

1 Basic probability

1.0.1 Inference by enumeration

$$P(\text{Effect} \mid \text{Cause}) = \frac{P(\text{Effect} \wedge \text{Cause})}{P(\text{Cause})} = \alpha P(\text{Effect}, \text{Cause})$$

α = Normalization constant

1.0.2 Bayes theorem

$$P(\text{Cause} \mid \text{Effect}) = \frac{P(\text{Effect} \mid \text{Cause})P(\text{Cause})}{P(\text{Effect})} = \alpha P(\text{Effect} \mid \text{Cause})P(\text{Cause})$$

2 Bayesian Networks

2.1 Flow of probabilistic influence

- **X direct cause** of Y: $X \rightarrow Y$
- **X direct effect** of Y: $X \leftarrow Y$
- **Causal trail** from X to Y: $X \rightarrow Z \rightarrow Y$
- **Evidential trail** from X to Y: $X \leftarrow Z \leftarrow Y$
- **Z common cause** of X and Y: $X \leftarrow Z \rightarrow Y$
- **Z common effect** of X and Y: $X \rightarrow Z \leftarrow Y$

2.1.1 Active trail

Let Z be a subset of observed variables. The trail $X_{i-1} \rightleftharpoons X_i \rightleftharpoons X_{i+1}$ is active given Z if

- $\forall X_{i-1} \rightarrow X_i \leftarrow X_{i+1}$, X_i or one of its descendants are in Z
- No other node along the trail is in Z

2.1.2 Direct separation

To determine if $P \models (X \perp Y \mid Z)$ (X and Y are independent given Z):

1. Traverse the graph bottom-up marking all nodes in Z or having descendants in given Z
2. Traverse the graph from X to Y , stopping if we get to a blocked node*
3. If we can't reach Y , then X and Y are independent

*A node is blocked if either the middle of an unmarked v-structure, or in Z , but not both

2.2 Reasoning Patterns

- **Causal** (or predictive) reasoning: $P(\text{Descendant} \mid \text{Ancestor})$
- **Evidential** (or explanatory) reasoning: $P(\text{Ancestor} \mid \text{Descendant})$
- **Intercausal** reasoning (or explaining away): $P(Y \mid X)$ where X is connected to Y via an active trail which passes through a common cause and/or a common effect