$$\int_a^b f(x)\cdot g'(x)\,dx = \left[f(x)\cdot g(x)\right]_a^b - \int_a^b f'(x)\cdot g(x)\,dx$$

$$\int \sin(\alpha x)\,dx = -\frac{1}{\alpha}\cos(\alpha x) + C$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\overbrace{(x+\pi)^2}^{f}}{4}\underbrace{\sin(kx)}_{g'}\,dx$$

$$= \boxed{\frac{1}{4\pi}\left[(x+\pi)^2\cdot\left(-\frac{1}{k}\cos(kx)\right)\right]_{-\pi}^{\pi}} - \boxed{\frac{1}{4\pi}\int_{-\pi}^{\pi} 2(x+\pi)\cdot\left(-\frac{1}{k}\cos(kx)\right)dx}$$

$$= \boxed{\frac{1}{4\pi}\left\{(\pi+\pi)^2\cdot\left(-\frac{1}{k}\cos(k\pi)\right) - (-\pi+\pi)^2\cdot\left(-\frac{1}{k}\cos(k(-\pi))\right)\right\}}$$

$$+ \boxed{\frac{1}{2k\pi}\int_{-\pi}^{\pi}\underbrace{(x+\pi)}_{f}\underbrace{\cos(kx)}_{g'}\,dx}$$

$$\int_a^b f(x)\cdot g'(x)\,dx = \left[f(x)\cdot g(x)\right]_a^b - \int_a^b f'(x)\cdot g(x)\,dx$$

$$= \boxed{\frac{1}{4\pi}\left\{(\pi+\pi)^2\cdot\left(-\frac{1}{k}\cos(k\pi)\right) - \cancel{(-\pi+\pi)^2\cdot\left(-\frac{1}{k}\cos(k(-\pi))\right)}\right\}}$$

$$+\boxed{\frac{1}{2k\pi}\int_{-\pi}^{\pi}\underbrace{(x+\pi)}_{f}\underbrace{\cos(kx)}_{g'}\,dx}$$

$$\int\cos(\alpha x)dx = \frac{1}{\alpha}\sin(\alpha x)+C$$

$$\cos(k\pi) = \cos(-k\pi) = (-1)^k$$

$$= \boxed{-\frac{\pi}{k}(-1)^k} + \boxed{\frac{1}{2k\pi}\int_{-\pi}^{\pi}\underbrace{(x+\pi)}_{f}\underbrace{\cos(kx)}_{g'}\,dx}$$

$$\sin(k\pi) = 0$$

$$\int\sin(\alpha x)dx = -\frac{1}{\alpha}\cos(\alpha x)+C$$

$$= -\frac{\pi}{k}(-1)^k + \frac{1}{2k\pi}\left[\cancel{(x+\pi)\frac{1}{k}\sin(kx)}\right]_{-\pi}^{\pi} - \frac{1}{2k\pi}\int_{-\pi}^{\pi}\frac{1}{k}\sin(kx)\,dx$$

$$= \frac{\pi}{k}(-1)^{k+1} - \cancel{\frac{1}{2k^2\pi}\left[-\frac{1}{k}\cos(kx)\right]_{-\pi}^{\pi}} = \frac{\pi}{k}(-1)^{k+1} = b_k$$

Fourier–Reihe von f

$$f \sim \frac{\pi^2}{3} + \sum_{k=1}^{\infty} (-1)^k \left(\frac{1}{k^2} \cos(k\,x) - \frac{\pi}{k} \sin(kx) \right)$$

$$x = \pi \Rightarrow f(\pi) = \frac{\pi^2}{2} = \frac{\pi^2}{3} + \sum_{k=1}^{\infty} \frac{1}{k^2} \qquad \boxed{\Rightarrow \frac{\pi^2}{6} = \sum_{k=1}^{\infty} \frac{1}{k^2}}$$

$$x = 0 \Rightarrow f(0) = \frac{\pi^2}{4} = \frac{\pi^2}{3} + \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \qquad \boxed{\Rightarrow \frac{\pi^2}{12} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}}$$