Path-based depth-first search for strong and biconnected components

Author of the paper: Harold N. Gabow

Reported by: T.T. Liu D.P. Xu B.Y. Chen

May 27, 2017

Outline

Characterastics of Gabow's Algorithms

- One-pass algorithm. But for the algorithm of strong components, what we have learned from the textbook is a two-pass algorithm, by which we must traverse the whole graph twice.

Several Questions

- LOWPOINT?
- Ear decomposition?
- Compele version?
- Robbin's Theorem?

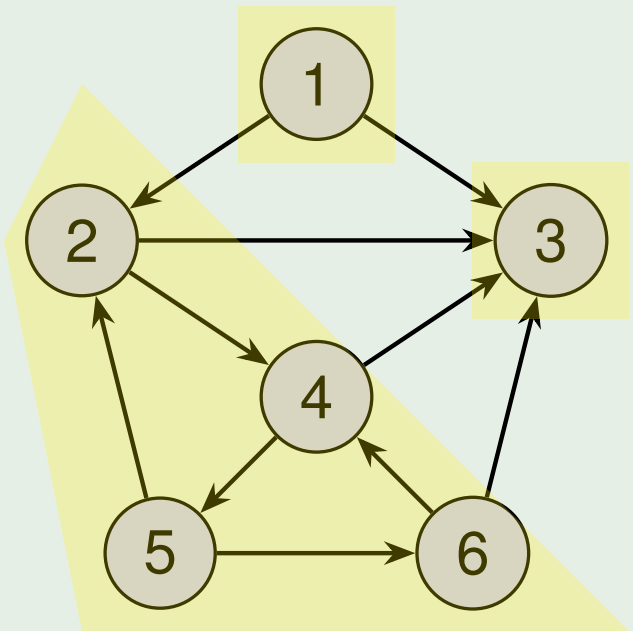
Outline

Review: What have we learned from the textbook?

Concepts of Strong Components

- Two mutually reachable vertices are in the same *strong component*.

Example



Review: What have we learned from the textbook?

Algorithms to Find Strong Components

- Idea: Run DFS twice. Once on the original graph G, once on the *tranposition* of G^T.
- Trick: Using *finishing times* of each vertex computed by the first DFS.
- Linear time complexity: $O(V + E)$

Outline

Pseudo-Code

```
1  H = G;
2  while H still has a vertex v
3     start a new path P = (v);
4     while P is not empty
5        if the last vertex v_k of P has an edge (v_k, w)
6           if w belongs to P
7              contract the cycle v_i(w), ... , v_k, both
                   in H and in P; /* w and v_i are
                   identical. */
8           else
9              add w to P, as the new last vertex of P;
10          end if
```

- Note that *contracting* means selecting one vertex as a representation and merging others rather than deleting them.

Pseudo-Code (Continued)

```
11          else
12              output v_k as a vertex of the strong component
                    graph;
13              delete v_k from both H and P;
14          end if
15      end
16  end
```

Assessment

- The time consumption of each statement in the pseudo-code is clear. Total time complexity is linear. except the statement in line 7:

```
7  contract the cycle v_i(w), ... , v_k, both in H and in
       P; /* w and v_i are identical. */
```

- Problem is how to merge in linear time while keeping the next time accessing this vertex still in constant time.
- Therefore, a good data structure for disjoint set merging is needed.

Outline

His Contribution

- He gave a simple list-based implementation that achieves linear time.
- Use only stacks and arrays as data structure.

New algorithm to discover strong components

Procedure 1: STRONG(G)

empty stacks S and B;
for $v \in V$ **do**
  $I[v] = 0$;
$c = n$;
for $v \in V$ **do**
  if $I[v] = 0$ **then**
    DFS(v);

New algorithm to discover strong components

Procedure 2: DFS(v)

```
PUSH(v,S);  I[v]=TOP(S);  PUSH(I[v],B);
/* add v to the end of P                                          */
for egdes(v, w) ∈ E do
    if I[v] = 0 then
        DFS(w);
    else /* contract if necessary                                 */
        while I[w] < B[TOP[B]] do
            POP(B);
    if I[v] = B[TOP(B)] then
        /* number vertices of the next strong component           */
        POP(B);
        increase c by 1;
        while I[v] ≤ TOP[S] do
            I[POP(S)]=c;
```

