

REVIEW

CH 1

- 1.1 - Set of operations (thm 1.1.22)
- 1.2 - 1.3 - propositions, truth tables, DNF, CNF
- 1.4 - 1.6 - predicates & quantifiers

CH 2

- 2.1 - 2.3 - Basic Proof techniques
- 2.4 - Induction
- 2.5 - Q-R thm.

CH 3

- 3.1 - functions (inj., surj., bij., inverse, $f \circ g$)
- 3.2 - Sequences, Strings (Σ^*), recursive sequences
- 3.3 - 3.4 - Binary Relations (Reflexive, transitive, anti-sym., equivalence relation, equivalence classes, partial / total orders)
- 3.5 - Matrices (defn. only)

CH 5

- 5.1, 5.3 - divisors, gcd, lcm, Euclidean algo.
- 5.2 - Counting bases

CH 6

- 6.1 - Multiplication Principle, Addition Principle, I-E
- 6.2 - permutation, choose operator
- 6.7 - Binomial Thm, Pascal triangle
- 6.8 - Pidgeon Hole Principle

CH 8

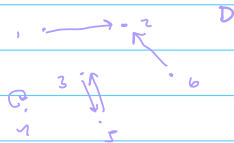
- 8.1 - Graph Basics (Vertices, Edges, undirected / directed, paths, cycles, degree)
- 8.2 - Euler cycles / paths,
- 8.3 - Hamiltonian cycles / paths
- 8.6 - Graph Isomorphism

CH 7

- 7.2 - solving recurrence relations
 - a) Algo ($S_n = C, S_{n-1}, C, S_{1-2}, S_0 = A_0, S_1 = A_1$)
 - b) repeated iteration + verify by induction

BINARY RELATIONS

Binary relation R on set D is $R \subseteq D \times D$



$$R = \{(1,2), (2,3), (3,4), (4,5), (5,6), (3,3)\}$$

Let D_1 be a set, R_1 relation on D_1

R is reflexive if $\forall x. xRx$ ("all loops")

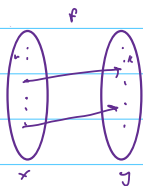
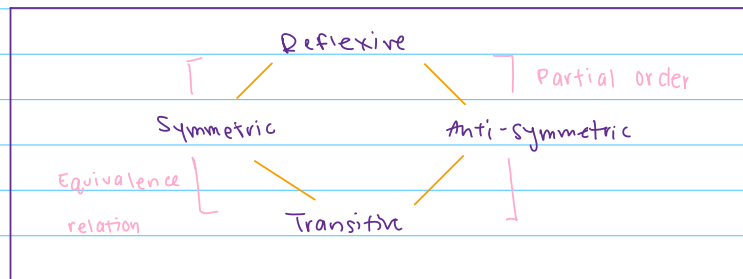
R is Symmetric if $\forall x, y \in D. (xRy) \rightarrow (yRx)$

R is Anti-Symmetric $\forall x, y \in D [(xRy) \wedge (yRx)] \rightarrow x=y$ ("no edges")

R is Transitive $\forall x, y, z \in D. [(xRy) \wedge (yRz)] \rightarrow (xRz)$

Equivalence Relation - Ref., Sym., Transitive

Partial / Total order - Ref, anti-sym, Transitive



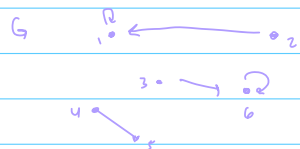
$$|X| = n \quad |Y| = k \quad n > k \quad ; \quad f - \text{not-injective}$$

GRAPH ISOMORPHISM

Let $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$

$G_1 \cong G_2$ iff there is a bijection $b: V_1 \rightarrow V_2$ s.t.

$$(v_i, v_j) \in E_1 \Leftrightarrow (b(v_i), b(v_j)) \in E_2$$



Find isomorphism b from G to itself $\forall x, b(x) \neq x$

look for bijection $\{1, 2, 3, 4, 5, 6\} \rightarrow \{1, 2, 3, 4, 5, 6\}$



s.t. $\forall x, b(x) \neq x$ and it is an isomorphism



1	2	3	4	5	6
6	3	2	5	4	1

RECURRENCE RELATIONS

Solve $a_n = 6a_{n-1} - 9a_{n-2}$

$a_0 = a_1 = 1$ $1, 1, -3, -27, -135, \dots$

① Solve: $t^2 - 6t + 9 = 0$ $r_1 = r_2 = 3$
 $(t - 3)^2 = 0$

② General Solution: $a_n = B3^n + Dn3^n$

③ $n=0$

$$a_0 = B3^0 + D(0)3^0 = B = 1$$

$n=1$

$$a_1 = B \cdot 3 + D \cdot 1 \cdot 3 = 3B + 3D = 1$$

$$D = -2/3$$

Solution: $a_n = 3^n - \frac{2}{3}n \cdot 3^n = 3^n - 2n3^{n-1}$

$$a_0 = 1 - 0 = 1$$

$$a_1 = 3 - 2(1) = 1$$

$$a_2 = 3^2 - 2 \cdot 2 \cdot 3 = 9 - 12 = -3$$

$$a_3 = 3^3 - 2 \cdot 3 \cdot 3^2 = 27 - 54 = -27$$

$$a_4 = 3^4 - 2 \cdot 4 \cdot 3^3 = 81 - 72 = 9$$

matches sequence ✓