Workshop on R and movement ecology:

Hong Kong University, Jan 2018

Eric Dougherty, Dana Seidel, Wayne Getz

Lecture 7

Epidemiological models

DEPARTMENT *of* ENVIRONMENTAL SCIENCE, POLICY, AND MANAGEMENT

Berkeley UNIVERSITY OF CALIFORNIA College of Natural Resources

Continuous time susceptible SEIR disease state model

S: suceptible

E: exposed
(infected but not infectious)

I: infectious

R=V+D:
(V: immune, D: dead)

λ: recruitment

μ: natural mortality

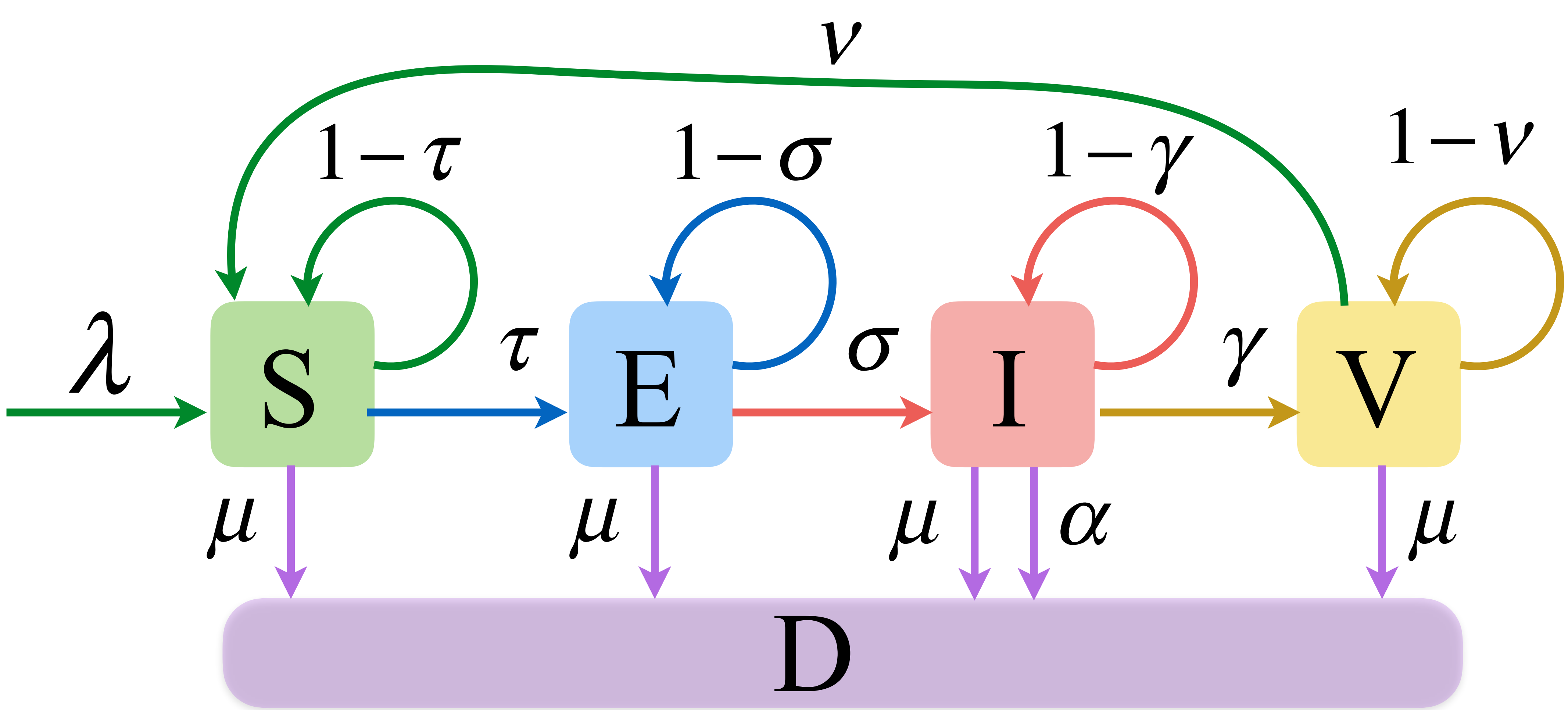
α: disease-induced mortality

τ: per-capita S transmission rate

$1/\sigma$: latent-period

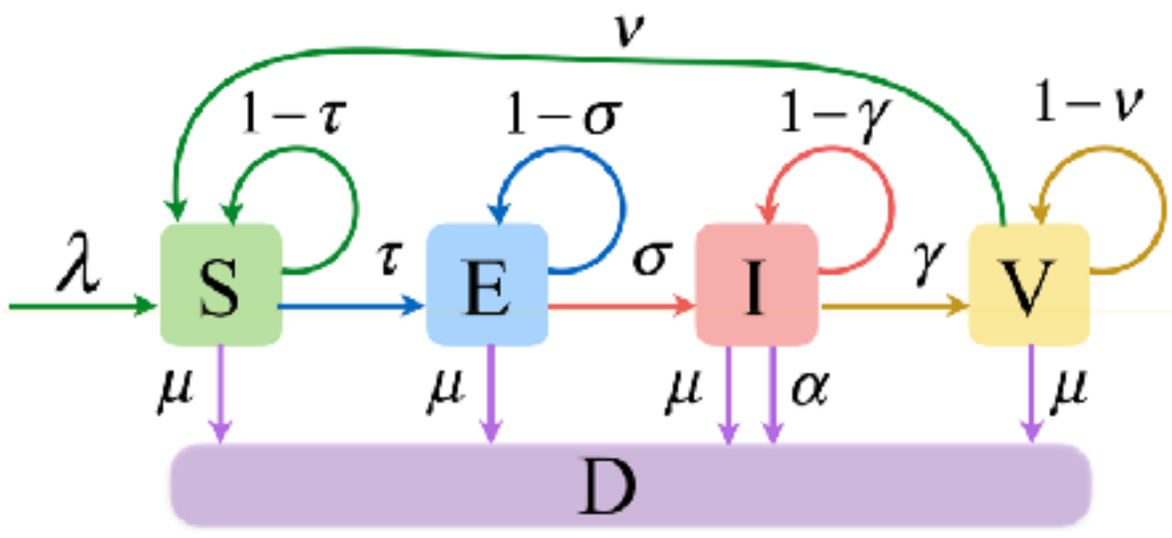
$1/\gamma$: infectious-period

$1/\nu$: immune-period



Continuous time susceptible SEIR disease state model

$$\begin{aligned}
\frac{dS}{dt} &= \lambda(t) + \nu V - (\tau(I,N) + \mu)S \\
\frac{dE}{dt} &= \tau(I,N)S - (\sigma + \mu)E \\
\frac{dI}{dt} &= \sigma E - (\gamma + \alpha + \mu)I \\
\frac{dV}{dt} &= \gamma I - (\nu + \mu)V
\end{aligned}$$



$$\tau(I,N) = \frac{\beta I}{N}, \quad \text{where} \quad N = S + E + I + V$$

Expected number of new infections from index case

$$R_0 = \frac{\beta\sigma}{(\sigma+\mu)(\gamma+\mu+\alpha)}$$

Outbreak threshold

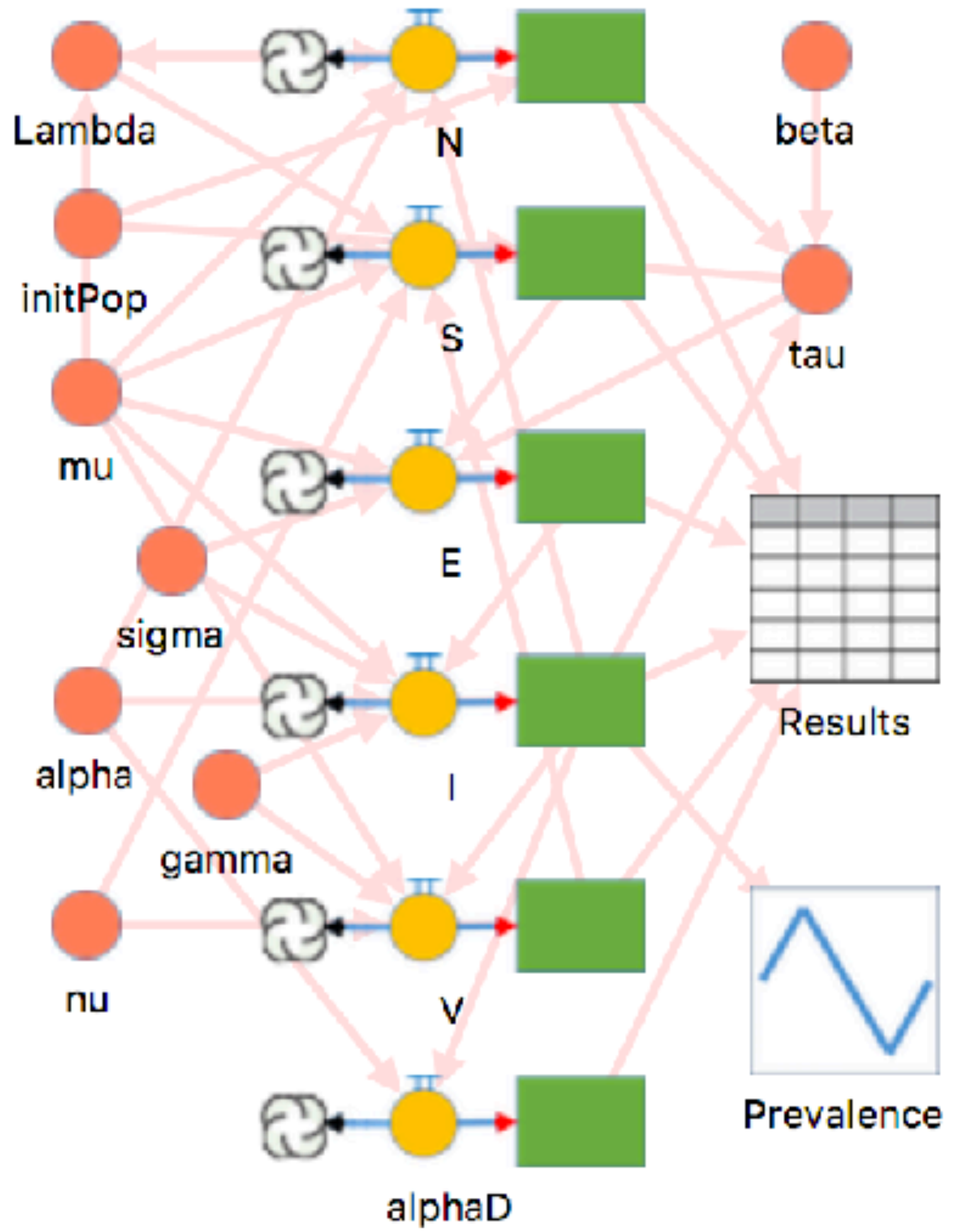
$$\beta > \frac{(\sigma+\mu)(\gamma+\mu+\alpha)}{\sigma}$$

