Quiz #1 Week 4 : Eigen values and Eigen vectors

Question 1

Select the characteristic polynomial for the given matrix.

$$\begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix}$$

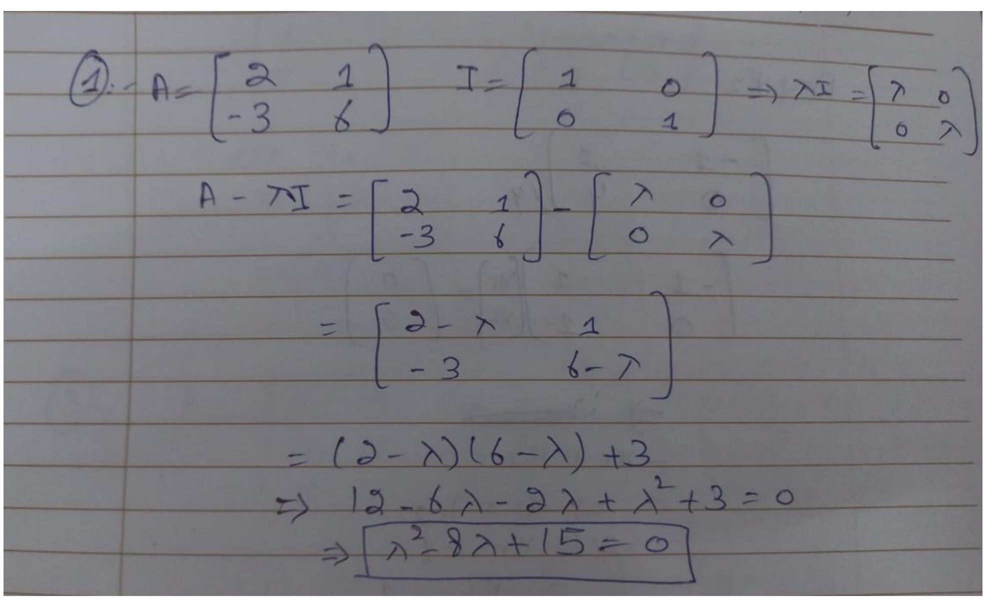
a):- λ^3 - 8 λ + 15

b):- λ^2 - 8 λ - 1

c):- λ^2 + 8 λ + 15

d):- λ^2 - 8 λ + 15

Answer:- d



Question 2

Select the eigenvectors for the previous matrix in Q1, as given below:

$$\begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix}$$

a):- $\begin{matrix}\mathbf{1}\\\mathbf{1}\end{matrix}, \begin{matrix}\mathbf{1}\\\mathbf{1}\end{matrix}$

b):- $\begin{matrix}\mathbf{1}\\\mathbf{3}\end{matrix}, \begin{matrix}\mathbf{1}\\\mathbf{3}\end{matrix}$

c):- $\begin{matrix}\mathbf{1}\\\mathbf{3}\end{matrix}, \begin{matrix}\mathbf{1}\\\mathbf{1}\end{matrix}$

d):- $\begin{matrix}\mathbf{1}\\\mathbf{0}\end{matrix}, \begin{matrix}\mathbf{0}\\\mathbf{1}\end{matrix}$

Answer:- c

②:- for eigen vectors we need to calculate the eigen values first.

$$\lambda^2 - 5\lambda - 3\lambda + 15 = 0$$

$$\lambda(\lambda - 5) - 3(\lambda - 5) = 0$$

$$\Rightarrow \boxed{\lambda = 3} \ \& \ \boxed{\lambda = 5}$$

for $\lambda = 3$

$$(A - \lambda I)x = 0$$

$$\begin{bmatrix} 2-3 & 1 \\ -3 & 6-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

let's reduce the matrix in the row echelon form:-

$R_1 + R_2$

$$\begin{bmatrix} -1 & 1 \\ 0 & 4 \end{bmatrix} /4$$

$$\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \cancel{x_2 = 0}$

$-x_1 + x_2 = 0$

$\Rightarrow x_1 = x_2$

let $x_1 = 1$

$\Rightarrow x_2 = 1$

So eigen vector $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

for $\lambda = 5$

$$\begin{bmatrix} 2-5 & 1 \\ -3 & 6-5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$R_1 - R_2$ $$\begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

as one row is zero so we can put it equal to free variable

i.e. $x_2 = t$

$-3x_1 + x_2 = 0$

let $t = 1 \Rightarrow -3x_1 = -1 \Rightarrow x_1 = 1/3$ $\Rightarrow \vec{V_2} = \begin{pmatrix} 1/3 \end{pmatrix}$

$$\begin{bmatrix} 1 & -1/3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

let $x_2 = t$

$x_1 - \frac{1}{3} t = 0$

$\Rightarrow \boxed{x_1 = 1/3\, t}$

Question 3

Which of the following is an eigenvalue for the given identity matrix.

