$$f(x)=\begin{cases}\frac{\pi}{2}, & -\pi<x<-\frac{\pi}{2}\\ |x|, & -\frac{\pi}{2}\le x\le\frac{\pi}{2}\\ \frac{\pi}{2}, & \frac{\pi}{2}<x\le\pi\end{cases}$$

$$a_k=\frac{2}{\pi}\int_0^{\pi} f(x)\cos(kx)\,dx$$

$$=\frac{2}{\pi}\left(\int_0^{\frac{\pi}{2}}\underbrace{x}_{f}\underbrace{\cos(kx)}_{g'}dx+\boxed{\int_{\frac{\pi}{2}}^{\pi}\frac{\pi}{2}\cos(kx)\,dx}\right)$$

$$\int_a^b f(x)\cdot g'(x)\,dx=\left[f(x)\cdot g(x)\right]_a^b-\int_a^b f'(x)\cdot g(x)\,dx$$

$$\int\cos(\alpha x)dx=\frac{1}{\alpha}\sin(\alpha x)+C$$

$$=\frac{2}{\pi}\left(\left[x\cdot\frac{1}{k}\sin(kx)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\frac{1}{k}\sin(kx)\right)+\boxed{\frac{2}{\pi}\left[\frac{\pi}{2}\frac{1}{k}\sin(kx)\right]_{\frac{\pi}{2}}^{\pi}}$$

$$\int\sin(\alpha x)dx=-\frac{1}{\alpha}\cos(\alpha x)+C$$

$$=\boxed{\frac{2}{\pi}\left(\left[x\cdot\frac{1}{k}\sin(kx)\right]_0^{\frac{\pi}{2}}\right.}-\boxed{\int_0^{\frac{\pi}{2}}\frac{1}{k}\sin(kx)}\Bigg)+\frac{2}{\pi}\left[\frac{\pi}{2}\frac{1}{k}\sin(kx)\right]_{\frac{\pi}{2}}^{\pi}$$

$$=\boxed{\frac{2}{\pi}\left(\frac{\pi}{2}\cdot\frac{1}{k}\sin\left(k\frac{\pi}{2}\right)-0\cdot\cancel{\frac{1}{k}\sin(k\cdot 0)}\right)}$$

$$\sin(k\pi)=0$$

$$\boxed{-\frac{2}{\pi}\left[\frac{1}{k}\cdot\left(-\frac{1}{k}\cos(kx)\right)\right]_0^{\frac{\pi}{2}}}$$

$$+\frac{2}{\pi}\left(\frac{\pi}{2}\cancel{\frac{1}{k}\sin(k\pi)}-\frac{\pi}{2}\frac{1}{k}\sin\left(k\frac{\pi}{2}\right)\right)$$

$$a_k = -\frac{2}{\pi}\left[\frac{1}{k}\cdot\left(-\frac{1}{k}\cos(kx)\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{2}{\pi k}\left(\frac{1}{k}\cdot\cos\left(k\,\frac{\pi}{2}\right) - \frac{1}{k}\underbrace{\cos(0)}_{=1}\right)$$

$$\cos\left(k\,\frac{\pi}{2}\right) = \begin{cases} 0, k & \text{ungerade} \\ (-1)^{\frac{k}{2}}, k & \text{gerade} \end{cases}$$

$$= \frac{2}{\pi k^2}\left(\cos\left(k\,\frac{\pi}{2}\right) - 1\right)$$

$$\Rightarrow a_k = \frac{2}{\pi k^2}\begin{cases} 0-1, k \text{ ungerade} \\ 1-1, k \text{ gerade, Vielfaches von 4} \\ -1-1, k \text{ gerade, keinVielfaches von 4} \end{cases} \quad \begin{cases} k = 1,3,5,\ldots \\ k = 0,4,8,\ldots \\ k = 2,6,10,\ldots \end{cases}$$

$$\Rightarrow a_k = \frac{2}{\pi k^2}\begin{cases} -1, k \equiv 1 \pmod 2 \\ 0, k \equiv 0 \pmod 4 \\ -2, k \equiv 2 \pmod 4 \end{cases}$$