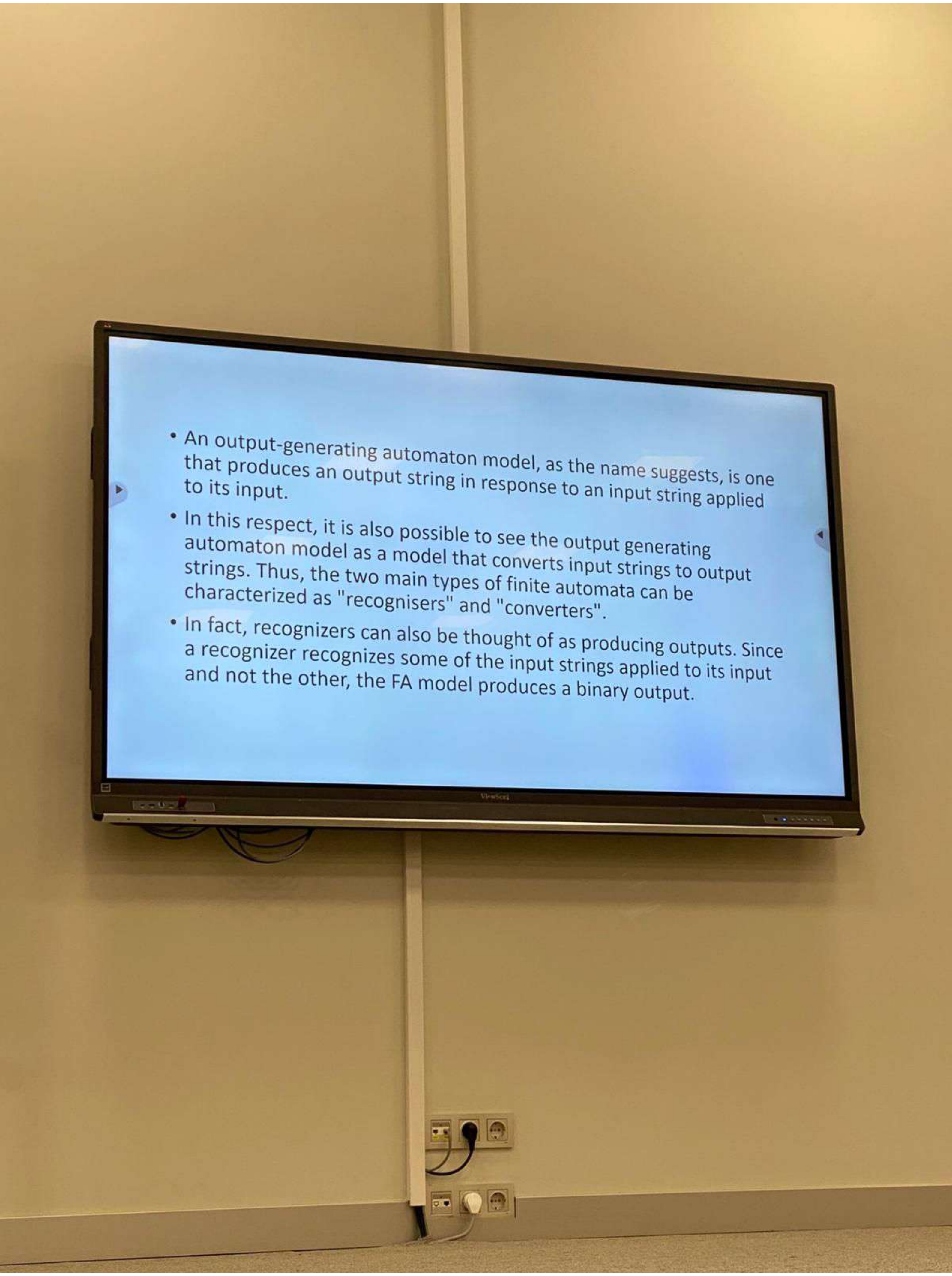
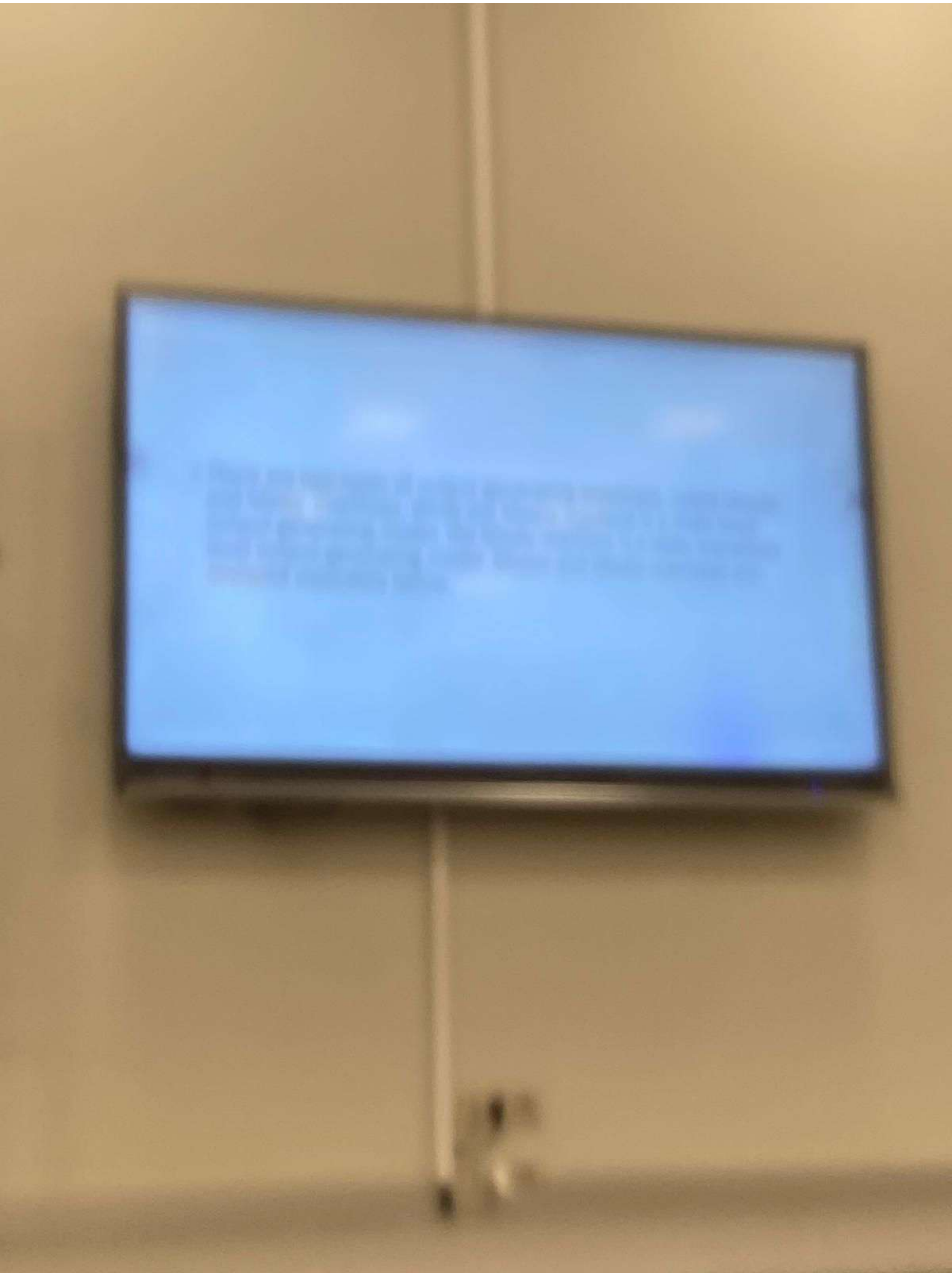
- FA model can be viewed as a binary output generating model. The outputs produced by the model can be, for example, “1: accepted” for final states and “0 : not accepted” for other states. From this point of view, the output generating finite automata model can be seen as a broader model, including the recognition model.


• An output-generating automaton model, as the name suggests, is one that produces an output string in response to an input string applied to its input.
• In this respect, it is also possible to see the output generating automaton model as a model that converts input strings to output strings. Thus, the two main types of finite automata can be characterized as "recognisers" and "converters".
• In fact, recognizers can also be thought of as producing outputs. Since a recognizer recognizes some of the input strings applied to its input and not the other, the FA model produces a binary output.



- There are two types of output-generating automata, called Moore and Mealy machines. While the Moore machine is a state-level output-generating model, the Mealy-machine is a state-transition-level output-generating model. Moore and Mealy machines are analyzed separately below.

3.1 Moore Machine

The Moore Machine is defined as a six.

$$M =< Q, \Sigma, \Delta, \delta, \lambda, q_0 >$$

Q: is finite state of internal states

Σ: input alphabet (finite set)

Δ: output alphabet (finite set)

δ: it is a transition funtion from $(Q\ x\Sigma)$ to Q

λ : it is a output funtion from Q to Δ

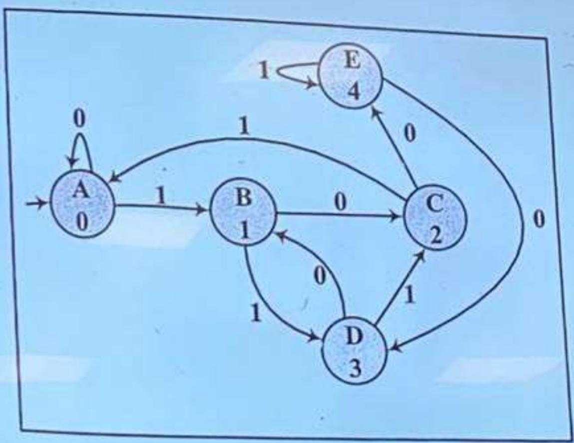
- The Moore machine can be seen as a generalization of the DFA model. In other words, the DFA model can be thought of as a special Moore machine.

