Additivity rules:

For any events A, B:

$$\boxed{\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)} \qquad \text{(Event Union)}$$

Conditional Probability

$$\boxed{\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}} \qquad \text{(Conditional Probability Def)}$$

For finite equally likely outcomes, can be written as follows:

$$\boxed{\mathbb{P}(A \mid B) = \frac{\text{number of elements of } A \cup B}{\text{number of elements of } B}} \qquad (1)$$

Product Rule

- With events D_1 to D_n where $D_1 > D_2 > \cdots > D_n$ (D_1 largest, D_n smallest):

$$\boxed{\mathbb{P}(D_n) = \mathbb{P}(D_1)\mathbb{P}(D_2 \mid D_1)\mathbb{P}(D_3 \mid D_2)\ldots\mathbb{P}(D_n \mid D_{n-1}} \qquad \text{(Product Rule 1)}$$

- With events A_1 to A_n with non-empty intersection, let $D_k = A_1 \cap A_2 \cap \cdots \cap A_k$, then $D_1 > D_2 > \cdots > D_n$:

$$\boxed{\mathbb{P}(A_1 \cap \cdots \cap A_n) = \mathbb{P}(A_1)\mathbb{P}(A_2 \mid A_1)\mathbb{P}(A_3 \mid A_1 \cap A_2)\ldots\mathbb{P}(A_n \mid A_1 \cap \ldots)} \qquad \text{(Product Rule 2)}$$

Total Probability Theorem, Bayes, & Independence

$$\boxed{\mathbb{P}(B) = \mathbb{P}(B \mid C_1)\mathbb{P}(C_1) + \mathbb{P}(B \mid C_2)\mathbb{P}(C_2) + \cdots + \mathbb{P}(B \mid C_n)\mathbb{P}(C_n)} \qquad \text{(Total Probability Theorem)}$$

If $A_1, A_2, \ldots, A_n$ events that partition Ω, nonzero $\mathbb{P}(A)$:

$$\boxed{\mathbb{P}(A_k \mid B) = \frac{\mathbb{P}(B \mid A_k)P(A_k)}{\mathbb{P}(B \mid A_1)\mathbb{P}(A_1) + \cdots + \mathbb{P}(B \mid A_n)}} \qquad \text{(Bayes' Law)}$$

$$\boxed{\begin{array}{ll} \mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) & \text{or} \\ \mathbb{P}(A \mid B) = \mathbb{P}(A), \mathbb{P}(B) > 0 & \end{array}} \qquad \text{(Independence Def)}$$

Conditional Dependence

$$\boxed{\begin{array}{ll} \mathbb{P}(A \cap B \mid C) = \mathbb{P}(A \mid C)\mathbb{P}(B \mid C) & \text{or} \\ \mathbb{P}(A \mid B \cap C) = \mathbb{P}(A \mid C), \mathbb{P}(B \cap C) > 0 & \end{array}} \qquad \text{(Conditional Independence Def)}$$

Discrete Random Variables

- **Discrete uniform pmf of interval** $a \le k \le b$, $a, b \in \mathbb{N}$:

$$p_X(k) = \begin{cases} \frac{1}{b-a+1} & \text{when } a \le k \le b \\ 0 & \text{all over } k \end{cases}$$

- Let $p \in [0, 1]$; the **Bernoulli p pmf** be defined by:

$$p_X(k) = \begin{cases} p & \text{when } k = 1 \\ 1-p & \text{when } k = 0 \end{cases}$$

- Given positive integer n, some $p \in [0, 1]$, the **Binomial(n,p) pmf** is defined as:

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad 0 \le k \le n$$

- Given $p \in (0, 1)$ the **geometric pmf** defined by:

$$p_X(k) = p(1-p)^{k-1} \quad \text{for all } 1 \le k \le \infty \text{positive integers}$$

- Poisson(X):

$$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!} \quad 0 \le k \le \infty (k \in \mathbb{N})$$

Expectation, Variance

$$\boxed{\mathbb{E}(X) = \sum_{x \in X} x p_X(x)} \qquad \text{(Expected Value Definition)}$$

$$\boxed{\mathbb{E}(Y) = \sum_{x \in X} g(X) p_X(x)} \qquad \text{(Expected Value Rule)}$$