M/M/1 Queuing Server Simulation Analysis

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This report presents the analysis of an M/M/1 queuing system simulation implemented using event-driven simulation techniques. The simulation examines how the average number of customers in the system varies with different server loads (ρ) and how the number of simulated customers affects the accuracy of the results.

1 Results and Analysis

1.1 Effect of Sample Size

The simulation was run with varying numbers of customers (100, 1,000, 5,000, 10,000, and 50,000) to analyze how sample size affects simulation accuracy. Figure 1 shows the results for 50,000 customers, demonstrating the closest match to theoretical predictions.

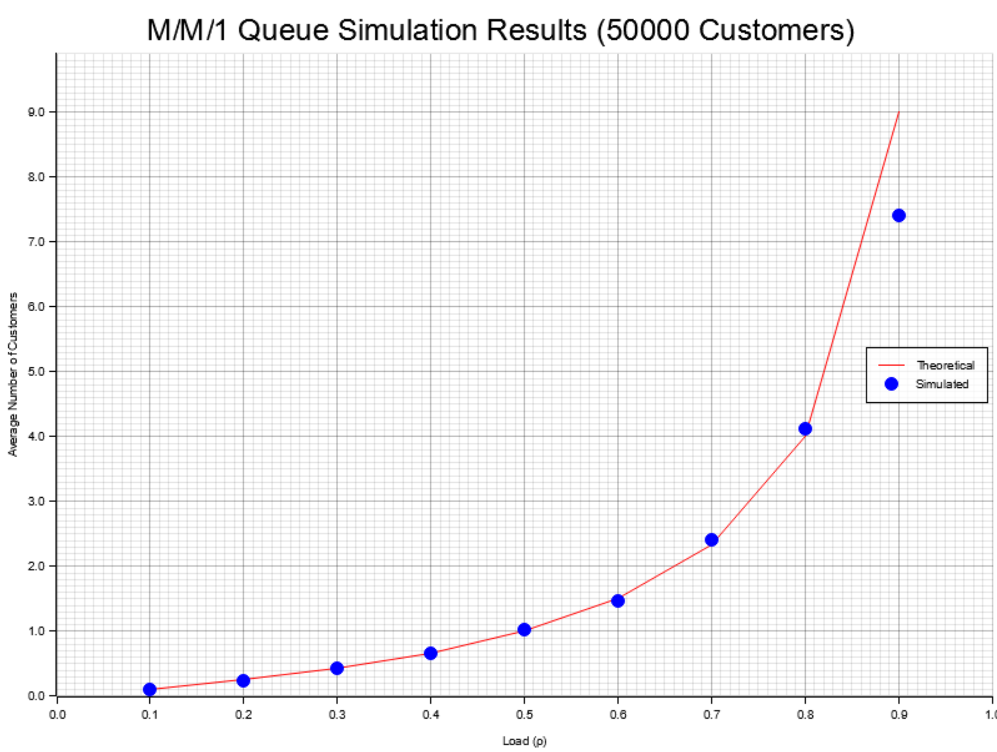

Figure 1: M/M/1 Queue Simulation Results (50,000 Customers)

Key observations from varying the sample size:

1. Small Sample Effects (100-1,000 customers):
   - Higher variance in results, particularly at high loads
   - Significant deviation from theoretical values when $\rho > 0.7$
   - Unstable behavior at $\rho = 0.9$
2. Large Sample Benefits (5,000-50,000 customers):
   - Improved convergence to theoretical values
   - More consistent behavior across all loads
   - Better stability at high loads

1.2 Load Impact Analysis

The simulation reveals several key characteristics about system behavior under different loads:

- Low Load ($\rho < 0.5$):
  - System behavior is highly predictable
  - Simulation closely matches theoretical values
  - Minimal variance across different sample sizes
- Medium Load ($0.5 \leq \rho \leq 0.7$):
  - Slight increase in variance
  - Good agreement with theory
  - Sample size begins to affect accuracy
- High Load ($\rho > 0.7$):
  - Increased sensitivity to sample size
  - Greater deviation from theoretical values
  - More pronounced queuing effects

2 Conclusions

The simulation demonstrates that accurate modeling of M/M/1 queue behavior requires careful consideration of sample size, particularly at high loads. While small samples can capture general system behavior, they may not reliably represent the system's steady-state characteristics, especially when ρ approaches 1.

The results confirm that:

- Larger sample sizes provide more reliable estimates of system behavior
- System performance becomes increasingly sensitive to load as ρ increases
- Theoretical predictions are most accurate at low to medium loads
- High-load scenarios require larger samples for accurate simulation

These findings align with queuing theory predictions and highlight the importance of appropriate sample sizing in discrete event simulations of queuing systems.

