ECE 3100 - PSet 3

Stephen Chin

February 20

Answers

1. Show that $\mathbb{P}(A_1^c \cup A_2^c \cup \cdots \cup A_n^c) = 1 - \mathbb{P}(A_1)\mathbb{P}(A_2)\ldots\mathbb{P}(A_n)$.

   Well, if we take the complement of $A_1^c \cup A_2^c \cup \cdots \cup A_n^c$, then we are left with the set of all outcomes that are not in the complement of any of the events A_1 to A_n. Thus, we are left with $A_1 \cap A_2 \cap \cdots \cap A_n$. By definition of independence, we can rewrite this as a product of all the events A_1 through A_n, bringing us to our goal.

$$
\begin{aligned}
\mathbb{P}(A_1^c \cup A_2^c \cup \cdots \cup A_n^c) &= 1 - \mathbb{P}((A_1^c \cup A_2^c \cup \cdots \cup A_n^c)^c) \\
&= 1 - \mathbb{P}(A_1 \cap A_2 \cap \cdots \cap A_n) \\
&= 1 - \mathbb{P}(A_1)\mathbb{P}(A_2)\ldots\mathbb{P}(A_n)
\end{aligned}
$$

2. Rangers and Canadiens

   Let us define the events $A =$ event that the Canadiens win and $B =$ the event that the rangers win. We know that there can only be one winner, so $\mathbb{P}(A \cap B) = 0$.

   Not enough time to complete question.

3. Communication System Transmissions

   (a) Let $A_i =$ the event that the i-th transistor sends a message. $\mathbb{P}(\text{a successful transmission}) = p(1-p)^{n-1}$.

   (b) The derivative of $p(1-p)^{n-1}$ is $(1-p)^{(-2+n)}(1-np)$, which has zeros at the values 1 and $1/n$. Because the range of possible values of p is $[0, 1]$, we also check the value of p at 0. At $p = 1$ and $p = 0$, the probability of a successful transmission is 0. At $p = 1/n$, the probability of a successful trasmission is $\frac{1}{n}(1 - \frac{1}{n})^{n-1}$. This is the relative maximum for p on the interval $[0, 1]$.

   (c) The $\lim_{x\to\infty} \frac{1}{n}(1-\frac{1}{n})^{n-1} = 0$. This makes sense, because as we get more and more transmitters in our system, we are more likely to have two transmitters that send their transmissions at the same time. The more tranmitters we have, the less likely we send out a successful transmission.

   (d) Not enough time to complete question.

4. Communication System w/ Time Slots

   (a) The probability that a successful transmission is sent within k time slots is equal to one minus the probability that no successful transmissions are sent within k time slots. Let $F = \{\text{the event that no successful transmissions are sent in the first } k \text{ time slots}\}$. $\mathbb{P}(F) = k(1 - p(1-p)^{n-1})$, because we would need to have no successful transmissions sent k times.

   (b) Not enough time to complete question.

(c) Not enough time to complete question.
(d) Not enough time to complete question.

5. Banking Password

   Not enough time to complete question.