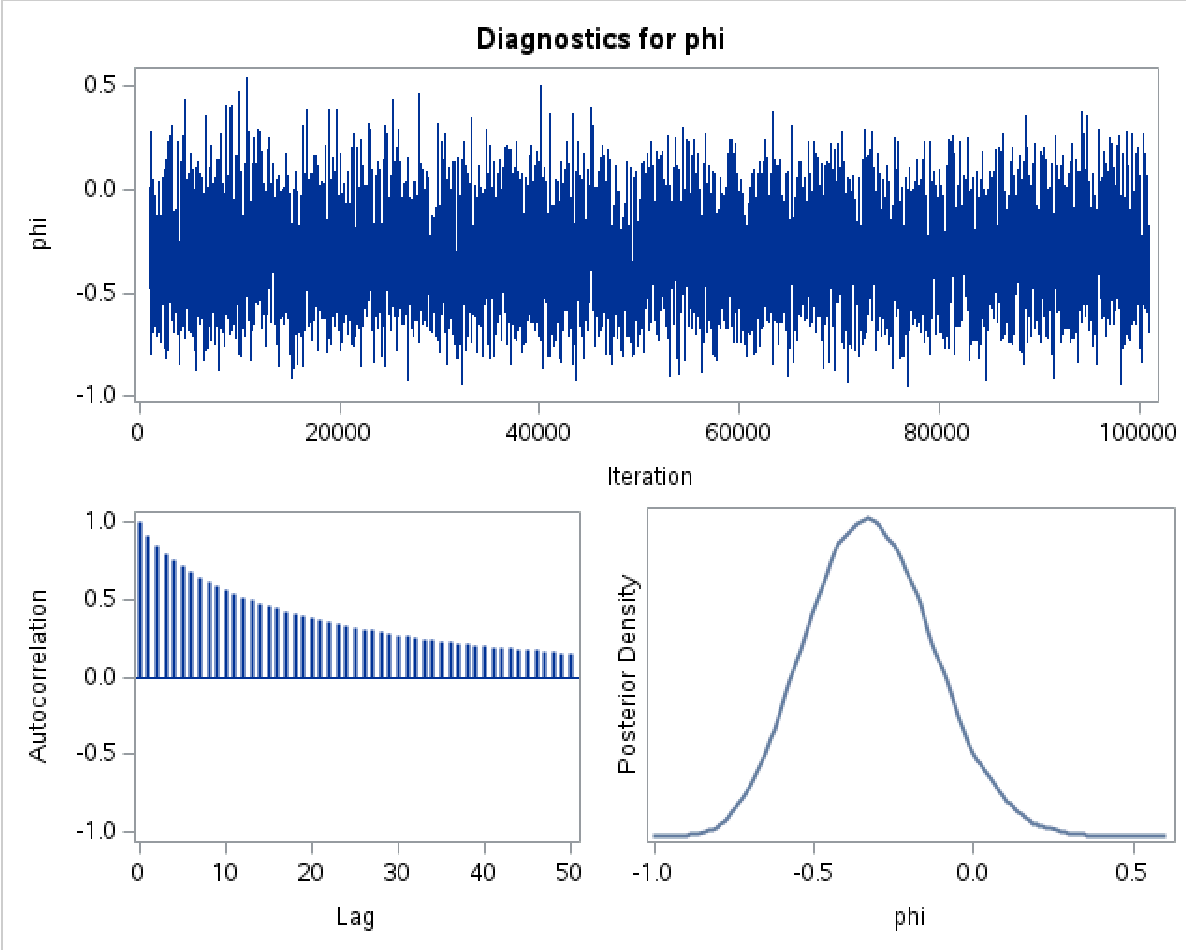

Diagnostics for theta2
theta2
Iteration
Autocorrelation
Lag
Posterior Density
theta2


Diagnostics for theta3
theta3
Iteration
Autocorrelation
Lag
Posterior Density
theta3