3 Appendix

3.1 Additional Simulation Results

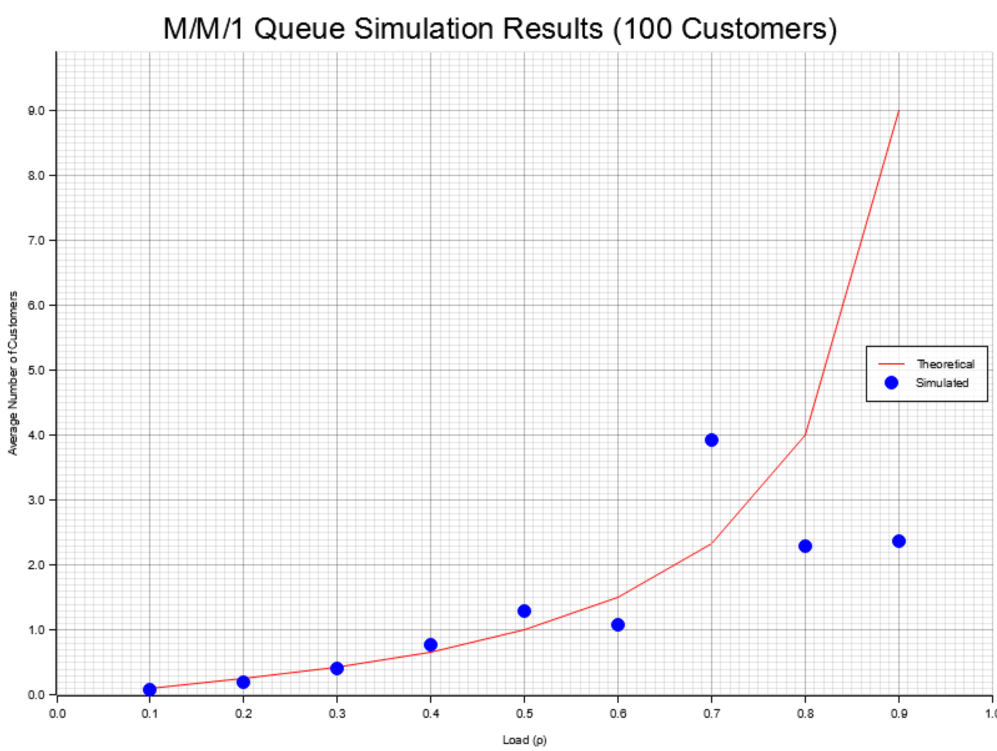

Figure 2: M/M/1 Queue Simulation Results (100 Customers)

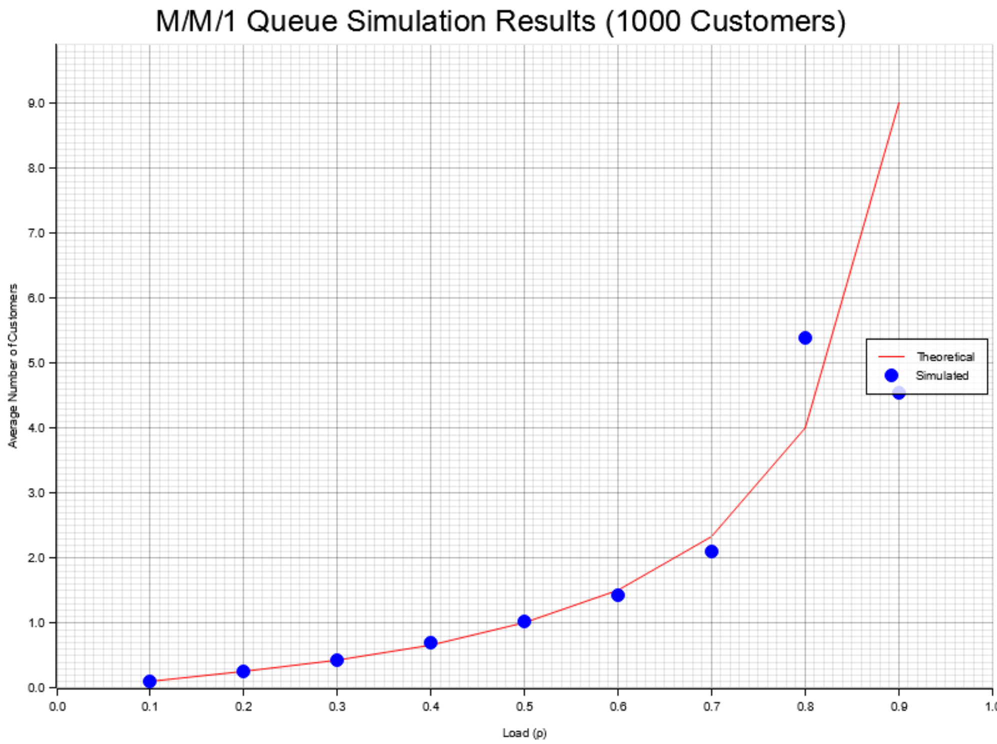

Figure 3: M/M/1 Queue Simulation Results (1,000 Customers)

