Weak Instruments Assignment

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Question 1

For this question, we analyse the rejection frequency of a classical t-statistic, with values simulated from the following model:

$$Y = X\beta + \varepsilon$$
$$X = Z\Pi + V$$
$$\begin{pmatrix} \varepsilon_i \\ V_i \end{pmatrix} \sim N(0, \Sigma), \quad \Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \quad \Pi = a \cdot e_{11}$$

The values used for a and ρ are:
$a = (1.5\ 0.7\ 0.5\ 0.3\ 0.15\ 0.07\ 0.03\ 0)$ and $\rho = (0\ 0.1\ 0.2\ 0.3\ 0.4\ 0.5\ 0.6\ 0.7\ 0.9\ 0.95)$

From the graph, we observe that for all values of s the size of the t-statistic moves towards values around 5% for uncorrelated errors ($\rho = 0$). However, if the explaining variables are endogenous, the size of the t-statistic increases rapidly (if weak instruments are used which implies a is close to 0). This shows that the t-statistic is not suitable when weak instruments are used, as the size should remain around 55%. The 2SLS estimate becomes:

$$\hat{\beta} = \left(X'P_Z X\right)^{-1} X'P_Z Y$$
$$P_Z = Z\left(Z'Z\right)^{-1} Z'$$

Question 2

Figure 2 displays the 95% critical value of the LR statistic as a function of $r(\beta_0)$ for k=11. We see that the critical value converges to 19.6751 for $r(\beta_0)$ close to zero, this is the critical value of a chi-squared distribution with k=11 degrees of freedom. For $r(\beta_0)$ is large, the critical value converges to 3.8415 which is the critical value of a chi-squared distribution with 1 degree of freedom.

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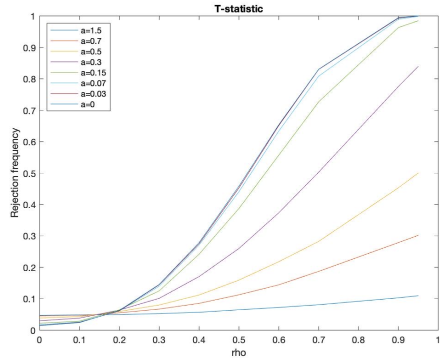


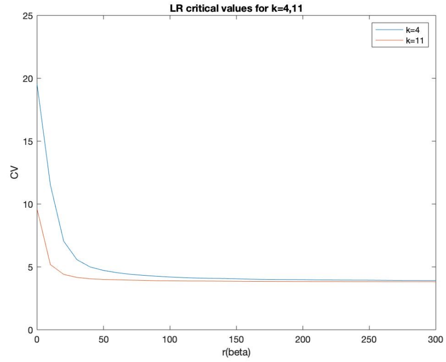
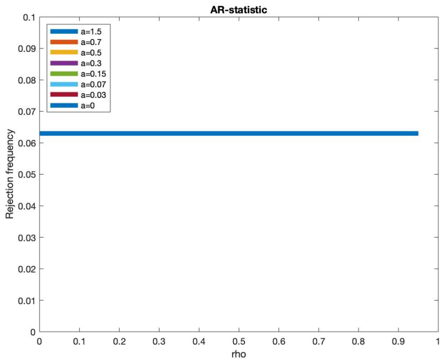


Figure 1: Rejection frequency t-statistic

Figure 2: Critical value LR statistic

Question 3

Again, the rejection frequencies for different ρ and a values are analyzed. The AR, Score and LR statistics are robust to weak instruments. Figures 4-6 show the results of respectively the AR, LM and LR test statistics rejection frequencies for the previously stated ρ and a values. Clear from these graphs is that there is a big difference between the behavior of these statistics compared to the previously analyzed t-statistic rejection frequencies. The AR statistic in figure 3 is independent of ρ and a, which explains why its rejection frequency is a horizontal line. The sizes of the different statistics remain around the 5% value for the various ρ and a values, proving that these three statistics are robust to weakness of the instruments.



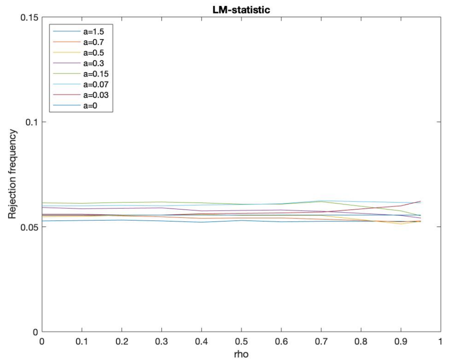


Figure 3: AR statistic

Figure 4: LM statistic

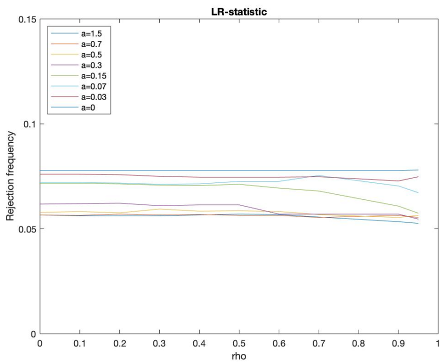


Figure 5: LR statistic

Question 4

Figure 2 also displays the 95% critical value function of the LR statistic for k=4 as a function of $r(\beta_0)$. This graph is comparable to figure 2, where k was larger. For k=4 we see the critical value converges to 9.4877 for low $r(\beta_0)$ and again to 3.8415 for high $r(\beta_0)$, as is expected for an LR statistic, which is the combination of an AR and an LM statistic.

