

## Quiz #4 : The Rank of matrix

### Question 1

You are a lead engineer at Stark Industries working on robotics special projects. You stumble upon the old schematics of the Iron Man suit and decide to take on an ambitious project. As a savvy engineer, you realize the potential of upgrading the exoskeleton to help people walk after a neurological injury.

To ensure your exoskeleton is affordable and slim, you generate a new composite structure combining fiberglass, aluminum, and carbon nanotube materials. Ultimately, you need to assess the price of each material.

1st iteration: You use 7 units of fiberglass, 5 units of aluminum, and 3 units of carbon nanotubes, which cost \$120.

2nd iteration: You engineer a less wasteful process that uses 3 units of fiberglass, 2 units of aluminum, and 5 units of carbon nanotubes to produce the same amount of composite, the total cost is \$70.

3rd iteration: You combine electrostimulation delivery, which cuts down the cost of the suit by using only 1 unit of fiberglass, 2 units of aluminum, and 1 unit of carbon nanotubes, which cost \$20.

Which of the following represents the correct system of equations?

a):-

$$\begin{cases} f + a + 3c = 100 \\ 3f + 2a + 5c = 20 \\ f + 5a + c = 50 \end{cases}$$

b):-

$$\begin{cases} 7a + 5f + 3c = 120 \\ 2f + 3a + 5c = 70 \\ 2c + a + f = 20 \end{cases}$$

c):-

$$\begin{cases} 7f + 5a + 3c = 120 \\ 3f + 2a + 5c = 70 \\ f + 2a + c = 20 \end{cases}$$

d):-

$$\begin{cases} 7f + 5a + 3c = 120 \\ 3f + 2a + 5c = 70 \end{cases}$$

Answer:- c

Quiz:-

①:- first iteration:-  
 $7f + 5a + 3c = 120 \rightarrow ①$

2nd iteration:-  
 $3f + 2a + 5c = 70 \rightarrow ②$

3rd iteration:-  
 $f + 2a + c = 20 \rightarrow ③$

in matrix form:-

$$\begin{cases} 7f + 5a + 3c = 120 \\ 3f + 2a + 5c = 70 \\ f + 2a + c = 20 \end{cases}$$

### Question 2

Which of the following steps can you take to solve the system of equations? Select all that apply.

a):- Subtract the second row from the first row.

b):- Multiply the first equation by 3 and subtract it from equation 2.

c):- Isolate one variable and substitute into the next equation to find the other variable.

d):- Multiply by a scalar and add the two rows.

e):- Divide the first equation by 7.

Answer:- a,b,c,d,e (all choices)

### Question 3

Which of the following information can you extract from the given system of equations?

a):- Row-reduced echelon form

b):- Whether the matrix is singular or non-singular.

c):- Number of linearly (in)dependent rows and columns.

d):- The rank of the matrix.

e):- The cost of each material.

f):- The weight and shape of each material.

Answer:- a,b,c,d,e

### Question 4

Which of the following matrices represents the system of sentences in Q1 for all three iterations?

a):- 
$$\begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

b):- 
$$\begin{bmatrix} 7 & 5 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\text{c):- } \begin{bmatrix} 7 & 2 & 1 \\ 5 & 3 & 2 \end{bmatrix}$$

$$\text{d):- } \begin{bmatrix} 7 & 2 & 1 \\ 5 & 3 & 2 \\ 3 & 5 & 1 \end{bmatrix}$$

**Answer:- a**

### Question 5

Calculate the cost of each material by solving the system of equations.

Hint: You can use the method of substitution, or row reducing the matrix to a simpler form.

