Sequences & Strings

A sequence is a function
$A \to Y$ s.t. $A \subseteq \mathbb{Z}$

Ex: $S: \{-5, -3, 0, 4, 2\} \to \{a, b, c, d\}$

$S(-5) = S_{-5} = b$

$S_{-3} = c$ $\qquad\qquad S = b, c, c, a, d$

$S_4 = d$

$S_0 = c$

$S_2 = a$

A sequence $d: \mathbb{Z} \to \mathbb{Z}$ $d_i = 2i$

$d_1 = 2$

$d_{-3} = -6$

$d_0 = 0$

$d_5 = 10$

A sequence $S: A \to Y$ is called a "String" if A, Y are finite sets.
Y is called the "alphabeit"

$S: \{0, 1, 2, 5, 7\} \to \{$alpha numeric characters$\}$

$S_0 = h$ $\qquad\qquad S$: hello

$S_1 = e$

$S_2 = l$

$S_5 = l$

$S_7 = o$

Sequence $d: \mathbb{Z} \to \mathbb{Z}$ $d = 2i$

SUM: $\sum_{i=-2}^{2} d_i \cdot 3 = (d_{-2} \cdot 3) + (d_{-1} \cdot 3) + (d_0 \cdot 3) + (d_1 \cdot 3) + (d_2 \cdot 3)$
$= -12 - 6 + 0 + 6 + 12 = 0$

$\prod_{i=1}^{3} \frac{d_i}{2} = \frac{d_1}{2} \cdot \frac{d_2}{2} \cdot \frac{d_3}{2} = 1 \cdot 2 \cdot 3 = 6$

Let $n \in \mathbb{N}$, $n \geq 1$

$\sum_{i=1}^{n} (2i - 1) = 1 + 3 + 5 + \dots + (2n-1) = n^2$

More Integers

The quotient - Remainder Theorem

Let $d>0$, n integers, there exist $q, r \in \mathbb{Z}$ s.t.

$n = q \cdot d + r$ and $0 \le r < d$ q, u, r are unique

$5 = \underset{q}{1} \cdot 3 + \underset{r}{2}$ $-5 = \underset{q}{-2} \cdot 3 + \underset{r}{1}$

$7 = \underset{q}{0} \cdot 9 + \underset{r}{7}$ $-12 = \underset{q}{-4} \cdot 3 + \underset{r}{0}$

$0 = \underset{q}{0} \cdot 3 + \underset{r}{0}$ $-2 = \underset{q}{-1} \cdot 3 + \underset{r}{1}$

Prime factorization:

10	5
2	2
1	

28	7
4	2
2	2
1	

28	2
14	7
2	2
1	

Let m be an integer

$m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdot \ldots \cdot p_k^{a_k}$

where $p_1, p_2, p_3 \ldots$ are prime numbers

and $a_1, a_2, a_3 \ldots$ are integers $a_i \ge 1$

if $p_1 < p_2 < p_3 \ldots < p_k$

then this factorization is unique

$28 = 2^2 \cdot 7$ $\underset{m}{36} = 2 \cdot 2 \cdot 3 \cdot 3$ $\underset{n}{150} = 2 \cdot 3 \cdot 5 \cdot 5$

Greatest Common Divisor

Let m, n be $\mathbb{N}$, the $\gcd(m,n) = g$ are all ints st.

$g \mid m \wedge g \mid n$ and g is the largest int that has this property

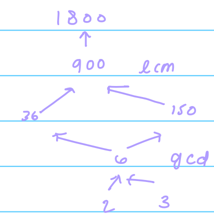
$\gcd(36, 150) = 2^1 \cdot 3^1 \cdot 5^0 = 6$

$36 = 2^2 \cdot 3^2 \cdot 5^0$ $150 = 2^1 \cdot 3^1 \cdot 5^2$

Least Common Multiple

Let m, n be $\mathbb{N}$, the $lcm(m,n) = \ell$ are all ints st.
$m | \ell$ and ℓ is the smallest int that has this property

$lcm(36, 150) = 2^2 \cdot 3^2 \cdot 5^2 = 4 \cdot 9 \cdot 25 = 900$



$36 = 2^2 \cdot 3^2 \cdot 5^0 \quad 150 = 2^1 \cdot 3^1 \cdot 5^2$

$1426 = 2 \cdot 713 = 2 \cdot 23 \cdot 31$ is 713 prime?

Let n be an int, then if $xy = n$ (x, y integers) then ($x \leq \sqrt{n}$ or $y \leq \sqrt{n}$).

proof by contradiction

The Euclidean Algorithm for gcd

Lemma: Let $a, b \in \mathbb{N}$ ($a > b$), then $gcd(a,b) = gcd(b,r)$ where $r = a \% b$

Apply lemma till $r = 0$, then return b

$gcd(36, 6) = gcd(6, 0) = 6$

36
72
108
144
150

$gcd(315, 825) = gcd(315, 195) = gcd(195, 120)$
$= gcd(120, 75) = gcd(75, 45) = gcd(45, 30)$
$= gcd(30, 15) = gcd(15, 0)$

Why does it work?

Say $d \in \mathbb{N}$, show $[d|a, d|b] \longleftrightarrow [d|b \wedge d|r]$

say $d|a \wedge d|b$

by q-r thm. there exists q, r integers $0 \leq r < d$

s.t. $a = q \cdot b + r$ so:

$d \cdot k = q \cdot (dj) + r$, so $dk - q(dj) = r \Rightarrow d(k - qj) = r$ by defn. of "|" we say $d|r$

Notice: let m, n be ints

$gcd(m,n) \cdot lcm(m,n) = m \cdot n$

Ch 5.2

Decimal

10_{10}

1×0

1×10

37049

9×10^0

4×10^1

0×10^2

7×10^3

3×10^4

Binary

0, 1

$1\,0_2$

0×2^0

1×2^1

$10110_2 = 16 + 4 + 2 = 22$

0×2^0

1×2^1 2

1×2^2 4

0×2^3

1×2^4 16

$26_{10} = \underline{11010}_2$

R

$26 \% 2 = 0$ | $26/2 = 13$

$13 \% 2 = 1$ | $(13-1)/2 = 6$

$6 \% 2 = 0$ | $6/2 = 3$

$3 \% 2 = 1$ | $2/2 = 1$

$1 \% 2 = 1$ | $1 - 1/2 = 0$

Hexadecimal (16)

0 1 2 3 4 5 6 7 8 9

A	B	C	D	E	F
10	11	12	13	14	15

$1C4_{16} = \underline{256 + 192 + 4}_{10} = 452_{10}$

$4 \times 16^0 = 4$

$12 \times 16^1 = 192$

$1 \times 16^2 = 256$

$$\begin{array}{r} 1C5_{16} \\ +\ 24B_{16} \\ \hline 410_{16} \end{array}$$

$$\begin{array}{r} 234_8 \\ +\ 576_8 \\ \hline 1032 \end{array}$$