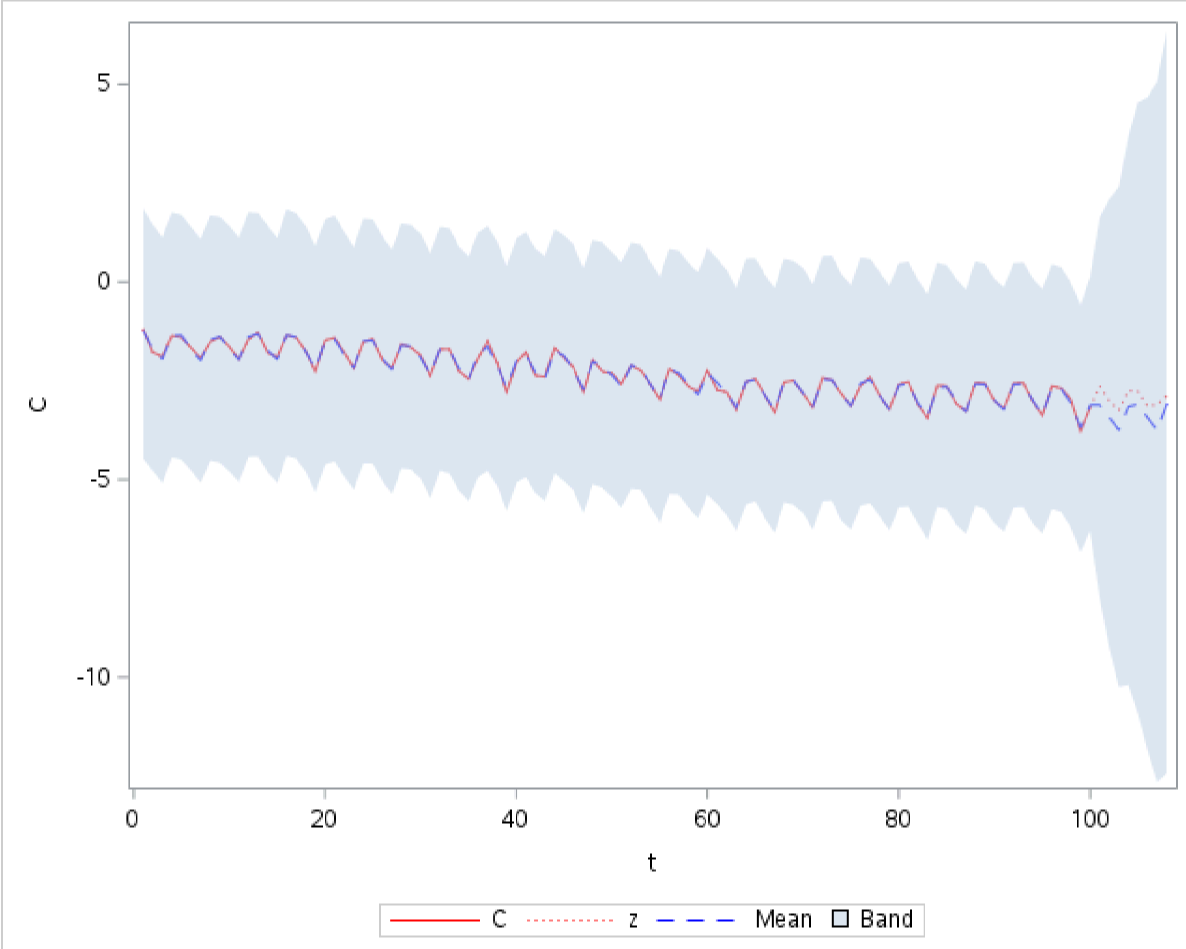

Diagnostics for theta4
theta4
Iteration
Autocorrelation
Lag
Posterior Density
theta4



Diagnostics for theta_phi
theta_phi
Iteration
Autocorrelation
Lag
Posterior Density
theta_phi



Diagnostics for phi
phi
0.5
0.0
-0.5
-1.0
0
20000
40000
60000
80000
100000
Iteration
Autocorrelation
1.0
0.5
0.0
-0.5
-1.0
0
10
20
30
40
50
Lag
Posterior Density
-1.0
-0.5
0.0
0.5
phi



The CONTENTS Procedure

Data Set Name	WORK.POSTERIOR	**Observations**	100000
Member Type	DATA	**Variables**	245
Engine	V9	**Indexes**	0
Created	04/15/2025 14:24:43	**Observation Length**	1960
Last Modified	04/15/2025 14:24:43	**Deleted Observations**	0
Protection		**Compressed**	NO
Data Set Type		**Sorted**	NO
Label			
Data Representation	SOLARIS_X86_64, LINUX_X86_64, ALPHA_TRU64, LINUX_IA64, LINUX_POWER_64		
Encoding	utf-8 Unicode (UTF-8)		

Engine/Host Dependent Information	
Data Set Page Size	131072

Engine/Host Dependent Information	
Number of Data Set Pages	1516
First Data Page	1
Max Obs per Page	66
Obs in First Data Page	50
Number of Data Set Repairs	0
Filename	/opt/sas/viya/config/var/tmp/compsrv/default/93dd6aec-4b16-49c0-9d4d-cb4d015701c2/SAS_workEEB80000020B_sas-compute-server-a5564084-adbf-4dd6-816f-56d632996a06-16126/posterior.sas7bdat
Release Created	V.0400M0
Host Created	Linux
Inode Number	35007704
Access Permission	rw-r--r--
Owner Name	UNKNOWN
File Size	190MB
File Size (bytes)	198836224

