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Trends and Cycles in Macroeconomic Time Series

A. C. Harvey
London School of Economics, Houghton St., London WC2A 2AE, England

Two structural time series models for annual observations are constructed in terms of trend, cycle, and irregular components. The models are then estimated via the Kalman filter using data on five U.S. macroeconomic time series. The results provide some interesting insights into the dynamic structure of the series, particularly with respect to cyclical behavior. At the same time, they illustrate the development of a model selection strategy for structural time series models.

KEY WORDS: Autoregressive integrated moving average model; Kalman filter; Model selection; State-space model; Structural time series model; Unobserved components.

1. INTRODUCTION

The decomposition of economic time series into trend and cyclical components plays an important role in much of macroeconomics. For example, it is common practice to regard the trend as a deterministic function of time and the cyclical component as a stationary process that exhibits transitory movements around the trend. This view leads economists to analyze detrended data and construct models relating different variables from such data. If the trend components in economic time series are not deterministic, however, Nelson and Kang (1981, 1984) showed that such an approach can lead to very misleading inferences being drawn.

Nelson and Plosser (1982) provided some empirical evidence on the properties of U.S. macroeconomic time series. They set up two models: the trend stationary (TS) model in which

$$y_t = \alpha + \beta t + w_t, \qquad (1.1)$$

where y_t is the natural logarithm of the observations and w_t is a stationary and invertible autoregressive moving average (ARMA) process, and the difference stationary (DS) model

$$\Delta y_t = \beta + v_t, \qquad (1.2)$$

where v_t is a stationary and invertible ARMA process (see also Beveridge and Nelson 1981). The evidence from the testing procedure of Dickey and Fuller (1979) indicates that for no series is it possible to confidently reject the null hypothesis of the DS model against the alternative of a TS model. As Nelson and Plosser were careful to point out, however, acceptance of the null hypothesis is not disproof of the alternative. They therefore examined the correlograms to see what these suggest about the nature of the series. For most of the series, the correlograms of first differences (of the logarithms) have sample autocorrelations that are positive and significant at lag one but not significant at higher lags. Nelson and Plosser argued that this constitutes strong evidence in favor of the DS model. They then argued further that if the autocorrelation function of the first differences of a process exhibits a positive first-order autocorrelation and zero higher-order autocorrelations, this seems to preclude the possibility of constructing a sensible unobserved components model in which y_t is the sum of a *stochastic* trend and a stationary process.

The aim of this article is to examine these questions further by adopting a somewhat different methodological stance. Nelson and Plosser followed the approach pioneered by Box and Jenkins (1976) in which the observations themselves are used to identify a suitably parsimonious model from the class of autoregressive integrated moving average (ARIMA) processes. Thus they effectively identified an ARIMA(0, 1, 1) process as being appropriate for virtually all of their series, and they then drew conclusions about the nature of these series on the basis of the properties of this process. The view taken here is that the ARIMA class is a very broad one and that attempts to identify suitable models on the basis of the data alone can lead to unsatisfactory results. In some cases, as argued in Harvey and Todd (1983), the models may yield poor forecasts. More generally, there is no guarantee that an ARIMA model identified from the data will have the kind of properties that a particular economic time series is postulated to exhibit. As a rule, there will be several ARIMA models that are consistent with a given set of data, in the sense that they exhibit a good fit, or to put it another way, have an autocorrelation function (at the appropriate degree of differencing) that has similar properties to the observed correlogram. It is often difficult to see exactly what properties these different ARIMA specifications will have, particularly with regard to potential decompositions into trend, cycle, and seasonal components. An alternative approach is therefore to work with a class of unobserved components models that have these properties explicitly built into their structure.

The plan of this article is as follows: After a preliminary discussion of the data in Section 2, the evidence in the correlogram is reexamined in Section 3 in the light of various formulations of trend plus cycle models, of which (1.1) is a special case. An alternative class of models in which the cycle is incorporated into the trend is also proposed. Both classes of models, which in the terminology of Harvey and Todd (1983) and Engle (1978) are "structural" models, are shown to be consistent with correlograms found for the series in question. In Section 4, the statistical properties of these structural models are discussed. The series are analyzed in Sections 5 and 6, and estimates of the unknown structural parameters are computed by a maximum likelihood procedure based on the Kalman filter. Conclusions and extensions are presented in Section 7.

2. THE DATA

The data used by Nelson and Plosser (1982) consist of 14 U.S. macroeconomic time series. The series are annual, with starting dates from 1860 to 1909. The final year is 1970 in all cases. In this study attention is concentrated on just five of these series, primarily because many of the series—for example real GNP, nominal GNP, and real per capita GNP—display fairly similar characteristics. The series considered are real GNP, industrial production, the unemployment rate, consumer prices, and common stock prices. Details of sources can be found in Nelson and Plosser (1982, p. 146).

The figures for real GNP are of particular interest in a study of cycles. Figure 1 shows the natural logarithm of real GNP and the first differences of the logarithms. It is apparent from these graphs that there is a fairly dramatic change in the series after about 1947. As one might expect, the movements in postwar GNP are much smoother. (The years 1946 and 1947 represent a settling down period after the war). This shows up clearly in the correlograms of the logs of first differences for the period up to and including 1947 and the post-1947 period (see Figure 2). The correlogram for the earlier period is very similar to the correlogram that Nelson and Plosser (1982, table 3) gave for the whole period. The correlogram for the later period is not. In particular, it does not display a high positive value for the autocorrelation at lag one. Thus the arguments put forward by Nelson and Plosser on the nature of cyclical decompositions are only valid for the data up to and including 1947. A study of the other series yields similar conclusions (see the correlograms in Table 1). All of them were therefore split at 1947.

