

Integration Tutorial Sheet

Sample Solutions

Q1. a) $\int x^0 \cdot dx = \frac{x^1}{1} + C$

b) $\int x^{1/2} \cdot dx = \frac{x^{3/2}}{3/2} + C = \frac{2x^{3/2}}{3} + C$

c) $\int \frac{6}{x} \cdot dx = 6 \int \frac{1}{x} \cdot dx = 6 \ln x + C$

d) $\int x^{-3} \cdot dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

Q2. a) $\int (2x^8 - 5x^2 + 6x - 9) dx$

$$= \frac{2x^4}{4} - \frac{5x^3}{3} + \frac{6x^2}{2} - 9x + C$$

$$= \frac{x^4}{2} - \frac{5x^3}{3} + 3x^2 - 9x + C$$

$$\begin{aligned}
 b) \int (4x^8 - 6x^2 - 16x + 4) dx \\
 &= \frac{4x^4}{4} - \frac{6x^3}{3} - \frac{16x^2}{2} + 4x + C \\
 &= x^4 - 2x^3 - 8x^2 + 4x + C
 \end{aligned}$$

$$\begin{aligned}
 c) \int (8x^3 + 6x^2 - 5x + 4) dx \\
 &= \frac{8x^4}{4} + \frac{6x^3}{3} - \frac{5x^2}{2} + 4x + C \\
 &= 2x^4 + 2x^3 - \frac{5x^2}{2} + 4x + C
 \end{aligned}$$

$$\begin{aligned}
 Q3. \quad a) \int_0^4 (x^2 - 3x + 4) dx \\
 &= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 4x \right]_0^4 \\
 &= \left[\frac{4^3}{3} - \frac{3(4)^2}{2} + 4(4) \right] - \left[\frac{0^3}{3} - \frac{3(0)^2}{2} + 4(0) \right] \\
 &= \left[\frac{64}{3} - \frac{48}{2} + 16 \right] - [0] \\
 &= \frac{64}{3} - 24 + 16 = \frac{64}{3} - 8 = \frac{64}{3} - \frac{24}{3} = \frac{40}{3} \\
 &= 13.333 \text{ square units}
 \end{aligned}$$

$$b) \int_{-2}^3 (3x^2 + 5) dx = \left[\frac{8x^3}{3} + 5x \right]_{-2}^3$$

$$= [27 + 15] - [-8 - 10]$$

= 60 square units.

$$c) \int_0^2 (-3x^2 + 6x + 5) dx$$

$$= \left[-\frac{3x^3}{3} + \frac{6x^2}{2} + 5x \right]_0^2$$

$$= \left[-x^3 + 3x^2 + 5x \right]_0^2$$

$$= [-(2)^3 + 3(2)^2 + 5(2)] - [0 + 0 + 0]$$

$$= [-8 + 12 + 10]$$

= 14 square units.

$$d) \int_{-1}^4 (-3x^2 + 12x + 10) dx$$
$$= \left[-\frac{3x^3}{3} + \frac{12x^2}{2} + 10x \right]_1^4$$

$$= \left[-x^3 + 6x^2 + 10x \right]_1^4$$

$$= \left[-(4)^3 + 6(4)^2 + 10(4) \right] - \left[-(1)^3 + 6(1)^2 + 10(1) \right]$$

$$= \left[-64 + 96 + 40 \right] - \left[-1 + 6 + 10 \right]$$

$$= 72 - 15$$

$$= 57 \text{ square units.}$$