Differentiation Tutorial

Average Rate of Change

Question 1.

If two variables *x* and *y* are related by the equation $y = 3x^2 - 2x$ calculate the **average rate of change** of *y* with respect to *x* as *x* varies from 1 to 2.

Question 2.

If two variables *x* and *y* are related by the equation $y = 5x^2 - x$ calculate the **average rate of change** of *y* with respect to *x* as *x* varies from 0.5 to 1.

Question 3.

The number of staff, *n*, working on a computer project is related to the time that the project has been running, *t*, measured in weeks, by the relationship $n = 40t - 6t^2$. Find the **average rate of change** in staffing for the project between 2 and 4 weeks.

Differentiation Using the Rules

Question 4.

Differentiate the following with respect to x:

a. $y = 3x^5 + 7x^4 + 2x^3 + 11x^2 + 4x + 9$

b. $s = 5x^2 - 7x + 11$

c. $r = 3 - 2x - 4x^2 - 10x^3$

d. $y = 2\sqrt{x}$

Question 5.

A curve is described by the equation $y = 2x^3 - 3x$. Find the slope (gradient) of the **tangent** to this curve at (2, 10).

Question 6.

Show that the **tangent** to the curve $y = x^2 + 2x - 3$ at the point (-2, -3) is parallel to the line given by $2y + 4x - 3 = 0$.

Question 7.

Write down $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for each of the following:

a. $y = 2x^2 - 3x + 6$

b. $y = 4x^4 - 2x^2$

Question 8.

Differentiate $y = (5x+7)(2x+11)$ with respect to *x*, using the product rule.

Question 9

Differentiate $\dfrac{3x-5}{x^2+1}$ with respect to *x* (use quotient rule).

Question 10.

Differentiate $y = (x^2 - x - 5)^3$. Hence find the slope of the tangent to the curve when $x = 3$.

Question 11.

A circuit is designed so that it outputs an alternating current (I) whose value in milliamps at any time t seconds after the circuit is activated is given by: $1 + 30t - 5t^2$.

Find,

a. The current 1 second after the circuit is activated.
b. The rate at which the current is changing after 2 seconds.
c. The time taken for the current to reach its first peak value.

Question 12.

Determine the position of any maximum and minimum points of the function $y = x^2 + 1$.

Question 13.

Determine the position of any maximum and minimum points of the function $y = 2x - x^2$.

Question 14.

Given the function $y = x^3 + 3x^2 - 9x + 5$. Use the second derivative test to find the coordinates of the local maximum or minimum turning points of the function.