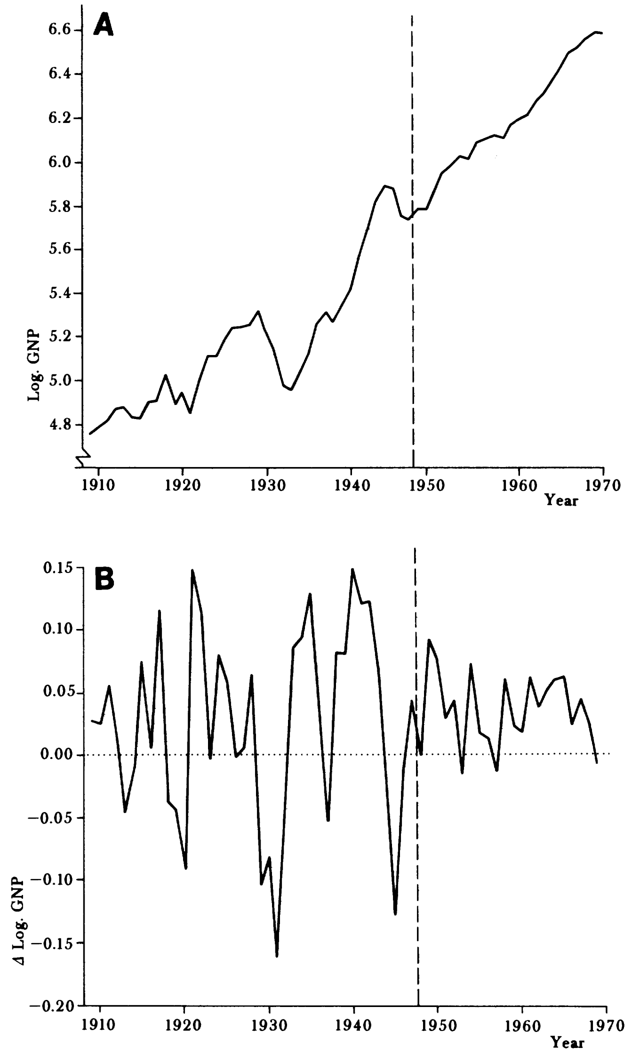

Figure 1. A: Logarithm of U.S. Real GNP, 1909–1970. B: First Differences of Logarithms of U.S. Real GNP, 1909–1970.

3. TREND AND CYCLE MODELS

The traditional formulation of an annual economic time series is

$$y_t = \mu_t + \psi_t + \epsilon_t, \qquad t = 1, \ldots, T, \tag{3.1}$$

where y_t is the logarithm of the observed value, μ_t is a trend, ψ_t is a cycle, and ϵ_t is an irregular component. In what follows, ψ_t will always be a stationary linear process, ϵ_t will denote a white noise disturbance term with variance σ^2, and all of the components will be assumed to be uncorrelated with each other.

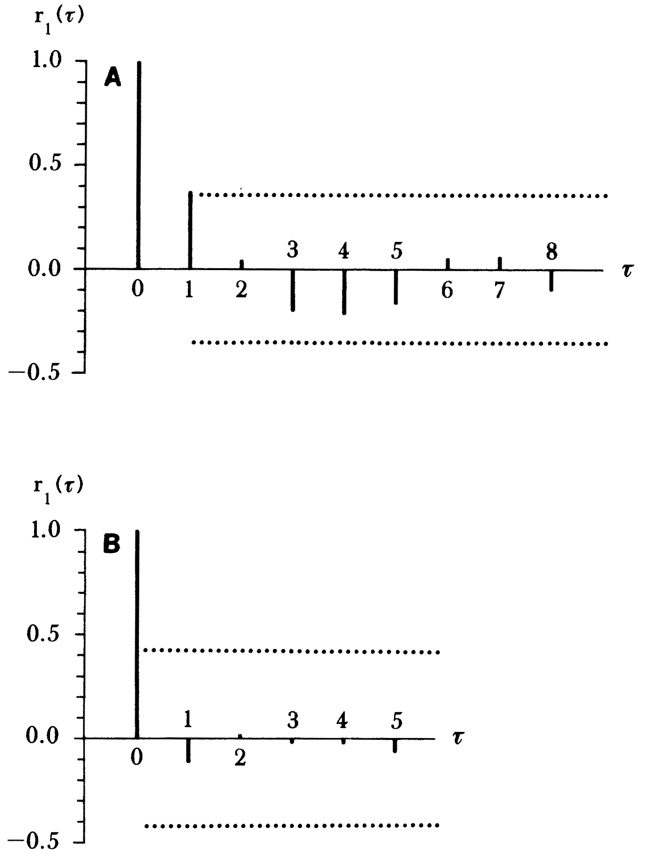

Figure 2. Correlograms of First Differences of Logarithms of U.S. Real GNP. A: 1909–1947. B: 1948–1970.

A stochastic linear trend can be modeled as

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \tag{3.2a}$$

$$\beta_t = \beta_{t-1} + \zeta_t, \tag{3.2b}$$

where η_t and ζ_t are uncorrelated white noise disturbance terms with variances σ_η^2 and σ_ζ^2, respectively (cf. Harvey and Todd 1983 and Harrison and Stevens 1976). If $\sigma_\eta^2 = \sigma_\zeta^2 = 0$, then μ_t reduces to a deterministic linear trend. Provided that ψ_t is a stationary linear process, the model is then of the TS form (1.1).

When $\sigma_\zeta^2 = 0$, (3.1) is stationary in first differences; and provided that $\sigma_\eta^2 > 0$, it is of the DS form (1.2), with $\beta_t = \beta$. In the rest of this section it will be assumed that $\sigma_\zeta^2 = 0$. This assumption will be relaxed in Section 4, but since it appears that σ_ζ^2 is quite small in practice, any conclusions reached under the assumption that it is zero are unlikely to be misleading.

3.1 The Correlogram of First Differences

Let $\rho_d(\tau)$ denote the autocorrelation at lag τ from the dth difference of a stochastic process and $r_d(\tau)$ denote the corresponding sample autocorrelation. Nelson and Plosser (1982) found that $r_1(1)$ is strongly positive and that most of the higher-order autocorrelations are statistically insignificant at the 5% level. This is illustrated by the correlogram in Figure 2A, where the dotted lines indicate ±2 standard errors on the assumption that $\rho_1(1) = r_1(1)$ and $\rho_1(\tau) = 0$ for $\tau \geq 2$. If the data were, in fact, generated by a process in which $\rho_1(\tau) = 0$ for $\tau \geq 2$, this would imply that $\psi_t = 0$ in (3.1). Furthermore, a positive $\rho_1(1)$ means that (3.1) must be ruled out completely unless there is a strong negative correlation between ϵ_t and η_t (see Nelson and Plosser 1982, pp. 152–158). If ϵ_t and η_t are uncorrelated, then $\rho_1(1)$ must be less than or equal to zero.

The above conclusions rest on the assumption that $\rho(\tau) = 0$ for $\tau \geq 2$, that is, that the process generating the series is ARIMA(0, 1, 1) with a constant. Although a standard application of the Box–Jenkins methodology suggests this model as a prime candidate on the grounds of parsimony, it might well be considered unacceptable, since it is unable to generate cyclical behavior of the kind that is plausible from an inspection of Figure 1A. Once the assumption that $\rho_1(\tau) = 0$ for $\tau \geq 2$ is relaxed, the fact that $\rho_1(1)$ is positive is still consistent with a wide range of stationary stochastic processes for ψ_t. In fact for any stationary stochastic process, it follows directly that

$$\gamma_1(1) = E[(y_t - y_{t-1})(y_{t-1} - y_{t-2})]$$
$$= 2\gamma_0(1) - \gamma_0(0) - \gamma_0(2),$$

and so $\gamma_1(1)$, and hence $\rho_1(1)$, will be positive if

$$2\rho_0(1) > 1 + \rho_0(2). \tag{3.3}$$

Suppose for the sake of simplicity that $\sigma_\eta^2 = \sigma^2 = 0$; that is, the model is of the TS form (1.1), with $w_t = \psi_t$. If ψ_t follows an AR(2) (autoregressive) process—that is, if $\psi_t = \phi_1\psi_{t-1} + \phi_2\psi_{t-2} + \omega_t$, where ω_t is white noise—

