Digital Signal Processing

Labwork 1

Frequency & Time Representation

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1. Introduction

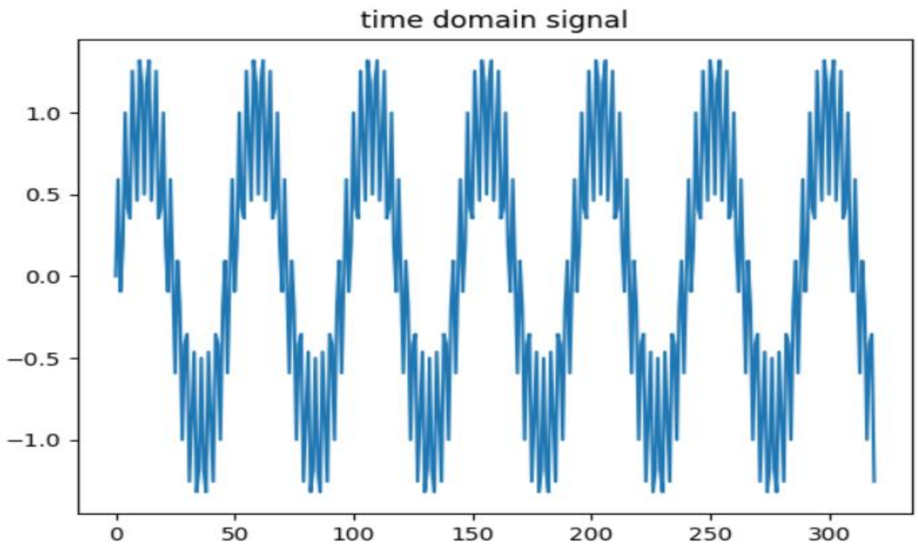
This labwork deals with the changing between time domain and frequency domain of several signals and systems.

Time domain and frequency domain refer to two different ways of looking at a signal. In the time domain, a signal is a wave that varies in amplitude (y-axis) over time (x-axis) while in the frequency domain, a signal is represented as a series of frequencies (x-axis) that each has an associated power (y-axis)

This is essential for students to imagine what would it be when applying knowledge, formulas in lecture time into pratical coding sessions.

1 Signal and System

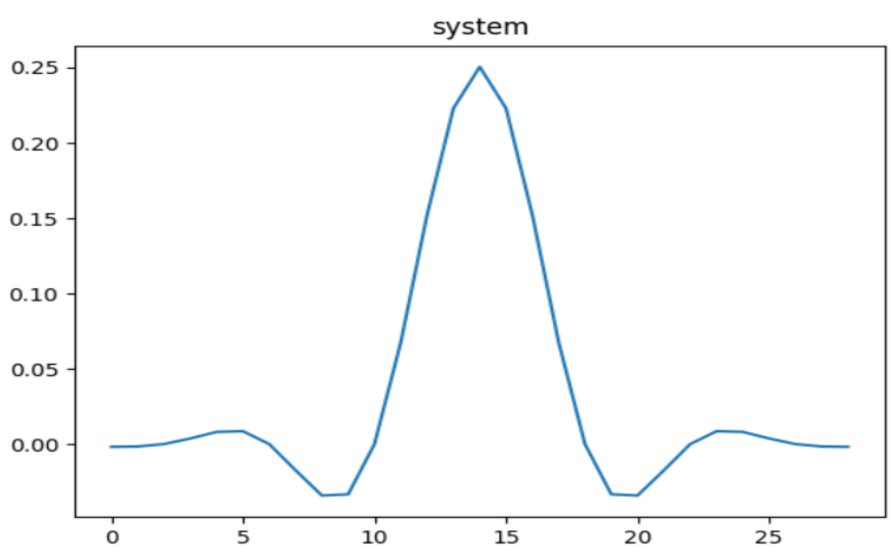
a) Time domain



- Signal : x(n)= Input_1kHz_15kHz

-The x axis is transferred to the domain from 0 to 2pi

-A time-domain graph shows how a signal changes over time.

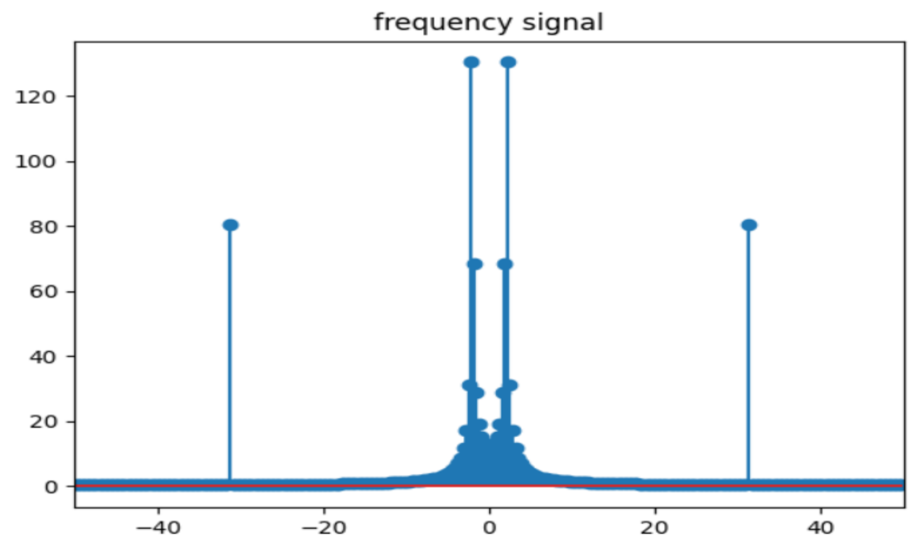


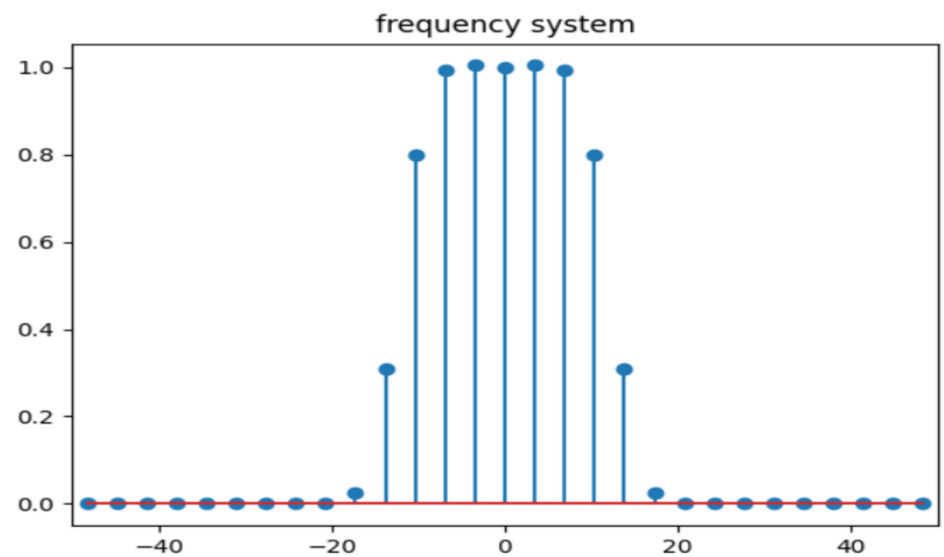
-System : `h(n) = Impulse_response`

-The x axis is transferred to the domain from 0 to 2pi

- Using DTFT as LTI system

b) Frequency





-The x axis is transferred to the domain from -pi to pi.(-40 to 40)