- The state transition function (δ) and the output function (λ) of the Moore machine can be represented by a table or diagram. The table created to describe the Moore machine is often called the "State Table", and the diagram is often called the "State Diagram".

Example 3.1 The M_8 machine is defined as a Moore machine that produces output $z = Mod(X, 5)$ if the binary number applied to its input is X.

$$M_8 =< Q, \Sigma, \Delta, \delta, \lambda, q_0 >$$
$$Q = \{A, B, C, D, E\}$$
$$\Sigma = \{0,1\}$$
$$\Delta = \{0,1,2,3,4\}$$
$$q_0 = A$$

The definition of δ and λ is in the State Table and State Diagram in Figure 3.1.



ŞD	SD x = 0	SD x = 1	z
→ A	A	B	0
B	C	D	1
C	E	A	2
D	B	C	3
E	D	E	4

Figure 3.1 State diagram and State table

For example, when the input string w=1011110 is applied to the M_8 machine, the machine's states and outputs will change as follows.

$$\begin{array}{llllllllllllllll}
\text{Input:} & & 1 & & 0 & & 1 & & 1 & & 1 & & 1 & & 0 & \\
\text{State:} & A & \rightarrow & B & \rightarrow & C & \rightarrow & A & \rightarrow & B & \rightarrow & D & \rightarrow & C & \rightarrow & E \\
\text{Output:} & 0 & & 1 & & 2 & & 0 & & 1 & & 3 & & 2 & & 4
\end{array}$$

3.2 Mealy Machine

Like the Moore machine, the Mealy machine is defined as a six.

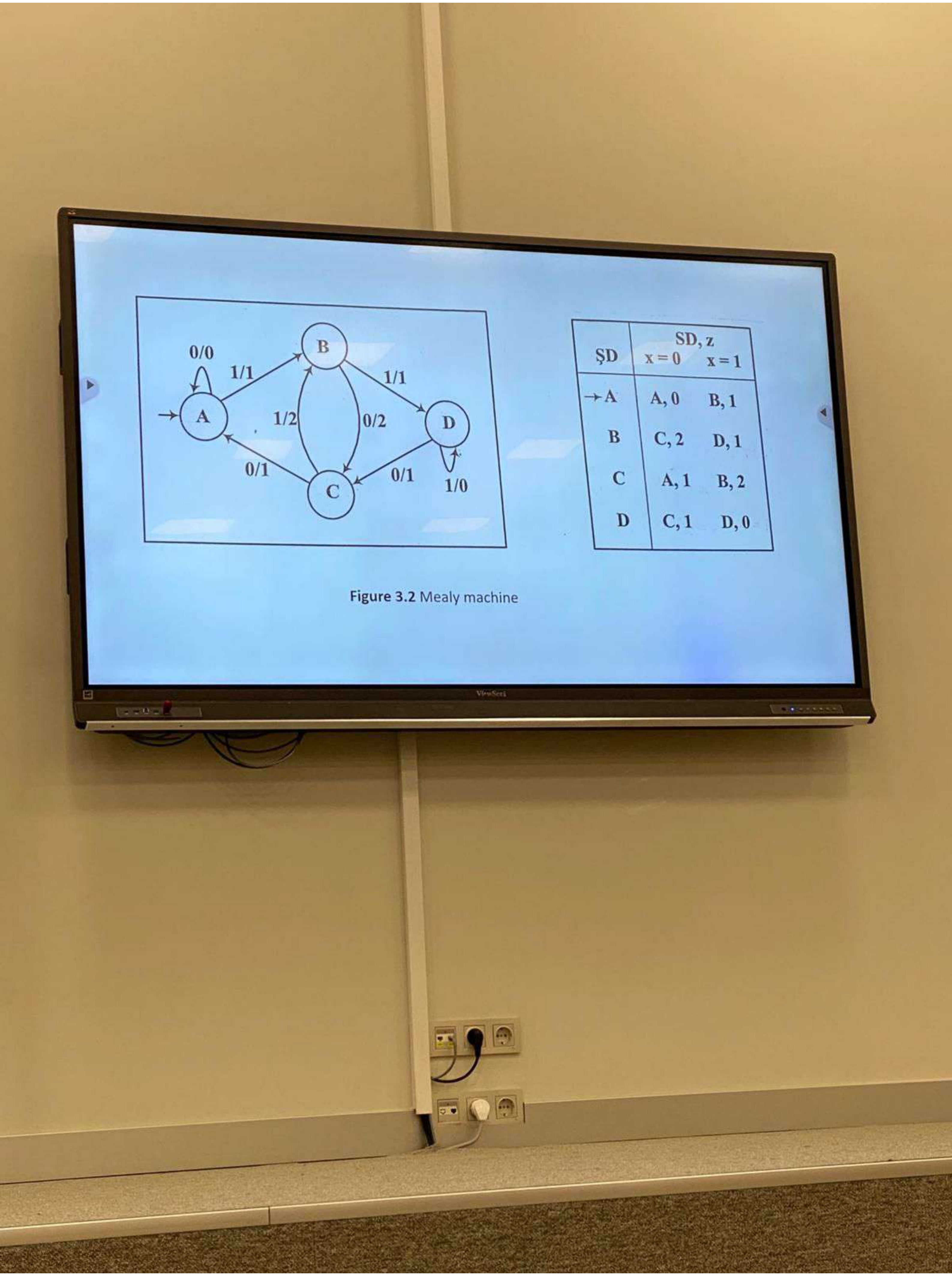
$$M =< Q, \Sigma, \Delta, \delta, \lambda, q_0 >$$

The definition of the elements are the same as in the Moore machine except the output function (λ). In other words, the only difference between Moore and Mealy machines is in the output function. The output function, defined as mapping from Q to Δ for Moore machines and for Mealy machines, is defined as a mapping from ($Q\ x\Sigma$) to Δ.

Example 3.2 The M_9 machine is defined as the Mealy machine with the input alphabet {0, 1} and the output alphabet {0, 1, 2} and showing how many of the last two input symbols are different from the previous one with the output it produces. When determining the first 2 output values that the machine will produce, the two input values before the initial state will be assumed to be 00.

$$M_6 =< Q, \Sigma, \Delta, \delta, \lambda, q_0 >$$
$$Q = \{A, B, C, D\}$$
$$\Sigma = \{0,1\}$$
$$\Delta = \{0,1,2\}$$
$$q_0 = A$$

δ and λ are defined by state table and state diagram in figure 3.2. According to the design, the states of the machine are A, B, C or D, respectively, according to the previous two input symbol values being 00, 01, 10 or 11.

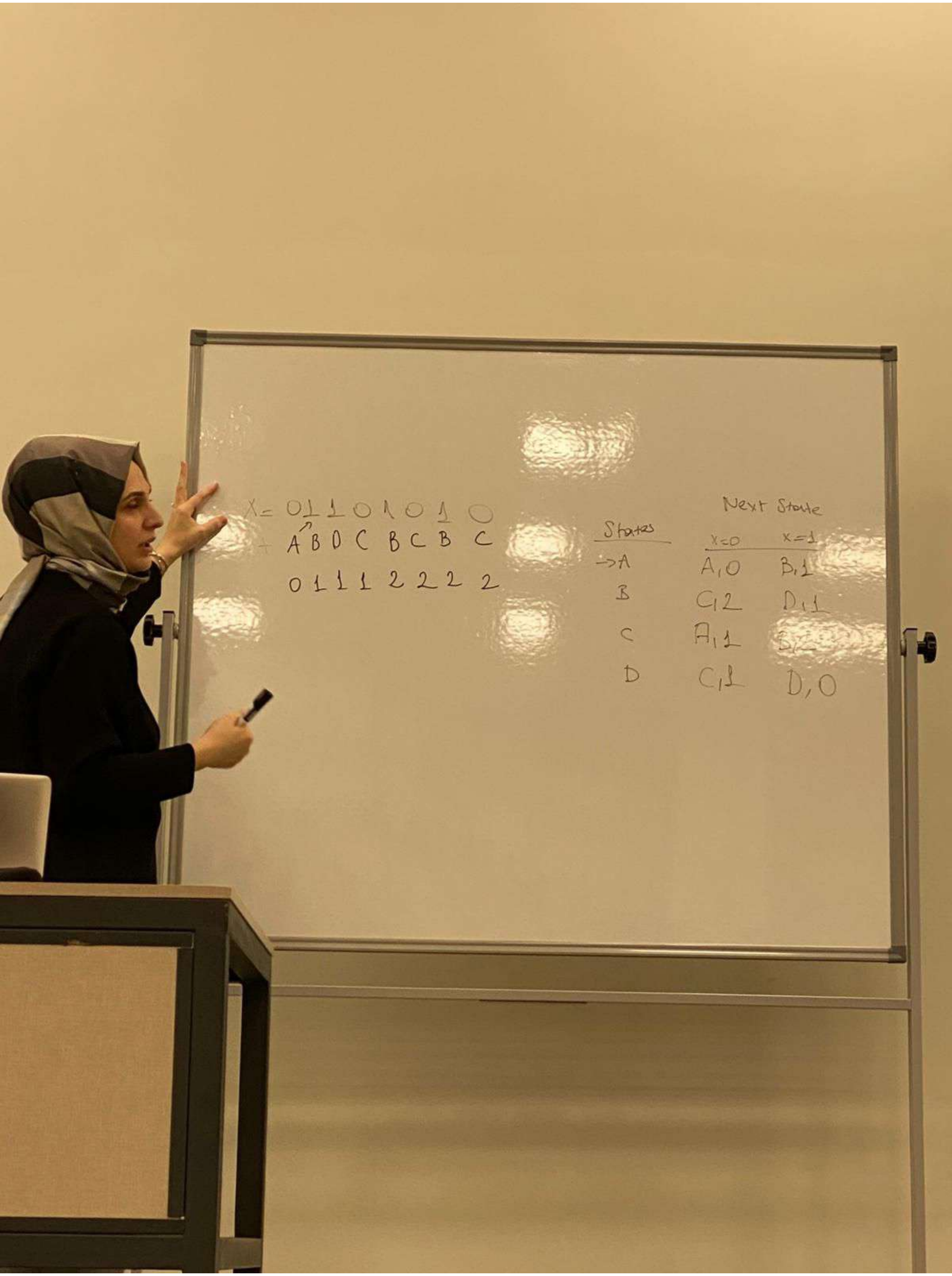


ŞD	SD, z x = 0	SD, z x = 1
→A	A, 0	B, 1
B	C, 2	D, 1
C	A, 1	B, 2
D	C, 1	D, 0

An example input string and the output string that the machine should generate in response to this input string are as follows:

