Linear Algebra 1

Exercise Number 1

1) Use induction to prove:

a)

$$\frac{1^2}{1\cdot 3}+\frac{2^2}{3\cdot 5}+\ldots+\frac{n^2}{(2n-1)(2n+1)}=\frac{n(n+1)}{2(2n+1)}$$

b)

$$2+\frac{3}{(1\cdot 2)^2}+\frac{5}{(2\cdot 3)^2}+\ldots+\frac{2n+1}{n^2(n+1)^2}=3-\frac{1}{(n+1)^2}$$

2) In the set of complex numbers $\mathbf{C}$ define the following numbers $z_1=2-i$, $z_2=3i$, $z_3=-1+3i$ and $z_4=5$. Compute the following numbers: $4z_1-5z_2+2\bar{z}_3-z_4$, $z_1\bar{z}_2$, $\frac{z_3}{z_1}$, z_1^2, $\frac{1}{z_3}$, $\mathrm{Im}(z_2-3z_1)$, $\mathrm{Re}(z_2-3z_4)$, $|z_1+z_2|$, $|z_1-z_2|$.

3) Prove the following:

a) $|\frac{z_1}{z_2}|=\frac{|z_1|}{|z_2|}$ b) $|z_1z_2|=|z_1||z_2|$ c) $z\bar{z}=|z|^2$

d) $\overline{z_1\pm z_2}=\bar{z_1}\pm\bar{z}_2$ e) $\bar{\bar{z}}=z$ f) $z+\bar{z}=2\mathrm{Re}(z)$

4) Use induction to prove that $|z^n|=|z|^n$ for all $z\in\mathbf{C}$ and all $n\in\mathbf{N}$.

5) a) Prove that $|z|^2=(\mathrm{Re}z)^2+(\mathrm{Im}z)^2$.

b) Use part a) to prove that $|\mathrm{Re}z|\le|z|$ and $|\mathrm{Im}z|\le|z|$.