# Linear Algebra 1

## Exercise Number 7

**1)** a) If possible, express the vector $(7, 3, 6, 14)$ as a linear combination of the three vectors $(0, 2, 0, 0); (1, 0, 0, 0); (2, 1, 3, 7)$.
b) Express the polynomial $q(x) = 2x^4+3x^2+3$ as a linear combination of the two polynomials $p_1(x) = x^4 + 3x$ and $p_2(x) = x^2 - 2x + 1$.

**2)** Find a spanning set for the space of solutions of the linear system

$$\begin{aligned} 2x + 3y + 4z &= 0 \\ x - y \ \ + z \ \ &= 0 \end{aligned}$$

**3)** Given the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{pmatrix}$, find a spanning set for the space of solutions for the linear system $Ax = 0$.

**4)** Can the space $Sp\{(1, 2, 0, -1), (2, 4, 0, 2), (-3, -6, 0, 3)\}$ be spanned with less than three vectors?

**5)** a) Assume that $U = Sp\{u_1, u_2, \ldots, u_r\}$ and $W = Sp\{u_2, \ldots, u_r\}$. Prove that $U = W$ if and only if the vector $u_1$ is a linear combination of $u_2, \ldots, u_r$.
b) Let $A$ be a reduced echelon matrix of size $m \times n$. Consider the nonzero rows of $A$ as vectors $v_1, \ldots, v_r$ in the vector space $F^n$. Let $v_{i_1}, \ldots, v_{i_l}$ be a proper subset of the set of vectors $v_1, \ldots v_r$. Prove that it is impossible that

$$Sp\{v_{i_1}, \ldots, v_{i_l}\} = Sp\{v_1, \ldots, v_r\}$$

**6)** Let

$$Sp\{(1, -1, 2, 3), (-1, 0, 1, 1), (2, 0, -1, 1)\} \subset \mathbf{R}^4$$

What are the conditions on the numbers $a, b, c, d$ in $\mathbf{R}$ so that $(a, b, c, d) \in W$?