

# BASIC DATABASES

## Relational Model of Data

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# Objectives

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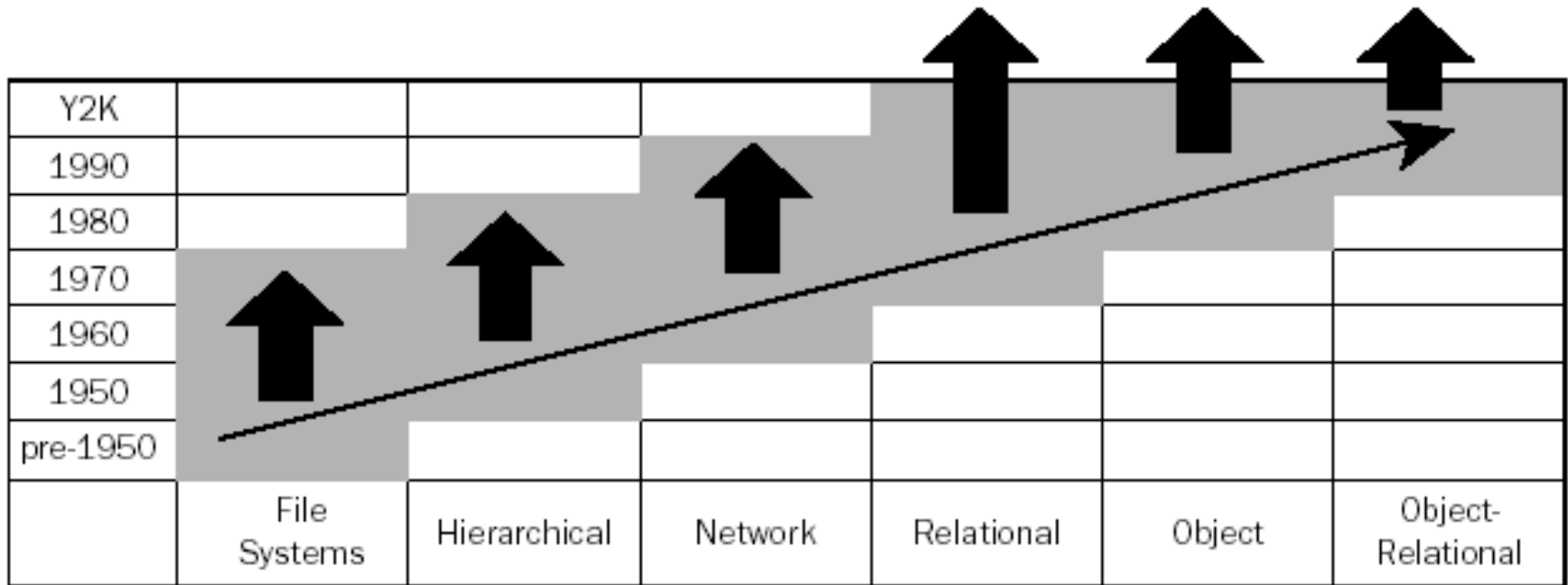
- Understand concepts of
  - Data models
  - Relational data model
    - Structure
    - Operations
    - Constraints

# What is a Data Model?

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- Mathematical representation of data.
  - Relational model = tables;
  - Semi-structured model = trees/graphs.
- A model consists of:
  - Structure of data
  - Operations on data.
  - Constraints.

# Data Models in history



The evolution of database modeling techniques.

# Why Relation?

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- Very simple model.
- *Often* matches how we think about data.
- Abstract model that underlies SQL, the most important database language today.

Reference: E. F. Codd, “A relational model for large shared data banks,” Comm.ACM 13:6, pp. 377-387, 1970.

# STRUCTURE OF RELATIONAL MODEL

# Relational Model in brief

Attribute


Tuple

<i>title</i>	<i>year</i>	<i>length</i>	<i>genre</i>
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

- The relational model represents data as a 2-dimensional table (called a relation)

# Attributes

Attribute



<i>title</i>	<i>year</i>	<i>length</i>	<i>genre</i>
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

- Each column represent a property of film and also called a “**attribute**”: title, year, length, genre



# Domains

- The relational model requires that each component of each tuple must be atomic, that is, it must be of some elementary type such as INTEGER or STRING
- It is **not** permitted for a value to be a record structure, set, list, array or any type that can have its values broken into smaller components
- A Domain is a particular elementary type of an attribute
  - More general: a set of values for an attribute
  - What are domains of title, year, length, genre?

