Lösungen

1 a. • 1.+2.: $P_A(\lambda) = \begin{vmatrix} 6-\lambda & 2 \\ 2 & 9-\lambda \end{vmatrix} = (6-\lambda)(9-\lambda) - 4 = 54 - 9\lambda - 6\lambda + \lambda^2 - 4$

$= \lambda^2 - 15\lambda + 50$

3.: $P_A(\lambda) = 0$

$\lambda^2 - 15\lambda + 50 = 0$

$\lambda_{1,2} = \frac{15}{2} \pm \sqrt{\left(\frac{15}{2}\right)^2 - 50}$

$= \frac{15}{2} \pm \sqrt{\frac{225}{4} - \frac{200}{4}}$

$= \frac{15}{2} \pm \sqrt{\frac{25}{4}} = \frac{15}{2} \pm \frac{5}{2}$

$\lambda_1 = \frac{20}{2} = 10$, $\lambda_2 = \frac{10}{2} = 5$ $\rightarrow$ Eigenwerte
spek(A) = {5, 10}

4.: $\underline{\lambda_1 = 10}$: $(A - \lambda_1 E)\vec{x} = \vec{0}$

$\left(\begin{array}{cc|c} 6-10 & 2 & 0 \\ 2 & 9-10 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} -4 & 2 & 0 \\ 2 & -1 & 0 \end{array}\right) \left(\tfrac{1}{2}\right)$

$\left(\begin{array}{cc|c} -4 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad x_2 = t, \; t \neq 0$
$-4x_1 + 2t = 0$
$-4x_1 = -2t$
$x_1 = \frac{1}{2}t$

$\left(10, \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} t\right), t \in \mathbb{R}\setminus\{0\}$
Eigenpaar von A

$\underline{\lambda_2 = 5}$: $(A - \lambda_2 E)\vec{x} = \vec{0}$

$\left(\begin{array}{cc|c} 6-5 & 2 & 0 \\ 2 & 9-5 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \end{array}\right) (-2)$

$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad x_2 = t, \; t \neq 0$
$x_1 + 2t = 0$
$x_1 = -2t$

$\left(5, \begin{pmatrix} -2 \\ 1 \end{pmatrix} t\right), t \in \mathbb{R}\setminus\{0\}$
Eigenpaar von A

$\downarrow$

ebenfalls Eigenpaare von A^T, da spek(A) = spek(A^T)

spur(A) = 6 + 9 = 15 = $\lambda_1 + \lambda_2$ = 10 + 5 ✓

• Eigenwerte von A^{-1} : A^{-1} existiert $\Leftrightarrow$ A regulär
$\forall i \; \lambda_i \neq 0 \Longrightarrow A^{-1}$ existiert

oder $\det(A) \neq 0$: $\begin{vmatrix} 6 & 2 \\ 2 & 9 \end{vmatrix} = 54 - 4 = 50 \neq 0$

$\rightarrow$ A invertierbar und $\left(10, \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} t\right), \left(5, \begin{pmatrix} -2 \\ 1 \end{pmatrix} t\right)$ Eigenpaare von A

$\Rightarrow \left(\frac{1}{10}, \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} t\right), \left(\frac{1}{5}, \begin{pmatrix} -2 \\ 1 \end{pmatrix} t\right)$ Eigenpaare von A^{-1}

b. • 1.+2. :

$$P_A(\lambda) = \begin{vmatrix} 3-\lambda & -2 & 5 \\ 1 & 0-\lambda & 7 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 3-\lambda & -2 \\ 1 & -\lambda \end{vmatrix}$$

$$= (2-\lambda)\left((3-\lambda)(-\lambda)+2\right)$$

$$= (2-\lambda)(-3\lambda+\lambda^2+2) = -(\lambda-2)(\lambda^2-3\lambda+2)$$

3. : $P_A(\lambda) = -(\lambda-2)(\lambda^2-3\lambda+2) = 0$

$\Leftrightarrow$ ① $(\lambda-2)=0$

$\lambda_1 = 2$

② $(\lambda^2-3\lambda+2)=0$

$\lambda_{2,3} = \frac{3}{2} \pm \sqrt{\frac{9}{4}-2}$

$= \frac{3}{2} \pm \sqrt{\frac{1}{4}}$

$= \frac{3}{2} \pm \frac{1}{2}$

$\underbrace{\lambda_2 = 2}_{=\lambda_1}$, $\lambda_3 = 1$

$\to$ doppelter Eigenwert

4. $\lambda=\lambda_1=2$: $(A-\lambda_1 E)\vec{x}=\vec{0}$

$$\left(\begin{array}{ccc|c} 3-2 & -2 & 5 & 0 \\ 1 & 0-2 & 7 & 0 \\ 0 & 0 & 2-2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 5 & 0 \\ 1 & -2 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) (-1)$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 5 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad 2x_3 = 0 \quad x_3 = 0 \qquad x_2 = t,\ t \neq 0$$

