Sample Solutions - Matrices Part 1.

Q1 Rank / size

i) $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \\ 2 & 4 \\ 1 & 6 \end{bmatrix}$ 4 rows x 2 cols

r x c

size $A = 4 \times 2$.

ii) $B = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ 2 rows x 2 cols

size $B = 2 \times 2$

iii) $C = \begin{bmatrix} 1 & 4 & 8 \\ 2 & -5 & 2 \\ 3 & 6 & -1 \end{bmatrix}$ ← row 1, ← row 2, ← row 3

col1 col2 col3

3 rows x 3 col

size $C = 3 \times 3$

iv) $D = \begin{bmatrix} -5 & 1 & 2 \\ 4 & -1 & 4 \\ 3 & 0 & 5 \\ 2 & 1 & -2 \end{bmatrix}$ ← r1, ← r2, ← r3, ← r4

C1 C2 C3

4 rows(r) x 3 cols(c

r x c

size $D = 4 \times 3$

v) $E = \begin{bmatrix} 1 & 4 & 6 & 3 \\ 0 & 5 & 9 & 5 \\ 7 & -1 & 6 & -5 \\ 12 & 6 & 8 & 4 \\ -2 & 3 & 4 & 0 \end{bmatrix}$

$E = 5 \times 4$

Q2 elements of a Matrix a_{ij}

Name

row col

col 2

row 1

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \\ 2 & 4 \\ 1 & 6 \end{bmatrix}$$

a_{12} = 3 (row 1, col 2, Matrix A)

$a_{12} = 3$

$a_{31} = 2$, $a_{22} = 5$, $a_{42} = 6$

$$B = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

$b_{22} = 4$ (row 2, col 2, Matrix B), $b_{12} = -2$.

$$C = \begin{bmatrix} 1 & 4 & 8 \\ 2 & -5 & 2 \\ 3 & 6 & -1 \end{bmatrix}$$

$C_{23} = 2$ (row 2, col 3, Matrix C)

$C_{23} = 2$

$C_{32} = 6$, $C_{13} = 8$.

c_2

$$D = \begin{bmatrix} -5 & 1 & 2 \\ 4 & -1 & 4 \\ 3 & 0 & 5 \\ 2 & 1 & -2 \end{bmatrix}$$

r2, r4, col 1

$d_{22} = -1$

$d_{41} = 2$, $d_{33} = 5$, $d_{23} = 4$

$$E = \begin{matrix} r1 \\ \\ \\ \\ r5 \end{matrix} \begin{bmatrix} 1 & 4 & 6 & 3 \\ 0 & 5 & 9 & 5 \\ 7 & -1 & 6 & -5 \\ 12 & 6 & 8 & 4 \\ -2 & 3 & 4 & 0 \end{bmatrix} \quad \begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \begin{matrix} c4 \\ \\ \\ \\ \\ \end{matrix}$$

$$\begin{matrix} \\ \\ \\ \\ c3 \end{matrix}$$

$e_{14} = 3$

$e_{23} = 9$, $e_{43} = 8$, $e_{53} = 4$, $e_{11} = 1$, $e_{44} = 4$

Q3 addition and subtraction of matrices – must be the same size, add/subtract corresponding elements

i) $\begin{bmatrix} 2 & 3 \\ -4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 2+1 & 3+2 \\ -4+5 & 3+4 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 7 \end{bmatrix}$

i.e. $\begin{bmatrix} r_1c_1 & r_1c_2 \\ r_2c_1 & r_2c_2 \end{bmatrix} + \begin{bmatrix} r_1c_1 & r_1c_2 \\ r_2c_1 & r_2c_2 \end{bmatrix} = \begin{bmatrix} r_1c_1+r_1c_1 & r_1c_2+r_1c_2 \\ r_2c_1+r_2c_1 & r_2c_2+r_2c_2 \end{bmatrix}$

r - row
c - column.