# Tuples

<i>title</i>	<i>year</i>	<i>length</i>	<i>genre</i>
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

Tuple →

- A row of a relation is called a tuple (or record)
  - Eg: Each row represents a film
- When we want to write a tuple in isolation, not as part of a relation, we normally use commas to separate components
  - Eg: (Star Wars, 1977, 124, sciFi)

# Equivalent representation of a relation

- Relation is a **bag** (extended set) of tuples, not a list of tuples
  - Order of tuples is not important

<i>title</i>	<i>year</i>	<i>length</i>	<i>genre</i>
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

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<i>title</i>	<i>year</i>	<i>length</i>	<i>genre</i>
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

- There could be identical tuples
- Question:
  - What is/are the difference(s) between a set and a list?
  - What is/are the difference(s) between a set and a bag?

# Relation instances

- A relation about **Movies** is not static but changing over time:
  - We want to insert tuples for new **Movies** as these appear
  - We want to edit existing tuples if we get corrected information about a **Movies**
  - We want to delete a tuple from the relation
- Sometime, we also want to add or delete attributes, this lead to the changing of the schema. So, a set of tuples for a given relation is called an instance of that relation

# Relation instances example

R

A	B
1	2
3	4

R

A	B
1	3
2	5

R

A	B
3	5
4	6

# Keys of relations

- A set of attributes forms a key for a relation if we don't allow 2 tuples in a relation instance to have the same values in all the attributes of the key
- Eg: The key of relation  
People(ID, name, address) is ID

# Schemas

- Relation schema:
  - Name
  - Set of attributes.
    - Order of attributes is arbitrary. In practice we assume the order given in the relation schema
  - Other structure infor: keys...
  - Eg: **Movies** (Id, title, year, length, genre)
- Database schema: set of relation schema
  - **Movies** (Id, title, year, length, genre)
  - **Producers** (name, address, country)

# DB Schema about Movies

```
Movies(  
  title:string,  
  year:integer,  
  length:integer,  
  genre:string,  
  studioName:string,  
  producerC#:integer  
)  
MovieStar(  
  name:string,  
  address:string,  
  gender:char,  
  birthdate:date  
)
```

```
StarsIn(  
  movieTitle:string,  
  movieYear:integer,  
  starName:string  
)  
MovieExec(  
  name:string,  
  address:string,  
  cert#:integer,  
  netWorth:integer  
)  
Studio(  
  name:string,  
  address:string,  
  presC#:integer  
)
```



# OPERATIONS OF RELATIONAL MODEL

# Relational Algebra (RA)

- Algebra is Mathematical system consisting of:
  - *Operands* --- variables or values from which new values can be constructed.
  - *Operators* --- symbols denoting procedures that construct new values from given values.
- Relational Algebra: an offshoot of algebra of **bags**
  - Operands:
    - Variables that stand for relations
    - Constants, which are finite relations
  - Operators work with relation(s) to create a new relation

# Notations

Operation	My HTML	Symbol
Projection	PROJECT	$\pi$
Selection	SELECT	$\sigma$
Renaming	RENAME	$\rho$
Union	UNION	$\cup$
Intersection	INTERSECTION	$\cap$
Assignment	$\leftarrow$	$\leftarrow$

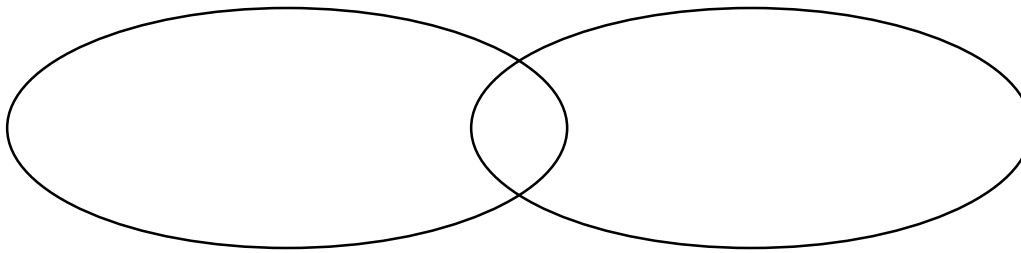
Operation	My HTML	Symbol
Cartesian product	X	$\times$
Join	JOIN	$\bowtie$
Left outer join	LEFT OUTER JOIN	$\ltimes$
Right outer join	RIGHT OUTER JOIN	$\rtimes$
Full outer join	FULL OUTER JOIN	$\ltimes\rtimes$
Semijoin	SEMIJOIN	$\ltimes$