Table 1. Correlograms of First Differences of Logarithms

Series	Period	Number of observations	Sample autocorrelations, $r_1(\tau)$: 1	2	3	4	5	6	7	8	Q statistic[a]	Variance × 10^4
Real GNP	1909–1947	39	.37	.04	−.20	−.21	−.17	.04	.05	−.09	10.73	62.2
	1948–1970	23	−.11	−.00	−.02	−.01	−.06				.43	8.87
Industrial production	1860–1947	88	−.06	−.11	−.03	−.12	−.27	.07	.14	−.01	9.74	122
	1948–1970	23	−.30	−.06	.07	−.03	−.04				2.57	34.4
Unemployment rate	1890–1947	58	.11	−.30	.02	−.06	−.18	.06	.16	−.17	8.58	3120
	1948–1970	23	−.13	−.24	−.16	.17	.08				3.70	670
Consumer prices	1860–1947	88	.57	.16	.01	−.03	.02	.01	.00	.02	31.71	39.9
	1948–1970	23	.32	−.13	−.05	−.09	.00				3.33	4.19
Common stock prices	1871–1947	77	.21	−.16	−.10	−.27	−.32	.00	.13	.01	20.90	295
	1948–1970	23	.03	−.21	−.04	.41	.11				6.54	126

[a] Q denotes the Box–Ljung form of the portmanteau test statistic [see (4.1)], with $P = 5$ df; 5% significance value = 11.07; 10% = 9.24.

it follows from (3.3) that $\rho_1(1)$ will be positive if $\phi_1(2 - \phi_1) > 1 - \phi_2^2$. It is not difficult to find parameters within the stationarity region that satisfy this constraint. In fact for real GNP in the pre-1948 period, regressing y_t on y_{t-1} and y_{t-2} together with time t gives estimates of these parameters of $\hat{\phi}_1 = 1.27$ and $\hat{\phi}_2 = -.48$. The autocorrelation function of the differenced observations can be calculated by noting that the following equation can be derived from (3.3):

$$\rho_1(\tau) = \frac{2\rho_0(\tau) - \rho_0(\tau - 1) - \rho_0(\tau + 1)}{2(1 - \rho_0(1))}, \quad \tau = 0, 1, 2, \ldots . \tag{3.4}$$

The autocorrelation function of Δy_t satisfies the same difference equation as that of ψ_t, but with starting values at $\tau = 1$ and 2. For the parameter estimates just given, it exhibits a damped cycle with a period of 15.3 years. This is shown in Figure 3, and it can be seen that the pattern is not dissimilar to that of the observed correlogram in Figure 2A. Relaxing the assumption that σ_η^2 and σ^2 are zero can only lead to an autocorrelation function of Δy_t that is even closer to the observed correlogram.

The conclusion therefore is that although the sample autocorrelations at lags higher than one are not statistically significant (at the 5% level), they are not negligible, and the overall pattern is not inconsistent with trend plus cycle models of the form (3.1). Even a deterministic trend, that is, the TS model, cannot be ruled out. Similar results can be found for many of the other series besides GNP.

3.2 The Cyclical Trend Model

Given the findings in Section 3.1, the next question is whether there is an alternative class of models, also reflecting the cyclical behavior of the series, that can be set against the trend plus cycle models of (3.1). One possibility is to incorporate the cycle within the trend. Thus (3.1) is replaced by

$$y_t = \mu_t + \epsilon_t, \tag{3.6}$$

and

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \psi_{t-1} + \eta_t, \tag{3.7}$$

and ψ_t is a stationary linear stochastic process. Equation (3.2b) remains the same, but when $\sigma_\zeta^2 = 0$, the model corresponds to the DS form of (1.2).

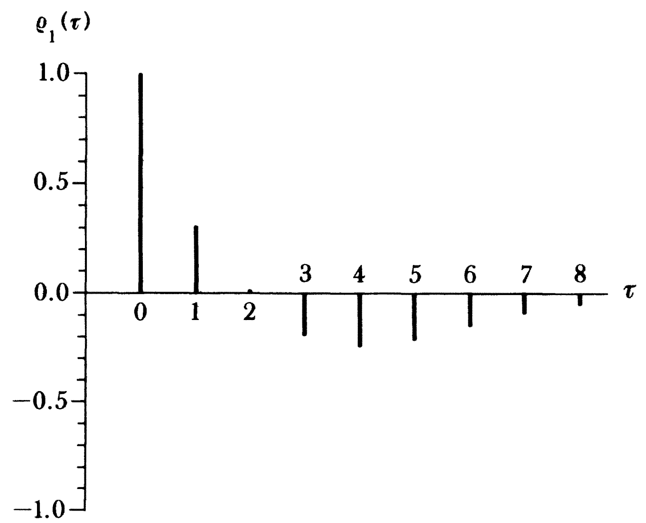

Figure 3. Autocorrelation Function for First Differences From a Deterministic Linear Trend With AR(2) Disturbances.

3.3 Modeling the Cyclical Component

So far the two general models, (3.1) and (3.6), have left the form of ψ_t unspecified, apart from a requirement that it should be a stationary linear process capable of displaying pseudocyclical behavior. The AR(2) process was used as an illustration simply because it is the best-known process of this kind and relatively easy to handle. An alternative specification, which has certain attractions within the context of the more general formulations of (3.1) and (3.6), involves modeling the cyclical process explicitly. It takes the form

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix}, \tag{3.8}$$

where ψ_t is the cyclical component, ω_t and ω_t^* are uncorrelated white noise disturbance terms with variances σ_ω^2 and σ_ω^{*2}, respectively, and ψ_t^* appears by construction (cf. Harrison and Akram 1983). The parameters $0 \leq \lambda \leq \pi$ and $0 \leq \rho \leq 1$ have a direct interpretation as the frequency of the cycle and the damping factor on the amplitude, respectively. The disturbances make the cycle stochastic rather than deterministic, and if ρ is strictly less than one, the process is stationary. The model can be written as

$$\psi_t = \frac{(1 - \rho\cos\lambda \cdot L)\omega_t + (\rho\sin\lambda \cdot L)\omega_t^*}{1 - 2\rho\cos\lambda \cdot L + \rho^2 \cdot L^2}, \tag{3.9}$$

where L is the lag operator. This reveals it to be an ARMA(2, 1) process; but if $\sigma_\omega^2 = 0$, it becomes AR(2). It is a rather special case of an AR(2), however, in that when $0 < \lambda < \pi$, the parameters are constrained to lie within the region corresponding to complex roots. (The modulus of these roots is ρ.) Since the aim is to model a stochastic cycle, this is a desirable constraint to impose. Note that the AR(1) model is obtained when λ is equal to 0 or π.