X: 0 1 1 0 1 0 1 0 1 1 1 1 0 1 0 0 1 1 1 0 0 0 0 1

Z: 0 1 1 1 2 2 2 2 2 1 0 0 1 2 2 1 1 1 0 1 1 0 0 1



X = 0 1 1 0 1 0 1 0
A B D C B C B C
0 1 1 1 2 2 2 2
Next State
States
X=0
X=1
A A,0 B,1
B C,2 D,1
C A,1
D C,1 D,0

3.3 The Equivalence of Mealy and Moore Machine

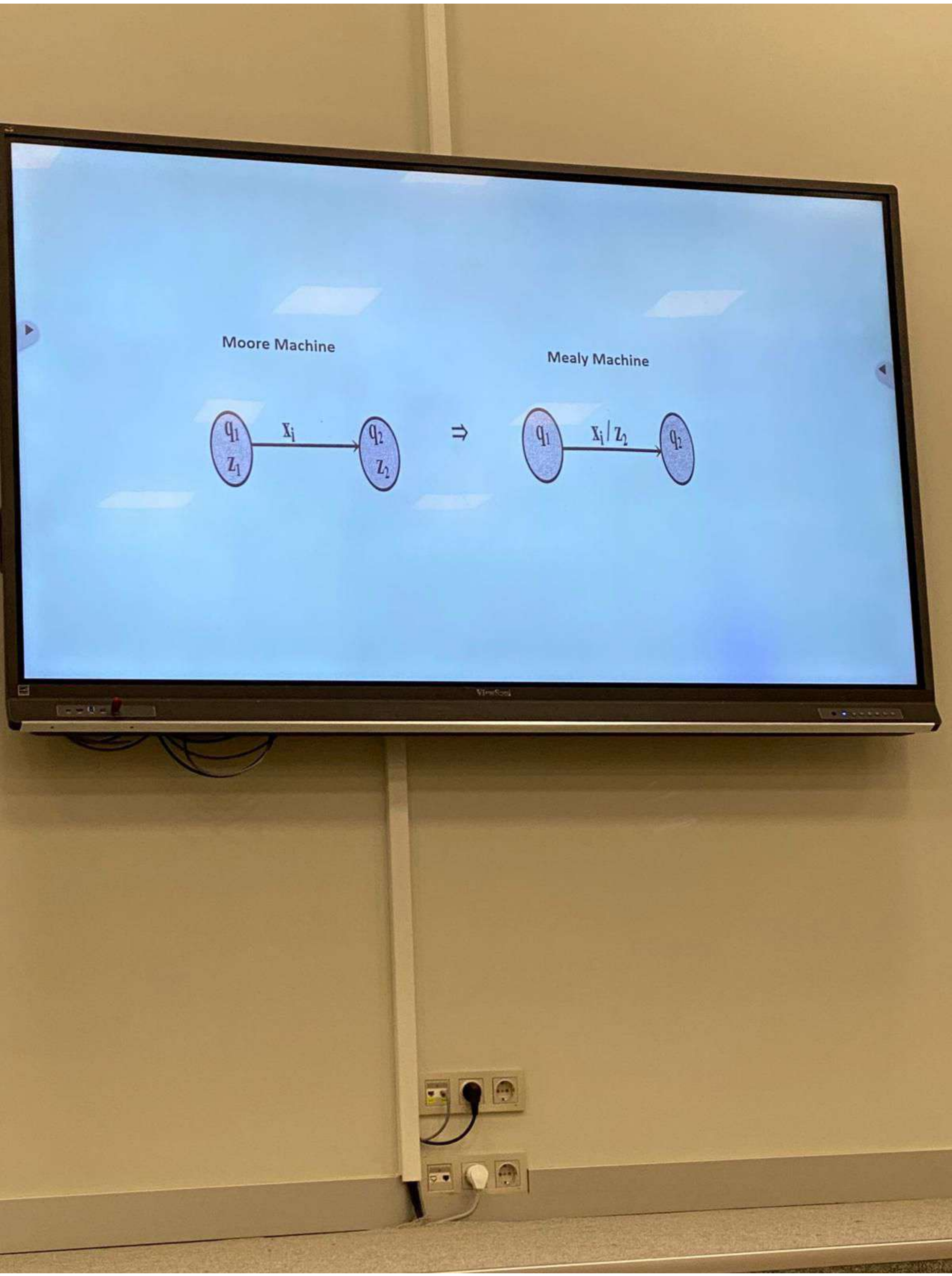
- Both Moore and Mealy machines are finite automata models that produce output.
- In Moore and Mealy models, which are defined as a sextet, 5 elements of the six are common. It is the output function that separates the Moore and Mealy models.
- In Moore's model, the machine generates an exit symbol for each state. When an input string consisting of n input symbols is applied to a Moore machine in a given state, the machine will generate an output string of length (n+1). In the Mealy model, the machine generates an output symbol for each state transition. When an input string of n input symbols is applied to a Mealy machine in a particular state, the machine will generate an output string of length n.
- If the initial output symbol produced by the Moore machine is excluded, equivalent Mealy and Moore machines will produce the same output string, regardless of the input string applied.

3.3.1 Finding the Mealy Machine Equivalent to the Moore Machine

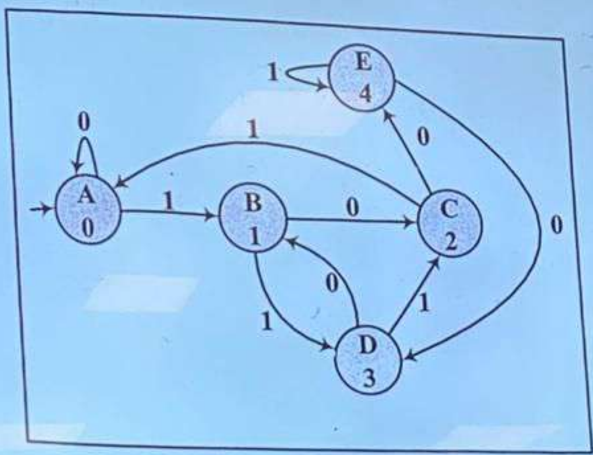
Lemma 3.1 Let $M_1 =< Q, \Sigma, \Delta, \delta, \lambda, q_0 >$ is a Moore machine. The Mealy machine equivalent to M_1 is found as follows:

$$M_2 =< Q, \Sigma, \Delta, \delta, \lambda', q_0 >, \quad \lambda'(q, a) = \lambda\big(\delta(q, a)\big)$$

When we try to find Mealy machine equivalent to a Moore machine, the output value mapped to each state transition is equal to the output value mapped on the Moore machine to the next state at the end of the state transition. It is possible to summarize this rule with the following drawing.

