```
In [1]:
#help function
def set_spines():
    ax = plt.gca()  # gca stands for 'get current axis'
    ax.spines['right'].set_color('none')
    ax.spines['top'].set_color('none')
    ax.xaxis.set_ticks_position('bottom')
    ax.spines['bottom'].set_position(('data',0))
    ax.yaxis.set_ticks_position('left')
    ax.spines['left'].set_position(('data',0))
```

```
In [40]:
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
import math
from mpl_toolkits.mplot3d import Axes3D
from random import random
#from mpl_toolkits.mplot3d import axes3d
import matplotlib.pyplot as plt
import pandas as pd
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.linear_model import LinearRegression
```

Linear Regression

In statistic, LA is an linear approach to modeling the realationship between one or more explanatory variables (independent or dependent).The case of one explanatory variable is called Simple Linear Regression for more then one is called Multiple Linear Regression.In Linear Regression the relationships are modeled using linear predictors function whose unknown parameters are estimated from data.

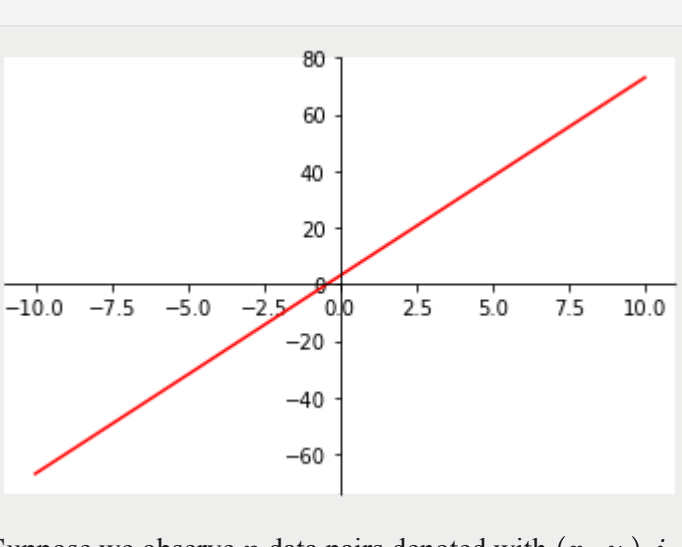
Simple linear regression

Simple Linear regression has a single explanatory variable.It concerns two-dimensional sample points with one dependent and one independent variable. (Conventionally x and y data point in Cartesian coordinate system).The adjective simple refers to the fact that the outcome variable is related to a single predictor.

Fitting the regression line with least-squared

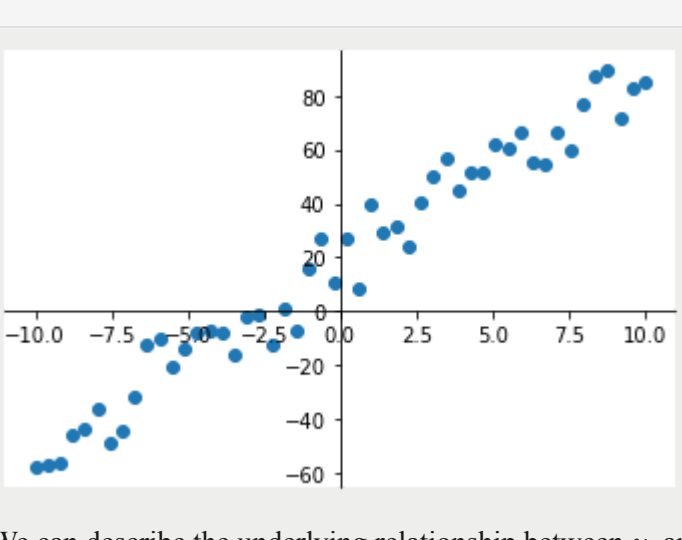
Consider the model function $y = \alpha + \beta x$ which describes a line with slope β and intercept α.
example : let $\alpha = 7$ and $\beta = 3$, geometrically $y = 7 + 3x$ appears to be as below graphic.

```
In [3]:
x = np.linspace(-10,10)
y = 7*x + 3
#plt.scatter(x,y)
plt.plot(x,y,color='r')
set_spines()
```