# Primitive operations

- In any algebra, some operators are primitive and the others, being definable in terms of the primitive ones, are derived
- The six primitive operators of relational algebra are:
  - the SELECTION,
  - the PROJECTION,
  - the CARTESIAN PRODUCT (also called the *cross product* or *cross join*),
  - the SET UNION,
  - the SET DIFFERENCE, and
  - the RENAME

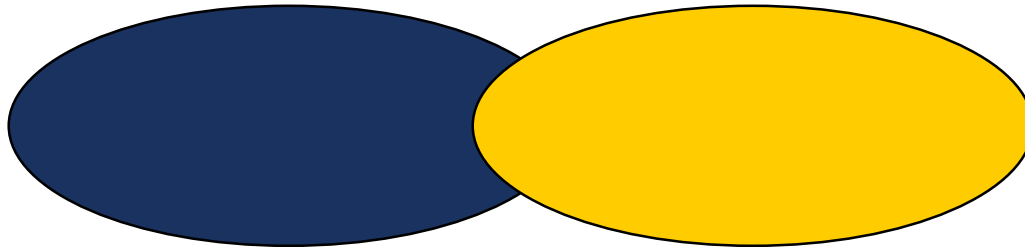
# Operations on Relations - Bag Union

- Example:  $\{1,2,1\} \cup \{1,1,2,3,1\} = \{1,1,1,1,1,2,2,3\}$



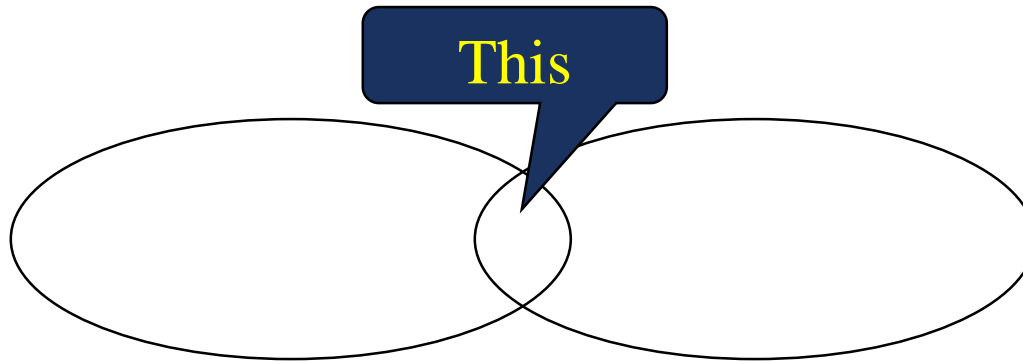
# Operations on Relations - Bag Difference

- **Example:**  $\{1,2,1,1\} - \{1,2,3\} = \{1,1\}$ .



# Set Operations on Relations - Bag Intersection

- **Example:**  $\{1,2,1,1\} \cap \{1,2,1,3\} = \{1,1,2\}$ .



# Projection

- $R1 := \pi_L(R2)$ 
  - $L$  is a list of attributes from the schema of  $R2$ .
  - $R1$  is constructed by looking at each tuple of  $R2$ , extracting the attributes on list  $L$ , in the order specified, and creating from those components a tuple for  $R1$ .
  - Eliminate duplicate tuples, if any.



# Projection Example

Relation Sells:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Miller	3.00

Prices :=  $\pi_{\text{beer, price}}(\text{Sells})$ :

beer	price
Bud	2.50
Miller	2.75
Miller	3.00

# Extended Projection

- Using the same  $\Pi_L$  operator, we allow the list  $L$  to contain arbitrary expressions involving attributes:
  - Arithmetic on attributes, e.g.,  $A+B \rightarrow C$ .
  - Duplicate occurrences of the same attribute.

