Probability Part 1 - Tutorial Sheet
Sample Solutions

1 Roll a dice $P[3] = 1/6$

2 $P[2] = 1/6$

3 $P[4] = 1/12$ (12-sided dice)

4 SUM

+	1	2	3	4	5	6
1	(2)	(3)	4	5	6	7
2	(3)	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P[SUM<4] = \frac{3}{36} = \frac{1}{12}$

5. PRODUCT

X	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	(8)	(10)	(12)
3	3	6	(9)	(12)	(15)	(18)
4	4	(8)	(12)	(16)	(20)	(24)
5	5	(10)	(15)	(20)	(25)	(30)
6	6	(12)	(18)	(24)	(30)	(36)

$P[Product > 7] = \frac{22}{36} = \frac{11}{18}$

6 sum

+	1	2	3	4	5	6
1	(2)	(3)	(4)	(5)	6	7
2	(3)	(4)	(5)	6	7	8
3	(4)	(5)	6	7	8	9
4	(5)	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P[\text{sum} < 6] = \frac{10}{36}$

$\Rightarrow P[\text{sum} \geq 6] = 1 - P[\text{sum} < 6]$

$= 1 - \frac{10}{36}$

$= \frac{36}{36} - \frac{10}{36}$

$\therefore P[\text{sum} \geq 6] = \frac{26}{36} = \frac{13}{18}$

7. 15 people to choose a number from 1 to 80. what is the probability that two or more pick the same number?

→ Simpler to answer the equivalent question,
- what is the probability that no two pick the same number?

Let S = the event that ~~one~~ two or more of the numbers are the same

N = the event that none are the same.

∴ Using Complement Rule: $P[S] = 1 - P[N]$

Find $P[N] = \frac{\text{no two pick same}}{\text{total possible (with no restrictions)}}$

- Total possible number of outcomes (no restrictions) $= 80^{15}$

  i.e. each person has a choice of 80 numbers.

- number of outcomes to satisfy the criteria for the event N - that no two pick the same

$$= 80 \times 79 \times 78 \times 77 \times 76 \times 75 \times 74 \times 73 \times 72 \times 71 \times 70 \times 69 \times 68 \times 67 \times 66$$
$$= 8.68 \times 10^{27}$$

$$\Rightarrow P[N] = \frac{80 \times 79 \times \ldots \times 66}{80^{15}} = 0.247$$

$$\therefore P[S] = 1 - 0.247 = 0.753$$

$\sim 75\%$ chance that two or more people pick the same number

8. Two events A and B
A is throwing a 4 with a fair dice
B is drawing a Queen from a full deck of cards

What is the ~~outcome~~ probability of the occurrence of both events?

Independent Events so use

$$P[A \text{ and } B] = P[A] \times P[B]$$
$$= \frac{1}{6} \times \frac{4}{52} = \frac{4}{312} = \frac{1}{78}$$

$$P[A \text{ and } B] = 0.013.$$

9. Two dice Rolled - Probability of getting a 2 and a 6?

$$\frac{2}{1/6} \text{ and } \frac{6}{1/6} = \frac{1}{36} \quad \underline{OR} \quad \frac{6}{1/6} \text{ and } \frac{2}{1/6} = \frac{1}{36}$$

$$\frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$$

10. Choose 4 counties from 6. $\boxed{\begin{matrix} n=6 \\ r=4 \end{matrix}}$

a) order Matters - Permutation

$${}^{n}P_r = \frac{n!}{(n-r)!} = {}^{6}P_4 = \frac{6!}{(6-4)!}$$

$$= \frac{6\times5\times4\times3\times\cancel{2\times1}}{\cancel{2\times1}}$$

$${}^{6}P_4 = 360$$

b) order does not Matter - Combinations

$${}^{n}C_r = \frac{n!}{r!(n-r)!} = {}^{6}C_4 = \frac{6!}{4!(6-4)!}$$

$$= \frac{6\times5\times4\times3\times\cancel{2\times1}}{4\times3\times2\times1(\cancel{2\times1})}$$

$${}^{6}C_4 = 360/24 = 15$$

11. 5 subjects out of 9. - How many different combinations?

$n = 9$, $r = 5$

$${}^{n}C_{r} = {}^{9}C_{5} = \frac{9!}{5!(9-5)!} = \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1\,(4\times3\times2\times1)}$$

$$= \frac{15{,}120}{120}$$

$${}^{9}C_{5} = 126$$

12. 4 letter codes constructed from first 10 letters.... no letter repeated.

$10\times9\times8\times7 = 5040$ or ${}^{10}P_{4} = \frac{10!}{(10-4)!}$

$$= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{6\times5\times4\times3\times2\times1}$$

$= 5040.$

13. user-codes : 2 letters, 2 digits, 1 letter eg. XY12A.

a) user-codes if repetition allowed.

$26\times26\times10\times10\times26 = 1{,}757{,}600$

b) where digit 0 occurs at least once

$26\times26\times9\times9\times26 = 1{,}423{,}656$

$\Rightarrow 1757600 - 1423656 = 333{,}944$

c) no repetition in letters or digits.

$26 \times 25 \times 10 \times 9 \times 24 = 1,404,000$

or $\quad {}^{26}P_3 \times {}^{10}P_2$

$$\frac{26!}{23!} \times \frac{10!}{8!} = 15,600 \times 90 = 1,404,000$$