Suppose we observe n data pairs denoted with $(x_i, y_i), i = 1, 2, \ldots, n$.

```
In [4]:
f = lambda x : 7*x + 3 + np.random.uniform(0,30)
y =[f(i) for i in x]
plt.scatter(x,y)
set_spines()
```



We can describe the underlying relationship between y_i and x_i invloving the error term ε_i in linear equation $y_i = \alpha + \beta x_i + \varepsilon_i$.The ε_i could be intepreted as the noise of the data or in some sense some type of randomness originated from various reasons.Also it is know as 'error term' or 'residual'. Our target is to find estimated values α and β which would provide the best fit in some sense for data points. In order to find the 'best' fit we will use the least-squared approach.From mathematical view point the issue involved the minimization of Lost function respect to α, β.

$$1) L = \sum_i (y_i - \alpha - \beta x_i)^2$$

. L is called *Lost* function the expression $(y_i - \alpha - \beta x_i)^2$ is called *Lost* function.This function is appropiative because of It is a diferencianal everywhere and gives us a measure of diference between dependent value y_i and predicted value of $y'(\alpha, \beta) = \alpha + \beta x_i$.
In order to find $min_{\alpha,\beta} L(\alpha, \beta, x_{i,y} i)$ we will use Gradient Descent method.

Gradient Descent over simple linear regression

The methods for finding the maximum and minumum (extrema) have been created with developing of mathematical analysis(calculus).Gradient Descent is numerical approach to local minumum of first-oreder differentiable function.The idea is to take repeaded steps on the oposite direction of the gradient the function in current point.

Our pupose is to find the value of x where the function gets the minumum value. The GD algorithm is difined by :

$$\alpha = \alpha - \nabla_\alpha L(\alpha, \beta) * h$$
$$\beta = \beta - \nabla_\beta L(\alpha, \beta) * h$$

where h is called Learning rate or step size. Applying GD algorithm over our lost function 1) we will get :

$$\nabla_\alpha (J(a,b) = \frac{\partial L}{\partial \alpha} = \frac{\partial \sum_i (y_i - \alpha - \beta x_i)^2}{\partial \alpha} = -\frac{2}{n} \sum_{n=1}^{n} (y_i - a * x_i - b) * x_i$$

$$\nabla_\beta (J(a,b) = \frac{\partial L}{\partial \beta} = \frac{\partial \sum_i (y_i - \alpha - \beta x_i)^2}{\partial \beta} = -\frac{2}{n} \sum_{n=1}^{n} (y_i - a * x_i - b)$$

applying above to gradient descent forumula related to simple linear regression we achieved :

$$\alpha = \alpha + \frac{2}{n} \sum_{n=1}^{n} (y_i - a * x_i - b) * x_i * h$$

$$\beta = \beta + \frac{2}{n} \sum_{n=1}^{n} (y_i - a * x_i - b) * h$$

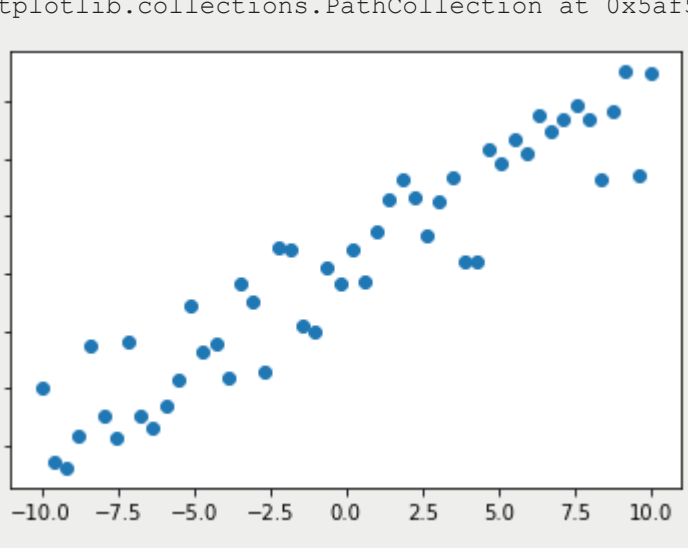
```
In [ ]:
```

```
In [5]:
#implementation cost function
def lost(X,Y,alpha,betta):
    '''
    Calculation of cost (error) function of simple linear regression

    Parrameters :
    X(array or number) : X arguments, independent variable
    Y (array or number) : Y arguments, actual dependent variable
    Returns :
      number of cost function
  '''

    return np.sum((Y - (alpha - betta*X ))**2 )
```

```
In [6]:
def gradient_descent(X,Y,C=0.1,E=100):
    '''

    Calculation of cost (error) function of simple linear regression

    Parrameters :
    X(array or number) : X arguments, independent variable
    Y (array or number) : Y arguments, actual dependent variable
    C (number) :  learning rate
    I(int) : number of iteration(Epoch)
    Returns :
      number of cost function
   '''

    a = 30
    b = 10
    a_args = []
    b_args = []
    n = X.size

    for i in range(E):
        a = a + 2/n*(np.sum((Y - a*X - b)*X))*C
        b =  b + 2/n*(np.sum((Y - a*X -b)))*C
        a_args.append(a)
        b_args.append(b)
    return a,b,a_args,b_args
```

```
In [7]:
x = np.linspace(-10,10)
f = lambda x : 7*x + 3 + np.random.uniform(-30,30)
y =[f(i) for i in x]
plt.scatter(x,y)
```

```
Out[7]: <matplotlib.collections.PathCollection at 0x5af59aaa60>
```