$R =$

$A$	$B$
1	2
3	4

$$\Pi_{A+B \rightarrow C, A, A}(R) =$$

$C$	$A1$	$A2$
3	1	1
7	3	3

# Exercises - I

- Suppose relation  $R(A,B,C)$  has the tuples:

- Compute the projection  $\pi_{C,B}(R)$

A	B	C
1	2	3
4	2	3
4	5	6
2	5	3
1	2	6

# Selection

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$$R1 := \sigma_C(R2)$$

- $C$  is a condition (as in “if” statements) that refers to attributes of  $R2$ .
- $R1$  is all those tuples of  $R2$  that satisfy  $C$ .

# Selection Example

Relation Sells:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Miller	3.00

JoeMenu :=  $\sigma_{\text{bar}=\text{"Joe's"}}(\text{Sells})$ :

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75

# Renaming

- The  $\rho$  operator gives a new schema to a relation.
- $R1 := \rho_{R1(A1, \dots, An)}(R2)$  makes R1 be a relation with attributes  $A1, \dots, An$  and the same tuples as R2.
- Simplified notation:  $R1(A1, \dots, An) := R2$ .

# Renaming Example

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

$R(\text{bar}, \text{addr}) := \text{Bars}$

R(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

# Product

- $R3 := R1 \times R2$ 
  - Pair each tuple  $t1$  of  $R1$  with each tuple  $t2$  of  $R2$ .
  - Concatenation  $t1t2$  is a tuple of  $R3$ .
  - Schema of  $R3$  is the attributes of  $R1$  and then  $R2$ , in order.
  - But beware attribute  $A$  of the same name in  $R1$  and  $R2$ : use  $R1.A$  and  $R2.A$ .



$$R3 = R1 \times R2$$

R1(

A,	B)
1	2
3	4

R2(

B,	C)
5	6
7	8
9	10

R3(

A,	R1.B,	R2.B,	C)
1	2	5	6
1	2	7	8
1	2	9	10
3	4	5	6
3	4	7	8
3	4	9	10

$$R3 = R1 \times R2$$

The table E (for EMPLOYEE)

enr	ename	dept
1	Bill	A
2	Sarah	C
3	John	A

The table D (for DEPARTMENT)

dnr	dname
A	Marketing
B	Sales
C	Legal

SQL	Result					Relational algebra
select * from E, D	enr	ename	dept	dnr	dname	E X D
	1	Bill	A	A	Marketing	
	1	Bill	A	B	Sales	
	1	Bill	A	C	Legal	
	2	Sarah	C	A	Marketing	
	2	Sarah	C	B	Sales	
	2	Sarah	C	C	Legal	
	3	John	A	A	Marketing	
	3	John	A	B	Sales	
	3	John	A	C	Legal	

# Join

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- Types
  - Theta join
  - Natural join
  - Outer join:
    - Left outer join
    - Right outer join
    - Full outer join

# Theta-Join

- $R3 := R1 \bowtie_C R2$ 
  - Take the product  $R1 \times R2$ .
  - Then apply  $\sigma_C$  to the result.
- As for  $\sigma$ ,  $C$  can be any boolean-valued condition.
  - Historic versions of this operator allowed only  $A \theta B$ , where  $\theta$  is  $=, <$ , etc.; hence the name “theta-join.”

# Theta-Join Example

Sells(

bar,	beer,	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

)

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells  $\bowtie_{\text{Sells.bar} = \text{Bars.name}}$  Bars

BarInfo(

bar,	beer,	price,	name,	addr
Joe's	Bud	2.50	Joe's	Maple St.
Joe's	Miller	2.75	Joe's	Maple St.
Sue's	Bud	2.50	Sue's	River Rd.
Sue's	Coors	3.00	Sue's	River Rd.

)

# Natural Join

- A useful join variant (*natural* join) connects two relations by:
  - Equating attributes of the same name, and
  - Projecting out one copy of each pair of equated attributes.
- Denoted  $R3 := R1 \bowtie R2$ .

# Eg: Natural Join

Sells(

bar,	beer,	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

)

Bars(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells ⋈ Bars

**Note:** Bars.name has become Bars.bar to make the natural join “work.”

BarInfo(

bar,	beer,	price,	addr
Joe's	Bud	2.50	Maple St.
Joe's	Milller	2.75	Maple St.
Sue's	Bud	2.50	River Rd.
Sue's	Coors	3.00	River Rd.

