Sample Solutions - Tutorial 6

Probability Distributions.

Q1 All numbers for $P[x]$ (Probability distribution) should add up to 1.

$\Rightarrow$ The sum of the numbers given:

$0.01 + 0.10 + 0.26 + 0.33 + 0.18 + 0.06 + 0.03 = 0.97$

The missing value is: $1 - 0.97 = \underline{0.03}$

Binomial Distribution

Q2. a) $n = 3$ $\quad p = 0.75$

$r = 2$ $\quad q = 1 - p = 0.25$

$$P[2 \text{ in } 3 \text{ trials}] = {}^{3}C_{2}\, p^{2}\, q^{3-2}$$

$$= \frac{3 \times 2 \times 1}{2 \times 1 (1)} \times 0.75^{2} \times 0.25^{1}$$

$$= 3 \times 0.5625 \times 0.25$$

$$= \cancel{0.04}\ \boxed{0.422}$$

b) $P[0 \text{ in } 3 \text{ trials}] = {}^{3}C_{0}\, p^{0} q^{3-0}$

$$= \frac{3 \times 2 \times 1}{0!\,(3 \times 2 \times 1)} \times 0.75^{0} \times 0.25^{3}$$

NOTE: $0! = 1$

$$= \frac{3 \times 2 \times 1}{1\,(3 \times 2 \times 1)} \times \overset{1}{\cancel{0.75^{0}}} \times 0.25^{3}$$

$$= \boxed{0.016}$$

$P[1 \text{ in } 3 \text{ trials}] = {}^{3}C_{1}\, p^{1} q^{3-1}$

$$= \frac{3 \times 2 \times 1}{1\,(2 \times 1)} \times 0.75^{1} \times 0.25^{2}$$

$$= \boxed{0.141}$$

$P[2 \text{ in } 3 \text{ trials}] = \boxed{0.422}$ (From part a))

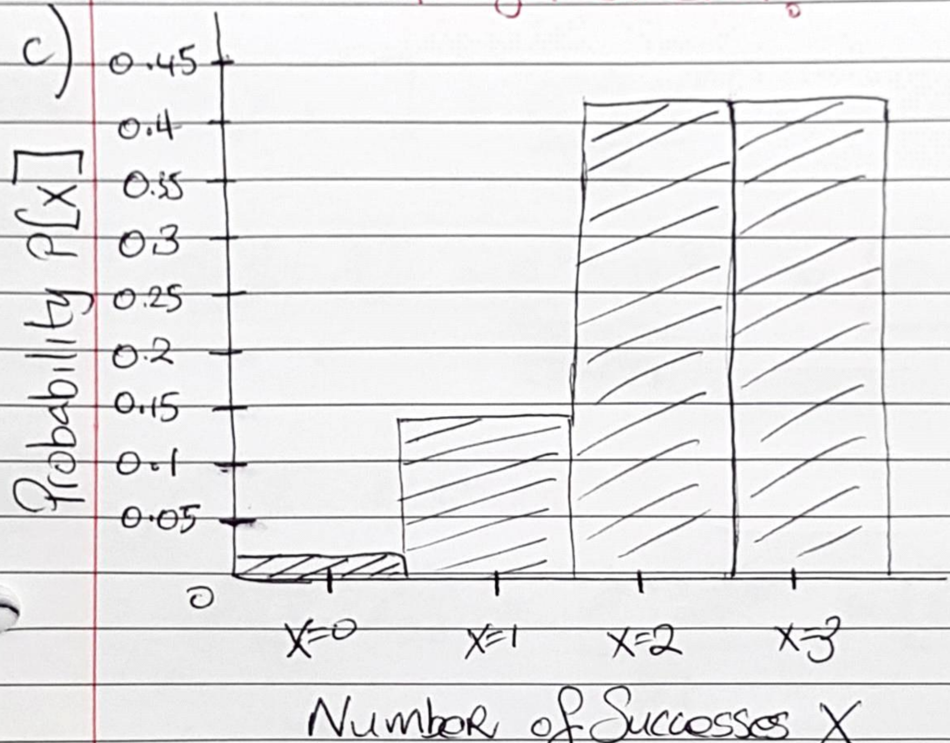
$P[3 \text{ in } 3 \text{ trials}] = {}^{3}C_{3}\, p^{3} q^{3-3}$

$$= \frac{3 \times 2 \times 1}{3 \times 2 \times 1\,(1)} \times 0.75^{3} \times 0.25^{0}$$

$$= \boxed{0.422}$$

Rough Sketch!

