Discrete-time Signals & Systems

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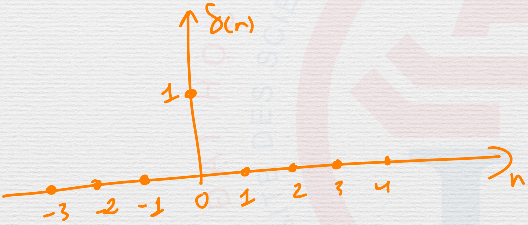
Delta Signal

Delta Signal

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$



Delta Signal

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0. \end{cases}$$

Delta Signal

$\delta(n-7)$

$$\delta(n-n_0) = \begin{cases} 1 & \text{if } n = n_0 \\ 0 & \text{if } n \neq n_0. \end{cases}$$

Unit Step Signal

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$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$

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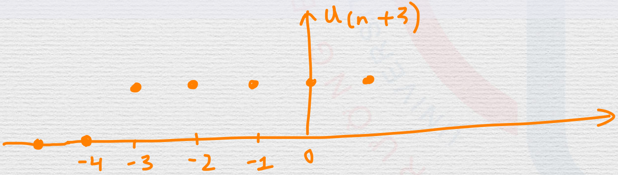
Unit Step Signal

$$u(n + \cancel{n_0}\,3) = \begin{cases} 1 & \text{if } n \geq \cancel{n_0}\,-3 \\ 0 & \text{if } n < \cancel{n_0}\,-3. \end{cases}$$

$u(n+3)$

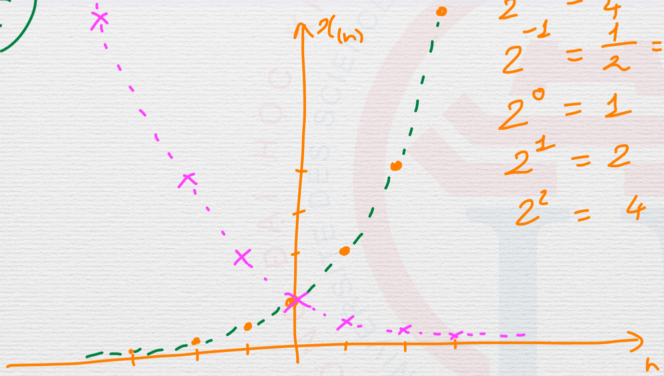


Exponential Signal

Real-valued Exponential Signal

$$x(n) = a^n$$

$$\left(\frac{1}{2}\right)^n$$



$$x(n) = 2^n$$

$$2^{-2} = \frac{1}{4} = 0.25$$

$$2^{-1} = \frac{1}{2} = 0.5$$

$$2^0 = 1$$

$$2^1 = 2$$

$$2^2 = 4$$

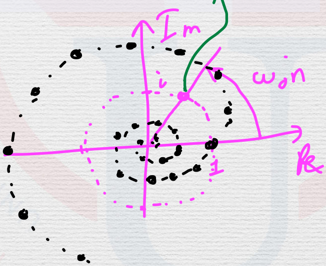
Exponential Signal

Real-valued Exponential Signal

$$x(n) = a^n$$

Complex-valued Exponential Signal

$$x(n) = e^{(\sigma + j\omega_0)n} = e^{\sigma n} \cdot \boxed{e^{j\omega_0 n}}$$



Exponential Signal

Real-valued Exponential Signal

$$x(n) = a^n$$

Complex-valued Exponential Signal

$$x(n) = e^{(\sigma + j\omega_0)n}$$

Periodic Signal

$$x(n) = x(n + N)$$

Operations (1)

Signal Addition

$$\{x_1(n)\} + \{x_2(n)\} = \{x_1(n) + x_2(n)\}$$

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$$\{x_1(n)\} + \{x_2(n)\} = \{x_1(n) + x_2(n)\}$$

Signal Multiplication

$$\{x_1(n)\}.\{x_2(n)\} = \{x_1(n)x_2(n)\}$$

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Signal Multiplication

$$\{x_1(n)\}.\{x_2(n)\} = \{x_1(n)x_2(n)\}$$

Signal Scaling

$$\alpha\{x(n)\} = \{\alpha x(n)\}$$

Operations (2)

Signal Shifting

$$y(n) = \{x(n-k)\}$$

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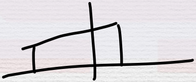

What about?

$$y(n) = \{x(3-n)\}$$

Operations (3)

Signal Energy

$$E_x = \sum_{-\infty}^{+\infty} |x(n)|^2$$

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Signal Energy

$$E_x = \sum_{-\infty}^{+\infty} |x(n)|^2$$

Signal Power of a periodic $x(n)$

$$P_x = \frac{1}{N} \sum_{0}^{N-1} |x(n)|^2$$

Some Useful Results

Delta

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\delta(n-k)$$

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Delta

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Even and odd synthesis

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

The Geometric Series

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$$

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The Geometric Series

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Correlations

$$r_{x,y}(l) = \sum_{n=-\infty}^{+\infty} x(n)y(n-l)$$

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Exercise

A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \frac{n}{3}, & -3 \leq n \leq -1 \\ 1, & 0 \leq n \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine its values and sketch the signal $x(n)$.

(b) Sketch the signals that result if we:

1. First fold $x(n)$ and then delay the resulting signal by four samples.
2. First delay $x(n)$ by four samples and then fold the resulting signal.

(c) Sketch the signal $x(-n+4)$.

(d) Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

(e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

Linear Systems (1)

Linearity

A discrete system $L[\cdot]$ is linear iff

$$L[a_1 x_1(n) + a_2 x_2(n)] = a_1 L[x_1(n)] + a_2 L[x_2(n)]$$

$$L[x_1(n) + x_2(n)] = L[x_1(n)] + L[x_2(n)]$$

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Time-invariant

$$y(n) = L[x(n)] \rightarrow L[x(n-k)] = y(n-k)$$

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Time-invariant

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Linear Time-invariant

Impulse response?

Linear Systems (2)

Stability - BIBO

$$|x(n)| < \infty \rightarrow |y(n)| < \infty$$

$$\Leftrightarrow \sum_{n=-\infty}^{+\infty} |h(n)| < \infty$$

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Stability - BIBO

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Causality

$$h(n) = 0, n < 0$$

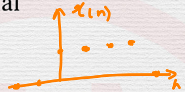
Exercise

Consider the system

$$y(n) = \mathcal{T}[x(n)] = x(n^2)$$

(a) Determine if the system is time invariant.

(b) To clarify the result in part (a) assume that the signal



$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is applied into the system.

(1) Sketch the signal $x(n)$.

(2) Determine and sketch the signal $y(n) = \mathcal{T}[x(n)]$.

(3) Sketch the signal $y_2'(n) = y(n-2)$.

(4) Determine and sketch the signal $x_2(n) = x(n-2)$.

(5) Determine and sketch the signal $y_2(n) = \mathcal{T}[x_2(n)]$.

(6) Compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion?

Convolution

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

Convolution Sum

Convolution

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

Correlations

$$r_{x,h}(n) = \sum_{k=-\infty}^{+\infty} x(k)h(k-n)$$

Exercise

(a) If $y(n) = x(n) * h(n)$, show that $\sum_y = \sum_x \sum_h$, where $\sum_x = \sum_{n=-\infty}^{\infty} x(n)$.

(b) Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test in (a).

(1) $x(n) = \{1, 2, 4\}, h(n) = \{1, 1, 1, 1, 1\}$

(2) $x(n) = \{1, 2, -1\}, h(n) = x(n)$

(3) $x(n) = \{0, 1, -2, 3, -4\}, h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$