Aufgaben 3

11. Aufgabe

$$\int_1^e \frac{\ln(x)}{x \cdot \sqrt{1+\left(\ln(x)\right)^2}}\ dx$$

$$\int \frac{\ln(x)}{x \cdot \sqrt{1+\left(\ln(x)\right)^2}}\ dx \qquad u = \ln(x),\ du = \frac{1}{x}$$

$$\int \frac{u}{\sqrt{1+u^2}}\ du$$

$$= \sqrt{1+u^2}$$

$$\sqrt{1+\left(\ln(x)\right)^2}$$

$$\int_1^e \frac{\ln(x)}{x \cdot \sqrt{1+\left(\ln(x)\right)^2}}\ dx = \left[\sqrt{1+\left(\ln(x)\right)^2}\right]_1^e$$

$$= \sqrt{1+\left(\ln(e)\right)^2} - \sqrt{1+\left(\ln(1)\right)^2}$$

$$= \sqrt{1+1^2} - \sqrt{1+0^2}$$

$$= \sqrt{2} - \sqrt{1}$$

$$= \sqrt{2} - 1 \approx 0.4142$$

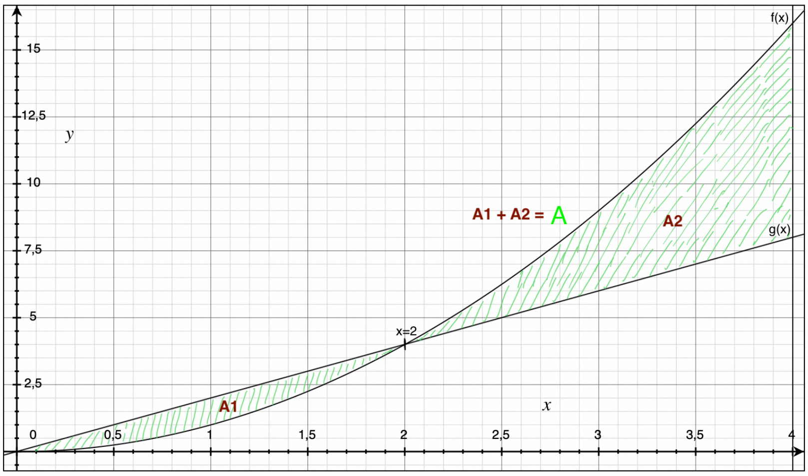
12. Aufgabe

$$f(x) = x^2, \quad g(x) = 2x \qquad [0,\ 4]$$

$$A = A_1 + A_2$$

$$A_1 = \int_0^2 \left(g(x) - f(x)\right)\ dx$$

$$A_2 = \int_2^4 \left(f(x) - g(x)\right)\ dx$$



$$A_1 = \int_0^2 \left(g(x) - f(x)\right)\ dx$$

$$= \int_0^2 \left(2x - x^2\right)\ dx$$

$$= \int_0^2 -x^2 + 2x\ dx$$

$$\int -x^2 + 2x\ dx$$

$$= -\frac{1}{3}x^3 + x^2$$

$$= \left[-\frac{1}{3}x^3 + x^2\right]_0^2$$

$$= \left(-\frac{1}{3}2^3 + 2^2\right) - \left(-\frac{1}{3}0^3 + 0^2\right)$$

$$= \left(-\frac{8}{3} + 4\right) - (0 + 0)$$

$$= -\frac{8}{3} + \frac{12}{3} = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$$

$$A_2 = \int_2^4 \left(f(x) - g(x)\right)\ dx$$

$$= \int_2^4 x^2 - 2x\ dx$$

$$\int x^2 - 2x\ dx$$

$$= \frac{1}{3}x^3 - x^2$$

$$= \left[\frac{1}{3}x^3 - x^2\right]_2^4$$

$$= \left(\frac{1}{3}4^3 - 4^2\right) - \left(\frac{1}{3}2^3 - 2^2\right)$$

$$= \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 4\right)$$

$$= \left(\frac{64}{3} - \frac{48}{3}\right) - \left(\frac{8}{3} - \frac{12}{3}\right)$$

$$= \frac{16}{3} + \frac{4}{3} = \frac{20}{3}$$

$$A = A_1 + A_2 = \frac{4}{3} + \frac{20}{3} = \frac{24}{3} = 8$$

13. Aufgabe

$$\int_0^1 x \cdot \ln(x)\ dx$$

TODO: Konvergenz/Divergenz

$$\int x \cdot \ln(x)\ dx$$

$$= \frac{1}{2}x^2 \cdot \ln(x) - \frac{1}{4}x^2$$

$$= x^2\left(\frac{1}{2} \cdot \ln(x) - \frac{1}{4}\right)$$

$$= \left[x^2\left(\frac{1}{2} \cdot \ln(x) - \frac{1}{4}\right)\right]_0^1$$

$$= 1^2\left(\frac{1}{2} \cdot \ln(1) - \frac{1}{4}\right) - 0^2\left(\frac{1}{2} \cdot \ln(0) - \frac{1}{4}\right)$$

$$= 1\left(\frac{1}{2} \cdot \ln(1) - \frac{1}{4}\right) - 0\left(\frac{1}{2} \cdot \ln(0) - \frac{1}{4}\right)$$

$$= \frac{1}{2} \cdot \ln(1) - \frac{1}{4}$$

$$= \frac{1}{2} \cdot 0 - \frac{1}{4} = -\frac{1}{4}$$

14. Aufgabe

$$\int_{\frac{1}{2}}^{\infty} \frac{1}{\sqrt{x\left(1+x^2\right)}}\ dx$$

15. Aufgabe

$$f(x) = \begin{cases} 1, & -\pi < x \leq 0 \\ x, & 0 < x \leq \pi \end{cases}$$