**a):- fiberglass = \$15, aluminum = \$0, carbon nanotubes = \$5**

**b):- fiberglass = \$15, aluminum = \$5, carbon nanotubes = \$0**

**c):- each material = \$15**

**d):- fiberglass = \$5, aluminum = \$0, carbon nanotubes = \$5**

**Answer:- a**

Handwritten solution for Question 5:

$$\textcircled{5} \text{:- } \begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 120 \\ 70 \\ 20 \end{bmatrix}$$

$$A \cdot B = C$$

$$\Rightarrow B = A^{-1}C \rightarrow \textcircled{2} \text{:-}$$

Let's find  $A^{-1}$

$$\left[ \begin{array}{ccc|ccc} 7 & 5 & 3 & 1 & 0 & 0 \\ 3 & 2 & 5 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array} \right]$$

find pivot in first row and swap first row with 3rd row.

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 3 & 2 & 5 & 0 & 1 & 0 \\ 7 & 5 & 3 & 1 & 0 & 0 \end{array} \right]$$

eliminate first column.

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & -4 & 2 & 0 & 1 & -3 \\ 0 & -9 & -4 & 1 & 0 & -7 \end{array} \right]$$

÷ 2nd row by -4

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1/2 & 0 & -1/4 & 3/4 \\ 0 & -9 & -4 & 1 & 0 & -7 \end{array} \right]$$

eliminate 2nd column.

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1/2 & 0 & -1/4 & 3/4 \\ 0 & 0 & -17/2 & 1 & -9/4 & -11/4 \end{array} \right]$$

divide 3rd row by -17/2

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1/2 & -1/2 \\ 0 & 1 & -1/2 & 0 & -1/4 & 3/4 \\ 0 & 0 & 1 & -2/17 & 9/34 & 1/34 \end{array} \right]$$

eliminate 3rd column

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 4/17 & -1/34 & -19/34 \\ 0 & 1 & 0 & -1/17 & -2/17 & 13/17 \\ 0 & 0 & 1 & -2/17 & 9/34 & 1/34 \end{array} \right]$$

So

$$A^{-1} = \begin{bmatrix} 4/17 & -1/34 & -19/34 \\ -1/17 & -2/17 & 13/17 \\ -2/17 & 9/34 & 1/34 \end{bmatrix} \begin{bmatrix} 120 \\ 70 \\ 20 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 18 \\ 0 \\ 5 \end{bmatrix}$$

### Question 6

Use the determinant to find if the matrix is singular or non-singular. Is the matrix in Row-echelon form or Reduced row-echelon form?

$$\begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

a, b, and c are non-zero real numbers. If the matrix is non-singular, which of the following must be true:

- a):- -34, Non-singular, Neither
- b):- -30, Non-singular, Both
- c):- 34, Non-singular, Neither

d):- 0, Singular, Reduced row-echelon form

Answer:- a

⑥:- determinant:-

	+	-	+
	7	5	3
	3	2	5
	1	2	1

$$\begin{aligned} &= 7(2-10) - 5(3-5) + 3(6-2) \\ &= 7(-8) - 5(-2) + 3(4) \\ &= -56 + 10 + 12 \\ &= -46 + 12 \\ &= -34 \end{aligned}$$

determinant is non zero so system is non singular, given matrix in its standard form is neither in REF nor in RREF.

### Question 7

What is the rank in the above matrix?

a):- 2

b):- 1

c):- 3

d):- 0

Answer:- c

(7) 
$$\begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

for rank let's convert the matrix into row echelon form.

$$\begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ \textcircled{1} & 2 & 1 \end{bmatrix}$$

$R_2 - 3R_3$

$$\begin{bmatrix} 7 & 5 & 3 \\ \textcircled{3} & 2 & 5 \\ 0 & \textcircled{-4} & 2 \end{bmatrix}$$

$2R_2 + R_3$

$$\begin{bmatrix} 7 & 5 & 3 \\ 0 & 1 & -26 \\ 0 & -4 & 2 \end{bmatrix}$$

$3R_1 - 7R_2$

$$\begin{bmatrix} 7 & 5 & 3 \\ 0 & 1 & -26 \\ 0 & -4 & 2 \end{bmatrix}$$

div  $R_3$  by 2

$$\begin{bmatrix} 7 & 5 & 3 \\ 0 & 1 & -26 \\ 0 & \textcircled{-2} & 1 \end{bmatrix}$$

$2R_2 + R_3$

$$\begin{bmatrix} 7 & 5 & 3 \\ 0 & 1 & -26 \\ 0 & 0 & -51 \end{bmatrix}$$

$4 \times 5 = 20$   
 $12 \times 5 = 60$   
 $12 \times 5 = 60$

Date: / /

$$\begin{bmatrix} 7 & 5 & 3 \\ 0 & 1 & -26 \\ 0 & 0 & 1 \end{bmatrix}$$

The rank is equal to number of pivots i.e. 3 here.

