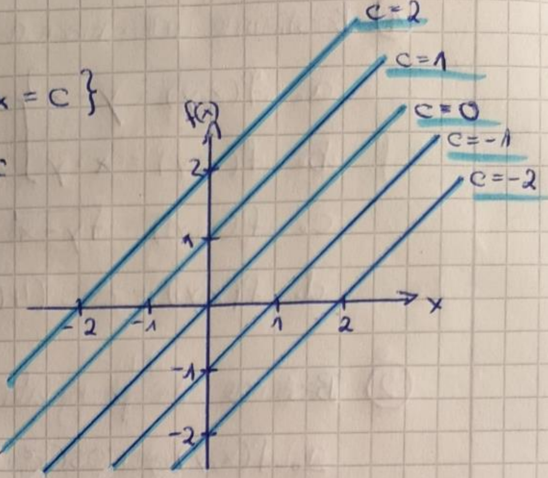
Lösungen

① **a** $f(x,y) = y - x = z$

$H_c = \{(x,y) \in \mathbb{R}^2 \mid y - x = c\}$

$y - x = c \iff y = x + c$

$c = -2: \quad y = x - 2$
$c = -1: \quad y = x - 1$
$c = 0: \quad y = x$
$c = 1: \quad y = x + 1$
$c = 2: \quad y = x + 2$

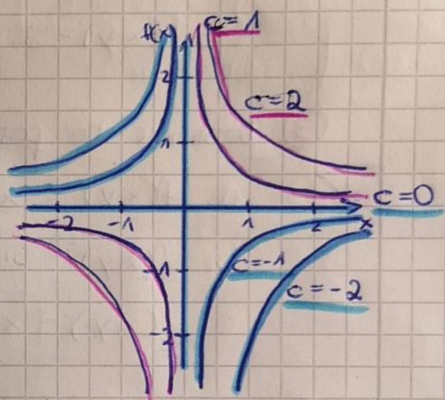


b $f(x,y) = x \cdot y = z$

$H_c = \{(x,y) \in \mathbb{R}^2 \mid x \cdot y = c\}$

$x \cdot y = c \iff y = \frac{c}{x} = c \cdot \frac{1}{x}$

$c = -2: \quad y = -2 \cdot \frac{1}{x}$
$c = -1: \quad y = -\frac{1}{x}$
$c = 0: \quad y = 0 \lor x = 0$
$c = 1: \quad y = \frac{1}{x}$
$c = 2: \quad y = 2 \cdot \frac{1}{x}$

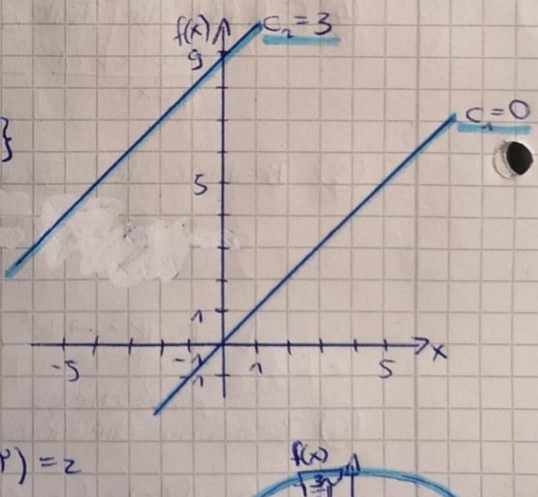


c $f(x,y) = \sqrt{y-x} = z$

$H_c = \{(x,y) \in \mathbb{R}^2 \mid \sqrt{y-x} = c\}$

$\sqrt{y-x} = c \mid ()^2$ da $c \geq 0$
$y - x = c^2$
$y = x + c^2$

$c_1 = 0: \quad y = x$
$c_2 = 3: \quad y = x + 3^2 = x + 9$



d $f(x,y) = \cos(4(x^2+y^2) - 2\pi) = z$

$H_c = \{(x,y) \in \mathbb{R}^2 \mid \cos(4(x^2+y^2) - 2\pi) = 1\}$

$\cos(\alpha) = 1 \iff \alpha = k \cdot 2\pi, \; k \in \mathbb{Z}$

$4(x^2+y^2) - 2\pi = k \cdot 2\pi$
$4(x^2+y^2) = k \cdot 2\pi + 2\pi$
$x^2 + y^2 = (k+1) 2\pi \cdot 1/4$
$x^2 + y^2 = (k+1) \cdot \pi/2, \; k \in \mathbb{Z}$

→ Kreise um den Ursprung mit Radius $r_k = \sqrt{(k+1) \cdot \pi/2}, \; k \in \mathbb{Z}$

