

MEC Day 4 Assignment Results

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Q1 Choo-Siow

In Choo-Siow set-up, the matching function is:

$$\mu_{xy} = \sqrt{\mu_{x0}\mu_{0y}} \exp\left(\frac{\alpha_{xy} + \gamma_{xy}}{2}\right)$$

Using IPFP algorithm, here are the results:

```
[1] "IPFP converged in 13 steps and 0.03900000000000015s."
[1] "Precision error is = 2.98404245091533e-07"
[1] "The total number of matched pairs = 30.2491899176455"
[1] "The average wage is = -5.10523972444432"
```

The average wage is negative, due to the scarcity of firms in this problem.

Q2 linear tax

Applying the following relationship between μ_{xy} and distance function:

$$\mu_{xy} = \exp(-D_{xy}(-\ln(\mu_{x0}), -\ln(\mu_{0y})))$$

Here with linear tax as in this case, the matching function is:

$$\mu_{xy} = \mu_{x0}^{\frac{1}{1.8}} \mu_{0y}^{\frac{0.8}{1.8}} \exp\left(\frac{\alpha_{xy} + 0.8\gamma_{xy}}{1.8}\right)$$

Using IFPF algorithm, here are the results:

```
[1] "Linear tax model"
[1] "IPFP converged in 13 steps and 0.7960000000000021s."
[1] "Precision error is = 7.66787443540196e-07"
[1] "The total number of matched pairs = 30.2497481017962"
[1] "The average wage is = -6.38152307611866"
[1] "The average payoff to workers is = -5.10521846089492"
```

Since the average wage is negative, hence a tax de-facto serves as a subsidy. With a subsidy, the payment firms receive increases (from 5.1 to 6.38), while payment from workers rarely changes (from 5.105 to 5.105). Benefit of subsidy mostly falls on firms side, which is also due to the scarcity of firms.

Q3 progressive tax

With progressive tax,

$$D_{xy} = \max_k D_{xy}^k(u, v)$$

Hence the matching function is:

$$\mu_{xy} = \min_k \mu_{xy}^k$$

where

$$\begin{aligned}\mu_{xy}^1 &= \sqrt{\mu_{x0}\mu_{0y}} \exp\left(\frac{\alpha_{xy} + \gamma_{xy}}{2}\right) \\ \mu_{xy}^2 &= \mu_{x0}^{\frac{1}{1.8}} \mu_{0y}^{\frac{0.8}{1.8}} \exp\left(\frac{\alpha_{xy} + 0.1 - 0.08 + 0.8\gamma_{xy}}{1.8}\right) \\ \mu_{xy}^3 &= \mu_{x0}^{\frac{1}{1.6}} \mu_{0y}^{\frac{0.6}{1.6}} \exp\left(\frac{\alpha_{xy} + 0.18 - 0.12 + 0.6\gamma_{xy}}{1.6}\right)\end{aligned}$$

Using IFPF algorithm, here are the results:

```
[1] "Progressive tax model"
[1] "IPFP converged in 13 steps and 4.43100000000001s."
[1] "Precision error is = 2.15244417844929e-07"
[1] "The total number of matched pairs = 30.466593050916"
[1] "The average wage is = NaN"
```

Wage is "NaN" because some μ are zero and $\log(0)-\log(0)$ creates "NaN". My code needs to be debugged..

Q4 Marital preference, private consumption and public goods

Conditional on g , define:

$$\begin{aligned}\alpha_{xy}^g &= 1 - 0.1d(x, y)^2 + 0.2g + \ln(10 - 2.5g) \\ \gamma_{xy}^g &= 1 - 0.2d(x, y)^2 + 0.1g + \ln(10 - 2.5g)\end{aligned}$$

Then the feasible set is:

$$\mathcal{F}_{xy}^g = \{(u, v) : \exp(u - \alpha_{xy}^g) + \exp(v - \gamma_{xy}^g) \leq 2\}$$

The distance function is:

$$D_{xy}^g(u, v) = \log\left(\frac{\exp(u - \alpha_{xy}^g) + \exp(v - \gamma_{xy}^g)}{2}\right)$$

This is an un-regularized problem, let me first solve the regularized one with $T = 1$, and the matching function is:

$$\mu_{xy}^g = \frac{2\mu_{x0}\mu_{0y} \exp(\alpha_{xy}^g + \gamma_{xy}^g)}{\mu_{x0} \exp(\alpha_{xy}^g) + \mu_{0y} \exp(\gamma_{xy}^g)}$$

(This has increasing economics of scale, not sure about the intuition..) The matching function for the general problem is:

$$\mu_{xy} = \max_g \mu_{xy}^g$$