- definitions
- examples
- notes

Rules of Inference (1.4)

↳ concluding

An argument is a list of propositions:

p_1
p_2
p_3
$\vdots$
p_n } premesis

$\therefore q$ } conclusion

an argument is **VALID** if whenever p_1, p_2, p_3, p_n are all true, then q **MUST** be true

that is;

"$(p_1 \land p_2 \land p_3 \land \ldots \land p_n) \to q$" is always true

ex. let r_1, r_2 be prop.

$p_1 : r_1 \to r_2$
$p_2 : r_1$

$\therefore r_2$

↓
VALID!

		p_1	p_2	q	$p_1 \land p_2$	$(p_1 \land p_2) \to q$
r_1	r_2	$r_1 \to r_2$	r_1	r_2	$(r_1 \to r_2) \land r_1$	$(r_1 \to r_2) \land r_1 \to r_2$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	F	F	F	T

VALID

ex. $r_1 \to r_2$
r_2

$\therefore r_1$

NOT VALID!

r_1	r_2	$r_1 \to r_2$	r_2	q	$p_1 \land p_2$	$(p_1 \land p_2) \to q$
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	$\boxed{F}$
F	F	T	F	F	F	T

:(

ex. $r_1 \lor r_2$
$\neg r_1$

$\therefore r_2$

VALID!

r_1	r_2	$r_1 \lor r_2$	$\neg r_1$	r_2	$(r_1 \lor r_2) \land \neg r_1$	$((r_1 \lor r_2) \land \neg r_1) \to r_2$
T	T	T	F	T	F	T
T	F	T	F	F	F	T
F	T	T	T	T	T	T
F	F	F	T	F	F	T

^ ^

ex. $r_1 \to r_2$
$\neg r_2$

$\therefore r_1$

NOT VALID!

r_1	r_2	$r_1 \to r_2$	$\neg r_2$	r_1	$(r_1 \to r_2) \land \neg r_2$	$(p_1 \land p_2) \to q$
T	T	T	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	F	F	T
F	F	T	T	F	T	F

:(

$r_1 \to r_2$
$\neg r_2$ ← VALID

$\therefore \neg r_1$

r_1, r_2, r_3 are prop

$r_1 \rightarrow r_2$

$r_2 \rightarrow r_3$

r_1

$\therefore r_3$

r_1	r_2	r_3	$r_1 \rightarrow r_2$	$r_2 \rightarrow r_3$	r_1	r_3	$((r_1 \rightarrow r_2) \wedge (r_2 \rightarrow r_3) \wedge (r_1)) \rightarrow r_3$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	F	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	F	T

VALID !

r_1

$\neg r_1$

$\therefore r_2$ ← is not shown in premesis

Contradiction ; will always prove to be VALID

HOW TO CONSTRUCT PROPOSITIONS

Predicates - takes input

BZ (before 0) - input of integer, returns true if input is bigger than 0

ex. BZ(5) = True   BZ(−2) = False

↑ proposition

A propositional function (predicate) P with domain D (set) is a statement

$P(x)$ ← for every $x \in D$, $P(x)$ is a proposition

- Let $BZ(x)$ be a predicate over domain $D = \mathbb{Z}$ to be true if x is bigger than 0, False otherwise

  $BZ(5)$ $BZ(-2)$ $BZ(0)$ $\neg BZ(0)$

- Define $BT(x,y)$ over $D = \mathbb{Z} \times \mathbb{Z}$

  to be true if x is bigger than y, False otherwise

ex. $BT(2,1)$ $\neg BT(1,2)$ $BT(2,3) \vee BT(3,1) =$ True

$(4,2) \in BT$ $BT(4,2)$

if input not in domain = undefined : $BT(1.4, 2)$

- $PLUS(x,y,z)$ over $D = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ to be true if the sum of x and y is z

  false otherwise.

  $PLUS(1,2,3)$ $\neg PLUS(1,3,2)$ $PLUS(1,3,2) \rightarrow BZ(4)$ = False

- Let $BZ_1(x)$ over $D = \{-1, 0, 1, 2, 3\}$ be true if x bigger than 0, False ow

ex: $-1 \to BZ(-1) = $ False $\quad 0 \to BZ(0) = F \quad 1 \to BZ(1) = T \quad 2 \to BZ(2) = T \quad 3 \to BZ(3) = T$

$BZ_1 \subseteq D \qquad BZ_1 = \{1, 2, 3\}$

- Let P be a predicate over domain D for every $x \in D$, happens 1 of 2

  ① $P(x)$ ② $\neg P(x)$

  $x \in P \qquad x \notin P \equiv \neg(x \in P)$

Quantifiers

- Let $P(x)$ be a predicate over domain,

  The proposition

  $\underbrace{\forall}_{\text{"for all"}} \underbrace{x.}_{\text{quantifier}}\ P(x)$ is true

  if $P(x)$ is true for every element in domain D.

- Domain $D = \mathbb{N}$

  $\forall x.\ BZ(x) \leftarrow$ true

  Domain $\mathbb{Z}$

  $\neg(\forall x.\ BZ(x))$

  Define $SZ(x)$ to be true if x less than 0, F ow

  $\forall x.\ (BZ(x) \lor SZ(x) \lor (x=0))$

  For every x bigger than 0 or smaller than 0

- Let $P(x)$ be a predicate over D

  Define

  $\underbrace{\exists}_{\text{"exists"}} x.\ P(x)$ is a prop. that is true if there is (at least one) $x \in D$ s.t. $P(x)$ is True, F ow

  $\exists x.\ (BZ(x)) \qquad \exists x.\ (SZ(x)) \qquad \neg \exists x.\ (BZ(x) \land SZ(x))$

  DOMAIN $\mathbb{Z}$

  $\neg(\forall x.\ BZ(x)) \equiv \exists x.\ \neg(BZ(x))$

  Domain $\mathbb{Z}^-$

  $\neg(\exists x.\ BZ(x)) \equiv \forall x.\ \neg(BZ(x))$

De Morgan Laws for Quantifiers

- Let $P(x)$ be a predicate over domain D then
  $\neg \forall x.\ P(x) \equiv \exists x.\ \neg P(x)$
  $\neg \exists x.\ P(x) \equiv \forall x.\ \neg P(x)$
- BT over $\mathbb{Z} \times \mathbb{Z}$
  $\underbrace{\forall x.\ \neg BT(x, x)}$ = True
  For every x, x is bigger than x is false
- BT w/ domain $\mathbb{Z} \times \mathbb{Z}$
  $\forall x. (\exists y. (BT(x, y)))$ = True
- BT w/ domain $\mathbb{N} \times \mathbb{N}$
  $\forall x. (\exists y.\ BT(x, y))$ = False
- Domain $D = \{3, 5, 7, 9\}$ Domain $D = D_1 \times D_1$
  $\forall x. \exists y. (x + y = 12)$ $\neg \exists x. \forall y. (x + y = 12)$
- $D_2 = \{0, 1, 2\}$
  Domain is $D_2 \times D_2$
  $\exists x. \forall y. (x + y = y)$ $\forall x. \forall y. (x + y < 5) \equiv$ $\forall_{(x, y)}. (x + y < 5)$
  $\exists_{(x, y)} (x + y < 3)$