THE UNIVERSITY OF BRITISH COLUMBIA

MATH 103 COURSE OUTLINE
Integral Calculus with Applications to Life Sciences

Academic Calendar Entry

MATH 103 (3) Integral Calculus with Applications to Life Sciences.

Antiderivatives, the definite integral, integration techniques, numerical integration, infinite series, applications of integration to differential equations and probability, linear algebra. Credit will be granted for only one of MATH 101, MATH 103, or MATH 142. [3-1-0]
Prerequisite: One of MATH 100, MATH 116.

Course Format

Lectures and labs.

- **Lecture Format:** Each week (excepting holidays) there will be a certain number of lectures (two or three). During a typical lecture, the instructor will present one or more concepts, cover examples, and answer questions. The focus will be on applications of the concepts, rather than proofs.

- **Lab Format:** Most weeks students will meet with their lab section and Teaching Assistant, and they will either review and practice course material or write a Lab Test.

Course Overview, Content, and Objectives

Course Overview: The proposed course will focus predominantly on single variable integration and its applications. While the list of topics aligns well with a general calculus course, most of the concepts will be presented through applications in life sciences including applications in Biology. The course will start off with integration. In this first chapter we will learn how to integrate using the substitution rule, find the area between two curves, and use integration to calculate cumulative rates of change and average values. In the second chapter, we will learn how to integrate using integration by parts, integrate by using partial fractions, find improper integrals, and integrate numerically. In the third chapter, we will learn how to use integrals to solve separable differential equations, and describe the behavior of solutions. In the fourth chapter, we will learn how to solve system of linear equations (2 equations and 2 unknowns), perform algebraic operations on 2 x 2 matrices, and find the eigenvalues and eigenvectors of 2 x 2 matrices. In chapter 5, we will learn how to calculate the sum of a geometric series, determine whether a given series is convergent or divergent by using the appropriate test(s), and find the Taylor series of a function. In the last chapter, we will learn how to calculate probabilities for continuous random variables, find the expected value, variance, and standard deviation of a continuous random variable, and find the linear regression line.

Contents: Topics include

- **Integration:** antiderivatives and indefinite integrals, area and the definite integral, the fundamental theorem of calculus, applications of integration (areas, net change, average values).
- **Techniques of Integration and Computational Methods:** integration by parts, integration using partial fractions, improper integrals, numerical integration (the midpoint rule and the trapezoidal rule).

- **Differential Equations:** solving first order separable differential equations, equilibria and their stability (compartment models, the Levins model, the Allee effect).
- **Linear Algebra:** systems of linear equations, matrix operations, eigenvectors and eigenvalues, an application: the Leslie matrix. (We restrict ourselves to 2 x 2 matrices.)
- **Infinite Series:** series (geometric series), Taylor series.
- **Continuous Probability Distributions and Linear Regression:** density functions, special probability density functions, linear regression.

Course Objectives:

- To acquire a working knowledge of a broad range of mathematical concepts which are foundational to biology and data analysis in biology.
- Specific objectives
    - to understand the essential calculus concepts and techniques at the same level as for the general calculus course.
    - to gain some exposure to more advanced calculus concepts from a computational viewpoint as opposed to a full mathematically rigorous treatment.
    - to appreciate the connection between biological phenomena and mathematical models by using linear algebra and differential equations.
    - to understand some basic statistical methods to understand a set of data.

Learning Outcomes

After completing this course, students will be able to:

- Understand the basic idea of integral calculus: finding the area under a curve.
- Approximate the area under a curve by using the left, right, or midpoint Riemann sums.
- Evaluate integrals by using basic antiderivative formulas.
- Evaluate integrals by using techniques such as substitution, integration by parts, and partial fraction decomposition.
- Use integrals to find the area between two curves, the cumulative/net change, and the average value of a function.
- Approximate the value of a definite integral using the midpoint rule and the trapezoidal rule.
- Determine whether an improper integral converges or diverges, and evaluate it if it is convergent.
- Solve and analyze first-order separable differential equations. Specifically, students should be able to:
    - Use integrals to solve separable differential equations;
    - Find equilibria and determine their stability graphically and analytically;
    - Describe the behavior of solutions of differential equations starting from different initial conditions.
    - Construct a differential equation modelling a quantity described in a problem.
- Compute the eigenvalues and eigenvectors of 2 x 2 matrices, and apply this to the study of Leslie matrices, which are used extensively in ecology to model the changes in population of organisms over a period of time.
- Find the sum of a geometric series or determine that a geometric series is divergent.
- Find the Taylor (or Maclaurin) series of a function.

- Calculate probabilities for continuous variables, find the expected values and the standard deviation, and find the linear regression line.

Evaluation Criteria and Grading

The final grade will be based on the following:

Written Assignments	15%
Lab Tests	35%
Final Exam	50%

Note:

- **Students MUST attain a grade of at least 40% on the Final Exam in order to pass the course.**
  - In the event a student does not get at least 40% on the final exam AND the final grade is less than 50%, the final grade will be recorded.
  - In the event a student does not attain at least 40% on the final exam AND the final grade is greater than or equal to 50%, the maximum grade of 47% will be recorded.
- **To pass the course,** students must attain a grade of at least 40% on the final exam AND their final grades have to be greater than or equal to 50%.

Written Assignments: There will be weekly written assignments throughout the semester, released at least a week in advance of the due date.

Lab Tests: Instead of midterms, students will have section tests (between 5 and 7 in total) that will take place during their labs.

Final Exam: There will be a 2.5-hours comprehensive final exam which will be written during the final examination period. The final exam will be cumulative.

Practice Problems: After every lecture, practice problems along with complete solutions will be posted on Canvas. While completion of these problems is essential for the success of students, they do not hand them in for grading.

Note: For assignments, lab tests, and final exam,

- Students must show all their work. The more shown, the better.
- Students may lose marks, or obtain no marks, if their final answer appears without justification.

Required Readings and Videos

Textbooks:

1. *Calculus for Biology and Medicine* by Claudia Neuhauser, third edition, Pearson, 2010, ISBN: 978-0321644688

2. *Integral Calculus with Applications to the Life Sciences* (free download) by Leah Edelstein-Keshet, Department of Mathematics, UBC, Vancouver.