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
233	C_101	Num	8	D8.	
234	C_102	Num	8	D8.	
235	C_103	Num	8	D8.	
236	C_104	Num	8	D8.	
237	C_105	Num	8	D8.	
238	C_106	Num	8	D8.	
239	C_107	Num	8	D8.	
240	C_108	Num	8	D8.	
1	Iteration	Num	8	8.	
244	LogLike	Num	8	D8.	Log-Likelihood Value
243	LogMiss	Num	8	D8.	Log Density for Missing Values
245	LogPost	Num	8	D8.	Log Posterior Density
241	LogPrior	Num	8	D8.	Log Prior Density
242	LogReff	Num	8	D8.	Log Random-Effects Prior Density

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
2	alpha0	Num	8	D8.	
13	alpha_1	Num	8	D8.	
14	alpha_2	Num	8	D8.	
15	alpha_3	Num	8	D8.	
16	alpha_4	Num	8	D8.	
17	alpha_5	Num	8	D8.	
18	alpha_6	Num	8	D8.	
19	alpha_7	Num	8	D8.	
20	alpha_8	Num	8	D8.	
21	alpha_9	Num	8	D8.	
22	alpha_10	Num	8	D8.	
23	alpha_11	Num	8	D8.	
24	alpha_12	Num	8	D8.	
25	alpha_13	Num	8	D8.	
26	alpha_14	Num	8	D8.	
27	alpha_15	Num	8	D8.	
28	alpha_16	Num	8	D8.	
29	alpha_17	Num	8	D8.	
30	alpha_18	Num	8	D8.	
31	alpha_19	Num	8	D8.	
32	alpha_20	Num	8	D8.	
33	alpha_21	Num	8	D8.	
34	alpha_22	Num	8	D8.	
35	alpha_23	Num	8	D8.	
36	alpha_24	Num	8	D8.	
37	alpha_25	Num	8	D8.	
38	alpha_26	Num	8	D8.	
39	alpha_27	Num	8	D8.	
40	alpha_28	Num	8	D8.	
41	alpha_29	Num	8	D8.	
42	alpha_30	Num	8	D8.	
43	alpha_31	Num	8	D8.	
44	alpha_32	Num	8	D8.	
45	alpha_33	Num	8	D8.	

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
46	alpha_34	Num	8	D8.	
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48	alpha_36	Num	8	D8.	
49	alpha_37	Num	8	D8.	
50	alpha_38	Num	8	D8.	
51	alpha_39	Num	8	D8.	
52	alpha_40	Num	8	D8.	
53	alpha_41	Num	8	D8.	
54	alpha_42	Num	8	D8.	
55	alpha_43	Num	8	D8.	
56	alpha_44	Num	8	D8.	
57	alpha_45	Num	8	D8.	
58	alpha_46	Num	8	D8.	
59	alpha_47	Num	8	D8.	
60	alpha_48	Num	8	D8.	
61	alpha_49	Num	8	D8.	
62	alpha_50	Num	8	D8.	
63	alpha_51	Num	8	D8.	
64	alpha_52	Num	8	D8.	
65	alpha_53	Num	8	D8.	
66	alpha_54	Num	8	D8.	
67	alpha_55	Num	8	D8.	
68	alpha_56	Num	8	D8.	
69	alpha_57	Num	8	D8.	
70	alpha_58	Num	8	D8.	
71	alpha_59	Num	8	D8.	
72	alpha_60	Num	8	D8.	
73	alpha_61	Num	8	D8.	
74	alpha_62	Num	8	D8.	
75	alpha_63	Num	8	D8.	
76	alpha_64	Num	8	D8.	
77	alpha_65	Num	8	D8.	
78	alpha_66	Num	8	D8.	
79	alpha_67	Num	8	D8.	