-Sampling rate, or number of measurements per second are: 100

-To convert a signal in time domain to frequency domain, we should use the fast Fourier transform algorithm for computing the discrete Fourier transform. By using this, the signal in frequency domain becomes discrete.

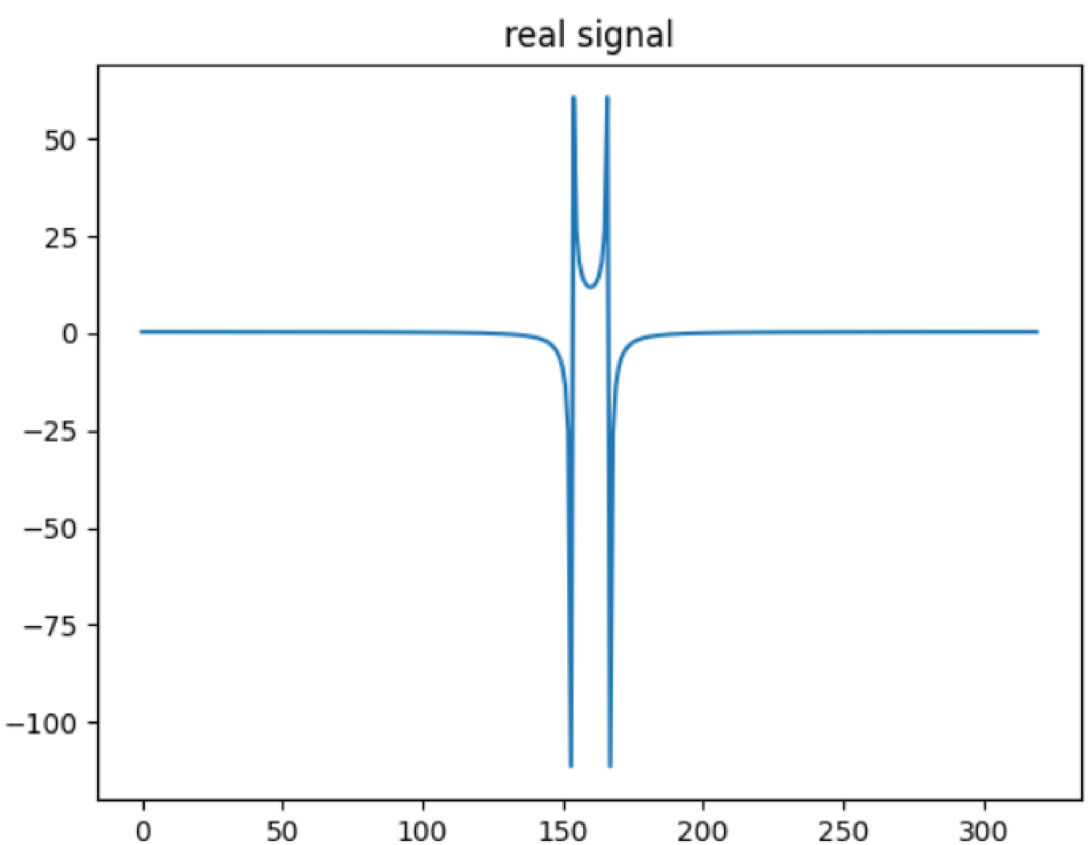
-The Fourier transform takes us from the time to the frequency domain.The fast Fourier transform (FFT) is an algorithm for computing the DFT. FFT depend on the fact that $e^{-j2\pi/N}$

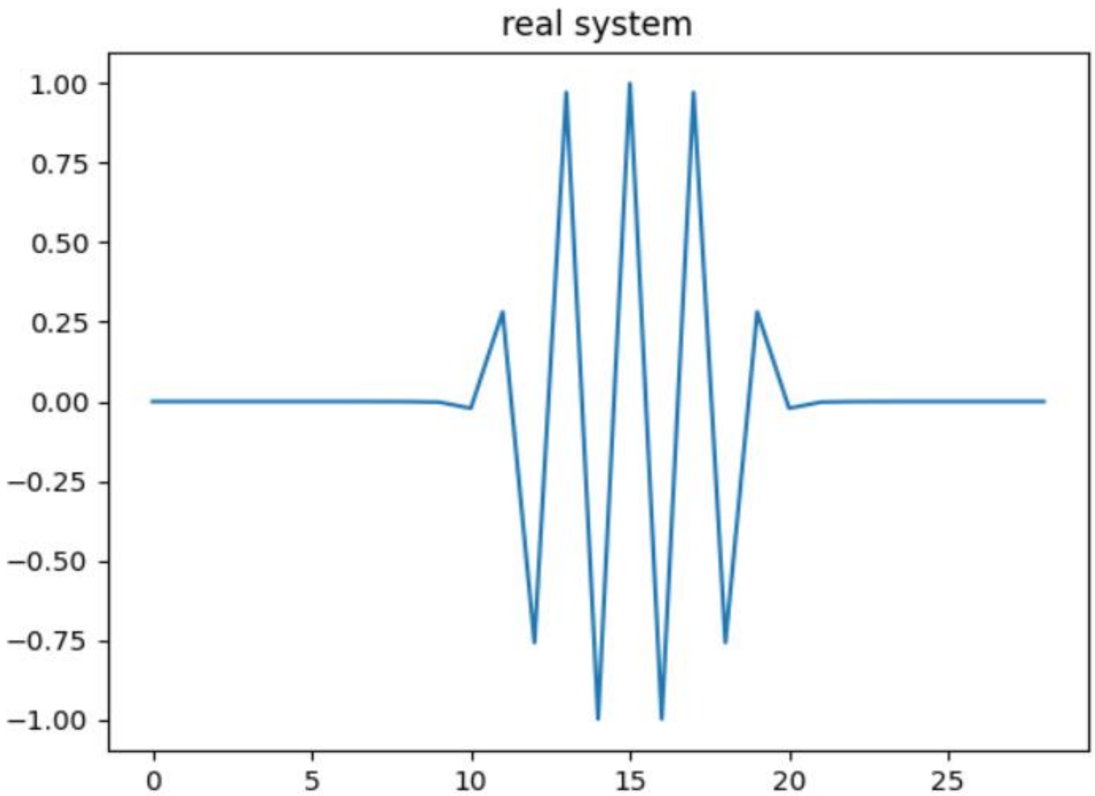
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$$

