Tutorial 5 – Probability

Section A: Probability

1. What is the probability of getting a 3 if you roll a fair dice (6-sided)?
2. What is the probability of getting a 2 if you roll a fair dice (6-sided)?
3. What is the probability of getting a 4 if you roll a fair 12-sided die?

The Probability *P* of an Event *E* is given by:

$$P[E] = \frac{Number\ of\ outcomes\ in\ E}{Total\ number\ of\ possible\ outcomes}$$

4. What is the probability of the <u>sum</u> of two (6-sided) fair dice rolls being less than 4?

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

SUM – Event Space

Build an event space for SUM – example given above. Where the values in the event space are the summation of the possible outcomes for 2 dice, represented in red and blue in the given example. Count up the number of outcomes < 4. Two dice = 36 total possible outcomes.

5. What is the probability of the <u>product</u> of two (6-sided) fair dice rolls being greater than 7?

Build an event space for PRODUCT (multiplication). Where the values in the event space are a result of multiplication of the possible outcomes for 2 dice. Count up the number of outcomes > 7

6. If two (6-sided) dice are rolled, what is the probability that the sum of the numbers is below 6? Using your answer, calculate the probability that the sum is equal to or greater than 6.

Use the SUM event space to work out *P[SUM < 6].* Then use the complement rule to work out *P[SUM ≥ 6].*

7. 15 people are asked to choose a number from 1 to 80. What is the probability that two or more people pick the same number?

To work on this question, it is simpler to look at the equivalent question
– find the probability that <u>no two people pick the same number</u>

Let:

P[N] = the probability that now two people pick the same number
P[S] = the probability that two or more people pick the same number
Then:
P[S] = 1 – P[N] (*see the example in the lecture notes – Probability part 1, slides 14-17)*

8. Consider two events A and B, where A is throwing a four with a fair (6-sided) dice, and B is drawing a Queen from a full deck of cards. Determine the probability of the occurrence of both events.

Independent Events: *P[A and B] = P[A]×P[B]. Let A = throwing a 4 with a fair dice and B = drawing a queen from a full deck of cards (52 cards in a deck, 4 Queens in a deck)*

9. If two fair (6-sided) dice are rolled, calculate the probability of getting a 2 and a 6?

Hint: 2 dice, can get a 2 and a 6 OR and 6 and a 2

10. There are six counties in a list. In how many ways can four counties be chosen:
    a. if the order matters?
    b. if the order doesn't matter?

If the order matters, use Permutations ${}^{n}P_r = \frac{n!}{(n-r)!}$

If the order doesn't matter, use Combinations ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

11. A student taking a degree course in computing has to choose five from a possible nine subjects: Computer Networks, Network Design and Management, Web Mining, Intelligent Computing, Games Development, Applied Language Engineering, Data Science, Text Analysis or Parallel Computing. How many different combinations of 5 subjects are there?

12. How many 4 letter codes can be constructed using the first 10 letters of the alphabet – if no letter can be repeated?

13. The user-codes on a certain device consist of 2 letters, followed by 2 digits, followed by a letter, for example *XY12A*. (Assume there is no distinction made between uppercase and lowercase letters)
    a. How many different user-codes can be constructed altogether if repetition is allowed?
    b. In how many of these user-codes does the digit 0 occur at least once?
    c. If neither the letters nor digits are allowed to repeat, how many different user-codes are there?

Section B: Probability Distributions

1. A random variable X represents the number of calls per hour to a call center in the last month. The probability distribution for X is given below. What is the missing value in the table?

X	0	1	2	3	4	5	6	7
$P[X]$	0.01	0.10	0.26	0.33	0.18	~~0.6~~	?	0.03

0.06

Hint: $P[E1] + P[E2] + \ldots + P[En] = 1$

Binomial Distribution

2. A surgery has a success rate of 75%. Suppose that the surgery is performed on 3 patients.

   a) What is the probability that the surgery is successful on exactly 2 patients?
   b) If X is the number of successes, possible values of X are 0, 1, 2, 3. Create the probability distribution for X *(hint: we already know it for $P[X = 2]$ from part a)*.
   c) Graph the probability distribution of X using a histogram.

Use $P(r \text{ successes in } n \text{ trials}) = {}^nC_r\ p^r\ q^{n-r} = \frac{n!}{(n-r)!\,r!}\ p^r\ q^{n-r}$ where p = the probability of success, q = the probability of failure, n = the number of trials (sample size), r = the number of successes.

