Differentiation Tutorial Sheet 1

use

- Average Rate of change : $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$

Q1 $y = 3x^2 - 2x$ $\quad x_1 = 1 \quad x_2 = 2$

when $x_1 = 1$ $\quad y_1 = 3(1)^2 - 2(1)$

$= 3 - 2$

$y_1 = 1$

$(1, 1)$

when $x_2 = 2$ $\quad y_2 = 3(2)^2 - 2(2)$

$= 12 - 4$

$y_2 = 8$

$(2, 8)$

$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-1}{2-1} = \boxed{7}$ avg. rate of change

Q2 $y = 5x^2 - x$

$x_1 = 0.5 \quad y_1 = 5(0.5)^2 - 0.5$

$= 0.75$

$(0.5, 0.75)$

$x_2 = 1 \quad y_2 = 5(1)^2 - 1$

$= 4$

$(1, 4)$

$m = \frac{4 - 0.75}{1 - 0.5} = \frac{3.25}{0.5} = \boxed{6.5}$ avg. rate of change.

Q3. $r = 40t - 6t^2$

$t_1 = 2$ $\quad r_1 = (40)(2) - 6(2)^2$
$= 80 - 24$
$r_1 = 56$

$t_2 = 4$ $\quad r_2 = 40(4) - 6(4)^2$
$= 160 - 96$
$r_2 = 64$

$M = \dfrac{64-56}{4-2} = \dfrac{8}{2} = \boxed{4}$ avg. rate of change.

Q4 Differentiation using Powe Rule $\boxed{\begin{array}{c} y = x^n \\ \frac{dy}{dx} = nx^{n-1} \end{array}}$

a. $y = 3x^5 + 7x^4 + 2x^3 + 11x^2 + 4x + 9$

$(3)(5)x^{5-1} + (7)(4)x^{4-1} + (2)(3)x^{3-1} + (11)(2)x^{2-1} + (4)(1)x^{1-1}$

$\frac{dy}{dx} = 15x^4 + 28x^3 + 6x^2 + 22x + 4$

b. $s = 5x^2 - 7x + 11$

$\frac{ds}{dx} = (5)(2)x^{2-1} - (7)(1)x^{1-1} + 0$

$= 10x - 7$

c. $r = 3 - 2x - 4x^2 - 10x^3$

$\frac{dr}{dx} = -2 - 8x - 30x^2$

d. $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$

$$dy/dx = (2)\left(\tfrac{1}{2}\right)x^{\frac{1}{2}-1}$$

$$= x^{-1/2} = \frac{1}{\sqrt{x}}$$

Q5 $y = 2x^3 - 3x$ slope of tangent at point (2,10) (Derivative) use power Rule.

get $\frac{dy}{dx} = 6x^2 - 3$

(2,10)

sub into dy/dx for $x \Rightarrow dy/dx = 6(2)^2 - 3$

$= \boxed{21}$ slope of tangent (instantaneous rate of change — the derivative)

Q6. $y = x^2 + 2x - 3$ at the point (-2,-3)

get slope of tangent: $dy/dx = 2x + 2$

when $x = -2$: $dy/dx = 2(-2) + 2$

$= -4 + 2$

$dy/dx = -2$

$2y + 4x - 3 = 0$ use $y = mx + c$

$2y = -4x + 3$

$y = \frac{-4}{2}x + \frac{3}{2}$

$y = \boxed{-2}x + 3$ slope M

Parallel

Q7 i) $y = 2x^2 - 3x + 6$

$$dy/dx = 4x - 3$$

$$d^2y/dx^2 = 4$$

ii) $y = 4x^4 - 2x^2$

$$dy/dx = 16x^3 - 4x$$

$$d^2y/dx^2 = 48x^2 - 4$$

Q8. $y = \underbrace{(5x+7)}_{u}\underbrace{(2x+11)}_{v}$ use Product Rule $\frac{df}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$.

Let $u = 5x+7$ $du/dx = 5$

$v = 2x+11$ $dv/dx = 2$.

Sub all into the formula.

$$df/dx = (5x+7)(2) + (2x+11)(5)$$

$$= 10x + 14 + 10x + 55$$

$$= 20x + 69.$$

Q9 $$y = \frac{3x-5}{x^2+1} \quad \begin{matrix} u \\ v \end{matrix}$$

use Quotient Rule:

$$\frac{df}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Let $u = 3x-5$ $\quad du/dx = 3$

$v = x^2+1$ $\quad dv/dx = 2x$

Sub all into the formula:

$$\frac{df}{dx} = \frac{(x^2+1)(3) - (3x-5)(2x)}{(x^2+1)}$$

$$= \frac{3x^2+3 - 6x^2 + 10x}{(x^2+1)^2}$$

$$df/dx = \frac{-3x^2+10x+3}{(x^2+1)^2}.$$