Quiz 10/12/2023

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Contents

1 Local Min/Max & Saddles

1.1 First Derivative Test

If a differentiable function f has a local maximum or minimum at (a, b) then the following is true $(\nabla f)\Big|_{(a,b)} = \langle 0, 0 \rangle$

1.2 Second Derivative Test

If (a, b) is a critical point of f, meaning $(\nabla f)\Big|_{(a,b)} = \langle 0, 0 \rangle$, then the following statements are true for the second Derivative test, D:

$D = f_{xx}(a, b)f_{yy}(a, b) - (f_{xy}(a, b))^2$

- If$D > 0$ and $f_{xx}(a, b) > 0$ then $f(a, b)$ is a local minimum
- If $D > 0$ and $f_{xx}(a, b) < 0$ then $f(a, b)$ is a local maximum
- If $D < 0$ then $f(a, b)$ is a saddle point
- If $D = 0$ then the test is inconclusive

2 Directional Derivatives

The Directional Derivative of F in the direction u is denoted as follows:

$$D_u f = \nabla f \cdot \vec{u}$$

thus, at any point (x_0, y_0)

$$D_u f_{(x_0,y_0)} = f_x(x_0, y_0)u_x + f_y(x_0, y_0)u_y$$

a gradient / vector of three variables follows the same conventions

$$D_u f_{(x_0,y_0,z_0)} = f_x(x_0, y_0, z_0)u_x + f_y(x_0, y_0, z_0)u_y + f_z(x_0, y_0, z_0)u_z$$

3 Gradient Vector

the gradient vector of a differentiable function f is denoted as follows:

$$\nabla f = \langle f_x, f_y \rangle$$

3.1 Properties of the Gradient Vector

If γ is the angle between ∇f and $\vec{u}$ then the following is true:

$$D_u f = |\nabla f| cos(\gamma)$$

this gives us the following properties:

- The function f increases the fastest when $\vec{u}$ is in the same direction as ∇f, thus the maximum increase rate of f is $|\nabla f|$
- The function f decreases the fastest when $\vec{u}$ is in the opposite direction as ∇f, thus the maximum decrease rate of f is $-|\nabla f|$
- Since the function f does not change along level curve or surfaces, ($D_u f = 0$) then ∇f is perpendicular to the level curves or level surfaces

this means that if asked to find the direction of maximum increase at an arbitrary point (x_0, y_0), the answer would be the direction of the gradient vector at that point.

$\nabla f = \langle f_x, f_y \rangle$

(so to find any point just plug into this)