

Eliminating Possibilities

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Part 1

Problem 1: SQUARE ROOTS

1. *Square Root of 4356:*

The number 4356 ends in 6. When a square number ends in 6, its square root must end in either 4 or 6 (since $4^2 = 16$ and $6^2 = 36$).

Next, consider the first digits of 4356. The square root of 4000 is 63 and the square root of 4900 is 70. So, the square root of 4356 is a number between 63 and 70.

Combining both observations, the possible candidates are 64 and 66. ($64^2 = 4096$) and ($66^2 = 4356$).

Therefore, the square root of 4356 is 66.

2. *Square Root of 8464:*

The number 8464 ends in 4. Therefore, its square root must end in either 2 or 8 (since $2^2 = 4$ and $8^2 = 64$).

Considering the first digits, the square root of 8000 is around 89, and the square root of 9000 is 95. So, the square root of 8464 is a number between 89 and 95.

The possible candidates are 92 and 98. ($92^2 = 8464$) and ($98^2 = 9604$).

Hence, the square root of 8464 is 92.

Thus, the square roots are 66 for 4356 and 92 for 8464.

Problem 3: HOW MANY LINES?

1. *Divisible by 3 with a remainder of 2: This means the number is of the form $(3n + 2)$.*

Example numbers are 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, etc.

2. *Divisible by 5 with a remainder of 2: This means the number is of the form $(5m + 2)$.*

Example numbers are 2, 7, 12, 17, 22, 27, 32, 37, 42, 47, etc.

3. *Divisible by 7 with a remainder of 5: This means the number is of the form $(7k + 5)$.*

Example numbers are 5, 12, 19, 26, 33, 40, 47, etc.

The task is to find a number that appears in all three lists. We can start by identifying numbers from the second list (since it's sparser than the first) and check if they satisfy the other two conditions.

The number that satisfies all three conditions (divisible by 3 with a remainder of 2, divisible by 5 with a remainder of 2, and divisible by 7 with a remainder of 5) is 47.

Therefore, there were 47 lines on the page in Stu's book.

Problem 4: THE THREE SQUARES

1. *All cousins are between the ages of 11 and 30.*

2. *Bob is the oldest and Chris is the youngest.*

3. *The sum of their ages is 70.*

4. *The square of each of their ages uses all the digits from 1 to 9 exactly once.*

Let's start by listing the squares of all numbers between 11 and 30, as this will help us identify the unique combination of squares that uses each digit from 1 to 9 exactly once.

Then, we'll try to match these squares with the given conditions.

{11: 121, 12: 144, 13: 169, 14: 196, 15: 225, 16: 256, 17: 289, 18: 324, 19: 361, 20: 400, 21: 441, 22: 484, 23: 529, 24: 576, 25: 625, 26: 676, 27: 729, 28: 784, 29: 841, 30: 900}

We have the squares of all numbers between 11 and 30. Now, we need to find a combination of three of these squares such that:

1. *The squares use all digits from 1 to 9 exactly once.*
2. *The ages of Bob, Chris, and Phil add up to 70, with Bob being the oldest and Chris the youngest.*

Let's start by checking combinations of these squares to see which ones meet the criteria.

The ages that satisfy all the conditions are 19, 23, and 28. These ages add up to 70, and the squares of these ages (361, 529, and 784, respectively) use all the digits from 1 to 9 exactly once.

Given that Bob is the oldest and Chris is the youngest, their ages are as follows:

Bob is 28 years old.

Chris is 19 years old.

Phil, being the remaining cousin, is 23 years old.

Part 2

Problem 1: THE FISHING TRIP

1. *Marta's statement: "Mickey was first."*

Since this is false, it means Mickey was not first.

2. *Woody's statement: "I beat Sally."*

This being false means Woody did not beat Sally. So, Sally finished ahead of Woody.

3. *Sally's statement: "Marta beat Woody."*

Since this is false, it means Marta did not beat Woody. Woody finished ahead of Marta.

4. *Mickey's statement: "Woody was second."*

This being false means Woody was not second.

Given these falsehoods, we can deduce the following:

1. *Mickey wasn't first, so Mickey could be second, third, or fourth.*
2. *Woody didn't beat Sally, so Sally is ahead of Woody.*
3. *Marta didn't beat Woody, so Woody is ahead of Marta.*
4. *Woody wasn't second, so he could be first, third, or fourth.*

Now let's deduce their standings using these clues:

1. *Since Mickey wasn't first and Woody wasn't second, Sally or Marta must be first. However, since Marta didn't beat Woody, she can't be ahead of him. Therefore, Sally must be first.*
2. *If Sally is first, Woody cannot be second (as per Mickey's false statement). Thus, Mickey must be second.*
3. *Since Woody wasn't second and must finish ahead of Marta, Woody is third.*
4. *Marta, therefore, is fourth.*

So, the standings are:

Sally is first.

Mickey is second.

Woody is third.

Marta is fourth.

Problem 2: VOLLETBALL TEAM

1. *Elaine is not the setter.*

This means Elaine is either the middle blocker or the outside hitter.

2. *Kelly has been in school longer than the middle blocker.*

This means Kelly cannot be the middle blocker and must be in a higher year than the middle blocker.

3. *The middle blocker has been in school longer than the outside hitter.*

This indicates a ranking in years: the middle blocker is in a higher year than the outside hitter.

