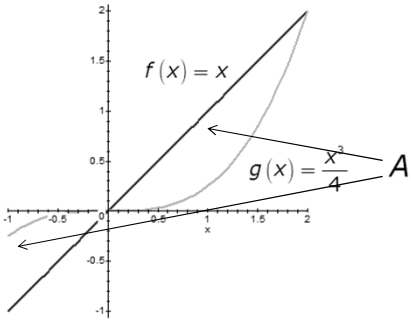
6. Aufgabe

Bestimmen Sie die Fläche *A*, welche von den Funktionen

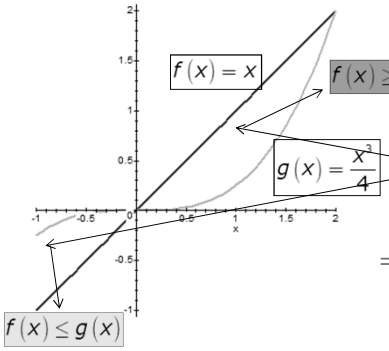
$$f(x) = x \text{ und } g(x) = \frac{x^3}{4}$$

im Intervall [-1,2] eingeschlossen wird.

Lösung



$$A = \int_{-1}^{2} \left| f(x) - g(x) \right| dx$$



$$A = \int_{-1}^{2} \left| f(x) - g(x) \right| dx$$

$$= \int_{-1}^{0} \left| f(x) - g(x) \right| dx + \int_{0}^{2} \left| f(x) - g(x) \right| dx$$

$$= \int_{-1}^{0} \left(g(x) - f(x) \right) dx + \int_{0}^{2} \left(f(x) - g(x) \right) dx$$

$$= \int_{-1}^{0} \left(\frac{x^3}{4} - x \right) dx + \int_{0}^{2} \left(x - \frac{x^3}{4} \right) dx = \left[\frac{x^4}{4 \cdot 4} - \frac{x^2}{2} \right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4 \cdot 4} \right]_{0}^{2}$$

$$= 0 - \underbrace{\left(\frac{(-1)^4}{16} - \frac{(-1)^2}{2} \right)}_{=\frac{-7}{16}} + \frac{2^2}{2} - \underbrace{\frac{2^4}{16}}_{=1} - 0 = \frac{23}{16}$$