-The DTFT of an impulse response with LTI system

$$H(e^{j\omega n}) \triangleq \mathcal{F}[h(n)] = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$$

b) Real





- The real DTFT and DFT transforms an N point time domain signal into two (N/2 + 1) point frequency domain signals/system. Points 0 through N/2 in the complex DTFT and DFT are the same as in the real DTFT and DFT . Even though the real DTFT and DFT uses only real numbers.

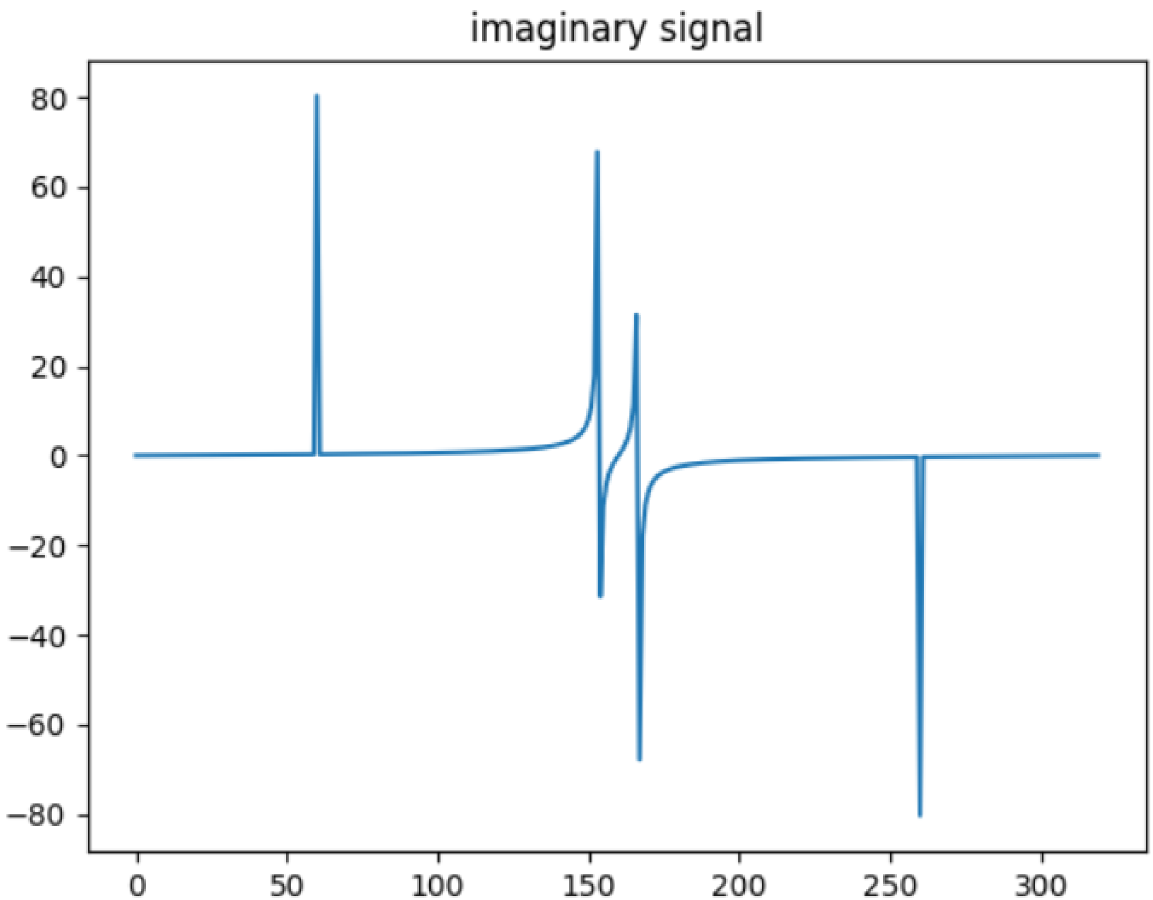
- Suppose you have an N point signal, and need to calculate the real DFT by means of the Complex DFT (using the FFT algorithm). First, move the N point signal into the real part of the complex DFT's time domain, and then set all of the samples in the imaginary part to zero. Calculation of the complex DFT results in a real signal /system in the frequency domain, each composed of N points. Samples 0 through N/2 of these signals correspond to the real DFT's spectrum. The N/2 even points are placed into the real part

