Mortgage Payments: A Comprehensive Analysis

Bank of Foster

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Executive Summary

Abstract

The following report discusses the importance of mortgage loans and details a model that can be used to calculate monthly mortgage payments based off of principal, loan term, and interest rate. The aim of this report is to help potential customers of *Bank of Foster* take out a mortgage loan that is right for them, and understand the details behind how their mortgage plan works.

Ultimately, this report defines a mortgage loan, sets up a hypothetical model to examine the machinations of a mortgage loan, describes a model and does calculations using this model to describe the borrower's monthly payments, does a thorough analysis of the model to find relationships between mortgage elements, and finally discusses why these are of concern to the borrower.

The main conclusions of this report are many. Not only is interest rate found to be of the most importance when shopping for a mortgage loan, but it also explains why banks need to stay so competitive in this realm. Also, loan term is found to be of import, as it describes how much money the bank ultimately makes off the borrower, and also can be adjusted to accommodate the borrower's financial needs. Lastly, the model, which is provided below for reference, detailed enough to make accurate calculations for your mortgage loan, but also not too convoluted, thus providing the borrower with details unimportant to their ultimate area of concern, their monthly mortgage payments and total mortgage loan cost.

$$\boldsymbol{x = \frac{P(1+i)^t i}{(1+i)^t - 1}}$$

Introduction

Description

For new homeowner's, the proposition of taking out a mortgage loan can be a daunting idea. The suggestion of a binding agreement with your bank isn't always a welcomed notion, especially when it involves many years of planning and thousands of dollars in investment. Mortgage loans, however, are an integral part of the collective economy. Not only is it important for prospective homeowner's to be savvy when choosing a mortgage loan for their unique situation, it is also important for them to be educated on exactly what they are getting into.

First, let's define that a mortgage loan is a loan taken out by a potential property owner, the borrower, wishing to purchase property they do not yet have the funds to purchase outright. The lender, usually a bank, loans the borrower an agreed upon amount of capital, with the understanding that the money will be paid back in full to the lender after a certain period of time, plus a payment to the lender for the service of providing the loan, called interest. The difference between a normal loan and a mortgage loan is that a mortgage loan is a legal mechanism acknowledging that the lender has the ability to foreclose or repossess the mortgaged property if the mortgage agreement is broken.

This comprehensive analysis provides details about finding the right mortgage loan for your circumstances, taking an empirical and model-based approach to mitigate possible nervousness or confusion when taking out your first mortgage loan or making your first mortgage payment, and is ultimately aimed at saving you money based on a cost-benefit perspective. It follows the conceptual mathematical modeling concept of *DAESI*: description, abstraction, equations for a mathematical model, solutions, interpretation of the modeling results. Current market data is reliable and readily available, allowing for easy comparison between like

mortgage loans. This report provides potential *Bank of Foster* customers with facts and aims at attracting potential homeowners for fixed-rate mortgage loans.

Abstraction

A comprehensive analysis of a mortgage loan must contain a definition, or abstraction, of the pertinent information on which our model will be based. The following table lists the factors which our mortgage loan model will be centered on:

Table 1: Mortgage Loan Elements

Name	Definition	Units	Symbol
Property	property in which loan is taken out for	-	-
Lender/Borrower	bank/customer	-	-
Principal	loan amount	dollars (lump)	P
Interest	payment to lender for service	percentage (monthly)	i
Loan Term	period of loan	time (months)	t
Payment	total payment from borrower to lender	dollars (monthly)	x
Down Payment	initial amount paid by borrower for property	dollars (lump)	d

For our mathematical model to make sense, we must make some basic assumptions on how the *Bank of Foster* takes out mortgage loans:

- While interest rates are usually provided on a percentage referencing the yearly rate, mortgage payments are made on a monthly basis. Any elements used in our model must be based on this yearly value; naturally, calculations are made to convert interest rate, loan term, and payment to monthly values.
- As the model considers the effect of principal, interest rate, and loan term on borrower payment, elements such as borrower credit rating, property type, etc. are not considers.

- Loans are taken out from *Bank of Foster* in San Diego, California and property is assumed to be in the Southern California region where *Bank of Foster* interest rates and mortgage terms apply.
- Interest rate is fixed.

