Inverse 3x3

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$$A = \begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix}$$

$\det(A) = -26$ $\quad A^{-1} = \frac{1}{\det(A)} C^T$

COFACTOR MATRIX

$C_{11} = -3 \quad C_{12} = 11 \quad C_{13} = 13$

where $C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

have these from when we worked out det(A)

a_{21}: $\begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} -3 & 1 \\ 2 & -1 \end{vmatrix}$

$ad - bc$

$= (-3)(-1) - (1)(2)$

$= 3 - 2$

$M_{21} = 1$

Minor $\quad ij$

$i = 2, \ j = 1$ odd.

$2 + 1 = 3$

$C_{21} = -M_{21} = \boxed{-1}$

a_{22}: $\begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = (2)(-1) - (1)(3)$

$= -2 - 3$

$i + j = 2 + 2 = 4$ even.

$M_{22} = -5$

$C_{22} = M_{22} = \boxed{-5}$

a_{23}: $\begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = (2)(2) - (-3)(3)$

$= 4 - (-9)$

$M_{23} = 4 + 9 = 13$

$ij \to 2 + 3 = 5$ odd

$C_{23} = -M_{23} = \boxed{-13}$

a_{31}: $\begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} -3 & 1 \\ -1 & 2 \end{vmatrix} = (-3)(2) - (1)(-1)$

$= -6 - (-1)$

$M_{31} = -6 + 1 = -5$

$i + j = 3 + 1 = 4$ even

$C_{31} = M_{31} = \boxed{-5}$

$\cdot 1$

$$C_{31} = M_{31} = \boxed{-5}$$

$$a_{32}: \begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} 2 & 1 \\ 5 & 2 \end{vmatrix} = (2)(2)-(1)(5)$$

$M_{32} = -1$

$i+j = 3+2 = 5$ - odd

$$C_{32} = -M_{32} = -(-1) = 1$$

$$C_{32} = \boxed{1}$$

$$a_{33}: \begin{bmatrix} 2 & -3 & 1 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \Rightarrow \begin{vmatrix} 2 & -3 \\ 5 & -1 \end{vmatrix} = (2)(-1)-(-3)(5)$$

i = row

j = col

$M_{33} = 13$

$i\ j = 3+3 = 6$ even

$$C_{33} = M_{33} = \boxed{13}$$

then $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -3 & 11 & 13 \\ -1 & -5 & -13 \\ -5 & 1 & 13 \end{bmatrix}$

$$C^T = \begin{bmatrix} -3 & -1 & -5 \\ 11 & -5 & 1 \\ 13 & -13 & 13 \end{bmatrix} \qquad A^{-1} = \frac{1}{\det A} C^T$$

$$A^{-1} = \frac{1}{-26} \begin{bmatrix} -3 & -1 & -5 \\ 11 & -5 & 1 \\ 13 & -13 & 13 \end{bmatrix} = \begin{bmatrix} \frac{3}{26} & \frac{1}{26} & \frac{5}{26} \\ -\frac{11}{26} & \frac{5}{26} & -\frac{1}{26} \\ -\frac{13}{26} & \frac{13}{26} & -\frac{13}{26} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{26} & \frac{1}{26} & \frac{5}{26} \\ -\frac{11}{26} & \frac{5}{26} & -\frac{1}{26} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$