$x_1 - 2t + 5\cdot 0 = 0$

$x_1 = 2t$

$\Rightarrow \left(2, \begin{pmatrix}2\\1\\0\end{pmatrix} t\right)$ Eigenpaar von A

$t \in \mathbb{R}\setminus\{0\}$

$\lambda_3 = 1$: $(A-\lambda_2 E)\vec{x}=\vec{0}$

$$\left(\begin{array}{ccc|c} 3-1 & -2 & 5 & 0 \\ 1 & 0-1 & 7 & 0 \\ 0 & 0 & 2-1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & -2 & 5 & 0 \\ 1 & -1 & 7 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) (-\tfrac{1}{2})$$

$$\left(\begin{array}{ccc|c} 2 & -2 & 5 & 0 \\ 0 & 0 & -\frac{9}{2} & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \quad -\tfrac{9}{2}x_3 = 0 \quad x_3 = 0 \qquad x_2 = t,\ t \neq 0$$

$2x_1 - 2t + 0\cdot 5 = 0$

$x_1 = t$

$\Rightarrow \left(1, \begin{pmatrix}1\\1\\0\end{pmatrix} t\right)$ Eigenpaar von A

$t \in \mathbb{R}\setminus\{0\}$

$\Rightarrow$ ebenfalls Eigenpaare von A^T

$spur(A) = 3+0+2 = 5 = \lambda_1+\lambda_2+\lambda_3 = 2+2+1 = 5$ ✓

• Eigenwerte von A^{-1} : A^{-1} existiert $\Leftrightarrow \det(A) \neq 0$ bzw. A regulär

da $\forall i \ \lambda_i \neq 0$

$\Rightarrow$ A regulär

$\rightarrow A^{-1}$ existiert

oder

$$\det(A) = \begin{vmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{vmatrix} = 2 \cdot \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix}$$

$$= 2 \cdot (0 + 2) = 4 \neq 0 \rightarrow A \text{ invertierbar}$$

und $\left(\frac{1}{2}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} t\right), \left(\frac{1}{1}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} t\right) = \left(1, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} t\right)$,

$t \in \mathbb{R} \setminus \{0\}$

Eigenpaare von A^{-1}

② a. • A ist Dreiecksmatrix $\rightarrow$ Diagonalelemente $\hat{=}$ Eigenwerte

$\lambda_1 = \lambda_2 = \lambda_3 = 2$ 3-faches Eigenwert $\rightarrow$ Vielfachheit von 3

(1.-3. nicht nötig)

4.: $\underline{\lambda_1 = \lambda_2 = \lambda_3 = 2}$: $(A - 2 \cdot E)\vec{x} = \vec{0}$

$$\left(\begin{array}{ccc|c} 2-2 & 1 & 0 & 0 \\ 0 & 2-2 & 1 & 0 \\ 0 & 0 & 2-2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_2 = 0$

$x_3 = 0$

$x_1 = t, \ t \in \mathbb{R} \setminus \{0\}$

$\Rightarrow \left(2, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} t\right)$ Eigenpaar von A

[• da $\forall i \ \lambda_i \neq 0 \rightarrow$ A regulär $\rightarrow$ A invertierbar

$\Rightarrow \left(\frac{1}{2}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} t\right)$ Eigenpaar von A^{-1}] nicht verlangt

• $P_A(\lambda) = (\lambda - 2)(\lambda - 2)(\lambda - 2) = (\lambda - 2)^3$

b. • A ist Dreiecksmatrix $\rightarrow$ Diagonalelemente $\hat{=}$ Eigenwerte

$\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 4$, $\lambda_4 = 8$

(1.-3. nicht nötig)

