

Quiz #2 : Partial Derivatives and Gradient Descent

Question 1

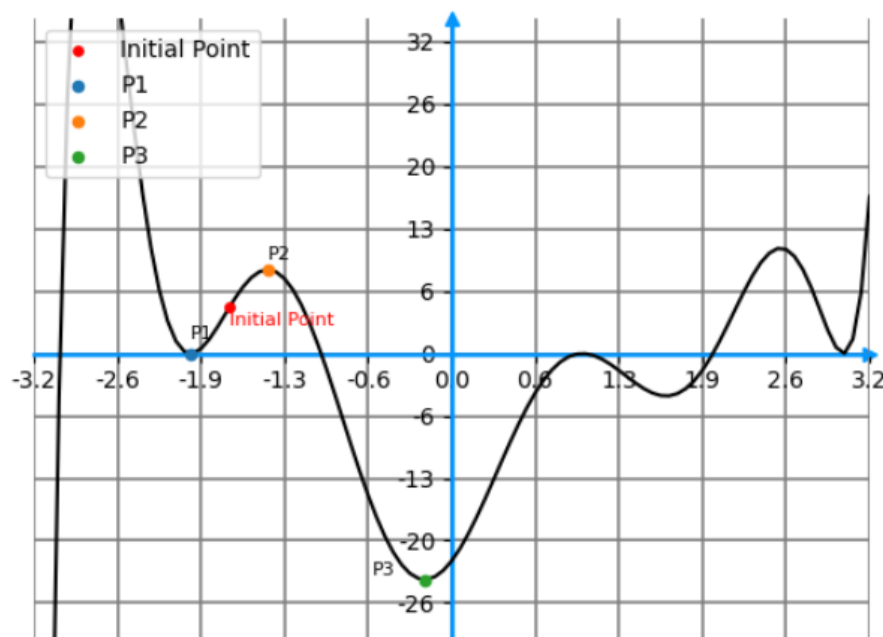
About the Gradient Descent method, choose all that are true:

- a):- It always converges to a local minimum.
- b):- The result may vary depending on the initial point.
- c):- If it converges, then it converges to a global minimum.
- d):- It only works for differentiable functions

Answer:- b and d

Question 2

Given the Initial Point on the following graph, to which point will the Gradient Descent method converge?



a):- P1

b):- P2

c):- P3

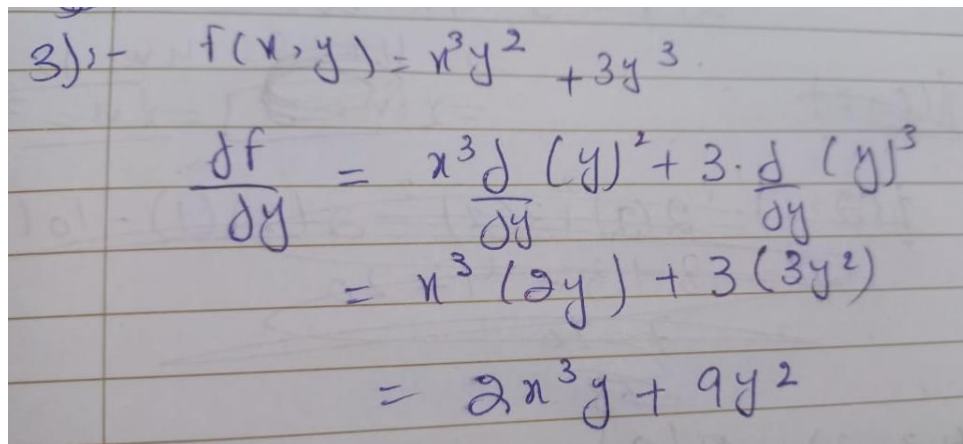
d):- it won't converge.

Answer:- a

P1 is the point. Gradient descent will reach the fastest.

Question 3

Given the $f(x,y) = x^3 y^2 + 3y^3$, find its derivative with respect to y , i.e., find $\partial f / \partial y$



3):- $f(x,y) = x^3 y^2 + 3y^3$

$$\begin{aligned}\frac{\partial f}{\partial y} &= x^3 \frac{\partial}{\partial y} (y)^2 + 3 \cdot \frac{\partial}{\partial y} (y)^3 \\ &= x^3 (2y) + 3(3y^2) \\ &= 2x^3 y + 9y^2\end{aligned}$$

Question 4

Given $f(x,y) = 2x^2 + 3y^2 - 2xy + 10x$, the minimum value of $f(x,y)$ is

a):- -15

b):- 3

c):- 1

Answer:- a

4).

$$f(x, y) = 2x^2 + 3y^2 - 2xy - 10x$$

$$\frac{\partial f}{\partial x} = 4x - 2y - 10$$

$$\frac{\partial f}{\partial y} = 6y - 2x$$

$$\Rightarrow \begin{aligned} 4x - 2y &= 10 \rightarrow \textcircled{1} \\ -2x + 6y &= 0 \rightarrow \textcircled{2} \end{aligned}$$

$$\textcircled{1} + 2\textcircled{2}$$

$$\begin{aligned} 4x - 2y &= 10 \\ -4x + 12y &= 0 \end{aligned}$$

$$10y = 10 \Rightarrow y = 1$$

$$4x = 12 \Rightarrow x = 3$$

$$\Rightarrow x = 3, y = 1$$

$$f(3, 1) = 2(3)^2 + 3(1)^2 - 2(3)(1) - 10(3)$$

$$= 18 + 3 - 6 - 30$$

$$= -15$$

Question 5

What are the parameters that the Gradient Descent algorithm has? (check all that apply)

a):- initial point

b):- final point

c):- learning rate

d):- number of iterations

Answer:- a,c,d

Question 6

Let $f(x, y) = x^2 + y^2 - 6x$ and $\nabla f(x, y) = \begin{bmatrix} 2x - 6 \\ 2y \end{bmatrix}$ and let the initial point $x_0 = (0, 1)$. Performing the gradient descent algorithm with learning rate = 0.1, the first iteration will lead us the point x_1 which is:

a):- $x_1 = (0.6, 0.8)$

b):- $x_1 = (-6, 2)$

c):- $(6, -1)$

d):- $(0, 1)$

Answer:-

$$b) \quad b(x, y) = x^2 + y^2 - 6xy.$$

$$\nabla b(x, y) = \begin{bmatrix} 2x - 6 \\ 2y \end{bmatrix}.$$

$$x^0 = (0, 1).$$

$$\alpha = 0.1$$

$$x_1 = x^0 - \alpha \cdot \frac{\partial f}{\partial x}$$

$$= 0 - 0.2(-6)$$

$$x_1 = 0.6$$

$$y_1 = y^0 - \alpha \cdot \frac{\partial f}{\partial y}$$

$$= 1 - 0.2(2)$$

$$= 0.8.$$

$$\Rightarrow (0.6, 0.8)$$