**1.** Let

$$x(t) = 7 - 3\cos\left(880\pi t - \frac{\pi}{13}\right) + 19\cos\left(2640\pi t + \frac{\pi}{7}\right) .$$

(a) $x(t)$ turns out to be periodic. What are its fundamental period $T_o$, fundamental frequency $f_o$, and fundamental angular frequency $\omega_o$?

(b) Find $N$ and $a_k$, $-N \le k \le N$, so that

$$x(t) = \sum_{k=-N}^{N} a_k e^{jk2\pi f_o t} .$$

(c) Graph $x(t)$'s spectrum (use $f$ as the horizontal axis and draw stems proportional to the magnitudes of the $a_k$ labeled with the values of the $a_k$).

(d) One of the following signals is periodic and one is not. Explain why the non-periodic one isn't periodic and for the periodic one repeat parts (a) through (c) of the problem with that signal in place of $x(t)$.

$$y_1(t) = x(t) + \cos\left(330\pi t + \frac{\pi}{3}\right)$$
$$y_2(t) = x(t) + \cos\left(330t + \frac{\pi}{3}\right)$$

**2.** A signal $x(t)$ has spectrum shown in Figure 1.

(a) Find $L$ and nonzero numbers $a_l$ and $f_l$ for $-L \le l \le l$, so that $f_{-l} = -f_l$ and

$$x(t) = \sum_{l=-L}^{L} a_l e^{j2\pi f_l t} .$$

(b) Write $x(t)$ as a constant plus a sum of phase-shifted cosines.

(c) As it happens, $x(t)$ is periodic. Find its fundamental frequency $f_o$ and find the Fourier series for $x(t)$ in the form

$$x(t) = \sum_{k=-N}^{N} a_k e^{jk2\pi f_o t} .$$

You can find all the $a_k$ without integrating, and some will be zero.

**3.** Problem P-3.26 in the textbook.

**4.** Define $x(t)$ by

$$x(t) = \left(13 - 7\cos\left(2\pi f_o t + \frac{\pi}{7}\right)\right)\cos(2\pi f_c t) ,$$

where $f_c >> f_o > 0$.

(a) Write $x(t)$ as the sum of three phase-shifted cosines.

(b) Graph the (two-sided) spectrum of $x(t)$ using $f$ for the horizontal axis in the usual way.

**5.** Let $x(t)$ be the periodic signal with fundamental period 5 and specification

$$x(t) = \begin{cases} 3 & 0 \le t < 4 \\ 0 & 4 \le t < 5 . \end{cases}$$

(a) Graph $x(t)$ vs. $t$ over the range $-20 \leq t \leq 20$.
(b) Find $a_0$, the "$k = 0$ Fourier coefficient" of $x(t)$.
(c) Find a formula in terms of $k$ for the $k$th Fourier coefficient $a_k$ of $x(t)$, i.e. simplify

$$a_k = \frac{1}{T_o} \int_0^{T_o} x(t) e^{-jk2\pi f_o t} dt \ ,$$

where $T_o$ is the fundamental period of $x(t)$ and $f_o$ is the fundamental frequency of $x(t)$.
(c) The signal $y(t) = x(t) - 37$ has the same fundamental period as $x(t)$. Find all of $y(t)$'s Fourier coefficients. (You've already done all the integration you need to do.)

**6.** Sometimes we cite things that, while true and plausibly so, are maybe not totally obviously true. Here's one of those: if $f(t)$ is a complex-valued function of $t$, then

$$\left( \int_a^b f(t) dt \right)^* = \int_a^b (f(t))^* dt \ ;$$

that is, the conjugate of an integral is the integral of the conjugate. Let's prove this in steps.

(a) Let

$$I = \int_a^b f(t) dt \ .$$

Write

$$f(t) = \text{Re}\{f(t)\} + j\text{Im}(f(t))$$

and integrate that from $a$ to $b$. From this you'll discover $\text{Re}\{I\}$ and $\text{Im}\{I\}$.
(b) Find $I^*$ using the result of (a).
(c) Write

$$(f(t))^* = \text{Re}\{f(t)\} - j\text{Im}(f(t))$$

and integrate that from $a$ to $b$. Compare with your answer to (b). There, you're done.

**7.** Use the previous problem to reassure yourself that the Fourier coefficients $a_k$ of a real-valued signal $x(t)$ do indeed have the conjugate-symmetric property we've noted in class, namely

$$a_{-k} = (a_k)^* \quad \text{for all} \quad k \ .$$