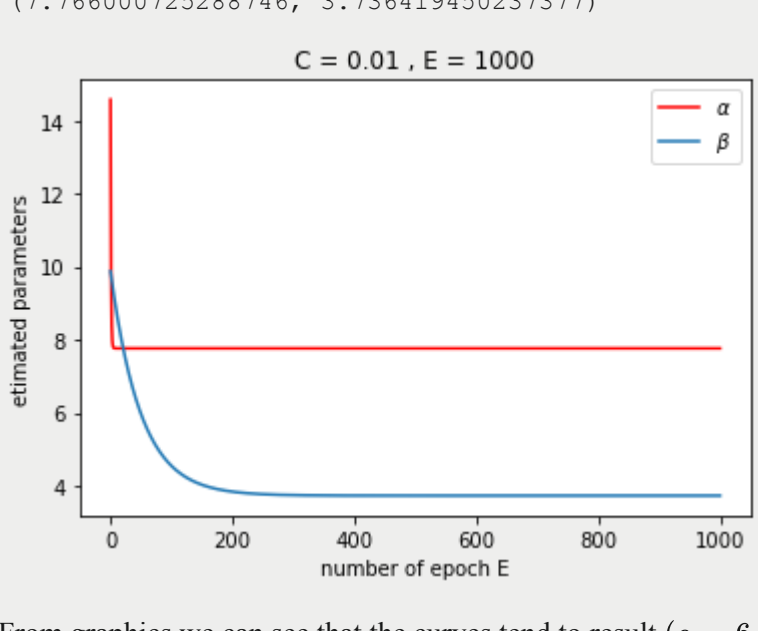


Let to apply gradient descent of above points.

```
In [8]:
a,b,a_r,b_r = gradient_descent(x,y,C=0.01,E=1000)
```

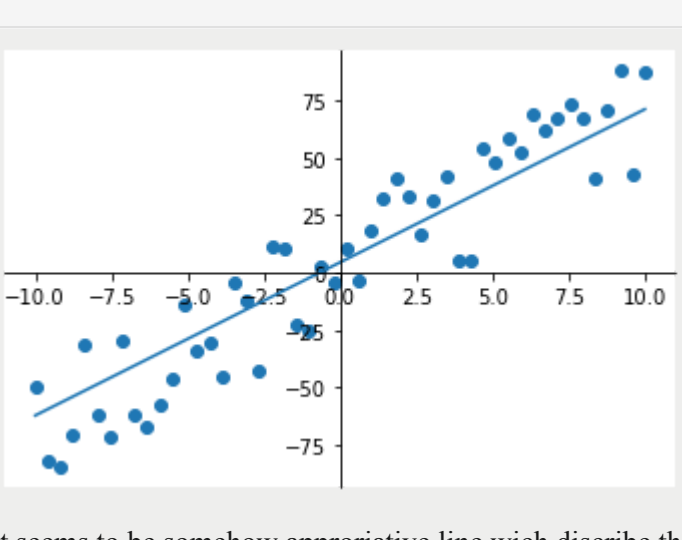
```
In [9]:
plt.plot(a_r,color='r',label=r'$\alpha $')
plt.plot(b_r,label=r'$\beta$')
plt.xlabel("number of epoch E")
plt.ylabel("etimated parameters")
plt.title('C = 0.01 , E = 1000')
plt.legend()
a,b
```

```
Out[9]: (7.766000725288746, 3.736419450237377)
```



