ity number measurements are briefly reviewed in Sec. 11.5. A summary table highlighting the suitability of each method for featuring basic textural properties is presented in Sec. 11.6. Bibliographical notes and references are given in Sec. 11.7.

11.1 Granulometry

We first introduce how the concept of granulometry known in materials science for characterising granular materials can be transposed to the field of digital image analysis. We then show its usefulness for texture classification and segmentation. We finally address technical issues related to the generation of discrete line segments and discs satisfying the properties of a granulometric analysis.

11.1.1 Principle

When analysing granular materials, a granulometry is performed by sieving a sample through sieves of increasing mesh size while measuring the mass retained by each sieve. A granulometric curve is a decreasing curve plotting the measured mass for each sieve size. Interestingly, the sieving of materials through a sieve shares all properties of an opening:

- *Anti-extensivity*: what is left in the sieve can only be a subsample of the input sample.
- *Increasingness*: when sieving a subsample of a larger sample, what remains in the sieve is a subsample of what would remain when sieving the whole sample.
- *Idempotence*: sieving a sample twice through the same sieve does not sieve further this sample.

However, because the granulometry process involves a series of sieves, it satisfies a stronger property than the idempotence called the ***absorption property***: what remains after sieving a sample through two sieves of arbitrary size is only driven by the size of the largest sieve. Consequently, the concept of granulometry can be transposed to image data by opening the image with a family of openings of increasing size λ while making sure that the absorption property is satisfied. That is, the composition of any two openings of the family must come down to applying the opening with the largest size:

$$\gamma_{\lambda_i}\gamma_{\lambda_j} = \gamma_{\lambda_j}\gamma_{\lambda_i} = \gamma_{\max(\lambda_i,\lambda_j)},$$

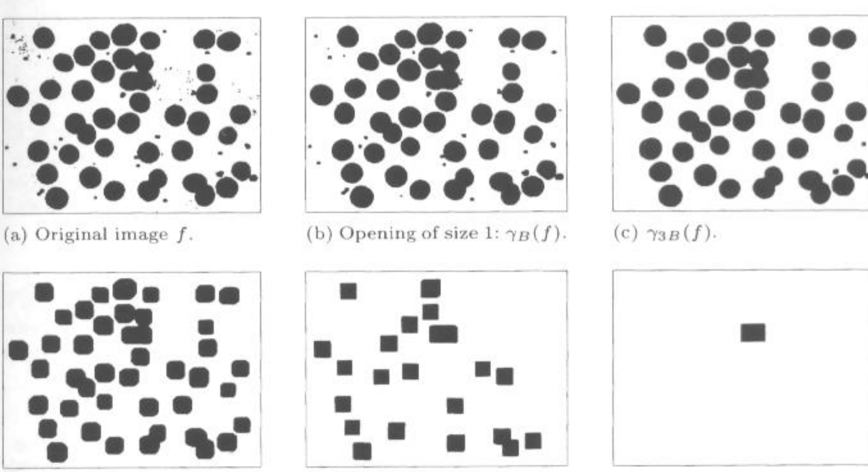
or, equivalently, the openings must satisfy the following ordering: $\gamma_{\lambda_i} \leq \gamma_{\lambda_j}$ if $\lambda_i \geq \lambda_j$. Structuring elements useful for practical applications and satisfying the property of a granulometry are mainly discs and line segments of increasing size. Their digital approximations satisfying the absorption property are discussed in Sec. 11.1.4.

A granulometric curve plots the sum of the pixel values (called *volume* V) of the opened image versus the size of the opening, i.e., $\mathsf{V}(\gamma_\lambda)$ versus λ or $\mathsf{V}(\gamma_\lambda)/\mathsf{V}(\mathsf{id})$ versus λ for a normalised granulometric curve. Unbiased measurements often require us to take the local knowledge property into account. For example, unbiased measurements of the normalised volume of the opening of an image f by a structuring element B can be obtained by restricting the volume measurements to the image definition domain eroded by $2B$:

$$\mathsf{V}\big[\gamma_B(f) \mid \varepsilon_{2B}(\mathcal{D}_f)\big] / \mathsf{V}\big[f \mid \varepsilon_{2B}(\mathcal{D}_f)\big].$$

In practice, granulometric curves are often interpreted by computing their discrete derivative, i.e., the loss of volume between two successive openings versus the size of the opening: $\mathsf{V}(\gamma_\lambda - \gamma_{\lambda+1})$ versus λ. The resulting curve is called *size distribution* or *pattern spectrum* because its peaks indicate the prevailing sizes of the image structures. Accordingly, the initial granulometric curve corresponds in fact to the inverse of the *cumulative* size distribution. In the binary case, the volume measurement comes down to the area measurement. More generally, volume measurements can be substituted by measurements revealing other texture features. For example, one may consider the evolution of the connectivity number or the number of connected components when opening the image by discs of increasing size.

