(IV) Bestimme zu jedem Eigenwert λ alle NICHTTRIVIALEN Lösungen des homogenen LGS $(\mathbf{A}-\lambda\mathbf{E})\vec{x}=\vec{0}$

$$\mathbf{A}=\begin{pmatrix}2&3&4\\2&3&3\\0&0&1\end{pmatrix}$$

$\lambda_1=1$

$$(\mathbf{A}-\lambda_1\mathbf{E})\overrightarrow{x^{(1)}}=\begin{pmatrix}1&3&4\\2&2&3\\\cancel{0}&\cancel{0}&\cancel{0}\end{pmatrix}\overrightarrow{x^{(1)}}=\vec{0}\quad\cdot(-2)\quad\to\begin{pmatrix}\boxed{1\quad 3\quad 4}\\\boxed{0\quad -4\quad -5}\end{pmatrix}\overrightarrow{x^{(1)}}=\vec{0}$$

$$\Rightarrow\boxed{-4x_2=5x_3,x_3=t,t\in\mathbb{R}\Rightarrow x_2=-\frac{5}{4}t}$$

$$\Rightarrow\boxed{x_1=-3x_2-4x_3=-\frac{1}{4}t}$$

$$\Rightarrow\overrightarrow{x^{(1)}}=t\begin{pmatrix}-\frac{1}{4}\\-\frac{5}{4}\\1\end{pmatrix},t\in\mathbb{R}\backslash\{0\}$$

$\lambda_2=0$

$$\mathbf{A}=\begin{pmatrix}2&3&4\\2&3&3\\0&0&1\end{pmatrix}$$

$$(\mathbf{A}-\lambda_2\mathbf{E})\overrightarrow{x^{(2)}}=\begin{pmatrix}\cancel{2}&\cancel{3}&\cancel{4}\\2&3&3\\0&0&1\end{pmatrix}\overrightarrow{x^{(2)}}=\vec{0}$$

$\Longrightarrow 2x_1+3x_2=0$

$\Longrightarrow x_3=0$

$$\Rightarrow\overrightarrow{x^{(2)}}=t\begin{pmatrix}-\frac{3}{2}\\1\\0\end{pmatrix},t\in\mathbb{R}\backslash\{0\}$$

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 4 \\ 2 & 3 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$\lambda_3 = 5$

$$\left(\mathbf{A} - \lambda_3 \mathbf{E}\right) \overrightarrow{x^{(3)}} = \begin{pmatrix} \cancel{-3} & \cancel{3} & \cancel{4} \\ 2 & -2 & 3 \\ 0 & 0 & -4 \end{pmatrix} \overrightarrow{x^{(3)}} = \vec{0} \quad \Longrightarrow \quad 2x_1 - 2x_2 = 0$$

$$\Longrightarrow x_3 = 0$$

$$\Rightarrow \overrightarrow{x^{(3)}} = t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, t \in \mathbb{R} \setminus \{0\}$$