Exercise 1

Build a knowledge base in which the following knowledge is represented: Father, Mother, GrandMother, GrandFather, Aunt, Uncle, Niece, Nephew, Mother of at least 3 sons, Father of at most 2 Daugthers.

First Order Logic

$$parent(Fabrizio, Simone)$$
$$parent(Rossella, Simone)$$
$$woman(Rossella)$$
$$man(Fabrizio)$$
$$parent(X, Y) \land man(X) \rightarrow father(X, Y)$$
$$parent(X, Y) \land woman(X) \rightarrow mother(X, Y)$$
$$parent(X, Y) \land parent(Y, Z) \land woman(X) \rightarrow grandmother(X, Z)$$
$$parent(X, Y) \land sister(Z, Y) \rightarrow aunt(Z, X)$$
$$parent(X, Y) \land brother(Z, Y) \rightarrow uncle(Z, X)$$
$$sibling(X, Y) \land man(X) \rightarrow brother(X, Y)$$
$$sibling(X, Y) \land woman(X) \rightarrow sister(X, Y)$$
$$(uncle(X, Z) \lor aunt(X, Z)) \land woman(Z) \rightarrow niece(Z, X)$$
$$(uncle(X, Z) \lor aunt(X, Z)) \land man(Z) \rightarrow nephew(Z, X)$$
$$mother(X, A) \land mother(X, B) \land mother(X, C) \land (A \neq B \neq C) \land$$
$$man(A) \land man(B) \land man(C) \rightarrow mother_3(X)$$
$$father(X, A) \land father(X, B) \land (A \neq B) \land woman(A) \land woman(B) \rightarrow father_2(X)$$

ALC

$$Father \equiv Parent \sqcap Man$$
$$Son \equiv (Man \sqcap \exists hasParent.\, Person)$$
$$Daughter \equiv (Woman \sqcap \exists hasParent.\, Person)$$
$$Mother \equiv Parent \sqcap Woman$$
$$GrandMother \equiv (Mother \sqcap \exists hasChild.\, Parent)$$
$$GrandFather \equiv (Father \sqcap \exists hasChild.\, Parent)$$
$$Aunt \equiv (Woman \sqcap \exists hasSibling.\, Parent)$$
$$Uncle \equiv (Man \sqcap \exists hasSibling.\, Parent)$$
$$Sibling \equiv (Brother \sqcup Sister)$$
$$Niece \equiv (Woman \sqcap \exists Parent.\, Sibling)$$
$$Nephew \equiv (Man \sqcap \exists Parent.\, Sibling)$$
$$Mother_3 \equiv (Woman \sqcap\, \geq 3 Son)$$
$$Father_2 \equiv (Man \sqcap\, \leq 2 Daughter)$$

Exercise 2

Build a knowledge base in which the following knowledge is represented: All humans are mammals; all mammals are warm blooded. All dogs are mammals. Humans own animals. There are animals that are not warm blooded. All mammals are animals. A human cannot own another human.

First Order Logic

$$human(X) \rightarrow mammal(X)$$
$$mammal(X) \rightarrow warmblooded(X)$$
$$dog(X) \rightarrow mammal(X)$$
$$own(X, Y) \rightarrow human(X), animal(Y)$$
$$\exists X. (animal(X) \wedge \neg warmblooded(X))$$
$$mammal(X) \rightarrow animal(X)$$
$$own(X, Y) \rightarrow human(X) \wedge \neg human(Y)$$

ALC

$$Human \sqsubseteq Mammal$$
$$Mammal \sqsubseteq WarmBlooded$$
$$Dog \sqsubseteq Mammal$$
$$AnimalOwner \equiv Human \sqcap \exists owns. Animal$$
$$ColdBloodedAnimal \equiv Animal \sqcap \neg WarmBlooded$$
$$Mammal \sqsubseteq Animal$$
$$(Human \sqcap \exists owns. Human). \bot$$

Exercise 3

First Order Logic

$student(X) \rightarrow smart(X)$

$\exists X. student(X)$

$\exists X. (student(X) \wedge smart(X))$

$\forall X \exists Y. (student(X) \wedge student(Y) \wedge loves(X, Y))$

$\forall X \exists Y. (student(X) \wedge student(Y) \wedge loves(X, Y) \wedge X \neq Y)$

$\exists X. (\forall Y. student(X) \wedge student(Y) \wedge loves(Y, X))$

$student(mark)$

$student(paul)$

$takes(mark, analysis) \leftrightarrow \neg takes(mark, geometry)$

$\neg takes(mark, analysis) \leftrightarrow takes(mark, geometry)$

$takes(paul, analysis) \wedge takes(paul, geometry)$

$\neg takes(mark, analysis)$

$\forall Y.\,(student(Y) \rightarrow \neg loves(Y, paul))$

ALC

$Student \sqsubseteq Smart$

$SmartStudent \equiv Student \sqcap Smart$

$(Student \sqcap \exists loves.\,Student) \equiv Student$

Sentence 5 is not encodable in ALC (I think)

$Student \sqcap \forall loves.\,Student$

$Mark \sqsubseteq Student$

$Paul \sqsubseteq Student$

$Mark \sqsubseteq ((\exists takes.\,GeometryExam \sqcap \neg\exists takes.\,AnalysisExam) \sqcup$

$(\exists takes.\,AnalysisExam \sqcap \neg\exists takes.\,GeometryExam))$

$Paul \sqsubseteq (\exists takes.\,GeometryExam \sqcap \exists takes.\,AnalysisExam)$

$Mark \sqsubseteq \neg\exists takes.\,AnalysisExam$

$(Student \sqcap \exists loves.\,Paul).\,\bot$