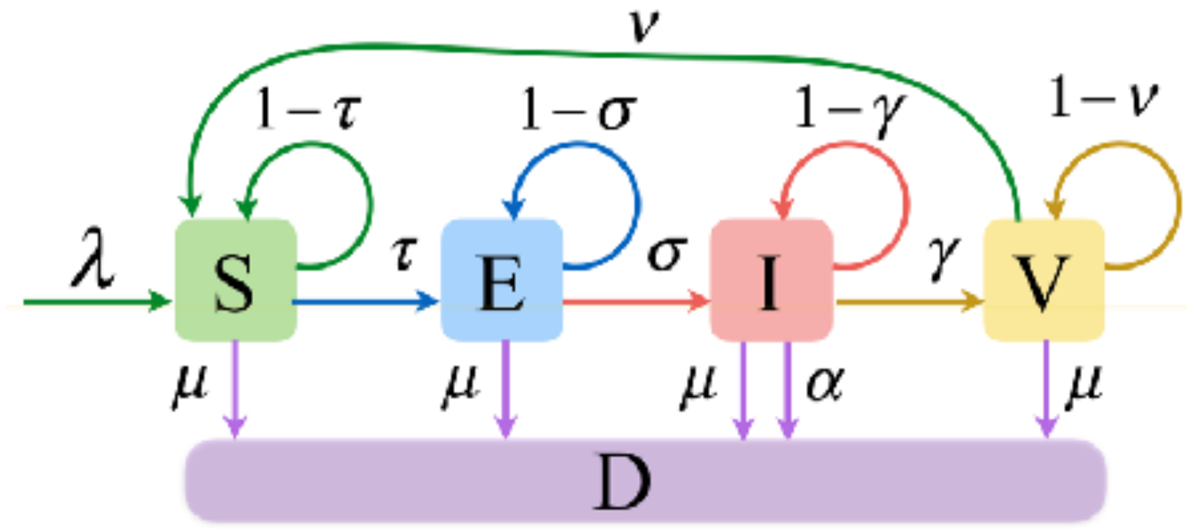
Deaths: natural & disease

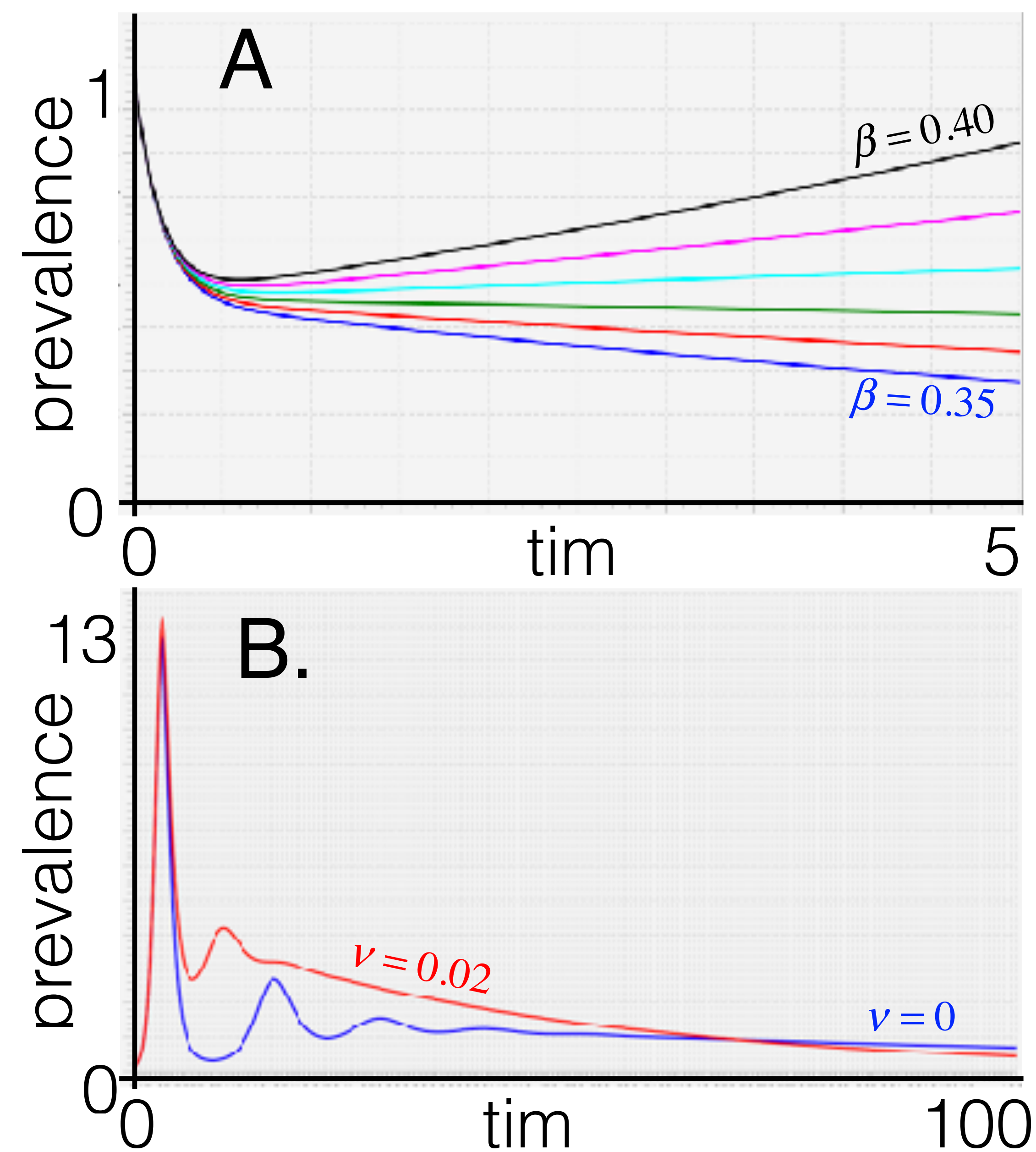
$$\begin{aligned}
D^{\mu}(t) &= \int_0^t \mu N(z)dz \\
D^{\alpha}(t) &= \int_0^t \alpha I(z)dz
\end{aligned}$$

Continuous time susceptible SEIR disease state model

Numerus Model Builder Implementation

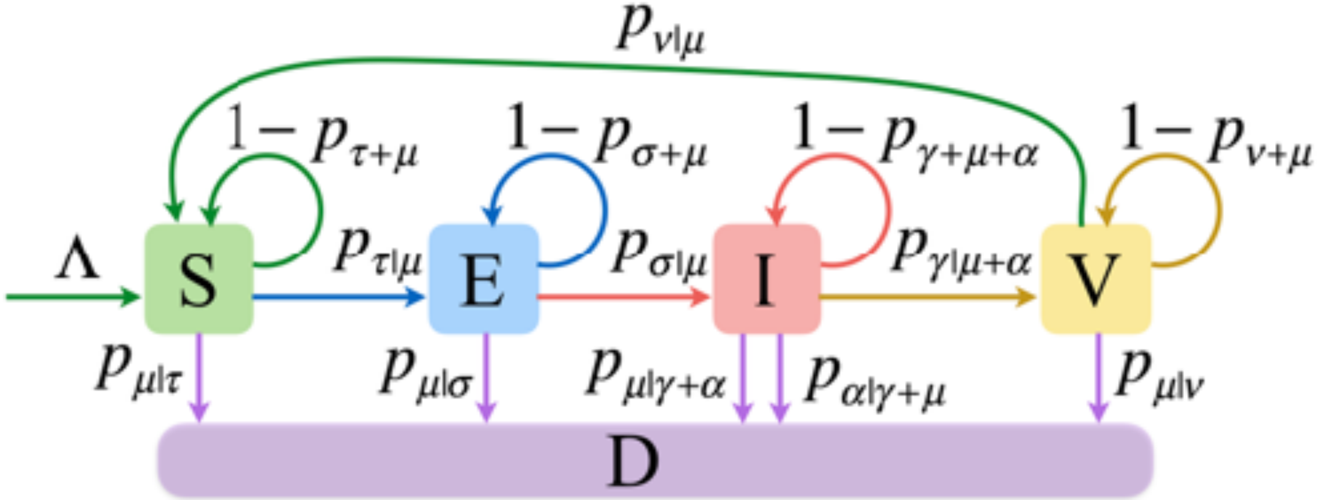


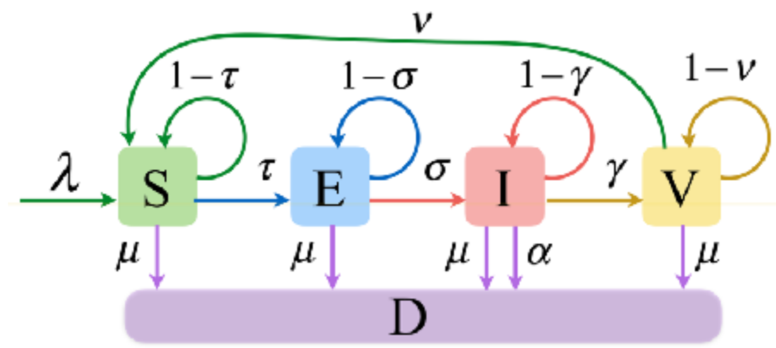




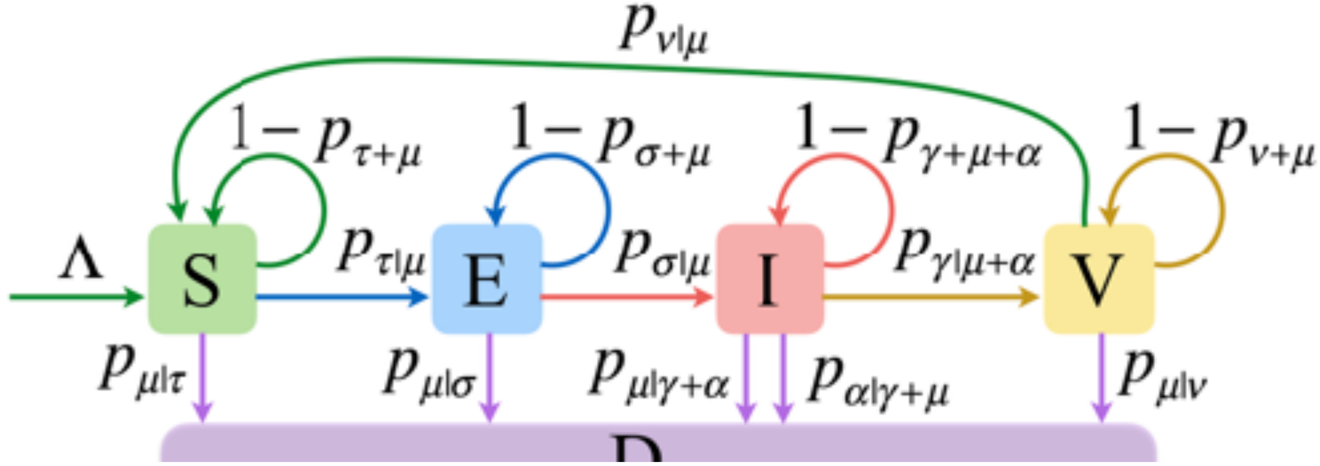
$S(0) = 999$, $E(0) = 0$, $I(0) = 1$, and $V = 0$ for the case $\mu = 0.01$, $\alpha = 0.05$, $\sigma = \gamma = 0.3$, under the assumption that $\lambda(t) = \mu N(t)$. In addition: in A. $\nu = 0$ and β varies from 0.35 to 0.40 (in steps of 0.01); and in B. $\beta = 1$ and $\nu = 0.02$ (red) and $\nu = 0$ (blue).

