3. Aufgabe

Bestimmen Sie das Lot und die orthogonale Projektion des Vektors

$$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \in \mathbb{R}^3$$

in den Vektor

$$\vec{b} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} \in \mathbb{R}^3$$

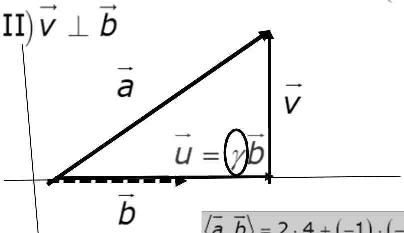
Lösung

$$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$$

Lot und Projektion

$$\vec{a} = \vec{u} + \vec{v} \quad (\mathrm{I})\,\vec{u} = \boxed{\gamma \vec{b}} \quad (\mathrm{II})\,\vec{v} \perp \vec{b}$$

$$\vec{a} = \boxed{\gamma \vec{b}} + \vec{v}$$



$$\left\langle \vec{a}, \vec{b} \right\rangle = 2 \cdot 4 + (-1) \cdot (-1) + 3 \cdot 2 = 15$$

$$\left\| \vec{b} \right\| = \sqrt{4^2 + (-1)^2 + 2^2} = \sqrt{21}$$

$$\left\langle \vec{a}, \vec{b} \right\rangle = \left\langle \gamma \vec{b} + \vec{v}, \vec{b} \right\rangle$$

$$= \left\langle \gamma \vec{b}, \vec{b} \right\rangle + \left\langle \vec{v}, \vec{b} \right\rangle$$

$$= \gamma \left\langle \vec{b}, \vec{b} \right\rangle + \left\langle \vec{v}, \vec{b} \right\rangle = \gamma \left\| \vec{b} \right\|^2 + \underbrace{\left\langle \vec{v}, \vec{b} \right\rangle}_{=0} \Rightarrow \gamma = \frac{\left\langle \vec{a}, \vec{b} \right\rangle}{\left\| \vec{b} \right\|^2} = \frac{15}{21} = \frac{5}{7}$$

$$\Rightarrow \vec{u} = \frac{5}{7} \vec{b} = \frac{1}{7} \begin{pmatrix} 20 \\ -5 \\ 10 \end{pmatrix}$$

$$\vec{v} = \vec{a} - \gamma \vec{b} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \frac{1}{7} \begin{pmatrix} 20 \\ -5 \\ 10 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -6 \\ -2 \\ 11 \end{pmatrix}$$

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