)

# Inner Join

The table E (for EMPLOYEE)

enr	ename	dept
1	Bill	A
2	Sarah	C
3	John	A

The table D (for DEPARTMENT)

dnr	dname
A	Marketing
B	Sales
C	Legal

SQL	Result					Relational algebra
select * from E, D where dept = dnr	enr	ename	dept	dnr	dname	SELECT <sub>dept = dnr</sub> (E X D)
	1	Bill	A	A	Marketing	<i>or, using the equivalent join operation</i>  E JOIN <sub>dept = dnr</sub> D
	2	Sarah	C	C	Legal	
	3	John	A	A	Marketing	



# Outer join

The table E (for EMPLOYEE)

enr	ename	dept
1	Bill	A
2	Sarah	C
3	John	A

The table D (for DEPARTMENT)

dnr	dname
A	Marketing
B	Sales
C	Legal

SQL	Result					Relational algebra
<pre>select * from (E right outer join D on edept = dnr)</pre>	enr	ename	dept	dnr	dname	$E \text{ RIGHT OUTER JOIN}_{\text{edept} = \text{dnr}} D$
	1	Bill	A	A	Marketing	
	2	Sarah	C	C	Legal	
	3	John	A	A	Marketing	
	null	null	null	B	Sales	

# Building Complex Expressions

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- Combine operators with parentheses and precedence rules.
- Three notations, just as in arithmetic:
  - Sequences of assignment statements.
  - Expressions with several operators.
  - Expression trees.

# Sequences of Assignments

- Create temporary relation names.
- Renaming can be implied by giving relations a list of attributes.
- **Example:**  $R3 := R1 \bowtie_c R2$  can be written:

$R4 := R1 \times R2$

$R3 := \sigma_c(R4)$

# Expressions in a Single Assignment

- **Example:** the theta-join  $R3 := R1 \bowtie_c R2$  can be written:

$$R3 := \sigma_c (R1 \times R2)$$

- Precedence of relational operators:
  1.  $[\sigma, \pi, \rho]$  (highest).
  2.  $[x, \bowtie]$ .
  3.  $\cap$ .
  4.  $[\cup, -]$

# Expression Trees

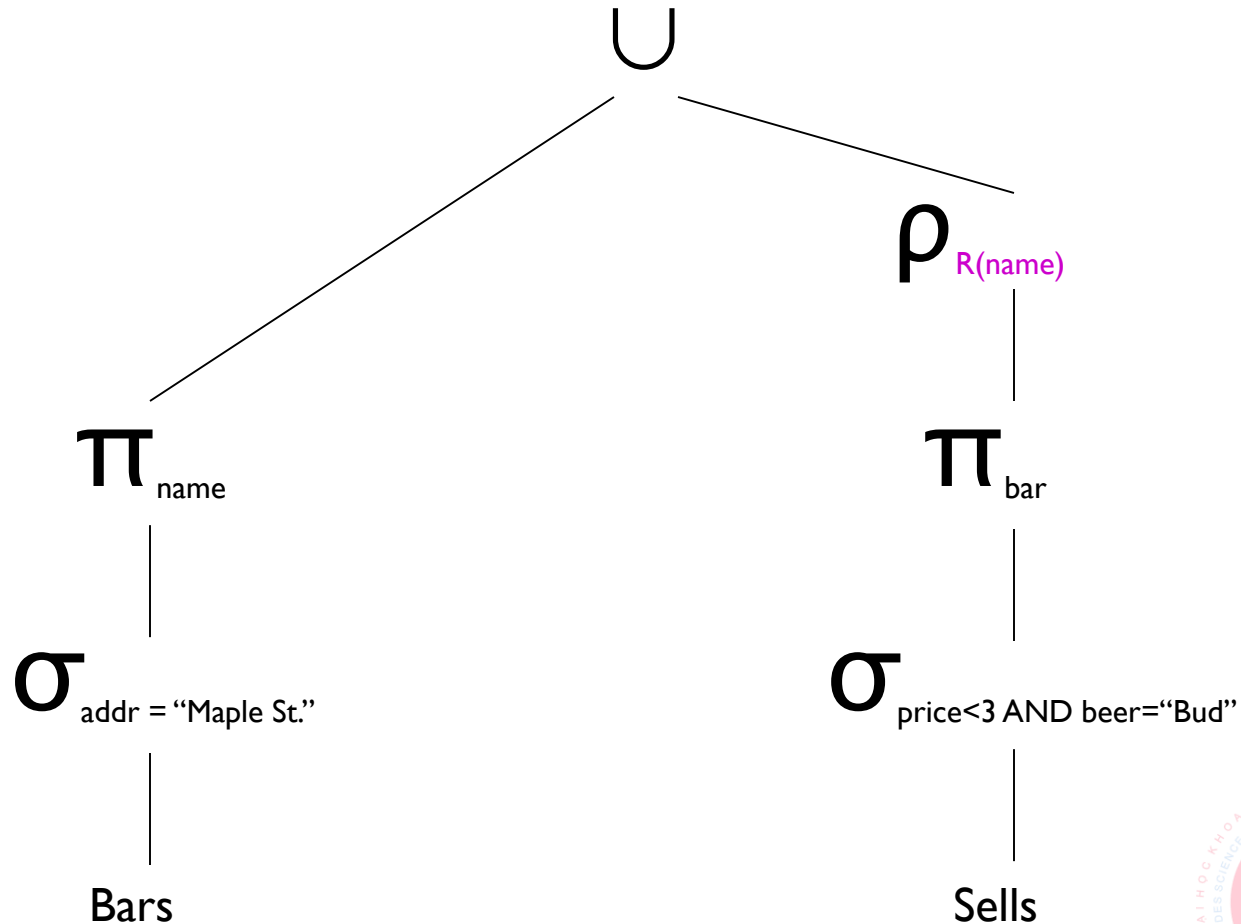
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- Leaves are operands --- either variables standing for relations or particular, constant relations.
- Interior nodes are operators, applied to their child or children.