Figure 11.1 depicts a binary granulometry with a family of squares of increasing size. Note that, in contrast to what happens when sieving materials, the size distribution by opening does not require the particles to be disconnected to reveal their actual size. Granulometric curves associated with the granulometry presented in Fig. 11.1 are displayed in Fig. 11.2.



(a) Original image f. (b) Opening of size 1: $\gamma_B(f)$. (c) $\gamma_{3B}(f)$.

(d) $\gamma_{9B}(f)$. (e) $\gamma_{13B}(f)$. (f) $\gamma_{15B}(f)$.

Fig. 11.1. Successive openings of a binary image of blood cells or granulometry (using square SEs of increasing size).

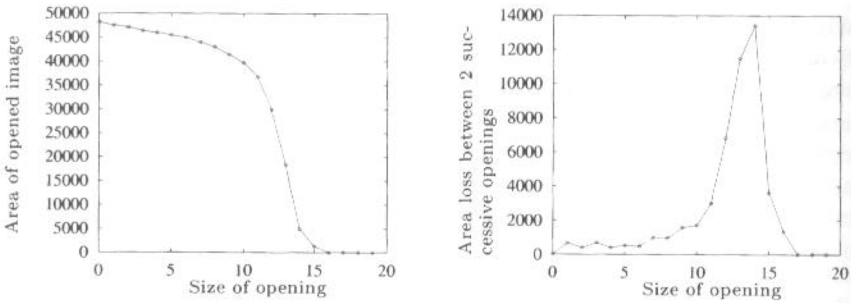
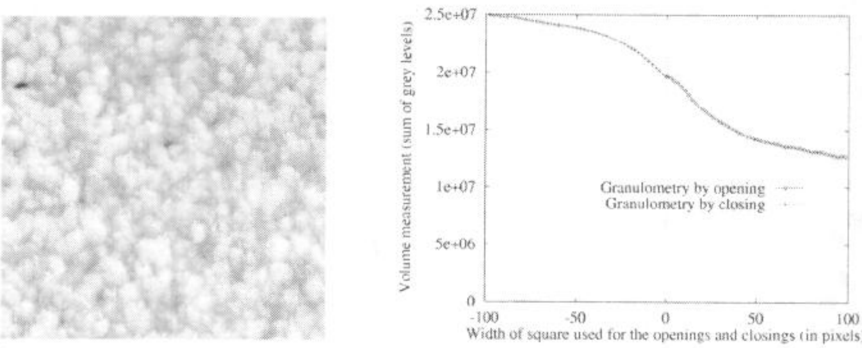


Fig. 11.2. Granulometric curves corresponding to the granulometry displayed in Fig. 11.1. *Left:* inverse of the cumulative size distribution. *Right:* size distribution or pattern spectrum. The high peak observed in this pattern spectrum indicates that most cells of Fig. 11.1a occur at this size.

A granulometry by opening produces information concerning image structures brighter than their neighbourhood. Indeed, when performing an opening, we test only whether the structuring element fits the foreground structures. Therefore, information about the arrangement of structures which appear darker than their neighbourhood is obtained by closing the image with the same family of structuring elements. Because closings are extensive rather than anti-extensive, this process is referred to as an ***anti-granulometry*** or simply a ***granulometry by closing***. Both granulometries can be collated into a unique curve with closings versus size on the left side and openings versus size on the right side of the diagram. Depending on whether or not the volume measurements have been normalised, the value at the origin equals one or the volume of the input image. The granulometries by opening and closing of a grey scale textured image are shown in Fig. 11.3.



(a) Roughness image (b) Granulometry of *(a)*

Fig. 11.3. Grey scale granulometry by closing and opening of an image representing the roughness of a cylinder used for producing metal sheets.

11.1.2 Texture classification using global granulometries

Granulometries provide us with useful texture features because they reflect information regarding the shape and size of the patterns defining ordered textures as well as the degree of granularity of disordered texture. For example, Fig. 11.4 shows two types of texture occurring in satellite images of forest stands together with their granulometries by opening with squares of increasing size. These granulometries can be used for discriminating the input textures because they highlight that the upper texture is coarser than the lower texture. Indeed, the size at which the maximum of the derivative of the granulometry occurs is significantly larger for the coarser than the finer texture.

