

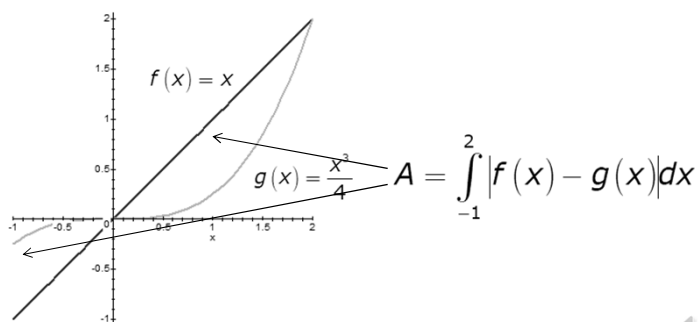
6. Aufgabe

Bestimmen Sie die Fläche A , welche von den Funktionen

$$f(x) = x \text{ und } g(x) = \frac{x^3}{4}$$

im Intervall $[-1, 2]$ eingeschlossen wird.

Lösung



Prof. Dr. Hans-Jürgen Dobner, HTWK Leipzig, MNZ

The solution shows the area A calculated by splitting the integral at $x=0$, where the functions intersect. The area is given by:

$$\begin{aligned}
 A &= \int_{-1}^2 |f(x) - g(x)| dx \\
 &= \int_{-1}^0 |f(x) - g(x)| dx + \int_0^2 |f(x) - g(x)| dx \\
 &= \int_{-1}^0 (g(x) - f(x)) dx + \int_0^2 (f(x) - g(x)) dx \\
 &= \int_{-1}^0 \left(\frac{x^3}{4} - x \right) dx + \int_0^2 \left(x - \frac{x^3}{4} \right) dx \\
 &= \left[\frac{x^4}{4 \cdot 4} - \frac{x^2}{2} \right]_{-1}^0 + \left[\frac{x^2}{2} - \frac{x^4}{4 \cdot 4} \right]_0^2 \\
 &= 0 - \left(\frac{(-1)^4}{16} - \frac{(-1)^2}{2} \right) + \left(\frac{2^2}{2} - \frac{2^4}{16} \right) - 0 = \frac{23}{16}
 \end{aligned}$$

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