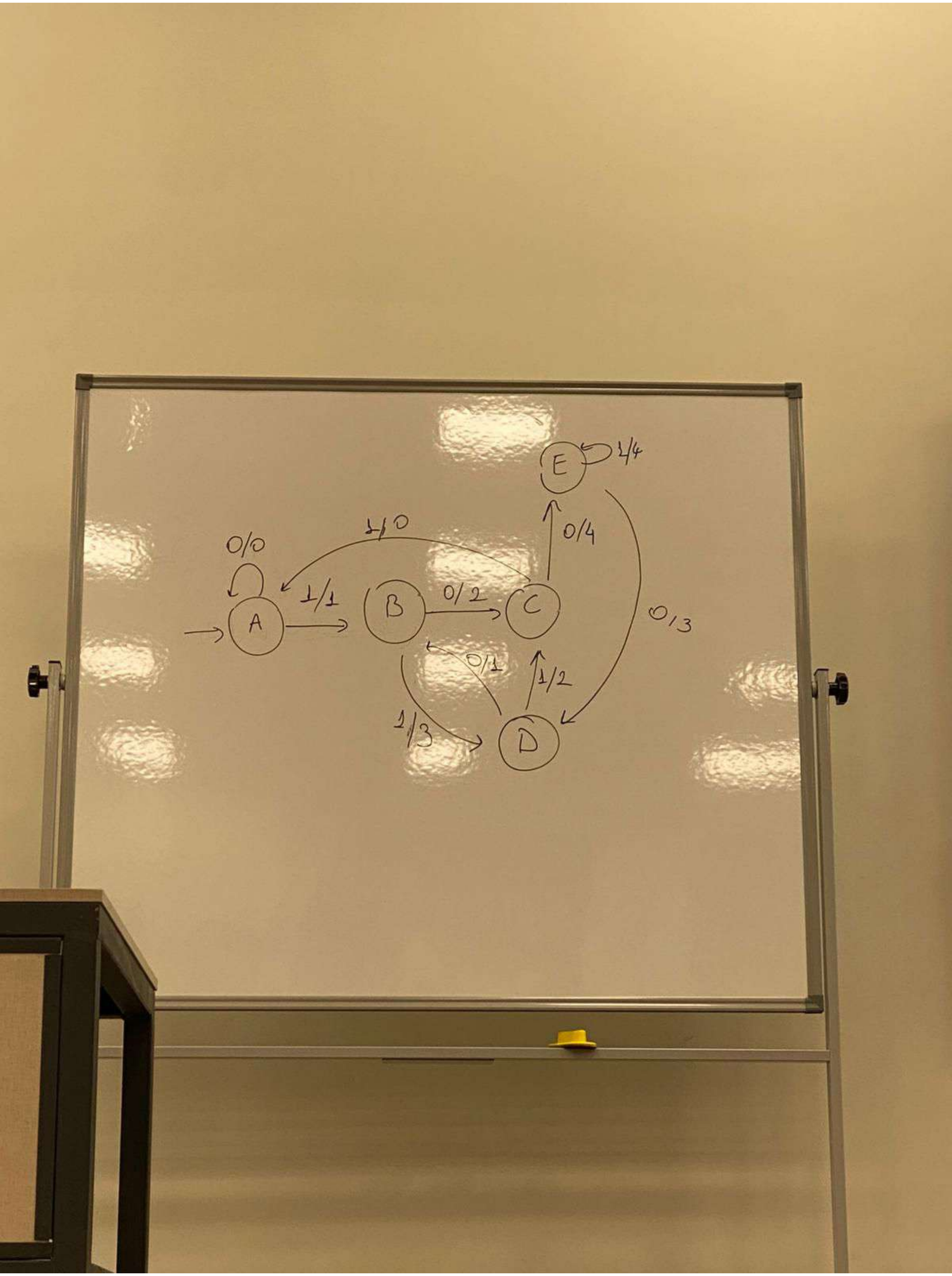


Moore Machine
Mealy Machine
q_1
z_1
x_i
q_2
z_2
⇒
q_1
x_i / z_2
q_2



ŞD	SD x = 0	x = 1	z
→ A	A	B	0
B	C	D	1
C	E	A	2
D	B	C	3
E	D	E	4

Figure 3.1 State diagram and State table



1/4
E
0/4
1/0
0/0
1/1
A
B
0/2
C
0/3
0/1
1/2
1/3
D

3.3.2 Finding the Moore Machine Equivalent to the Mealy Machine

Lemma 3.2 Let $M_2 =< Q, \Sigma, \Delta, \delta, \lambda, q_0 >$ is a Mealy machine. The Moore machine equivalent to M_1 is found as follows:

$M_1 =< Q', \Sigma, \Delta, \delta', \lambda', q_0' >$

$Q' = Qx\Delta$

$q_0' = [q_0, z_j]$ z_j: one of the output symbol

$\delta'([q_i, z_k], x_j) = [\delta(q_i, x_j), \lambda(q_i, x_j)]$

$\lambda'([q_i, z_k]) = z_k$

- If in the Mealy machine the output symbol z_j is generated during the transition from state q_1 to state q_2, then in the equivalent Moore machine, a transition from each state with the first element q_1 to the state with the first element q_2 and the second element z_j with the input symbol x_i is created. Accordingly, if the output alphabet has the output symbol m, for each state transition in the Mealy machine, the equivalent Moore machine has m transition.

Example 3.3 M_{10} Mealy machine is as follows:

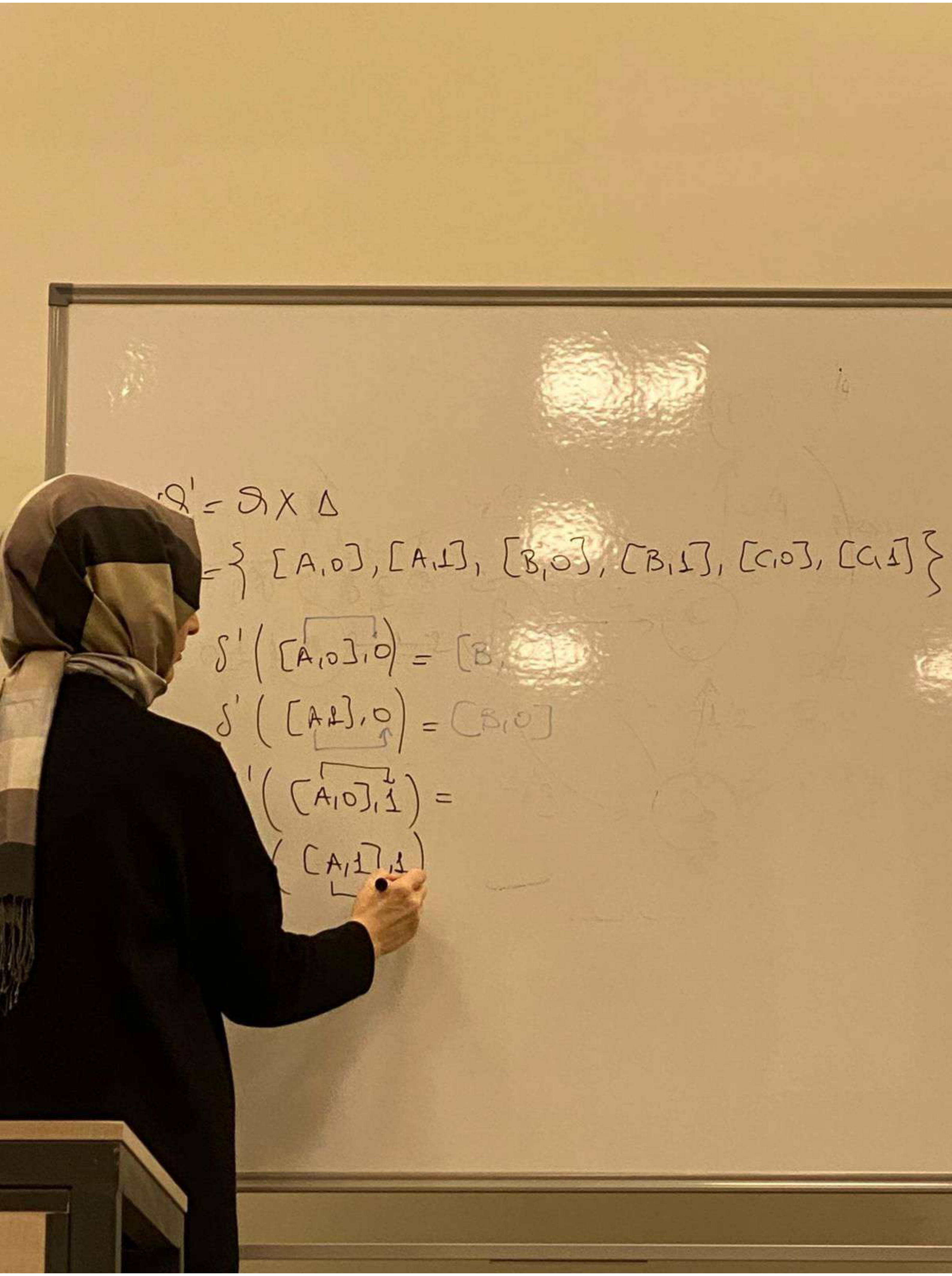
$$M_{10} =< Q, \Sigma, \Delta, \delta, \lambda, q_0 >$$
$$Q = \{A, B, C\}$$
$$\Sigma = \{0,1\}$$
$$\Delta = \{0,1\}$$
$$q_0 = A$$

$\delta(A,0) = B$	$\lambda(A,0) = 0$
$\delta(A,1) = A$	$\lambda(A,1) = 1$
$\delta(B,0) = B$	$\lambda(B,0) = 1$
$\delta(B,1) = C$	$\lambda(B,1) = 1$
$\delta(C,0) = A$	$\lambda(C,0) = 0$
$\delta(C,1) = C$	$\lambda(C,1) = 0$

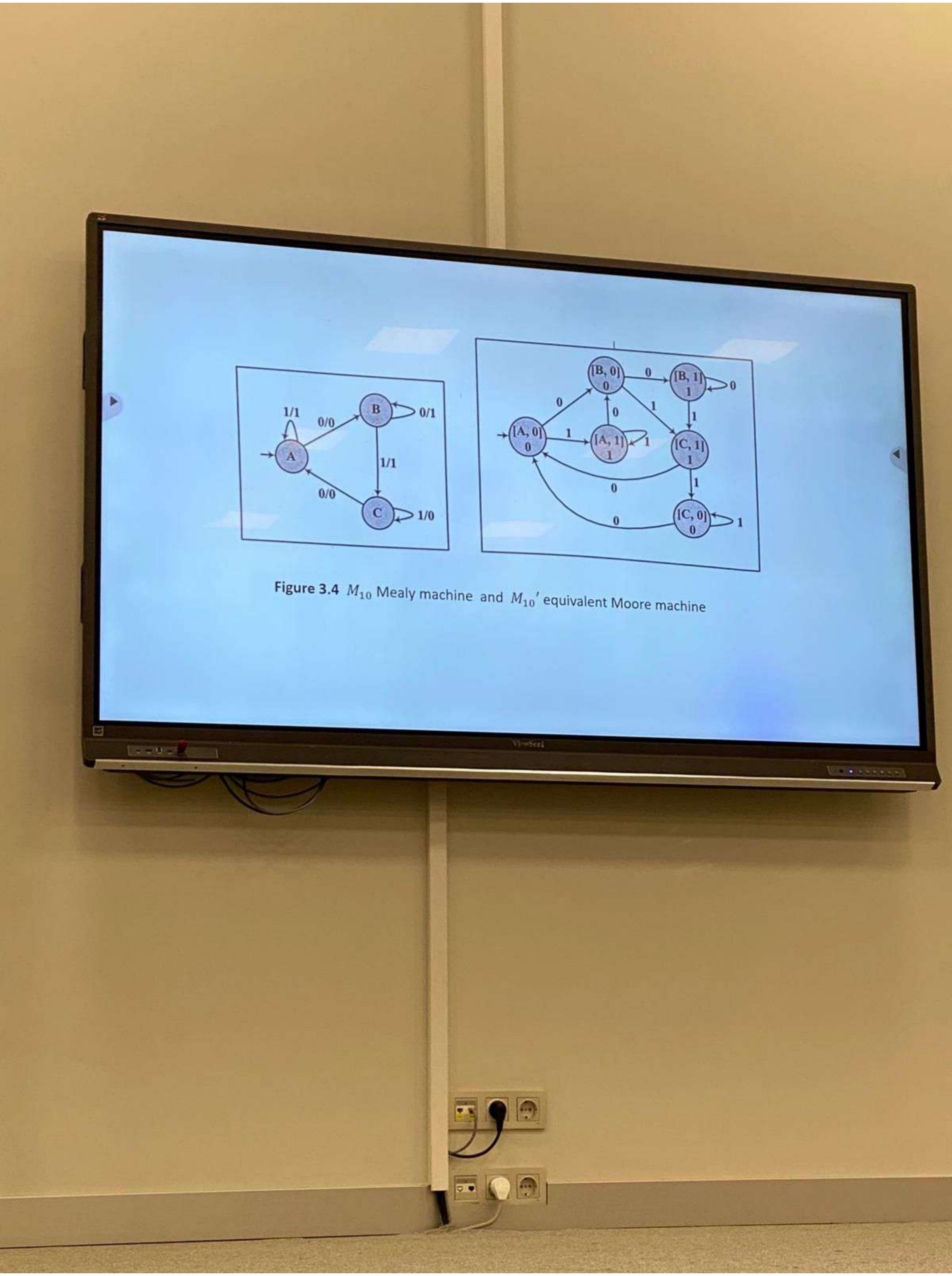
We can find the moore machine which is equivalent to M_{10} Mealy machine.