From graphics we can see that the curves tend to result $(\alpha = 6.6603, \beta = 4.431)$.
The fit line will be :

```
In [10]:
#7.047696929660597, 1.081292922517757
plt.scatter(x,y)   #7.892385161777225, 5.6459662769234535
plt.plot(x,6.6603*x +  4.431)
set_spines()
```



It seems to be somehow approriative line wich discribe the data well, but our origin values of $$ are $(\alpha = 7, \beta = 1)$ not $(\alpha = 6.6603, \beta = 4.431)$.In below we will examine how will chaged the (α, β) resprect to different Learnig rate C and nuber of epoch E(itarations).

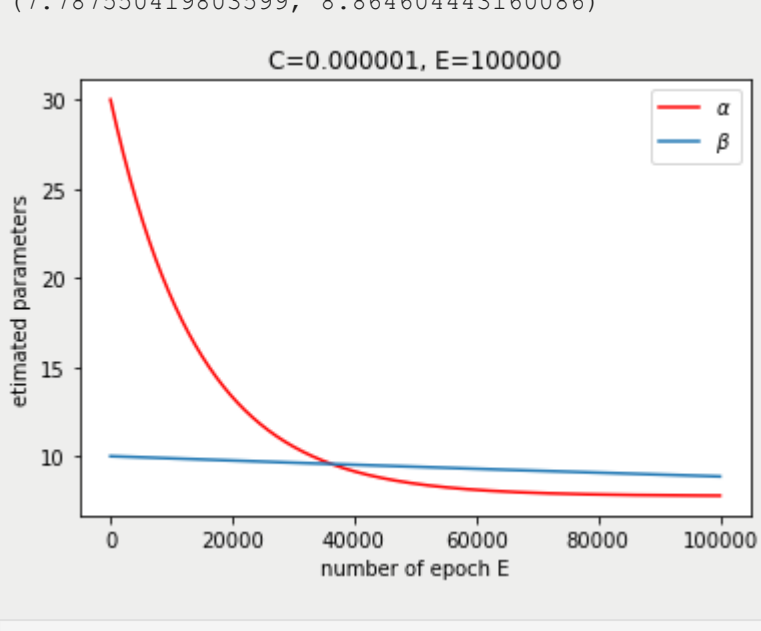
1.3 Effect of different values for learning rate

The Learning rate C is the tunning parameter in an optimization algorithm that determines the step size at each iteration while moving toward a minimum of cost function.While the gradient direction is usually determined from the gradient of the cost function, the learning rate determines how big a step is taken in that direction.The too hight learning rate will make the learning jump over minima but too low leaning rate will either take too long to converge or get stuck in an undesirable local minimum.In order to achieve faster convergence, prevent oscillations and getting stuck in undesirable local minima the learning rate is often varied during training either in accordance to a learning rate schedule or by using an adaptive learning rate

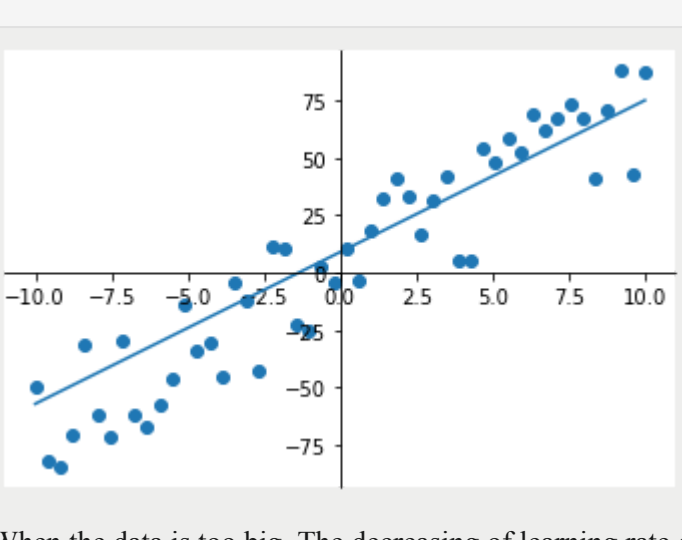
let's apply different learnning rate to our example: $C = 0.000001$

```
In [11]:
a,b,a_r,b_r = gradient_descent(x,y,C=0.000001,E=100000)
plt.plot(a_r,color='r',label=r'$\alpha $')
plt.plot(b_r,label=r'$\beta$')
plt.xlabel("number of epoch E")
plt.ylabel("etimated parameters")
plt.title('C=0.000001, E=100000')
plt.legend()
a,b
```

```
Out[11]: (7.787550419803599, 8.864604443160086)
```



