$$\int f(x)\cdot g'(x)\,dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx$$

4. Aufgabe

Berechnen Sie mittels partieller Integration

$$\int x^7 \ln(x)\,dx$$

Lösung

$$\int \underbrace{x^7}_{g'(x)} \underbrace{\ln(x)}_{f(x)}\,dx$$

$$f(x) = \ln(x) \quad \Rightarrow f'(x) = \frac{1}{x}$$

$$g'(x) = x^7 \Rightarrow g(x) = \frac{1}{8}x^8$$

$$\int x^7 \ln(x)\,dx = \ln(x)\cdot\frac{1}{8}x^8 - \int \frac{1}{x}\cdot\frac{1}{8}x^8 dx = \ln(x)\cdot\frac{1}{8}x^8 - \int \frac{1}{8}x^7 dx$$

$$= \ln(x)\cdot\frac{1}{8}x^8 - \frac{1}{8}\cdot\frac{1}{8}x^8 = \ln(x)\cdot\frac{1}{8}x^8 - \frac{1}{64}x^8 + C$$

5. Aufgabe

Berechnen Sie mittels der Substitution $u=3x^3-1$ das unbestimmte Integral

$$\int \underline{8x^2}\boxed{\left(3x^3-1\right)^{16}}\underline{dx}$$

Lösung

Substitution $\boxed{u=3x^3-1}$

$$\Rightarrow \frac{du}{dx} = g'(x) = 9x^2 \Rightarrow du = 9x^2 dx \Rightarrow \underline{\frac{8}{9}du = 8x^2 dx}$$

$$\int 8x^2\left(3x^3-1\right)^{16} dx = \int u^{16}\frac{8}{9}du = \frac{8}{9}\int u^{16}\,du = \frac{8}{9}\cdot\frac{1}{17}\boxed{u^{17}} + C$$

$$= \frac{8}{153}\left(3x^3-1\right)^{17} + C$$

Rücksubstitution nicht vergessen!!!