4: $\underline{\lambda_1 = 1}$

$$\left(\begin{array}{cccc|c} 1-1 & 3 & 9 & 27 & 0 \\ 0 & 2-1 & 12 & 54 & 0 \\ 0 & 0 & 4-1 & 36 & 0 \\ 0 & 0 & 0 & 8-1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 0 & 3 & 9 & 27 & 0 \\ 0 & 1 & 12 & 54 & 0 \\ 0 & 0 & 3 & 36 & 0 \\ 0 & 0 & 0 & 7 & 0 \end{array}\right)$$

$x_1 = t, \ t \neq 0$

$x_2 = x_3 = x_4 = 0$ $\Rightarrow \left(1, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} t\right)$ Eigenpaar von A

$t \in \mathbb{R} \setminus \{0\}$

$\underline{\lambda_2 = 2}$

$$\left(\begin{array}{cccc|c} 1-2 & 3 & 9 & 27 & 0 \\ 0 & 2-2 & 12 & 54 & 0 \\ 0 & 0 & 4-2 & 36 & 0 \\ 0 & 0 & 0 & 8-2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} -1 & 3 & 9 & 27 & 0 \\ 0 & 0 & 12 & 54 & 0 \\ 0 & 0 & 2 & 36 & 0 \\ 0 & 0 & 0 & 6 & 0 \end{array}\right)$$

$x_2 = t$

$x_3 = x_4 = 0$

$-x_1 + 3t + 0 + 0 = 0$

$x_1 = 3t$

$$\Rightarrow \left(2, \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} t\right) \text{ Eigenpaar von A}, \; t \in \mathbb{R} \setminus \{0\}$$

$\underline{\lambda_3 = 4}$

$$\left(\begin{array}{cccc|c} 1-4 & 3 & 9 & 27 & 0 \\ 0 & 2-4 & 12 & 54 & 0 \\ 0 & 0 & 4-4 & 36 & 0 \\ 0 & 0 & 0 & 8-4 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} -3 & 3 & 9 & 27 & 0 \\ 0 & -2 & 12 & 54 & 0 \\ 0 & 0 & 0 & 36 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right)$$

$x_4 = 0$

$x_3 = t$

$-2x_2 + 12t + 0 = 0$

$x_2 = 6t$

$-3x_1 + 6 \cdot 3t + 9t + 0 = 0$

$-3x_1 = -27t$

$x_1 = 9t$

$$\Rightarrow \left(4, \begin{pmatrix} 9 \\ 6 \\ 1 \\ 0 \end{pmatrix} t\right), \; t \in \mathbb{R} \setminus \{0\} \text{ Eigenpaar von A}$$

$\lambda_4 = 8$

$$\left(\begin{array}{cccc|c} 1-8 & 3 & 9 & 27 & 0 \\ 0 & 2-8 & 12 & 54 & 0 \\ 0 & 0 & 4-8 & 36 & 0 \\ 0 & 0 & 0 & 8-8 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} -7 & 3 & 9 & 27 & 0 \\ 0 & -6 & 12 & 54 & 0 \\ 0 & 0 & -4 & 36 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x_4 = t$

$-4x_3 + 36t = 0$

$x_3 = 9t$

$-6x_2 + 12 \cdot 9t + 54t = 0$

$x_2 = 27t$

$-7x_1 + 3 \cdot 27t + 9 \cdot 9t + 27t = 0$

$x_1 = 27t$

$$\Rightarrow \left(8, \begin{pmatrix} 27 \\ 27 \\ 9 \\ 1 \end{pmatrix} t\right), \; t \in \mathbb{R} \setminus \{0\} \text{ Eigenpaar von A}$$

$$P_A(\lambda) = (\lambda-1)(\lambda-2)(\lambda-4)(\lambda-8)$$

c. 1.+2.: $P_A(\lambda) = \begin{vmatrix} 5-\lambda & 4 \\ -4 & 5-\lambda \end{vmatrix} = (5-\lambda)(5-\lambda) + 16 = 25 - 5\lambda - 5\lambda + \lambda^2 + 16$

$= \lambda^2 - 10\lambda + 41$

3.: $P_A(\lambda) = 0 \Leftrightarrow \lambda^2 - 10\lambda + 41 = 0$

$\lambda_{1,2} = 5 \pm \sqrt{25-41} = 5 \pm \sqrt{-16} = 5 \pm \sqrt{(-1) \cdot 16}$

