Determinant 3x3

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$$A = \begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix}$$

$$\det(A) = a_{11}\underbrace{C_{11}}_{\text{Cofactor.}} + a_{12}C_{12} + a_{13}C_{13}$$

$$= (2)(-3) + (-3)(11) + (1)(13)$$

$$= \boxed{-26}$$

get minors i.e. M_{11}, M_{12}, M_{13} → then use these to give the cofactors C_{11}, C_{12}, C_{13}.

a_{11}:

$$\begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \overset{ad-bc}{\begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix}} = (-1)(-1) - (2)(2)$$

$M_{11} = -3$

$i\ j \quad 1+1 = 2$ even

$C_{11} = M_{11} = \boxed{-3}$

a_{12}:

$$\begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} 5 & 2 \\ 3 & -1 \end{vmatrix} = (-1)(5) - (2)(3)$$

$M_{12} = -11$

$i\ j \Rightarrow 1+2 = 3$ odd

$\therefore C_{12} = -(M_{12}) = -(-11)$

$C_{12} = \boxed{11}$

a_{13}:

$$\begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix} = (5)(2) - (-1)(3) =$$

$10 - (-3) \Rightarrow 10+3$

$M_{13} = 13$

$i\ j \Rightarrow 1+3 = 4$ even

$\therefore C_{13} = M_{13} = 13$

$C_{13} = \boxed{13}$