The preceding cyclical model was estimated both with and without the constraint that $\sigma_\omega^2 = \sigma_\omega^{*2}$. It appeared that very little, if anything, was lost in terms of goodness of fit by imposing this constraint. From the point of view of numerical optimization, having one parameter rather than two was found to be a considerable advantage. Hence in the results reported in the Section 4, this constraint was always imposed, and estimates are given for a single parameter, σ_ω^2. A theoretical rationale for the constraint, in terms of an underlying continuous time model, can be found in Harvey (1983).

3.4 ARIMA Models and the Reduced Form

Consider the trend plus cycle model (3.1), with trend (3.2) and cycle (3.8). When $\sigma_\zeta^2 = 0$, taking first differences yields the reduced or canonical form

$$\Delta y_t = \beta + \eta_t + \Delta\psi_t + \Delta\epsilon_t. \tag{3.10}$$

Since $\psi_t \sim$ ARMA(2, 1), it follows that the right side of (3.10) is ARMA(2, 3). It is an ARMA model, however, in which the AR and MA parameters are subject to strong restrictions. These restrictions fulfill two important roles. In the first place, they ensure that the model displays the desired characteristics. In the second place, provided that $\rho > 0$, they ensure that there are no common factors in the AR and MA polynomials; that is, the model is locally identifiable. (If the constraint $\sigma_\omega^2 = \sigma_\omega^{*2}$ is not imposed in the cycle, the model can also cease to be identifiable in the special case when $\lambda = 0$ or π. This provides yet another reason for imposing it.)

The cyclical trend model can also be expressed as an ARIMA(2, 1, 3) model when $\sigma_\zeta^2 = 0$, and the condition that $\rho > 0$ is again necessary for identifiability.

If $\sigma_\zeta^2 > 0$, both (3.1) and (3.6) can be expressed as ARIMA(2, 2, 4) models without a constant term.

4. ESTIMATION, TESTING, AND MODEL EVALUATION

Unobserved components models can be estimated in a number of ways (see Nerlove et al. 1979 and Harvey and Peters 1984). In this article, direct estimation of the structural parameters is carried out in the time domain by casting the model in state-space form. The same state-space form can be used to make predictions of future observations and to construct optimal estimates of the various unobserved components in the model by smoothing.

4.1 Maximum Likelihood Estimation

Both the trend plus cycle and the cyclical trend models can be written in state-space form very easily by defining the state vector to be $\alpha_t = (\mu_t, \beta_t, \psi_t, \psi_t^*)'$. The Kalman filter is then initiated by setting the mean squared errors of the initial estimators of μ_t and β_t equal to large but finite numbers. Since ψ_t is stationary, the initial mean squared error matrix of $(\psi_t, \psi_t^*)'$ is equal to the unconditional covariance matrix of $(\psi_t, \psi_t^*)'$. The summations in the prediction error decomposition form of the likelihood function then run from $t = 3$ to T. Maximization of this likelihood function with respect to the unknown parameters σ^2, σ_η^2, σ_ζ^2, σ_ω^2, λ, and ρ can be carried out numerically as in Harvey and Todd (1983).

4.2 Tests and Model Evaluation

Asymptotic standard errors of the maximum likelihood (ML) estimates can be computed by evaluating the Hessian numerically. This enables confidence intervals to be constructed for ρ and λ. Tests can also be carried out, although it should be remembered that if $\rho = 0$, the model ceases to be identifiable. Tests that any of the variance components σ_η^2, σ_ζ^2, and σ_ω^2 are zero are also subject to problems, though for a different reason—namely that the usual asymptotic normality of the ML estimators does not hold under the null hypothesis. (This difficulty does not arise with σ_η^2 if $\sigma_\zeta^2 > 0$.) Within the ARIMA framework, hypotheses of this kind involve the degree of differencing and are not normally tested (cf. Plosser and Schwert 1977 and Sargan and Bhargava 1983). It is possible to construct exact tests, as in Franzini and Harvey (1983), but it is difficult to apply such an approach here because of the possible presence of a cyclical component.

Comparison of various models, including those such as (3.1) and (3.6), that are nonnested can be made on the basis of the maximized likelihood function. The prediction error variance $\bar{\sigma}_p^2$, defined as the steady-state variance of the one-step ahead prediction error, can also be used to give an indication of goodness of fit.

Diagnostic checking tests could, in theory, be constructed using the Lagrange multiplier (LM) principle. Since both (3.1) and (3.6) are relatively rich models anyway, however, there is no obvious alternative hypothesis. In any case, LM tests for unobserved components models are best formulated in the frequency domain (see Harvey and Hotta 1982), and a detailed discussion of this is beyond the scope of this article. An alternative approach to diagnostic checking is to use the more conventional Box–Ljung Q statistic

$$Q = T^*(T^* + 2) \sum_{\tau=1}^{P} (T^* - \tau)^{-1} r^2(\tau), \tag{4.1}$$

where T^* is the number of residuals (usually $T - 2$) and $r(\tau)$ is the τth autocorrelation in the residuals. If the model contains n parameters, it may be conjectured that Q has a χ^2 distribution with $P - n + 1$ df under the null hypothesis.

A simple diagnostic test for heteroscedasticity can also be constructed from the residuals. Suppose that m is $T^*/3$ or the nearest integer to it. The test statistic is

$$H = \left[\sum_{t=T-m+1}^{T} (\nu_t^2/f_t)\right] \bigg/ \sum_{t=k+1}^{m+k} (\nu_t^2/f_t), \tag{4.2}$$

where $k = T - T^*$. A naive test could be carried out by comparing H with the appropriate significance point of an $F_{m,m}$ distribution. Of course, this is not strictly valid unless all of the parameters, apart from one of the variances, are known. Nevertheless, it may be preferable to the large sample alternative of treating mH as χ_m^2 under the null hypothesis.

Subject to satisfactory diagnostics, a fitted model will be evaluated not only on its goodness of fit but also on the basis of the numerical values of estimated parameters. Thus as in model building in econometrics, the question of statistical significance (at an arbitrary significance level of 5%) is not necessarily as important as the consideration of whether the estimated parameters give a model with a sensible interpretation.

4.3 Preliminary Analysis

If there is some doubt as to whether a cyclical model is appropriate, it may be wise to conduct a preliminary analysis of the data. One reason for doing this is to avoid unnecessary computing, particularly as (3.1) and (3.6) are not identifiable when $\rho = 0$.

