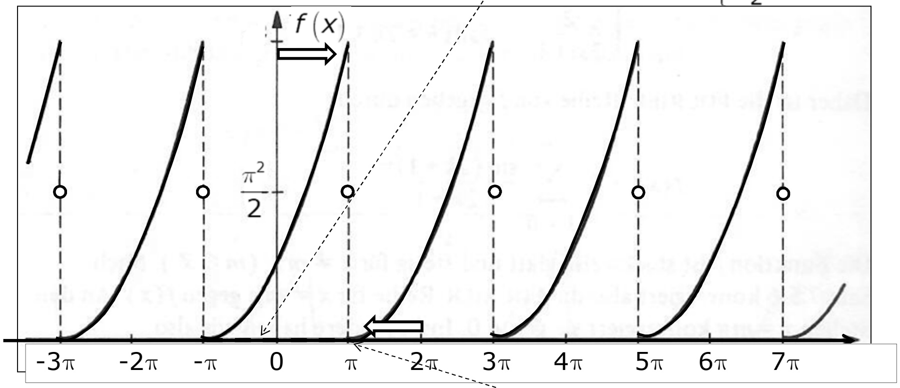
③ Mittelwerteigenschaft (MWE)

$$f(x)=\begin{cases}\left(\dfrac{x+\pi}{2}\right)^2 & ,-\pi<x<\pi\\[2mm] \dfrac{\pi^2}{2} & ,x=\pi\end{cases}$$

erfüllt an allen Stellen, an denen *f* stetig ist.



zu überprüfen, an den Unstetigkeitsstellen

$$\lim_{h\to 0}\frac{f(\pi+h)+f(\pi-h)}{2}=\frac{1}{2}\left(0+\left(\frac{\pi+\pi}{2}\right)^2\right)=\frac{1}{2}\left(\pi^2\right)$$ MWE erfüllt

④ Integrationsbereich festlegen

$$f(x)=\begin{cases}\left(\dfrac{x+\pi}{2}\right)^2 & ,-\pi<x<\pi\\[2mm] \boxed{\dfrac{\pi^2}{2} \quad ,x=\pi} \end{cases}$$

$$-\pi..\pi$$

⑤ Berechnung der Fourier-Koeffizienten

$$a_0=\frac{1}{2\pi}\int_{\boxed{-\pi}}^{\pi}\underbrace{\left(\frac{x+\pi}{2}\right)^2}_{\frac{(x+\pi)^2}{4}}dx=\frac{1}{2\pi}\left[\frac{1}{4}\frac{(x+\pi)^3}{3}\right]_{-\pi}^{\pi}$$

$$=\frac{1}{2\pi}\cdot\frac{1}{4}\frac{(2\pi)^3}{3}-\frac{1}{2\pi}\cdot\cancel{\frac{1}{4}\frac{(-\pi+\pi)^3}{3}}=\frac{\pi^2}{3}$$

$$a_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos(kx)\,dx \qquad b_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin(kx)\,dx$$

Zweimalige partielle Integration

$$f(x)=\begin{cases}\left(\frac{x+\pi}{2}\right)^2 &, -\pi < x < \pi\\ \frac{\pi^2}{2} &, x=\pi\end{cases}$$

$$\int_a^b f(x)\cdot g'(x)\,dx = \left[f(x)\cdot g(x)\right]_a^b - \int_a^b f'(x)\cdot g(x)\,dx$$

$$\int \cos(\alpha x)dx = \frac{1}{\alpha}\sin(\alpha x)+C$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\overbrace{(x+\pi)^2}^{f}}{4}\underbrace{\cos(kx)}_{g'}dx$$

$$\sin(k\pi)=0$$

$$=\frac{1}{4\pi}\left[\cancel{(x+\pi)^2\frac{1}{k}\sin(kx)}\right]_{-\pi}^{\pi}-\frac{1}{4\pi}\int_{-\pi}^{\pi}2(x+\pi)\cdot\frac{1}{k}\sin(kx)\,dx$$

$$\int \sin(\alpha x)dx = -\frac{1}{\alpha}\cos(\alpha x)+C$$

$$=-\frac{1}{2k\pi}\int_{-\pi}^{\pi}\underbrace{(x+\pi)}_{f}\underbrace{\sin(kx)}_{g'}dx$$

$$=-\frac{1}{2k\pi}\left\{\left[(x+\pi)\cdot\left(-\frac{1}{k}\cos(kx)\right)\right]_{-\pi}^{\pi}-\int_{-\pi}^{\pi}1\cdot\left(-\frac{1}{k}\right)\cos(kx)\,dx\right\}$$

$$=-\frac{1}{2k\pi}\left\{\boxed{\left[(x+\pi)\cdot\left(-\frac{1}{k}\cos(kx)\right)\right]_{-\pi}^{\pi}}-\boxed{\int_{-\pi}^{\pi}1\cdot\left(-\frac{1}{k}\right)\cos(kx)\,dx}\right\}$$

$$=\boxed{-\frac{1}{2k\pi}\left((\pi+\pi)\cdot\left(-\frac{1}{k}\cos(k\pi)\right)-\left(\cancel{(-\pi+\pi)}\cdot\left(-\frac{1}{k}\cos(k(-\pi))\right)\right)\right)}$$

$$\int \cos(\alpha x)dx = \frac{1}{\alpha}\sin(\alpha x)+C \qquad \boxed{-\frac{1}{2k^2\pi}\int_{-\pi}^{\pi}\cos(kx)\,dx}$$

$$\sin(k\pi)=0$$

$$=\boxed{\frac{1}{k^2}\cos(k\pi)}-\boxed{\frac{1}{2k^2\pi}\left[\cancel{\frac{1}{k}\sin(kx)}\right]_{-\pi}^{\pi}}=\frac{(-1)^k}{k^2}$$

$$\cos(k\pi)=\cos(-k\pi)=(-1)^k$$