Formulary

Languages and Algorithms for Artificial Intelligence

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1 Propositional logic

$\Gamma = F_1, \dots, F_n$ (*promises*); F *conclusion*; $\Gamma \vDash F \iff F$ is a **logical consequence** of Γ.
Valid formula $\iff$ Tautology $\iff$ True in any circumstance.
Inconsistent formula $\iff$ Unsatisfiable formula $\iff$ False in any circumstance.
G valid $\iff \neg G$ inconsistent.

Truth table of main logical connectives						
P_1	P_2	$\neg P_1$	$P_1 \land P_2$	$P_1 \lor P_2$	$P_1 \to P_2$	$P_1 \leftrightarrow P_2$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

F logically equivalent to $G \iff F \equiv G \iff (F \vDash G$ and $G \vDash F)$.

Logical equivalence rules			
$P \land Q$	$\equiv$	$Q \land P$	Commutativity of AND
$P \lor Q$	$\equiv$	$Q \lor P$	Commutativity of OR
$(P \land Q) \land R$	$\equiv$	$P \land (Q \land R)$	Associativity of AND
$(P \lor Q) \lor R$	$\equiv$	$P \lor (Q \lor R)$	Associativity of OR
$\neg(\neg P)$	$\equiv$	P	Double-negation elimination
$P \to Q$	$\equiv$	$\neg P \to \neg Q$	Contraposition
$P \to Q$	$\equiv$	$\neg P \land Q$	Implication elimination
$P \leftrightarrow Q$	$\equiv$	$(P \to Q) \land (Q \to P)$	Biconditional elimination
$\neg(P \land Q)$	$\equiv$	$\neg P \lor \neg Q$	De Morgan
$\neg(P \lor Q)$	$\equiv$	$\neg P \land \neg Q$	De Morgan
$P \land (Q \lor R)$	$\equiv$	$(P \land Q) \lor (P \land R)$	Distributivity of AND over OR
$P \lor (Q \land R)$	$\equiv$	$(P \lor Q) \land (P \lor R)$	Distributivity of OR over AND

Conjunctive Normal Form (CNF): $F = F_1 \land F_2 \land ... \land F_n$ where $F_i = F_{i1} \lor F_{i2} \lor ... \lor F_{ik}$.
Disjunctive Normal Form (DNF): $F = F_1 \lor F_2 \lor ... \lor F_n$ where $F_i = F_{i1} \land F_{i2} \land ... \land F_{ik}$.
Deduction Theorem: $(F_1 \land F_2 \land ... \land F_n) \vDash G \iff (\vDash (F_1 \land F_2 \land ... \land F_n)) \to G$
Proof by refutation: $(F_1 \land F_2 \land ... \land F_n) \vDash G \iff F_1 \land F_2 \land ... \land F_n \land \neg G$ inconsistent.

1.1 Natural deduction

AND: $\dfrac{\varphi \quad \theta}{\varphi \land \theta} \land I \qquad \dfrac{\varphi \land \theta}{\varphi} \land E \quad \dfrac{\varphi \land \theta}{\varphi} \land E$

OR: $\dfrac{\varphi}{\varphi \lor \theta} \lor I \quad \dfrac{\theta}{\varphi \lor \theta} \lor I$

IMPLIES: $\dfrac{\begin{matrix}[\varphi] \\ \vdots \\ \theta\end{matrix}}{\varphi \to \theta} \to I \qquad \dfrac{\varphi \quad \varphi \to \theta}{\theta} \to E$

Ex falso sequitur quodlibet: $\dfrac{\perp}{\varphi}\perp$.

Reduction ad absurdum: $\dfrac{\begin{array}{c}[\neg\varphi]\\ \vdots \\ \perp\end{array}}{\varphi}RAA$.