# Example: Tree for a Query

- Using the relations **Bars(name, addr)** and **Sells(bar, beer, price)**, find the names of all the bars that are either on Maple St. or sell Bud for less than \$3.

# As a Tree:

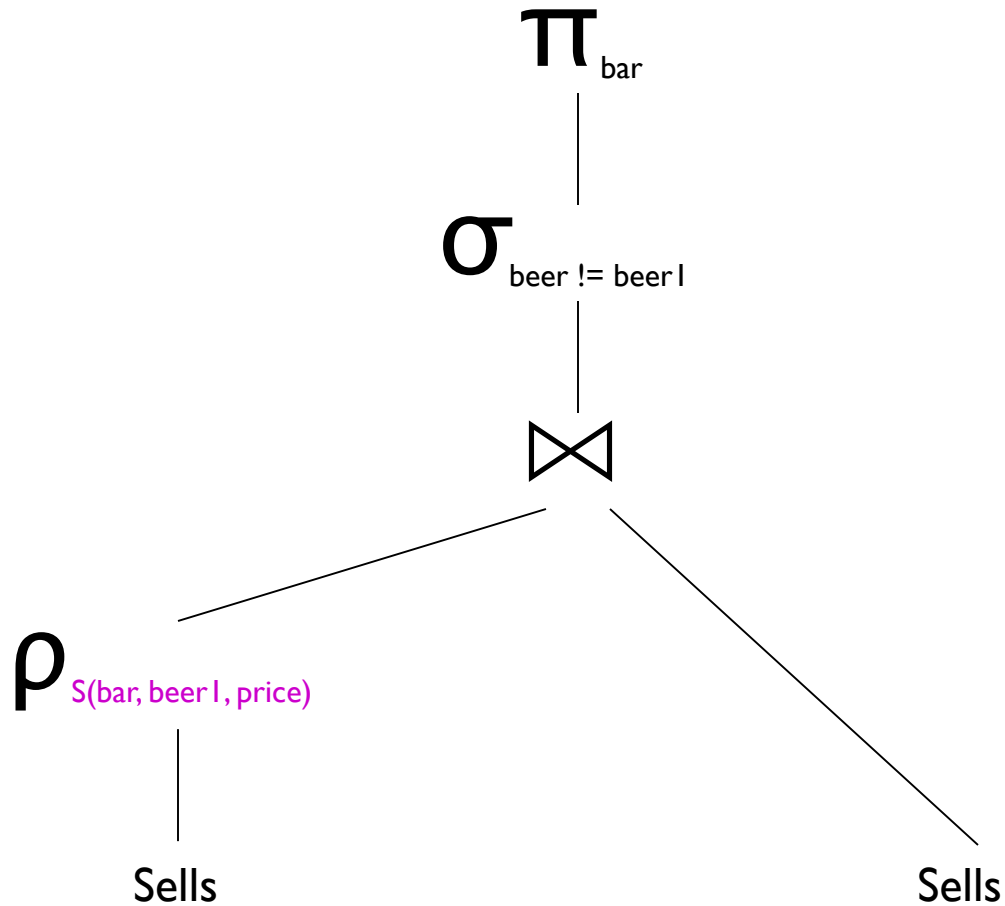


# Example: Self-Join

- Using  $\text{Sells}(\text{bar}, \text{beer}, \text{price})$ , find the bars that sell two different beers at the same price.
- **Strategy:** by renaming, define a copy of Sells, called  $\text{S}(\text{bar}, \text{beer