Aufgaben 2

6. Aufgabe

a) $$\sum_{k=0}^{\infty} \left(\frac{4k-1}{7k+3}\right)^k$$

$$\lim_{k\to\infty} \sqrt[k]{\left|\left(\frac{4k-1}{7k+3}\right)^k\right|}$$

$$= \lim_{k\to\infty} \sqrt[k]{\left|\frac{4k-1}{7k+3}\right|^k}$$

$$= \lim_{k\to\infty} \left|\frac{4k-1}{7k+3}\right|$$

$$= \lim_{k\to\infty} \frac{|4k-1|}{|7k+3|} \qquad \to \text{Typ } \frac{\infty}{\infty}$$

$$= \lim_{k\to\infty} \frac{\left(\frac{4(4k-1)}{|-4k+1|}\right)}{\left(\frac{7(7k+3)}{|7k+3|}\right)} \qquad \to \text{Typ } \frac{4}{7}$$

$$= \frac{4}{7} \approx 0.5714 < 1 \to \text{Konvergenz}$$

b) $$\sum_{k=0}^{\infty} \frac{(-2)^k}{1+2^{2k}}$$

$$a_k = \frac{(-2)^k}{1+2^{2k}}$$

$$|a_k| = \frac{2^k}{1+2^{2k}}$$

$$\leq \frac{2^k}{2^{2k}} = \frac{2^k}{4^k}$$

$$\sum_{k=0}^{\infty} \frac{2^k}{4^k} = 2 \to \text{divergent}$$

7. Aufgabe

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} (x-2)^k$$

$$= \lim_{k\to\infty} \sqrt[k]{\left| (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} (x-2)^k \right|}$$

$$= \left| x-2 \right| \cdot \lim_{k\to\infty} \sqrt[k]{\left| (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} \right|}$$

$$\lim_{k\to\infty} \sqrt[k]{\left| (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} \right|} = 1$$

$\left| x-2 \right| < 1 \rightarrow$ Konvergenz

$$1 < x < 3$$

Die Potenzreihe konvergiert für alle reellen Zahlen x mit $1 < x < 3$.

8. Aufgabe

$$\sum_{k=1}^{\infty} \left(2 - \frac{1}{k}\right)^k (x+3)^k$$

$$\lim_{k\to\infty} \sqrt[k]{\left| \left(2 - \frac{1}{k}\right)^k (x+3)^k \right|}$$

$$= \left| x+3 \right| \cdot \lim_{k\to\infty} \sqrt[k]{\left| \left(2 - \frac{1}{k}\right)^k \right|}$$

$$= \left| x+3 \right| \cdot \lim_{k\to\infty} \sqrt[k]{\left| 2 - \frac{1}{k} \right|^k}$$

$$= \left| x+3 \right| \cdot \lim_{k\to\infty} \left| 2 - \frac{1}{k} \right|$$

$$\rho = \frac{1}{\lim_{k\to\infty}\left|2-\frac{1}{k}\right|}$$

$$\rho = \frac{1}{4}$$

$$x = -\frac{7}{2}$$

$$\left|-\frac{7}{2}+3\right| = \left|3-3.5\right| = \left|-0.5\right| = 0.5 > 0.25 \rightarrow \text{divergent}$$

$$x = -3$$

$$\left|-3+3\right| = \left|3-3\right| = \left|0\right| = 0 < 0.25 \rightarrow \text{absolut konvergent}$$

9. Aufgabe

$$\int x^2e^x\ dx = G(x)$$

$$\int x^2e^x\ dx = \int e^x x^2\ dx \qquad f'(x) = e^x \rightarrow f(x) = e^x$$

$$g(x) = x^2 \rightarrow g'(x) = 2x$$

$$\int f'(x)g(x)\ dx = f(x)g(x) - \int f(x)g'(x)\ dx$$

$$\int e^x x^2\ dx = e^x x^2 - \int e^x 2x\ dx$$

$$\int e^x 2x\ dx \qquad f'(x) = e^x \rightarrow f(x) = e^x$$

$$g(x) = 2x \rightarrow g'(x) = 2$$

$$\int e^x 2x\ dx = e^x 2x - \int e^x 2\ dx$$

$$= e^x 2x - 2\int e^x\ dx$$

$$= e^x 2x - 2e^x + C$$

$$= e^x x^2 - \left(e^x 2x - 2e^x + C\right)$$

$$= e^x x^2 - e^x 2x + 2e^x + C$$

$$= e^x\left(x^2 - 2x + 2\right) + C$$

$G(x) = e^x \left(x^2 - 2x + 2\right)$

10. Aufgabe

$$f(x) = \int \frac{\cos\left(\ln(x)\right)}{x}\ dx,\ x > 0$$

$$t = \ln(x),\ dt = \frac{1}{x}$$

$$\int \frac{\cos(t)}{x}\frac{1}{x}\ dt$$

$$= \int \cos(t),\ dt$$

$$= \sin(t) + C$$

$$F(x) = \sin\left(\ln(x)\right) + C$$