As was demonstrated in Section 3, a good deal of information can be obtained by looking at the correlogram of Δy_t. A cyclical pattern in the sample autocorrelations suggests the possibility of either (3.1) or (3.6). In the case of (3.6) it can be shown that when $\sigma_\zeta^2 = \sigma^2 = 0$, the LM test for $\rho = 0$ is based simply on $r_1(1)$ (see the Appendix). The LM principle can be extended to carry out tests of $\rho = 0$ in more complicated models. Thus if no clear pattern emerges from the correlogram of Δy_t—perhaps because σ_ζ^2 is positive—a stochastic trend, (3.2), can be fitted to the data and an LM test of $\rho = 0$ carried out. As already indicated, such tests are best developed in the frequency domain, and a detailed consideration of them is beyond the scope of this particular article. Instead, the Box–Ljung test was used to give an indication of model inadequacy in such cases.

Returning to the correlogram of Δy_t, it should be borne in mind that when this indicates white noise, the appropriate model is random walk plus drift; that is,

$$y_t = y_{t-1} + \beta + \eta_t, \qquad t = 2, \ldots, T. \qquad (4.3)$$

Thus in terms of (3.1) and (3.6), all of the parameters apart from σ_η^2 are zero. This model is known to give a reasonable fit to many economic time series, and in Harvey (1984) it is used as a yardstick in the R_D^2 measure of goodness of fit:

$$R_D^2 = 1 - T^* \tilde{\sigma}_p^2 \Big/ \sum_{t=2}^{T} (\Delta y_t - \Delta \bar{y})^2. \qquad (4.4)$$

4.4 Prediction

Given estimates of the unknown parameters σ_η^2, σ_ζ^2, σ_ω^2, σ^2, λ, and ρ, the Kalman filter produces the minimum mean squared estimators (MMSE's) of the elements in the state vector at time T. The estimator of β_T is the current estimate of the long-run growth rate, that is, the slope of the final forecast function for the log of the variable in question. For both (3.1) and (3.6), the forecast function exhibits damped cyclical behavior before settling down to a linear trend. The difference is that in (3.6) the intercept term in the final forecast function depends on the cyclical component, whereas in (3.1) it does not.

5. EMPIRICAL RESULTS FOR SERIES BEFORE 1948

The ML estimates for the following models are presented in Table 2: (a) the stochastic trend model, that is, (3.1) and (3.2), but without the cyclical component, ψ_t; (b) the trend plus cycle model, (3.1), (3.2), and (3.8); and (c) the cyclical trend model, (3.6), (3.7), and (3.8). The series contained observations up to and including 1947. A detailed analysis of the results obtained for each series is given in Sections 5.1–5.5.

The following points should be noted:

1. The period of a cycle corresponding to a frequency of λ radians is $2\pi/\lambda$ years. This is also, by definition, the period of the damped cycle in the autocorrelation and forecast functions. The peak in the power spectrum is usually at a slightly different fre-

Table 2. Maximum Likelihood Estimates of Parameters for (a) Stochastic Trend, (b) Trend Plus Cycle, and (c) Cyclical Trend for Pre-1948 Period

		Estimates ($\times 10^4$)											
Series	Model	σ_η^2	σ_ζ^2	σ_ω^2	σ^2	ρ	λ	$2\pi/\lambda$	Log L	$\tilde{\sigma}_p^2$ ($\times 10^3$)	Q^a	H^b	R_D^2
GNP	(a)	62.2	.0	—	.0	—	—	—	73.66	6.22	19.15* (10)	1.62	.00
($P = 10$, $m = 12$)	(b)	23.7	6.1	3.3	.0	.97	.90	7.0	76.49	5.88	5.10 (6)	.82	.05
	(c)	.0	.0	24.3	4.9	.73	.72	8.7	77.83	4.98	5.90 (7)	.87	.20
Industrial	(a)	122	.0	—	.0	—	—	—	144.5	122	14.48 (9)	5.88*	.00
production	(b)	39.2	.0	52.9	.0	.79	.45	14.0	146.3	116	14.20* (6)	6.67*	.05
($P = 10$, $m = 28$)	(c)	106.0	.0	.3	.0	1.00	.99	6.3	147.5	115	12.6* (6)	5.00*	.06
Unemployment	(a)	3120	.0	—	.0	—	—	—	2.59	312	15.55 (10)	.90	.00
rate	(b)	1810	.0	500	.0	.77	.91	6.9	3.81	297	13.78 (7)	1.00	.05
($P = 10$, $m = 18$)	(c)	.0	.0	2140	.0	.56	1.38	4.5	5.58	283	7.81 (8)	1.11	.09
Consumer	(a)	.0	32.4	—	.0	—	—	—	203.4	3.24	11.70 (9)	.88	.19
prices	(b)	.0	5.6	6.8	.0	.87	.77	8.2	209.1	2.84	7.01 (6)	.83	.29
($P = 9$, $m = 28$)	(c)	.0	2.1	15.6	.0	.69	.79	8.0	212.3	2.62	2.76 (6)	.98	.34
Common stock	(a)	295.0	.0	—	.0	—	—	—	92.4	295	21.80* (8)	4.76*	.00
prices	(b)	.0	.0	176	.0	.83	.47	13.3	99.3	234	8.85 (6)	5.88*	.21
($P = 8$, $m = 25$)	(c)	229	.0	1.8	.0	1.00	.69	9.1	97.8	259	13.30* (5)	3.65*	.12

[a] Box–Ljung Q statistic: degrees of freedom (i.e., $P - n + 1$) are in parentheses; values of P are given under the series name; asterisks in this column indicate a significant value at the 5% level.

[b] Heteroscedasticity statistic: values of m are given under the series name; asterisks in this column indicate significant values at the 10% level for a two-sided test based on the F distribution of (m, m) df.

quency. This is, of course, similar to the situation that arises with a pure AR(2) process (see Box and Jenkins 1976, p. 63).

2. In assessing the degrees of freedom for the Q statistic, it is suggested that a degree of freedom not be deducted when a parameter is estimated as zero. If a degree of freedom is deducted, the Box–Ljung tests may give contradictory results. An example is provided by the fitting of model (a) to GNP. Since the estimates of σ_ζ^2 and σ^2 are both zero, the correlogram of residuals is identical to the correlogram of first differences, as given in Table 1. Thus the degrees of freedom of the Q statistic are best taken as P rather than $P - 2$. Of course, numerical values close to but not equal to zero present something of a dilemma in this respect, but there is obviously no clear-cut solution to the problem.

5.1 Real GNP

As already noted, the correlogram of first differences shows evidence of a cyclical movement, and an examination of the series itself indicates that a cycle with a period of, say, six to nine years might not be unreasonable. Models (b) and (c) give a cyclical component with a period within this range, and both are satisfactory with respect to the diagnostics. The estimate of ρ, however, is close to unity for the trend plus cycle model, and this indicates that the model may be inappropriate. The superiority of the cylical trend is confirmed by its better goodness of fit.