Question 5

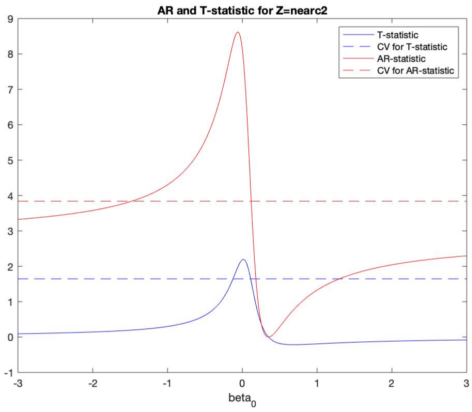


Figure 6: AR and t-statistic

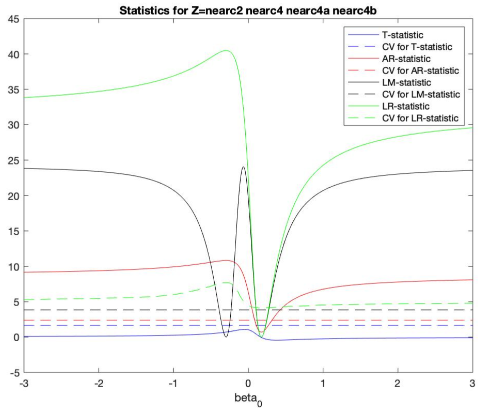


Figure 7: AR, LM, LR and t-statistic

5a

Figure 6 depicts the 95% confidence set for the return on education, using the proximity to a 2-year college (nearc2) as an instrument, calculated using the 2SLS t-statistic and the AR statistic. The confidence set for the 2SLS t-statistic is $(0.0198; 0.6798)$ and for the AR statistic it is $(-\infty; -1.47) \cup (0.11; \infty)$

5b

As the confidence sets for the AR statistic is of the form $(-\infty; \beta_{lower}) \cup (\beta_{upper}; \infty)$, it is unbounded and disjoint indicating that the instruments are weak. The AR statistic is a weak identification robust statistic meaning that its limiting distribution remains unchanged when the instruments are weak. The 2SLS t-statistic, however, is not of is this class and therefore over-rejects the confidence set, resulting in this difference.

5c

The first stage F-statistic of the 2SLS estimation for the 2-year instrument equals 2.8105, which is a lot lower than the required value of 10 shown by Stock and Yogo (2005) to be able to assume a standard normally distributed 2SLS t-statistic.

The AR statistic is defined as follows:

$$AR(\beta_0) = \frac{(y - X\beta_0)' P_Z (y - X\beta_0)/k}{y - X\beta_0)' M_Z (y - X\beta_0)/(N-k)}$$

Figure 6 shows that the AR statistic converges to about 2.8 when β is large.

5d

No, we did not use the LM and LR statistic. The LR statistic can be written as a combination of the of AR and LM statistic and therefore the AR statistic could be seen as a combination of the LM and LR statistic. However, this is not used in 5a), as this requires the estimation of the LM and LR statistic first which takes a lot more time and computation than estimating it using formula in 5c).

5e

Figure 7 depicts the 95% confidence set for the return on education, using the proximity to a 2-year college (nearc2), the proximity to a 4-year college (nearc4), the proximity to a 4-year community college (nearc4a) and the proximity to a 4-year private college (nearc4b) as instruments, calculated using the 2SLS t-statistic, AR, LM and LR statistic.

These 4 statistics result in 4 different confidence sets, namely:

2SLS t-statistic: $(0.1055; 0.2251)$

AR statistic: $(0.09; 0.31)$

LM statistic: $(-0.39; -0.24) \cup (0.11; 0.27)$

LR statistic: $(0.1; 0.28)$

5f

Again, there is a difference between the confidence sets. Due to the form of the confidence set of the AR, LM and LR statistic, one can conclude that the instruments are weak and it is therefore straightforward that the 2SLS t-statistic differs from the others. As the LM statistic is based on extremum estimation by ML, it has two minima. One represents the maximum estimator and the other minimum represents the minimum estimator. The LR statistic can be written as a combination of the LM and AR statistic and uses the preferred one when necessary, it is obvious that the three statistics can't be the same and the LR statistic has the smallest confidence set of the three "correct" statistics.

5g

The minimum value of the AR statistic is 0.7007, this is way smaller than the 95% critical value of a chi-squared distribution with 3 degrees of freedom (7.8147) that we have to use for the over-identification test. The degrees of freedom are computed by $m - k$, where m is the number of instruments (columns in Z, 4) and k is the number of possible endogenous variables (columns in X, 1). The null hypothesis can not be rejected by this test, so assume the instruments to be exogenous.