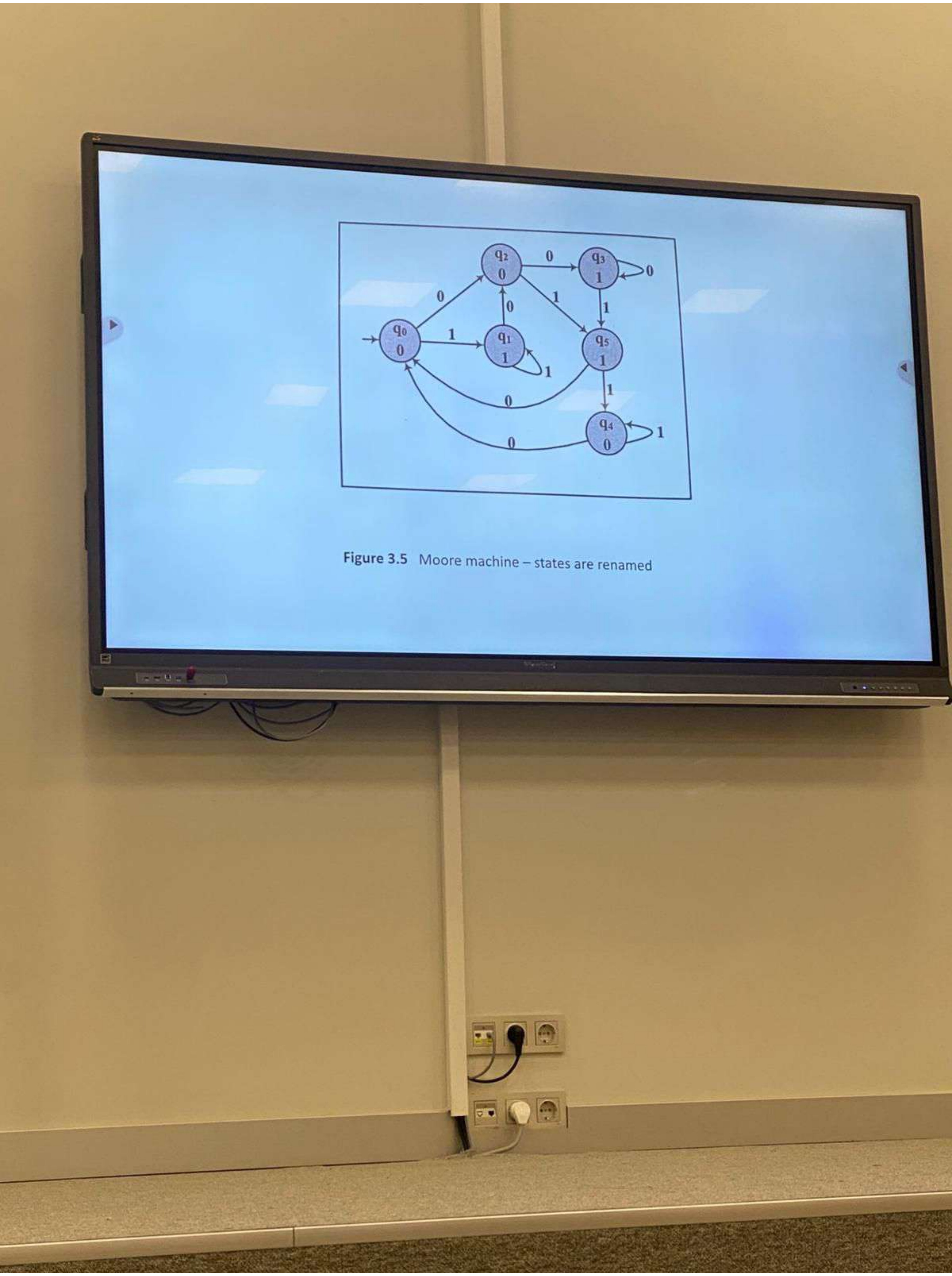
$$M_{10}{}' = < Q', \Sigma, \Delta, \delta', \lambda', q_0{}' >$$
$$Q' = \{[A,0],[A,1],[B,0],[B,1],[C,0],[C,1]\}$$
$$q_0' = [A,0]$$

$\delta'([A,0],0) = [B,0]$ $\quad \lambda([A,0]) = 0$
$\delta'([A,1],0) = [B,0]$ $\quad \lambda([A,1]) = 1$
$\delta'([A,0],1) = [A,1]$ $\quad \lambda([B,0]) = 0$
$\delta'([A,1],1) = [A,1]$ $\quad \lambda([B,1]) = 1$
$\delta'([B,0],0) = [B,1]$ $\quad \lambda([C,0]) = 0$
$\delta'([B,1],0) = [B,1]$ $\quad \lambda([C,1]) = 0$
$\delta'([B,0],1) = [C,1]$
$\delta'([B,1],1) = [C,1]$
$\delta'([C,0],0) = [A,0]$
$\delta'([C,1],0) = [A,0]$
$\delta'([C,0],1) = [C,0]$
$\delta'([C,1],1) = [C,0]$


Q' = Q X Δ
= { [A,0], [A,1], [B,0], [B,1], [C,0], [C,1] }
δ' ([A,0], 0) = [B,
δ' ([A,1], 0) = [B,0]
' ([A,0], 1) =
([A,1], 1)

- Figure 3.4 shows the state diagram of the M_{10} Mealy machine and the equivalent Moore machine. According to Lemme 3.2, when finding a Moore machine equivalent to a Mealy machine, the states of the Moore machine should be represented by the pairs [state name, output symbol].


1/1
0/0
B
0/1
A
1/1
0/0
C
1/0
[B, 0]
0
[B, 1]
1
[A, 0]
0
[A, 1]
1
[C, 1]
1
[C, 0]
0
Figure 3.4 M_{10} Mealy machine and M_{10}' equivalent Moore machine