In addition, *Foster Consulting* has provided hypothetical values for our base model. These values will be strategically changed later on to extenuate relationships in the model and describe the functionality of the model using the *DAESI* approach. Below is a table defining the values used in our based model:

Table 2: Model Quantities		
Name	Symbol	Value
Principal	P	***$800,000.00***
Interest	i	4.5% yearly / 12 months = ***0.375% monthly***
Loan Term	t	25 years * 12 months = ***300 months***
Monthly Payment	x	***unknown***
Down Payment	d	***$50,000.00***

With these values provided and the explanatory model forthcoming, we can see that we have four known values and one unknown value. Our model will be interested in determining a monthly payment value which would be of highest priority to a new homeowner (borrower) wishing to purchase a property. Hopefully the model will provide insight as to why *Bank of Foster* provides customer-friendly mortgage loans at affordable interest rates.

Data and Method

Equations for a Mathematical Model

To formulate a mathematical model that can be used to computer monthly mortgage payments, we first consider the down payment (*d*), which must be subtracted from the initial principal (*P*), or cost of the house being mortgaged.

$$P = P - d = \$800{,}000.00 - \$50{,}000.00 = \$\mathbf{750{,}000.00}$$

so, after subtracting the borrower's down payment on the property from the initial cost of the real estate being mortgaged, we are left with a principal (*P*) of $750,000.00.

Next, we try to decipher the model that will compute the monthly payment for the mortgage loan. We start out by focusing on the first month of payment, where the total cost owed to the bank is the principal (*P*), or $750,000.00. After the first month, the principal can be redefined as follows:

$$P_1 = P + Pi - x = P(1 + i) - x$$

as you have already paid the first monthly payment (*x*) at the end of the first month, and owed money to the lender is now principal (*P*) added to principal (*P*) multiplied by the monthly interest rate (which is the payment to the bank for providing the mortgaging service).

Now we can apply a recursive approach to examine the money owed to the bank after the 2nd and 3rd months, respectively:

$$P_2 = P_1(1 + i) - x$$

$$P_2 = (P(1 + i) - x)(1 + i) - x = P(1 + i)^2 - (1 + i)x - x$$

$$P_3 = P_2(1 + i) - x$$

$$P_3 = (P(1 + i)^2 - (1 + i)x - x)(1 + i) - x = P(1 + i)^3 - (1 + i)^2x - (1 + i)x - x$$

where we can substitute the principal for the previous month into the remaining payment calculation for the current month. As can be seen, the first month's equation can be used as a

model for subsequent months, and a recursive model is shown above, which can be generalized for the remaining payment after the kth month of the mortgage loan.

After the kth month, the remaining payment can be modeled as follows (noticing the recursive pattern):

$$P_k = P(1+i)^k - (1+i)^{(k-1)}x \ldots (1+i)^2x - (1+i)x - x$$

and because this resembles the recurrence for a geometric series, we can use its corresponding summation formula ($\frac{1-a^k}{1-a}$):

$$P_k = P(1+i)^k - x\frac{1-(1+i)^k}{1-(1+i)} = P(1+i)^k - x\frac{1-(1+i)^k}{i}$$

As we are interested in finding the monthly payment over the total course of the mortgage, our dummy index of summation (k) and our loan term (t) should be equal; that is, $k = t$. Using this information, we can substitute t for k so that the model, after solving for variable of interest x, shows as:

$$\boldsymbol{x = \frac{P(1+i)^t i}{(1+i)^t - 1}}$$

Solutions

With our model found, we can solve for the monthly payment of the hypothetical situation where a borrower is signing on for a mortgage loan with the *Bank of Foster*. Below is the equation solving for this monthly payment, with the variables principal (*P=$750,000.00*), interest (*i=0.375%*), and loan term (*t=300*) defined previously:

$$x = \frac{750{,}000.00(1+0.00375)^{300}0.00375}{(1+0.00375)^{300}-1} = \$\mathbf{4,168.75}$$

After the entirety of the 300 month mortgage payment (a loan term of 25 years), the principal will be paid in full to the bank with a monthly payment (x) of $4,168.75, and gives us a solution to our initial question: How do I find the monthly payment for my mortgage loan? Our

model went a long way in describing the mortgage loan process and will continue to give us insight in the next section.