The standard errors of the estimators of ρ and λ in the cyclical trend model were estimated numerically as .15 and .16, respectively. If the distribution of the ML estimator of λ is taken to be approximately normal, then a 95% confidence interval for λ translates into a confidence interval of 6.0–15.7 years for the period. It may well be, however, that the distribution of the estimator of the period is closer to normality, in which case the confidence interval would be different. This question clearly requires further investigation, as indeed does that of the distribution of the estimators of the variance parameters.

The cyclical trend model has the estimate of σ_ζ^2 equal to zero. Taken together with σ^2 equal to zero, this suggests that the model can also be interpreted as ARIMA(2, 1, 2). Furthermore, the fact that the estimated value of σ_ω^2 is much greater than that of σ_η^2 indicates that an ARIMA(2, 1, 1) or even an ARIMA(2, 1, 0) model may not be a bad approximation. Fitting the second of these models by ordinary least squares gave a prediction error variance of 5.28×10^{-3}, which is not much more than that of the full model. The estimated parameters were $\hat{\phi}_1 = .417$ and $\hat{\phi}_2 = -.115$, and these yield a damped cyclical autocorrelation function with a period of 7.0 years. The model therefore displays some of the desirable characteristics for this series. The interesting question is whether the standard ARIMA-model selection methodology would lead to its being selected. As already noted, Nelson and Plosser (1982) felt that the correlogram indicated an ARIMA(0, 1, 1) model. (Fitting this model gives $\tilde{\sigma}_p^2 = 5.40 \times 10^{-3}$.) It is conceivable, however, that it could have been taken to indicate an ARIMA(2, 1, 0) model. But since the t statistic associated with ϕ_2 is only $-.65$, many researchers would have then dropped this term and estimated an ARIMA(1, 1, 0) model. On reexamining the correlogram, they might then have been led back to the choice of ARIMA(0, 1, 1), since the correlogram is certainly not consistent with the autocorrelation function of a first-order AR process.

5.2 Industrial Production

The correlogram of first differences given in Table 1 shows no obviously discernible pattern. This suggests that a random walk with drift, (4.3), may be the best one can do. The estimates for the more general models reported in Table 2 seem to confirm that this is the case. The stochastic trend model collapses directly to the random walk plus drift model, and the cyclical trend model has only a 6% better fit, coupled with an estimate of ρ equal to unity and a very small estimate of the cyclical variance σ_ω^2. This apparently indicates something close to a deterministic cycle. The estimates of the cyclical parameters in model (b) are more acceptable, although the period is 14 years and there is no improvement on model (c) in terms of goodness of fit. Furthermore, the value of the Q statistic is much the same as when no cycle is fitted.

The major problem with all of the models fitted lies in the unacceptably high values for the H statistic. This indicates increased volatility of the observations in the later years, a fact that is not immediately apparent from a casual inspection of the graph. In order to investigate this matter further, the series was split at 1909 (since the first observation on GNP is at 1909). The correlograms of first differences showed the following results: for 1860–1908, $Q(5) = 5.72$, $H(15) = 1.79$, $\tilde{\sigma}_p^2 = 57 \times 10^{-4}$; and for 1909–1947, $Q(5) = 5.14$, $H(12) = 1.45$, $\tilde{\sigma}_p^2 = 206 \times 10^{-4}$, where the figures in parentheses after Q and H denote P and m, respectively. There is still a slight indication of heteroscedasticity, but it is not significant, nor is the Box–Ljung statistic. The random walk plus drift model therefore seems appropriate for both parts of the series. It is interesting that when the cyclical trend model, (c), was fitted to the period 1909–1947, it collapsed to the random walk plus drift model. Thus the cycle found in GNP during that period apparently cannot be found in industrial production.

5.3 Unemployment Rate

Both cyclical models give sensible results, but the Q statistic for model (c) is lower, and the goodness of fit is better. The period of the cycle in model (c) is only

4.5 years, but it must be remembered that the series contains nineteen more observations than the GNP series.

Fitting an AR(2) process to first differences gives an almost identical fit to model (c), with the period of the implied cycle being 4.4 years.

5.4 Consumer Price Index

The interesting point about the consumer price series is that all three models have σ_ζ^2 positive. This is presumably because the series is much more volatile than real series, such as GNP and industrial production. A positive σ_ζ^2 indicates that second differencing is needed to render the series stationary, although this would not be apparent from the correlogram of first differences, which dies away in classical fashion. Presumably, part of the explanation lies in the exceptionally high value of .57 for $r_1(1)$. In any case, the results show that the correlogram can display some ambiguity in this respect. Having said that, it must be conceded that the obvious Box–Jenkins choice, namely ARIMA(0, 1, 1) plus constant, actually gives a better fit than either of the cyclical models. Specifically: $\tilde{\theta} = .69$ (.08), $\tilde{\beta} = .011$ (.009), $\tilde{\sigma}_p^2 = 2.47 \times 10^{-3}$, $R_D^2 = .38$, $Q = 4.38$, and $H = 1.08$, where β is the constant term, the figures in parentheses indicate asymptotic standard errors, the Q statistic is based on 8 df, and $m = 29$ in the H statistic.

Of the two cyclical models, the cyclical trend is to be preferred on the grounds of better goodness of fit and less residual serial correlation. The fact that the ARIMA(0, 1, 1) model gives a slightly better fit than the cyclical trend model and would clearly be selected on grounds of parsimony is perhaps, however, a little disconcerting. Nevertheless, model (c) does have a cyclical component that is remarkably consistent with the cycle found for GNP.

5.5 Stock Prices

This series is very erratic, and the results from fitting time series models, both structural and ARIMA, are not very convincing. The correlogram in Table 1 displays no clearly discernible pattern, yet the random walk plus drift model appears to be ruled out by the high Q statistic. Fitting the cyclical trend model, (c), is unsatisfactory because of the estimate of ρ equal to unity and the high value of Q. Curiously enough, the trend plus cycle model, (b), fares best, and the zero estimates for σ_η^2 and σ_ζ^2 actually seem to indicate that the trend is deterministic! An examination of a graph of the series shows that this is a plausible result, although it is also clear that such a deterministic trend could not possibly carry over into the post-1947 period.

Finally, observe that the H statistic indicates strong heteroscedasticity for all models. This seems to be due to the violent fluctuations in stock prices of the 1920s and 1930s, and it provides further evidence that fitting univariate time series models to the full period is not satisfactory.

5.6 General Comments

The following general comments can be made on the basis of the analysis of the five series discussed in Sections 5.1–5.5.