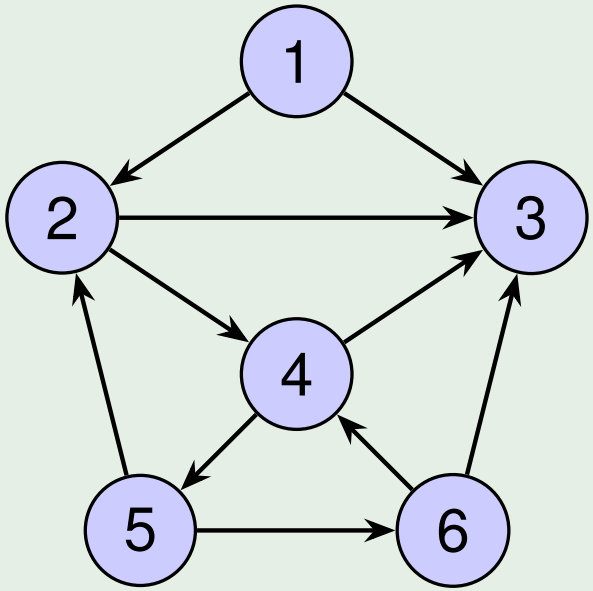
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
		1	0
		2	0
		3	0
		4	0
		5	0
		6	0

Graph H



- Call Stack: STRONG()
- This state is the first after initialized. DFS(1) is going to be called.

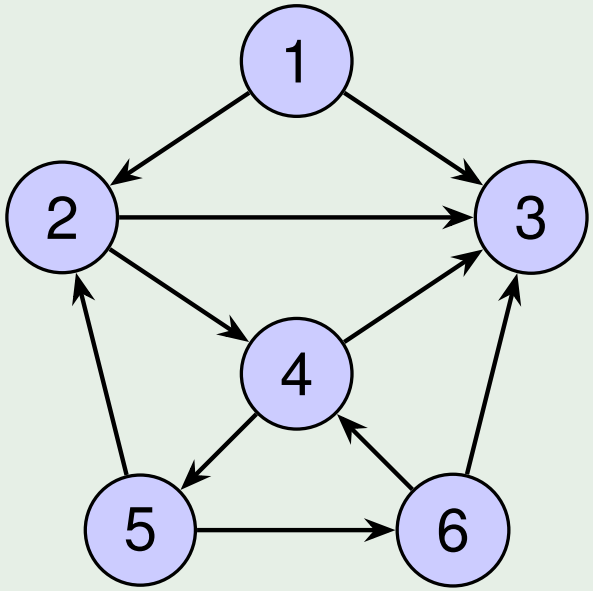
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
		2	0
		3	0
		4	0
		5	0
		6	0

Graph H



- Call Stack: STRONG()→DFS(1)
- Code: `for edges(v,w)∈E do ...`
- $w = 2$.

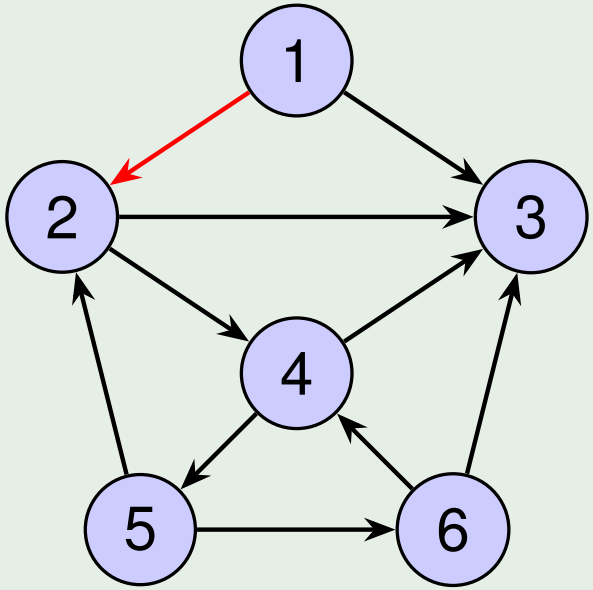
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
2	2	2	2
		3	0
		4	0
		5	0
		6	0

Graph H



- Call Stack: STRONG()→DFS(1)→DFS(2)
- Code: `for edges(v,w)∈E do ...`
- $w = 3$.