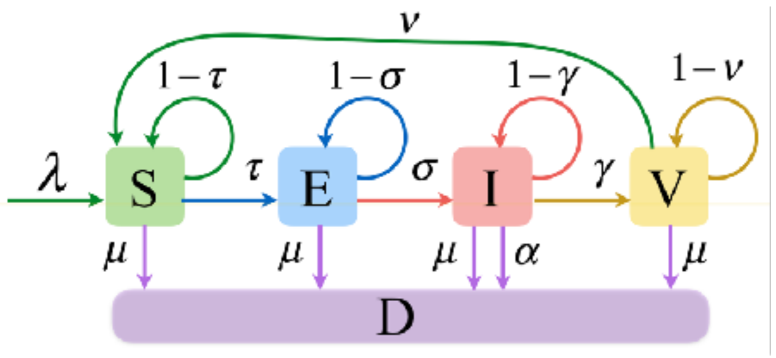
Discrete time susceptible SEIR disease state model





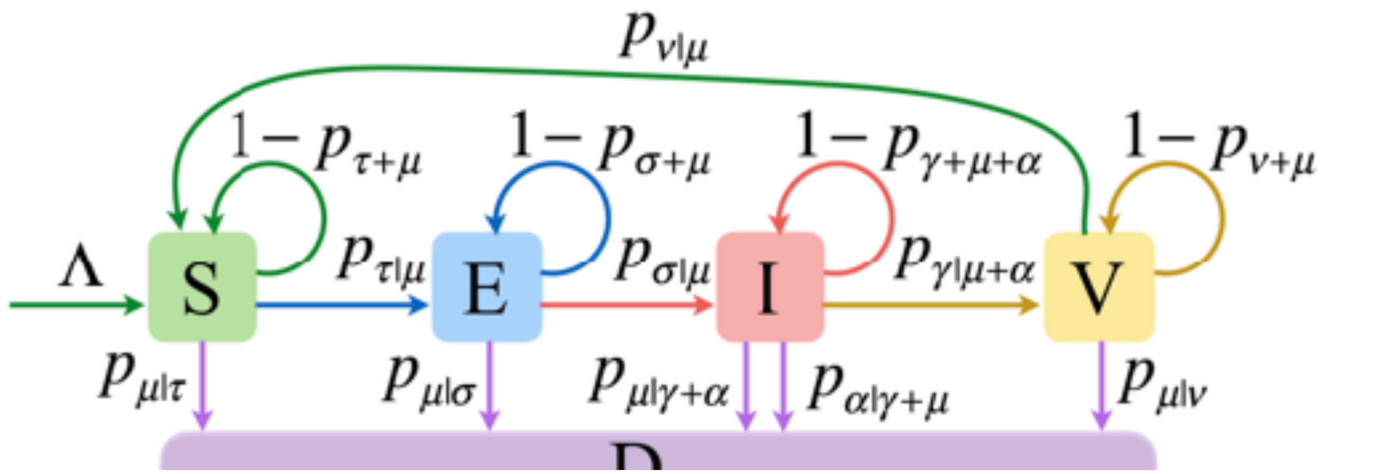
Discrete time susceptible SEIR disease state model

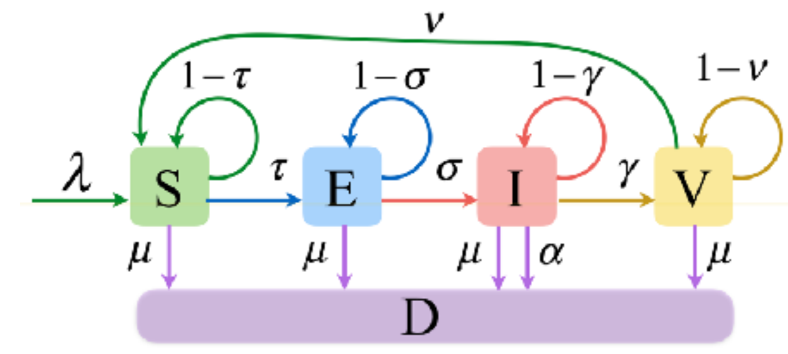




$$\Lambda_t = \int_t^{t+1} \lambda(s) ds \quad \text{and} \quad \tau_t = \tau(I(t), N(t))$$

Discrete time susceptible SEIR disease state model

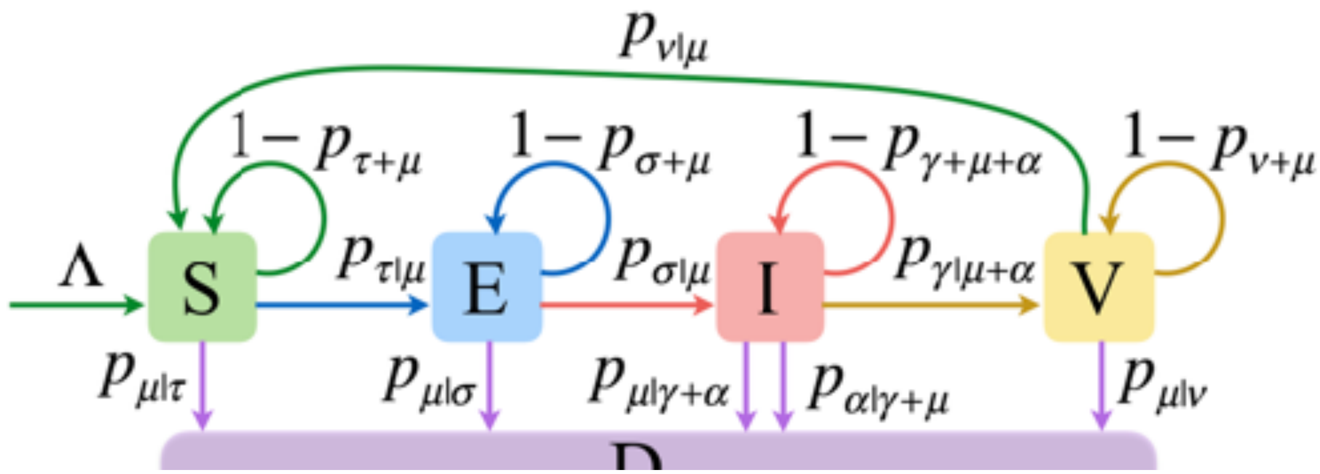


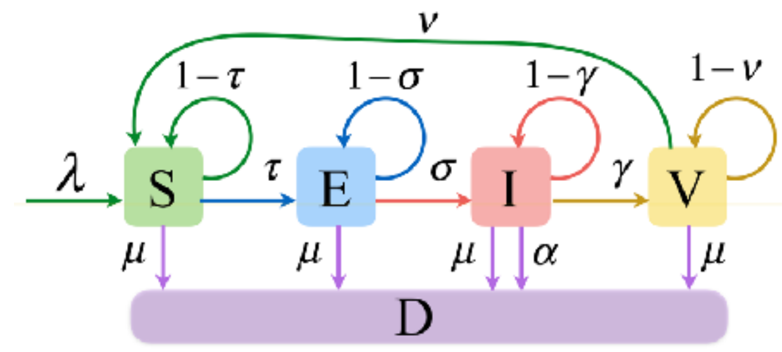


$$\Lambda_t = \int_t^{t+1} \lambda(s)ds \quad \text{and} \quad \tau_t = \tau(I(t), N(t))$$

$$\begin{aligned}
S(t+1) &= \Lambda_t + p_{\nu|\mu}V(t) + S(t)(1 - p_{\tau_t+\mu}) \\
E(t+1) &= p_{\tau_t|\mu}S(t) + E(t)(1 - p_{\sigma+\mu}) \\
I(t+1) &= p_{\sigma|\mu}E(t) + I(t)(1 - p_{\gamma+\alpha+\mu}) \\
V(t+1) &= p_{\gamma|\mu+\alpha}I(t) + V(t)(1 - p_{\nu+\mu})
\end{aligned}$$

Discrete time susceptible SEIR disease state model





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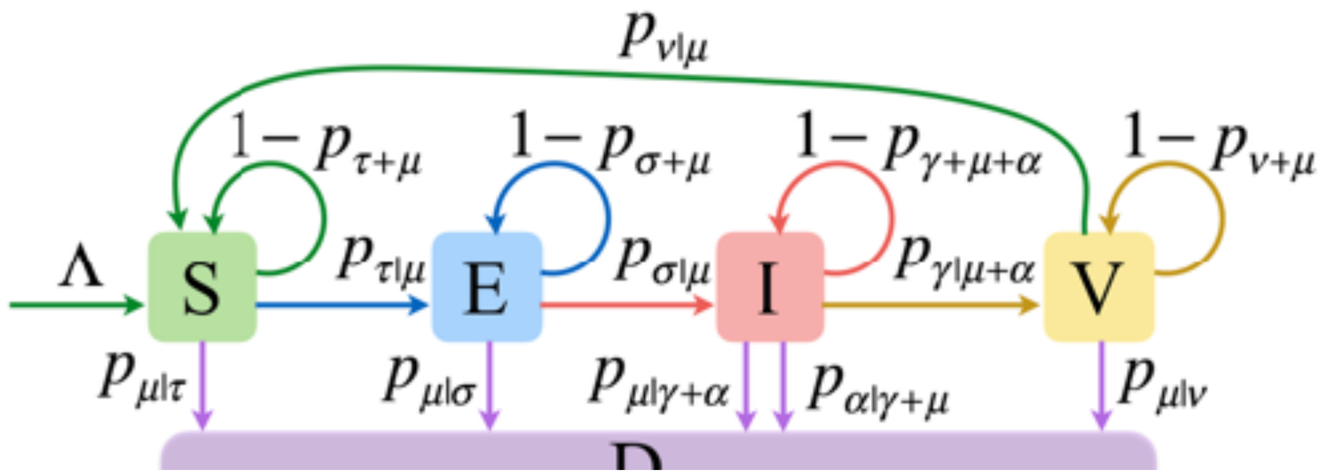
V(t+1) &= p_{\gamma|\mu+\alpha}I(t) + V(t)(1 - p_{\nu+\mu})
\end{aligned}$$

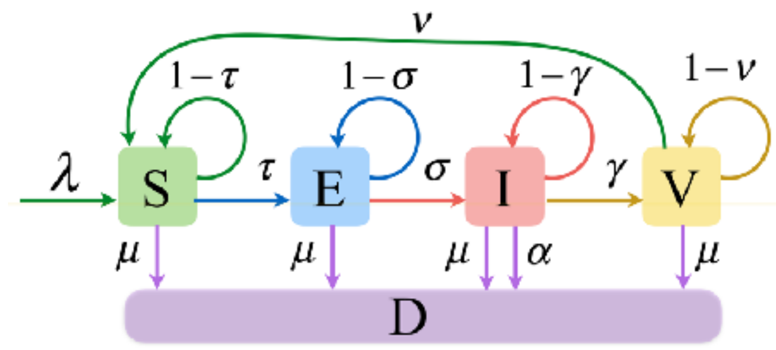
$$\begin{aligned}
p_{\gamma|\alpha+\mu} &= \frac{\gamma\left(1 - e^{-(\gamma+\alpha+\mu)}\right)}{\gamma+\alpha+\mu} \\
p_{\alpha|\gamma+\mu} &= \frac{\alpha\left(1 - e^{-(\gamma+\alpha+\mu)}\right)}{\gamma+\alpha+\mu} \\
p_{\mu|\gamma+\alpha} &= \frac{\mu\left(1 - e^{-(\gamma+\alpha+\mu)}\right)}{\gamma+\alpha+\mu}
\end{aligned}$$

Competing rates formulation and so on for rest of p's

$$p_{\gamma|\alpha+\mu} + p_{\alpha|\gamma+\mu} + p_{\mu|\gamma+\alpha} = p_{\gamma+\alpha+\mu}$$

