1 Error measurement

Absolute error $E_x = \tilde{x} - x$
Relative error $R_x = \dfrac{\tilde{x} - x}{x}$

2 Linear Algebra

2.1 Matrices

$A \in \mathbb{R}^{nxn}$; $\vec{x}$ right eigenvector $\Leftrightarrow A\vec{x} = \lambda\vec{x}$; $\vec{x}$ left eigenvector $\Leftrightarrow A\vec{x} = \lambda\vec{x}$. λ are eigenvalues.
A triangular or symmetric $\Rightarrow$ eigenvalues are on the main diagonal.
Spectrum $\sigma(A) = \{\vec{x} : \vec{x}$ eigenvector of $A\}$. Spectral norm $\rho(A) = max(\lambda)$
$C \in \mathbb{R}^{nxn}$ singular $\Leftrightarrow \det(C) = 0$
Similarity transformation: $A, C \in \mathbb{R}^{nxn}$, C non-singular $\Rightarrow A$ and $C^{-1}AC$ are similar (same spectrum and eigenvalues).
TODO
$A \in \mathbb{R}^{mxn} \Rightarrow A^TA \in \mathbb{R}^{nxn}$ is positive semi-definite.
$A \in \mathbb{R}^{mxn}$ with maximum rank $(rk(A) = min(m, n)) \Rightarrow A^TA \in \mathbb{R}^{nxn}$ is positive definite.
Spectral theorem: $A \in \mathbb{R}^{nxn}$ symmetric $\Rightarrow$ eigenvalues are real, eigenvectors create an orthogonal basis.

2.2 Projections

TODO

3 Matrix decompositions / factorizations

3.1 LU decomposition

$A \in \mathbb{R}^{nxn}$ non-singular $(det(A) \neq 0)$ with all principal minors non-singular $\Rightarrow A = LU$ with $L \in \mathbb{R}^{nxn}$ lower triangular and $U \in \mathbb{R}^{nxn}$ upper triangular.

3.2 Cholesky factorization

$A \in \mathbb{R}^{nxn}$ positive definite $\Rightarrow A = LL^T$ with $L \in \mathbb{R}^{nxn}$ lower triangular.

3.3 Singular Value Decomposition (SVD)

$A \in \mathbb{R}^{mxn}, r = rk(A) \in [0, min(m, n)] \Rightarrow A = U\Sigma V^T$ with

- $U \in \mathbb{R}^{mxm}$ orthogonal.
- $V \in \mathbb{R}^{nxn}$ orthogonal.
- $\Sigma \in \mathbb{R}^{mxn}$ with $\Sigma_{ii} = \sigma_i$ ("singular value") and $i \neq f \Rightarrow \Sigma_i j = 0$.

$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > \sigma_{r+1} = \sigma_{r+2} = \ldots = \sigma_n = 0$.
$\lambda_i(A)$ is the i-th eigenvalue of A by value. $\sigma_i = \sqrt{\lambda_i(A^TA)}$.
$\sigma_1 = \sqrt{\rho(A^TA)} = \|A\|_2$. $|A^{-1}|_2 = \dfrac{1}{\sigma_r}$. $K_2(A) = \dfrac{\sigma_1}{\sigma_r}$.

3.3.1 Rank-k-approximation

$A = U\Sigma V^T = \sum_{i=1}^{r} \sigma_i u_i v_i^T = \sum_{i=1}^{r} \sigma_i A_i$ with $u_i \in \mathbb{R}^m$ column of U and $v_i \in \mathbb{R}^n$ column of V. $\hat{A}_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T = \sum_{i=1}^{k} \sigma_i A_i$ with $k < r$ is the rank-k-approximation of A.