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
80	alpha_68	Num	8	D8.	
81	alpha_69	Num	8	D8.	
82	alpha_70	Num	8	D8.	
83	alpha_71	Num	8	D8.	
84	alpha_72	Num	8	D8.	
85	alpha_73	Num	8	D8.	
86	alpha_74	Num	8	D8.	
87	alpha_75	Num	8	D8.	
88	alpha_76	Num	8	D8.	
89	alpha_77	Num	8	D8.	
90	alpha_78	Num	8	D8.	
91	alpha_79	Num	8	D8.	
92	alpha_80	Num	8	D8.	
93	alpha_81	Num	8	D8.	
94	alpha_82	Num	8	D8.	
95	alpha_83	Num	8	D8.	
96	alpha_84	Num	8	D8.	
97	alpha_85	Num	8	D8.	
98	alpha_86	Num	8	D8.	
99	alpha_87	Num	8	D8.	
100	alpha_88	Num	8	D8.	
101	alpha_89	Num	8	D8.	
102	alpha_90	Num	8	D8.	
103	alpha_91	Num	8	D8.	
104	alpha_92	Num	8	D8.	
105	alpha_93	Num	8	D8.	
106	alpha_94	Num	8	D8.	
107	alpha_95	Num	8	D8.	
108	alpha_96	Num	8	D8.	
109	alpha_97	Num	8	D8.	
110	alpha_98	Num	8	D8.	
111	alpha_99	Num	8	D8.	
112	alpha_100	Num	8	D8.	
113	alpha_101	Num	8	D8.	

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
114	alpha_102	Num	8	D8.	
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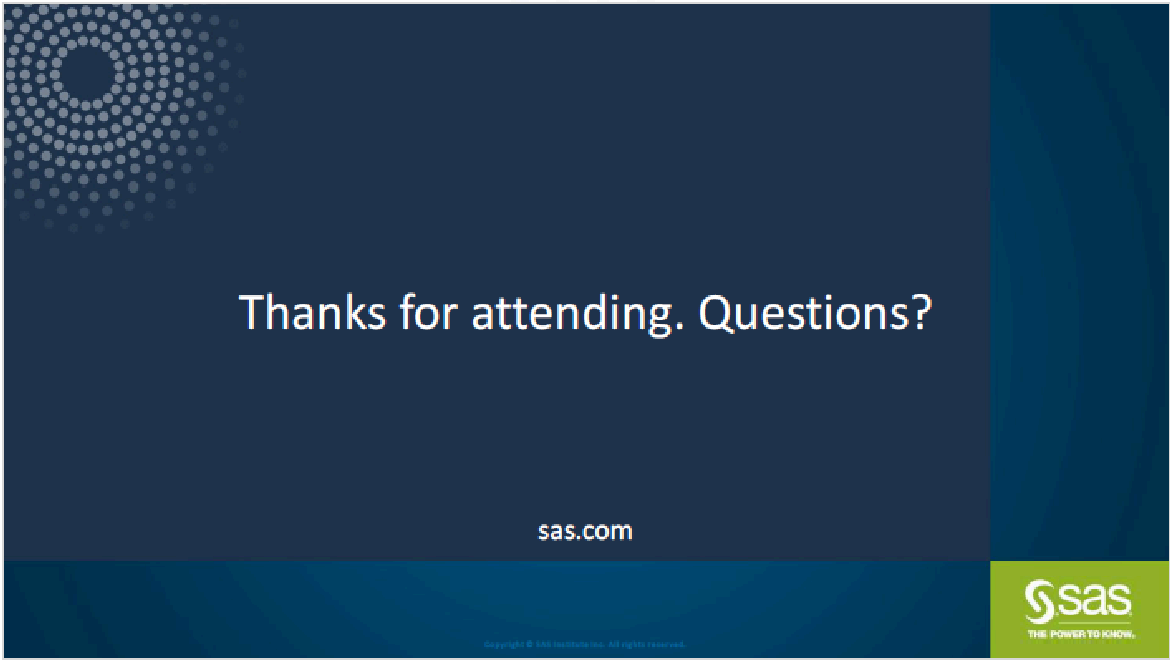
116	alpha_104	Num	8	D8.	
117	alpha_105	Num	8	D8.	
118	alpha_106	Num	8	D8.	
119	alpha_107	Num	8	D8.	
120	alpha_108	Num	8	D8.	
3	mu0	Num	8	D8.	
125	mu_1	Num	8	D8.	
126	mu_2	Num	8	D8.	
127	mu_3	Num	8	D8.	
128	mu_4	Num	8	D8.	
129	mu_5	Num	8	D8.	
130	mu_6	Num	8	D8.	
131	mu_7	Num	8	D8.	
132	mu_8	Num	8	D8.	
133	mu_9	Num	8	D8.	
134	mu_10	Num	8	D8.	
135	mu_11	Num	8	D8.	
136	mu_12	Num	8	D8.	
137	mu_13	Num	8	D8.	
138	mu_14	Num	8	D8.	
139	mu_15	Num	8	D8.	
140	mu_16	Num	8	D8.	
141	mu_17	Num	8	D8.	
142	mu_18	Num	8	D8.	
143	mu_19	Num	8	D8.	
144	mu_20	Num	8	D8.	
145	mu_21	Num	8	D8.	
146	mu_22	Num	8	D8.	
147	mu_23	Num	8	D8.	
148	mu_24	Num	8	D8.	
149	mu_25	Num	8	D8.	
150	mu_26	Num	8	D8.	