Discrete time susceptible SEIR disease state model





$$\Lambda_t = \int_t^{t+1} \lambda(s)ds \quad \text{and} \quad \tau_t = \tau(I(t), N(t))$$

$$\begin{aligned}
S(t+1) &= \Lambda_t + p_{\nu|\mu}V(t) + S(t)(1-p_{\tau_t+\mu}) \\
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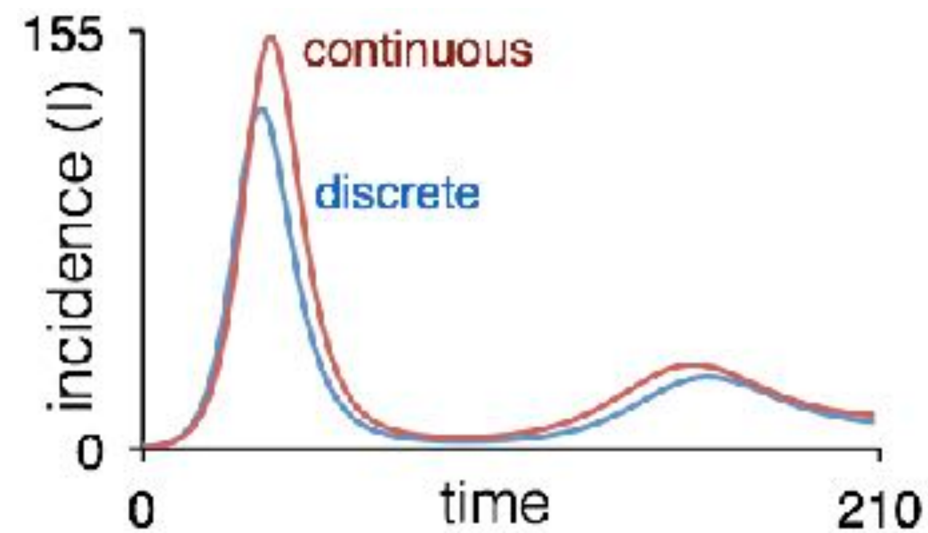
\end{aligned}$$



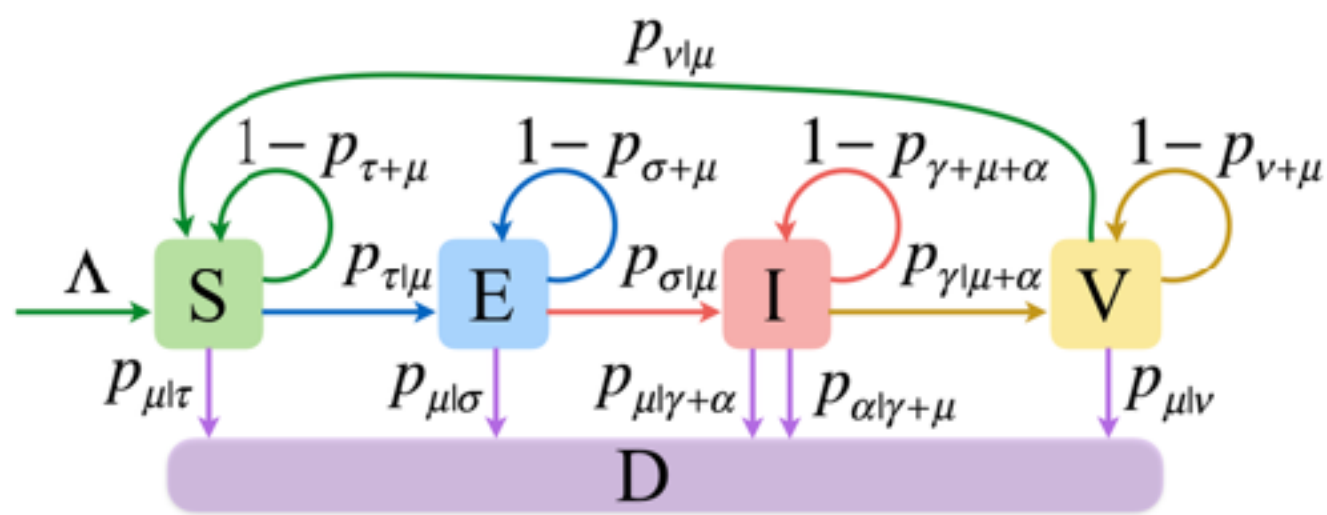
$$\begin{aligned}
p_{\gamma|\alpha+\mu} &= \frac{\gamma\left(1-e^{-(\gamma+\alpha+\mu)}\right)}{\gamma+\alpha+\mu} \\
p_{\alpha|\gamma+\mu} &= \frac{\alpha\left(1-e^{-(\gamma+\alpha+\mu)}\right)}{\gamma+\alpha+\mu} \\
p_{\mu|\gamma+\alpha} &= \frac{\mu\left(1-e^{-(\gamma+\alpha+\mu)}\right)}{\gamma+\alpha+\mu}
\end{aligned}$$

Competing rates formulation and so on for rest of p's

$$p_{\gamma|\alpha+\mu} + p_{\alpha|\gamma+\mu} + p_{\mu|\gamma+\alpha} = p_{\gamma+\alpha+\mu}$$

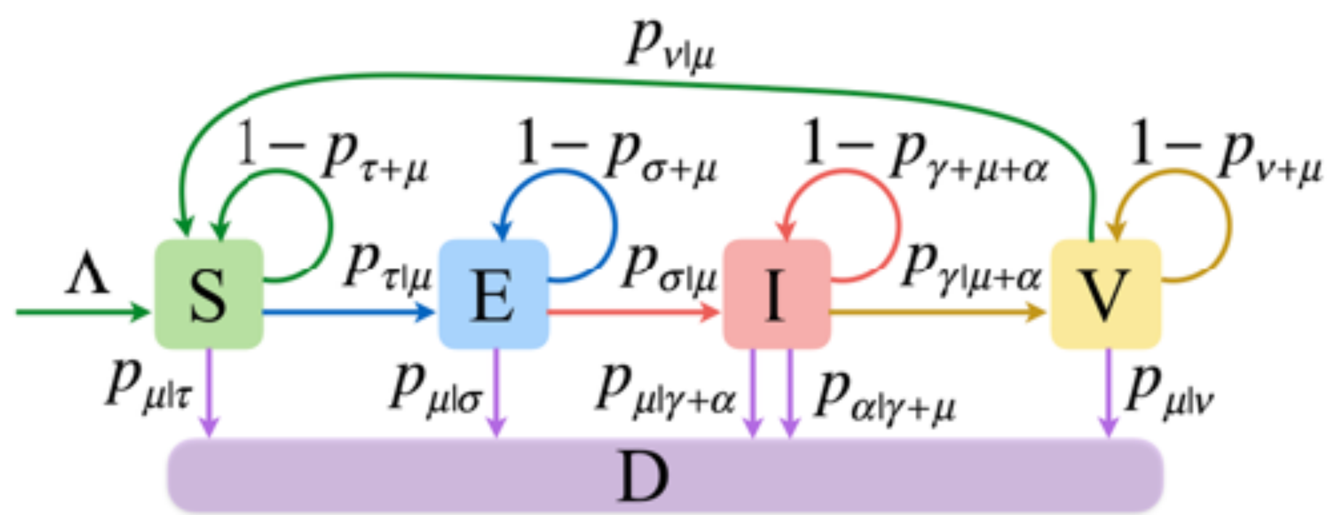
Stochastic time susceptible SEIR disease state model

$$
\begin{aligned}
S(t+1) &= \hat{S}(t) + \hat{\Lambda}_t + \hat{U}^{\mathrm{V}}(t) \\
&\quad - \left(\hat{T}(t) + \hat{M}^{\mathrm{S}}(t)\right) \\
E(t+1) &= \hat{E}(t) + \hat{T}(t) \\
&\quad - \left(\hat{U}^{\mathrm{E}}(t) + \hat{M}^{\mathrm{E}}(t)\right) \\
I(t+1) &= \hat{I}(t) + \hat{U}^{\mathrm{E}}(t) \\
&\quad - \left(\hat{U}^{\mathrm{I}}(t) + \hat{M}^{\mathrm{I}}(t) + \hat{D}(t)\right) \\
V(t+1) &= \hat{V}(t) + \hat{U}^{\mathrm{I}}(t) \\
&\quad - \left(\hat{U}^{\mathrm{V}}(t) + \hat{M}^{\mathrm{V}}(t)\right)
\end{aligned}
$$



Stochastic time susceptible SEIR disease state model

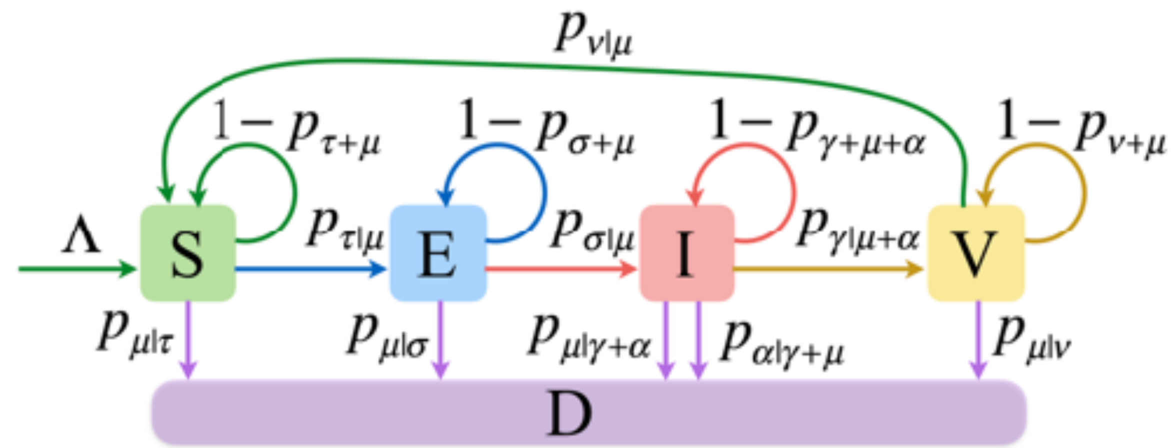
$$
\begin{aligned}
S(t+1) &= \hat{S}(t) + \hat{\Lambda}_t + \hat{U}^{\mathrm{V}}(t) \\
&\quad - \left(\hat{T}(t) + \hat{M}^{\mathrm{S}}(t)\right) \\
E(t+1) &= \hat{E}(t) + \hat{T}(t) \\
&\quad - \left(\hat{U}^{\mathrm{E}}(t) + \hat{M}^{\mathrm{E}}(t)\right) \\
I(t+1) &= \hat{I}(t) + \hat{U}^{\mathrm{E}}(t) \\
&\quad - \left(\hat{U}^{\mathrm{I}}(t) + \hat{M}^{\mathrm{I}}(t) + \hat{D}(t)\right) \\
V(t+1) &= \hat{V}(t) + \hat{U}^{\mathrm{I}}(t) \\
&\quad - \left(\hat{U}^{\mathrm{V}}(t) + \hat{M}^{\mathrm{V}}(t)\right)
\end{aligned}
$$

