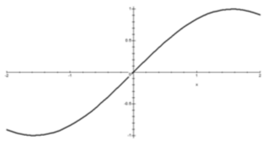
<u>4. Aufgabe</u>

f sei eine stetige, ungerade Funktion auf dem Intervall $[-a,a]$,

d.h. $\forall x \in [-a,a] : f(x) = -f(-x)$

Was gilt dann für das bestimmte Integral

$$\int_{-a}^{a} f(x)\,dx\ ?$$



<u>Lösung</u>

Rechnung analog zur 2.Aufgabe

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx = -\int_{0}^{-a} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

Substitution im ersten Integral $u=-x$

$$\Rightarrow \frac{du}{dx} = g'(x) = -1 \Rightarrow du = -\,dx$$

$$x = 0 \Rightarrow u = 0 \quad x = -a \Rightarrow u = a$$

$$= \int_{0}^{a} f(-u)\,du + \int_{0}^{a} f(x)\,dx$$

$$= \int_{0}^{a} f(-u)\cdot du + \int_{0}^{a} f(x)\,dx = \int_{0}^{a} -f(u)\cdot du + \int_{0}^{a} f(x)\,dx$$

f ungerade *f*(-*u*)=-*f*(*u*)

$$= -\int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(x)\,dx = 0$$

5. Aufgabe

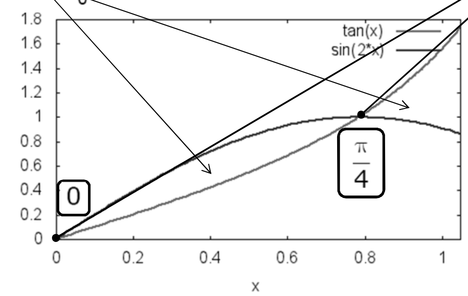
Bestimmen Sie die Fläche A, welche von den Funktionen

$$\tan(x) \text{ und } \sin(2x), 0 \le x \le \frac{\pi}{3}$$

eingeschlossen wird.

Lösung

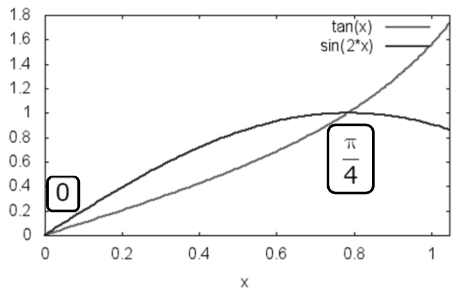
$$A = \int_0^{\frac{\pi}{3}} \left|\tan(x) - \sin(2x)\right| dx$$



(I) Zerlege $[a,b]$ an den Schnittpunkten von f-g in Teilintervalle → Bestimme $|f$-$g|$

(II) Integriere $|f$-$g|$ über jedem Teilintervall

(III) Addiere die Integrale



Die Funktionen schneiden sich bei $a = 0$ und $b = \frac{\pi}{4}$

$$\int \sin(2x)\,dx = -\frac{1}{2}\cos(2x) + C$$

$$\int \underbrace{\tan(x)}_{=\frac{\sin(x)}{\cos(x)}}\,dx = -\ln(|\cos(x)|) + C$$

Logarithmische Integration

$$A = \int_0^{\frac{\pi}{3}} \left|\tan(x) - \sin(2x)\right| dx$$

$$= \int_0^{\frac{\pi}{4}} \left\{\sin(2x) - \tan(x)\right\} dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left\{\tan(x) - \sin(2x)\right\} dx$$

$$= \left[-\frac{1}{2}\cos(2x) + \ln\left|\cos(x)\right|\right]_0^{\frac{\pi}{4}} + \left[-\ln\left|\cos(x)\right| + \frac{1}{2}\cos(2x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \left[-\frac{1}{2}\cos(2x) + \ln\left|\cos(x)\right|\right]_0^{\frac{\pi}{4}} + \left[-\ln\left|\cos(x)\right| + \frac{1}{2}\cos(2x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \boxed{-\frac{1}{2}\cos\left(2\frac{\pi}{4}\right) + \ln\left|\cos\left(\frac{\pi}{4}\right)\right| - \left(-\frac{1}{2}\cos(0) + \ln\left|\cos(0)\right|\right)}$$

$$\boxed{-\ln\left|\cos\left(\frac{\pi}{3}\right)\right| + \frac{1}{2}\cos\left(2\frac{\pi}{3}\right) - \left(-\ln\left|\cos\left(\frac{\pi}{4}\right)\right| + \frac{1}{2}\cos\left(2\frac{\pi}{4}\right)\right)}$$

$\cos(0) = 1,$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

$\cos\left(\frac{\pi}{2}\right) = 0$

$\ln(1) = 0$

$$= \boxed{0 + \underbrace{\ln\left(\frac{\sqrt{2}}{2}\right)}_{-\frac{1}{2}\ln(2)} + \frac{1}{2} - 0} \boxed{-\underbrace{\ln\left(\frac{1}{2}\right)}_{=\ln(1)-\ln(2)} + \frac{1}{2}\cdot\left(-\frac{1}{2}\right) + \underbrace{\ln\left(\frac{\sqrt{2}}{2}\right)}_{-\frac{1}{2}\ln(2)} - 0}$$

$$= \boxed{-\frac{1}{2}\ln(2) + \frac{1}{2}} + \boxed{\ln(2) - \frac{1}{4} - \frac{1}{2}\ln(2)} = \frac{1}{4}$$