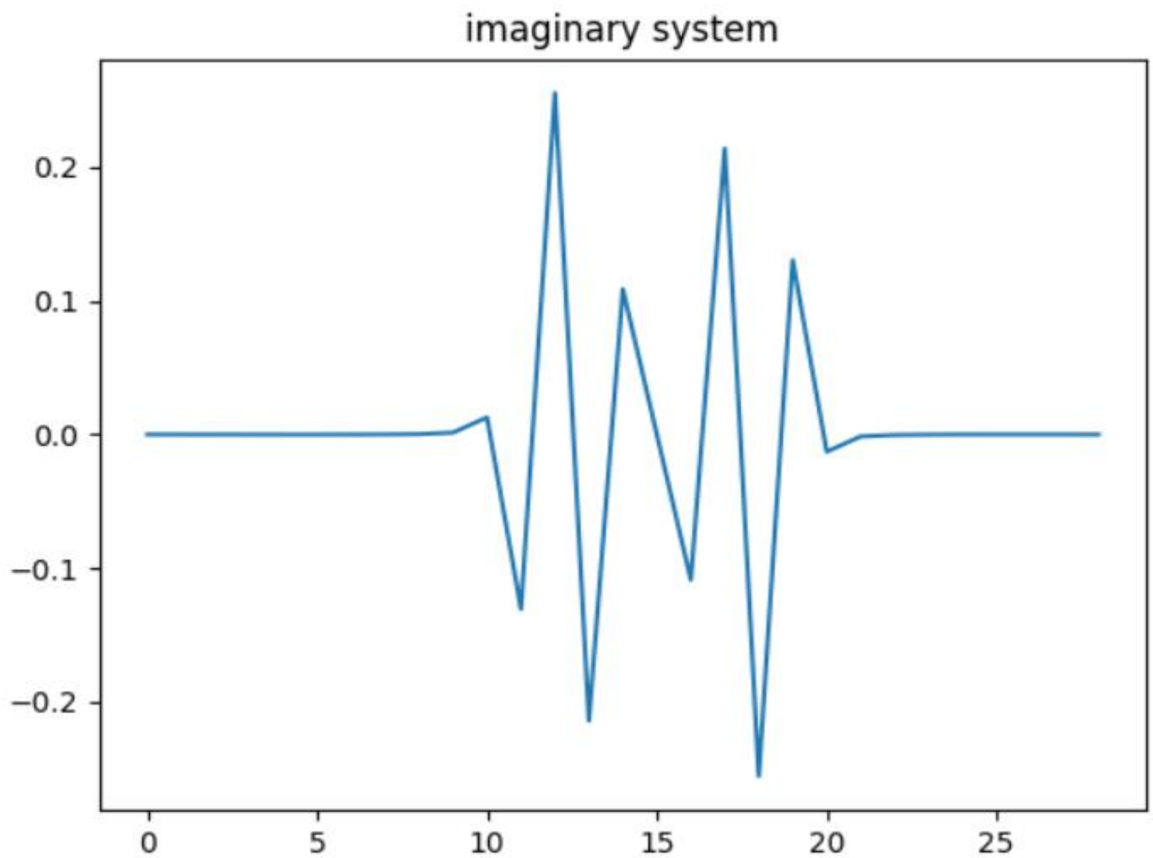
-The amplitudes of the cosine waves are contained in ReX[w] (signal) an H_R(w)(system):

$$\boldsymbol{X}(\boldsymbol{e}^{-\boldsymbol{jw}}) = \boldsymbol{X}^*(\boldsymbol{e}^{\boldsymbol{jw}})$$

$$ReX(\omega) = \sum_{n=-\infty}^{+\infty} x[n]\cos(\omega n) \qquad H_R(\omega) = \sum_{k=-\infty}^{\infty} h(k)\cos\omega k$$

e) Imaginary





-The imaginary DTFT and DFT transforms an N point time domain signal into two (N/2 + 1) point frequency domain signals. Points 0 through N/2 in the complex DTFT and DFT are the same as in the imaginary DTFT and DFT

-Do the same the real part.We have: the N/2 old points are placed into the imaginary part

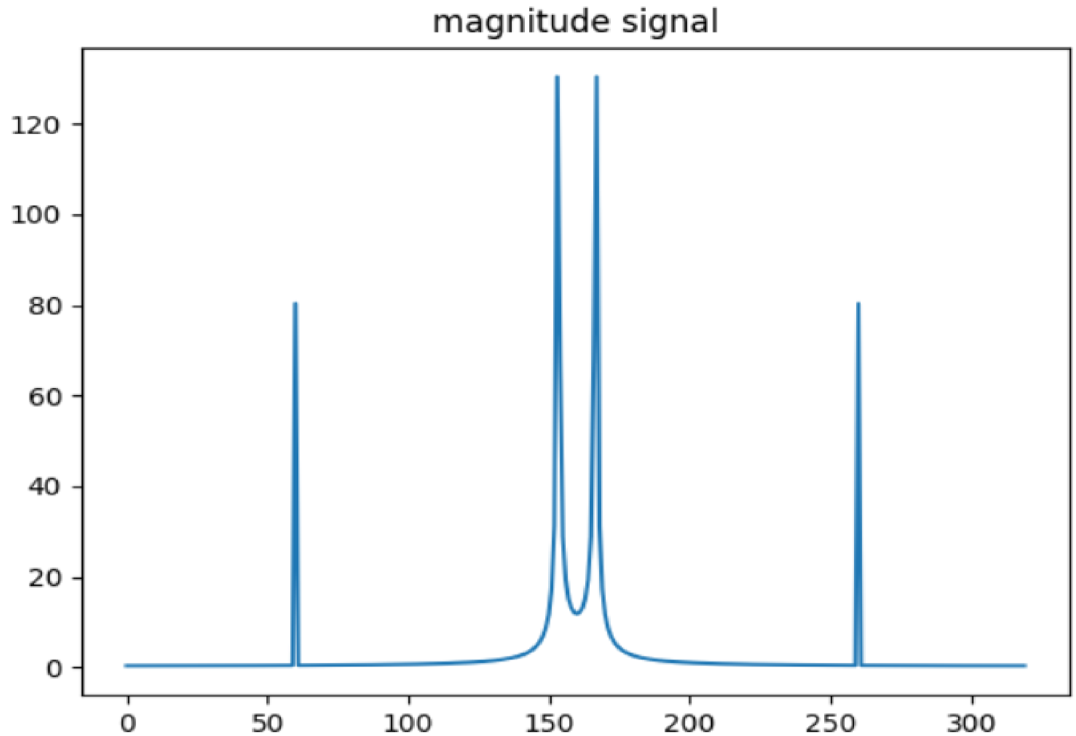
- The amplitudes of the sine waves are contained in ImX[w] (signal) an H_I(w)(system):

$$\boldsymbol{X}(-\boldsymbol{e}^{-\boldsymbol{jw}}) = \boldsymbol{X}^{*}(\boldsymbol{e}^{\boldsymbol{jw}})$$

