$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du \quad u = g(x)$

8. Aufgabe

Verwenden Sie die Substitution $\boxed{u} = \boxed{\ln(x)}$ zur Berechnung des unbestimmten Integrals

$$\int \frac{\ln(x)}{x \cdot \sqrt{1+(\ln(x))^2}}\,dx$$

Lösung

$$u = \ln(x) \Rightarrow du = \frac{1}{x}\,dx$$

$$\int \frac{\boxed{\ln(x)}}{x \cdot \sqrt{1+\left(\boxed{\ln(x)}\right)^2}}\,dx = \int \frac{u}{\sqrt{1+u^2}}\,du$$

Erneute Substitution

$$\int \frac{\ln(x)}{x \cdot \sqrt{1+(\ln(x))^2}}\,dx \text{ Substitution } \boxed{u = \ln(x)} \Rightarrow du = \frac{1}{x}\,dx$$

$$= \int \frac{u}{\sqrt{1+u^2}}\,du$$

Erneute Substitution

Substitution $w = \sqrt{1+u^2}$

$$\Rightarrow \frac{dw}{du} = \frac{u}{\sqrt{1+u^2}}$$

$$\Rightarrow dw = \frac{u}{\sqrt{1+u^2}}\,du$$

$$\Rightarrow \int dw = w + C$$

$$= \int \frac{u}{\sqrt{1+u^2}}\,du = \sqrt{1+\boxed{u^2}} + C$$

Rücksubstitution

$$\int \frac{\ln(x)}{x \cdot \sqrt{1+(\ln(x))^2}}\,dx = \sqrt{1+(\ln(x))^2} + C$$

$\int f(x) \cdot g'(x)\,dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x)\,dx$

9. Aufgabe

Bestimmen Sie mittels partieller Integration

$$\int x^2 \cdot \cos(x)\,dx$$

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \cos(x)\,dx = \sin(x) + C$

Lösung

$$\int \underbrace{x^2}_{f} \cdot \underbrace{\cos(x)}_{g'}\,dx = \underbrace{x^2}_{f} \cdot \underbrace{\sin(x)}_{g} - \int \underbrace{2x}_{f'} \cdot \underbrace{\sin(x)}_{g}\,dx$$

$$= x^2 \cdot \sin(x) - 2\boxed{\int x \cdot \sin(x)\,dx}$$

Cave!

Erneute partielle Integration

$$\boxed{\int \underbrace{x}_{f} \cdot \underbrace{\sin(x)}_{g'}\,dx} = \underbrace{x}_{f} \cdot \underbrace{\left[-\cos(x)\right]}_{g} - \int \underbrace{1}_{f'} \cdot \underbrace{\left[-\cos(x)\right]}_{g}\,dx$$

$$= -x \cdot \cos(x) + \int \cos(x)\,dx = \boxed{-x \cdot \cos(x) + \sin(x)}$$

$$\Rightarrow \int x^2 \cos(x)\,dx = x^2 \cdot \sin(x) + 2\,\boxed{\left[x \cdot \cos(x) - \sin(x)\right]} + C$$