ECE 3100 - PSet 6

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1 Answers

1. Bills w/ Gollum

   (a) Let rv X be the number of dollar bills in the envelope X. Let rv Y be the number of dollar bills in the envelope Y.

   $$p_X(7) = \sum_{z \in Z} p_Z(z) p_{X|Z}(x \mid z)$$
   $$= 0 + \cdots + 0 + p_Z(21) p_{X|Z}(7 \mid 21) + 0 + \cdots + 0$$
   $$= p(1-p)^{(21/3)-1}(0.5)$$
   $$= 0.5p(1-p)^6$$

   (b) Prob $X = 6$.

   $$p_X(6) = \sum_{z \in Z} p_Z(z) p_{X|Z}(x \mid z)$$
   $$= p_Z(9) p_{X|Z}(6 \mid 9) + p_Z(18) p_{X|Z}(6 \mid 18)$$
   $$= p(1-p)^{(9/3)-1}(0.5) + p(1-p)^{(18/3)-1}(0.5)$$
   $$= 0.5p(1-p)^2 + 0.5p(1-p)^5$$

   (c) If Gollum finds an odd number of bills in X, then he should take the contents of Y instead. The evelope with the two-thirds of the bills will always be even, so if the # of bills in X is odd, we know Y has more bills.

   (d)
   $$p_{Y|X}(2m \mid m) = \frac{p_{X,Y}(m, 2m)}{p_X(m)}$$

   (e)

2. Three-sided Dice

   (a) Find the joint pmf $p_{X,Y}(x, y)$.

   $$p_{X,Y}(x, y) = \mathbb{P}(\{X = x \cap \{Y = y\})$$
   $$= 1/9 \forall x, y \in \{1, 2, 3\} \times \{1, 2, 3\}$$

   (b) PMF of Z.

   $$p_Z(k) = \begin{cases} 1/9 & k = 2, 6 \\ 2/9 & k = 3, 5 \\ 1/3 & k = 4 \end{cases}$$

(c) Expectation and Variance of Z

$$\mathbb{E}(Z) = \sum_{x \in X} \sum_{y \in Y} g(x,y) p_{X,Y}(x,y)$$
$$= 2(1/9) + 3(2/9) + 4(1/3) + 5(2/9) + 6(1/9)$$
$$= 4$$

3. RV Joint PMF

(a) Find the constant c.

$$1 = c(1)(1) + c(3)(1) + c(4)(1) + c(1)(4) + c(3)(4) + c(4)(4) c = 1/40 = 0.025$$

(b) Find $\mathbb{P}(\{Y = 2\})$.

$$\mathbb{P}(\{Y = 2\}) = \sum_x p_{X,Y}(x,2) = 4/40 + 12/40 + 16/40 = 32/40 = 0.8$$

(c) Find marginal pmfs $p_X(x)$, $p_Y(y)$.

$$p_X(x) = \sum_y p_{X,Y}(x,y) = \begin{cases} 5/40 (0.125) & x = 1 \\ 15/40 (0.375) & x = 3 \\ 20/40 (0.5) & x = 4 \end{cases}$$

$$p_Y(y) = \sum_x p_{X,Y}(x,y) = \begin{cases} 8/40 (0.2) & y = 1 \\ 32/40 (0.8) & y = 2 \end{cases}$$

(d) Find $\mathbb{E}(X), \mathbb{E}(Y), \mathbb{E}(XY)$

$$\mathbb{E}(X) = \sum_x x p_X(x)$$
$$= (1)(5/40) + 3(15/40) + 4(20/40)$$
$$= 2.75$$

$$\mathbb{E}(Y) = \sum_y y p_Y(y)$$
$$= (1)(8/40) + (2)(32/40)$$
$$= 1.8$$

$$\mathbb{E}(XY) = \sum_x \sum_y (xy) p_{X,Y}(x,y)$$
$$= 1(1/40) + 3(3/40) + 4(4/40) + 2(4/40) + 6(12/40) + 8(16/40)$$
$$= 5.85$$

(e) Find $Var(X)$ and $Var(Y)$.