$= 5 \pm i \cdot 4$

$\lambda_1 = 5 + 4i, \; \lambda_2 = 5 - 4i$

4.: $\underline{\lambda_1 = 5 + 4i}$

$$\left(\begin{array}{cc|c} 5-(5+4i) & 4 & 0 \\ -4 & 5-(5+4i) & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} -4i & 4 & 0 \\ -4 & -4i & 0 \end{array}\right) (\cdot i)$$

$$\left(\begin{array}{cc|c} -4i & 4 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} x_2 = t \;,\; t \in \mathbb{R}\setminus\{0\} \\ -4i \cdot x_1 + 4t = 0 \\ x_1 = -4t \; : (-4i) \\ x_1 = \frac{1}{i} \cdot t = -it \end{array}$$

$$\Rightarrow \left(5+4i, \begin{pmatrix} -i \\ 1 \end{pmatrix} t\right)$$

$\underline{\lambda_2 = 5-4i}$

$$\left(\begin{array}{cc|c} 5-(5-4i) & 4 & 0 \\ -4 & 5-(5-4i) & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 4i & 4 & 0 \\ -4 & 4i & 0 \end{array}\right) \begin{array}{l} (-i) \\ \lrcorner + \end{array}$$

$$\left(\begin{array}{cc|c} 4i & 4 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad x_2 = t, \; t \in \mathbb{R}\setminus\{0\}$$

$$4i x_1 + 4t = 0$$
$$x_1 = \frac{-4t}{4i} = -\frac{1}{i} t = it$$

$$\Rightarrow \left(5-4i, \begin{pmatrix} i \\ 1 \end{pmatrix} t\right)$$

$$P_A(\lambda) = (\lambda-(5+4i))(\lambda-(5-4i)) \overset{\text{siehe vorher}}{=} \lambda^2 - 10\lambda + 41$$

③ a. • $\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1-4 = -3 \neq 0$ A regulär

$\rightarrow \forall_i \; \lambda_i \neq 0$

• $\underline{\lambda = 3}$ von A $\Leftrightarrow \vec{x} \neq \vec{0}$ existiert $\hat{=} \; \exists \vec{x} \neq 0$

$$\left(\begin{array}{cc|c} 1-3 & 2 & 0 \\ 2 & 1-3 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} -2 & 2 & 0 \\ 2 & -2 & 0 \end{array}\right) \begin{array}{l} (1) \\ \lrcorner \end{array} \Rightarrow \left(\begin{array}{cc|c} -2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$x_2 = t \;,\; t \in \mathbb{R}\setminus\{0\}$

$-2x_1 + 2t = 0$
$x_1 = t$

$\Rightarrow \vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} t$ nichttriviale Lösung

$\rightarrow \lambda = 3$ ist Eigenwert von A

alternativ

$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ alle Zeilensummen $= 3$

$\rightarrow 3$ Eigenwert von A

• $\mathrm{spur}(A) = 1+1 = \underline{2} = \lambda_1 + \lambda_2 = \underline{3 + \lambda_2}$

alternativ mit Determinante

$3 + \lambda_2 = 2$
$\lambda_2 = -1$

$\underline{\lambda_2 = -1}$

$$\left(\begin{array}{cc|c} 1+1 & 2 & 0 \\ 2 & 1+1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 2 & 2 & 0 \\ 2 & 2 & 0 \end{array}\right) \qquad \begin{array}{l} x_2 = t \;,\; t \in \mathbb{R}\setminus\{0\} \\ 2x_1 + 2t = 0 \\ x_1 = -1t \end{array}$$

$$\rightarrow \vec{x}^{(2)} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} t$$

$$P_A(\lambda) = (\lambda - 3)(\lambda + 1)$$

b. • $\begin{vmatrix} 4 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & -2 & 0 \end{vmatrix} = 3 \cdot \begin{vmatrix} 2 & 1 \\ -1 & -2 \end{vmatrix} = 3 \cdot (-4+1) = 3 \cdot (-3) = -9 \neq 0$

A regulär

$\rightarrow \forall i \; \lambda_i \neq 0$

• $\lambda_1 = \lambda_2 = 3$

$\text{spur}(A) = 4+1+0 = \underline{5} = \lambda_1 + \lambda_2 + \lambda_3 = \underline{3+3+\lambda_3}$

$6 + \lambda_3 = 5$

$\lambda_3 = -1$

$P_A(\lambda) = (\lambda-3)^2(\lambda+1)$

$\underline{\lambda_3 = -1}$

$\left(\begin{array}{ccc|c} 4+1 & 2 & 3 & 0 \\ 2 & 1+1 & 0 & 0 \\ -1 & -2 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 5 & 2 & 3 & 0 \\ 2 & 2 & 0 & 0 \\ -1 & -2 & 1 & 0 \end{array}\right) (2)(5)$