I}, \text{price})$ . The natural join of Sells and S consists of quadruples  $(\text{bar}, \text{beer}, \text{beer I}, \text{price})$  such that the bar sells both beers at this price.



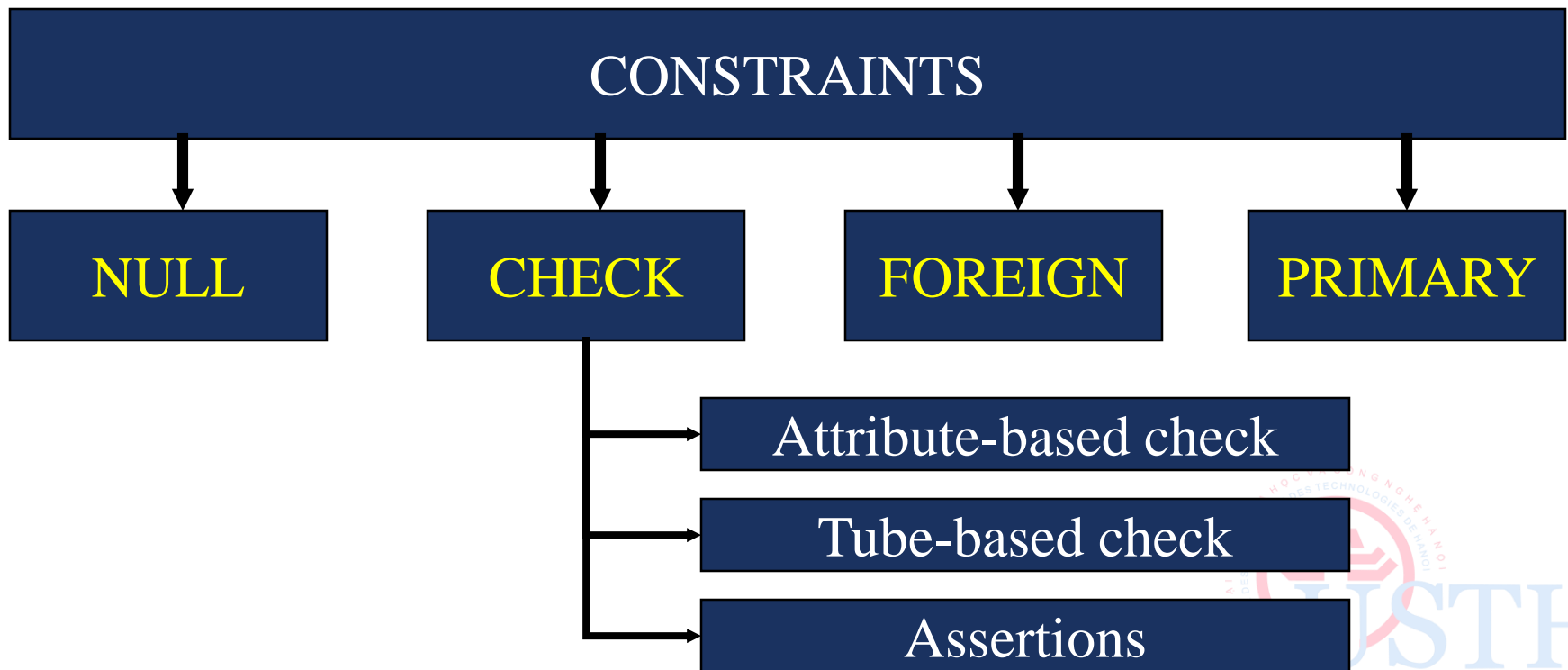
# The Tree



# CONSTRAINTS IN RELATIONAL MODEL

# Introduction

- A *constraint* is a relationship among data elements that the DBMS is required to enforce.
- → mechanism that may be used to limit the values entered into a relation.



