SET THEORIES

$A = \{3, -1, \heartsuit\}$ set

A variable name

member / element

$3 \in A$ = true
is a member of

$4 \notin A$ = false

$A_1 = \{\text{"cat"}, -1, 3\}$

$A_2 = \{3, -1, -1, \text{"cat"}\}$

$B_1 = \{1, 2\}$

$B_2 = \{1, 2, 3\}$

$D = \{1, 2\}$

$E = \{1, 2, 4\}$

$F = \{2, D\}$

$|D| = 2$ cardinality $|F| = 2$

$|D| = |F|$

$|D| \neq |E|$

$A = \{3, 4, \{6, 7\}\}$

$3 \in A$ ✓ $6 \in \{6, 7\}$ ✓

$6 \in A$ ✗

$B = \{6, 7\}$

$B \subseteq A$ ✗ $B \in A$ ✓

$\emptyset \subseteq B$ ✓

set - Collection of objects

$\in$ - is a member of

- boolean statement; evaluates to true

- unique, members are read once

$\subseteq$ - is subset of $B_1 \subseteq B_2$ (true) ; $B_1 \subseteq A_1$ (false)

$X \subseteq Y$ if every element in X, is also an element in Y

$X = Y$ if $X \subseteq Y$ AND $Y \subseteq X$ $A_1 = A_2$ true $B_1 \neq B_2$ true

$X \subset Y$ - is proper subset of $B_1 \subset B_2$

if $X \subseteq Y$ AND $X \neq Y$ $B_1 \not\subset B_1$

FOLLOWING STATEMENTS ARE TRUE:

- $D \subseteq E$
- $D \notin E$
- $1 \in F$
- $D \in E$
- $D \not\subseteq F$
- $\emptyset \subseteq D$

$| \, |$ - cardinality / size of - how many elements

$\{\} = \emptyset$ - empty set

repeating members are only counted once

ex: $|\{3, 3, 1, 1, 3\}| = 2$

$C = \{1, 2\}$ $D = \{'a', 'b', 'c'\}$

$C \times D = \{(1,'a'), (1,'b'), (1,'c'),$
$(2,'a'), (2,'b'), (2,'c')\}$

$D \times C = \{('a',1), ('a',2)$
$('b',1), ('b',2)$
$('c',1), ('c',2)\}$

Let $X = \{0, 3\}$ write

$\wp(X) = \{\varnothing, \{0\}, \{3\}, \{0,3\}\}$

$0 \in \wp(X)$ False

$X \in \wp(X)$ True

CROSS / CARTESIAN PRODUCT

Let X, Y be sets

$X \times Y$ is a set of all pairs (x, y) such that $x \in X$ and $y \in Y$

That is;

$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$

$|X \times Y| = |X| \cdot |Y|$

Let X be a set

$\wp(X)$ is the set of all subsets of X

$\wp$ - power set

$|X| = n$ $\wp(X) = 2^n$

Set Builder Notation

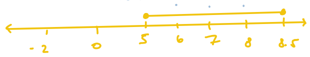
$\{x \underset{\text{such that}}{\mid} x^2 - 1 = 0\} = \{1, -1\}$

$\{a \mid a \text{ is an even number}\} = \{b \mid b \% 2 = 0\}$

$\{\underbrace{3t}_{\text{expression}} \mid \underbrace{t = 3 \text{ or } t = 4 \text{ or } t = -1}_{\text{condition}}\} = \{9, 12, -3\}$

$D \subseteq \mathbb{R}$. $D = \{|x| \; x > 5 \text{ and } x \leq 8.5\}$

$|D| = 3$



$D_1 \subseteq \mathbb{Z}$ $D_1 = \{x | \; x > 5 \text{ and } x \leq 8.5\}$

$|D| = \infty$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$
naturals

$\mathbb{Z} = \{-2, -1, 0, 1, 2, \ldots\}$
integers

$\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z} \text{ and } b \in \mathbb{Z} \text{ and } b \neq 0\}$
rationals

$\mathbb{R}$ = all numbers on the number line
reals

$\mathbb{C}$ = uses imaginary numbers
complex

$\mathbb{Z}^+$ = all positive integers

$\mathbb{Q}$ non-regular = positive rationals

$A = \{1, 2, 3\}$, $B = \{3, 4, 2\}$

$A \cup B = \{1, 2, 3, 4\}$

Let X, Y be sets

$X \cup Y = \{a \mid a \in X \text{ or } a \in Y\}$

all of the elements

$\mathbb{Z} \cup \mathbb{Q} = \mathbb{Q}$

$\mathbb{N} \cup \{1, 2, 3, \frac{1}{2}\} = \mathbb{N} \cup \{\frac{1}{2}\}$

$A = \{x \mid x \in \mathbb{Z} \text{ and } x > -24 \text{ a } x \leq 10$

$= \{-2, -1, 0, 1, 2, \ldots, 10\}$

$B = \{0, 1, 3\}$

$A \cup B = A$

$B \cup \emptyset = B$

Notice if $a \in A$ then $a \in A \cup B$.