$\left(\begin{array}{ccc|c} 0 & -8 & 8 & 0 \\ 0 & -2 & 2 & 0 \\ -1 & -2 & 1 & 0 \end{array}\right) (-4) \rightarrow \left(\begin{array}{ccc|c} \cancel{0} & \cancel{0} & \cancel{0} & 0 \\ 0 & -2 & 2 & 0 \\ -1 & -2 & 1 & 0 \end{array}\right)$

$x_3 = t$

$-2x_2 = -2t$

$x_2 = t$

$-x_1 - 2t + t = 0$

$x_1 = -t$

$\vec{x}^{(3)} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} t \Leftarrow$

• $\underline{\lambda_1 = \lambda_2 = 3}$

$\left(\begin{array}{ccc|c} 4-3 & 2 & 3 & 0 \\ 2 & 1-3 & 0 & 0 \\ -1 & -2 & 0-3 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & -2 & 0 & 0 \\ -1 & -2 & -3 & 0 \end{array}\right) (1)(-2) \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -6 & -6 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & 0 \end{array}\right)$

$x_3 = t, \; t \in \mathbb{R} \setminus \{0\}$

$-6x_2 - 6t = 0$

$x_2 = -t$

$x_1 + 2(-t) + 3t = 0$

$x_1 = -t$

$\Rightarrow \vec{x}^{(1)} = \vec{x}^{(2)} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} t$

$\rightarrow \lambda = 3$ ist Eigenwert von A

(4) $A = \begin{pmatrix} 1 & 3 & -2 \\ -1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix} \begin{array}{c} \leftarrow \\ \leftarrow \\ \leftarrow \end{array}$ alle Zeilensummen $= 2$

$\rightarrow$ Eigenwert $\lambda = 1$ ist Eigenwert von A

mit $\vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ Eigenvektor

(5) • $|\lambda| \leq \|A\|$ mit $\| \; \|_\infty$: $\|A\|_\infty = \max\limits_{i=1,2,3} \sum\limits_{k=1}^{3} |a_{ik}|$

$= \max \{ |1| + |-4| + |7|, \; |7| + |9| + |-12|, \; |3| + |4| + |-3| \}$

$= \max \{12, 28, 10\} = 28$

$|\lambda| \leq 28 \Leftarrow$

• alle Zeilensummen (ohne Betrag) sind 4 $\rightarrow \lambda = 4$ ist Eigenwert von A

(6) $n = 3$: $c_0 = \det(A) = \begin{vmatrix} 0 & 2 & 0 \\ 3 & 1 & -1 \\ -2 & 0 & 1 \end{vmatrix} = -2 \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = -2(3-2) = -2$

$c_{n-2} = c_1 = -\left(\begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} \right) = -(-6+0+1) = 5$

$c_{n-1} = c_2 = (-1)^{3-1} \cdot \text{spur}(A) = 1 \cdot (0+1+1) = 2$

$c_n = c_3 = (-1)^3 = -1$

$\longrightarrow \underline{P_A(\lambda) = (-1)\cdot\lambda^3 + 2\lambda^2 + 5\lambda - 2}$

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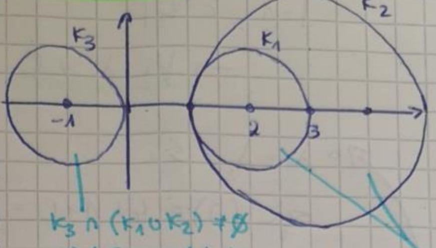
a. $A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 3 & 1 \\ 1 & 0 & -1 \end{pmatrix}$