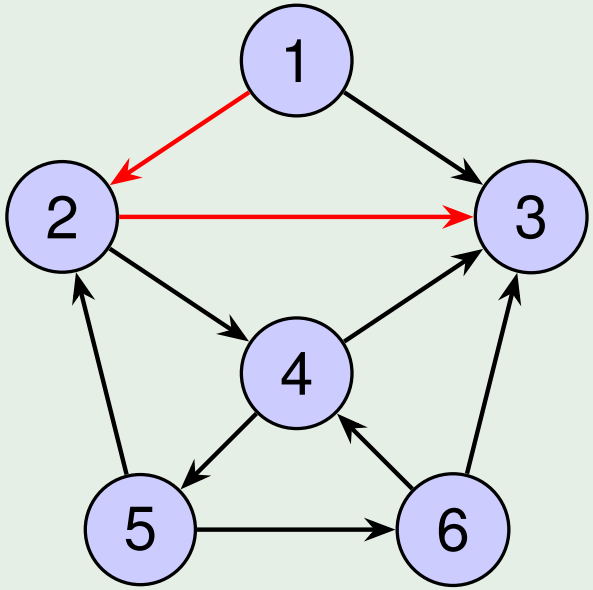
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
2	2	2	2
3	3	3	3
		4	0
		5	0
		6	0

Graph H



- Call Stack: STRONG()→DFS(1)→DFS(2)→DFS(3)
- Code: `if I[v]=B[TOP(B)] then ...`
- Go back.

Demo: Gabow's strong components algorithm

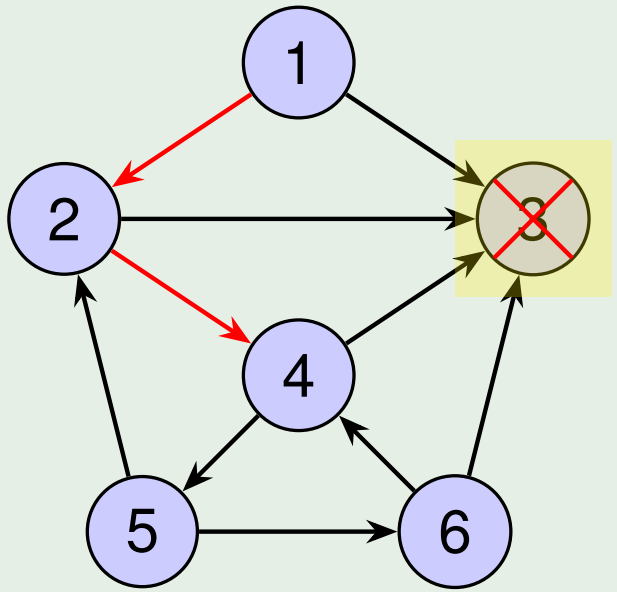
Data in memory

B and S: stack. I: array.

B	S
1	1
2	2
3	4

	I
1	1
2	2
3	7
4	3
5	0
6	0

Graph H



- Call Stack: STRONG()→DFS(1)→DFS(2)→DFS(4)
- Code: `for edges(v,w)∈E do ...`
- $w = 5$.

Demo: Gabow's strong components algorithm

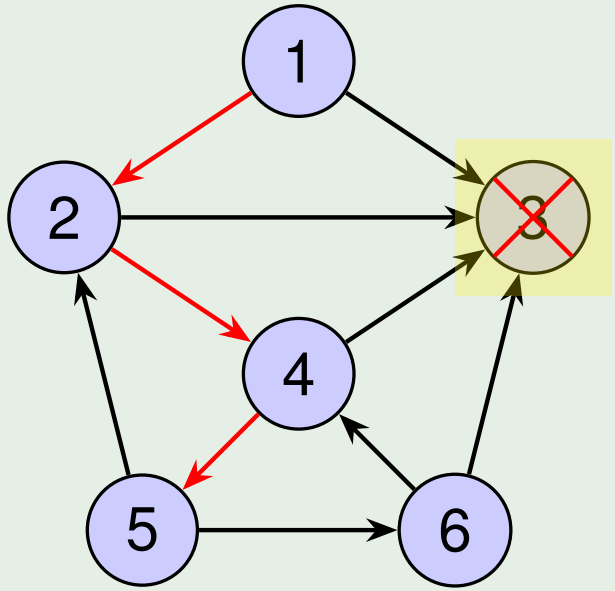
Data in memory

B and S: stack. I: array.

