1. In each case, find $A > 0$, $f_o > 0$, and $\phi \in (-\pi, \pi]$ so that $x(t) = A\cos(2\pi f_o t + \phi)$ for all t.

(a) $x(t) = -13\cos(7\pi t + (17\pi/3))$ for all t.
(b) $x(t) = 7\sin(26\pi t - (7\pi/3))$ for all t.
(c) You see from a graph of $x(t)$ that $x(t)$'s negative peak values are all -19; that $x(t)$'s first positive peak to the right of $t = 0$ occurs at $t = 3$; and that the first negative peak to the right of that peak occurs at $t = 6.5$.

2. Use one of Euler's Formulas to show that

$$(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta)$$

when θ is real. Then find $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)^{60}$.

3. Find the rectangular and polar forms of the following complex numbers.

(a) $5e^{j\pi/2} - 7e^{j\pi/3}$
(b) $(3 - j4)^4$
(c) $(3 - j4)^{-1}$
(d) $z_o^2 + 2z_o + 1$, where $z_o = -1 + 7j$

4. Suppose $f_o > 0$ is given. Use phasors to find $A > 0$ and $\phi \in (-\pi, \pi]$ so that $x(t) = A\cos(2\pi f_o t + \phi)$ for all t when

$$x(t) = 7\cos(2\pi f_o t) - 7\cos(2\pi f_o t + 2\pi/3) - 7\cos(2\pi f_o t - 2\pi/3) \quad \text{for all } t\ .$$

5. Find all real values of θ for which

$$\mathrm{Re}\left\{(1 - j)e^{j\theta}\right\} = \sqrt{\frac{3}{2}}\ .$$

6. This one comes from the book — it's Problem 2.28 parts (a) through (c). In a mobile radio system (e.g. cell phones), there is one type of degradation that can be modeled easily with sinusoids. This is the case of *multipath fading* caused by reflections of the radio waves interfering destructively at some locations. Suppose that a transmitting tower sends a sinusoidal signal, and a mobile user receives not one but two copies of the transmitted signal: a direct-path transmission and a reflected-path signal (e.g. from a large building). See accompanying diagram.

The received signal is the sum of the two copies, and since they travel different distances they have different time delays. If the transmitted signal is $s(t)$, then the received signal is

$$r(t) = s(t - t_1) + s(t - t_2)\ .$$

In a mobile phone scenario, the distance between the mobile user and the cell tower is always changing. If $d_t = 1000$m, the direct-path distance is

$$d_1 = \sqrt{x^2 + 10^6} \text{ (meters)}\ ,$$

where x is the position of a mobile user who is moving along the x-axis. Assume that the reflector is at $d_r = 55$m, so the reflected-path distance is

$$d_2 = \sqrt{(x-55)^2 + 10^6} + 55 \text{ (meters)} .$$

(a) The amount of delay (in seconds) can be computed for both propagation paths using the fact that the time delay is the distance divided by the speed of light (3×10^8 m/sec). Determine t_1 and t_2 as a function of the mobile's position x.

(b) Assume the transmitted signal is

$$x(t) = \cos(300 \times 10^6 \pi t) .$$

Determine the received signal when $x = 0$. Prove that the received signal is a sinusoid and find its amplitude, phase, and frequency when $x = 0$.

(c) The amplitude of the received signal is a measure of its strength. Show that as the mobile user moves, it is possible to find positions where the signal strength is zero. Find one such location.