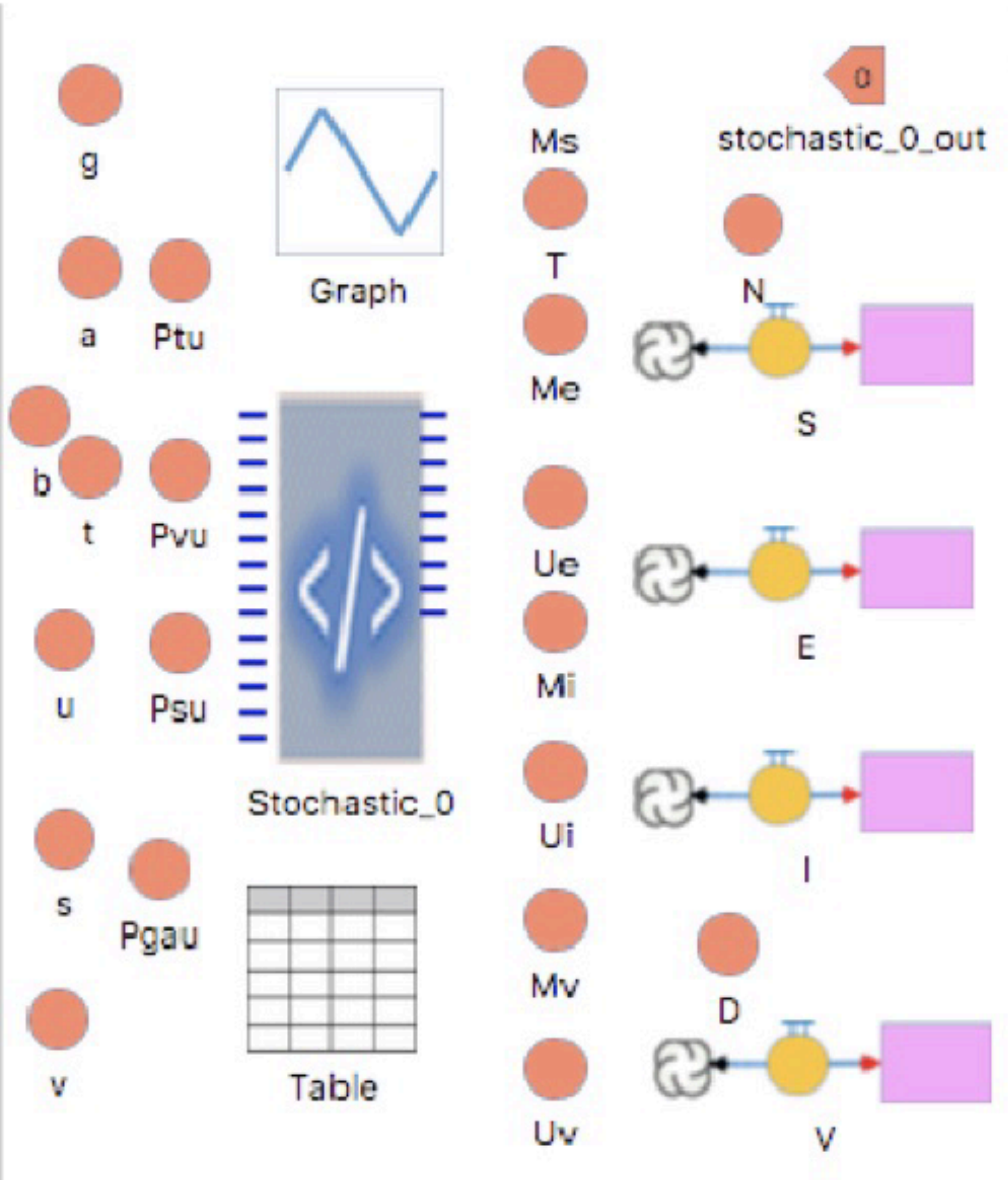


$$
\begin{aligned}
(\hat{S}(t), \hat{T}(t), \hat{M}^{\mathrm{S}}(t)) &:= \mathrm{MULTINOMIAL}\left[S(t); 1 - p_{\tau_t+\mu}, p_{\tau_t|\mu}, p_{\mu|\tau_t}\right] \\
(\hat{E}(t), \hat{U}^{\mathrm{E}}(t), \hat{M}^{\mathrm{E}}(t)) &:= \mathrm{MULTINOMIAL}\left[E(t); 1 - p_{\sigma+\mu}, p_{\sigma|\mu}, p_{\mu|\sigma}\right] \\
(\hat{I}(t), \hat{U}^{\mathrm{I}}(t), \hat{M}^{\mathrm{I}}(t), \hat{D}(t)) &:= \mathrm{MULTINOMIAL}\left[I(t); 1 - p_{\gamma+\alpha+\mu}, p_{\mu|\alpha+\mu}, p_{\gamma|\alpha+\mu}, p_{\alpha|\gamma+\mu}\right] \\
(\hat{V}(t), \hat{U}^{\mathrm{V}}(t), \hat{M}^{\mathrm{V}}(t)) &:= \mathrm{MULTINOMIAL}\left[V(t); 1 - p_{\nu+\mu}, p_{\mu|\nu}, p_{\nu|\mu}\right]
\end{aligned}
$$

Stochastic time susceptible SEIR disease state model

Numerus Model Builder Implementation

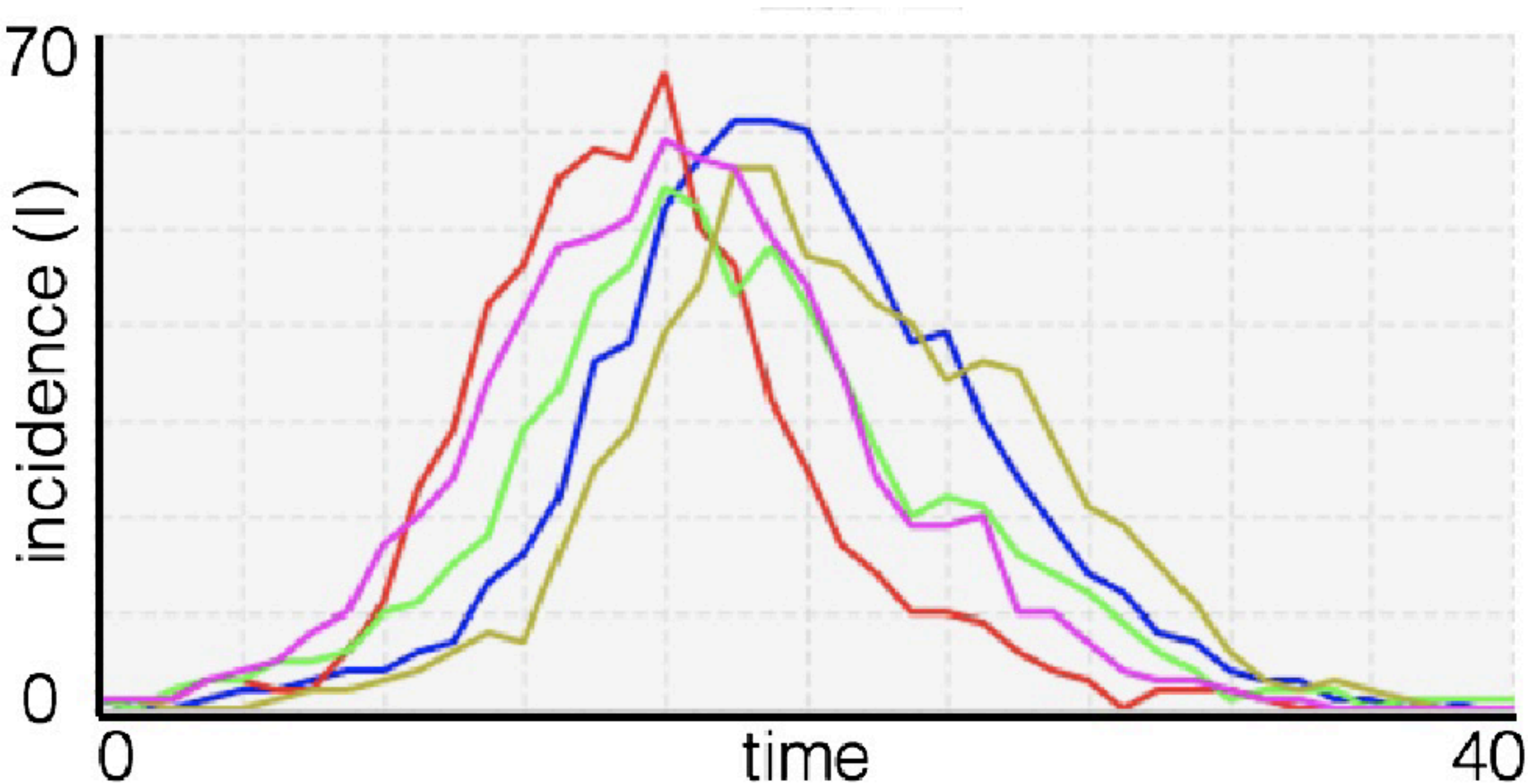





```
Sset = MULTINOMIAL(S,[(1-Ptu),
(t/(t+u))*Ptu,(u/(t+u))*Ptu])
T = Sset[1]
Ms = Sset[2]

Eset = MULTINOMIAL(E,[(1-Psu),
(s/(s+u))*Psu,(u/(s+u))*Psu])
Ue = Eset[1]
Me = Eset[2]

Iset = MULTINOMIAL(I,[(1-Pgau),
(g/(g+a+u))*Pgau,(u/(g+a+u))*Pgau,
(a/(g+a+u))*Pgau])
Ui = Iset[1]
Mi = Iset[2]
D = Iset[3]

Vset =  MULTINOMIAL(V,[(1-Pvu),
(v/(v+u))*Pvu,(u/(v+u))*Pvu])
Uv = Vset[1]
Mv = Vset[2]
```

Stochastic time susceptible SEIR disease state model

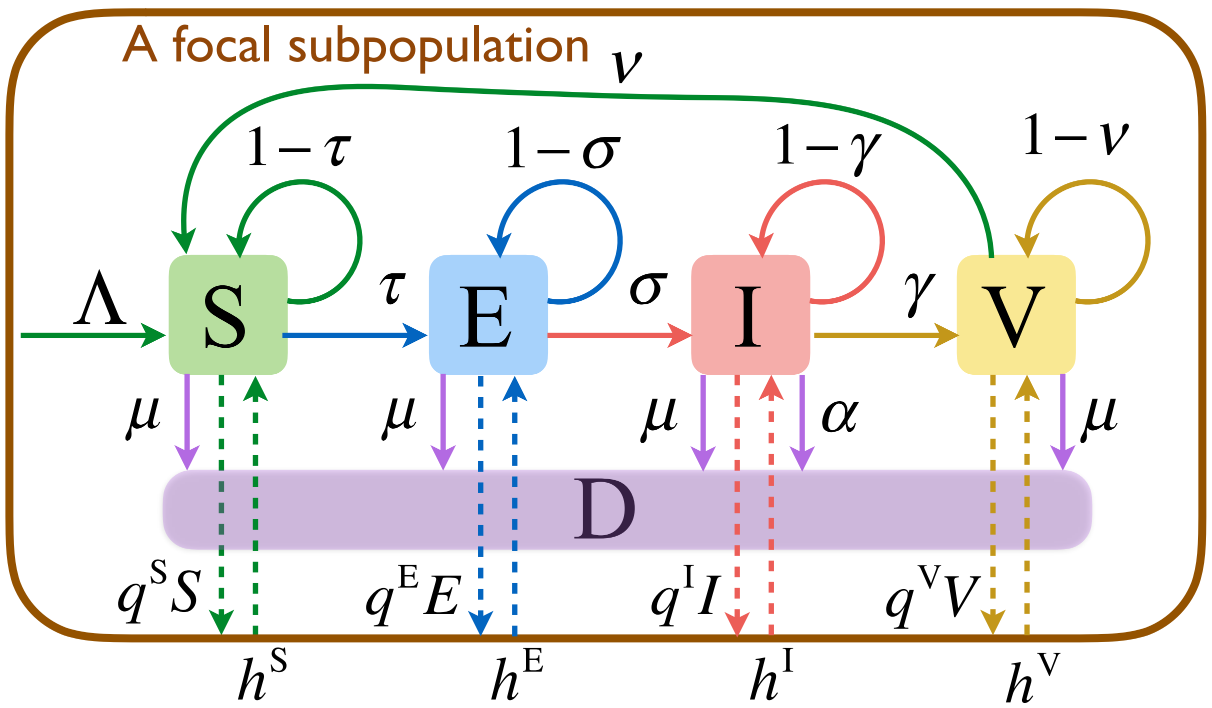
Stochastic solutions highly variable for small populations:
expectation very close to deterministic solution