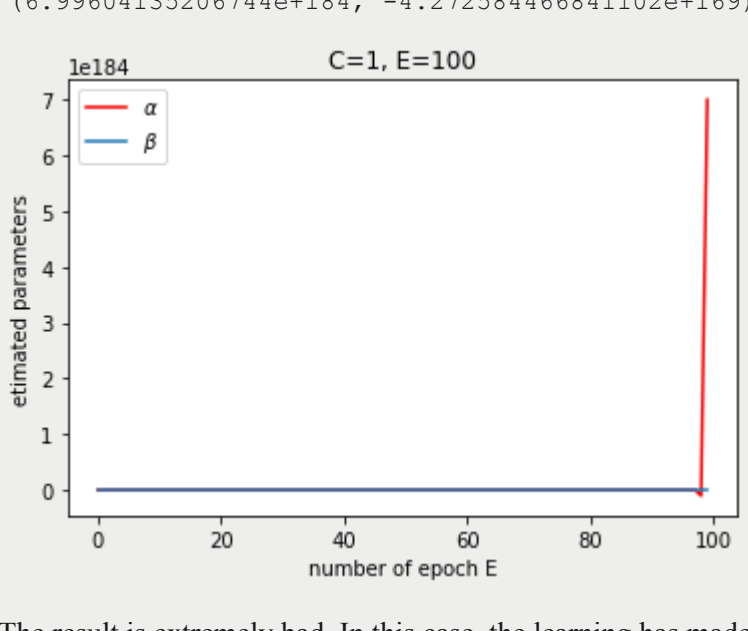

```
In [12]:
plt.scatter(x,y)
plt.plot(x,6.6*x + 8.9)
set_spines()
```



When the data is too big, The decreasing of learning rate can lead to enormous time execution,but in above the result is satisfying too.

```
In [13]:
a,b,a_r,b_r = gradient_descent(x,y,C=1,E=100)
plt.plot(a_r,color='r',label=r'$\alpha $')
plt.plot(b_r,label=r'$\beta$')
plt.xlabel("number of epoch E")
plt.ylabel("etimated parameters")
plt.title('C=1, E=100')
plt.legend()
a,b
```

```
Out[13]: (6.99604135206744e+184, -4.272584466841102e+169)
```



The result is extremely bad. In this case, the learning has made a learning jump, therefore the result is run away from minima.

2. Multiple Linear Regression

Multiple Linear Regression also known as multiple regression, is a statistical technique that uses several explanatory variable to predict the outcome of response variable.In essence, MLR is an extension of ordinary least-squared regression.The equation that we are searching for is defined by :

$$\hat{y}^i = h(x^i) = \vartheta_0 + \vartheta_1 x_1^i + \vartheta_2 x_2^i + \ldots \vartheta_p x_p^i$$

where , i is number of obeservation, y_i dependet(target) value, x_p feature values, ϑ_0 intercept, ϑ_p slope coeff. for each explanority var. ,

The above hypotesis can also be represented by

$$\hat{Y} = X\Theta^T$$

where $\Theta = \begin{bmatrix} \vartheta_0 \\ \vdots \\ \vartheta_p \end{bmatrix}$ $X = \begin{bmatrix} 1 & x_1^1 & x_1^2 & x_1^n \\ \vdots & \ddots & \vdots & \vdots \\ 1 & x_p^1 & \ldots & x_p^n \end{bmatrix}$ and $\hat{Y} = \begin{bmatrix} \hat{y}^0 \\ \vdots \\ \hat{y}^p \end{bmatrix}$

we've append column $\hat{Y} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ to X in order to be used in matrix multiplication directly.