The next section of this report analyzes and interprets different implications of this model and gives insight into how this model can help you, the investor, make a safe and knowledgeable decision about asking out a mortgage loan with *Bank of Foster*.

Table 3: Varying Interest Rate Comparison – Sensitivity Analysis	
Annual Interest Rate	Monthly Payment
4%	$3,957.13
4.25%	$4,062.20
4.5%	$4,168.75
4.75%	$4,275.71
5%	$4,384.08

Results

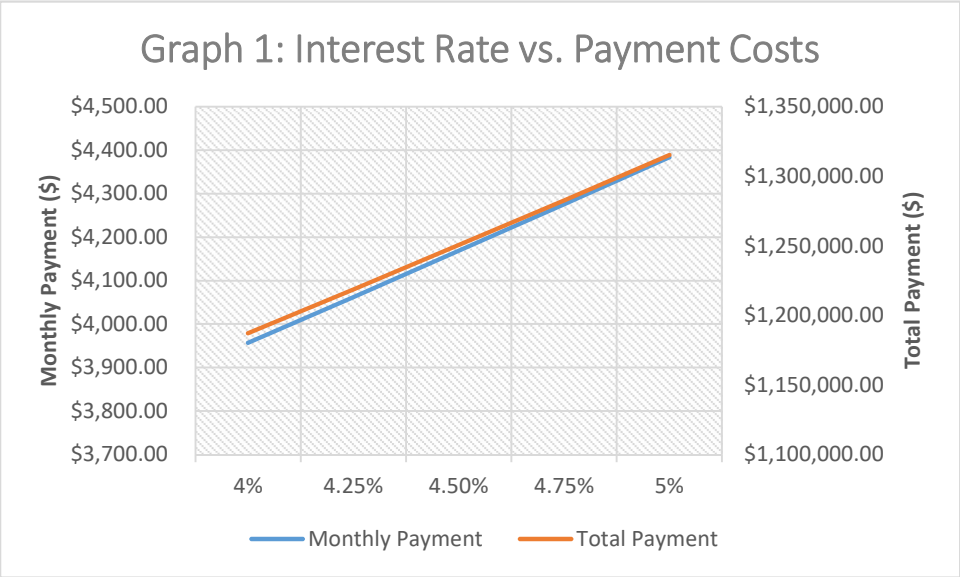
Solutions & Interpretation of Modeling Results

I will begin to present the results that the model brings by emphasizing a sensitivity analysis of the monthly payment to different common interest rates. This analysis can begin to show the effect that interest rates have on the amount of money a borrower may expect to pay their lender or bank.

As seen from the table in the last section, as the interest rate is increased, the monthly payment increases for the borrower and the lender's profit increases, showing the importance of finding a good interest rate. With variations of just 0.25% yearly in the interest rate, a change of $105.00-$110.00 per month is seen. Over the course of a 25 year mortgage loan, a 0.25% increase in interest could cost the property owner an extra ~$32,250.00 for a 25 year loan term ($107.50 x 12 months x 25 years = $32,250.00), a crazy increase in price! This shows the ability of a ruthless bank to increase interest rates and therefore take a substantial profit from the customer. Only a very small increase in interest can result in thousands of dollars in the pocket of the lender; which tells you, the customer, that interest rate shopping is the number one key to look for when examining mortgage loans with fixed interest rates.

Next, it is important to describe the relationships that this model suggests. First, it is relatively easy to see that principal (P) has a direct correlation to the monthly payment (x) if both interest rate and loan term remain the same. This means that if the total amount of the loan is increased (i.e. high real estate cost or lower down payment amount), the monthly payment will be effected linearly. Next, it is important to explain the relationship between the interest rate (i) and the monthly payment (x). Although the sensitivity analysis above does a good job of charting this relationship, below is a graph demonstrating the comparison visually, as well as showing the total cost of the loan by the end of the loan term for each noted interest rate. As you

can see from the graph, the relationship between interest rate (i) and monthly payment (x) increases at a non-linear rate, the total mortgage payment does as well, making a discouraging, but not shocking discovery that interest rate increase both monthly payment and total mortgage payment at a rate equal to that of the increase in interest rate.