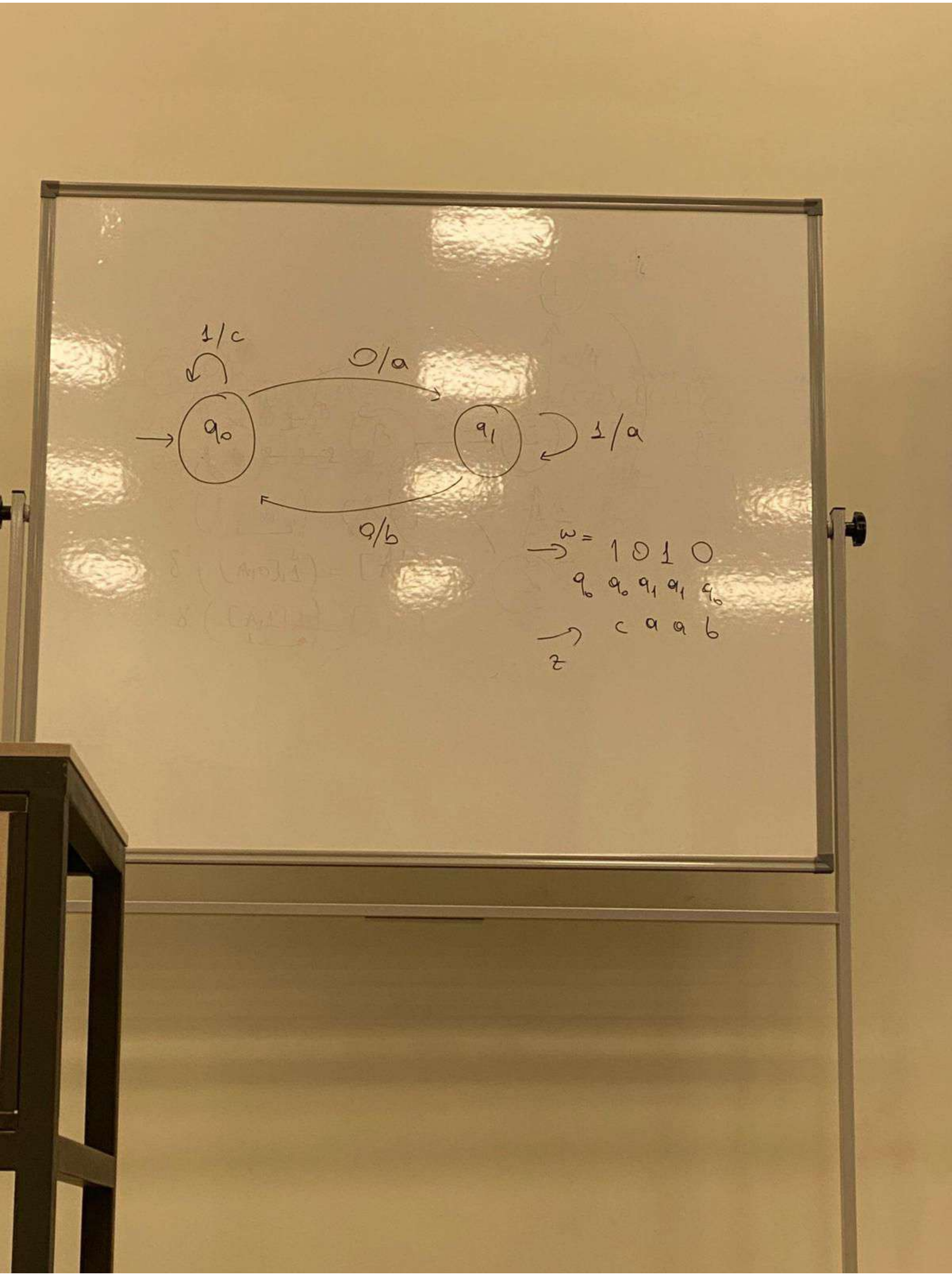

q0
q1
q2
q3
q4
q5
Figure 3.5 Moore machine – states are renamed

Example 3.4 The Mealy machine is defined as follows:

$$M = < Q, \Sigma, \Delta, \delta, \lambda, q_0 >$$
$$Q = \{q_0, q_1\}$$
$$\Sigma = \{0,1\}$$
$$\Delta = \{a, b, c\}$$

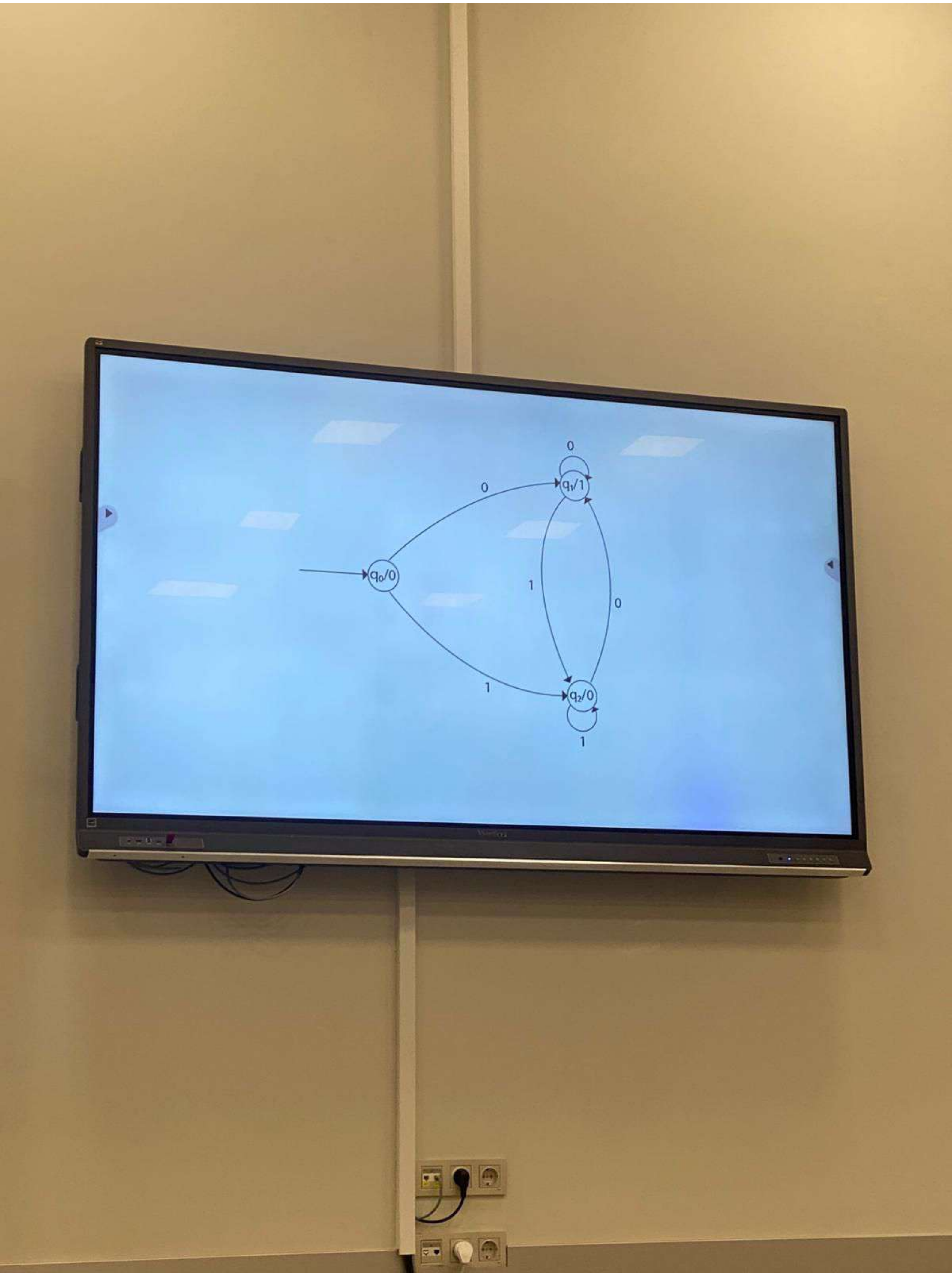
q_0 is the initial state

$$\begin{array}{ll} \delta(q_0, 0) = q_1 & \lambda(q_0, 0) = a \\ \delta(q_0, 1) = q_0 & \lambda(q_0, 1) = c \\ \delta(q_1, 0) = q_0 & \lambda(q_1, 0) = b \\ \delta(q_1, 1) = q_1 & \lambda(q_1, 1) = a \end{array}$$

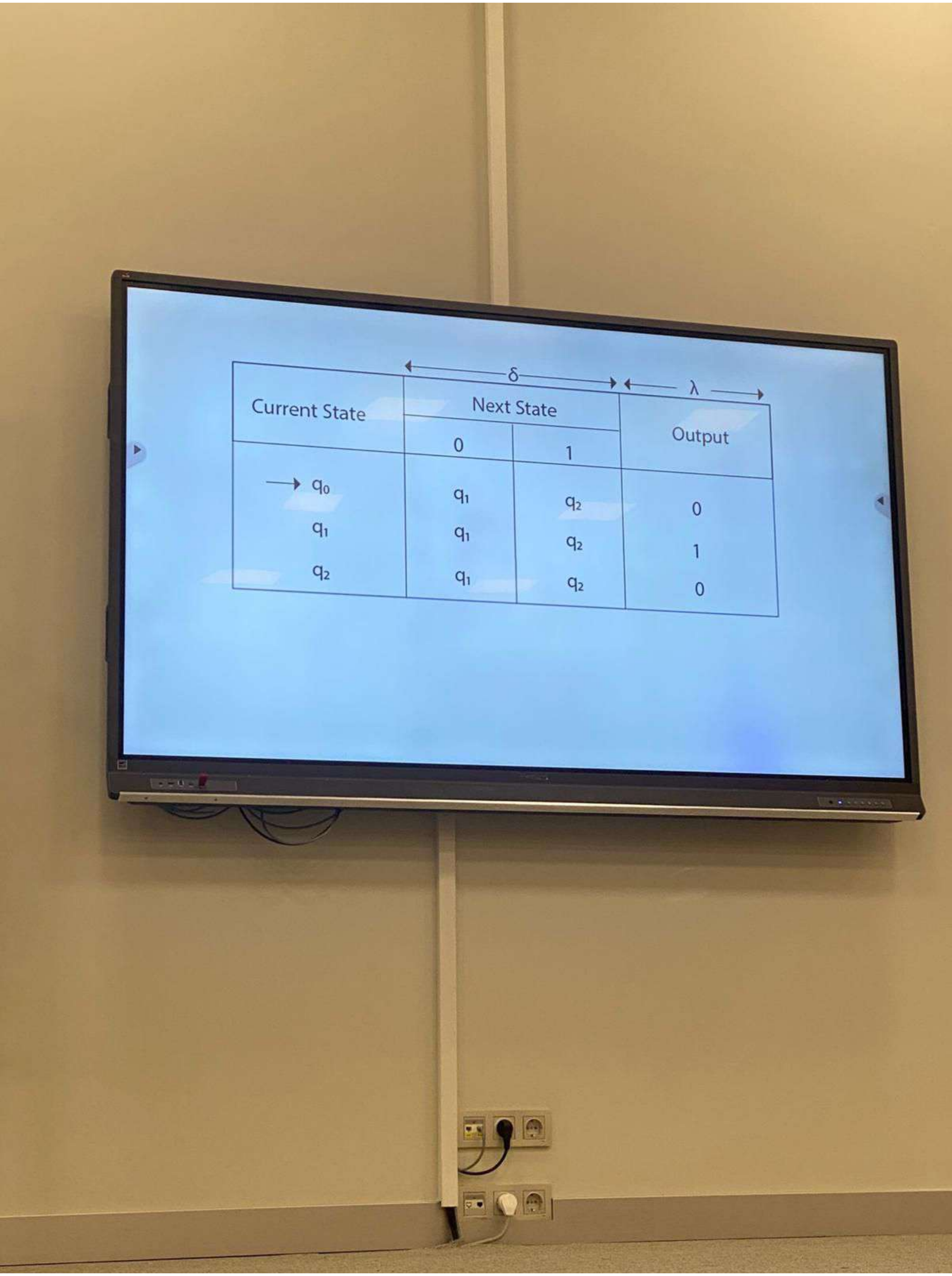

1/c
0/a
q0
q1
1/a
0/b
w = 1 0 1 0
q0 q0 q1 q1 q0
c a a b
z

Example 3.5 Design a Moore machine to generate 1's complement of a given binary number.

