Financial Econometrics
Prof Olivier Scaillet
TA Gaetan Bakalli

TP 4

CAPM (2 sessions)

Data

You have received an Excel file containing weekly data between the 12/11/1992 and the 14/11/2002. Included are the SP100 price index, the prices of its constituents, the SP500 price index, the returns on the 3-months T-Bill, and the Fama French data on six ME and BM/ME portfolios. The data has been downloaded from Datastream and E. Fama's website. Some values are missing on a small set of the SP100 constituents because the corresponding stocks did not always belong to the index. This is a usual problem with datastream as it does not indicate old compositions of the index.

Test of the CAPM: Time-Series Approach

1. The standard Sharpe-Lintner CAPM assumes the existence of a constant risk-free rate and is written as:

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f).$$

For each stock, estimate the betas on the 12/11/1992–15/8/2001 period, using the SP500 as a market proxy. Issue: how can you form the expectations?

2. A first method to test the CAPM is based on the following time-series regression:

$$z_{i,t} = \alpha_i + \beta_i z_{m,t} + \varepsilon_{i,t},$$

where $z_{i,t} = r_{i,t} - r_{f,t}$, and $\varepsilon_{i,t}$ is a zero mean noise. The null hypothesis $H_0 : \alpha_i = 0$ can then be tested for each asset separately using standard t-statistics.
Using the remaining data, perform this naïve test of the CAPM and check how well the market explains individual stock returns.

The CAPM is actually more restrictive and more sophisticated tests have focused on the following implications:

- All the α_i's are jointly zero.
- The beta completely captures the cross-sectional variation of expected returns.
- The market risk premium is positive.

Test of the CAPM: Cross-Section Approach (Fama MacBeth Procedure)

A second approach for testing the Sharpe-Lintner CAPM [FM73] is based on a two-stage procedure:

i. Estimate the betas β_i using the time-series dimension.

ii. For each period of time, run the following cross-sectional regression:

$$z_{i,t} = \psi_{0,t} + \psi_{1,t}\widehat{\beta}_i + u_{i,t}, \quad i = 1, \ldots, N, \quad u_{i,t} \text{ noise with mean zero.}$$

iii. Estimate ψ_0 and ψ_1 as the average of the cross-sectional regression estimates:

$$\widehat{\psi}_0 = \frac{1}{T}\sum_{t=1}^{T} \widehat{\psi}_{0,t}, \quad \widehat{\psi}_1 = \frac{1}{T}\sum_{t=1}^{T} \widehat{\psi}_{1,t}.$$

If the CAPM is valid, we should have $H_0 : \psi_0 = 0$ and $\psi_1 > 0$. ψ_1 should correspond to the market risk premium.

If the errors in the $\widehat{\beta}_i$ are not positively correlated, the $\widehat{\beta}$'s of portfolios can be much more precise estimates of true β's than the $\widehat{\beta}$'s for individual securities. To reduce the loss of information caused by using portfolios, a wide range of values of portfolio $\widehat{\beta}_p$'s is obtained by forming portfolios on the basis of ranked $\widehat{\beta}_i$'s for the individual securities. Furthermore, in order to reduce another regression problem, the portfolios are formed from ranked $\widehat{\beta}_i$'s computed from data for one period, and a subsequent period is used to recompute the $\widehat{\beta}_p$'s that are then used in the cross-sectional regression.

1. For each stock, estimate the betas on the 12/11/1992–14/11/1996 period.
2. Sort the stocks and split them into 10 portfolios, depending on the size of the betas.
3. Re-estimate the individual betas on the 21/11/1996–16/11/2000 period and compute the betas of your (equally-weighted) portfolios as the average of the individual betas. (Remember that with arithmetic returns, $R_{p,t} = \frac{1}{N}\sum R_{i,t}$.)
4. Run the cross-sectional regression using the remaining part of the data sample and check the validity of the CAPM. As a first try you can test the null hypothesis on the 23/11/2000. Then perform the test on the whole remaining sample by taking the mean of the estimates between the 23/11/2000 and the 14/11/2002, as explained above.

Alternatives to CAPM: The Fama-French (1992) Model

To deal with the shortcomings of the standard CAPM model, Fama and French proposed that, in addition to the excess market return, two additional factors may better explain excess stock returns [FF92]. These two factors are:

1. The difference in returns between "small" and "big" companies, according to their market capitalization.
2. The difference in returns between companies with "high" and "low" book-to-price ratios.

The choice of these factors is based on the empirically observed fact that small capitalization stocks tend to outperform large capitalization stocks, while so-called value stocks (i.e., stocks with high book-to-price ratios) tend to outperform growth stocks (i.e., stocks with low book-to-price ratios). The model is usually written in regression form as:

$$r_{i,t} - r_{f,t} = \alpha_i + b_i(r_{m,t} - r_{f,t}) + s_i SMB_t + h_{i,t} HML_t + \varepsilon_{i,t},$$

where SMB represents the return differential between the small and the big capitalization portfolios, while HML stands for the return difference between the high and low book-to-price portfolios.

Using the CRSP database, the original Fama and French paper took a large sample of stocks and sorted them into five classes according to their market capitalization (ME) and into another overlapping five classes according to their book-to-market ratios (BE/ME). This yields a total of 25 portfolios. In this exercise, we will only use two categories based on market capitalization and three categories based on book-to-price ratios, for a total of 6 portfolios.

1. To compute the SMB returns, construct a time series consisting of the mean return of the small portfolios minus the return of the big portfolios. Similarly, to compute the HML returns, construct another time series consisting of the mean return of the high BE/ME portfolios minus the low BE/ME portfolios.

2. For each of the six portfolios, estimate the parameters of the regression equation above for the 12/11/1992–15/8/2001 period. Is the Fama French 3-factor model better than the CAPM? Pay attention to the R^2 and adjusted R^2 statistics. Now, estimate the model again for the remainder of the sample. Do the regression parameters stay constant in time? What does this mean?

3. Now consider the above regression, without the SMB and HML variables, i.e., the standard CAPM. Estimate the model again for each of the six portfolios and compare the results with those of the previous section. What can you conclude?

References

[FF92] Eugene F. Fama and Kenneth R. French. The cross-section of expected stock returns. *The Journal of Finance*, 47(2):427–465, 1992.

[FM73] Eugene F. Fama and James D. MacBeth. Risk, return and equilibrium: Empirical tests. *The Journal of Political Economy*, 81:607–636, 1973.