Metapopulation Formulation

Metapopulation Formulation

A. Continuous SEIR in a metapopulation setting



Metapopulation Formulation

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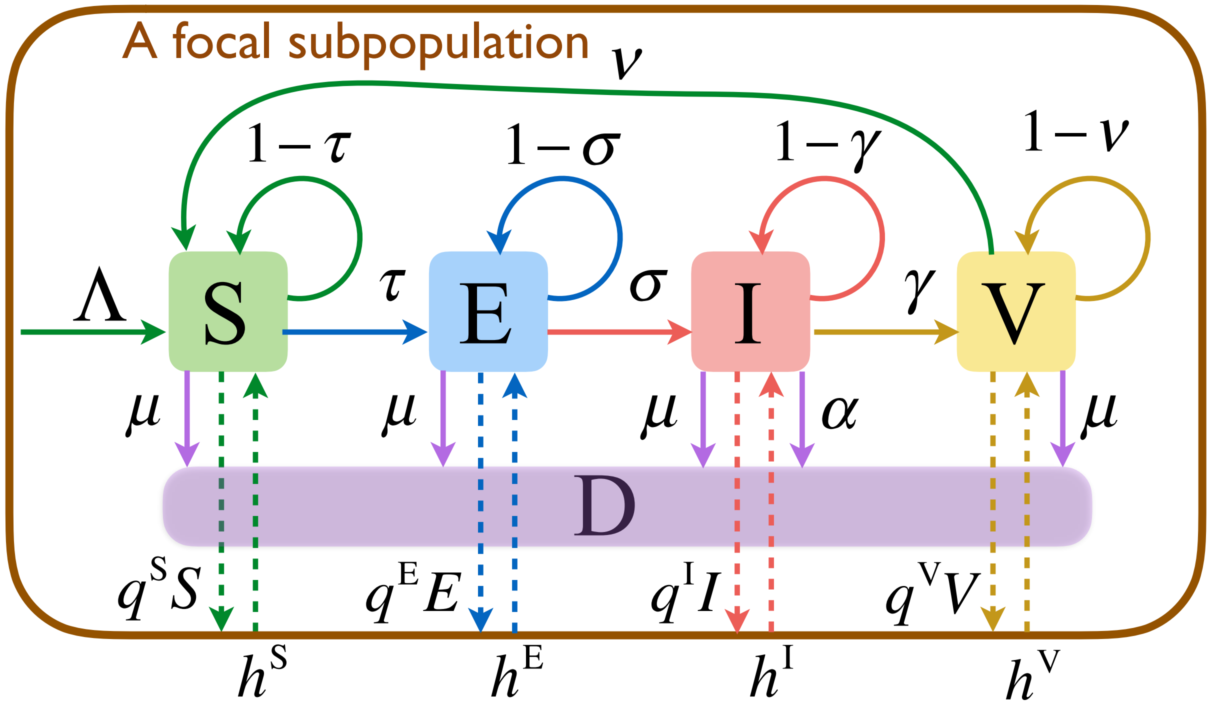
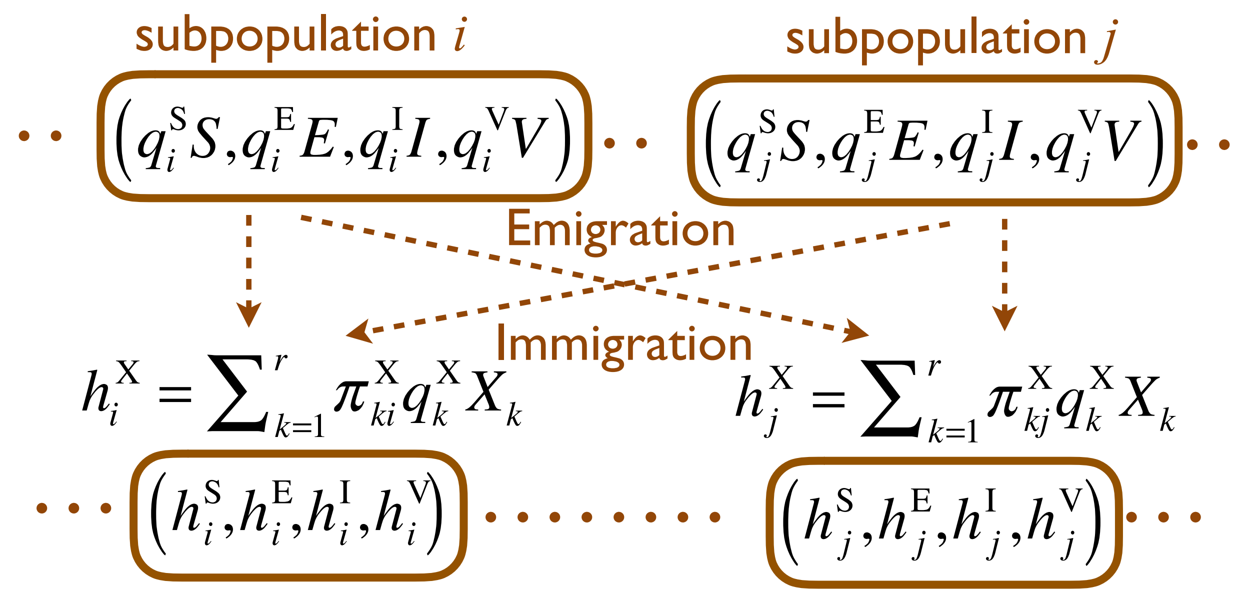


B. Metapopulation network



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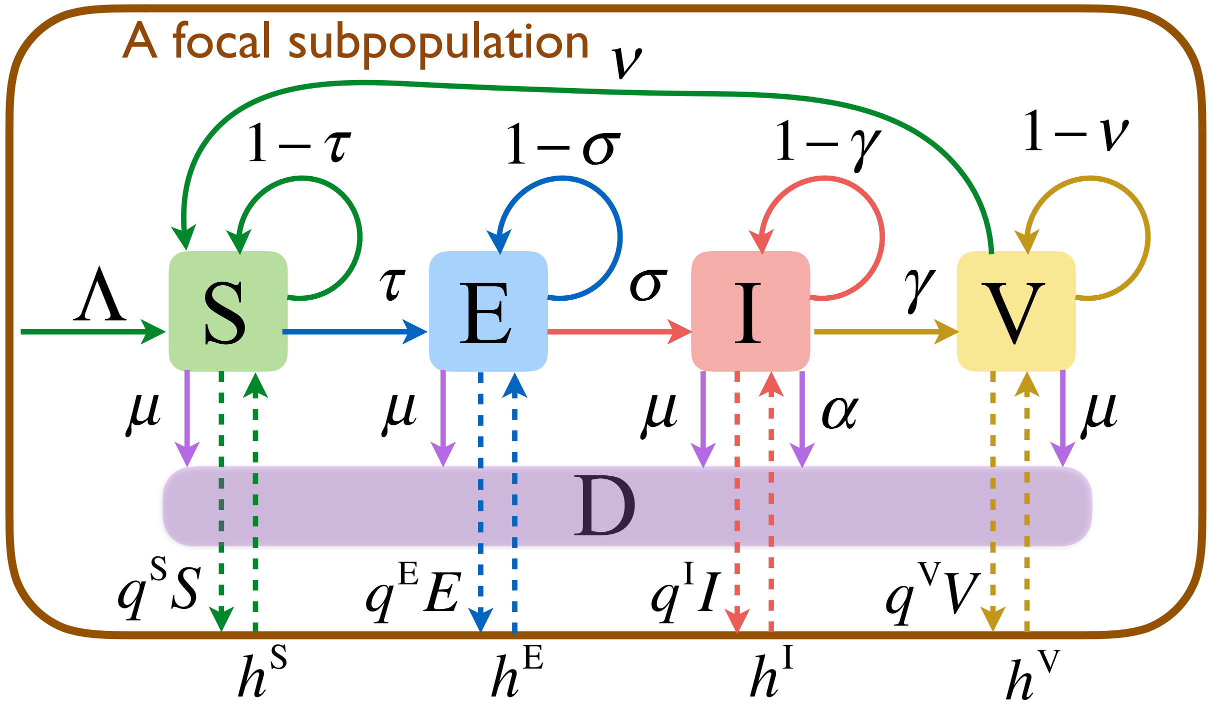