$C = \{-3, 11\}$

Let XY be sets

$X \cup Y = \{a \mid a \in X \text{ or } a \in Y\}$

union set

$\mathbb{Z} \cup \mathbb{Q} = \mathbb{Q}$

Notice: if $a \in A$ then $a \in A \cup B$

$B \cup (A \cup C) = A \cup C = (B \cup A) \cup C$

$B_1 = \{0, 1, 3, 17\}$

$B_1 \cup A = A \cup \{17\}$

Intersection

$\mathbb{Z} \cap \mathbb{Q} = \mathbb{Z}$ $\quad$ $A \cap B = B$

$A \cap Q = A$

$|\mathcal{P}(B_1 \cap B)| = 8$

$|\underbrace{|B \cup C|}_{\{0,1,3,-3,4\}} \times B| = 15$

$\boxed{A \cap \emptyset = \emptyset}$

Let X, Y be sets

$X \cap Y = \{a \mid a \in X \text{ and } a \in Y\}$

↑ intersection set

Let x be a set

$\overline{X} = X^c = \{a \mid a \notin X\}$

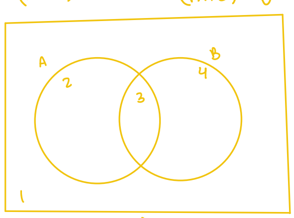
compliment set

$U = \{0, 1, 2, 3, 4 \dots, 10\}$

$A = \{0, 1, 2, 3, 7, 8, 10\}$

$\overline{A} = \{4, 5, 6, 9\}$

$\overline{(A \cup B)}$ $\quad$ $\overline{(A \cap B)}$ $\quad$ U



$A \setminus B = A \setminus (A \cap B)$

$\overline{A} \cup \overline{B} = \overline{(A \cap B)}$ $\quad$ $\overline{A} \cap \overline{B} = \overline{(A \cup B)}$

The universal set $\quad U$

- The set of all objects that are relevant to the problem

$A \cap \overline{B} = A \setminus B = A - B$ $\qquad$ $B \setminus A = B \cap \overline{A}$

difference set

$A_3 = \{0, 1, \dots, 10\}$

$A_4 = \{x \mid x \in \mathbb{Z} \text{ and } x < 4\}$

$A_3 \setminus A_4 = \{4, 5, 6, \dots, 10\}$

$A_4 \setminus A_3 = \{x \mid x\, \mathbb{Z} \text{ and } x \leq -1\}$

$\mathbb{N} \setminus \mathbb{Z} = \emptyset$

$\mathbb{Z} \setminus \mathbb{N}$ = all positive ints = $\mathbb{Z}^- \cup \{0\}$

$B_3 = \{\text{all even integers}\}$

$B_4 = \{x \mid x \in \mathbb{Z} \mid \text{ and } x \geq 2 \text{ and } x < 8\}$

$B_3 \cup B_4 = B_3 \cup \{3, 5, 7\}$

$B_3 \cap B_4 = \{2, 4, 6\}$

$B_3 \setminus B_4$ = all even integers except $\{2, 4, 6\} = B_3 \setminus \{B_3 \cap B_4\}$

$B_4 \setminus B_3 = \{3, 5, 7\}$

Partition

Let S be a (Family) set of sets

$\bigcup_S = \{x \mid x$ is in at least one element i S

$S_1 = \{ \{1,2,3\}, \{0,1\}, \{3,4\}$

$\bigcup_{S_1} = \{0,1,2,3,4\}$

$\bigcap_S = \{x \mid x$ is in every element of $S\}$

$\bigcap_S = \{\} = \emptyset$

A family of sets is Pairwise Disjoint if every two of its elements, has an empty intersection

S_n is NOT pairwise disjoint, b/c ie. $\{1,2,3\}$ and $\{0,1\}$ are not disjoint

Let T be a set $T \neq \emptyset$

A family of sets S is a partition of T if $\emptyset \notin S$ an $\bigcup_S = T$ and S is pairwise disjoint

$S' = \{ \{0,1\}, \{3\}, 3, 0\}$

Let $T = \{0,1,2,3,4\}$

$S_3 = \{ \{ \}, \{ \}, \{ \}\}$

St. $\emptyset \notin S_3$ and $\bigcup_{S_3} = T$ and S_3 is a pair wise disjoint

Solutions:

$T_1 = \{2,0\}$, $T_2 = \{1\}$, $T_3 = \{3,4\}$

$T_1 = \{0,1\}$, $T_2 \{2,3\}$ $T_3 = \{4\}$

$T_1 = \{0,2,1\}$ $T_2 = \{3,4\}$

$T_1 = \{0,1,2,3,4\}$

$\{ \{0,1\}, \{2\}, \{3\}, \{4\}\}$