Digital Signal Processing

TRAN Hoang Tung

time

$x(n) \longrightarrow \boxed{h(n)} \longrightarrow$

$y(n) = x(n) * h(n)$

Discrete-time Fourier Analysis

Systems

TRAN Hoang Tung

Information and Communication Technology (ICT) Department
University of Science and Technology of Hanoi (USTH)

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1. The Discrete-time Fourier Transform (DTFT)

2. Frequency Presentation of LTI Systems

3. Ideal Filters

$x(t)$ cont.	CTFT	$X(\omega)$
disc.	DTFT	$X(e^{j\omega})$
$x(n)$	DFT	$X(k)$

Definition

The Discrete-time Fourier Transform (DTFT)

$$X(e^{j\omega}) \triangleq \mathcal{F}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

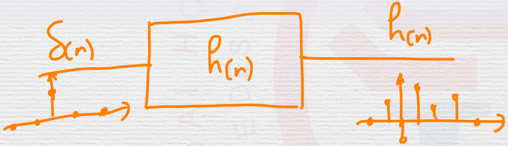
$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Frequency Response

Definition

The DTFT of an impulse response is called the frequency response (or transfer function) of an LTI system

$$H(e^{j\omega n}) \triangleq \mathcal{F}[h(n)] = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$$



Response to a Complex Exponential $e^{j\omega_0 n}$

const

LTI

$$x(n) = e^{j w_0 n} \longrightarrow \boxed{h(n)} \longrightarrow ?\; y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$x(n) * h(n) = \sum_{k=-\infty}^{+\infty} x(k) \cdot h(n-k)$$

$$= \sum h(k) \cdot e^{j w_0 (n-k)}$$

$$= \left[\sum h(k) \cdot e^{-j w_0 k}\right] e^{j w_0 n}$$

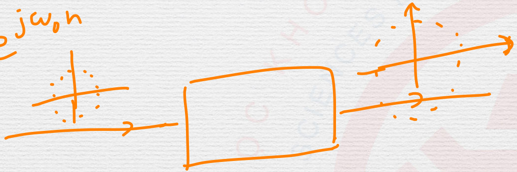
$$\boxed{y(n) = H(e^{j\omega})\Big|_{\omega=\omega_0} \cdot x(n)}$$

$\cancel{x(n)}$ $\cancel{e}^{j\omega_0 n} +$

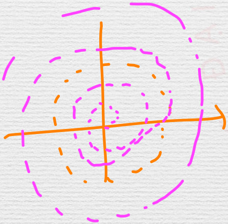
Response to a Complex Exponential $e^{j\omega_0 n}$

$$y(n) = \underline{H(e^{j\omega_0})}\underbrace{e^{j\omega_0 n}}_{x(n)}$$

$x(n) = e^{j\omega_0 n}$



$e^{s.n}$



Response to Sinusoidal $x(n) = cos(\omega_0 n)$

Hint: use Euler formula

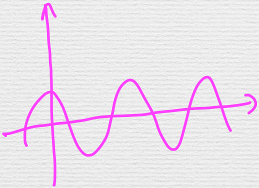
$$\cos(\omega_0 n) = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$$

$$\frac{H(e^{j\omega_0})}{2}$$

$\cos(\omega_0 n) \rightarrow \boxed{h(n)} \rightarrow$

real!

