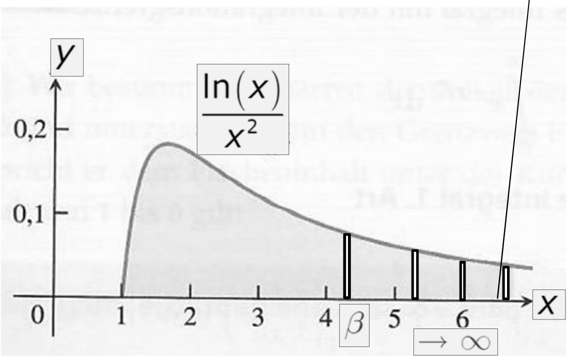
Beispiel 1

Existiert das uneigentliche Integral $\int\limits_1^{\infty} \frac{\ln(x)}{x^2}\,dx$?



$$\int\limits_1^{\beta} \frac{\ln(x)}{x^2}\,dx$$

$$\int f(x)\cdot g'(x)\,dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx$$

$$\int \frac{\ln(x)}{x^2}\,dx$$

Partielle Integration

$$\int \underbrace{\ln(x)}_{f}\underbrace{\frac{1}{x^2}}_{g'}\,dx \qquad f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$$

$$g'(x) = x^{-2} \Rightarrow g(x) = -x^{-1}$$

$$= \ln(x)\left(-\frac{1}{x}\right) - \int \frac{1}{x}\cdot\left(-\frac{1}{x}\right)dx \;= -\frac{\ln(x)}{x} - \left(\frac{1}{x}\right) + C$$

Bestimmtes Integral

$$\Rightarrow \int\limits_1^{\beta} \frac{\ln(x)}{x^2}\,dx = \left[-\frac{\ln(x)}{x} - \left(\frac{1}{x}\right)\right]_1^{\beta} = -\frac{\ln(\beta)}{\beta} - \frac{1}{\beta} - \left(-\frac{\ln(1)}{1} - \frac{1}{1}\right)$$

$$= -\frac{\ln(\beta)}{\beta} - \frac{1}{\beta} + 1$$

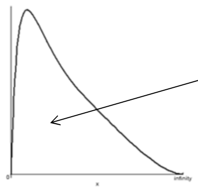
$$\int_1^\infty \frac{\ln(x)}{x^2}\,dx = \lim_{\beta\to\infty}\int_1^\beta \frac{\ln(x)}{x^2}\,dx = \lim_{\beta\to\infty}\left(-\frac{\ln(\beta)}{\beta}-\frac{1}{\beta}+1\right)$$

$$= \boxed{\boxed{-\lim_{\beta\to\infty}\frac{\ln(\beta)}{\beta}}} - \underbrace{\lim_{\beta\to\infty}\left(-\frac{1}{\beta}\right)}_{\to 0} + 1 \;= 1$$

Typ "$\frac{\infty}{\infty}$" $\to$ 2. L'Hospitalsche Regel

$$\boxed{\boxed{\lim_{\beta\to\infty}\frac{\ln(\beta)}{\beta} \underset{"\frac{\infty}{\infty}"}{=} \lim_{\beta\to\infty}\frac{\frac{1}{\beta}}{1} \;\to 0}}$$

Das uneigentliche Integral konvergiert und die Fläche hat den Wert 1.



Schreibweise

$$\int_1^\infty \frac{\ln(x)}{x^2}\,dx = 1$$