To define and measure the error of our model we define the cost function as the sum of the squares of the residuals. The cost function is denoted by :

$$3) \quad J(\Theta) = \frac{1}{2m} \sum_{i=0}^{m} (h(x^i) - y^i)^2$$

We have to initialize the model parameter with some random values(random initialization).To use Gradient Descent we need to measure how the cost function changes with change in it's parametes.Therefore we compute the partial derivatives of cost finction
4) $\quad J(\vartheta_0, \vartheta_1, \ldots, \vartheta_n)$

$$\frac{\partial J(\Theta)}{\partial \theta_j} = \frac{1}{m} \sum_{i=0}^{m} (h(x^i) - y^i) x_j^i$$

In more compatable form using matrix in order to be implemnted using NUMPY

Using the Eistein notaion we can rewrite eq. 4)

$$\frac{\partial J(\vartheta_k)}{\partial \theta_j} = \frac{1}{m} (\theta_p x_p^i - y^i) x_j^i$$

in matrix form

$$5) \quad \vec{\nabla} J(\Theta) = ((X\Theta^T - Y)^T . X)^T$$

applying the rule $(AB)^T$ the eq can be reformed

$$6) \quad \vec{\nabla} J(\Theta) = X^T (X\Theta^T - Y)$$

$$where \quad \vec{\nabla} = \frac{\partial}{\partial \theta_j \vec{e}_j}$$

eq. 6 is extremely comfortable because it can be implemented very simple in NUMPY which is many times faster than common Python

The Gradient Descent alg finaly will looks like that :

$$\theta = \theta - h \vec{\nabla} J(\Theta)$$

h is learnig rate

Calcaulation of accuracy of Multilinear regression by R^2

We will be using Root mean squared error(RMSE) of Determination(R^2 score) to evaluate our model.
RMSE is a square root of average of sum of suares of residualas. RMSE is difined by :

$$RMSE = \sqrt{\frac{1}{2} \sum_{i=1}^{m} (h(x^i) - y^i)}$$

R² score or the coefficient of determination explains how much the total variance of the dependent variable can be reduced by using the least square regression.
R^2 is determed by

$$R^2 = 1 - \frac{SS_r}{SS_t}$$

SS_t is the total sum of errors if we take the mean of the observed values as the predicted value.

$$SS_t = \sum_{i=1}^{m} (y^i - \bar{y})^2$$

$$SS_r = \sum_{i=1}^{m} (h(x^i) - y^i)^2$$

Implementation of gradient descent for Multiple Linear regression in NUMPY

```
In [19]:
class MultipleLinearRegression:
    """
    Multiple Linear regression

    Parameters
    ..........

    C : float
        learning rate
    I : int
        number of iterations (Epochs)
    Attributes
    ..........
     w_ : weights (Predictors)

    """

    def __init__(self, C=0.001, I=50):
        self.C = C
        self.I = I
        self.w_ = None

    def fit(self, X, Y):
        """Fit tthe training data

        Parameters
        ..........

        X : array, shape = [N_samples,n_features]
            Training samples
        Y  : array, shape = [n_samples, n_target_values]

        Returns
        .......
        self : object

        """

        X, Y = self._validate(X, Y)
        self.w_ = np.ones((X.shape[1], 1))  # inicialized predictors
        M = X.shape[0]  # numbers of samples
        for _ in range(self.I):
            self.w_ =self.w_ -  self.C * (1 / M) * X.T.dot((X.dot(self.w_) - Y))
            #self.w -= self.C * (1 / M) * X.T.dot((X.dot(self.w) - Y))


        return self

    def predict(self, x):
        """ Predicts the value after the model has been trained.
        Parameters
        ----------
        x : array-like, shape = [n_samples, n_features]
            Test samples
        Returns
        -------
        Predicted value
        """
        x = np.append(np.ones((x.shape[0], 1)), x, axis=1)
        return np.dot(x, self.w_)

    def score(self,X, Y):
        """Calculation of accuracy using (R^2 score)
             x : array-like, shape = [n_samples, n_features]
             y : array-like, shape = [n_samples, n_features]
            Test samples
        Returns
        float : score
        -------
        Predicted value

        """
        #X = np.append(np.ones((X.shape[0], 1)), X, axis=1)
        #Y = np.array(Y).reshape(X.shape[0], 1)
        X,Y = self._validate(X,Y)
        y_prime = np.dot(X, self.w_)
        ssr = np.sum((y_prime - Y) ** 2)
        sst = np.sum((Y - np.mean(y_prime)) ** 2)
        r2_score = 1 - (ssr / sst)
        return r2_score

    def _validate(self,X,y):
        '''Added tow with one number to X data
        reshape Y data


        :param X: array-like, shape = [n_samples, n_features]
        :param y: array-like, shape = [n_samples, n_target_values]
        :return: validated X,Y
        '''
        return   np.append(np.ones((X.shape[0], 1)), X, axis=1),\
                 np.array(y).reshape(y.shape[0], 1)
```