ii) $\begin{bmatrix} 5 & -1 \\ 6 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 7 & -2 \end{bmatrix} = \begin{bmatrix} 5-4 & -1-(-1) \\ 6-7 & 2-(-2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$

iii) $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & -3 \\ -1 & 0 & -2 \end{bmatrix} - \begin{bmatrix} -2 & 3 & 5 \\ 5 & 1 & -3 \\ 3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3-(-2) & 2-3 & 5-5 \\ 4-5 & 1-1 & -3-(-3) \\ -1-3 & 0-2 & -2-(-1) \end{bmatrix}$

$$= \begin{bmatrix} 5 & -1 & 0 \\ -1 & 0 & 0 \\ -4 & -2 & -1 \end{bmatrix}$$

iv) $\begin{bmatrix} 1 & 4 & 0 \\ 2 & 8 & -2 \\ -3 & 5 & -5 \end{bmatrix} - \begin{bmatrix} -1 & 4 & 3 \\ -2 & -5 & -2 \\ 5 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1-(-1) & 4-4 & 0-3 \\ 2-(-2) & 8-(-5) & -2-(-2) \\ -3-5 & 5-0 & -5-(-1) \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & -3 \\ 4 & 13 & 0 \\ -8 & 5 & -4 \end{bmatrix}$

v) $\begin{bmatrix} 1 & 3 & 9 & 6 \\ 2 & -1 & 4 & 7 \\ 3 & -2 & 0 & 9 \\ 4 & 6 & 7 & 2 \\ -3 & 0 & 9 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 & 1 & 6 \\ 2 & -4 & 0 & 2 \\ 3 & 9 & -1 & 4 \\ 0 & 7 & -3 & 8 \\ 7 & 0 & 4 & -6 \end{bmatrix}$

$= \begin{bmatrix} 1-3 & 3-(-2) & 9-1 & 6-6 \\ 2-2 & -1-(-4) & 4-0 & 7-2 \\ 3-3 & -2-9 & 0-(-1) & 9-4 \\ 4-0 & 6-7 & 7-(-3) & 2-8 \\ -3-7 & 0-0 & 9-4 & 1-(-6) \end{bmatrix}$

$= \begin{bmatrix} -2 & 5 & 8 & 0 \\ 0 & 3 & 4 & 5 \\ 0 & -11 & 1 & 5 \\ 4 & -1 & 10 & -6 \\ -10 & 0 & 5 & 7 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 3 & -4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 & -2 \\ 1 & -1 & 0 \\ 4 & -3 & -2 \end{bmatrix}$$

$3 \times 3 \quad 3 \times 3$

size of Resulting Matrix

r_1 c_1 → Multiply the corresponding elements and add.

Multiply row x col

$r_i \times c_i$ The answer goes in position $r_i c_i$ in the resulting matrix

Repeat for each row in the first matrix by each column in the second matrix

$$AB = \begin{bmatrix} r_1c_1 \times r_1c_1 + r_1c_2 \times r_2c_1 + r_1c_3 \times r_3c_1 & r_1c_1 \times r_1c_2 + r_1c_2 \times r_2c_2 + r_1c_3 \times r_3c_2 & r_1c_1 \times r_1c_3 + r_1c_2 \times r_2c_3 + r_1c_3 \times r_3c_3 \\ r_2c_1 \times r_1c_1 + r_2c_2 \times r_2c_1 + r_2c_3 \times r_3c_1 & r_2c_1 \times r_1c_2 + r_2c_2 \times r_2c_2 + r_2c_3 \times r_3c_2 & r_2c_1 \times r_1c_3 + r_2c_2 \times r_2c_3 + r_2c_3 \times r_3c_3 \\ r_3c_1 \times r_1c_1 + r_3c_2 \times r_2c_1 + r_2c_3 \times r_3c_1 & r_3c_1 \times r_1c_3 + r_3c_2 \times r_2c_2 + r_3c_3 \times r_3c_2 & r_3c_1 \times r_1c_3 + r_3c_2 \times r_2c_3 + r_3c_3 \times r_3c_3 \end{bmatrix}$$

$$= \begin{bmatrix} r_1c_1 & r_1c_2 & r_1c_3 \\ r_2c_1 & r_2c_2 & r_2c_3 \\ r_3c_1 & r_3c_2 & r_3c_3 \end{bmatrix} = \begin{bmatrix} (2\times0+1\times1+-1\times4) & (2\times2+1\times-1+-1\times-3) & (2\times-2+1\times0+-1\times-2) \\ (0\times0+2\times1+1\times4) & (0\times2+2\times-1+1\times-3) & (0\times-2+2\times0+1\times-2) \\ (3\times0+-4\times1+-2\times4) & (3\times2+-4\times-1+-2\times-3) & (3\times-2+-4\times0+-2\times-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0+1-4 & 4-1+3 & -4+0+2 \\ 0+2+4 & 0-2-3 & 0+0-2 \\ 0-4-8 & 6+4+6 & -6+0+4 \end{bmatrix} = \begin{bmatrix} -3 & 6 & -2 \\ 6 & -5 & -2 \\ -12 & 16 & -2 \end{bmatrix}$$

