Pdf, Cdf Def

$$\mathbb{P}(a \le X \le b) = \int_a^b f_X(x)dx$$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt \ \underline{\textbf{cont}}$$

$$f_X(x) = \frac{d}{dx}F_X(x)$$

$$F_X(x) = \mathbb{P}(\{X \le x\}) \ \underline{\textbf{discr}}$$

$$\mathbb{P}(\{X \in V\}) = \int_V f_X(x)dx$$

$$F_X(x) = \sum_{\{x_k | x_k \le x\}} p_X(x_k)$$

$$p_X(x_k) = F_X(x_k) - F_X(x_{k-1})$$

Mean and Variance: Continuous

$$\mathbb{E}(X) = \int_{-\infty}^{+\infty} x f_X(x)dx$$

$$Var(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$$

Conditional Prob Def

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

Types of continuous rvs

- **X uniform on [a,b]:**

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{when } x \in [a,b] \\ 0 & \text{else} \end{cases}$$

$$F_X(x) = \begin{cases} 0 & \text{when } x < a \\ \frac{x-a}{b-a} & \text{when } a \le x \le b \\ 1 & \text{when } x > b \end{cases}$$

$$\mathbb{E}[X] = \frac{b+a}{2} \qquad Var(X) = \frac{(b-a)^2}{12}$$

- **Gaussian rv:**

$$f_X(x) = \frac{1}{\sqrt{2x\sigma^2}} \exp\Big(-\frac{(x-\mu)^2}{2\sigma^2}\Big)$$

$$F_X(x) = \frac{1}{\sqrt{2x\sigma^2}} \int_{-\infty}^{x} \exp\Big(-\frac{(t-\mu)^2}{2\sigma^2}\Big)dt$$

$$\mathbb{E}(X) = \mu \qquad Var(X) = \sigma^2$$

- **X exponential(λ):**

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{when } x \ge 0 \quad \forall \lambda > 0 \\ 0 & \text{when } x < 0 \end{cases}$$

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} & \text{when } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}[X] = \frac{1}{\lambda} \qquad Var(X) = \frac{1}{\lambda^2}$$

- **standard normal:**

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad \therefore \mathbb{E}(X) = 0,\ Var = 1$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}dt$$

$$\mathbb{P}(\{X > x\}) = \mathbb{P}(\{\sigma + Y + M > x\}) = \mathbb{P}\Big(Y > \frac{x-M}{\sigma}\Big)$$

Joint Pdf

$$\mathbb{P}(\{(X,Y) \in V\}) = \iint_{-V} f_{X,Y}(x,y)dxdy$$

$$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)dy$$

$$\mathbb{E}[g(X,Y)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(X,Y)f_{X,Y}(x,y)dxdy$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)dx$$

$$F_{X,Y}(x,y) = \mathbb{P}(X \le x, Y \le y)$$

$$f_{X,Y}(x,y) = \frac{\delta F_{X,Y}}{\delta x \delta y}(x,y)$$

Indpendence: Continuous

- $f_{X,Y}(x,y) = f_X(x)f_Y(y) \quad \forall x,y$
- $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y)$
- $\mathbb{E}(g(X)h(Y)) = \mathbb{E}(g(X))\mathbb{E}(h(Y))$
- $Var(X+Y) = Var(X) + Var(Y)$

Conditioning on Event

$$\mathbb{P}(\{X \in V\} \mid A) = \int_V f_{X|A}(x)dx$$

$$f_{X|A}(x) = \begin{cases} \frac{f_X(x)}{\mathbb{P}(\{X \in W\})} & \text{when } X \in W \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}(X \mid A) = \int_{-\infty}^{+\infty} x f_{X|A}(x)dx$$

Conditioning on rv

$$f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_X(x) = \int_{-\infty}^{+\infty} f_Y(y) f_{X|Y}(x \mid y)dy$$

$$\mathbb{E}[X \mid Y = y] = \int_{-\infty}^{+\infty} x f_{X|Y}(x \mid y)dx$$