Discrete-time Fourier Analysis

Signals

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March 31, 2021

Definition

time freq

discrete disrete ? cont

$x(\frac{1}{2})$ $x(1), x(2) \dots$

The Discrete-time Fourier Transform (DTFT)

$$X(e^{j\omega}) \triangleq \mathcal{F}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

cont dis. $z = e^{j\omega}$

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

DTFT

time $\leftrightarrow$ freq

$x(n)$ $X(e^{j\omega})$

DFT:

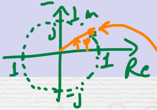
time $\leftrightarrow$ freq

$x(n)$ $X(k)$

$$X(z) = \sum_{-\infty}^{+\infty} x(n) \cdot z^{-n}$$

Definition

$e^{j2\pi k} = 1$



$j^2 = -1$

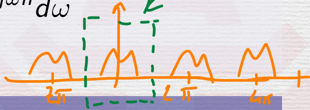
The Discrete-time Fourier Transform (DTFT)

$e^{j\psi}$

$$X(e^{j\omega}) \triangleq \mathcal{F}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$[-\pi, \pi]$

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$



Periodicity

$$X(e^{j\omega}) \overset{?}{=} X(e^{j[\omega+2\pi]})$$

$e^{-j\omega n} \cdot e^{-j2\pi n}$

Implication: We need only one period of $X(e^{j\omega})$

$X(e^{j(\omega+2\pi)}) = \sum x(n) \cdot e^{-j(\omega+2\pi)\cdot n}$

Two Properties

$X(e^{j\omega}) = \sum_{-\infty}^{+\infty} x(n) \cdot e^{-jn\omega}$

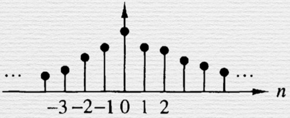
Symmetry

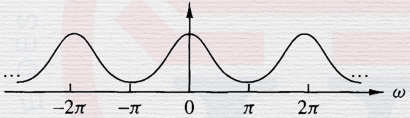
For real-valued $x(n)$

$$X(e^{-j\omega}) = X^*(e^{j\omega})$$

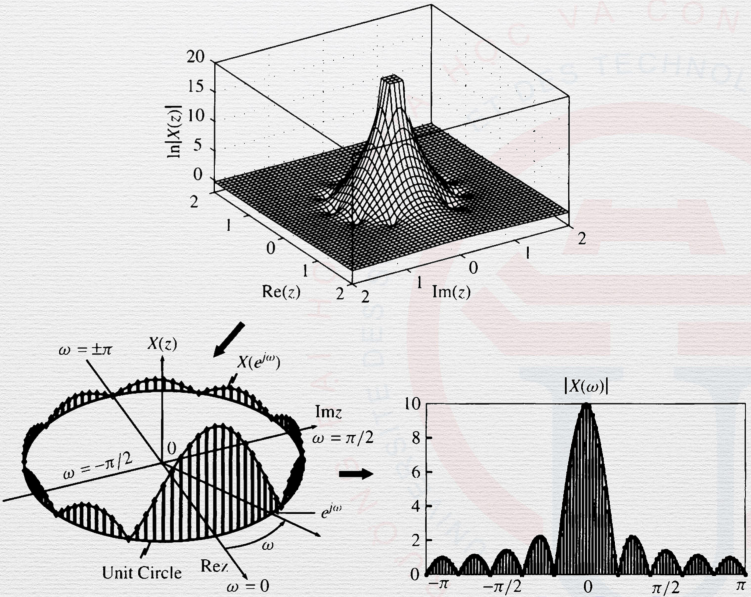
Im $z = r.e^{j\varphi}$ φ $-\varphi$ z^* Re $= r.e^{-j\varphi}$

Implication: We now need to consider only a half period of $X(e^{j\omega})$





Relationship with z-Transform



Properties (1)

Linearity

$$\mathcal{F}[a_1x_1(n) + a_2x_2(n)] = a_1\mathcal{F}[x_1(n)] + a_2\mathcal{F}[x_2(n)]$$

Properties (1)

Linearity

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Time-shifting

$$\mathcal{F}[x(n-k)] = X(e^{j\omega})e^{-j\omega k}$$

Properties (1)