1. In all but one instance, the variance of the irregular component was found to be zero.
2. The variance parameter, σ_ζ^2, was found to be zero for all series except consumer prices (cf. the results of Harvey and Todd 1983).
3. The stochastic trend model, (a), reduced to a random walk plus drift model for all series, apart from consumer prices.
4. With the exception of the stock price series, the cyclical trend model is to be preferred to the trend plus cycle model. This means that the trend and cycle components cannot be separated. Instead, the growth rate of the trend decomposes into a long-run component, a transitory cyclical component, and a random component. In the case of GNP, the fact that σ_η^2 and σ_ζ^2 are both zero means that the growth rate in the trend is equal to a constant term plus the cyclical component.
5. The fact that the cyclical trend model is preferred in most cases is, in itself, evidence against the TS model of (1.1). Furthermore, when the trend plus cycle model is fitted, the possibility of a deterministic trend is effectively ruled out by a clear positive estimate of σ_η^2 or σ_ζ^2. The only exception is stock prices, but as already noted, the model is not really very convincing.
6. The assumption that after appropriate differencing, a series is stationary throughout its length is not one that can be taken for granted. For industrial production and stock prices, the lack of homogeneity is indicated by the diagnostics. Once this has been noticed, this finding also becomes apparent from a close inspection of the graphs of levels and first differences.
7. The structural models are more convincing for the real series—namely GNP, industrial production, and unemployment—than they are for consumer and stock prices.

6. EMPIRICAL RESULTS FOR THE POSTWAR PERIOD

The correlograms presented in Table 1 show the properties of the series in the postwar period (or more precisely, 1948–1970) to be very different from the properties of the same series before 1948. Unfortunately, the relatively small number of observations makes time series modeling difficult. Of course, the data set could be extended by including observations beyond 1970 or obtaining observations on a quarterly basis, or both. Doing this, however would have widened the scope of this article considerably and introduced a number of new issues.

Table 3. Maximum Likelihood Estimates of Parameters for (a) Stochastic Trend, (b) Trend Plus Cycle, and (c) Cyclical Trend for 1948–1970

Series	Model	Estimates ($\times 10^4$) σ_η^2	σ_ζ^2	σ_ω^2	σ^2	Estimates ρ	λ	$2\pi/\lambda$	Log L	$\tilde{\sigma}_p^2$ ($\times 10^{-4}$)	Q^a	H^b	R_D^2
GNP	(a)	7.52	.0	—	.65	—	—	—	61.74	8.77	9.26	.58	.01
	(c)	6.89	.0	1.68	.0	.37	3.14	2.0	61.80	8.77	9.00	.58	.01
Industrial production	(a)	12.6	.0	—	10.4	—	—	—	48.55	29.8	10.33	.88	.13
Unemployment rate	(a)	311	.0	—	188	—	—	—	16.58	632	4.95	.33	.06
	(b)	217	.0	99	49	.72	1.34	4.7	17.45	576	7.88	.45	.14
	(c)	24	.0	207	126	.68	1.43	4.4	17.38	578	7.66	.45	.14
Consumer prices	(a)	2.5	.73	—	.0	—	—	—	70.68	4.19	6.66	.06*	.00
	(c)	.0	.28	2.1	.0	.60	1.05	6.0	72.28	3.64	10.33	.13*	.13
Common stock prices	(a)	25.1	4.4	.0	44	—	—	—	34.36	126	12.55	1.06	.00
	(c)	.6	.68	8.09	202	.94	1.48	4.3	37.16	103	1.53	.81	.18

[a] For all series $P = 8$.
[b] For all series $m = 7$; asterisks indicate significant values at the 10% level.

Table 3 presents the ML estimates of the same models as Table 2 except that the series contain data from 1948 to 1970.

6.1 Real GNP

The correlogram of first differences in Table 1 indicates a random walk plus drift model, with the Q statistic taking the very low value of .43. Fitting the stochastic trend model (a) does, however, give a positive value for σ^2, as well as for σ_η^2. Although it is nice to note that the positive value is consistent with the negative (but insignificant) value of $r_1(1)$, the overall improvement in goodness of fit is very small.

6.2 Industrial Production

The sample autocorrelations of first differences are all small except for the one at lag one, which is $-.30$. This suggests a stochastic trend model with $\sigma^2 > 0$. The results show that such a model gives a reasonable fit with $R_D^2 = .13$. As with real GNP, no improvement in goodness of fit could be obtained by including a cyclical component in the model.

6.3 Unemployment Rate

The unemployment series shows evidence of a cycle, albeit a fairly weak one. The period of between four and five years is not unreasonable, and there is no strong indication of residual serial correlation or heteroscedasticity.

6.4 Consumer Prices

As with the pre-1948 price series, the estimate of σ_ζ^2 is positive for both (a) and (c). The period of the cycle for (c), however, is perhaps too big to make the model attractive, and in terms of goodness of fit, (a) shows no gain over the random walk plus drift model. Model (b), as usual, was even less attractive than (c). As with the pre-1948 observations, an ARIMA(0, 1, 1) model gives a better fit than the structural cyclical models. The extremely strong heteroscedasticity (with high variances at the beginning) casts doubt, however, on the validity of any of these models.

6.5 Common Stock Prices

For this series the message in the correlogram of first differences is not at all clear. The Q statistic is not significant at the 5% level of significance, but $r_1(2) = -.21$ and $r_1(4) = .41$. The stochastic trend model, (a), and the trend plus cycle model, (b), give the same result. There is no improvement, however, over the random walk plus drift model in terms of goodness of fit. The cyclical trend model, on the other hand, has $R_D^2 = .18$, and the estimated period of 4.3 years suggests that the model may provide a reasonable description of the series. Nevertheless one can hardly be enthusiastic about it on grounds of parsimony.

6.6 General Remarks

For most of the series it is possible to estimate models that are somewhat more complex than the random walk plus drift model and have a positive R_D^2. On a statistical criterion, however, such as the Akaike information criterion (AIC), the random walk plus drift model is to be preferred in all cases. On the other hand, it must be remembered that the sample size is small. Furthermore, the fact that for several series, the additional parameters estimated have a sensible interpretation is not without interest.

Regarding a deterministic trend, it is again very clear that the TS model of (1.1) cannot possibly hold.

Finally, as with the pre-1948 data, the structural models seem relatively less convincing when applied to consumer and stock prices.

7. CONCLUSIONS AND EXTENSIONS

The results of this research have implications on two fronts. In the first place, they provide useful information on the nature of economic time series. Second, they demonstrate the development of a new methodology for analyzing and modeling time series.

7.1 Statistical Methodology

In the prevailing Box–Jenkins approach, the idea is to select a parsimonious model from the class of ARIMA processes. This is done on the basis of the data themselves, with statistical tools such as the correlogram playing a prominent role in model selection. The structural approach, on the other hand, is based on a class of models containing unobserved components that have a direct interpretation. These include trend, seasonal, and cycle components. All are known to exist, to a greater or lesser extent, in economic time series, and so strong consideration is given to actually including such components in the model at the outset. The result is a change in emphasis in model selection. The reduced form of a structural model is still an ARIMA model, but it is subject to restrictions imposed by a priori considerations. Thus the "flexibility" of the Box–Jenkins approach to select models that may have unacceptable properties is lost. This does not mean that the information in the data, as reflected, say, in the correlogram, should be ignored; and there is certainly no case for fitting a structural model that is inconsistent with the correlogram. The argument is, perhaps, the other way round, however, in the sense that the correlogram may be very difficult to interpret in the absence of some prior notion of the sort of model that may be reasonable. The correlogram in Figure 2A provides an excellent example.