Solution. To generate 1's complement of a given binary number the simple logic is that if the input is 0 then the output will be 1 and if the input is 1 then the output will be 0. That means there are three states. One state is start state. The second state is for taking 0's as input and produces output as 1. The third state is for taking 1's as input and producing output as 0.



q₀/0
q₁/1
q₂/0
0
0
1
1
0
1

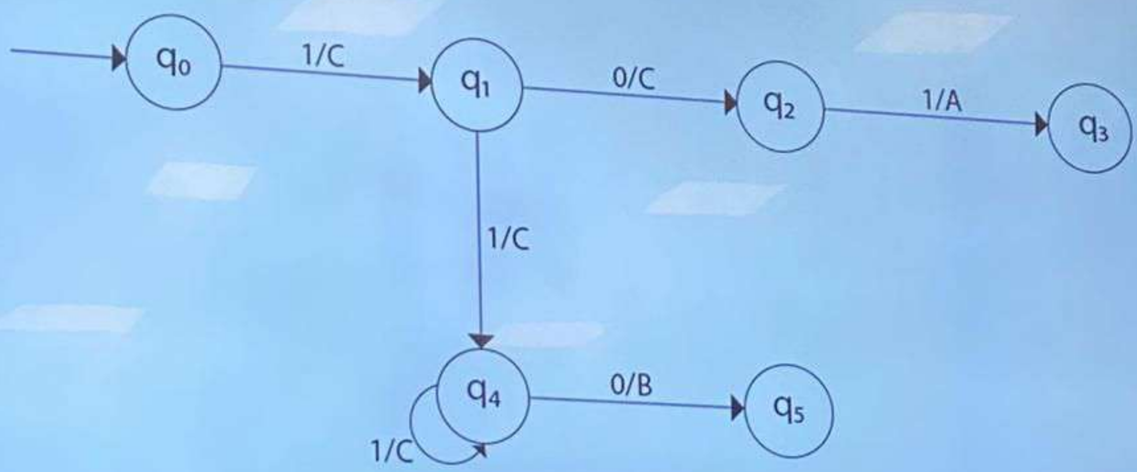


Current State	Next State (δ)		Output (λ)
	0	1	
$\rightarrow q_0$	q_1	q_2	0
q_1	q_1	q_2	1
q_2	q_1	q_2	0

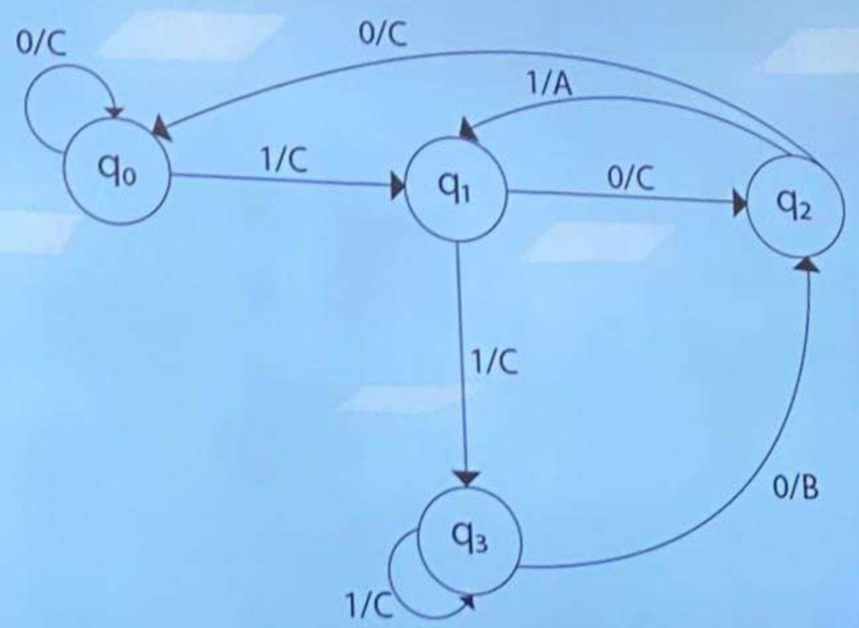
Example 3.6 Design a Mealy machine for a binary input sequence such that if it has a substring 101, the machine output A, if the input has substring 110, it outputs B otherwise it outputs C.

Solution. For designing such a machine, we will check two conditions, and those are 101 and 110. If we get 101, the output will be A. If we recognize 110, the output will be B. For other strings the output will be C.

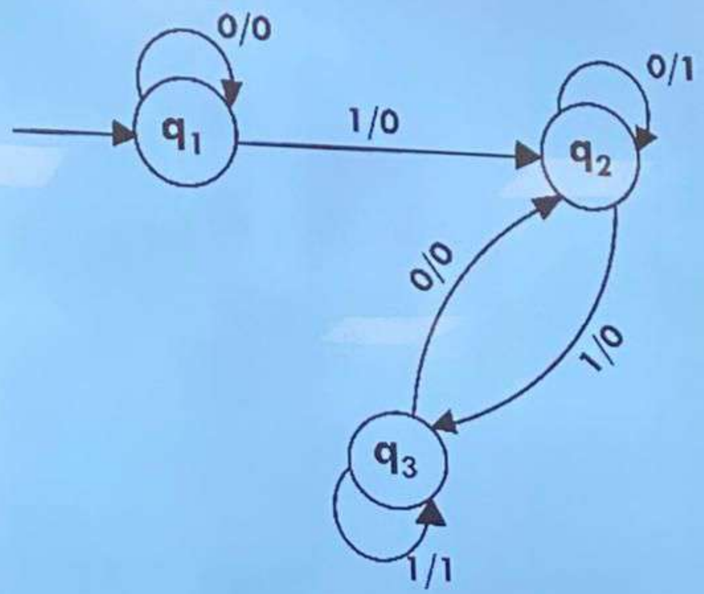
The partial diagram will be:



Now we will insert the possibilities of 0's and 1's for each state. Thus the Mealy machine becomes:



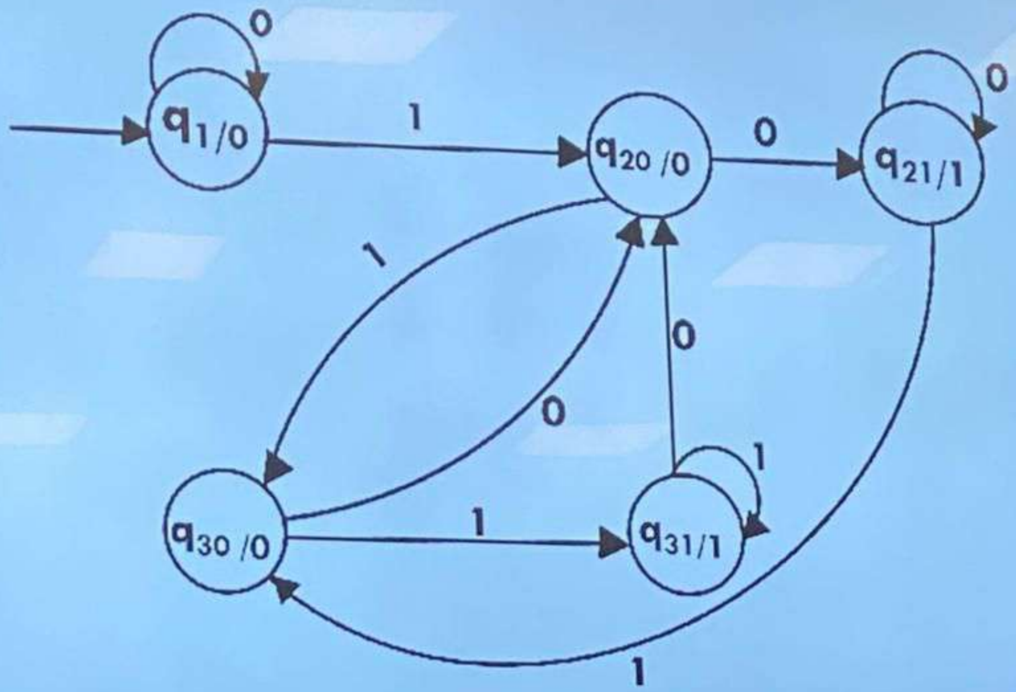
Example 3.7 Convert the following Mealy machine into equivalent Moore machine.



