

Aufgaben 2

6. Aufgabe

a) $\sum_{k=0}^{\infty} \left(\frac{4k-1}{7k+3} \right)^k$

$$\lim_{k \rightarrow \infty} \sqrt[k]{\left| \left(\frac{4k-1}{7k+3} \right)^k \right|}$$

$$= \lim_{k \rightarrow \infty} \sqrt[k]{\left| \frac{4k-1}{7k+3} \right|^k}$$

$$= \lim_{k \rightarrow \infty} \left| \frac{4k-1}{7k+3} \right|$$

$$= \lim_{k \rightarrow \infty} \frac{|4k-1|}{|7k+3|} \quad \rightarrow \text{Typ } \frac{\infty}{\infty}$$

$$= \lim_{k \rightarrow \infty} \frac{\left(\frac{4(4k-1)}{|-4k+1|} \right)}{\left(\frac{7(7k+3)}{|7k+3|} \right)} \quad \rightarrow \text{Typ } \frac{4}{7}$$

$$= \frac{4}{7} \approx 0.5714 < 1 \rightarrow \text{Konvergenz}$$

b) $\sum_{k=0}^{\infty} \frac{(-2)^k}{1+2^{2k}}$

7. Aufgabe

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} (x-2)^k$$

$$= \lim_{k \rightarrow \infty} \sqrt[k]{\left| (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} (x-2)^k \right|}$$

$$= |x - 2| \cdot \lim_{k \rightarrow \infty} \sqrt[k]{\left| (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} \right|}$$

$$\lim_{k \rightarrow \infty} \sqrt[k]{\left| (-1)^{k+1} \frac{3}{\sqrt[5]{k^3}} \right|} = 1$$

$$|x - 2| < 1 \rightarrow \text{Konvergenz}$$

$$1 < x < 3$$

Die Potenzreihe konvergiert für alle reellen Zahlen x mit $1 < x < 3$.

8. Aufgabe

$$\sum_{k=1}^{\infty} \left(2 - \frac{1}{k}\right)^k (x + 3)^k$$

$$\lim_{k \rightarrow \infty} \sqrt[k]{\left| \left(2 - \frac{1}{k}\right)^k (x + 3)^k \right|}$$

$$= |x + 3| \cdot \lim_{k \rightarrow \infty} \sqrt[k]{\left| \left(2 - \frac{1}{k}\right)^k \right|}$$

$$= |x + 3| \cdot \lim_{k \rightarrow \infty} \sqrt[k]{\left| 2 - \frac{1}{k} \right|^k}$$

$$= |x + 3| \cdot \lim_{k \rightarrow \infty} \left| 2 - \frac{1}{k} \right|$$

$$\rho = \frac{1}{\lim_{k \rightarrow \infty} \left| 2 - \frac{1}{k} \right|}$$

$$\rho = \frac{1}{4}$$

$$x = -\frac{7}{2}$$

$$\left| -\frac{7}{2} + 3 \right| = |3 - 3.5| = |-0.5| = 0.5 > 0.25 \rightarrow \text{divergent}$$

$$x = -3$$

$$|-3 + 3| = |3 - 3| = |0| = 0 < 0.25 \rightarrow \text{absolut konvergent}$$

9. Aufgabe

$$\int x^2 e^x dx = G(x)$$

$$\int x^2 e^x dx = \int e^x x^2 dx$$

$$f'(x) = e^x \rightarrow f(x) = e^x$$

$$g(x) = x^2 \rightarrow g'(x) = 2x$$

$$\int f'(x)g(x) dx = f(x)g(x) - \int f(x)g'(x) dx$$

$$\int e^x x^2 dx = e^x x^2 - \int e^x 2x dx$$

$$\int e^x 2x dx$$

$$f'(x) = e^x \rightarrow f(x) = e^x$$

$$g(x) = 2x \rightarrow g'(x) = 2$$

$$\int e^x 2x dx = e^x 2x - \int e^x 2 dx$$

$$= e^x 2x - 2 \int e^x dx$$

$$= e^x 2x - 2e^x + C$$

$$= e^x x^2 - (e^x 2x - 2e^x + C)$$

$$= e^x x^2 - e^x 2x + 2e^x + C$$

$$= e^x (x^2 - 2x + 2) + C$$

$$G(x) = e^x (x^2 - 2x + 2)$$

10. Aufgabe

$$f(x) = \int \frac{\cos(\ln(x))}{x} dx, x > 0$$

$$t = \ln(x), \, dt = \frac{1}{x}$$

$$\int \frac{\cos(t)}{x} \frac{1}{x} \, dt$$

$$= \int \cos(t) \, dt$$

$$= \sin(t) + C$$

$$F(x) = \sin(\ln(x)) + C$$