1 Propositional logic

$\Gamma = F_1, \dots, F_n$ (*promises*); F *conclusion*; $\Gamma \vDash F \iff F$ is a **logical consequence** of Γ.
Valid formula $\iff$ Tautology $\iff$ True in any circumstance.
Inconsistent formula $\iff$ Unsatisfiable formula $\iff$ False in any circumstance.
G valid $\iff \neg G$ inconsistent.

Truth table of main logical connectives						
P_1	P_2	$\neg P_1$	$P_1 \wedge P_2$	$P_1 \vee P_2$	$P_1 \to P_2$	$P_1 \leftrightarrow P_2$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

F logically equivalent to $G \iff F \equiv G \iff (F \vDash G$ and $G \vDash F)$.

Logical equivalence rules			
$P \wedge Q$	$\equiv$	$Q \wedge P$	Commutativity of AND
$P \vee Q$	$\equiv$	$Q \vee P$	Commutativity of OR
$(P \wedge Q) \wedge R$	$\equiv$	$P \wedge (Q \wedge R)$	Associativity of AND
$(P \vee Q) \vee R$	$\equiv$	$P \vee (Q \vee R)$	Associativity of OR
$\neg(\neg P)$	$\equiv$	P	Double-negation elimination
$P \to Q$	$\equiv$	$\neg P \to \neg Q$	Contraposition
$P \to Q$	$\equiv$	$\neg P \wedge Q$	Implication elimination
$P \leftrightarrow Q$	$\equiv$	$(P \to Q) \wedge (Q \to P)$	Biconditional elimination
$\neg(P \wedge Q)$	$\equiv$	$\neg P \vee \neg Q$	De Morgan
$\neg(P \vee Q)$	$\equiv$	$\neg P \wedge \neg Q$	De Morgan
$P \wedge (Q \vee R)$	$\equiv$	$(P \wedge Q) \vee (P \wedge R)$	Distributivity of AND over OR
$P \vee (Q \wedge R)$	$\equiv$	$(P \vee Q) \wedge (P \vee R)$	Distributivity of OR over AND

Conjunctive Normal Form (CNF): $F = F_1 \wedge F_2 \wedge ... \wedge F_n$ where $F_i = F_{i1} \vee F_{i2} \vee ... \vee F_{ik}$.
Disjunctive Normal Form (DNF): $F = F_1 \vee F_2 \vee ... \vee F_n$ where $F_i = F_{i1} \wedge F_{i2} \wedge ... \wedge F_{ik}$.
Deduction Theorem: $(F_1 \wedge F_2 \wedge ... \wedge F_n) \vDash G \iff (\vDash (F_1 \wedge F_2 \wedge ... \wedge F_n)) \to G$
Proof by refutation: $(F_1 \wedge F_2 \wedge ... \wedge F_n) \vDash G \iff F_1 \wedge F_2 \wedge ... \wedge F_n \wedge \neg G$ inconsistent.

1.1 Natural deduction

AND: $\dfrac{\varphi \quad \theta}{\varphi \wedge \theta} \wedge I$ $\qquad \dfrac{\varphi \wedge \theta}{\varphi} \wedge E \quad \dfrac{\varphi \wedge \theta}{\varphi} \wedge E.$

OR: $\dfrac{\varphi}{\varphi \vee \theta} \vee I \quad \dfrac{\theta}{\varphi \vee \theta} \vee I$ $\qquad .$

IMPLIES: $\dfrac{\begin{matrix}[\varphi] \\ \vdots \\ \theta\end{matrix}}{\varphi \to \theta} \to I$ $\qquad \dfrac{\varphi \quad \varphi \to \theta}{\theta} \to E.$

Ex falso sequitur quodlibet: $\dfrac{\perp}{\varphi}\perp$. Reduction ad absurdum: $\dfrac{\begin{matrix}[\neg\varphi] \\ \vdots \\ \perp\end{matrix}}{\varphi} RAA$.
Completeness theorem: $\Gamma \vdash \varphi \Rightarrow \Gamma \vDash \varphi$. Soundness theorem $\Gamma \vdash \varphi \Leftarrow \Gamma \vDash \varphi$.