Aufgaben 2

6. Aufgabe

a) $$\sum_{k=0}^{\infty} \left(\frac{4k-1}{7k+3} \right)^k$$

$$\lim_{k\to\infty} \frac{1}{\sqrt[k]{\left| \left(\frac{4k-1}{7k+3} \right)^k \right|}}$$

$$= \lim_{k\to\infty} \frac{1}{\sqrt[k]{\left| \frac{4k-1}{7k+3} \right|^k}}$$

$$= \lim_{k\to\infty} \frac{1}{\left| \frac{4k-1}{7k+3} \right|}$$

$$= \lim_{k\to\infty} \frac{7k+3}{4k-1}$$ → Typ $\frac{\infty}{\infty}$

$= \frac{7}{4} > 1 \rightarrow$ divergent

b) $$\sum_{k=0}^{\infty} \frac{(-2)^k}{1+2^{2k}}$$

$$a_k = \frac{(-2)^k}{1+2^{2k}}$$

$$\left| a_k \right| = \frac{2^k}{1+2^{2k}}$$

$$\leq \frac{2^k}{2^{2k}} = \frac{2^k}{4^k}$$

$\sum_{k=0}^{\infty} \frac{2^k}{4^k} = 2 \rightarrow$ divergent

7. Aufgabe

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} (x-2)^k$$

$$= \lim_{k\to\infty} \frac{1}{\sqrt[k]{\left|(-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} (x-2)^k\right|}}$$

$$= \lim_{k\to\infty} \frac{1}{\left|x-2\right| \cdot \sqrt[k]{\left|(-1)^{k+1} \frac{3}{\sqrt[5]{k^3}}\right|}}$$

$$= \left|x-2\right| \cdot \lim_{k\to\infty} \frac{1}{\sqrt[k]{\left|(-1) \cdot (-1)^{k} \frac{3}{\sqrt[5]{k^3}}\right|}}$$

$$= \left|x-2\right| \cdot \lim_{k\to\infty} \frac{1}{\left|-1\right| \cdot \sqrt[k]{\left|(-1) \cdot \frac{3}{\sqrt[5]{k^3}}\right|}}$$

$$= \left|x-2\right| \cdot \lim_{k\to\infty} \frac{1}{\sqrt[k]{\frac{3}{\sqrt[5]{k^3}}}} \qquad \to \text{Typ } \frac{1}{1}$$

$$= \left|x-2\right| \cdot 1$$

$\left|x-2\right| < 1$ → Konvergenz

$$1 < x < 3$$

Die Potenzreihe konvergiert für alle reellen Zahlen x mit $1 < x < 3$.

8. Aufgabe

$$\sum_{k=1}^{\infty}\left(2-\frac{1}{k}\right)^k (x+3)^k$$

$$\lim_{k\to\infty}\frac{1}{\sqrt[k]{\left|\left(2-\frac{1}{k}\right)^k (x+3)^k\right|}}$$

$$= \left|x+3\right| \cdot \lim_{k\to\infty}\frac{1}{\sqrt[k]{\left|\left(2-\frac{1}{k}\right)^k\right|}}$$

$$= \left|x+3\right| \cdot \lim_{k\to\infty}\frac{1}{\sqrt[k]{\left|2-\frac{1}{k}\right|^k}}$$

$$= \left|x+3\right| \cdot \lim_{k\to\infty}\frac{1}{\left|2-\frac{1}{k}\right|}$$

$$\rho = \lim_{k\to\infty}\frac{1}{\left|2-\frac{1}{k}\right|}$$

$$\rho = \frac{1}{2}$$

$$x = -\frac{7}{2}$$

$\left|-\frac{7}{2}+3\right| = \left|3-3.5\right| = \left|-0.5\right| = 0.5 = \rho \to$ Keine Aussage möglich

$\to$ Randpunkt gesondert untersuchen

$$\sum_{k=1}^{\infty}\left(2-\frac{1}{k}\right)^k\left(-\frac{7}{2}+3\right)^k = \sum_{k=1}^{\infty}\left(2-\frac{1}{k}\right)^k\left(-\frac{1}{2}\right)^k$$

$$= \lim_{k\to\infty}\left|\left(-1+\frac{1}{2k}\right)^k\right|$$

$$= \lim_{k\to\infty}\left|(-1)^k\right| \cdot \left|\left(1-\frac{1}{2k}\right)^k\right|$$

$$= \lim_{k\to\infty} \left(1 - \frac{1}{2}\cdot\frac{1}{k}\right)^k$$

$$= \lim_{k\to\infty} \left(1 + \frac{\frac{1}{2}}{k}\right)^k$$

$= e^{\frac{1}{2}} \neq 0 \rightarrow$ divergent (Nullfolgekriterium)

$x = -3$

$\left|-3+3\right| = \left|3-3\right| = \left|0\right| = 0 < \frac{1}{2} \rightarrow$ absolut konvergent

9. Aufgabe

$$\int x^2 e^x \, dx = G(x)$$

$$\int x^2 e^x \, dx = \int e^x x^2 \, dx$$

$$f'(x) = e^x \rightarrow f(x) = e^x$$

$$g(x) = x^2 \rightarrow g'(x) = 2x$$

$$\int f'(x)g(x) \, dx = f(x)g(x) - \int f(x)g'(x) \, dx$$

$$\int e^x x^2 \, dx = e^x x^2 - \int e^x 2x \, dx$$

$$\int e^x 2x \, dx$$

$$f'(x) = e^x \rightarrow f(x) = e^x$$

$$g(x) = 2x \rightarrow g'(x) = 2$$

$$\int e^x 2x \, dx = e^x 2x - \int e^x 2 \, dx$$

$$= e^x 2x - 2\int e^x \, dx$$

$$= e^x 2x - 2e^x + C$$

$$= e^x x^2 - \left(e^x 2x - 2e^x + C\right)$$

$$= e^x x^2 - e^x 2x + 2e^x + C$$

$$= e^x\left(x^2 - 2x + 2\right) + C$$

$$G(x) = e^x\left(x^2 - 2x + 2\right)$$

10. Aufgabe

$$f(x) = \int \frac{\cos\left(\ln(x)\right)}{x}\ dx,\ x > 0$$

$$t\ = \ln(x),\ dt\ = \frac{1}{x}$$

$$\int \frac{\cos(t)}{x}\frac{1}{x}\ dt$$

$$= \int \cos(t),\ dt$$

$$= \sin(t) + C$$

$$F(x) = \sin\left(\ln(x)\right) + C$$