84 Matrix Multiplication

2) $AD \Rightarrow \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 3 & -4 & -2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 3 \\ 1 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix}$

$3\times3 \quad\quad 3\times3$ ✓

Size of Resulting Matrix.

$$= \begin{bmatrix} (2\times2+1\times1+-1\times-1) & (2\times0+1\times2+-1\times1) & (2\times3+1\times-4+-1\times-2) \\ (0\times2+2\times1+1\times-1) & (0\times0+2\times2+1\times1) & (0\times3+2\times-4+1\times-2) \\ (3\times2+-4\times1+-2\times-1) & (3\times0+-4\times2+-2\times1) & (3\times3+-4\times-4+-2\times-2) \end{bmatrix}$$

$$= \begin{bmatrix} (4+1+1) & (0+2-1) & (6-4+2) \\ (0+2-1) & (0+4+1) & (0-8-2) \\ (6-4+2) & (0+8+2) & (9+16+4) \end{bmatrix} = \begin{bmatrix} 6 & 1 & 4 \\ 1 & 5 & -10 \\ 4 & -10 & 29 \end{bmatrix}$$

3) $BE \Rightarrow \begin{bmatrix} 0 & 2 & -2 \\ 1 & -1 & 0 \\ 4 & -3 & -2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 4 \\ 2 & -1 & -3 \\ -2 & 0 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 0\times0+2\times2+-2\times-2 & 0\times1+2\times-1+-2\times0 & 0\times4+2\times-3+-2\times-2 \\ 1\times0+-1\times2+0\times-2 & 1\times1+-1\times-1+0\times0 & 1\times4+-1\times-3+0\times-2 \\ 4\times0+-3\times2+-2\times-2 & 4\times1+-3\times-1+-2\times0 & 4\times4+-3\times-3+-2\times-2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4+4 & 0+(-2)+0 & 0-6+4 \\ 0-2+0 & 1+1+0 & 4+3+0 \\ 0-6+4 & 4+3+0 & 16+9+4 \end{bmatrix} = \begin{bmatrix} 8 & -2 & -2 \\ -2 & 2 & 7 \\ -2 & 7 & 29 \end{bmatrix}$$

4) $BF = \begin{bmatrix} 0 & 2 & -2 \\ 1 & -1 & 0 \\ 4 & -3 & -2 \end{bmatrix} \times \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

$3 \times \boxed{3 \quad 3} \times 1$ ✓

Size of Resulting Matrix = 3×1 i.e.

B F
$r1 \times C1$
$r2 \times C1$
$r3 \times C1$
only 1 column in F.

$$BF = \begin{bmatrix} 0\times3+2\times-2+-2\times1 \\ 1\times3+-1\times-2+0\times1 \\ 4\times3+-3\times-2+-2\times1 \end{bmatrix} = \begin{bmatrix} 0-4-2 \\ 3+2+0 \\ 12+6-2 \end{bmatrix} = \begin{bmatrix} -6 \\ 5 \\ 16 \end{bmatrix}$$

5) $(A+B)F$ Do what is in the brackets first

$$(A+B) = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 3 & -4 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -2 \\ 1 & -1 & 0 \\ 4 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -3 \\ 1 & 1 & 1 \\ 7 & -7 & -4 \end{bmatrix}$$

$$\therefore \underbrace{\begin{bmatrix} 2 & 3 & -3 \\ 1 & 1 & 1 \\ 7 & -7 & -4 \end{bmatrix}}_{(A+B)} \times \underbrace{\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}}_{F} = \begin{bmatrix} 2\times3+3\times-2+-3\times1 \\ 1\times3+1\times-2+1\times1 \\ 7\times3+-7\times-2+-4\times1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 31 \end{bmatrix}$$

6) AI I = the Identity Matrix. A is 3×3 therefore the corresponding Identity Matrix I is also 3×3.

$$\underbrace{\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 3 & -4 & -2 \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I} = \begin{bmatrix} 2+0+0 & 0+1+0 & 0+0+(-1) \\ 0+0+0 & 0+2+0 & 0+0+1 \\ 3+0+0 & 0-4+0 & 0+0-2 \end{bmatrix}$$

over page →

Matrix A

$$AI = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 3 & -4 & -2 \end{bmatrix}$$

the Identity Matrix I is the equivalent for matrices as the number 1 in ordinary numbers i.e. $3 \times 1 = 3$

$\therefore AI = A$.

