Quiz #1 : Week 1 - Practice Quiz

Question 1

1. In a room, there are 200 people.

- 30 of them like only soccer
- 100 of them like only basketball
- 70 of them like both soccer and basketball

What is the probability of a randomly selected person likes basketball **given that** they like soccer?

- ○ $\frac{7}{20}$
- ◉ $\frac{7}{10}$
- ○ $\frac{3}{7}$
- ○ $\frac{1}{2}$

Answer:- b

As events are dependent:

$$P(A \cap B) = P(A) \cdot P(B|A) \rightarrow (1)$$

Let $A \rightarrow$ Basketball

$B \rightarrow$ Soccer

$$P(A|B) = ?$$

$$P(A \cap B) = \frac{70}{200} \Rightarrow P(A \cap B) = 0.35$$

$$P(A) = \frac{100}{200} \Rightarrow P(A) = 0.5$$

$\therefore$ (1) becomes:-

$$0.35 = (0.5)\, P(B|A)$$

$$\Rightarrow P(B|A) = 0.7 = \frac{7}{10}$$

Question 2

2. Consider the following experiment:

You roll a dice. If the result is less than 4 (excluding 4), you roll two dice and sum the results. If the result is greater than 4, you roll only one dice and use the result.

What is the probability of getting a final result of 6 after this experiment?

- (selected) $\frac{11}{72}$
- $\frac{5}{72}$
- $\frac{1}{6}$
- $\frac{5}{36}$

Answer:- a

Correct! If we define $E_{<4}$ as the event of getting a number less than 4 in the first throw and $E_{\geq 4}$ the event of getting a number greater or equal to 4 in the first throw, then

$$P(\text{getting a }6) = P(\text{getting a }6 \mid E_{<4}) + P(\text{getting a }6 \mid E_{\geq 4})$$

If the first dice is less than 4, we throw two dice, thus the probability of getting a 6 is $\frac{5}{36}$, because the possible values for the dice are $(1,5), (2,4), (3,3), (4,2), (5,1)$. The probability of getting a number less than 4 is $\frac{3}{6}$. If the first dice is greater or equal to 4, then we just throw a new dice and get the result, therefore to get 6, there is a chance of $\frac{1}{6}$.

Therefore

$$P(\text{getting a }6) = \frac{5}{36} \cdot \frac{3}{6} + \frac{1}{6} \cdot \frac{3}{6} = \frac{11}{72}$$

Question 3

3. Suppose there is a disease that affects 1% of the population. Researchers developed a diagnostic test for this disease. The test has a sensitivity of 95% (meaning it correctly identifies 95% of people with the disease) and a specificity of 90% (meaning it correctly identifies 90% of people without the disease). If a person tests positive for the disease, what is the probability that they actually have the disease, according to Bayes Theorem?

- ○ 15.58%
- ◉ 8.76%
- ○ 90%
- ○ 42.76%

Answer:- b

(3) disease affects 1% population.

Sensitivity = 95%

Specificity = 90%

P(sick | diagnosed sick) = ?

using Baye's theorem:

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} \mid \text{not sick})} \longrightarrow (1)$$

$P(\text{sick}) = 0.01$

$P(\text{not sick}) = 1 - 0.01 = 0.99$

$P(\text{diagnosed sick} \mid \text{sick}) = 0.95$

$P(\text{diagnosed sick} \mid \text{not sick}) = \cancel{0.9}\ 1 - 0.90$

$= \cancel{0.1}\ 0.$

$\therefore$ (1) becomes.

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{(0.01)(0.95)}{(0.01)(0.95) + (0.99)(0.9)}$$

$$= \frac{0.0095}{0.0095 + 0.891}$$

$$= \frac{0.0095}{0.9005}$$

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$\therefore$ (1) becomes:

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{(0.01)(0.95)}{(0.01)(0.95) + (0.99)(\cancel{0.05})(0.1)}$$

$$= \frac{0.0095}{0.0095 + \cancel{0.0495}\ \ 0.099}$$

$$= \frac{0.0095}{0.059} = \cancel{16\%}\ \ 8.76\%$$

Question 4

4. Consider the following experiment:

You flip a coin 10 times.

What is the probability of getting at least 2 heads?

Hint: You can use the Binomial Distribution to model this experiment. Also, in this case, it might be easier to use the complement rule $P(X \geq 2) = 1 - P(X < 2) = 1 - (P(X = 0) + P(X = 1))$.

- ○ $\frac{2^{10} - 1}{2^{10}}$
- ○ $\frac{2^{10} - 10}{2^{10}}$
- ◉ $\frac{2^{10} - 11}{2^{10}}$
- ○ $\frac{1}{2^{10}}$

Answer:- c

(4). $n = 10$, $P = ?$ $x = 2$, $2^{10} =$
Probability of getting atleast 2 heads $= ?$

→ using formula of binomial distribution

$$PX(x) = \binom{n}{k} (P)^k (1-P)^{n-k} \longrightarrow ①$$

using compliment rule:-

$$P(X \geq 2) = 1 - P(X < 2)$$
$$= 1 - [P(X = 0) + P(X = 1)] \longrightarrow ②$$

$P(X = 0) = ?$

$$Px(x) = p^x \cdot (1-P)^{5-x}$$

for $x = 0$

$P(\cancel{X=0}) =$ 'when head $= 0$ then there will be all tails so $p = \frac{1}{2^{10}}$

$P(X=1)$ means 1 head and 9 tails.