# Key of a relation

- A set attributes  $\{A_1, A_2, \dots, A_n\}$  is called a **key** of the relation R if:
  - 1. Those attributes determine all other attributes.  
→ It is impossible for 2 tuples of R to agree on all of  $\{A_1, A_2, \dots, A_n\}$
  - 2. No subset of  $\{A_1, A_2, \dots, A_n\}$  determines all other attributes
  - → A Key must be minimal
- Example
  - AccademicResults (StudentID, SubjectID, grade, comment)

# Different key types of a relation

- Super key
  - Set of attributes (columns) to uniquely identify rows
- Key or candidate key
  - Minimal super key
- Primary key
  - One selected from candidate keys
- Alternate key
  - Candidate key other than PK
- Foreign key
  - Attribute refers to a PK of another relation

# Express Primary Key Constraints by RA

- Suppose we have a schema  $R(A_1, A_2, A_3)$
- So, we have  $A_1$  is the primary key of  $R$  if:

$$\delta_{R1.A1=R2.A1 \text{ AND } R1.A2 \neq R2.A2 \text{ AND } R1.A3 \neq R2.A3} (R \times R) = \Phi$$

# Example: Express Primary Key Constraints

Order ID	Customer
10248	Vins et alcools Chevalier
10249	Toms Spezialitäten
10250	Hanari Carnes
10251	Victuailles en stock
10252	Suprêmes délices
10253	Hanari Carnes
10254	Chop-suey Chinese
10255	Richter Supermarkt
10256	Wellington Importadora
10257	HILARIÓN-Abastos
10258	Ernst Handel
10259	Centro comercial Moctezuma
10260	Ottilies Käseladen
10261	Que Delícia
10262	Rattlesnake Canyon Grocery
10263	Ernst Handel

■  $\delta_{R1.OrderId = R2.OrderId \text{ AND } R1.Customer \neq R2.Customer} (R \times R) = \Phi$

# Express Foreign Key Constraints

- Suppose we have 2 relations R and S.  
R contains an attribute A  
S contains an attribute B
- “R references to S via A and B” if:  $\Pi_A(R) - \Pi_B(S) = \Phi$



# Example: Express Foreign Key Constraints

Sells( bar, beer, price )

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

Bars( bar, addr )

bar	addr
Joe's	Maple St.
Sue's	River Rd.

$$\diamond \Pi_{\text{Bar}}(\text{Sells}) - \Pi_{\text{Bar}}(\text{Bars}) = \Phi$$

# Express Check Constraints

- Give a schema:  
EMPLOYEE(Empld, Name, Sex)
- If we want to specify that the only legal values for SEX attribute are 'F' and 'M', we can write:

$\text{SELECT}_{\text{sex} \neq 'F' \text{ and } \text{sex} \neq 'M'} (\text{EMPLOYEE}) = \emptyset$

# Exercises I

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- Give a schema:  
PC (Model, Speed, RAM, HDD, Price)
- Use Relational Algebra to express following constraints:

A PC with a processor speed less than 3.00 must not sell for more than \$800

## Exercises 2

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- Give a schema:  
LAPTOP (Model, speed, RAM, HDD, Screen, Price)
- Use Relational Algebra to express following constraints:

A laptop with a screen size less than 15.4 inches must have at least a 120GB hard disk or sell for less than \$1000

# Exercises 3

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- PC (Model, Speed, RAM, HDD, Price)
- LAPTOP (Model, speed, RAM, HDD, Screen, Price)
- Use Relational Algebra to express following constraints:

If a laptop has a larger RAM than a PC, then the laptop must also have a higher price than the PC

# Exercises 4

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- PC (Maker, Model)
- PRINTER (Maker, Model)
- Use Relational Algebra to express following constraints:

No manufacturer of PC's may also make printers

# Exercises 5

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- PRODUCT (Maker, model)
- PC (Model)
- PRINTER (Model)
- Use Relational Algebra to express following constraints:

No manufacturer of PC's may also make printers

# Exercises 6

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- PC (Model, Speed, RAM, HDD, Price)
- Use Relational Algebra to express following constraints:
  - Higher model, higher price
  - With the same model, higher Speed and RAM and HDD, higher price