$AI = A$

7) $2A = 2\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 3 & -4 & -2 \end{bmatrix} = \begin{bmatrix} 2(2) & 2(1) & 2(-1) \\ 2(0) & 2(2) & 2(1) \\ 2(3) & 2(-4) & 2(-2) \end{bmatrix}$

↑ Multiply Matrix A by the scalar 2.

↘ multiply each element in A by 2.

$$= \begin{bmatrix} 4 & 2 & -2 \\ 0 & 4 & 2 \\ 6 & -8 & -4 \end{bmatrix}$$

8) $A - E = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 3 & -4 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 4 \\ 2 & -1 & -3 \\ -2 & 0 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 2-0 & 1-1 & -1-4 \\ 0-2 & 2-(-1) & 1-(-3) \\ 3-(-2) & -4-0 & -2-(-2) \end{bmatrix} = \begin{bmatrix} 2 & 0 & -5 \\ -2 & 3 & 4 \\ 5 & -4 & 0 \end{bmatrix}$$

9) $\frac{B}{2}$ ← cannot do division directly in Matrix, invert and multiply (i.e. 2 inverted is $\frac{1}{2}$)

i.e. $B \times \frac{1}{2} = \begin{bmatrix} 0 & 1 & -1 \\ 1/2 & -1/2 & 0 \\ 2 & -3/2 & -1 \end{bmatrix}$

Q5 Matrix multiplication.

i) $AB = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}$

$2\times2 \quad 2\times2$ ✓

2×2.

$= \begin{bmatrix} 1\times2+3\times-1 & 1\times0+3\times1 \\ -1\times2+3\times-1 & -1\times0+3\times1 \end{bmatrix} = \begin{bmatrix} 2-3 & 0+3 \\ -2-3 & 0+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ -5 & 3 \end{bmatrix}$

2×2

ii) $AB = \begin{bmatrix} 1 & 3 \\ -5 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & -2 \end{bmatrix}$

$2\times2 \quad 2\times3$ ✓

Ans: 2×3.

$= \begin{bmatrix} 1\times1+3\times-1 & 1\times0+3\times2 & 1\times-1+3\times-2 \\ -5\times1+3\times-1 & -5\times0+3\times2 & -5\times-1+3\times-2 \end{bmatrix}$

$= \begin{bmatrix} 1-3 & 0+6 & -1-6 \\ -5-3 & 0+6 & 5-6 \end{bmatrix} = \begin{bmatrix} -2 & 6 & -7 \\ -8 & 6 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} -2 & 6 & -7 \\ -8 & 6 & -1 \end{bmatrix}$

2×3

iii) $AB = \begin{bmatrix} 1 & 2 \\ -4 & 2 \end{bmatrix} \times \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times -1 + 2\times 2 \\ -4\times -1 + 2\times 2 \end{bmatrix} = \begin{bmatrix} -1+4 \\ 4+4 \end{bmatrix}$

$2\times 2 \quad \checkmark \quad 2\times 1$

ANS: 2×1

$= \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

2×1

iv) $AB = \begin{bmatrix} 0 & 1 & -2 \\ 2 & -1 & -1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -4 & 3 \\ 2 & 0 \end{bmatrix}$

$2\times 3 \quad \checkmark \quad 3\times 2$

ANS: 2×2.

$= \begin{bmatrix} 0\times 3 + 1\times -4 + -2\times 2 & 0\times 1 + 1\times 3 + -2\times 0 \\ 2\times 3 + -1\times -4 + -1\times 2 & 2\times 1 + -1\times 3 + -1\times 0 \end{bmatrix}$

$= \begin{bmatrix} 0-4-4 & 0+3+0 \\ 6+4-2 & 2-3+0 \end{bmatrix}$

$= \begin{bmatrix} -8 & 3 \\ 8 & -1 \end{bmatrix}$

2×2