In addition to the two comparisons detailed above, it is important to explore the effect that loan term (t) has on the expected monthly payment. Below is a table detailing the monthly payment (x) when the loan term is varied straddling the already discussed 25 years. Interest rate (i) is kept at the aforementioned 4.5% based on current market data, and principal (P) is also left untouched with a property value of $800,000.00 minus the disclosed down payment (d) of $50,000.00.

Table 4: Varying Loan Term Comparison	
Loan Term	Monthly Payment
15 years	$5,737.45
20 years	$4,744.88
25 years	$4,168.75
30 years	$3,800.14
35 years	$3,549.43

The table above clearly shows the strong influence that loan term has on monthly payment (x). For a large principal (P) such as the one used in this example (which is a normal price for a family home in San Diego), it is evident that a longer loan term (t) ultimately decreases the monthly payment (x). In this way, we can say that loan term and monthly payment values for a mortgage loan have an inverse relationship, with longer loan terms making more sense for larger principals, and vice versa. If we compare the 15 year loan term with the 35 year loan term, we can start to see this relationship. Although the shorter loan term decreases the total cost of the mortgage and the amount paid to the bank, the amount paid monthly may not be feasible for families or individuals making an inadequate monthly income. On the other hand, the longer loan term boasts a much friendly monthly payment of almost $2,000.00 dollars less, but the total cost of the mortgage is almost $500,000.00. Unfortunately, though, many families are forced to take longer loan terms as they must consider the big picture and make certain they are able to pay their mortgage and not lose their investment.

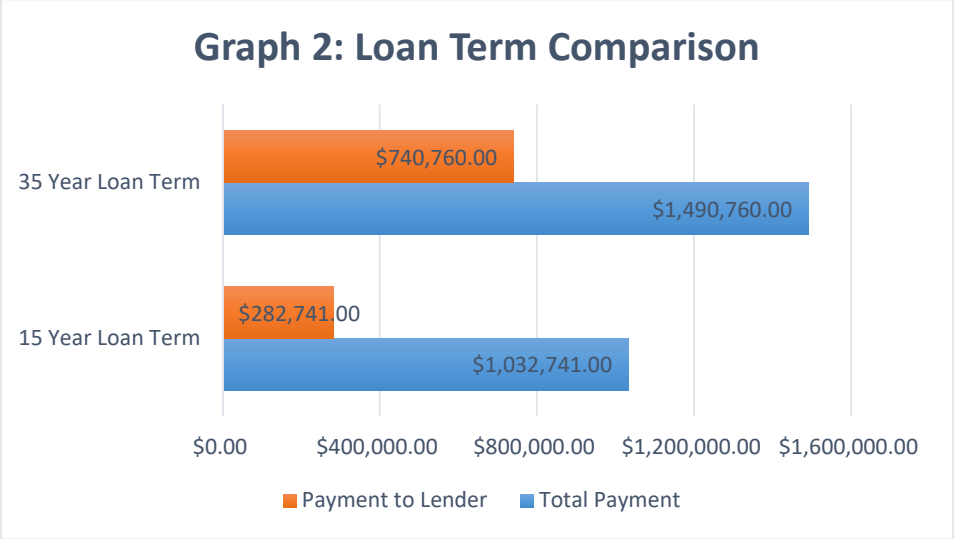
$$\textbf{15 } \boldsymbol{year\ loan\ term} = \$5{,}737.45\ monthly\ payment = \$1{,}032{,}741.00\ total\ property\ cost$$

$$\$1{,}032{,}741.00 - \$750{,}000.00\ principal = \$282{,}741.00\ paid\ to\ bank$$

$$\textbf{35 } \boldsymbol{year\ loan\ term} = \$3{,}549.43\ monthly\ payment = \$1{,}490{,}760.00\ total\ property\ cost$$

$$\$1{,}490{,}760.00 - \$750{,}000.00\ principal = \$740{,}760.00\ paid\ to\ bank$$

Included below is a pie chart showing the relationship between these loan terms:



This graph does a great job illustrating that as loan term (t) increases, the total payment is increased by nearly 50%, while the payment to the lender (or bank) is increased by more than 250%. The rate at which these variables increase/decrease is not linear, showing the importance of making a smart decision when choosing a loan term for your mortgage. Banks are aware of this relationship with fixed rate mortgages and loan terms, and will often push longer loan terms over shorter ones.