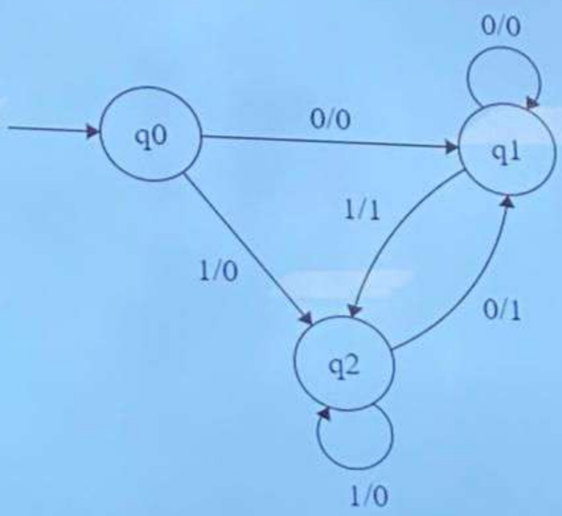
	$x=0$	$x=1$
$\rightarrow q_1$	$q_1, 0$	$q_2, 0$
q_2	$q_2, 1$	$q_3, 0$
q_3	$q_2, 0$	$q_3, 1$

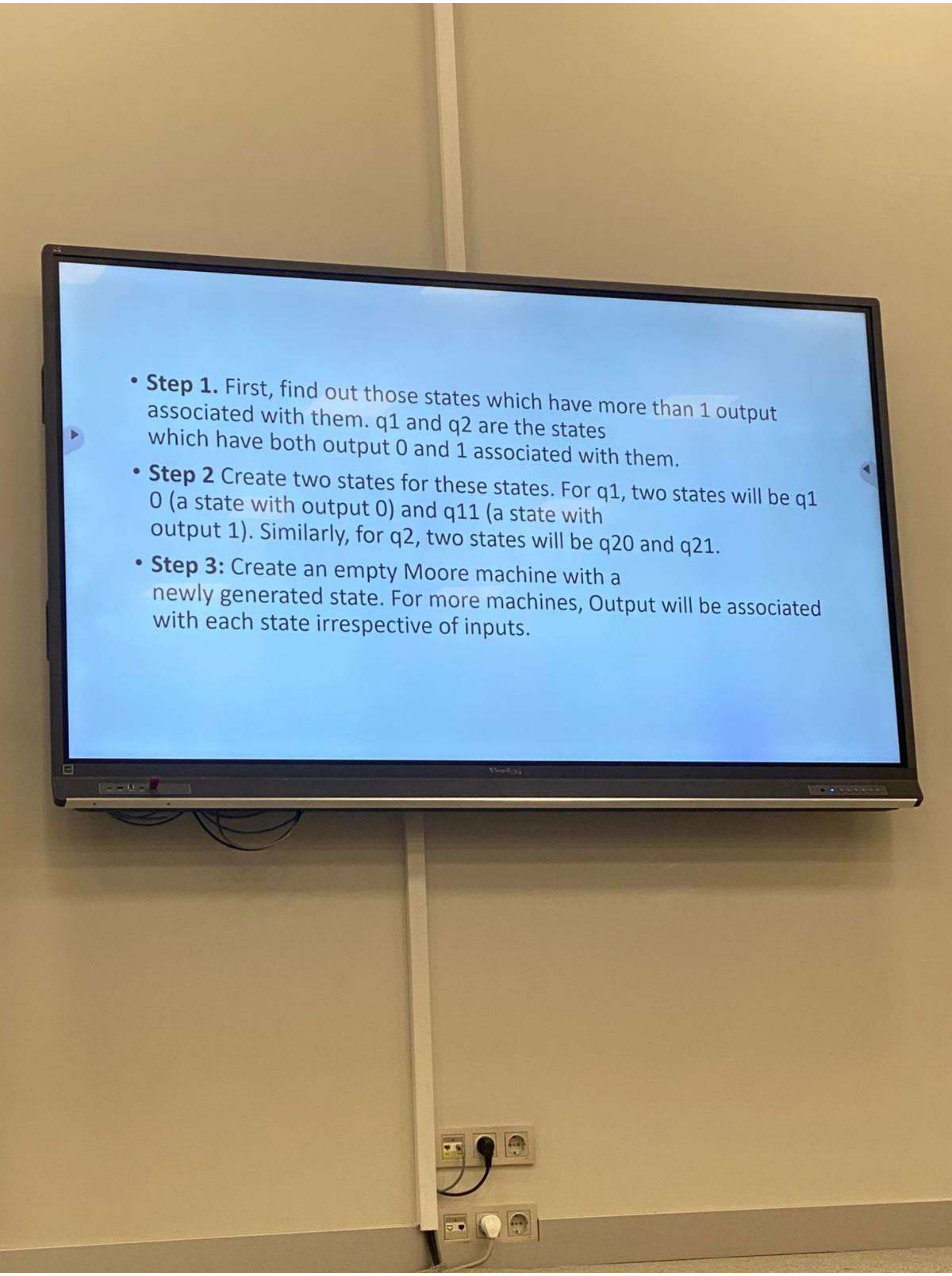
	$x=0$	$x=1$	Outputs
q_{10}	q_{10}	q_{20}	0
q_{20}	q_{21}	q_{30}	0
q_{21}	q_{21}	q_{30}	1
q_{30}	q_{20}	q_{31}	0
q_{31}	q_{20}	q_{31}	1

Transition diagram for Moore machine will be:



Example 3.8 Convert the following Mealy machine into equivalent Moore machine.

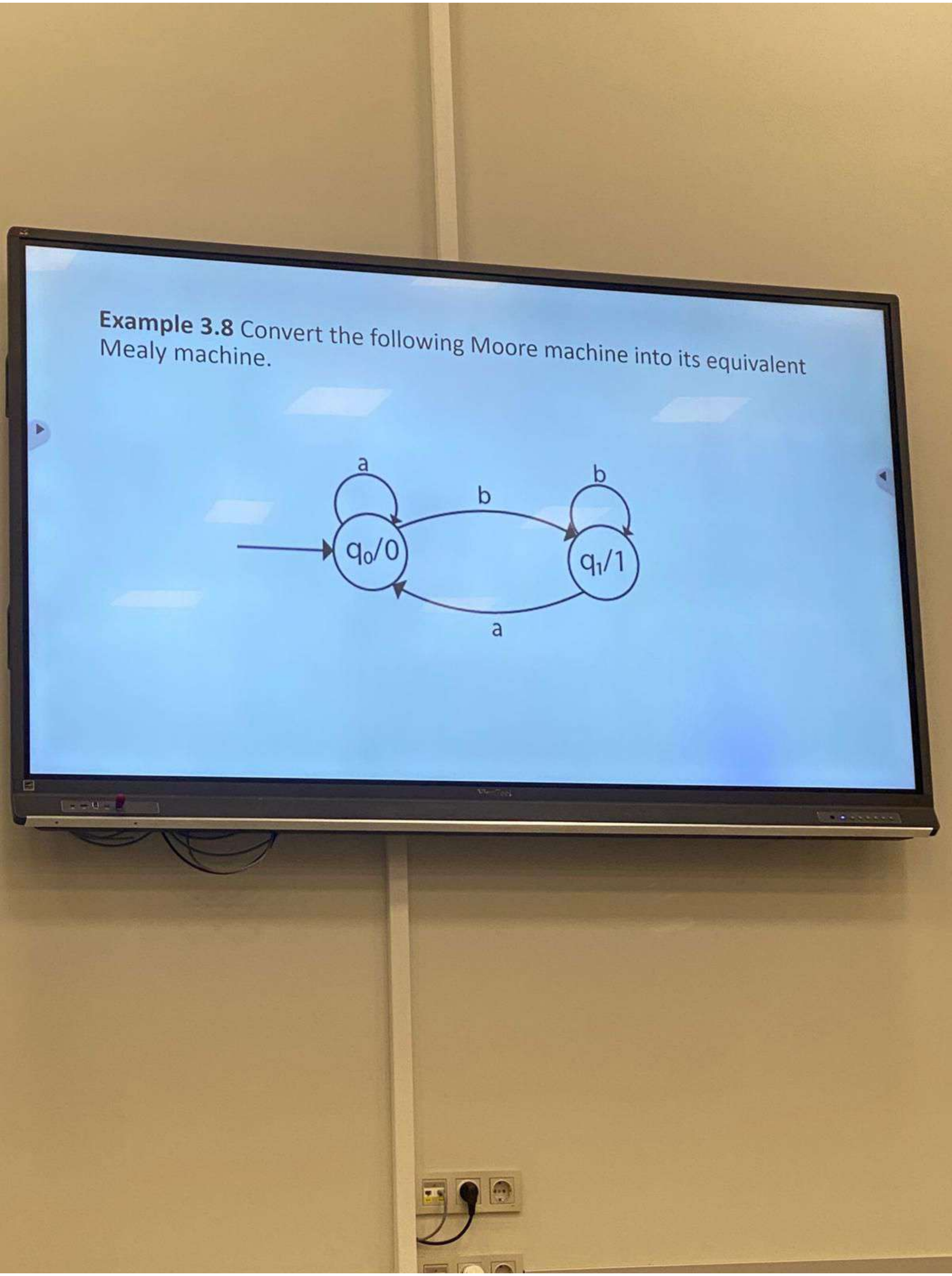



• **Step 1.** First, find out those states which have more than 1 output associated with them. q1 and q2 are the states which have both output 0 and 1 associated with them.
• **Step 2** Create two states for these states. For q1, two states will be q1 0 (a state with output 0) and q11 (a state with output 1). Similarly, for q2, two states will be q20 and q21.
• **Step 3:** Create an empty Moore machine with a newly generated state. For more machines, Output will be associated with each state irrespective of inputs.

- **Step 4:** Fill in the entries of the next state using the mealy machine transition table shown in Table. For q0 on input 0, the next state is q10 (q1 with output 0). Similarly, for q0 on input 1, the next state is q20 (q2 with output 0). For q1 (both q10 and q11) on input 0, the next state is q10. Similarly, for q1(both q10 and q11), next state is q21. For q10, the output will be 0 and for q11, the output will be 1. Similarly, other entries can be filled.

	$x=0$	$x=1$
$\rightarrow q_0$	$q_{1,0}$	$q_{2,0}$
q_1	$q_{1,0}$	$q_{2,1}$
q_2	$q_{1,1}$	$q_{2,0}$

	$x=0$	$x=1$	Output
q_0	q_{10}	q_{20}	0
q_{10}	q_{10}	q_{21}	0
q_{11}	q_{10}	q_{21}	1
q_{20}	q_{11}	q_{20}	0
q_{21}	q_{11}	q_{20}	1


Example 3.8 Convert the following Moore machine into its equivalent Mealy machine.
a
b
b
q0/0
q1/1
a

The equivalent Mealy machine will be,