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

a):- λ=2

b):- λ=-1

c):- $\lambda = 1$

Answer:- c

3):- $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \lambda I \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$

$= 1-\lambda\left[(1-\lambda)(1-\lambda)\right] - 0(-) + 0(-)$

$= (1-\lambda)(1-\lambda-\lambda+\lambda^2)$

$= (\lambda^2 - 2\lambda + 1)(-\lambda + 1) = 0$

$= -\lambda^3 + 2\lambda^2 - \lambda + \lambda^2 - 2\lambda + 1 = 0$

$\Rightarrow -\lambda^3 + 3\lambda^2 - 3\lambda + 1 = 0$

$\boxed{\lambda = 1}$

all three eigen values are same and equal to 1

Question 4

Find the eigenvalues of matrix A·B where:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hint: What type of matrix is B? Does it change the output when multiplied with A? If not, focus only on one of the matrices to find the eigenvalues.

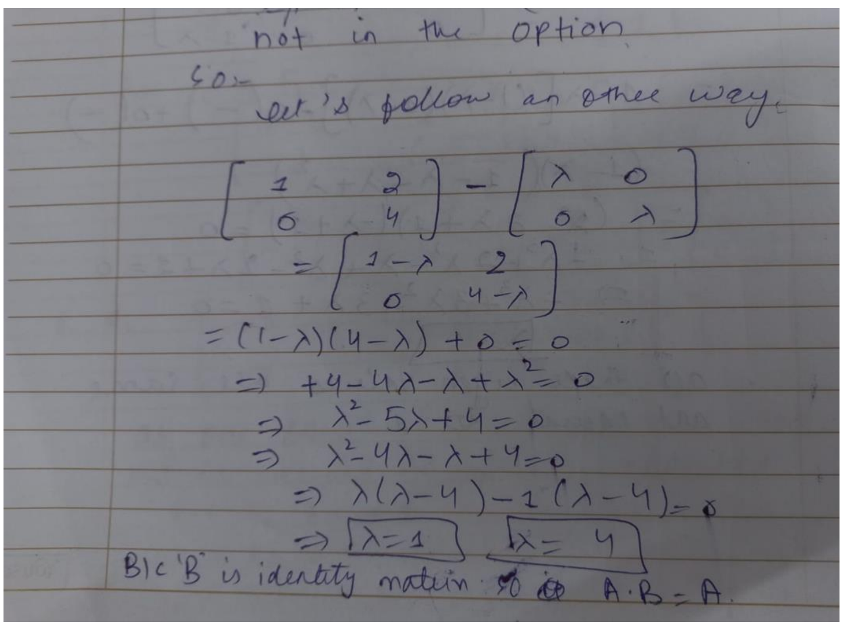
a):- $\boldsymbol{\lambda_1 = 4\,, \lambda_2 = 2}$

b):- $\boldsymbol{\lambda_1 = 4\,, \lambda_2 = 1}$

c):- Eigen values can not be determined

d):- $\boldsymbol{\lambda_1 = 3\,, \lambda_2 = 1}$

Answer:- b



not in the option

so:-

let's follow an other way.

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 \\ 0 & 4-\lambda \end{bmatrix}$$

$$= (1-\lambda)(4-\lambda) + 0 = 0$$

$$\Rightarrow +4 - 4\lambda - \lambda + \lambda^2 = 0$$

$$\Rightarrow \lambda^2 - 5\lambda + 4 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda - \lambda + 4 = 0$$

$$\Rightarrow \lambda(\lambda - 4) - 1(\lambda - 4) = 0$$

$$\Rightarrow \boxed{\lambda = 1} \quad \boxed{\lambda = 4}$$

B/c 'B' is identity matrix so $A \cdot B = A$.

Question 5

Select the eigenvectors, using the eigenvalues you found for the above matrix A·B in Q4.

a):- $\vec{v}_1$= (2,3); $\vec{v}_2$= (1,0);

b):- $\vec{v}_1$= (2,0); $\vec{v}_2$= (1,0);

c):- $\vec{v}_1$= (2,3); $\vec{v}_2$= (2,3);

d):- $\vec{v}_1$= (1,3); $\vec{v}_2$= (1,0);

Answer:- a

(5). $\lambda = 1, \lambda = 4$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

for $\lambda = 1$

$$\begin{bmatrix} 1-1 & 2 \\ 0 & 4-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$3R_1 - 2R_2$
$$\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

let $x_2 = t$

and $x_1 = 0$.