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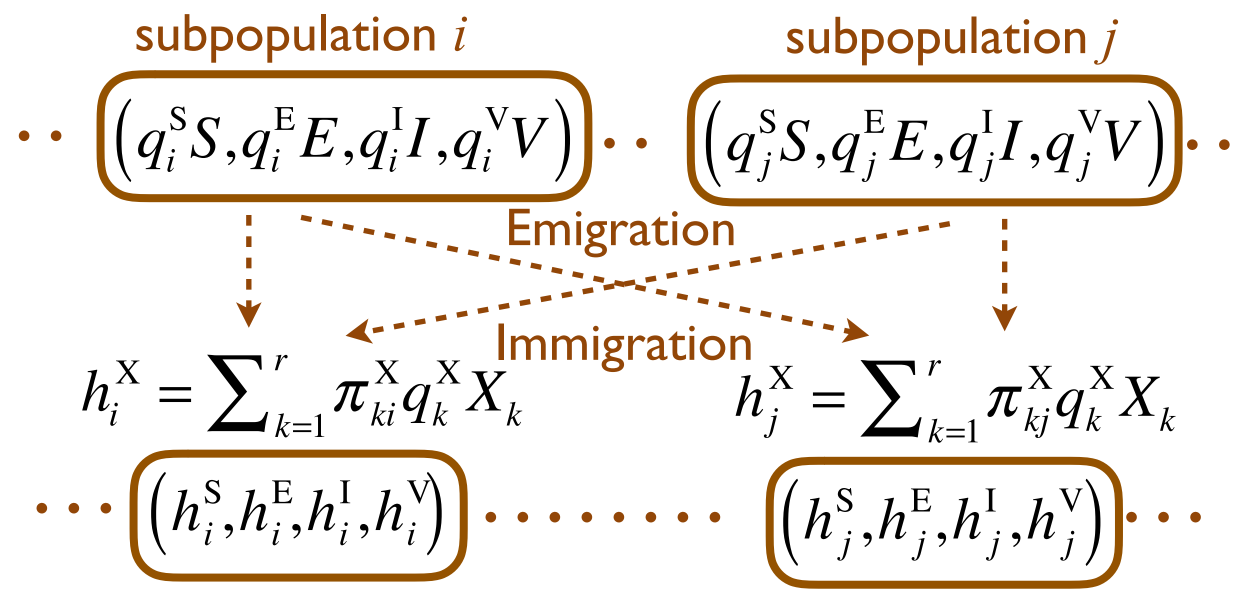
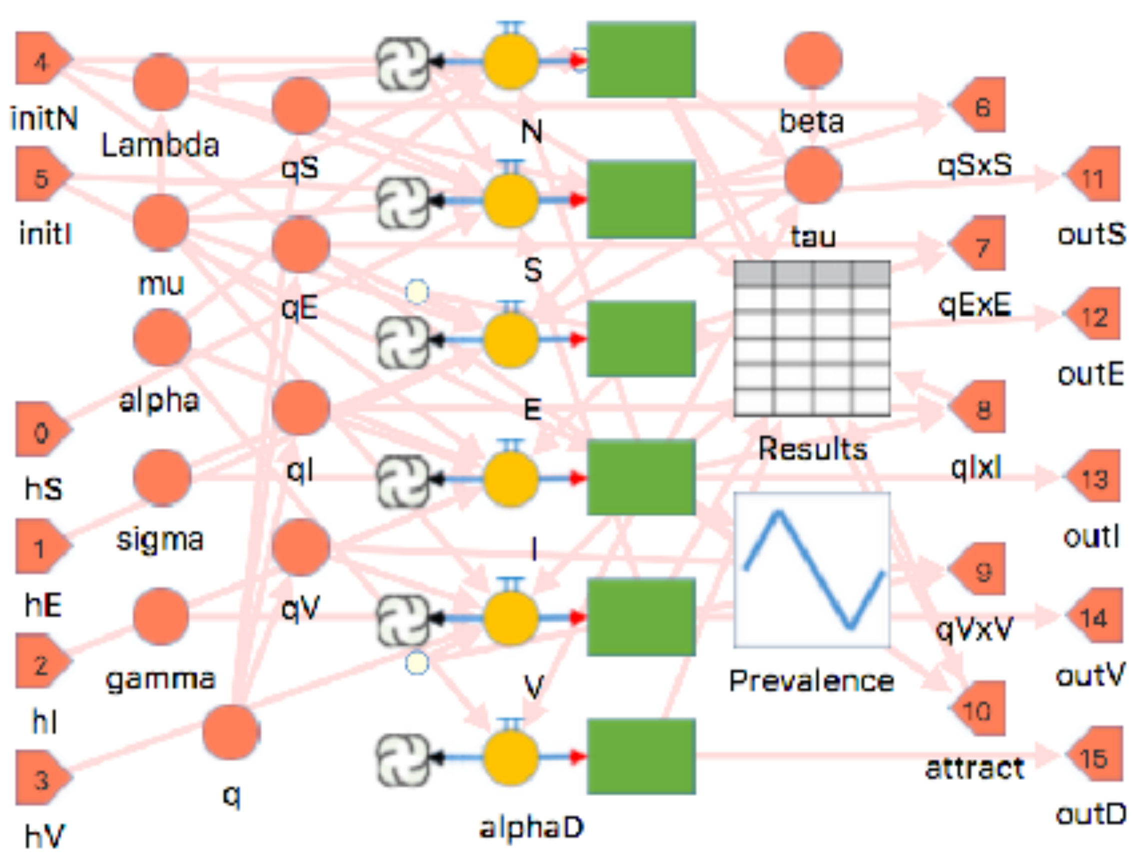
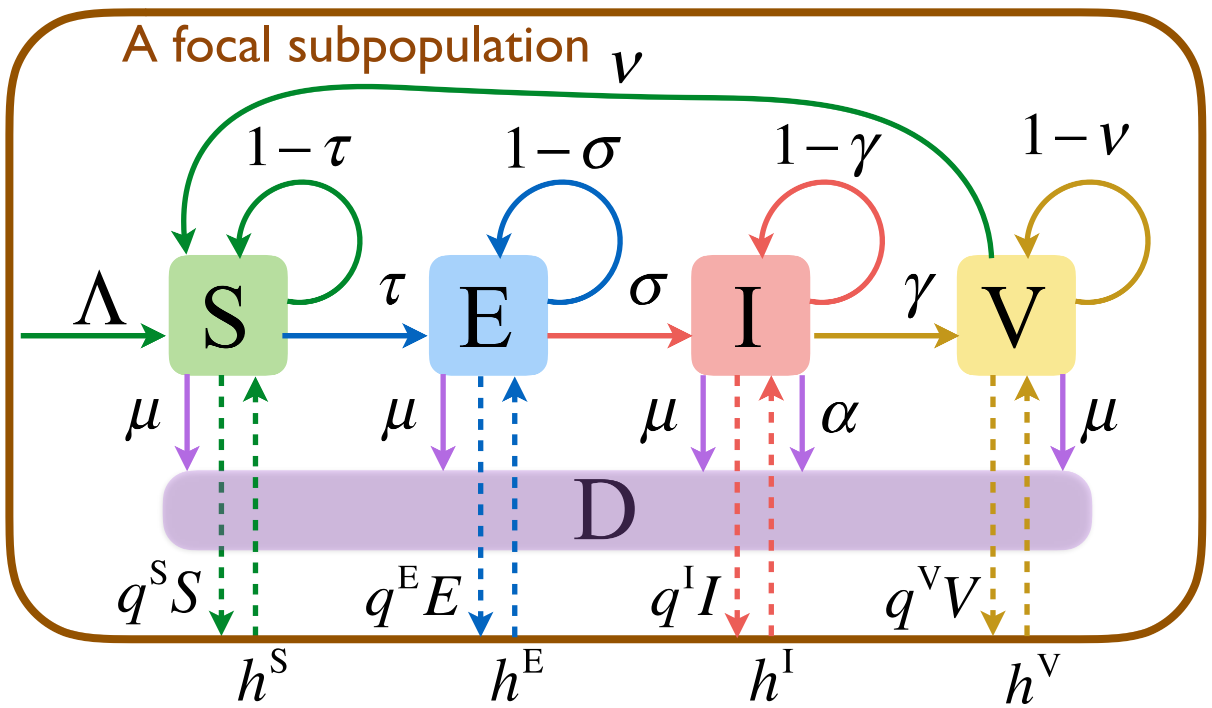


C. Nova Homogeneous SEIR Model with pins



Metapopulation Formulation

A. Continuous SEIR in a metapopulation setting



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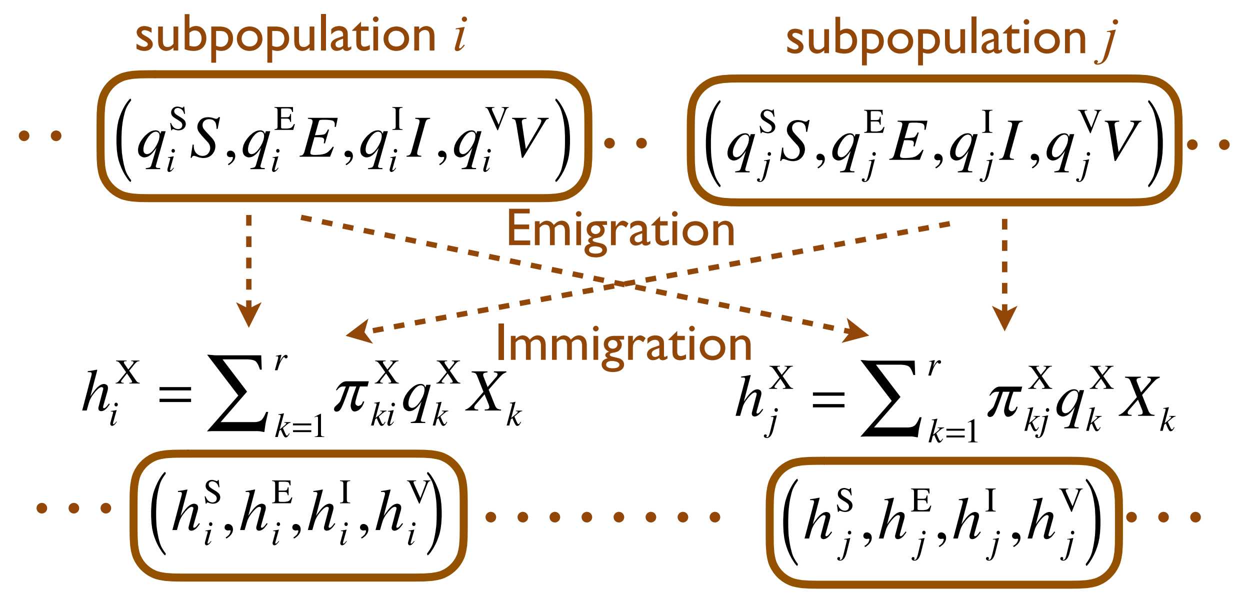

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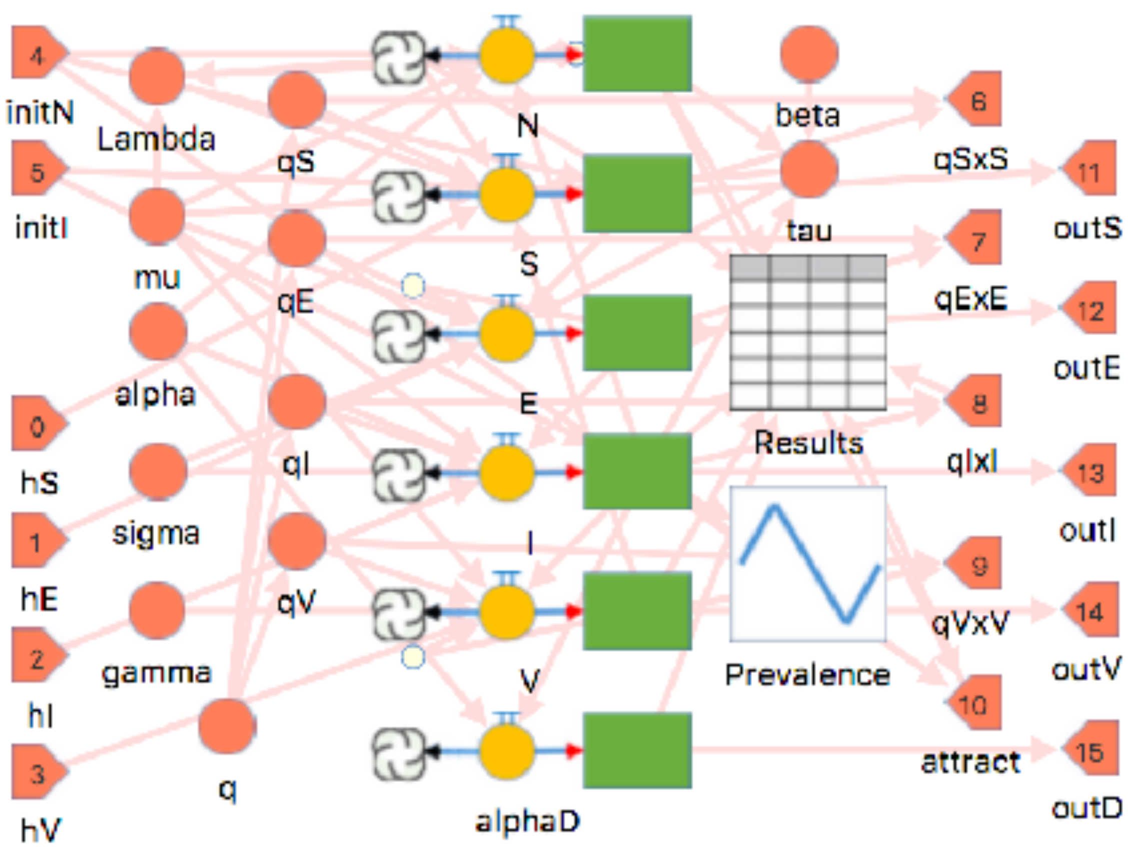
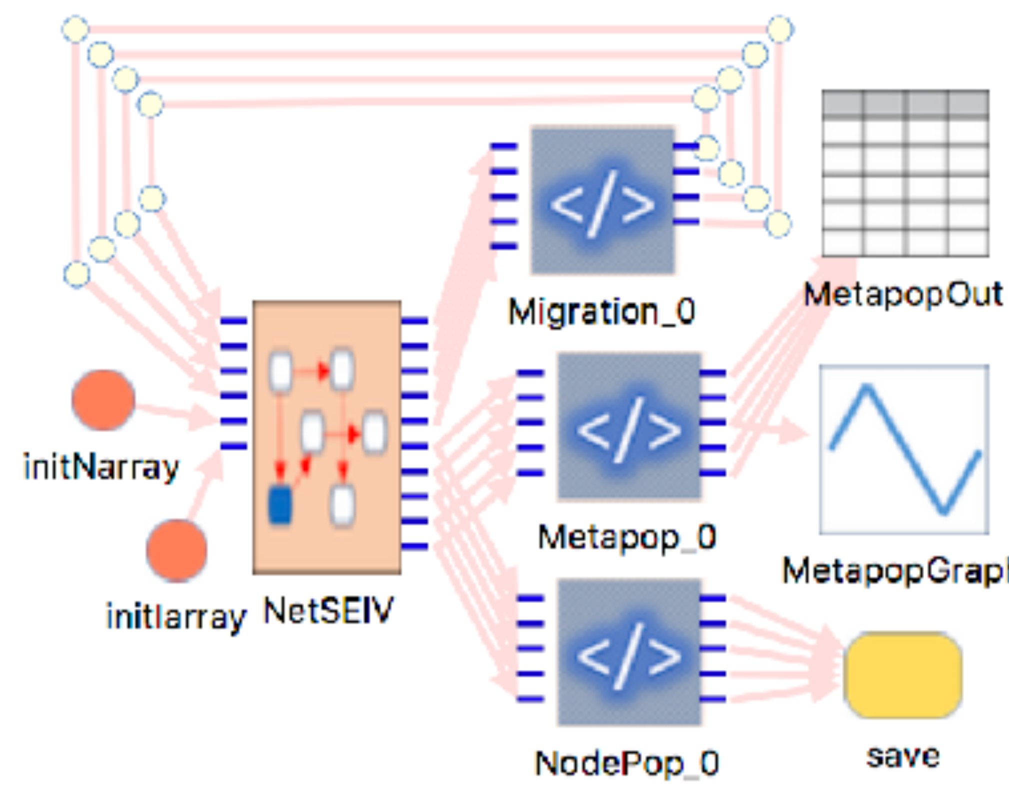
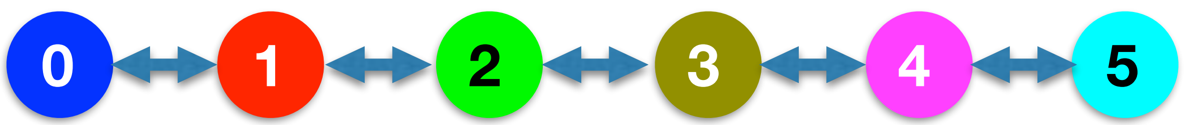