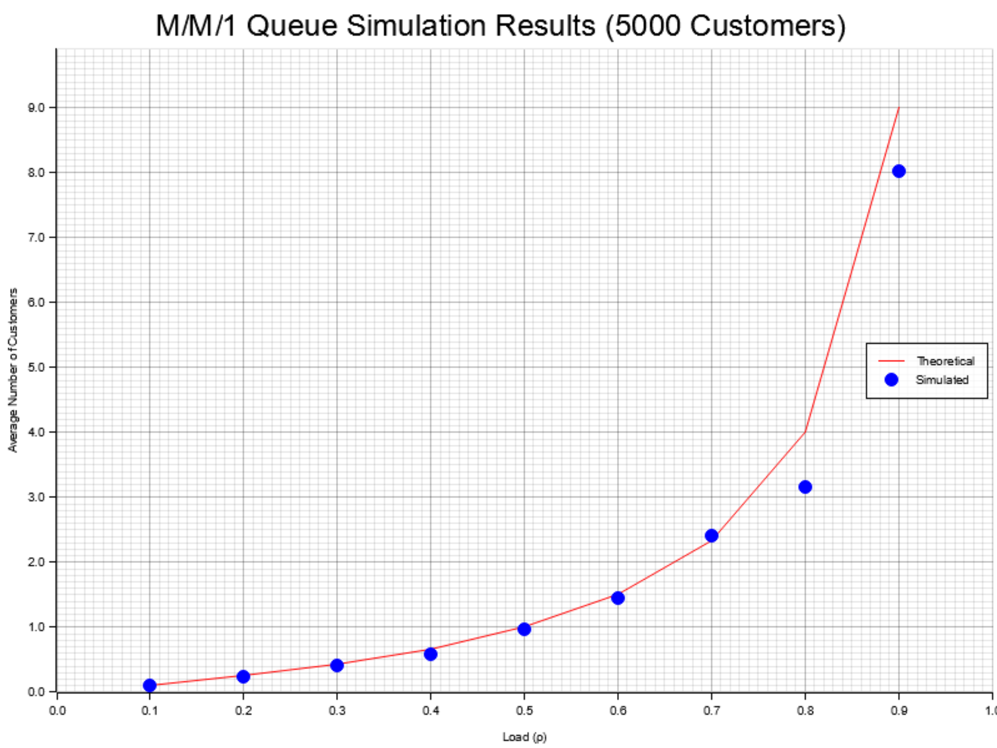

Figure 4: M/M/1 Queue Simulation Results (5,000 Customers)

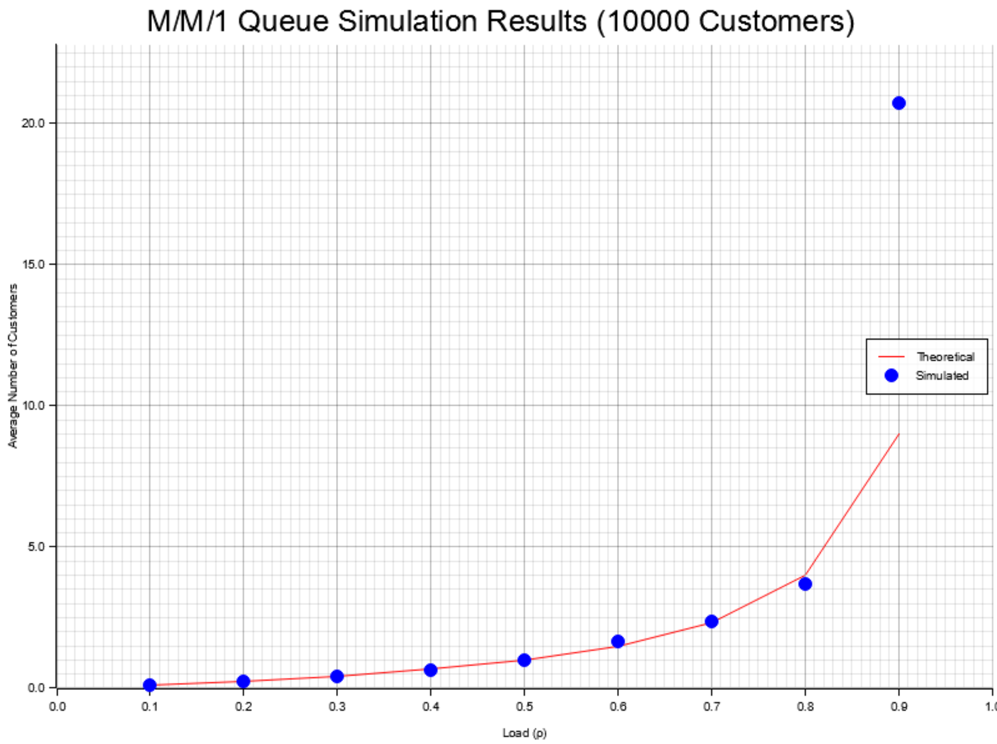

Figure 5: M/M/1 Queue Simulation Results (10,000 Customers)