$K_1 = \{ z \in \mathbb{C} \mid |z-2| \leq 1 \}$

$K_2 = \{ z \in \mathbb{C} \mid |z-3| \leq 2 \}$

$K_3 = \{ z \in \mathbb{C} \mid |z+1| \leq 1 \}$



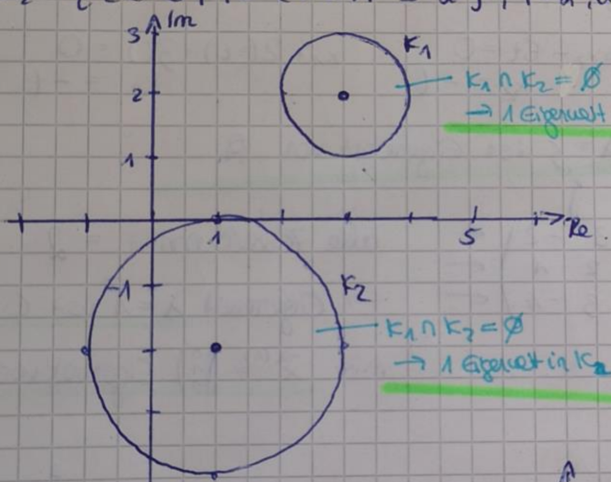
$K_3 \cap (K_1 \cup K_2) = \emptyset$
$\rightarrow$ 1 Eigenwert in K_3

$(K_1 \cup K_2) \cap K_3 = \emptyset$
$\rightarrow$ 2 Eigenwerte in $K_1 \cup K_2$
(nicht zwingend 1 λ in K_1!!)

b. $A = \begin{pmatrix} 3+2i & i \\ -2i & 1-2i \end{pmatrix}$

$K_1 = \{ z \in \mathbb{C} \mid |z-(3+2i)| \leq 1 \}$, $r=1$, da $|i| = \sqrt{0^2+1^2} = 1$

$K_2 = \{ z \in \mathbb{C} \mid |z-(1-2i)| \leq 2 \}$, $r=2$, da $|2i| = \sqrt{0^2+2^2} = 2$



$K_1 \cap K_2 = \emptyset$
$\rightarrow$ 1 Eigenwert in K_1

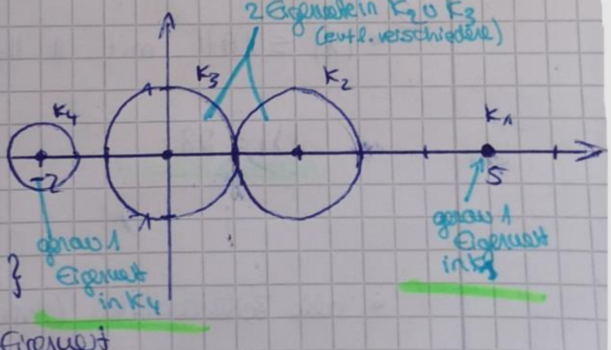
$K_1 \cap K_2 = \emptyset$
$\rightarrow$ 1 Eigenwert in K_2

c. $K_1 = \{ z \in \mathbb{C} \mid |z-5| \leq 0 \}$

$K_2 = \{ z \in \mathbb{C} \mid |z-2| \leq 1 \}$

$K_3 = \{ z \in \mathbb{C} \mid |z| \leq 1 \}$

$K_4 = \{ z \in \mathbb{C} \mid |z+2| \leq 1/2 \}$



2 Eigenwerte in $K_2 \cup K_3$ (evtl. verschiedene)

genau 1 Eigenwert in K_4

genau 1 Eigenwert in K_1

$\underline{\lambda_{max} = 5}$ ist betragsgrößter Eigenwert

$$\left(\begin{array}{cccc|c} 5-5 & 0 & 0 & 0 & 0 \\ 0 & 2-5 & -1 & 0 & 0 \\ 0 & -1 & 0-5 & 0 & 0 \\ 1/4 & 1/4 & 0 & -2-5 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & -3 & -1 & 0 & 0 \\ 0 & -1 & -5 & 0 & 0 \\ 1/4 & 1/4 & 0 & -7 & 0 \end{array}\right) (-3) \Rightarrow \left(\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 14 & 0 & 0 \\ 0 & -1 & -5 & 0 & 0 \\ 1/4 & 1/4 & 0 & -7 & 0 \end{array}\right) \quad \begin{array}{l} x_3 = 0 \\ \\ x_4 = t \end{array}$$

$-x_2 - 5\cdot 0 = 0$
$x_2 = 0$

$1/4\, x_1 + 0 + 0 - 7t = 0$
$x_1 = 28t$

$\Rightarrow \vec{x} = \begin{pmatrix} 28 \\ 0 \\ 0 \\ 1 \end{pmatrix} t \qquad t \in \mathbb{R} \setminus \{0\}$

$\rightarrow \vec{x}_1 = \begin{pmatrix} 28 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$\vec{x}_1$ ist 1 Eigenvektor