ECE 3100 - Functions, Formulas, and Definitions

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1 Pre - Prelim 1

1.1 Lecture 1 - What is Probability?

Probability is a way of mathematically modelling situations involving uncertainty with the goal of making predications decisions and models. Probability can be understood in many ways, such as:

1. Frequency of Occurence: Or percentage of successes in a moderately large number of similar situations.

2. Subjective belief: Or ceratinty based on other understood facts about a claim.

For our Probability Models, we define the set of all outcomes to be Ω, better known as the **sample space** of an experiment. All subsets of Ω are called **events**. These are both sets and can be understood using default set notation.

1.2 Lecture 2 - Probability Law

Given Ω chosen, a **probability law** on Ω is a mapping $\mathbb{P}$ that assings a number for every event such that:

$$\boxed{\begin{array}{ll} \mathbb{P}(A) \geq 0 & \text{for every event A} \\ \mathbb{P}(\Omega) = 1 & \text{(normalization)} \end{array}} \tag{Kolmogorov's Axioms}$$

Additivity rules:

- If $A \cap B = \varnothing$, (A, B) events, then:

$$\boxed{\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)} \tag{1}$$

- If events $A_1, A_2, \ldots$ are all disjoint, then:

$$\boxed{\mathbb{P}(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mathbb{P}(A_n)} \tag{2}$$

By these rules, we can surmise that $\boxed{\mathbb{P}(\varnothing) = 0}$.
For any events A, B:

$$\boxed{\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)} \tag{Event Union}$$

When we have a probability law on a finite Ω with all outcomes equally likely (i.e. $\mathbb{P}(\{s\}) = 1/size(\Omega)$), we call this probability law $\mathbb{P}$ a **(discrete uniform probability law**.

1.3 Lecture 3 - Conditional Prob & Product Rule

Conditional Probability is defined $\mathbb{P}(A \mid B) =$ "Probability of A given B". It is understood as the likelyhood that event A occurs, given that B also occurs.

$$\boxed{\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}} \qquad \text{(Conditional Probability Definition)}$$