Differentiation Part 2 Overview

Q1 (Product Rule) and **Q3** (Quotient Rule) are on the Differentiation part 1 overview document on the Moodle page.

Question 2

Find the value of the derivative of $\underbrace{(1-x^2)}_{u}\underbrace{(x^3-2x-5)}_{v}$ at the point (1, 0).

product Rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = (1-x^2) \quad \frac{du}{dx} = -2x$

$V = (x^3-2x-5) \quad \frac{dv}{dx} = 3x^2 - 2$

sub in $x=1$ into dy/dx

Question 4

Differentiate $y = (\underbrace{x^2 - x - 5}_{u})^3$. Hence find the slope of the tangent to the curve when $x = 3$.

use chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$u = (x^2 - x - 5) \quad$ so $y = u^3$

$\frac{du}{dx} = 2x - 1 \quad \frac{dy}{du} = 3u^2$

Question 5

A circuit is designed so that it outputs an alternating current (I) whose value in milliamps at any time t seconds after the circuit is activated is given by: $1 + 30t - 5t^2$.

Find,

a. The current 1 second after the circuit is activated.
b. The rate at which the current is changing after 2 seconds.
c. The time taken for the current to reach its first peak value.

a) $t = 1$ sub in t in $1+30t-5t^2$ and solve.

b) rate $\frac{dI}{dt} =$
sub in $t = 2$

c) $dI/dt = 0$ solve for t.

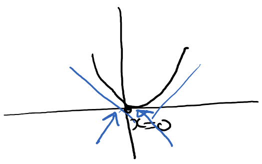
Question 6

Determine the position of any maximum and minimum points of the function $y = x^2 + 1$.

First Derivate test:

- Get dy/dx
- Let dy/dx = 0 and solve for x
- Take a point to the left of x and a point to the right of x. Sub these values into dy/dx and solve
- If the slope is positive to the left and negative to the right then it is a Maximum turning point
- If the slope is negative to the left and positive to the right then it is a Minimum turning point

$\frac{dy}{dx} = 2x$



$dy/dx = 0$

$2x = 0$

$x = 0/2$

$x = 0$

let $x = -0.1$

$\frac{dy}{dx} = 2(-0.1) = -0.2$

NEG

let $x = 0.1$

$\frac{dy}{dx} = 2(0.1) = 0.2$

POS

$\Rightarrow$ Minimum turning point

Question 8

Determine the position of any maximum and minimum points of the function

$y = \frac{t^3}{3} - \frac{t^2}{2} - 2t + 3.$

Second Derivative test:

Get dy/dx
Let dy/dx = 0 and solve for x
Get d^2y/dx^2
Sub in the value(s) for x into the 2nd derivative d^2y/dx^2
Where $d^2y/dx^2 < 0$ it is a Maximum turning point
Where $d^2y/dx^2 > 0$ it is a Minimum turning point

$\frac{dy}{dt} = \frac{\cancel{3}t^2}{\cancel{3}} - \frac{\cancel{2}t}{\cancel{2}} - 2 = t^2 - t - 2 = 0$

$$\frac{dy}{dx} = \frac{\cancel{3}t^2}{\cancel{3}} - \frac{\cancel{2}t}{\cancel{2}} - 2 = t^2 - t - 2 = 0$$

$$(t+1)(t-2) = 0$$

$$\therefore \quad t+1=0 \quad \text{or} \quad t-2=0$$

$$t=-1 \qquad\qquad t=2$$

$$\frac{d^2y}{dx^2} = 2t - 1$$

sub in $t=-1$

$$\frac{d^2y}{dx^2} = 2(-1) - 1 = -3$$

< 0 therefore at $t= -1$ there is a Max turning point

sub in $t=2$

$$2(2)-1 = 3 \Rightarrow \frac{d^2y}{dx^2} > 0 \therefore \text{MIN POINT.}$$

Find the coordinates of the MAX & MIN points:

$$t=-1 \Rightarrow y = \frac{-1^3}{3} - \frac{(-1)^2}{2} - 2(-1) + 3$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 + 3$$

$$= \frac{-2}{6} - \frac{3}{6} + \frac{12}{6} + \frac{18}{6}$$

$$= \frac{25}{6}$$

point : $(-1, 25/6)$

$$t=2 \Rightarrow y = \frac{2^3}{3} - \frac{2^2}{2} - 2(2) + 3$$

$$= -1/3$$

point : $(2, -1/3)$