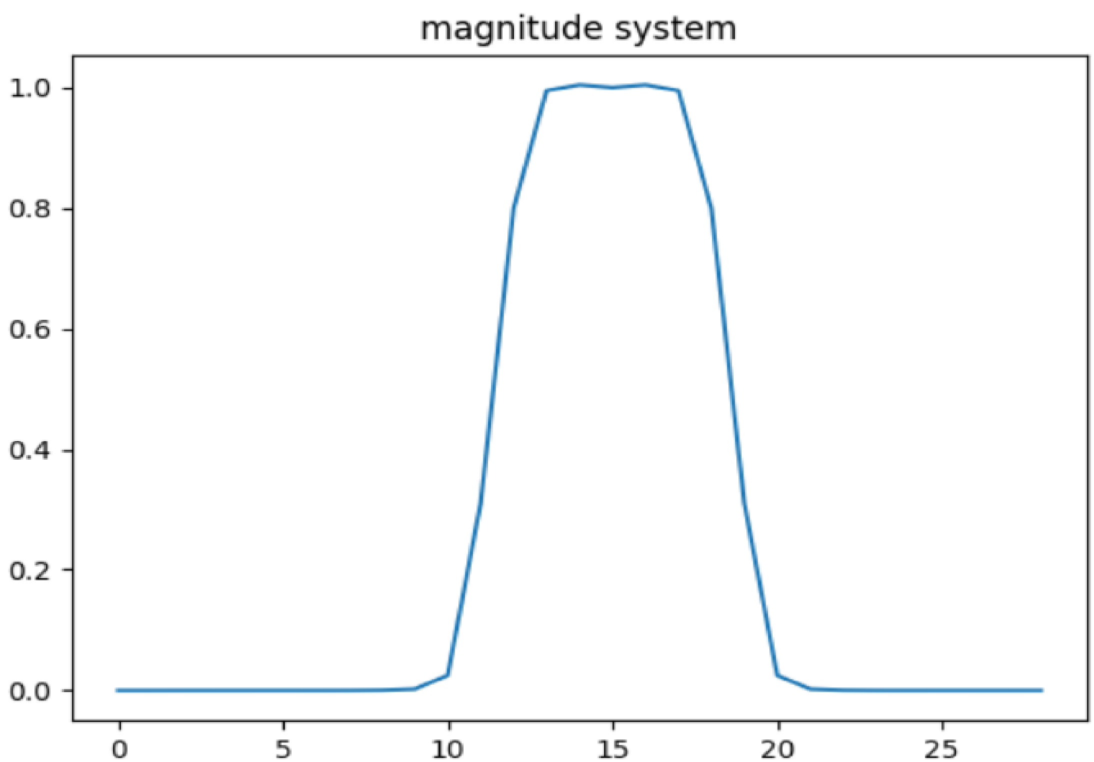
$$ImX(\omega) = -\sum_{n=-\infty}^{+\infty} x[n]\sin(\omega n)$$

$$H_I(\omega) = -\sum_{k=-\infty}^{\infty} h(k)\sin\omega k$$

e) Magnitude



magnitude signal
120
100
80
60
40
20
0
0
50
100
150
200
250
300



magnitude system
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25

-With DFT as Linear Transformation:

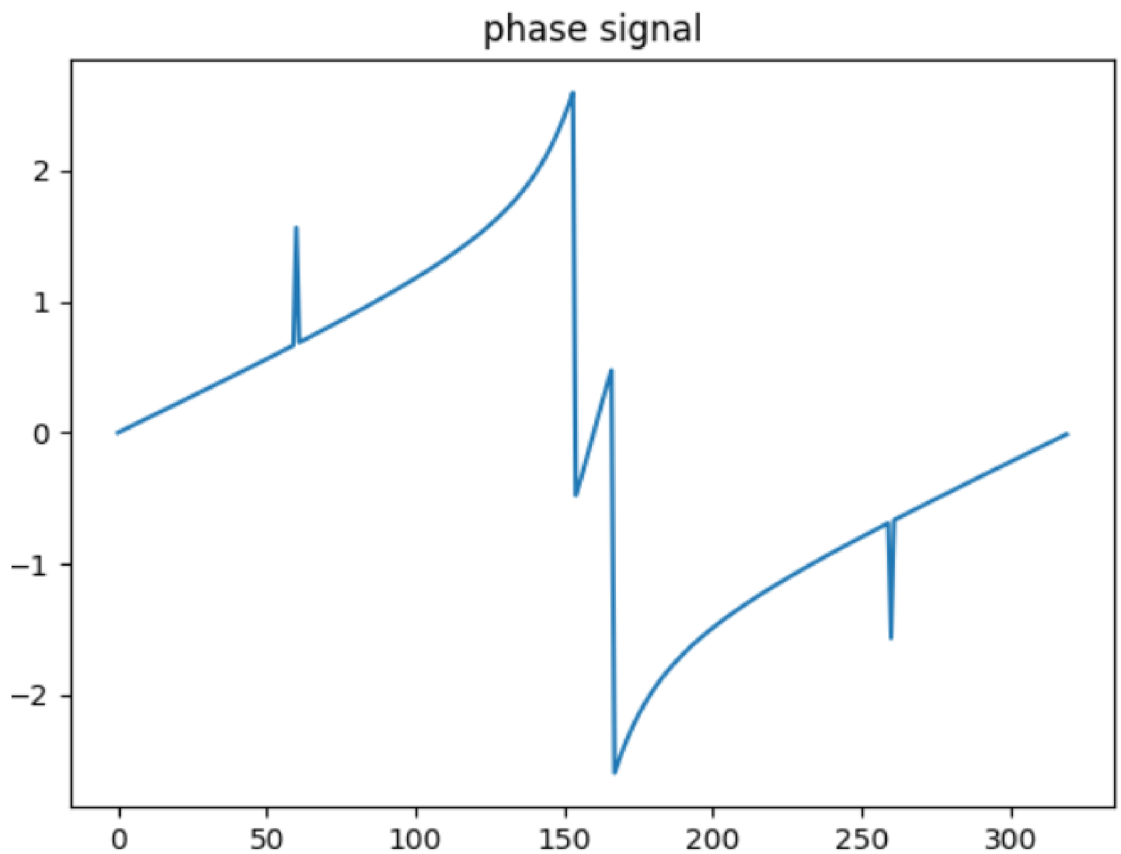
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

