<u>Beispiel 2</u> (unstetiger aber beschränkter Integrand)

Was tun, wenn Unstetigkeitsstelle nicht am Intervallende?

f(*x*) stetig im Intervall [*a*,*b*] mit Ausnahme der Stelle *c*, wobei *a*<*c*<*b*, z. B.

$$f(x) = \begin{cases} 1, & 0 \le x < \frac{3}{2} \\ 2, & \frac{3}{2} \le x \le 3 \end{cases}$$

→ Aufsplitten des uneigentlichen Integrals in zwei uneigentliche Integrale, so dass Unstetigkeit jeweils am Intervallende

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

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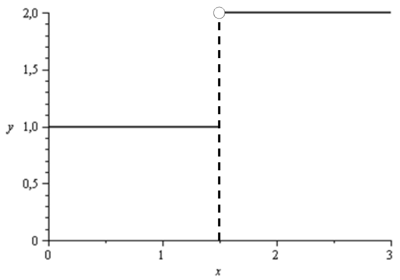

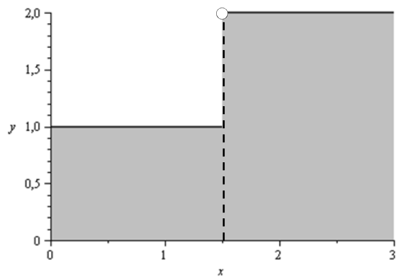

$$f(x) = \begin{cases} 1, & 0 \le x < \frac{3}{2} \\ 2, & \frac{3}{2} \le x \le 3 \end{cases}$$

$$\int_0^3 f(x)\,dx$$

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$$\int_0^3 f(x)\,dx = \int_0^{\frac{3}{2}} f(x)\,dx + \int_{\frac{3}{2}}^3 f(x)\,dx$$

$$f(x) = \begin{cases} 1, & 0 \le x < \frac{3}{2} \\ 2, & \frac{3}{2} \le x \le 3 \end{cases}$$

$$[0,\beta]: \int_0^\beta \underbrace{f(x)}_{=1}dx,\; 0 < \beta < \frac{3}{2}$$

$$= [x]_0^\beta = \beta - 0$$

$$\int_0^{\frac{3}{2}} \underbrace{f(x)}_{=1}dx = \lim_{\beta\to\frac{3}{2},\beta<\frac{3}{2}} \int_0^\beta \underbrace{f(x)}_{=1}dx$$

$$= \lim_{\beta\to\frac{3}{2},\beta<b} \beta = \frac{3}{2}$$

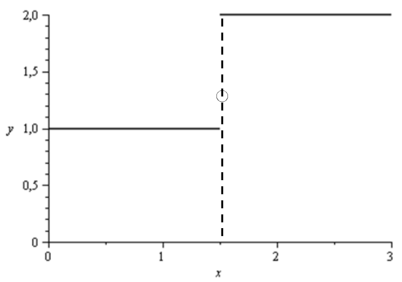
$$[\alpha,3]: \int_\alpha^3 \underbrace{f(x)}_{=2}dx,\; \frac{3}{2} < \alpha < 3$$

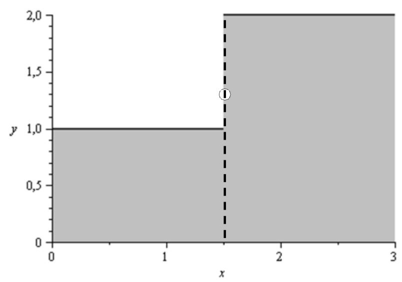
$$= [2x]_\alpha^3 = 2\cdot 3 - 2\cdot\alpha = 6 - 2\alpha$$

$$\int_{\frac{3}{2}}^3 \underbrace{f(x)}_{=2}dx = \lim_{\alpha\to\frac{3}{2},\alpha>\frac{3}{2}} \int_\alpha^3 \underbrace{f(x)}_{=2}dx$$

$$= 6 - \lim_{\alpha\to\frac{3}{2},\alpha>\frac{3}{2}} 2\alpha = 3$$

$$\Rightarrow \int_0^{\frac{3}{2}} f(x)\,dx + \int_{\frac{3}{2}}^3 f(x)\,dx = \frac{9}{2}$$





$$u(x) = \begin{cases} 1, & 0 \le x < \frac{3}{2} \\ 1.3, & x = \frac{3}{2} \\ 2, & \frac{3}{2} < x \le 3 \end{cases}$$

$$\int_0^3 u(x)\,dx$$

$$u(x) = \begin{cases} 1, & 0 \le x < \frac{3}{2} \\ 1.3, & x = \frac{3}{2} \\ 2, & \frac{3}{2} < x \le 3 \end{cases}$$

$$\int_0^3 u(x)\,dx = \int_0^{\frac{3}{2}} u(x)\,dx + \int_{\frac{3}{2}}^3 u(x)\,dx$$

$$[0,\beta]: \ \int_0^\beta \underbrace{u(x)}_{=1} dx, 0 < \beta < \frac{3}{2}$$

$$= [x]_0^\beta = \beta - 0$$

$$\int_0^{\frac{3}{2}} \underbrace{u(x)}_{=1} dx = \lim_{\beta \to \frac{3}{2}, \beta < \frac{3}{2}} \int_0^\beta \underbrace{u(x)}_{=1} dx$$

$$= \lim_{\beta \to \frac{3}{2}, \beta < b} \beta = \frac{3}{2}$$

$$[\alpha,3]: \ \int_\alpha^3 \underbrace{u(x)}_{=2} dx, \frac{3}{2} < \alpha < 3$$

$$= [2x]_\alpha^3 = 2 \cdot 3 - 2 \cdot \alpha = 6 - 2\alpha$$

$$\int_{\frac{3}{2}}^3 \underbrace{u(x)}_{=2} dx = \lim_{\alpha \to \frac{3}{2}, \alpha > \frac{3}{2}} \int_\alpha^3 \underbrace{u(x)}_{=2} dx$$

$$= 6 - \lim_{\alpha \to \frac{3}{2}, \alpha > \frac{3}{2}} 2\alpha = 3$$

Der Wert an der Stelle $\frac{3}{2}$ spielt keine Rolle

$$\Rightarrow \int_0^{\frac{3}{2}} u(x)\,dx + \int_{\frac{3}{2}}^3 u(x)\,dx = \frac{9}{2}$$