

TP 6

Risk Analysis (VaR)

The Excel and .mat files provided contain weekly returns for ten stocks. The VaRs and sensitivities have to be computed for a one-week horizon.

Quantile estimation

For one series of your choice, compute the empirical quantile y_q with q equal to 1% and 5%:

$$\Pr(y_t < y_q) = q,$$

where y_t stands for stock returns. Hint: you are asked to identify the cutoff returns such that the probabilities of exceeding those values are 99% and 95% respectively.

VaR estimation

1. Form a portfolio of your choice and compute the first two empirical moments (mean and var-cov matrix) of the multivariate distribution of returns. Also look at the skewness and kurtosis of the portfolio returns.
2. Estimate the VaR under the Gaussian hypothesis at the 5% level.
3. Estimate the VaR for the general case (no parametric assumption but gaussian kernel) at the same level. Hint: use the Excel solver or the Matlab optimization toolbox to compute:

$$\text{VaR}(a, \alpha) = \arg \min \left(\frac{1}{T} \sum_{t=1}^T \Phi \left(\frac{z_t - \text{VaR}(a, \alpha)}{h} \right) - \alpha \right)^2.$$

Does the Gaussian hypothesis under or over estimate the VaR?

4. *Optional*: Try to form three portfolios with kurtosis less than/equal to/more than three and go back to questions 2 and 3.

Sensitivities of VaR

1. Estimate the sensitivities in the gaussian case.
2. Estimate the sensitivities in the general case. Comment.

Comparison with mean-variance optimization

In this question, you can assume that returns are normally distributed.

1. Using the stock series, find the mean-variance efficient portfolio leading to a 5% expected return.
2. Is this portfolio mean-VaR efficient? Hint: search for the portfolio which provides a 5% return and minimizes the VaR at the 5% level.