### Question 8

To assist you with your design choices, your AI assistant compiles a few matrices with different combinations of materials. Since your experiments are not free, you want to try the option that gives you the highest amount of information.

Sort the matrices from the one that provides the lowest amount of information to the highest ( from the lowest rank to the highest rank).



$$\begin{array}{ccc} 0 & 1 & 1 \\ \text{a):-} & 2 & 4 & 2, \\ 1 & 2 & 1 \end{array} \quad \begin{array}{ccc} 7.5 & 5 & 12.5 \\ \text{b):-} & 3 & 2 & 5, \\ 0 & 0 & 0 \end{array} \quad \begin{array}{ccc} 7 & 5 & 3 \\ \text{c):-} & 3 & 2 & 5 \\ 1 & 2 & 1 \end{array}$$

Hint: To help you get started, determine which matrices have linearly dependent rows. You've already found the rank of the third matrix!

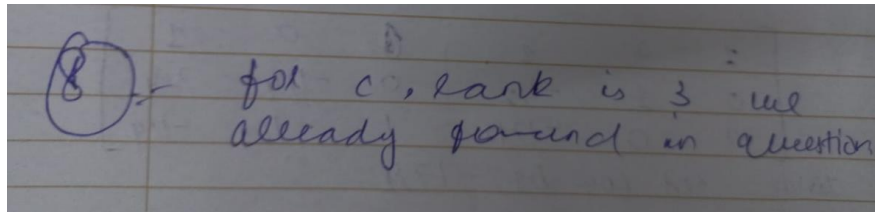
a):- b,a,c

b):- a,b,c

c):- b,c,a

d):- c,a,b

Answer:- a



for  $a = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 4 & 2 \\ \textcircled{1} & 2 & 1 \end{bmatrix}$

$R_2 \rightarrow 2R_3$

$$\begin{bmatrix} 0 & 1 & 1 \\ \textcircled{2} & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 2 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

let's swap  $R_1$  and  $R_2$

$$\begin{bmatrix} 2 & 4 & 2 \\ 0 & 1 & 1 \\ \textcircled{2} & 2 & 1 \end{bmatrix}$$

$\odot R_1 \rightarrow 2R_3$

$$\begin{bmatrix} 2 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2 is rank b/c 2 elements are in pivot:-

for b matrix

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$$b = \begin{bmatrix} 7.5 & 5 & 12.5 \\ \textcircled{3} & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$3R_1 - 7.5R_2$

$$\begin{bmatrix} 7.5 & 5 & 12.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$15 - 15$

$37.5 - 37.5$

Rank is 1

So answer is

b, a, c

### Question 9

To further optimize the cost of materials, you finally reduce your number of iterations to only 2 tries, where you now obtain a 2x2 matrix with rank 1.

**Which of the following is your matrix?**

**Hint: Which of the following 2x2 matrices have rank = 1?**

a):-  $\begin{bmatrix} 5 & 2 \\ 10 & 3 \end{bmatrix}$

b):-  $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

c):-  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

**Answer:- b**

⑨:-

let's find rank of each one of the given matrices.

$$\begin{bmatrix} 5 & 2 \\ 10 & 3 \end{bmatrix} \Rightarrow 2R_1 - R_2 \Rightarrow \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2/5 \\ 0 & 1 \end{bmatrix}$$

rank is 2

$$b):- \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \Rightarrow 2R_1 - R_2 \Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$  rank is 1.

$$c):- \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \text{ rank is } 0.$$

answer is b.

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