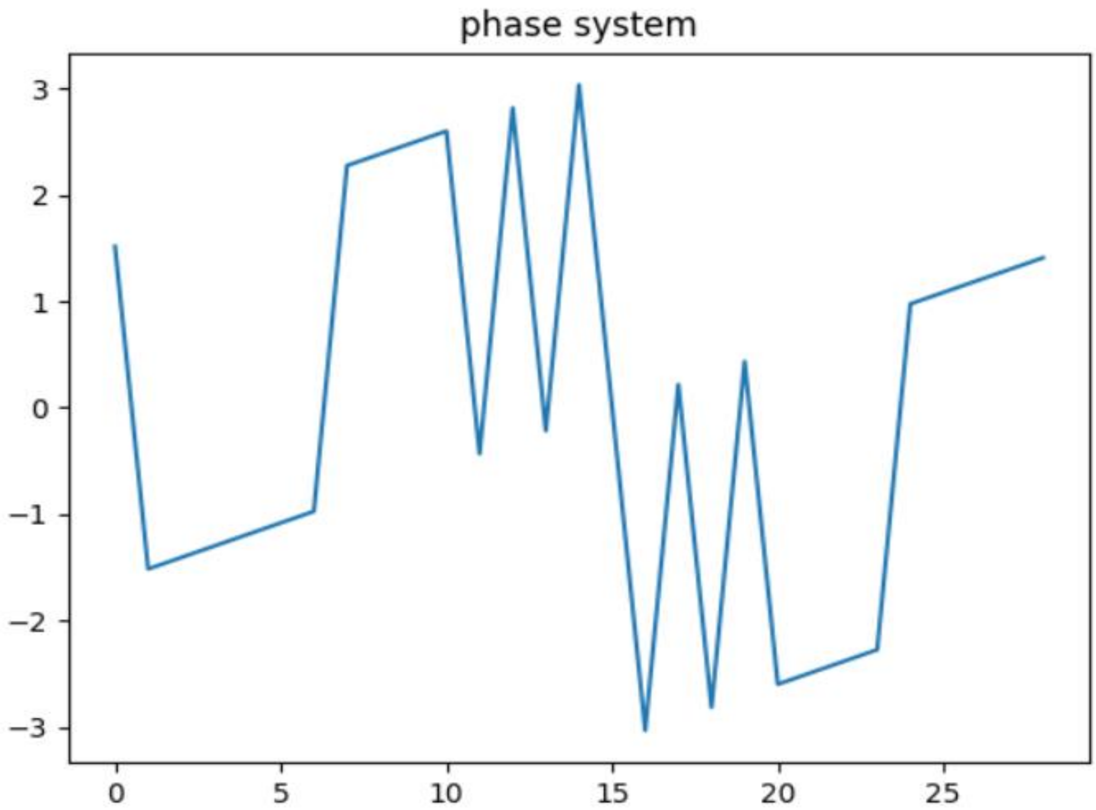
we defined $W_N = e^{-j2\pi/N}$.

-Its magnitude is always maintained at uniform frequency domain.

-Magnitude represents the magnitude of the frequencies as they change over time.

f) Phase

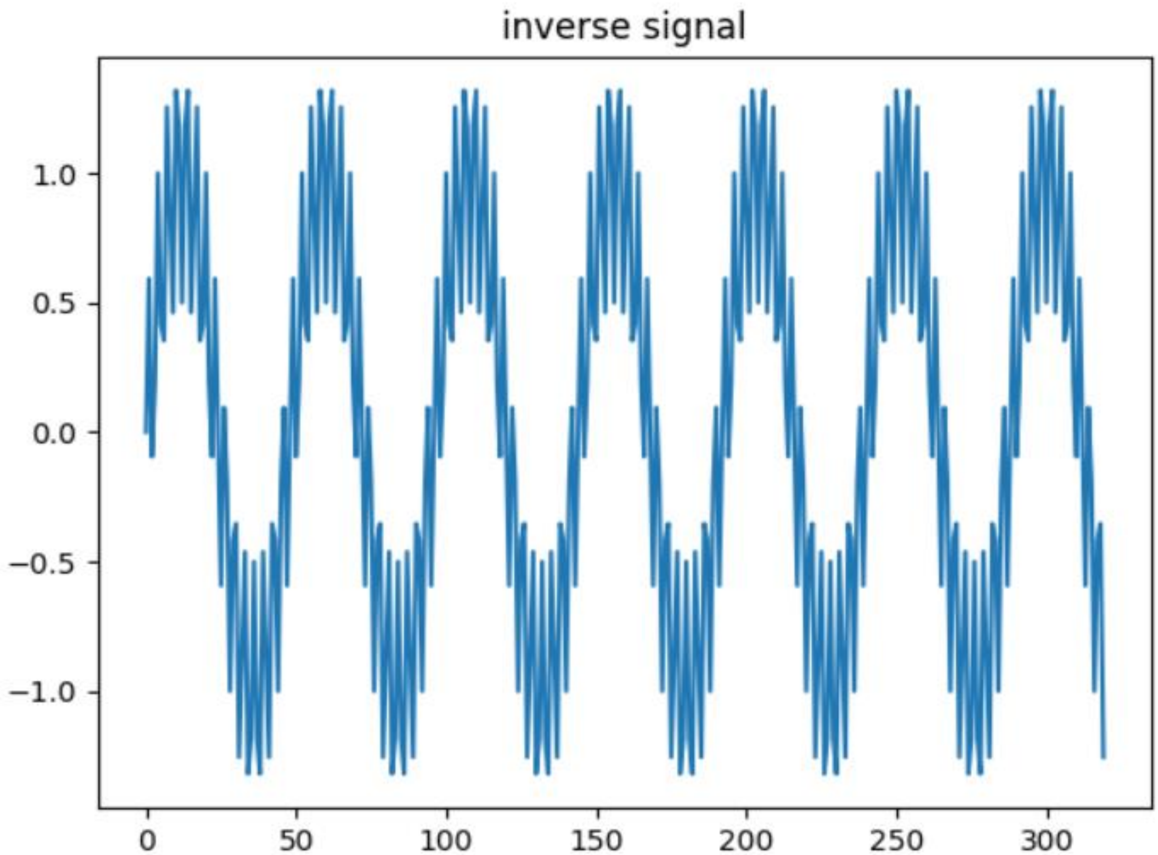




- Phase is an angle-like quantity representing the number of periods spanned by that variable.