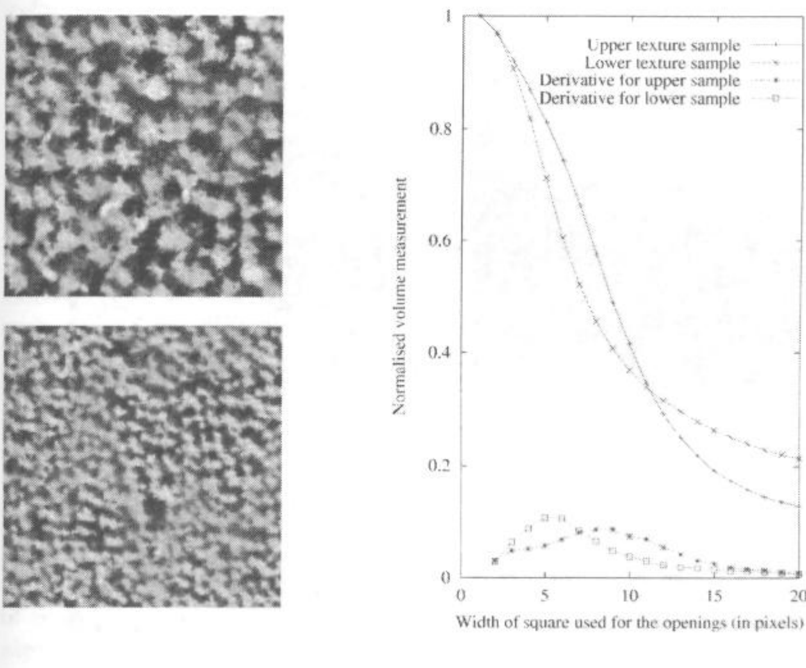
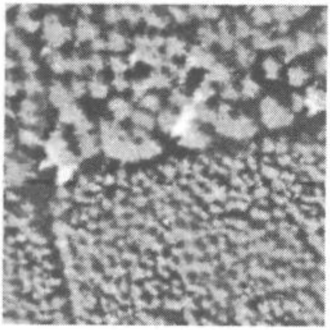


Fig. 11.4. *Left:* two samples of a 1 m resolution panchromatic IKONOS satellite image showing two forest stands of different age (crown size). *Right:* their granulometric curves by opening using squares of increasing width.

Similarly, granulometries computed on binary shapes provide signatures that can be used for pattern recognition or classification purposes. A comparison of the amount of concavities and convexities can be obtained by comparing the sum of the pattern spectra by closing and opening respectively. For example, the pattern spectrum by closing of a convex set has zero values for all SE sizes. A symmetric figure such as that shown in Fig. 2.3b contains identical amounts of concavities and convexities. Its pattern spectra by opening and closing are therefore equivalent.

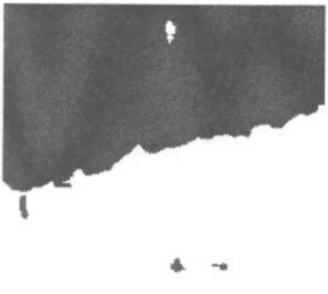
11.1.3 Texture segmentation using local granulometries

Assuming that textures appearing in an image have discriminating granulometric curves, a segmentation of the image into regions of homogeneous texture can be achieved by computing a granulometry for each image pixel and within a window of fixed shape and size. A new image, whose grey values are statistics resulting from the local size distributions, is then created. This image is then segmented assuming that regions of homogeneous texture have similar statistics (e.g., similar pattern spectrum variance). An application to the segmentation of forest stands of different age is presented in Fig. 11.5.

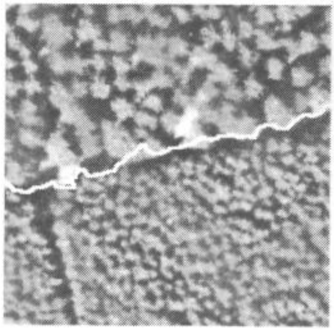


(a) Input 1 m resolution panchromatic IKONOS satellite image of two forest stands of different age (crown size).

(b) Image of the size at which the maximum of the derivative occurs (local granulometries are computed within 60×60 windows).



(c) Global threshold: small crowns for size ≤ 5 pixels appear in white.



(d) Boundary of filtered threshold overlaid on input satellite image.

Fig. 11.5. Local granulometries for texture segmentation. The granulometric curves of the two types of textures occurring in the input image are displayed in Fig. 11.4.