Q2 part a)

$p = 75\% = \frac{75}{100} = 0.75$

$q = 1-p = 1-0.75 = 0.25$

$n = 3$

$r = 2$

$$
\begin{aligned}
P(2 \text{ successes in } 3 \text{ trials}) &= \frac{3!}{(3-2)!\,2!} \times p^2 \times q^{3-2} \\
&= \frac{3\times2\times1}{(2\times1)(1)} \times 0.75^2 \times 0.25^1 \\
&= 3 \times 0.5625 \times 0.25 \\
&= 0.422
\end{aligned}
$$

For:

Part b) work out *P(0 successes in 3 trails), P(1 successes in 3 trials)* and *P(3 successes in 3 trials)*.

Note: 0! = 1

Part c) use your answers from part a) and part b) to draw a histogram of *Number of Successes X* versus *Probability P[X].*

3. Dublin typically has rain on about 48% of days in October. What is the probably that it will rain on exactly 5 days in October. (Let us assume weather, rainy days, is quite close to being independent from day to day so that we can use the binomial distribution.)

Use Binomial distribution formula. Work out *p*, *q*, *n*, *r,* sub them into the formula and calculate.

Hint: how many days in October!

Poisson Distribution

4. The average number of houses sold by an estate agent is 2 houses per week. What is the probability that exactly 3 houses will be sold next week?

Use $P[R=r]=\frac{e^{-\mu}\mu^r}{r!}$ where μ = average, and *e* is a constant (Euler's number) and on your calculator.

Q4
μ = 2
r = 3

$$P[R=3] = \frac{e^{-2}\times 2^3}{3\times 2\times 1}$$
$$= 0.180 = \text{~}18\%$$

5. In a town, the average number of power cuts is 3 per year. Find the probability that in any given year, there will be exactly 5 power cuts.

6. A report presented to the quality control manager shows that 4% of computer parts being produced by a manufacturing company are defective. In a random sample of 100 parts, what is the probability that 7 of them will be defective? (use Poisson distribution)

   Hint: *μ = n×p*

7. The average number of patients arriving at an A&E department is 6 per hour. A patient arrives to the department at 11.30am. What is the probability that the next patient arrives before 12pm?

   Hint: The average number of patients per hour = 6. Therefore, the average number per half hour = 6/2 = 3

Normal Distribution

8. *X* is a random variable which is *normally distributed* with a mean of 1.98 and a standard deviation of 0.012. What is the probability that a value of *X*, chosen at random, is greater than 2?

   Use the following Steps - Step 1: Standardise the problem; Step 2: Look up the z-table; Step 3: Calculate the probability

   Q8:

   Use: $Z=\frac{x-\mu}{\sigma}$ to standardise, where μ = 1.98 and σ = 0.012

   Step 1: Standardise the problem

$$\frac{X-\mu}{\sigma} = \frac{2-1.98}{0.012} = 1.67\ (round\ to\ 2\ decimal\ places)$$

   Step 2: Look up the z-table
   P[X>2] = P[Z>1.67]
   Step 3: Calculate the probability
   P[X>2] = PZ>1.67] = 0.0475 = ~4.75%

9. X is a random variable, which is *normally distributed* with a mean of 25.6 and a standard deviation of 2.4. What is the probability that a value of X, chosen at random, is less than 28? What is the probability that it is between 25 and 27?

Follow the same steps as Q8.
Step 1: Standardise the problem
Step 2: Look up the z-table
Step 3: Calculate the probability

However, you will need to use *symmetry* for *P[X < a] i.e. P[X < a] = 1- P[X > a]* and *ranges* for *P[a < X < b]* (see lecture notes)

Probability Distributions:

$$P[X = r] = {}^nC_r\, p^r q^{n-r} = \frac{n!}{(n-r)!r!} p^r q^{n-r}$$ (Binomial distribution)

$$P[X = r] = \frac{e^{-\mu}\mu^r}{r!}$$, where $\mu = np$ (Poisson distribution)

$$a = \frac{x-\mu}{\sigma}$$ (Normal distribution)

Z-table on the last page.

Table of Probabilities – $P[Z > a]$, where Z is the standard normal variable.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000