c)



Q3. Rain 48% of days in October, (31 days in Oct) what is the probability that it will rain on exactly 5 days?

$$P[5 \text{ in } 31 \text{ trials}] = {}^{n}C_r\, p^r q^{n-r} = \frac{n!}{(n-r)!\,r!}\, p^r q^{n-r}.$$

$n = 31$

$r = 5$

$p = \frac{48}{100} = 0.48$

$q = 1 - p = 1 - 0.48 = 0.52$

$$= {}^{31}C_5 \times 0.48^5 \times 0.52^{26}$$

$$= \frac{31!}{(31-5)!\,5!} \times 0.48^5 \times 0.52^{26}$$

$$= \boxed{0.00018}$$

Poisson Distribution

Q4 $P[R=r] = \dfrac{e^{-\mu}\mu^r}{r!}$ $\mu=2$ $r=3$.

$P[R=3] = \dfrac{e^{-2}2^3}{3\times2\times1} = \boxed{0.180}$

Q5 $\mu=3$ $r=5$

$P[R=5] = \dfrac{e^{-3}3^5}{5!} = \boxed{0.101}$

Q6 $p = \dfrac{4}{100} = 0.04$ $q = 1-p = 0.96$

$n=100$ $\mu = np = 100\times0.04$

$r=7$ $\Rightarrow \mu = 4$

$P[R=7] = \dfrac{e^{-4}4^7}{7!} = \boxed{0.06}$

Q7 avg. number of patients arriving → 6 per hour.
A patient arrives at 11:30am. What is the probability that the next patient arrives before 12pm?
– what we are expecting is that <u>1 or MORE</u> patients arrive.

R = the number of patients arriving per half hour.
so $\mu = 6/2 = 3$.

$r \geq 1$

$P[R \geq 1] = 1 - P[R=0]$

$$P[R=0] = \frac{e^{-3}3^0}{0!} = 0.049787$$

$$P[R \geq 1] = 1 - 0.049787$$

$$= \boxed{0.95}$$

Normal Distribution

Q8 $\mu = 1.98$
$\sigma = 0.012$

$$\left[a = \frac{x-\mu}{\sigma}\right]$$ Standardise

$$P[X > 2] \Rightarrow \underbrace{\frac{2 - 1.98}{0.012}}_{a} = 1.67$$

Round to 2 decimal places

So $P[X>2] = P[Z > 1.67]$

look up $P[Z > 1.67]$ is the Z table (table for standard normal distribution).

$$P(X>2] = \boxed{0.0475}$$

Q9 $\mu = 25.6$ $\quad Z = \frac{X-\mu}{\sigma}$

$\sigma = 2.4$

$P[X < 28]$?

$$Z = \frac{28 - 25.6}{2.4} = 1.0 \quad \text{(Standardise...)}$$

$P[X < 28] = P[Z < 1.0] = 1 - P[Z > 1.0]$

look up Table.

$= 1 - 0.1587$

$P[X < 28] = \boxed{0.8413}$

$P[25 < X < 27]$?

$P[25 < X < 27] = P[X > 25] - P[X > 27]$

$P[X > 25]$: $Z = \frac{25 - 25.6}{2.4} = -0.25$ negative so use symmetry

symmetry $\rightarrow$ $P[Z > -a] = P[Z < a]$

so $P[Z > -0.25] = P[Z < 0.25]$

$P[Z < 0.25] = 1 - P[Z > 0.25]$ — look up Table.

$= 1 - 0.4013$

$\therefore$ $P[X > 25] = 0.5987$

$P[X > 27]$: $z = \dfrac{27 - 25.6}{2.4} = 0.58$ ← look up Table.

$\underline{P[X > 27] = 0.2810}$.

$\therefore P[25 < X < 27] = P[X > 25] - P[X > 27]$

$= 0.5987 - 0.2810$

$= \boxed{0.3177}$