② a $f(x,y) = \cos(x^2+y)$

$f_x(x,y) = -\sin(x^2+y) \cdot 2x$

$f_y(x,y) = -\sin(x^2+y) \cdot 1$

c $f(x,y) = \sin(x^2 \cdot e^{3y})$

$f_x(x,y) = 2xe^{3y}\cos(x^2 \cdot e^{3y})$

$f_y(x,y) = x^2 \cdot 3 \cdot e^{3y} \cos(x^2 \cdot e^{3y})$

b $f_x(x,y,z) = e^{x^2y} \cdot 2xy^3$

$f_y(x,y,z) = x^2e^{x^2y}y^2 + 2ye^{x^2y}$

$f_z(x,y,z) = 1$

$f_{xy}(x,y,z) = e^{x^2y} \cdot x^2 \cdot 2xy^3 + e^{x^2y} \cdot 6xy^2$
$= f_{yx}(x,y,z)$

$f_{xz}(x,y,z) = 0$
$= f_{zx}(x,y,z)$

$f_{yz}(x,y,z) = 0$
$= f_{zy}(x,y,z)$

③ a $f(x,y) = x^2 3y + x^3$

$f_x(x,y) = 6xy + 3x^2$

$f_y(x,y) = 3x^2$

$f_{xy}(x,y) = 6x$
$= f_{yx}(x,y)$

$f_{xx}(x,y) = 6y + 6x$

$f_{yy}(x,y) = 0$

b $f(x,y) = \sin(x^2 \cdot 3y)$

$f_x(x,y) = 6xy\cos(3x^2y)$

$f_y(x,y) = 3x^2\cos(3x^2y)$

$f_{xy}(x,y) = 6x\cos(3x^2y)$
$-18x^3y = f_{yx}(x,y)$

$f_{xx}(x,y) = 6y\cos(3x^2y) - 36x^2y^2\sin(3x^2y)$

$f_{yy}(x,y) = -9x^4\sin(3x^2y)$

c $f(x,y) = xe^{xy}$

$f_x(x,y) = e^{xy} + xye^{xy}$

$f_y(x,y) = x^2e^{xy}$

$f_{xy}(x,y) = 2xe^{xy} + yx^2e^{xy} = f_{yx}(x,y)$

$f_{xx}(x,y) = ye^{xy} + ye^{xy} + xy^2e^{xy}$

$f_{yy}(x,y) = x^3e^{xy}$

④ $\frac{\partial^2 \omega(x,y,z)}{\partial x^2}$, $\frac{\partial^2 \omega(x,y,z)}{\partial z^2}$, $\frac{\partial^2 \omega(x,y,z)}{\partial y \partial z}$ berechnen

$= \omega_{xx}(x,y,z)$ $= \omega_{zz}(x,y,z)$ $= \omega_{yz}(x,y,z) = \omega_{zy}(x,y,z)$
da stetig

$\omega_x(x,y,z) = 2x$, $\omega_{xx}(x,y,z) = 2$

$\omega_z(x,y,z) = y$, $\omega_{zz}(x,y,z) = 0$

$\omega_{zy}(x,y,z) = 1$

$\omega_{xx} - (\omega_{zz} + \omega_{yz}) = 0$

$2 - (0 + 1) = 0$

$1 \neq 0$ → Gleichung nicht erfüllt

⑤ a $f_1(x_1, x_2) = x_1 - x_2$

$f_2(x_1, x_2) = 3x_1 + x_2 - 1$

$f_3(x_1, x_2) = x_1^2 + 7x_2$

$$D_f(\vec{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(\vec{x}) & \frac{\partial f_1}{\partial x_2}(\vec{x}) \\ \frac{\partial f_2}{\partial x_1}(\vec{x}) & \frac{\partial f_2}{\partial x_2}(\vec{x}) \\ \frac{\partial f_3}{\partial x_1}(\vec{x}) & \frac{\partial f_3}{\partial x_2}(\vec{x}) \end{pmatrix}$$

$$D_f(\vec{x}) = \begin{pmatrix} 1 & -1 \\ 3 & 1 \\ 2x_1 & 7 \end{pmatrix}, \quad D_f\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 3 & 1 \\ 2 & 7 \end{pmatrix}$$

b $f_1(x_1, x_2, x_3) = x_1^2 - \sqrt{x_2} \cdot x_3^3 = x_1^2 - x_2^{1/2} \cdot x_3^3$

