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Mathematik für Informatiker (MfI) II

Seminar KW 20

$$\int_a^b f(x)\,dx$$

Thema:

Eigentliches Integral	Uneigentliches Integral
f stetig in $[a,b]$	f stetig in $[a,b)$/(a,b]

$$\int_a^b f(x)\cdot g'(x)\,dx = \left[f(x)\cdot g(x)\right]_a^b - \int_a^b f'(x)\cdot g(x)\,dx$$

1. Aufgabe

Bestimmen Sie mittels partieller Integration

$$\int \sin(x)\,dx = -\cos(x)+C$$
$$\int \cos(x)\,dx = \sin(x)+C$$

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,dx$$

Lösung

Berechnen des bestimmten Integrals mithilfe partieller Integration und sofort die Integrationsgrenzen einsetzen.

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,dx = \int_0^{\frac{\pi}{2}} \underbrace{\sin^{n-1}(x)}_{f(x)}\underbrace{\sin(x)}_{g'(x)}\,dx$$

$$= \left[-\sin^{n-1}(x)\cdot\cos(x)\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \underbrace{\boxed{(n-1)\sin^{n-2}(x)\cdot\cos(x)}}_{=f'(x)}\cos(x)\,dx$$

$I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,dx$

$$I_n = \left[-\sin^{n-1}(x)\cdot\cos(x)\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} (n-1)\sin^{n-2}(x)\cdot\cos(x)\cos(x)\,dx$$

$$= \underbrace{-\sin^{n-1}\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{2}\right) - \left(-\sin^{n-1}(0)\cdot\cos(0)\right)}_{=0} + \int_0^{\frac{\pi}{2}} (n-1)\sin^{n-2}(x)\cdot\cos^2(x)\,dx$$

$\sin^2(x)+\cos^2(x)=1$

$$= (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}(x)\cdot\left(1-\sin^2(x)\right)dx$$

$$= (n-1)\underbrace{\int_0^{\frac{\pi}{2}} \sin^{n-2}(x)\,dx}_{I_{n\text{-}2}} - (n-1)\underbrace{\int_0^{\frac{\pi}{2}} \sin^n(x)\,dx}_{I_n}$$

$I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,dx$

$$I_n = (n-1)\underbrace{\int_0^{\frac{\pi}{2}} \sin^{n-2}(x)\,dx}_{I_{n\text{-}2}} - (n-1)\underbrace{\int_0^{\frac{\pi}{2}} \sin^n(x)\,dx}_{I_n}$$

$$I_n = (n-1)I_{n-2} - (n-1)I_n$$

$\int \sin(x)\,dx = -\cos(x)+C$

$\cos\left(\frac{\pi}{2}\right)=0, \cos(0)=1$

$\Rightarrow$ Rekursionsformel für I_n

$$I_n = \frac{n-1}{n} I_{n-2}, n = 2,3,\ldots$$

$$I_0 = \int_0^{\frac{\pi}{2}} 1(x)\,dx = \frac{\pi}{2}, I_1 = \int_0^{\frac{\pi}{2}} \sin(x)\,dx = 1$$