Linear Algebra 1

Exercise Number 2

1) Solve the following linear systems over the real numbers:

$$\begin{aligned} x_1 - x_2 + x_3 &= 2 \\ 3x_1 - x_2 + 2x_3 &= -6 \\ 3x_1 + x_2 + x_3 &= -18 \end{aligned} \qquad \begin{aligned} x_1 - x_2 + x_3 + x_4 &= -2 \\ x_1 + x_2 - x_3 + 2x_4 &= 1 \\ 3x_1 - x_2 + x_3 + 2x_4 &= 2 \end{aligned}$$

$$\begin{aligned} x_1+x_2+x_3+x_4 &= 1 \\ x_1+x_2+x_3-x_4 &= 2 \\ x_1+x_2-x_3-x_4 &= 3 \\ x_1-x_2-x_3-x_4 &= 4 \end{aligned}$$

2) Find for what values of t the following linear system has a unique solution, no solution, infinite number of solutions. Work over the real numbers. You dont need to find the explicit sets of solutions.

$$\begin{aligned} tx + 3y - z &= 1 \\ x + 2y - z &= 2 \\ -tx + y + 2z &= -1 \end{aligned}$$

3) Let A be the matrix that represents a linear system with n unknowns. Let B be the reduced echelon matrix obtained from A using elementary row operations. Show that the number of free variables is n minus the number of leading variables in B.

4) Solve the following linear system over the complex numbers

$$\begin{aligned} ix+(1+i)y+2z &= 0 \\ 2x-(1-i)y+iz &= 0 \end{aligned}$$

5) Suppose that linear system

$$ax+by=0$$
$$cx+dy=0$$

has a unique solution. Prove that for any choice of numbers c_1 and c_2, the linear system

$$ax+by=c_1$$
$$cx+dy=c_2$$

has a unique solution.

6) Given a linear system, denote by A the corresponding reduced echelon matrix.

a) If the system is homogenous, and if A has five leading terms, what is the minimal number of variables that the system can possibly have?

b) If the system is homogenous and has 5 variables and a unique solution, what is the minimal and maximal number of leading terms in the matrix A

c) Is it possible that the leading term which appears in the fifth row of A, be located in the sixth column of A? in its fourth column? Explain!

d) If A has five columns, what is the minimal number of leading terms the matrix A can have? Explain!

7) From all the citizens of a certain country, which start the year in this country, 80 percent stay and 20 percent leave the country. From all the citizens which live outside the country, 90 percent stay outside and 10 percent return to the country. If at the end of the year there were 200 million people in the country, and 30 millions live outside the country, write down a system of linear equations to find the number of citizens leaving in the country and outside it, at the beginning of the year. No need to solve the system.