$$\Theta(\omega) = \tan^{-1} \frac{H_I(\omega)}{H_R(\omega)}$$

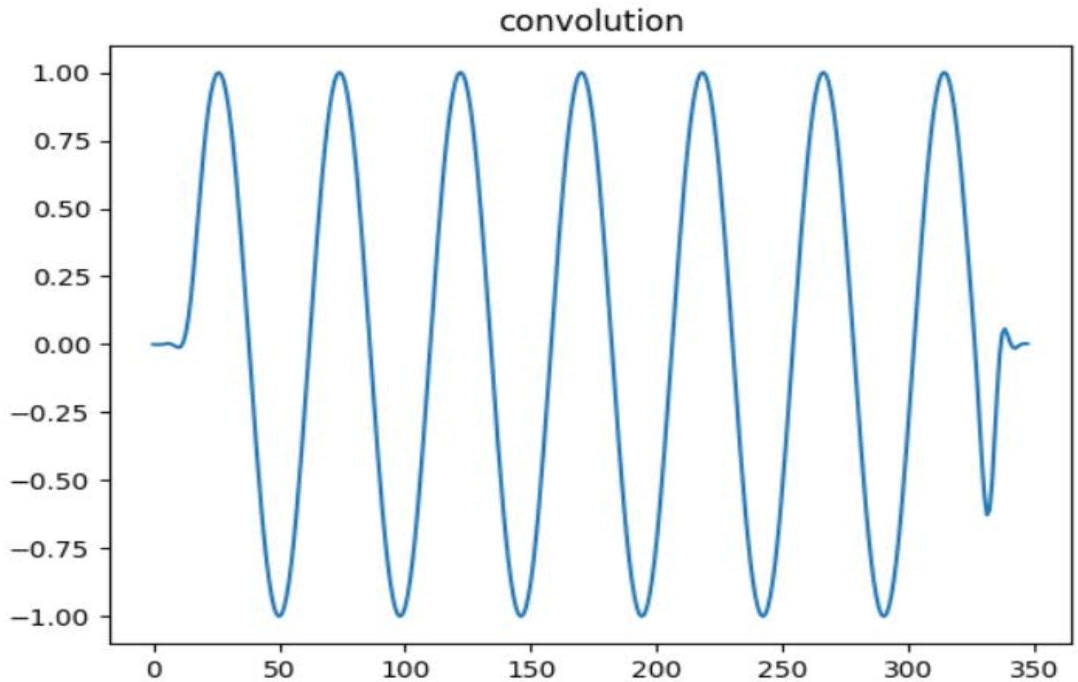
g) Inverse-FFT

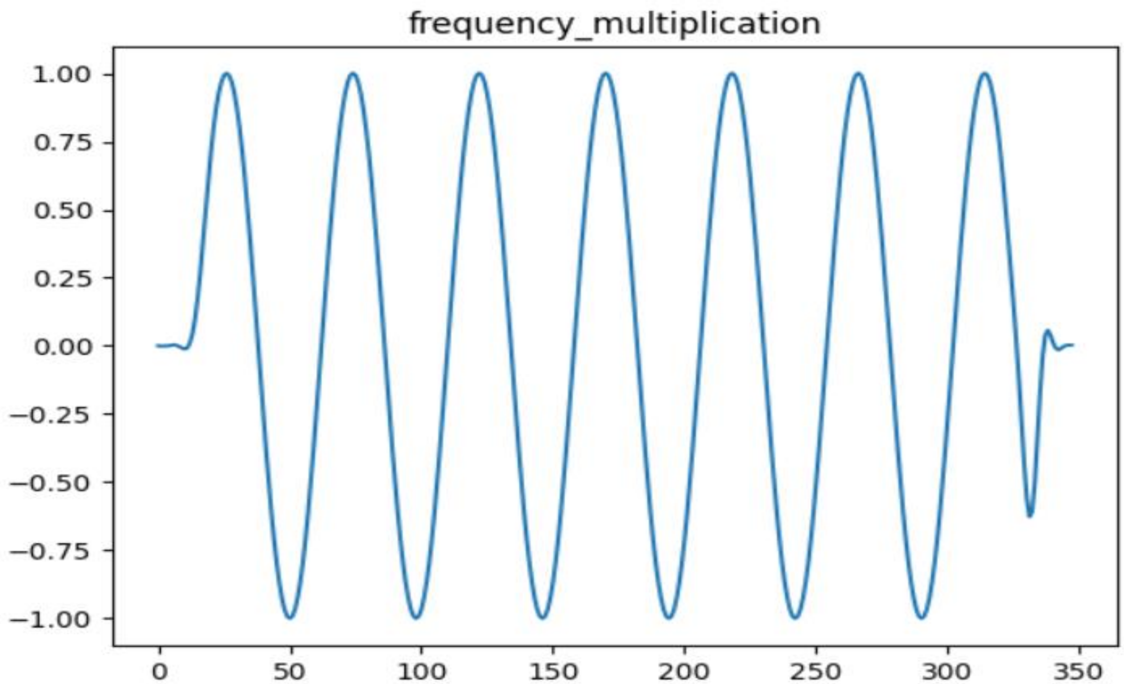


-The inverse transform is the same as the forward transform with the real and imaginary parts swapped for both input and output, up to a normalization

=> Inverse same input time domain

h) Convolution and Frequency Multiplicaition





-Since we already have the signal in time domain and impulse response, we can find out the output using time convolution: y(s) = x(s) * h(s)

- Convolution in time domain equals multiplication in frequency domain

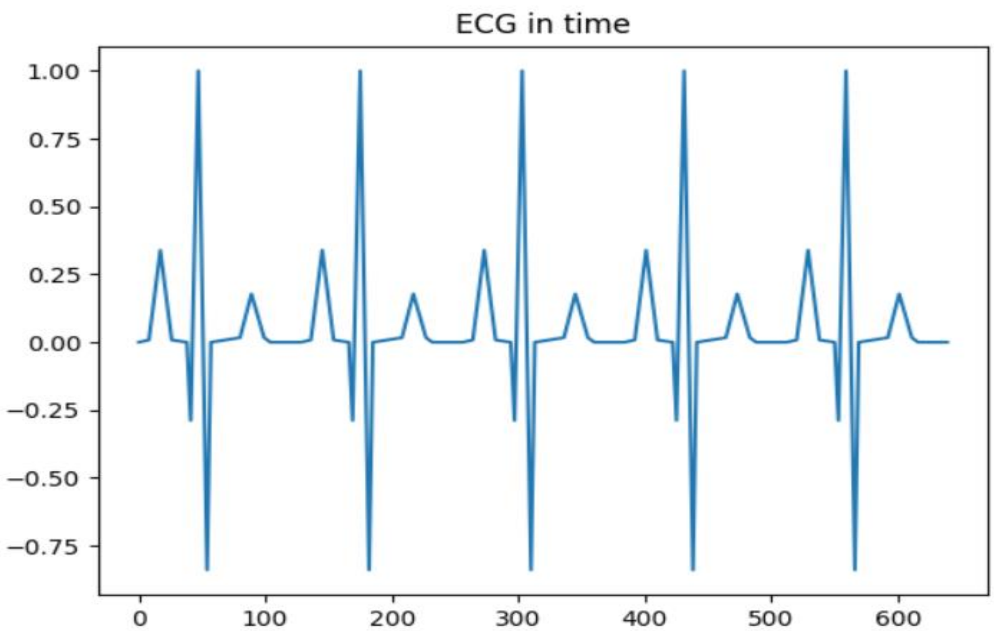
$$F\{x(t)*y(t)\}(f)=F\{x(t)\}(f) \cdot F\{y(t)\}(f)$$

where $F\{x(t)\}(f)$F{x(t)}(f) denotes the Fourier transform of $x(t)$x(t), f evaluated at the frequency

- The convolution theorem can be used to perform convolution via multiplication in the time domain.

