

1 Propositional logic

1.1 Semantics

$\Gamma = F_1, \dots, F_n$ (*promises*); F *conclusion*; $\Gamma \models F \iff F$ is a **logical consequence** of Γ .

Valid formula \iff Tautology \iff True in any circumstance.

Inconsistent formula \iff Unsatisfiable formula \iff False in any circumstance.

G valid $\iff \neg G$ inconsistent.

Truth table of main logical connectives						
P_1	P_2	$\neg P_1$	$P_1 \wedge P_2$	$P_1 \vee P_2$	$P_1 \rightarrow P_2$	$P_1 \leftrightarrow P_2$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

F logically equivalent to $G \iff F \equiv G \iff (F \models G \text{ and } G \models F)$.

1.2 Calculus

Logical equivalence rules		
$P \wedge Q \equiv Q \wedge P$		Commutativity of AND
$P \vee Q \equiv Q \vee P$		Commutativity of OR
$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$		Associativity of AND
$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$		Associativity of OR
$\neg(\neg P) \equiv P$		Double-negation elimination
$P \rightarrow Q \equiv \neg P \rightarrow \neg Q$		Contraposition
$P \rightarrow Q \equiv \neg P \wedge Q$		Implication elimination
$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$		Biconditional elimination
$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$		De Morgan
$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$		De Morgan
$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$		Distributivity of AND over OR
$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$		Distributivity of OR over AND

Conjunctive Normal Form: $F = F_1 \wedge F_2 \wedge \dots \wedge F_n$ where $F_i = F_{i1} \vee F_{i2} \vee \dots \vee F_{ik}$ (disjunction of atoms).

Disjunctive Normal Form: $F = F_1 \vee F_2 \vee \dots \vee F_n$ where $F_i = F_{i1} \wedge F_{i2} \wedge \dots \wedge F_{ik}$ (conjunction of atoms).

Deduction Theorem: $(F_1 \wedge F_2 \wedge \dots \wedge F_n) \models G \iff (\models (F_1 \wedge F_2 \wedge \dots \wedge F_n)) \rightarrow G$

Proof by refutation: $(F_1 \wedge F_2 \wedge \dots \wedge F_n) \models G \iff F_1 \wedge F_2 \wedge \dots \wedge F_n \wedge \neg G$ inconsistent.

1.2.1 Natural deduction

AND: $\frac{\varphi \quad \theta}{\varphi \wedge \theta} \wedge I \quad \frac{\varphi \wedge \theta}{\varphi} \wedge E \quad \frac{\varphi \wedge \theta}{\theta} \wedge E.$

OR: $\frac{\varphi}{\varphi \vee \theta} \vee I \quad \frac{\theta}{\varphi \vee \theta} \vee I \quad .$

IMPLIES: $\frac{[\varphi] \quad \vdots \quad \theta}{\varphi \rightarrow \theta} \rightarrow I \quad \frac{\varphi \quad \varphi \rightarrow \theta}{\theta} \rightarrow E.$

Ex falso sequitur quodlibet: $\frac{}{\bot} \bot.$ Reduction ad absurdum: $\frac{[\neg\varphi] \quad \vdots \quad \bot}{\varphi} RAA.$

$\Gamma \vdash \varphi \iff \Gamma \models \varphi$ (Completeness theorem: $\Gamma \vdash \varphi \Rightarrow \Gamma \models \varphi$; Soundness theorem: $\Gamma \vdash \varphi \Leftarrow \Gamma \models \varphi$).

1.2.2 Resolution

$$\frac{R \vee A \quad R' \vee \perp A}{R \vee R'}$$

2 First Order Logic

TODO