Einige wichtige unbestimmte Integrale

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, C \in \mathbb{R}, n = 0,1,2,\cdots$$

$$\int x^{-n} dx = \frac{1}{(1-n)x^{n-1}} + C, C \in \mathbb{R}, n = 2,3,4,\cdots, x \neq 0$$

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C, C \in \mathbb{R}, a \neq -1, x > 0$$

$$\int a^x dx = \frac{1}{\ln(a)} a^x + C, a > 0 \land a \neq 1, C \in \mathbb{R},$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, a \neq 0, C \in \mathbb{R},$$

$$\int \frac{1}{ax} dx = \frac{1}{a} \ln|ax| + C, a \neq 0, x \neq 0, C \in \mathbb{R}$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C, a \neq 0, C \in \mathbb{R}$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C, a \neq 0, C \in \mathbb{R}$$

$$\int \frac{1}{\sin^2(ax)} dx = -\frac{1}{a} \cot(x) + C, x \neq n\pi, a \neq 0, C \in \mathbb{R}$$

$$\int \frac{1}{\cos^2(ax)} dx = \frac{1}{a} \tan(ax) + C, x \neq \left(n + \frac{1}{2}\right)\pi, a \neq 0, C \in \mathbb{R}$$

§26 Die Berechnung von Stammfunktionen

(I) Summen– und Faktorregel

$$\int \big(f(x) \pm g(x)\big)dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int \alpha f(x)\,dx = \alpha \int f(x)\,dx \ , \ \alpha \in \mathbb{R}$$

Beispiel 1

$$\int \big(3x^4 + 5x^2 + 2\big)dx = 3\int x^4 dx + 5\int x^2 dx + 2\int 1\,dx$$

$$= 3\left(\frac{1}{5}x^5 + C_1\right) + 5\left(\frac{1}{3}x^3 + C_2\right) + 2\big(x + C_3\big)$$

$$= \frac{3}{5}x^5 + \frac{5}{3}x^3 + 2x + \underbrace{3C_1 + 5C_2 + 2C_3}$$

$$= \frac{3}{5}x^5 + \frac{5}{3}x^3 + 2x + C$$

Beispiel 2

$$\sin^2(x) + \cos^2(x) = 1$$

$$\int \tan^2(x)\,dx = \int \frac{\sin^2(x)}{\cos^2(x)}\,dx = \int \frac{1 - \cos^2(x)}{\cos^2(x)}\,dx$$

$$= \int \frac{1}{\cos^2(x)}\,dx - \int dx = \tan(x) - x + C$$