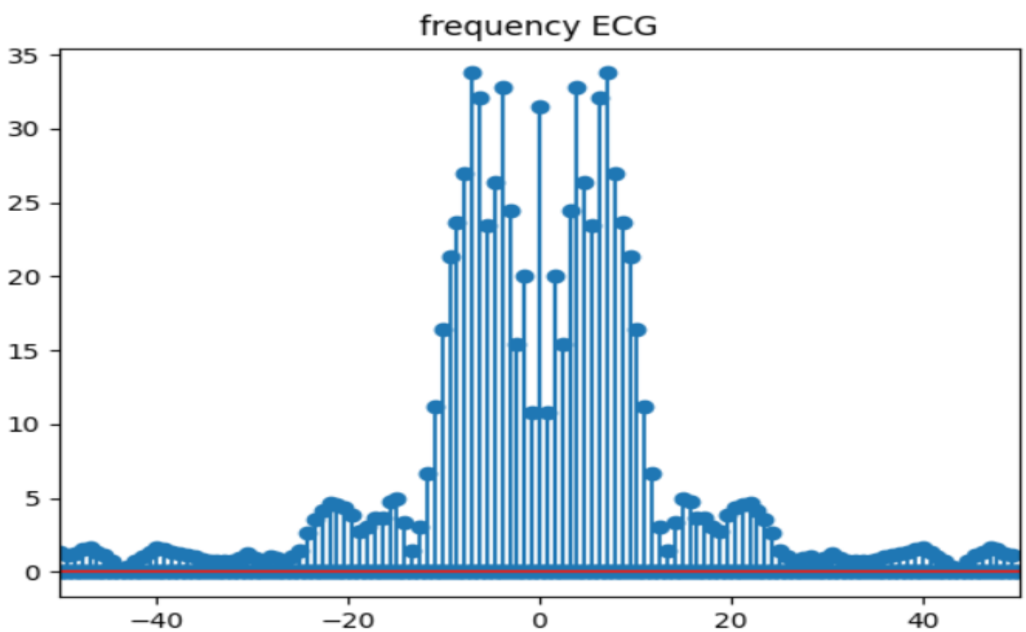
2.ECG

+ ECG in time

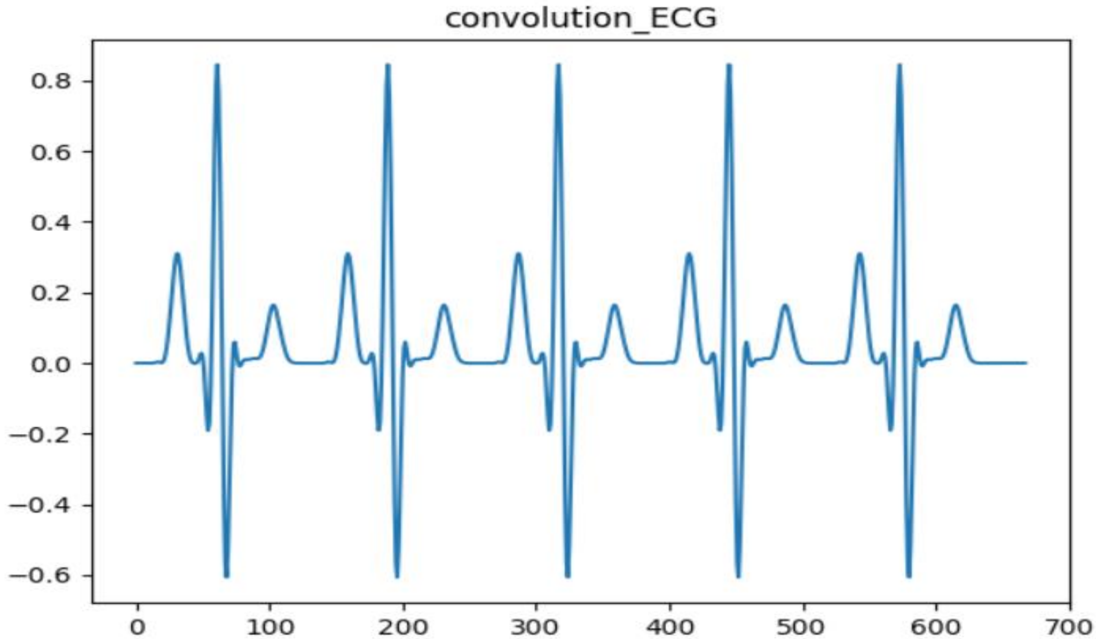


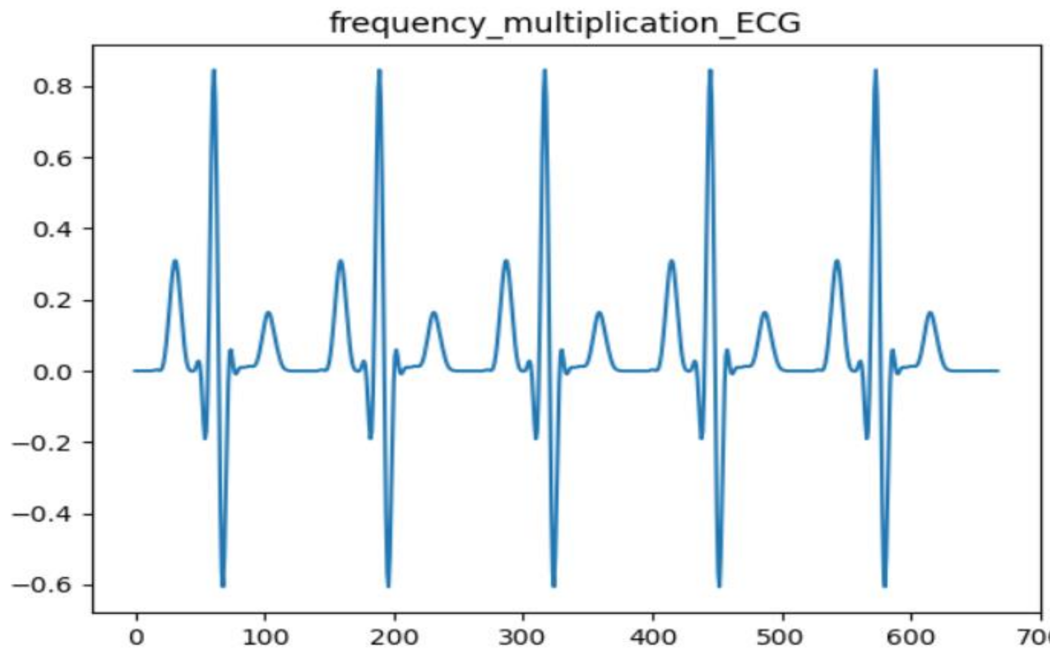
-x(n)=ECG

+ Frequency domain



+ Concolution and Frequency Multipication





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