

Recursion & Algorithms

Sample Sols

1. Recursion is a simple yet powerful concept in developing algorithms for solving complex problems. In programming it is the ability of a method to call itself. it is used in routines that solve problems by repeatedly processing the output of the same process - breaking a hard problem into smaller tasks.

2. a) $t(1) = 3$
 $t(n) = t(n-1) + 4 \quad (n > 1)$

$$\begin{aligned} t(2) &= t(2-1) + 4 & t(3) &= t(3-1) + 4 \\ &= t(1) + 4 & &= t(2) + 4 \\ &= 3 + 4 & &= 7 + 4 \end{aligned}$$

$$t(2) = 7$$

$$t(3) = 11$$

$$\begin{aligned} t(4) &= t(4-1) + 4 \\ &= t(3) + 4 \\ &= 11 + 4 \end{aligned}$$

$$t(4) = 15$$

$$b) \quad t(1) = 0$$

$$t(n) = 2t(n-1) + 1 \quad (n > 1)$$

$$t(2) = 2t(2-1) + 1$$

$$= 2t(1) + 1$$

$$= 2(0) + 1$$

$$t(2) = 1$$

$$t(3) = 2t(3-1) + 1$$

$$= 2t(2) + 1$$

$$= 2(1) + 1$$

$$t(3) = 3$$

$$t(4) = 2t(4-1) + 1$$

$$= 2t(3) + 1$$

$$= 2(3) + 1$$

$$t(4) = 7$$

3. a) non-Recursive.

2, 4, 6, 8, 10, 12, ...

$$t(n) = 2n$$

e.g. when $n=1$ $t(1) = 2(1) = 2$

$n=2$ $t(2) = 2(2) = 4$

$n=3$ $t(3) = 2(3) = 6$ and so on.

Recursive

$$t(1) = 2$$

$$t(n) = t(n-1) + 2 \quad (n > 1)$$

↖ contains the previous term / defined in terms of itself! (2)

b) 2, 5, 8, 11, 14, 17, ... Begin with 2 and go up in steps of 3.

non-Recursive

$$t(n) = 3n - 1$$

$$\text{e.g. } t(1) = 3(1) - 1 \\ = 2$$

$$t(2) = 3(2) - 1 \\ = 5$$

$$t(3) = 3(3) - 1 \\ = 8$$

and so on.

Recursive

$$t(1) = 2$$

$$t(n) = t(n-1) + 3 \quad (n > 1)$$

$$\text{e.g. } t(2) = t(2-1) + 3 \\ = t(1) + 3 \\ = 2 + 3 \\ = 5$$

$$t(3) = t(2) + 3 \\ = 5 + 3 \\ = 8$$

$$t(4) = t(3) + 3 \\ = 8 + 3 \\ = 11$$

and so on.

4. a) 1. Input m
2. $t \leftarrow 2$
3. Output t
4. For $n = 2$ to m do
 4.1 $t \leftarrow t + 2$
 4.2 Output t

b) 1. Input m
2. $t \leftarrow 2$
3. Output t
4. For $n = 2$ to m do
 4.1 $t \leftarrow t + 3$
 4.2 output t