$$\underline{A} \cdot \cos(\omega_0 n + \varphi)$$



Frequency Response from Difference Equations

Given an LTI system $\delta(n) \to \boxed{} \to h(n)$

$$y(n) + \sum_{l=1}^{N} a_l y(n-l) = \sum_{m=0}^{M} b_m x(n-m)$$

$$h(n) + 2h(n-1) - 7h(n-2) = \delta(n-1) + 3\delta(n-3)$$

$$h(0) + 2\cancel{h(-1)} - 7\cancel{h(-2)} = \delta(-1) + 3\delta(-3) = 0$$

$$h(1) + 2h(0) - 7\cancel{h(-1)} = \delta(0) + 3\delta(-2) = 1$$

$$h(2) + 2h(1) - 7h(0) = \delta(1) + 3\delta(-1) = 0$$

$$\vdots$$

at rest : no input $\to$ has no output

Frequency Response from Difference Equations

Given an LTI system

$$y(n) + \sum_{l=1}^{N} a_l y(n-l) = \sum_{m=0}^{M} b_m x(n-m)$$

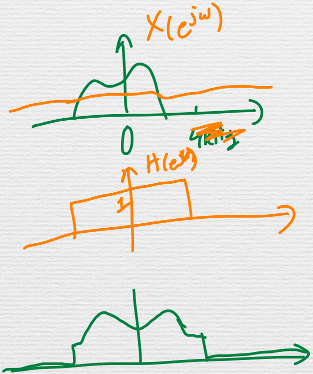
its frequency response is

$$H(e^{j\omega}) = \frac{\sum_{m=0}^{M} b_m e^{-j\omega m}}{1 + \sum_{l=1}^{N} a_l e^{-j\omega l}}$$

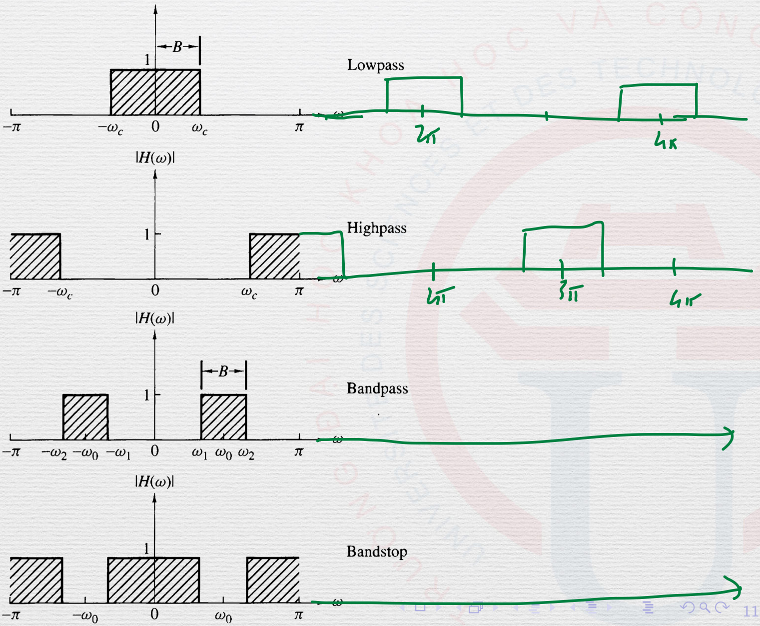
$$Y(e^{j\omega}) \ldots = X(e^{j\omega}) \ldots$$

$$X(e^{j\omega}) \rightarrow \boxed{H(e^{j\omega})} \rightarrow Y = X \cdot H$$

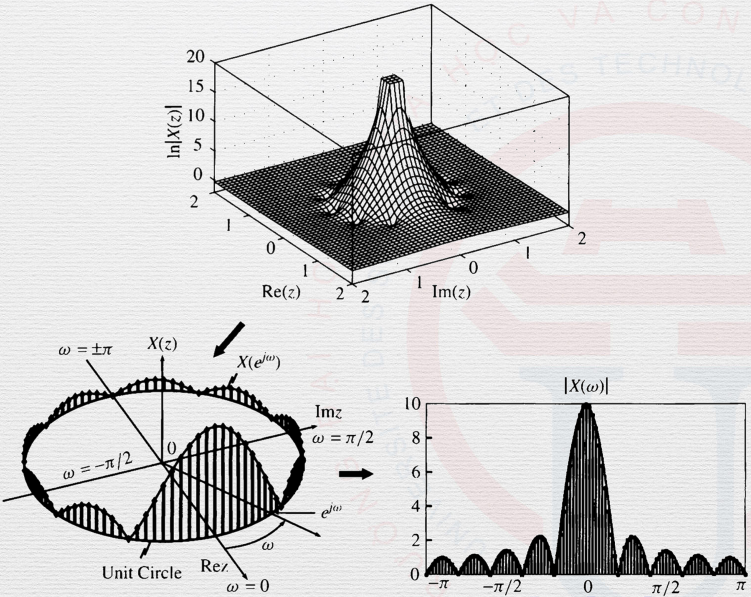
$$\rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$


X(e^jw)
0
H(e^jw)
1

Characteristics



Pole-zero patterns



Pole-zero patterns