

# 1 Propositional logic

## 1.1 Semantics

$\Gamma = F_1, \dots, F_n$  (*axioms/promises*);  $F$  *conclusion*;  $\Gamma \models F \iff F$  is a **logical consequence** of  $\Gamma$ .

**Valid** formula  $\iff$  **Tautology**  $\iff$  True in any circumstance.

**Inconsistent** formula  $\iff$  **Unsatisfiable** formula  $\iff$  False in any circumstance.

$G$  valid  $\iff \neg G$  inconsistent.

Table 1: Truth table of main logical connectives

		<b>Not</b>	<b>And</b>	<b>Or</b>	<b>Implication</b>	<b>Double implication</b>
$P_1$	$P_2$	$\neg P_1$	$P_1 \wedge P_2$	$P_1 \vee P_2$	$P_1 \rightarrow P_2$	$P_1 \leftrightarrow P_2$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

$F$  **logically equivalent** to  $G \iff F \equiv G \iff (F \models G \text{ and } G \models F)$ .

## 1.2 Calculus

Table 2: Logical equivalence rules

$P \wedge Q \equiv Q \wedge P$	Commutativity of AND
$P \vee Q \equiv Q \vee P$	Commutativity of OR
$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$	Associativity of AND
$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$	Associativity of OR
$\neg(\neg P) \equiv P$	Double-negation elimination
$P \rightarrow Q \equiv \neg P \rightarrow \neg Q$	Contraposition
$P \rightarrow Q \equiv \neg P \vee Q$	Implication elimination
$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$	Biconditional elimination
$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$	De Morgan's law *
$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$	De Morgan's law *
$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$	Distributivity of AND over OR
$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$	Distributivity of OR over AND

\* De Morgan's law can be generalized to any arbitrary finite number of connected variables.

**Conjunctive Normal Form:**  $F = F_1 \wedge F_2 \wedge \dots \wedge F_n$  where  $F_i = F_{i1} \vee F_{i2} \vee \dots \vee F_{ik}$  (disjunction of atoms).

**Disjunctive Normal Form:**  $F = F_1 \vee F_2 \vee \dots \vee F_n$  where  $F_i = F_{i1} \wedge F_{i2} \wedge \dots \wedge F_{ik}$  (conjunction of atoms).

**Deduction Theorem:**  $(F_1 \wedge F_2 \wedge \dots \wedge F_n) \models G \iff (\models (F_1 \wedge F_2 \wedge \dots \wedge F_n)) \rightarrow G$

**Proof by refutation:**  $(F_1 \wedge F_2 \wedge \dots \wedge F_n) \models G \iff F_1 \wedge F_2 \wedge \dots \wedge F_n \wedge \neg G$  inconsistent.

### 1.2.1 Natural deduction

Table 3: Introduction and elimination rules for main logical connectives

	<b>Introduction</b>	<b>Elimination</b>
<b>And</b>	$\frac{\varphi \quad \theta}{\varphi \wedge \theta} \wedge I$	$\frac{\varphi \wedge \theta}{\varphi} \wedge E \quad \frac{\varphi \wedge \theta}{\theta} \wedge E$
<b>Or</b>	$\frac{\varphi}{\varphi \vee \theta} \vee I \quad \frac{\theta}{\varphi \vee \theta} \vee I$	$\frac{\varphi \vee \theta \quad \varphi \rightarrow \psi \quad \theta \rightarrow \psi}{\psi} \vee E$
<b>Implication</b>	$\frac{[\varphi] \quad \dots \quad \theta}{\varphi \rightarrow \theta} \rightarrow I$	$\frac{\varphi \quad \varphi \rightarrow \theta}{\theta} \rightarrow E \text{ (modus ponens)}$

$[\neg\varphi]$

$\vdots$

$\perp$

Ex falso sequitur quodlibet:  $\frac{\perp}{\varphi} \perp$ . Reductio ad absurdum:  $\frac{\perp}{\varphi} RAA$ .

$\Gamma \models \varphi \iff \Gamma \models \varphi$  (Completeness theorem:  $\Gamma \models \varphi \Rightarrow \Gamma \models \varphi$ ; Soundness theorem:  $\Gamma \models \varphi \Leftarrow \Gamma \models \varphi$ ).

### 1.2.2 Resolution

Refutation theorem:  $\theta \models \psi \iff \not\models \psi \wedge \neg\theta$

Resolution: 
$$\frac{R \vee A \quad R' \vee \neg A'}{R \vee R'}$$

## 2 First Order Logic

TODO

## 3 Logic Programming

TODO

$A \doteq B \Leftrightarrow A$  unifiable with  $B$

Let  $A_1, \dots, A_k$  be atomic formulas; A unifier for the set  $A_1, \dots, A_k$  is a substitution  $\sigma$  such that  $A_1\sigma = A_2\sigma = \dots = A_k\sigma$ , where  $=$  represents the property of being the identical formula. A unifier  $\sigma$  is said to be a Most General Unifier (MGU) if, for every unifier  $\tau$  for the same set, there is a unifier  $\rho$  such that  $\tau = \sigma\rho$ . From the MGU you can derive any other unifier.