We will test our implementation over insurence.csv data set

```
In [21]:
df = pd.read_csv("../../../resources/data/insurance.csv")
df.head()
```

Out[21]:

	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

Converting Categories to Numbers. The linear regression can be performed only on numbers, so we should convert these categorical features into numbers. To do that, we can make use of a function called get_dummies. So let’s convert the “sex,” “smoker,” and “region” columns into numerically represented features.

```
In [22]:
cols = ['sex', 'smoker', 'region']
new_df = pd.get_dummies(df, cols, drop_first= True)
new_df.head()
```

Out[22]:

	age	bmi	children	charges	sex_male	smoker_yes	region_northwest	region_southeast	region_southwest
0	19	27.900	0	16884.92400	0	1	0	0	1
1	18	33.770	1	1725.55230	1	0	0	1	0
2	28	33.000	3	4449.46200	1	0	0	1	0
3	33	22.705	0	21984.47061	1	0	1	0	0
4	32	28.880	0	3866.85520	1	0	1	0	0

Now, let’s only select the features that are the most relevant. Feature selection is one of the important tasks in any machine learning project. You must know which features are most correlated with the targets (the “charges” column in our case) and must only use those features that have a high correlation with your target. This can be done through experimentation. For example, in this problem, I tried using the “sex” and “region” features to predict “charges” but didn’t find much of an improvement in the prediction performance of the model. So I decided to omit these features from the model. Through small experimentation like that, I found the “age,” “bmi,” and “smoker” columns to be most relevant when predicting insurance costs (the “charges” column in our data frame).

```
In [24]:
X = new_df[['age', 'bmi', 'smoker_yes']]
y = new_df['charges']
```

We will perfomr standarlization over X

```
In [28]:
scaler = StandardScaler()
```

```
In [29]:
X = scaler.fit_transform(X)
```

Splliting data to Train and Test

```
In [30]:
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.2)
```

End let to train the data using our implementation

```
In [32]:
model =  MultipleLinearRegression(C=0.01,I=1000)
z = model.fit(X_train,y_train)
```

```
In [ ]:
```

```
In [37]:
model.score(X_test,y_test
```

```
Out[37]: 0.7323173758003395
```

We've achived according to $R2$ score 73% accuracy

Let see how will be in standart LinearRegresion in sklearn in Python

```
In [41]:
lr = LinearRegression(fit_intercept=True)
lr.fit(X_train,y_train)
```

```
Out[41]: LinearRegression()
```

```
In [42]:
lr.score(X_test,y_test)
```

```
Out[42]: 0.7322470193784116
```

The score using imprlemntation in sklearn in Python is the same as our implementation , Let to compare the predicted valuea

```
In [92]:
our_predicted_data = model.predict(X_test)[:,0]
python_predicted_data = lr.predict(X_test)
pd.DataFrame(our_predicted_data ,python_predicted_data  )
```

Out[92]:

	0
26963.233582	26962.539696
32466.037854	32465.293221
16386.515466	16386.117349
10313.491299	10312.927540
4554.159257	4553.882338
...	...
6066.659315	6066.487660
1301.918644	1301.573708
9178.571257	9178.181162
6629.357965	6629.146516
28596.887404	28596.167084

268 rows × 1 columns

The diferences is too small,but we can perform $R^2 score$ over the both Python and Our predicted data

```
In [93]:
ssr = np.sum((our_predicted_data - python_predicted_data) ** 2)
sst = np.sum((our_predicted_data - np.mean(python_predicted_data)) ** 2)
r2_score = 1 - (ssr / sst)
r2_score
```

```
Out[93]: 0.9999999976969305
```

0.9999999976969305 shows that the difference is negligible