Differentiation Sample Solutions Q10 - Q14

Q10 $y = (x^2 - x - 5)^3$. Find slope when $x = 3$.

use chain Rule.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$u = (x^2 - x - 5)$ so $y = u^3$

$\frac{du}{dx} = 2x - 1$ $\frac{dy}{du} = 3u^2$

so $\frac{dy}{dx} = 3u^2(2x-1)$

$= 3(x^2 - x - 5)^2(2x - 1)$

$\frac{dy}{dx} = (6x - 3)(x^2 - x - 5)^2$

$x = 3 \Rightarrow (6(3) - 3)(3^2 - 3 - 5)^2 = 15(1) = 15.$

Q11 $1 + 30t - 5t^2$

a) sub in $t = 1 \Rightarrow 1 + 30(1) - 5(1)^2 = 26$

b) $\frac{dI}{dt} = 30 - 10t$ sub in $t = 2$

$= 30 - 10(2)$

$= 10$

c) time taken to reach its peak?

at peak $\frac{dy}{dx} = 0$.

so let $30 - 10t = 0$ and solve for t.

$10t = 30$

$t = \frac{30}{10}$

$t = 3$ seconds.

Q12. $y = x^2 + 1$ at max|min point $\frac{dy}{dx} = 0$.

Using 2^{nd} derivative test:

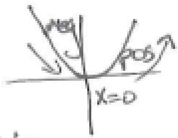
$$\frac{dy}{dx} = 2x \qquad \text{let } \frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} = 2 \; \ldots$$

$$2x = 0$$

$$x = \frac{0}{2}$$

$$x = 0.$$

Use 1^{st} derivative test



$x = 0$.

let $x = -0.1$ $\quad \frac{dy}{dx} = 2(-0.1) = -0.2$ NEG

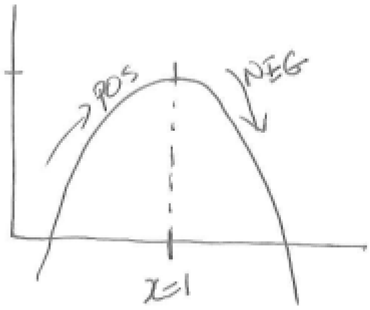
$x = 0.1$ $\quad \frac{dy}{dx} = 2(0.1) = 0.2$ POS

$\Rightarrow$ MINIMUM POINT 5

Q13 $y = 2x - x^2$

$\frac{dy}{dx} = 2 - 2x$ $\quad\quad$ $\frac{d^2y}{dx^2} = -2$

use 1st derivative test:



$\frac{dy}{dx} = 2 - 2x = 0$

$2x = 2$

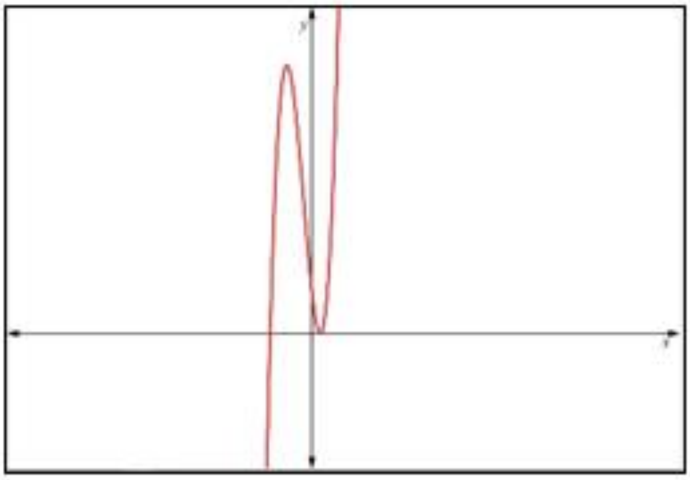
$x = 1$

Let $x = 0.9$: $\frac{dy}{dx} = 2 - 2(0.9) = 0.2$ POS

$x = 1.1$: $dy/dx = 2 - 2(1.1) = -0.2$ NEG

$\therefore$ MAXIMUM POINT.

Q14



$$y = x^3 + 3x^2 - 9x + 5$$

$$\frac{dy}{dx} = 3x^2 + 6x - 9$$

$$\frac{d^2y}{dx^2} = 6x + 6$$

if there is a max/min point / at a max/min point $\frac{dy}{dx} = 0$.

let $3x^2 + 6x - 9 = 0$ and solve for x.

$(3x - 3)(x + 3)$ by factorisation.

$\therefore 3x = 3 \quad x = -3.$

$x = 1$

that is there are max/min turning points where $x = 1$ and $x = -3$

Sub $x=1$ into the original eqn to find the corresponding y value : $y = (1)^3 + 3(1)^2 - 9(1) + 5$

$y = 0$

have the point $(1,0)$ min turning point see graph.

sub $x = -3$ into the original equation to find the corresponding y value:

$$y = (-3)^3 + 3(-3)^2 - 9(-3) + 5$$

$$y = 32$$

have the point $(-3, 32)$ max turning point
see graph.