$f_2(x_1, x_2, x_3) = 3\cos(x_1 + x_2) + \sqrt{x_3^3} = 3\cos(x_1 + x_2) + x_3^{3/2}$

$$D_f(\vec{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(\vec{x}) & \frac{\partial f_1}{\partial x_2}(\vec{x}) & \frac{\partial f_1}{\partial x_3}(\vec{x}) \\ \frac{\partial f_2}{\partial x_1}(\vec{x}) & \frac{\partial f_2}{\partial x_2}(\vec{x}) & \frac{\partial f_2}{\partial x_3}(\vec{x}) \end{pmatrix}$$

$$= \begin{pmatrix} 2x_1 & -\frac{1}{2} \cdot \frac{1}{\sqrt{x_2}} x_3^3 & -3\sqrt{x_2}\, x_3^2 \\ 3\cos(x_1 + x_2) & 3\cos(x_1 + x_2) & \frac{3}{2}\sqrt{x_3} \end{pmatrix}$$

$$D_f\begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 & -\frac{1}{2 \cdot \sqrt{6}} & -3\sqrt{6} \\ 3\cos(8) & 3\cos(8) & \frac{3}{2} \end{pmatrix}$$

$$D_f\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 & -\frac{27}{2} & -27 \\ 3 & 3 & \frac{3\sqrt{3}}{2} \end{pmatrix}$$

⑥

$X : [\alpha, \beta] \subseteq \mathbb{R} \to \mathbb{R}^2,\ t \mapsto X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$

a $f(x) = (x-3)^2,\ x \in [-1, 4]$

1. $x_1 = t$

$x_2 = f(x) = (t-3)^2$

$X : [-1, 4] \to \mathbb{R}^2,\ t \mapsto X(t) = \begin{pmatrix} t \\ (t-3)^2 \end{pmatrix}$

2. $t = x_1 - 3 \to x_1 = t + 3$

$x_2 = f(x) = t^2$

$X : [-1, 4] \to \mathbb{R}^2,\ t \mapsto X(t) = \begin{pmatrix} t+3 \\ t^2 \end{pmatrix}$

▽ Anfangspunkt : $x_1 = -1 = t$ $\Rightarrow \begin{pmatrix} -1 \\ 16 \end{pmatrix} = X(-1)$

oder Anfangspunkt : $x_1 = -1 \to t = -4 = x_1 - 3$ berechnen $\Rightarrow \begin{pmatrix} -1 \\ 16 \end{pmatrix} = X(-4)$

○ Endpunkt : $x_1 = 4 = t$ $\Rightarrow X(4) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

oder Endpunkt : $x_1 = 4 \to t = x_1 - 3 = 1$ $X(1) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

b $f(x) = m(x-\alpha)+b \quad , x \in [0,|\alpha|]$

<u>1.</u> $x_1 = t$

$x_2 = f(x) = m(t-\alpha)+b$

$X: [0,|\alpha|] \to \mathbb{R}^2, t \mapsto X(t) = \begin{pmatrix} t \\ m(t-\alpha)+b \end{pmatrix}$

<u>2.</u> $t = x_1 - \alpha \to x_1 = t+\alpha$

$x_2 = f(x) = mt+b$

$X: [0,|\alpha|] \to \mathbb{R}^2, \; t \mapsto X(t) = \begin{pmatrix} t+\alpha \\ mt+b \end{pmatrix}$

∇ <u>Anfangspunkt</u>: $x_1 = 0 = t$ mit <u>1.</u> : $X(0) = \begin{pmatrix} 0 \\ -\alpha m+b \end{pmatrix}$

oder

$x_1 = 0 \to t = 0-\alpha$ mit <u>2.</u> : $X(-\alpha) = \begin{pmatrix} -\alpha+\alpha \\ m\cdot(-\alpha)+b \end{pmatrix} = \begin{pmatrix} 0 \\ -\alpha m+b \end{pmatrix}$

$\circ$ <u>Endpunkt</u>: $x_1 = |\alpha| = t$ mit <u>1.</u> :

$$X(|\alpha|) = \begin{pmatrix} |\alpha| \\ m(|\alpha|-\alpha)+b \end{pmatrix} = \begin{cases} \begin{pmatrix} \alpha \\ b \end{pmatrix}, & \alpha \geq 0 \\ \begin{pmatrix} -\alpha \\ -2m\alpha+b \end{pmatrix}, & \alpha < 0 \end{cases}$$

oder

$x_1 = |\alpha| \to t = |\alpha| - \alpha = \begin{cases} 0, & \alpha \geq 0 \\ -2\alpha, & \alpha < 0 \end{cases}$ mit <u>2.</u> :

$$X(|\alpha|-\alpha) = \begin{cases} \begin{pmatrix} \alpha \\ b \end{pmatrix}, & \alpha \geq 0 \\ \begin{pmatrix} -\alpha \\ -2m\alpha+b \end{pmatrix}, & \alpha < 0 \end{cases}$$

<u>Hinweis</u>: beide Parameterdarstellungen beschreiben immer dieselbe Kurve, es ist also jeweils egal und bringt dasselbe Ergebnis, ob man mit 1. oder 2. Anfangs- und Endpunkt berechnet