Symbolic AI systems are based on logic reasoning; they can provide explanations for decisions. Sub-symbolic AI systems are based on machine learning and artificial neural networks; they cannot provide explanations for decisions.

1 Propositional logic

1.1 Semantics

$\Gamma = F_1, \ldots, F_n$ (*axioms/promises*); F *conclusion*; $\Gamma \models F \iff F$ is a **logical consequence** of Γ.
Valid formula $\iff$ **Tautology** $\iff$ True in any circumstance.
Inconsistent formula $\iff$ **Unsatisfiable** formula $\iff$ False in any circumstance.
G valid $\iff \neg G$ inconsistent.

Table 1: Truth table of main logical connectives

		Not	**And**	**Or**	**Implication**	**Double implication**
P_1	P_2	$\neg P_1$	$P_1 \wedge P_2$	$P_1 \vee P_2$	$P_1 \to P_2$	$P_1 \leftrightarrow P_2$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

F **logically equivalent** to $G \iff F \equiv G \iff (F \models G$ and $G \models F)$.

1.2 Calculus

Table 2: Logical equivalence rules

$P \wedge Q$	$\equiv$	$Q \wedge P$	Commutativity of AND
$P \vee Q$	$\equiv$	$Q \vee P$	Commutativity of OR
$(P \wedge Q) \wedge R$	$\equiv$	$P \wedge (Q \wedge R)$	Associativity of AND
$(P \vee Q) \vee R$	$\equiv$	$P \vee (Q \vee R)$	Associativity of OR
$\neg(\neg P)$	$\equiv$	P	Double-negation elimination
$P \to Q$	$\equiv$	$\neg Q \to \neg P$	Contraposition
$P \to Q$	$\equiv$	$\neg P \vee Q$	Implication elimination
$P \leftrightarrow Q$	$\equiv$	$(P \to Q) \wedge (Q \to P)$	Biconditional elimination
$\neg(P \wedge Q)$	$\equiv$	$\neg P \vee \neg Q$	De Morgan's law *
$\neg(P \vee Q)$	$\equiv$	$\neg P \wedge \neg Q$	De Morgan's law *
$P \wedge (Q \vee R)$	$\equiv$	$(P \wedge Q) \vee (P \wedge R)$	Distributivity of AND over OR
$P \vee (Q \wedge R)$	$\equiv$	$(P \vee Q) \wedge (P \vee R)$	Distributivity of OR over AND

* De Morgan's law can be generalized to any arbitrary finite number of connected variables.

Conjunctive Normal Form: $F = F_1 \wedge F_2 \wedge ... \wedge F_n$ where $F_i = F_{i1} \vee F_{i2} \vee ... \vee F_{ik}$ (disjunction of atoms).
Disjunctive Normal Form: $F = F_1 \vee F_2 \vee ... \vee F_n$ where $F_i = F_{i1} \wedge F_{i2} \wedge ... \wedge F_{ik}$ (conjunction of atoms).
Deduction Theorem: $(F_1 \wedge F_2 \wedge ... \wedge F_n) \models G \iff (\models (F_1 \wedge F_2 \wedge ... \wedge F_n)) \to G$
Proof by refutation: $(F_1 \wedge F_2 \wedge ... \wedge F_n) \models G \iff F_1 \wedge F_2 \wedge ... \wedge F_n \wedge \neg G$ inconsistent.

1.2.1 Natural deduction

Ex falso sequitur quodlibet: $\dfrac{\perp}{\varphi}\perp$. Reductio ad absurdum: $\dfrac{\begin{array}{c}[\neg\varphi] \\ \vdots \\ \perp\end{array}}{\varphi}RAA$.
$\Gamma \models \varphi \iff \Gamma \models \varphi$ (Completeness theorem: $\Gamma \models \varphi \Rightarrow \Gamma \models \varphi$; Soundness theorem: $\Gamma \models \varphi \Leftarrow \Gamma \models \varphi$).

1.2.2 Resolution

Refutation theorem: $\theta \models \psi \iff \not\models \psi \wedge \neg\theta$
Resolution: $\dfrac{R \vee A \quad R' \vee \neg A'}{R \vee R'}$

Table 3: Introduction and elimination rules for main logical connectives

	Introduction	**Elimination**
And	$\dfrac{\varphi \quad \theta}{\varphi \wedge \theta} \wedge I$	$\dfrac{\varphi \wedge \theta}{\varphi} \wedge E \qquad \dfrac{\varphi \wedge \theta}{\varphi} \wedge E$
Or	$\dfrac{\varphi}{\varphi \vee \theta} \vee I \qquad \dfrac{\theta}{\varphi \vee \theta} \vee I$	$\dfrac{\varphi \vee \theta \quad \varphi \to \psi \quad \theta \to \psi}{\psi} \vee E$
Implication	$\dfrac{\begin{array}{c}[\varphi] \\ \vdots \\ \theta\end{array}}{\varphi \to \theta} \to I$	$\dfrac{\varphi \quad \varphi \to \theta}{\theta} \to E$ (*modus ponens*)

2 First Order Logic

TODO

3 Logic Programming

TODO

$A \doteq B \Leftrightarrow A$ unifiable with B

Let $A_1, \dots, A_k$ be atomic formulas; A unifier for the set $A_1, \dots, A_k$ is a substitution σ such that $A_1\sigma = A_2\sigma = \dots = A_k\sigma$, where $=$ represents the property of being the identical formula. A unifier σ is said to be a Most General Unifier (MGU) if, for every unifier τ for the same set, there is a unifier ρ such that $\tau = \sigma\rho$. From the MGU you can derive any other unifier.