- definition
- examples
- notes

SYNTAX

Predicates $BZ(x)$ $SZ(x)$ over domain $\mathbb{Z}$, $BT(x,y)$ over $\mathbb{Z} \times \mathbb{Z}$ is true if $x > y$, F

- $\neg(\forall x.\ SZ(x)) \equiv \exists x.\ \neg SZ(x)$

  $\neg \exists x.\ (BZ(x) \wedge SZ(x)) \equiv \forall x.\ \neg(BZ(x) \wedge SZ(x)) \equiv \forall x.\ [\neg BZ(x) \vee \neg SZ(x)]$

- $\forall x.\ (\exists y.\ BT(x,y))$

  de morgan;
  switch x and y to move negatio

  $\neg \exists y.\ \forall x.\ (BT(x,y)) \equiv \forall y.\ \neg \forall x.\ (BT(y,x)) = \forall y.\ (\exists x.\ [\neg BT(y,x)])$

- $\neg \forall x.\ \exists y.\ (BT(x,y) \to SZ(x)) \equiv$

  $\exists x.\ \neg \exists y.\ \neg(BT(x,y)) \to SZ(x)) \equiv$

  $\exists x.\ \forall y.\ \neg(BT(x,y) \to SZ(x)) \equiv$

  $\exists x.\ \forall y.\ (BT(x,y) \wedge SZ(x))$

- $\neg \forall x.\ (BZ(x) \vee SZ(x) \vee (x=0)) \equiv$

  $\exists x.\ (\neg BZ(x) \wedge \neg SZ(x) \wedge \neg(x=0))$

PROOFS

Data $\longrightarrow$ conclusions

- Let n be an integer. If there exist an integer k such that $2k=n$. Then n is called <u>EVEN</u>

  Let $n \in \mathbb{Z}$ n is even if $\exists k.\ (k \in \mathbb{Z} \wedge 2k=n)$

- Prove: 6 is even

  Proof: 2 $3=6$ $3 \in \mathbb{Z}$ so $\exists k.\ (k \in \mathbb{Z} \wedge 2k=n)$

- Prove: 7 is <u>NOT</u> even

  proof: Show: $\neg \exists k.\ (k \in \mathbb{Z} \wedge 2k=7) \equiv$

  $\forall k.\ (k \notin \mathbb{Z} \vee 2k \neq 7)$

RULES OF INFERENCE FOR $\forall$, $\exists$

E·1:

$\exists x.\ P(x)$

$\therefore$ There is some d in the domain s.t. P(d)

E·2:

There is some $d \in D$ st $P(d)$

$\therefore \exists x.\ P(x)$

A–1

$\forall x.\ P(x)$

$\therefore$ P(d) is T for all elements $d \in D$

A-2

P(d) for every element in $d \in D$

$\therefore \forall x.\ P(x)$

A-3

$\forall x.\ P(x)$

$y \in D$

$\therefore P(y)$

- Let m, n be even

Sketch: $\underset{2\cdot4}{8} + \underset{2\cdot2}{4} = \underset{2\cdot6}{12}$ $\quad \underset{2\cdot8}{16} + \underset{2\cdot2}{4} = \underset{2\cdot10}{20}$

$\underset{2\cdot2}{2} + \underset{2\cdot3}{6} = \underset{2\cdot4}{8}$

Prove: m+n is even

Data: m is even
$\exists k.\ (2k = m)$
k is int
n is even
$\exists k.\ (2k = n)$
k is int

Conclusion: m+n is even

$\exists b.\ (2b = m+n)$; b is int

Proof: m is even, therefore there is some int k such that $2k = m$
n is even, therefore there is some int such that $2a = n$
Let us take $b = k + a$
The $2 \cdot b = 2(k+a) = 2k + 2a = m+n$
so $2b = m+n$ and b is an int b/c it is the sum of 2 ints