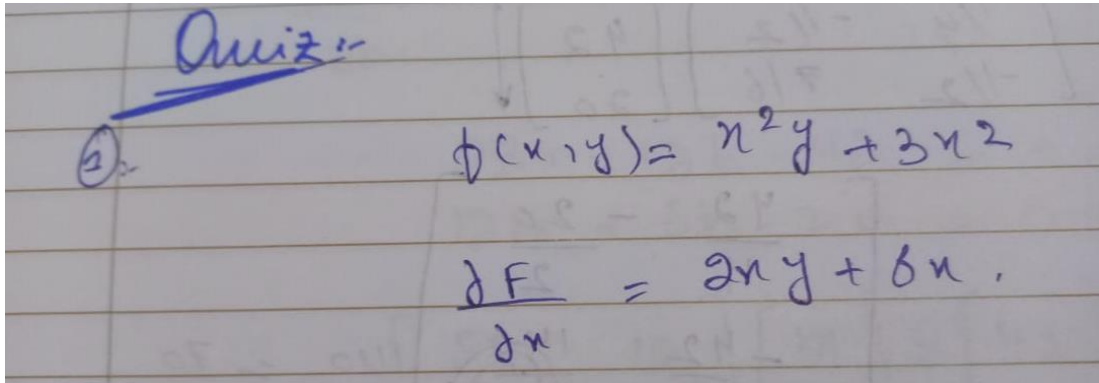


Quiz #1 : Partial Derivatives and Gradients

Question 1

Given the $f(x,y) = x^2 y + 3x^2$, find its derivative with respect to x , i.e., find $\partial f / \partial x$



A photograph of a handwritten solution on lined paper. At the top, 'Quiz:-' is written in blue ink and underlined. Below it, a circled '2' is written. To the right, the function $f(x,y) = x^2 y + 3x^2$ is written. Below that, the partial derivative $\frac{\partial F}{\partial x} = 2xy + 6x$ is written.

Question 2

Given that $f(x, y) = xy^2 + 2x + 3y$ its gradient, i.e., $\nabla f(x, y)$ is:

a):-

$$\begin{bmatrix} 2xy + 3 \\ y^2 + 2 \end{bmatrix}$$

b):-

$$\begin{bmatrix} 2xy \\ 2x + 3 \end{bmatrix}$$

c):-

$$\begin{bmatrix} y^2 + 2 \\ 2xy + 3 \end{bmatrix}$$

d):-

$$\begin{bmatrix} 2y \\ 0 \end{bmatrix}$$

Answer:- c

Q:-

$$f(x, y) = xy^2 + 2x + 3y$$

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \rightarrow (1)$$
~~$$\frac{\partial f}{\partial x} = 1(6xy + 2) \rightarrow (A)$$~~

$$\frac{\partial f}{\partial x} = y^2 + 2 \rightarrow (A)$$

$$\frac{\partial f}{\partial y} = 2xy + 3 \rightarrow (B)$$

Put (A) & (B) in (1):-

$$\nabla f(x, y) = \begin{bmatrix} y^2 + 2 \\ 2xy + 3 \end{bmatrix}$$

Question 3

Given the $f(x, y) = x^2 + 2y^2 + 8y$, The minimum value of f is:

Hint: The question asks for the minimum value that the function can output, and not the point (x, y) that gives it.

③ $f(x, y) = x^2 + 2y^2 + 8y \rightarrow \text{①}$

$$\frac{\partial f}{\partial x} = 2x = 0$$

$$\frac{\partial f}{\partial y} = 4y + 8 = 0$$

$$\Rightarrow \boxed{x = 0}$$

$$4y = -8$$

$$\Rightarrow \boxed{y = -2}$$

\Rightarrow minimum value of
will be at point $(0, -2)$
 \therefore ① becomes-

$$f_{\min} = 0 + 2(-2)^2 + 8(-2)$$

$$= 0 + 2(4) - 16$$

$$= 0 + 8 - 16$$

$$= -8$$

Question 4

The gradient of $f(x, y, z) = x^2 + z^2 + 2xyz$ is ,

a):-

$$\begin{bmatrix} 2x + 2yz \\ 2xz \\ 2xy + 2z \end{bmatrix}$$

b):-

$$\begin{bmatrix} 2x + 2xz \\ 2yz \\ 2xy + z \end{bmatrix}$$

c):-

$$\begin{bmatrix} 2x + 2yz \\ 2xy \\ 2xy + z \end{bmatrix}$$

d):-

$$\begin{bmatrix} 2yz + 2xz \\ 2z \\ 2x \end{bmatrix}$$

Answer:- a

4.

$$f(x, y, z) = x^2 + 2xyz + z^2 \rightarrow (1)$$

$$\nabla f(x, y, z) = \begin{bmatrix} \partial f / \partial x \\ \partial f / \partial y \\ \partial f / \partial z \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = 2x + 2yz \rightarrow (A)$$

$$\frac{\partial f}{\partial y} = 2xz \rightarrow (B)$$

$$\frac{\partial f}{\partial z} = 2xy + 2z \rightarrow (C)$$

→ (4)

$$\nabla f(x, y, z) = \begin{bmatrix} 2x + 2yz \\ 2xz \\ 2xy + 2z \end{bmatrix}$$