$\Rightarrow \vec{V_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

for $\lambda = 4$

$$\begin{bmatrix} -3 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

let $x_2 = t$

$-3x_1 + 2t = 0$

$\Rightarrow \cancel{-}3x_1 = \cancel{-}2t$

$\Rightarrow x_1 = \frac{2}{3}t$

for $x_2 = 3$

$\Rightarrow x_1 = 2$

$\Rightarrow \vec{V_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$

Question 6

Which of the vectors span the matrix

$$w = \begin{matrix} 2 & 3 & 0 \\ 1 & 2 & 5 \\ 3 & -2 & -1 \end{matrix}$$

a):- $\vec{v}_1 = \begin{matrix} 2 \\ 1 \\ 3 \end{matrix}\ \vec{v}_2 = \begin{matrix} 3 \\ 2 \\ -2 \end{matrix}\ \vec{v}_3 = \begin{matrix} 0 \\ 5 \\ -1 \end{matrix}$

b):- $\vec{v}_1 = \begin{matrix} 2 \\ 3 \\ 0 \end{matrix}\ \vec{v}_2 = \begin{matrix} 1 \\ 2 \\ 5 \end{matrix}\ \vec{v}_3 = \begin{matrix} 3 \\ -2 \\ -1 \end{matrix}$

Answer:- a

There are linearly independent columns that span the matrix, which individually form three vectors. These vectors span the matrix W.

Question 7

Given matrix P select the answer with the correct eigenbasis.

$$\boldsymbol{P} = \begin{matrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{matrix}$$

Hint: First compute the eigenvalues, eigenvectors and contrust the eigenbasis matrix with the spanning eigenvectors.

a):- $\boldsymbol{Eigenbasis} = \begin{matrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$

b):- $\boldsymbol{Eigenbasis} = \begin{matrix} 0 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$

c):- $\boldsymbol{Eigenbasis} = \begin{matrix} 0 & -1 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{matrix}$

Answer:- b

(7):-

$$P = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 0 & 0 \\ 1 & 2-\lambda & 1 \\ -1 & 0 & 1-\lambda \end{bmatrix}$$

$$= (2-\lambda)\left[(2-\lambda)(1-\lambda)\right] + 0$$

$$= (2-\lambda)(2 - 2\lambda - \lambda + \lambda^2) = 0$$

$$= (2-\lambda)(\lambda^2 - 3\lambda + 2) = 0$$

$$= 2\lambda^2 - 6\lambda + 4 - \lambda^3 + 3\lambda^2 - 2\lambda = 0$$

$$= -\lambda^3 + 5\lambda^2 - 8\lambda + 4 = 0$$

$$\cancel{\lambda = 1, 1, 2}$$

$$\lambda = 2, 2, 1$$

for $\lambda = 2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 + R_3$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

let $x_3 = t$, $x_1 = \mu$

$x_1 + x_3 = 0$

$\Rightarrow x_1 = -t$ $\quad \Rightarrow \vec{v} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

let $\lambda = 1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

let $x_3 = t$

$x_1 = 0$

$0 + x_2 + t = 0$

$\Rightarrow \boxed{x_2 = -t}$

$\Rightarrow \vec{V_2} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}.$

Question 8

Select the characteristic polynomial for the given matrix.

$$\begin{matrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ 2 & 1 & -1 \end{matrix}$$

a):- $\lambda^3 + 2\lambda^2 + 4\lambda - 5$

b):- $-\lambda^3 + 2\lambda^2 + 4\lambda - 5$

C):- $2\lambda^3 - \lambda^2 + 4\lambda - 5$

d):- $-\lambda^3 + 2\lambda^2 + 9$

Answer:- b

8:
$$\begin{bmatrix} 3-\lambda & 1 & +2 \\ 4 & -\lambda & 1 \\ 2 & 1 & -1-\lambda \end{bmatrix}$$

$$= (3-\lambda)\left[\lambda(\lambda+1)-1\right] - 1\left[-4-4\lambda-2\right] - 2\left[4+2\lambda\right.$$

$$= (3-\lambda)\left[\lambda^2+\lambda-1\right] - 1\left[-4\lambda-6\right] - 8 - 4\lambda$$

$$= 3\lambda^2 + 3\lambda - 3 - \lambda^3 - \lambda^2 + \lambda + 4\lambda + 6 - 8 - 4\lambda$$

$$\Rightarrow -\lambda^3 + 2\lambda^2 + 4\lambda - 5 = 0$$

Question 9

You are given a non-singular matrix A with real entries and eigenvalue $\boldsymbol{i}$.

Which of the following statements is correct?

a):- 1/*i* is an eigenvalue of A^{-1}

b):- *i* is an eigenvalue of $A^{-1} + A$

c):- *i* is an eigenvalue of $A^{-1}.A.I$

d):- $det(A.B)^{-1}$ can not be computed

Answer:- a