Lastly, it is important to view the model from a broad perspective and analyze the influence that the variables have when changed together, instead of independently. For example, if interest rate (i) and loan term (t) are increased together both monthly payment (x) and total mortgage cost increase. If interest rate is increased and loan term decreased, or vice versa, the monthly payment and total mortgage cost will increase or decrease based on the relative rate at which these numbers fluctuate. The examination done above can offer an inclination to this fact, and the relationship is quite predictable. If interest rate (i) is increased marginally while loan term (t) is decreased exponentially (both relative to their respective scales), loan term will

overpower interest rate and show a model more characteristic to that of a changing loan term. It is also important to now consider the effect that interest rate (i) has on the mortgage. In essence, it is the mortgage loan, and largely defines which bank you choose to loan from. The following attributes of your mortgage loan will most likely not change:

- Principal (P) – the choice of property would not influence which mortgage you choose
- Loan term (t) – individual situation (i.e. large income, small family, other investments, etc.) would have a small impact on your mortgage choice
- Down payment (d) – the amount you have saved for a down payment would not affect one mortgage as compared to another

With this information based off the model, it begs the question: Why not always choose the bank with the smallest interest rate? The answer to this question lies in personal preferences such as how much a customer likes a specific bank, how close the bank or lender is to the homeowner, or even benefits that the chosen lender offers in addition to the mortgage. Obviously, thousands of dollars in differences between mortgage loans because of differing interest rates can influence potential property owners more than any of the criteria listed above, but it's worth considering that this is why banks need to stay so competitive in their advertised interest rates. At the *Bank of Foster*, interest rates are not only competitive, but the knowledgeable staff, coupled with this report aimed at giving the customer a thorough examination of their investment, mortgage loans are just the beginning of a trusting relationship between our bank and you, the customer.

The conclusion that follows provides a closing statement on the information provided in the preceding sections.

Conclusion and Discussion

The model we found by following the *DAESI* approach was able to provide this report with valuable insight about mortgage loans and the relationship between interesting factors such as principal, interest rate, term period, and monthly payment. A potential borrower should now be able to make and educated and confident decision when choosing the mortgage loan that is right for them. There are, however, elements that factor into a mortgage loan that were not discussed here: taxes, varied interest rates/term periods, estimated property value, etc. These factors certainly could be a part of a separate model focusing on the gritty details, but the information provided here should encompass the important elements of a mortgage loan that any homeowner will encounter.

Other alternatives to the problem of taking out a mortgage loan can also be considered. What if a potential homeowner wants to invest? What is a real estate company wants to start a property business? What happens when people flip houses? These are complex scenarios that would take time to understand. The model proposed here is aimed at finding monthly payments for the average homeowner taking out a mortgage, and while there are many equations and models that could describe this, this model describes only the important elements, while maintaining real-world accuracy and pertinent data.

It is also imperative to describe the pros and cons of the approach we took. As stated earlier, this model does a good job of describing the important information in a concise way, without giving superficial or extraneous data that some mortgages simply ignore. It is aimed at giving a first time homeowner the information they need to take out a mortgage loan intelligently. The model doesn't, however, provide you with the details that may give you the exact monthly payment of your particular mortgage. For this reason, we recommend consulting

your bank, or in this case a trained professional at the *Bank of Foster*, to determine exactly what your mortgage plan will entail.

I now want to summarize the really crucial information that was found in this report, providing a closing statement for future reference:

1. Interest rate has the biggest effect on the amount of money the bank makes off your mortgage, and with all other factors being equal, has the largest effect on your monthly payments.
2. Loan term is important to consider based on your principal and down payment, but is often chosen based off the borrower's financial situation. Banks (lenders) would tend to sway towards longer terms, as they have more time to collect interest.
3. Because of the way a mortgage is structured, banks take more money in their pockets towards the start of the mortgage and less towards the end. In this way, the property owner is paying the bank towards the start of the loan, and isn't paying off most of their property value until at least the second half of the mortgage.
4. The consequences of breaking a mortgage agreement were not discussed here, as this is not a legal discussion. Repossession and/or foreclosure can be example consequences of defaulting on or breaking your mortgage agreement.

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