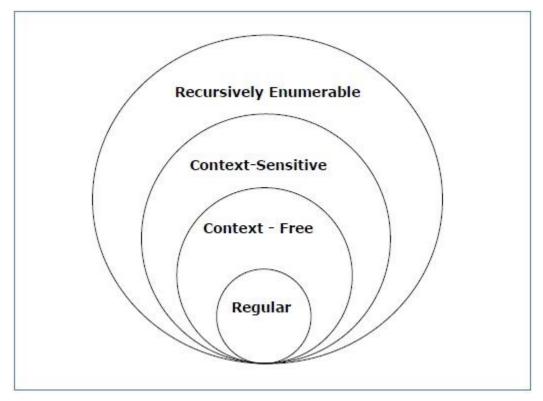
Classification of Grammars

According to Noam Chomosky, there are four types of grammars — Type 0, Type 1, Type 2, and Type 3. The following table shows how they differ from each other —

| Grammar Type | Grammar Accepted | Language Accepted | Automaton |
|-----------------|---------------------------|---------------------------------|--------------------------|
| Type 0 | Unrestricted grammar | Recursively enumerable language | Turing Machine |
| Type 1 | Context-sensitive grammar | Context-sensitive language | Linear-bounded automaton |
| Type 2 | Context-free grammar | Context-free language | Pushdown automaton |
| Type 3 | Regular grammar | Regular language | Finite state automaton |

Take a look at the following illustration. It shows the scope of each type of grammar -



Type - 3 Grammar

Type-3 grammars generate regular languages. Type-3 grammars must have a single non-terminal on the left-hand side and a right-hand side consisting of a single terminal or single terminal followed by a single non-terminal.

The productions must be in the form $X \rightarrow a$ or $X \rightarrow aY$

where $X, Y \in N$ (Non terminal)

and $a \in T$ (Terminal)

The rule $S \to \varepsilon$ is allowed if S does not appear on the right side of any rule.

Example

 $X \to \epsilon$

 $X \rightarrow a \mid aY$

 $Y \rightarrow b$

Type - 2 Grammar

Type-2 grammars generate context-free languages.

The productions must be in the form $A \rightarrow \gamma$

where $A \in N$ (Non terminal)

and $\gamma \in (T \cup N)^*$ (String of terminals and non-terminals).

These languages generated by these grammars are be recognized by a non-deterministic pushdown automaton.

Example

 $S \rightarrow X a$

 $X \rightarrow a$

 $X \rightarrow aX$

 $X \rightarrow abc$

 $X \rightarrow \epsilon$

Type - 1 Grammar

Type-1 grammars generate context-sensitive languages. The productions must be in the form

$$\alpha A \beta \rightarrow \alpha \gamma \beta$$

where $A \in \mathbb{N}$ (Non-terminal)

and α , β , $\gamma \in (T \cup N)^*$ (Strings of terminals and non-terminals)

The strings α and β may be empty, but γ must be non-empty.

The rule $S \to \varepsilon$ is allowed if S does not appear on the right side of any rule. The languages generated by these grammars are recognized by a linear bounded automaton.

Example

 $AB \rightarrow AbBc$ $A \rightarrow bcA$ $B \rightarrow b$

Type - 0 Grammar

Type-0 grammars generate recursively enumerable languages. The productions have no restrictions. They are any phase structure grammar including all formal grammars.

They generate the languages that are recognized by a Turing machine.

The productions can be in the form of $\alpha \to \beta$ where α is a string of terminals and nonterminals with at least one non-terminal and α cannot be null. β is a string of terminals and non-terminals.

Example

 $S \rightarrow ACaB$ $Bc \rightarrow acB$ $CB \rightarrow DB$ $aD \rightarrow Db$

Derive the string "aabbabba" for leftmost derivation and rightmost derivation using a CFG

$$S \rightarrow aB \mid bA$$

$$S \rightarrow a \mid aS \mid bAA \quad S \rightarrow b \mid aS \mid aBB$$

Leftmost derivation:

Rightmost derivation:

| 1. | S | |
|----|----------|---------------------|
| 2. | aB | $S \rightarrow aB$ |
| 3. | aaBB | $B \rightarrow aBB$ |
| 4. | aabB | $B \rightarrow b$ |
| 5. | aabbS | $B \rightarrow bS$ |
| 6. | aabbaB | $S \rightarrow aB$ |
| 7. | aabbabS | $B \rightarrow bS$ |
| 8. | aabbabbA | $S \rightarrow bA$ |
| 9. | aabbabba | $A \rightarrow a$ |
| | | |