1 Basic probability

1.0.1 Inference by enumeration

$$P(\text{ Effect } \mid \text{ Cause }) = \frac{P(\text{ Effect } \wedge \text{ Cause })}{P(\text{ Cause })} = \alpha P(\text{ Effect }, \text{ Cause })$$

α = Normalization constant

1.0.2 Bayes theorem

$$P(\text{ Cause} \mid \text{Effect }) = \frac{P(\text{ Effect} \mid \text{Cause })P(\text{ Cause })}{P(\text{ Effect })} = \alpha P(\text{ Effect} \mid \text{Cause })P(\text{ Cause })$$

2 Bayesian Networks

2.1 Flow of probabilistic infuence

- X **direct cause** of Y: $X \rightarrow Y$
- X **direct effect** of Y: $X \leftarrow Y$
- **Causal trail** from X to Y: $X \rightarrow Z \rightarrow Y$
- **Evidential trail** from X to Y: $X \leftarrow Z \leftarrow Y$
- **Common cause**: $X \leftarrow Z \rightarrow Y$
- **Common effect**: $X \rightarrow Z \leftarrow Y$

2.1.1 Active trail

Let Z be a subset of observed variables. The trail $X_{i-1} \leftrightharpoons X_i \leftrightharpoons X_{i+1}$ is active given Z if

- $\forall X_{i-1} \rightarrow Xi \leftarrow X_{i+1}$, X_i or one of its descendants are in Z
- No other node along the trail is in Z

2.1.2 Direct separation

To determine if $P \models (X \perp Y|Z)$ (X and Y are independent given Z):

1. Traverse the graph bottom-up marking all nodes in Z or having descendants in given Z
2. Traverse the graph from X to Y , stopping if we get to a blocked node*
3. If we can't reach Y, then X and Y are independent

*A node is blocked if either the middle of an unmarked v-structure, or in Z, but not both

2.2 Reasoning Patterns

- **Causal** (or predictive) reasoning: $P(\text{Descendant}|\text{Ancestor})$
- **Evidential** (or explanatory) reasoning: $P(\text{Ancestor}|\text{Descendant})$
- **Intercausal** reasoning (or explaining away): $P(Y|X)$ where X is connected to Y via an active trail which passes through a common cause and/or a common effect