

1. (15 points) Evaluate the following integrals. (5 points for each)

(a) $\int x\sqrt{2-5x} \, dx$

(b) $\int \frac{\cos(\pi/x)}{x^2} \, dx$

(c) $\int \frac{1}{\sqrt{x}\sqrt{x+x}} \, dx$

2. (10 points) The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm. Use differentials to estimate maximum error and the relative error in the calculated area of the disk.

3. Let $f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}} \, dt$, where $g(x) = \int_0^{\cos x} [1 + \sin(t^2)] \, dt$.

(a) (2 points) Find $g(\frac{\pi}{2})$

(b) (4 points) Find $g'(x)$

(c) (4 points) Find $f'(\frac{\pi}{2})$

4. (10 points) Find the volume of the solid generated by revolving the plane region enclosed by $y = 2x - x^2$ and $y = 0$ about the line $x = 4$.

5. (10 points) Prove that the equation $3x + 1 - \sin x = 0$ has exactly one real solution.

6. Let $f(x) = (x^3 + x^2)^{\frac{1}{3}}$

(a) (6 points) Find the intervals of increase or decrease.

(b) (6 points) Find the intervals of concavity.

(c) (2 points) Find the local maximum and minimum values.

(d) (1 points) Find the inflection points.

7. (10 points) A plane flies horizontally at an altitude of 5 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\frac{\pi}{3}$, this angle is decreasing at a rate of $\frac{\pi}{6}$ rad / min. How fast is the plane traveling at that time?

8. A curve $C: x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ on xy -plane. There is a point $P(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$ on the curve C .

(a) (5 points) Find the lines that are tangent to the curve C at the point P .

(b) (5 points) Find the lines that are normal to the curve C at the point P .

(c) (10 points) Find the arc length of the curve C from the point $P(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4})$ to the point $Q(1, 0)$ on the curve C .

9. (10 points) If a resistor of R ohms is connected across a battery of E volts with internal resistance r ohms, then the power (in watts) in the external resistor is

$$P = \frac{E^2 R}{(R + r)^2}$$

If E and r are fixed but R varies, what is the maximum value of the power?

10. (10 points) A curve $x = \sqrt{r^2 - y^2}$, $0 \leq y \leq \frac{r}{2}$ is rotated about y -axis. Please find the area of the resulting surface.

題目終於寫完了！ 請再仔細檢查一遍！