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
151	mu_27	Num	8	D8.	
152	mu_28	Num	8	D8.	
153	mu_29	Num	8	D8.	
154	mu_30	Num	8	D8.	
155	mu_31	Num	8	D8.	
156	mu_32	Num	8	D8.	
157	mu_33	Num	8	D8.	
158	mu_34	Num	8	D8.	
159	mu_35	Num	8	D8.	
160	mu_36	Num	8	D8.	
161	mu_37	Num	8	D8.	
162	mu_38	Num	8	D8.	
163	mu_39	Num	8	D8.	
164	mu_40	Num	8	D8.	
165	mu_41	Num	8	D8.	
166	mu_42	Num	8	D8.	
167	mu_43	Num	8	D8.	
168	mu_44	Num	8	D8.	
169	mu_45	Num	8	D8.	
170	mu_46	Num	8	D8.	
171	mu_47	Num	8	D8.	
172	mu_48	Num	8	D8.	
173	mu_49	Num	8	D8.	
174	mu_50	Num	8	D8.	
175	mu_51	Num	8	D8.	
176	mu_52	Num	8	D8.	
177	mu_53	Num	8	D8.	
178	mu_54	Num	8	D8.	
179	mu_55	Num	8	D8.	
180	mu_56	Num	8	D8.	
181	mu_57	Num	8	D8.	
182	mu_58	Num	8	D8.	
183	mu_59	Num	8	D8.	
184	mu_60	Num	8	D8.	

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
185	mu_61	Num	8	D8.	
186	mu_62	Num	8	D8.	
187	mu_63	Num	8	D8.	
188	mu_64	Num	8	D8.	
189	mu_65	Num	8	D8.	
190	mu_66	Num	8	D8.	
191	mu_67	Num	8	D8.	
192	mu_68	Num	8	D8.	
193	mu_69	Num	8	D8.	
194	mu_70	Num	8	D8.	
195	mu_71	Num	8	D8.	
196	mu_72	Num	8	D8.	
197	mu_73	Num	8	D8.	
198	mu_74	Num	8	D8.	
199	mu_75	Num	8	D8.	
200	mu_76	Num	8	D8.	
201	mu_77	Num	8	D8.	
202	mu_78	Num	8	D8.	
203	mu_79	Num	8	D8.	
204	mu_80	Num	8	D8.	
205	mu_81	Num	8	D8.	
206	mu_82	Num	8	D8.	
207	mu_83	Num	8	D8.	
208	mu_84	Num	8	D8.	
209	mu_85	Num	8	D8.	
210	mu_86	Num	8	D8.	
211	mu_87	Num	8	D8.	
212	mu_88	Num	8	D8.	
213	mu_89	Num	8	D8.	
214	mu_90	Num	8	D8.	
215	mu_91	Num	8	D8.	
216	mu_92	Num	8	D8.	
217	mu_93	Num	8	D8.	
218	mu_94	Num	8	D8.	

Alphabetic List of Variables and Attributes					
#	Variable	Type	Len	Format	Label
219	mu_95	Num	8	D8.	
220	mu_96	Num	8	D8.	
221	mu_97	Num	8	D8.	
222	mu_98	Num	8	D8.	
223	mu_99	Num	8	D8.	
224	mu_100	Num	8	D8.	
225	mu_101	Num	8	D8.	
226	mu_102	Num	8	D8.	
227	mu_103	Num	8	D8.	
228	mu_104	Num	8	D8.	
229	mu_105	Num	8	D8.	
230	mu_106	Num	8	D8.	
231	mu_107	Num	8	D8.	
232	mu_108	Num	8	D8.	
12	phi	Num	8	D8.	
4	s0	Num	8	D8.	
5	s1	Num	8	D8.	
6	s2	Num	8	D8.	
121	s_1	Num	8	D8.	
122	s_2	Num	8	D8.	
123	s_3	Num	8	D8.	
124	s_4	Num	8	D8.	
7	theta1	Num	8	D8.	
8	theta2	Num	8	D8.	
9	theta3	Num	8	D8.	
10	theta4	Num	8	D8.	
11	theta_phi	Num	8	D8.	


Thanks for attending. Questions?
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