Linearity

$$\mathcal{F}[a_1x_1(n) + a_2x_2(n)] = a_1\mathcal{F}[x_1(n)] + a_2\mathcal{F}[x_2(n)]$$

Time-shifting

$$\mathcal{F}[x(n-k)] = X(e^{j\omega})e^{-j\omega k}$$

Frequency-shifting

$$\mathcal{F}[x(n)e^{j\omega_0 n}] = X(e^{j(\omega-\omega_0)})$$

Properties (2)

Conjugation

$$\mathcal{F}[x^*(n)] = X^*(e^{-j\omega})$$

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Folding

$$\mathcal{F}[x(-n)] = X(e^{-j\omega})$$

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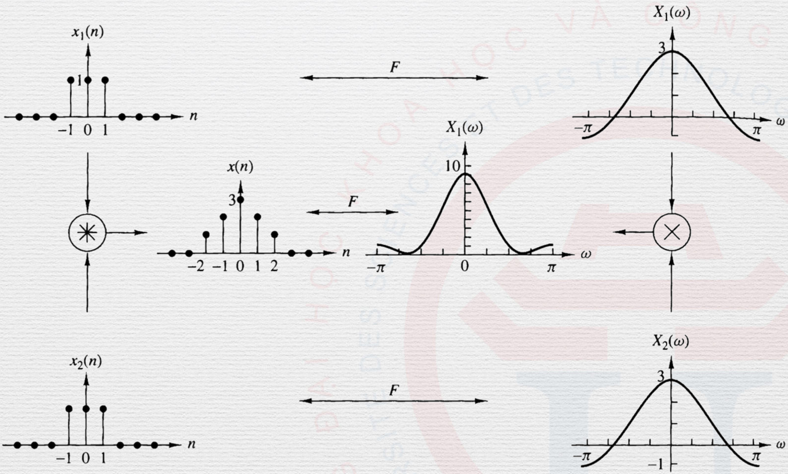
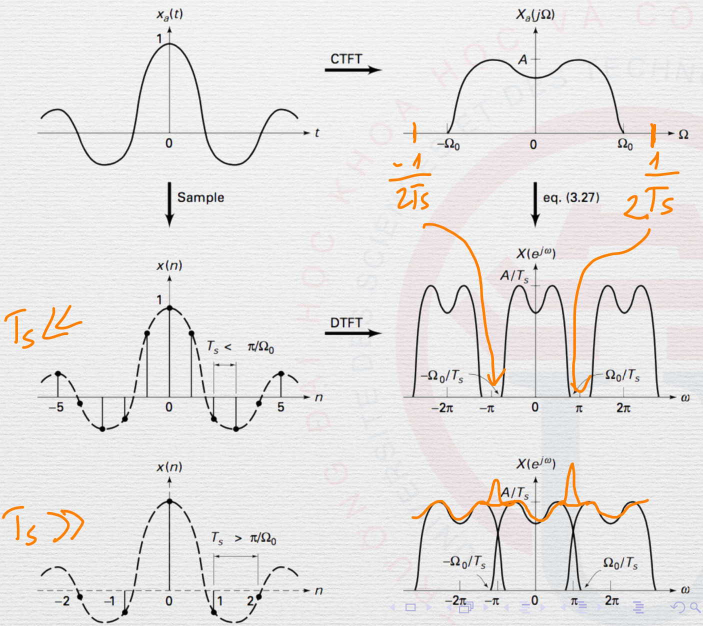
Convolution

$$\mathcal{F}[x_1(n) * x_2(n)] = \mathcal{F}[x_1(n)]\mathcal{F}[x_2(n)] = X_1(e^{j\omega})X_2(e^{j\omega})$$

Convolution



Sampling


$x_a(t)$
1
0
t
CTFT
$X_a(j\Omega)$
A
$-\Omega_0$
0
Ω_0
Ω
$-\frac{1}{2T_s}$
$\frac{1}{2T_s}$
Sample
eq. (3.27)
$x(n)$
$T_s \ll$
$T_s < \pi/\Omega_0$
-5
5
n
DTFT
$X(e^{j\omega})$
A/T_s
$-\Omega_0/T_s$
Ω_0/T_s
-2π
$-\pi$
π
2π
ω
$T_s \gg$
$T_s > \pi/\Omega_0$
-2
-1
1
2

Reconstruction