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10 possibilities.

$$P = \frac{10}{2^{10}}.$$

$\cancel{P(X=1)}$ $\therefore$ 2 becomes

$$P(X \geq 2) = 1 - \left[\frac{1}{2^{10}} + \frac{10}{2^{10}}\right]$$

$$= 1 - \left[\frac{11}{2^{10}}\right].$$

$$P(X \geq 2) = \frac{2^{10} - 11}{2^{10}}$$

Question 5

5. Suppose a random variable X is such that $X \sim Uniform(0, 1)$.

The value for $P(X \leq \frac{1}{2})$ is:

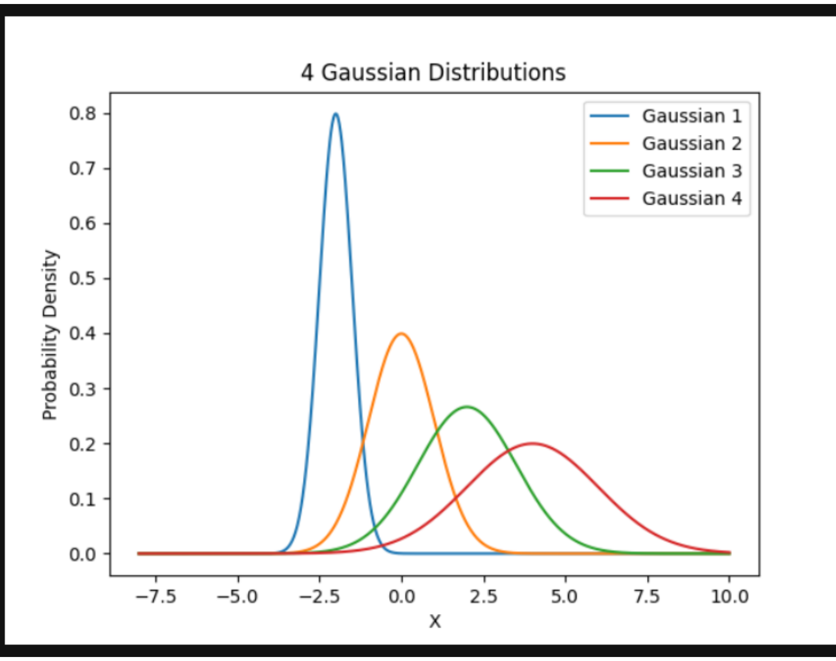
- ○ $\frac{1}{3}$
- ○ 1
- ○ 0
- ◉ $\frac{1}{2}$

Answer:- d

Correct! Since X is equally likely to have any value between 0 and 1, it has a probability of $\frac{1}{2}$ of being less than or equal to $\frac{1}{2}$.

Question 6

6.



About the 4 Gaussians in the graph above, it is correct to say (check all that apply).

- [ ] $\sigma_{\text{Gaussian 1}} > \sigma_{\text{Gaussian 2}}$
- [ ] $\mu_{\text{Gaussian 1}} > \mu_{\text{Gaussian 2}}$
- [x] $\mu_{\text{Gaussian 4}} > \mu_{\text{Gaussian 3}}$

Correct

Correct! The parameter μ controls the center of the distribution, therefore the higher the μ, the farther the center is from the origin.

- [x] $\sigma_{\text{Gaussian 3}} > \sigma_{\text{Gaussian 2}}$

Correct

Correct! The parameter σ controls the spread of the distribution, therefore the higher the σ, more spread the graph is around the center.

- [x] $\sigma_{\text{Gaussian 4}} > \sigma_{\text{Gaussian 1}}$

Correct

Correct! The parameter σ controls the spread of the distribution, therefore the higher the σ, more spread the graph is around the center.

(6):-

$\mu_{gaussian\,1} = -2.5$

$\mu_{gaussian\,2} = 0$

$\mu_{gaussian\,3} = 2.5$

$\mu_{gaussian\,4} = 5$

$\sigma_{gaussian\,1} \simeq 2.0$

$\sigma_{gaussian\,4} \simeq 3.5$

$\sigma_{gaussian\,2} \simeq 2.5$

$\sigma_{gaussian\,3} \simeq 3$

Question 7

7. You roll a dice 20 times and count how many times the number 4 appears.

If X is the number of times the number 4 appears, then $X \sim Binomial(n, p)$, where n and p are:

- [ ] $n = \frac{1}{6}, p = 20$
- [ ] $n = \frac{1}{2}, p = 4$
- [ ] $n = 4, p = \frac{1}{2}$
- [x] $n = 20, p = \frac{1}{6}$

Answer:- d

7.:- $n = 20$, ~~k =~~ ~~count how many~~ times

$k = 4$.

$n = 20$

Probability of getting a 4 $= 1/6$.

Question 8

8. You have to work with the following random variable: the height of people in a country. What is the best distribution to model this random variable from the options below?

( ) Uniform Distribution

(•) Normal Distribution

( ) Binomial Distribution

Correct

Correct! In this case it is reasonable to suppose that the random variable follows a normal distribution!

Answer:- b