4. *Either Kelly is the setter or Elaine is the middle blocker.*

Since Elaine cannot be the setter (from clue 1), if Kelly is not the setter, then Elaine must be the middle blocker. But if Kelly is the setter, Elaine could be either the middle blocker or the outside hitter.

Now let's use these clues to deduce the solution:

1. *Since Kelly has been in school longer than the middle blocker (clue 2) and the middle blocker has been in school longer than the outside hitter (clue 3), Kelly must be the fourth-year student.*
2. *Since Elaine is not the setter (clue 1), she must be either the middle blocker or the outside hitter.*
3. *From clue 4, if Kelly is the setter, Elaine could be the middle blocker or the outside hitter. But if Kelly is not the setter, Elaine must be the middle blocker.*
4. *Since Kelly is the fourth-year student, and the middle blocker is not the most senior, Kelly cannot be the middle blocker. So, Kelly is either the setter or the outside hitter.*

Let's consider the two possibilities:

1. *If Kelly is the setter:*

Elaine must be the middle blocker or the outside hitter. If Elaine is the middle blocker, then Shannon is the outside hitter. In this case, the middle blocker (Elaine) would be a first or second year, and the outside hitter (Shannon) would be the remaining year.

2. *If Kelly is not the setter:*

Then Elaine must be the middle blocker (from clue 4). Kelly would be the outside hitter, and Shannon would be the setter. In this case, Elaine (the middle blocker) would be a first or second year, and Shannon (the setter) would be the remaining year.

However, given that Kelly has been in school longer than the middle blocker (clue 2) and she is a fourth-year student, the only arrangement that fits all the clues is:

Kelly is the setter and is in her fourth year.

Elaine is the middle blocker and is in her second year.

Shannon is the outside hitter and is in her first year.

Problem 3: ATHLETES

1. *Hicks and Russ play on the same men's volleyball team.*

This means Russ's last name cannot be Hicks. Since they play men's volleyball, this also indicates that both Hicks and Russ are male.

2. *Drake and Braun have both set women's records in swimming.*

This means Drake and Braun are female. Therefore, Pamela and Yvonne must be one of these two.

3. *Yvonne and Drake both went to the same high school.*

This means Yvonne's last name is Drake.

Given this information, let's assign the names:

1. *Yvonne's last name is Drake (from clue 3).*
2. *Since Drake is taken and Drake and Braun are female (from clue 2), Pamela must be Braun.*
3. *Since Hicks and Russ are on the same men's volleyball team (from clue 1), and Yvonne and Pamela are accounted for, the remaining male names are Russ and Don. Therefore, one of them is Hicks and the other is Krieger.*

4. *Since Krieger has a full scholarship because he is a star in two different sports, and there is no indication that Russ is a star in two sports, Russ's last name cannot be Krieger. Thus, Russ is Hicks.*
5. *This leaves Don as Krieger.*

So, the full names are:

Russ Hicks

Don Krieger

Pamela Braun

Yvonne Drake

Problem 4: EXPERTS

1. *Bridget's expertise is not biological diversity.*

Bridget is either a water quality engineer, a soil contamination scientist, or an air pollution consultant.

2. *Abbie loves to garden, but her expertise has nothing to do with soil or air pollution.*

Abbie is either a water quality engineer or a biological diversity advocate.

3. *The air pollution expert, the water quality engineer, and Demi all met one another at a global warming conference.*

This implies that Demi is neither the air pollution expert nor the water quality engineer.

Therefore, Demi is either a soil contamination scientist or a biological diversity advocate.

4. *Bridget has never met the person who works on air pollution.*

Since Bridget has never met the air pollution consultant, she cannot be the water quality engineer (as they met each other at the conference). Therefore, Bridget is either a soil contamination scientist or a biological diversity advocate.

Based on these clues, let's deduce the professions:

1. *Since Abbie's expertise has nothing to do with soil or air pollution, and Bridget is not a biological diversity advocate (from clue 1), Abbie must be the biological diversity advocate.*
2. *As Abbie is the biological diversity advocate, Demi must be the soil contamination scientist (since Demi is neither the air pollution expert nor the water quality engineer, from clue 3).*
3. *Since Bridget is not the biological diversity advocate (from clue 1) and cannot be the water quality engineer (from clue 4), she must be the air pollution consultant.*
4. *This leaves Clare as the water quality engineer.*

So, the matches are:

Abbie: Biological Diversity Advocate

Bridget: Air Pollution Consultant

Clare: Water Quality Engineer

Demi: Soil Contamination Scientist

Part 3

Problem 1: TURKEYS

1. *Format of the Price:*

We know the total price is in the format "67.9", where the first and last digits are missing. This means the total price could potentially be anything from €067.90 to €967.99.

2. *Divisibility by 72:*

The total price for 72 turkeys must be divisible by 72. This is because if the total price is divisible by the number of items (in this case, turkeys), it ensures that the price per item is a whole number in cents.

3. *Checking Possible Prices:*

I iterated through all possible values for the missing digits (from 0 to 9 for both the first and last missing digits). For each possible total price, I checked if it's divisible by 72.

4. *Finding the Correct Total Price:*

The only value in the range that is divisible by 72 and fits the format "67.9" is **€367.92**.

5. *Calculating the Price per Turkey:*

To find the price of one turkey, I divided the total price (€367.92) by the number of turkeys (72), resulting in a price of **€5.11 per turkey**.