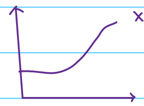
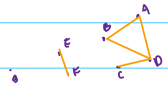


A Graph $G = (V, E)$ is V-vertices and edges - E



$V: \{A, B, C, D, E, F, g\}$

$E: \{\{A, B\}, (A, D), (C, D), (B, D), (E, F)\}$

A Path in graph G is a list

$V_0 - V_1 - V_2 - \cdots V_k$ st $V_i - V_{i+1}$

has an edge ; length of path = k

ex. B - A - D

B - A - D - C

A cycle is a path where $V_0 = V_k$

Cycle length = 0 ; it's a loop

The Degree of vertex v is the # of edges connected to v.

$d(A) = 2$ $d(E) = 1$ $d(g) = 0$

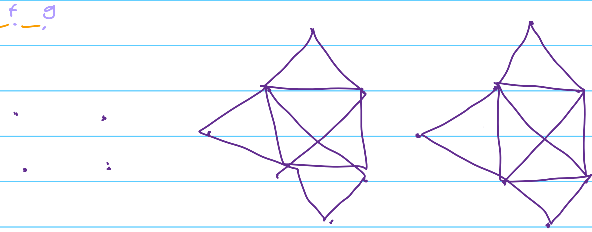
Vertex w/ degree 0 is "isolated"

A grach is "connected" if for every pair of (diff) vertices there is a path

SIMPLE GRAPHS

A simple graph has no loops and no double edges

A tree is a connected graph and has no cycles

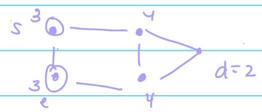


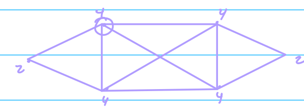
EULER PATHS & CYCLE

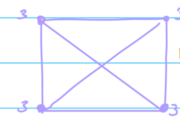
Path – graph that goes over every edge , exactly once

Cycle - Start & end at same point

A graph G has an Euler path iff all degrees are even, except (maybe) 2 verticer







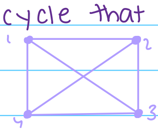
not a Euler path

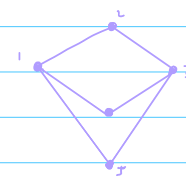
HAMILTONIAN PATHS & CYCLES

Path / cycle that visits every vertex once

ex: 2-1-4-3-2

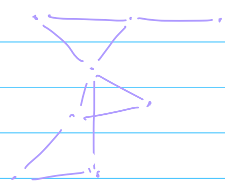
2-4-1-3-2





no hamiltonian path or cycle

Traveling sales person problem: difficult



REPRESENTING GRAPHS

Simple: NO loops

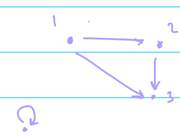
NO parallel edges

ADJACENCY MATRIX

Let R b a relation on {1, 2, 3, 4}

Can tell if a graph is symmetric

	1	2	3	4
1	·	1	1	·
2	·	·	1	·
3	·	·	·	·
4	·	·	·	1



How many reflective relations can we find?

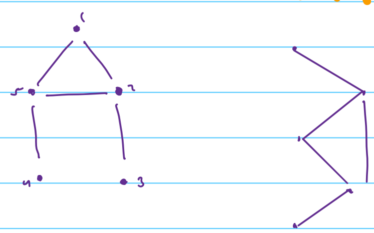
	1	2	3	4
1	1	·	·	·
2	·	1	·	·
3	·	·	1	·
4	·	·	·	1

by multiplication principle:

$\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdots 2}_{12}$

$= \boxed{2^{12}}$

GRAPH ISOMORPHISM

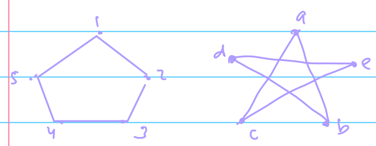


Let $G_1 = (V_1, E_1)$

$G_2 = (V_2, E_2)$ then G_1 is isomorphic to G_2

if we can find a bijection $b \; V_1 \to V_2$ and

$(v_i, v_j) \in E$ iff $(b(v_i), b(v_j)) \in E_2$



1 → a

2 → b

3 → d

4 → e

5 → c

all have edges ✓

SEQUENCES & STRINGS - RECURSIVE

Sequence - a function $A \to Y$ where $A \subseteq \mathbb{Z}$

$\{a\}_{i=0}^{\infty}$

$a_0 = 1$

$a_1 = 2$

$a_3 = 4$

$a_i = 2^i$

$a_3 = 8$

$a_4 = 16$

⋮

String - Sequence $A \to Y$, $A \subseteq \mathbb{Z}$ s.t. A, Y are finite sets

$|A|$ is length of string

alphabet

$Y = \{a, b, c\}$

empty string, length = 0

ex: a, b, ca, cca, λ

$|aaab| = 4$ $\quad ccbaaab = c^2ba^3b$

$\alpha = abcc$ $\quad \beta = bcabb$

Concatination: $\alpha\beta = abccbcabb$

Let X be a finite set.

$X^* = \{$ all strings over $X \}$ $\quad X^+ = X^* \setminus \lambda$

Let t be a string, then w is a substring of t iff there are string α, β

s.t. $t = \alpha w \beta$

$a_i = 2^i \rightarrow a: 2, 4, 8, 16, 32, \ldots$

$b_n = 2n \rightarrow b: 2, 4, 6, 8, 10, \ldots$

$\{a\}_1$ is a subsequence of $\{b\}_0$

$\{S\}_{i=0}^{1}: 0, 3, 6, 9, 12, \ldots$

$\{S'\}_{i=2}^{5}: \quad S'_2 = 6$

$S'_3 = 12$ $\quad$ S' is a subsequence of S

$S'_4 = 300$

$S'_5 = 330$

Let L be a set of Strings over the alphabet $\{a, b\}$

1. $\lambda \in L$
2. if $\alpha \in L$ then $a\alpha a \in L$
3. if $\alpha \in L$ the $b\alpha b \in L$

$L = \{\lambda, aa, bb, abba,$

[say $\alpha = \lambda$, then $\alpha \in L$ so $a\lambda a \in L$ then $aa \in L$] , $aaaa, baaaab, \ldots\}$

RECURSIVE SEQUENCES

$\{a\}_{i=1}^{\infty} \quad a_i = 2i$

$\{f\}_{n=1}^{\infty} \quad f_n = f_{n-1} + f_{n-2}$ for $f_1 = 1$, $f_2 = 1$

$1, 1, 2, 3, 5, 8, 13, 21, \ldots$

Fibonacci sequence

Recursive Formula $\rightarrow$ Explicit Formula

homogenious Linear Recursive formulas (degree 2)

Let $\{S\}_0^{\infty}$ be a sequence described by :

$S_n = C_1 S_{n-1} + C_2 S_{n-2}$ $\quad$ C_1, C_2 are constants

and $S_0 = A$, $S_1 = B$ $\quad$ A, B are constants

Apply algorithm to get Explicit Formula

1. Solve for t : $t^2 - C_1 t - C_2 = 0$

$t = r_1 \quad t_2 = r_2$

case 1 : if $r_1 \neq r_2$

explicit formula: $S_n = br_1^n + dr_2^n$ b, d are constants

case 2: $r_1 = r_2$ then the formula:

$S_n = br_1^n + dnr_1^n$

② use A, B to find b, d