

# Recursion & Algorithms

## Sample Solns

1. Recursion is a simple yet powerful concept in developing algorithms for solving complex problems. In programming it is the ability of a method to call itself. It is used in routines that solve problems by repeatedly processing the output of the same process - breaking a hard problem into smaller tasks.

2. a)  $t(1) = 3$

$$t(n) = t(n-1) + 4 \quad (n > 1)$$

$$t(2) = t(2-1) + 4$$

$$= t(1) + 4$$

$$= 3 + 4$$

$$t(3) = t(3-1) + 4$$

$$= t(2) + 4$$

$$= 7 + 4$$

$$t(2) = 7$$

$$t(3) = 11$$

$$t(4) = t(4-1) + 4$$

$$= t(3) + 4$$

$$= 11 + 4$$

$$t(4) = 15$$

$$b) t(1) = 0$$

$$t(n) = 2t(n-1) + 1 \quad (n > 1)$$

$$t(2) = 2t(2-1) + 1 \quad t(3) = 2t(3-1) + 1$$

$$= 2t(1) + 1 \quad = 2t(2) + 1$$

$$= 2(0) + 1 \quad = 2(1) + 1$$

$$t(2) = 1 \quad t(3) = 3$$

$$t(4) = 2t(4-1) + 1$$

$$= 2t(3) + 1$$

$$= 2(3) + 1$$

$$t(4) = 7$$

3. a) non-Recursive.  $2, 4, 6, 8, 10, 12, \dots$

$$t(n) = 2n$$

e.g. when  $n=1$   $t(1) = 2(1) = 2$

$n=2$   $t(2) = 2(2) = 4$

$n=3$   $t(3) = 2(3) = 6$  and so on.

Recursive

$$t(1) = 2$$

$$t(n) = t(n-1) + 2 \quad (n > 1)$$

☞ contains the previous term / defined in terms of itself! ②

b)  $2, 5, 8, 11, 14, 17, \dots$  Begin with 2 and go up in steps of 3.

Non-Recursive

$$t(n) = 3n - 1$$

$$\begin{aligned} \text{e.g. } t(1) &= 3(1) - 1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} t(2) &= 3(2) - 1 \\ &= 5 \end{aligned}$$

$$\begin{aligned} t(3) &= 3(3) - 1 \\ &= 8 \end{aligned}$$

and so on.

Recursive

$$t(1) = 2$$

$$t(n) = t(n-1) + 3 \quad (n > 1)$$

$$\begin{aligned} \text{e.g. } t(2) &= t(2-1) + 3 \\ &= t(1) + 3 \\ &= 2 + 3 \\ &= 5 \end{aligned}$$

$$\begin{aligned} t(3) &= t(2) + 3 \\ &= 5 + 3 \\ &= 8 \end{aligned}$$

$$\begin{aligned} t(4) &= t(3) + 3 \\ &= 8 + 3 \\ &= 11 \end{aligned}$$

and so on.

4. a)
1. Input m
  2.  $t \leftarrow 2$
  3. Output t
  4. For  $n = 2$  to m do
    - 4.1  $t \leftarrow t + 2$
    - 4.2 Output t

- b)
1. Input m
  2.  $t \leftarrow 2$
  3. Output t
  4. For  $n = 2$  to m do
    - 4.1  $t \leftarrow t + 3$
    - 4.2 Output t