The construction of a structural time series model has two purposes. The first is to make forecasts. The second is to provide a way of presenting the "stylized facts" associated with the movements of a particular economic time series (cf. Ashenfelter and Card 1982). The structural time series model is not intended to represent the underlying data generation process. Rather, it aims to present the "facts" about the series in terms of a decomposition into trend, cycle, seasonal, and irregular components. These quantities are of interest to economists in themselves. Furthermore, they highlight the features of a series that must be accounted for by a properly formulated behavioral econometric model (cf. the "encompassing principle" advocated for econometric model building by Hendry and Richard 1983 and Mizon 1984). Of course, the two roles of a structural model are not entirely independent. A model that reflects the main features of a series is also likely to be good for forecasting.

7.2 Macroeconomic Time Series

In their 1982 article, Nelson and Plosser concluded that the deterministic trend with a stationary disturbance term (or cycle) superimposed on it was not a tenable model for economic time series. This article questions the methodology used to arrive at that conclusion. In particular, objections have been raised against the way in which Nelson and Plosser used the principle of parsimony to effectively identify an ARIMA(0, 1, 1) model for the time series in question and against the fact that they assumed a constant structure throughout the whole time period. Nevertheless, the results obtained here using the structural methodology strongly support the conclusion reached by Nelson and Plosser.

For the data up to 1947, the most interesting conclusion is that a stochastic cycle is best modeled within the trend. In other words, the cycle is an intrinsic part of the trend rather than a separate component that can just be added on afterwards. For the period 1948–1970, a faint cycle can be detected in some series, but it is difficult to justify on statistical grounds the inclusion of a cyclical component anywhere in the model. A stochastic trend model is sufficient, and in many cases (e.g., real GNP) this model effectively reduced to a random walk plus drift model. The absence of a significant cycle explains why, for many postwar series, a trend plus seasonal plus irregular components model can be adopted for monthly and quarterly observations (e.g., see Harvey and Todd 1983). Within a Box–Jenkins framework, the widespread adoption of the airline model [i.e., ARIMA(0, 1, 1) $\times$ $(0, 1, 1)_s$] is a reflection of the same phenomenon. Of course, after 1970 many economic time series became more volatile again and it could be argued that the reintroduction of a cyclical component is desirable.

7.3 Extensions

The model building in this article has been based on annual data. The extension of the state-space model to handle quarterly or monthly observations, however, is relatively straightforward. Another interesting extension is to include more than one cyclical component in the model. If it is felt that there may be a 20-year cycle as well as a 5-year cycle, then both components may be modeled by (3.8) and incorporated into the overall structure as appropriate.

Structural models other than those described here may be appropriate in other circumstances, particularly when one moves away from economic time series. There is nothing necessarily restrictive about the structural class; the only requirement is that the model be set up in terms of components that have some kind of interpretation.

In the examples reported here, the Box–Ljung test was used as a diagnostic for serial correlation. As already noted, the LM principle may yield more satisfactory tests, and this is currently being investigated. A rather rough-and-ready heteroscedasticity test was also used. Heteroscedasticity is rarely tested for in time series modeling, and the original motivation for including it here was to check on the suitability of the logarithmic transformation. Further analysis prompted by significant values of the test statistic, however, revealed that the assumption of homoscedasticity cannot always be taken for granted in economic time series. This then raises the question of how heteroscedasticity is best modeled. One possibility is to develop the autoregressive conditional heteroscedasticity (ARCH) framework of Engle (1982) and Weiss (1984).

As Figures 1B and 2 reveal, the stationarity of an economic time series (after appropriate differencing) may not be a tenable assumption. Indeed, the remarkable thing about differenced economic time series is not that they are sometimes nonstationary, but rather that they are occasionally stationary! One of the attractions of the state-space approach in this context is that it provides a framework for incorporating changing structures into the model. The question of exactly how this should be done and how various types of change might be detected clearly warrants further research.

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APPENDIX: LAGRANGE MULTIPLIER TEST FOR A CYCLE

Consider model (3.6) with $\sigma^2 = \sigma_\zeta^2 = 0$ and with $\sigma_\omega^2 = \sigma_\omega^{2*}$ in the cycle (3.9). We wish to test H_0: $\rho = 0$ against H_1: $\rho > 0$. Since the model is not identifiable under H_0, it is also necessary to set $\sigma_\eta^2 = 0$ (cf. the use of a similar device in Poskitt and Tremayne 1980).

Although tests of this kind are generally best developed in the frequency domain (as in Harvey and Hotta 1982), the present test can also be derived in the time domain by noting that the canonical form of the model is

$$(1 - 2\rho \cdot \cos \lambda \cdot L + \rho^2 L^2)\Delta y_t^* = \xi_t + \theta(\rho)\xi_{t-1}, \quad \text{(A.1)}$$

where ξ_t is a white noise disturbance term and Δy_t^* is Δy_t in deviation from the mean form. The moving average parameter, $\theta(\rho)$, is a nonlinear function of ρ, and so $\partial\theta(\rho)/\partial\rho = 0$ when $\rho = 0$. Furthermore, it will be clear from (3.9) that when $\rho = 0$, $\xi_t = \omega_t$ and $\theta(\rho) = 0$.

Suppose that λ is regarded as being known. Although (A.1) is an ARMA(2, 1) process, it depends on a single unknown parameter, and a Lagrange multiplier test can be constructed by regressing ξ_t on $\partial\xi_t/\partial\rho$, with both evaluated at $\rho = 0$. This suggests a test statistic of TR^2, where R^2 is the coefficient of (multiple) correlation from a regression of $\hat{\xi}_t = \Delta y_t$ on $(-2 \cos \lambda)\Delta y_{t-1}^*$. The result is exactly the same, however, if Δy_t^* is simply regressed on Δy_{t-1}^*. This regression does not depend on λ. Hence it is valid for all λ, and so it is immaterial whether λ is known or not.

This argument suggests a test based on $Tr_1^2(1)$, with the distribution under the null hypothesis being χ_1^2. Alternatively, the von Neumann ratio may be used, although it must be remembered that the test is two-sided, since $\rho_1(1)$ may be negative or positive for (3.9). Note that in a somewhat wider context, the same argument can be used to show that the Durbin–Watson test is appropriate when the disturbance term in a static linear regression is assumed to follow (3.9).

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