D. Nova Metapopulation Implementation



Metapopulation simulation

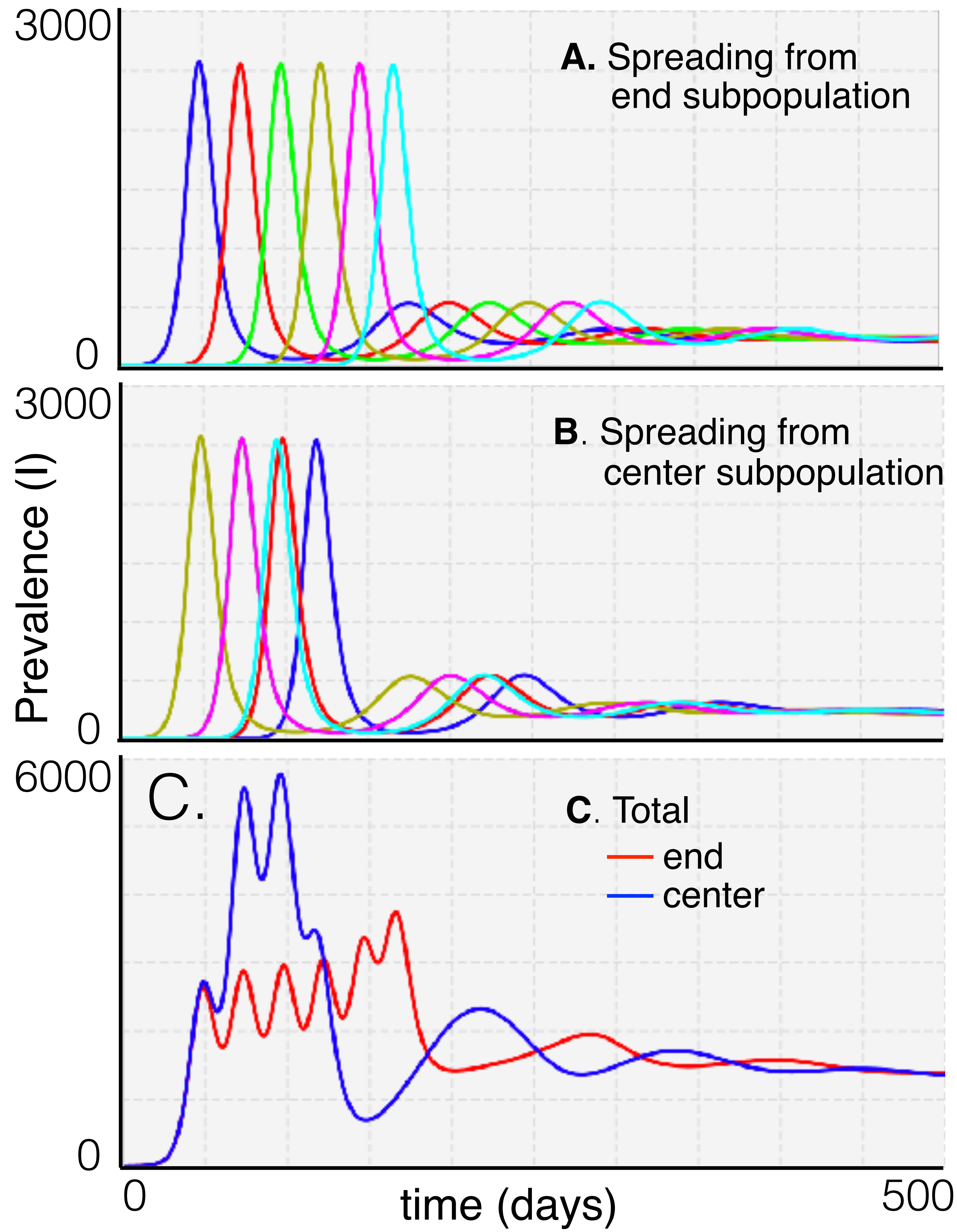


Prevalence plots (SEIV process at each node with migration: individuals flow only to subpopulations that are immediate neighbors.)

A. (index case in subpopulation 0)

B. (index case in subpopulation 2)

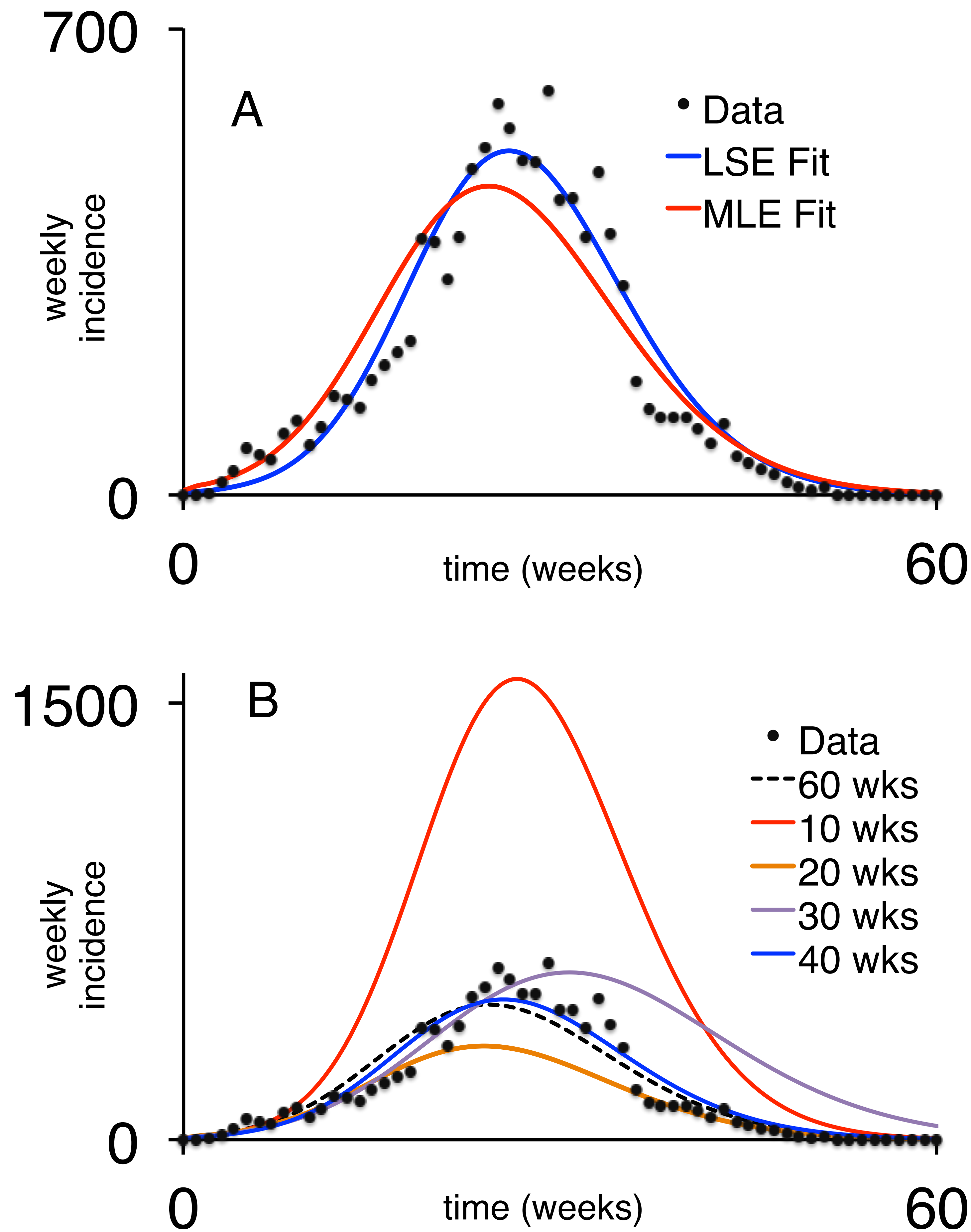
C. Total prevalence for two index cases that are either the end two (red plot, index cases on 0 and 5) or center two (blue plot,index cases in 2 and 3) subpopulations.



Fitting the SEIV model to Ebola data

A. The blue and red curves are the best fit least-square and maximum-likelihood estimate fits of the discrete-time SEIV model to Ebola incidence data from the Sierra Leone 2014

B. The black dotted line is the MLE fit, as in Panel A, with the red, orange, purple and blue plots, being simulations obtained after obtaining the best ML fits to the first 10, 20, 30 and 40 weeks of incidence respectively.



Access Models and Videos

Models available at: https://www.dropbox.com/home/NumerusMB_Epi_Models

Videos: search on **Youtube** using terms: **GetzEtAl SEIR**