B	S
1	1
2	2
3	4
4	5

	I
1	1
2	2
3	7
4	3
5	4
6	0

Graph H



- Call Stack: $\cdots \rightarrow$DFS(1)$\rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)
- Code: `for edges(v,w)∈E do ...`
- $w = 2$.

Demo: Gabow's strong components algorithm

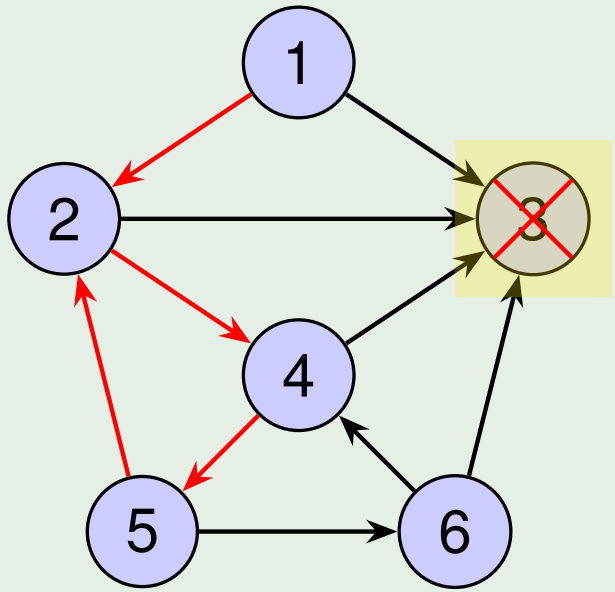
Data in memory

B and S: stack. I: array.

B	S
1	1
2	2
3	4
4	5

	I
1	1
2	2
3	7
4	3
5	4
6	0

Graph H



- Call Stack: $\cdots \rightarrow$DFS(1)$\rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)
- Code: `while I[w]<B[TOP(B)] do POP(B);`
- Now, $w = 2$, contract!

Demo: Gabow's strong components algorithm

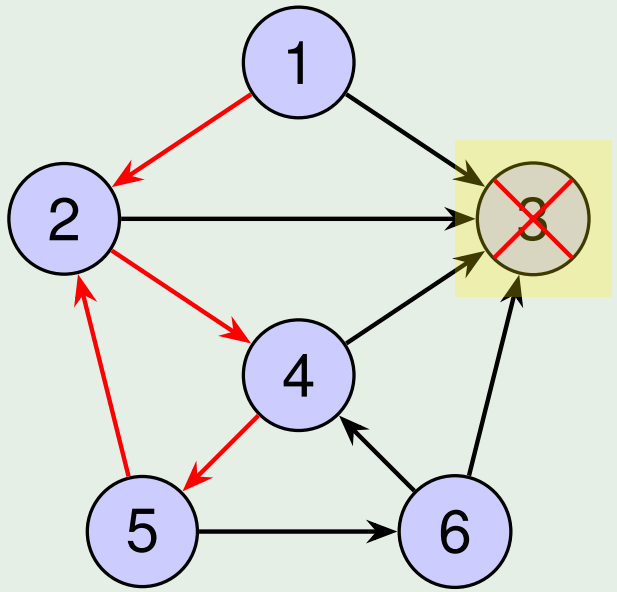
Data in memory

B and S: stack. I: array.

B	S
1	1
2	2
	4
	5

	I
1	1
2	2
3	7
4	3
5	4
6	0

Graph H



- Call Stack: $\cdots \rightarrow$DFS(1)$\rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)
- Code: `while I[w]<B[TOP(B)] do POP(B);`
- Now, $w = 2$, contract!

Demo: Gabow's strong components algorithm

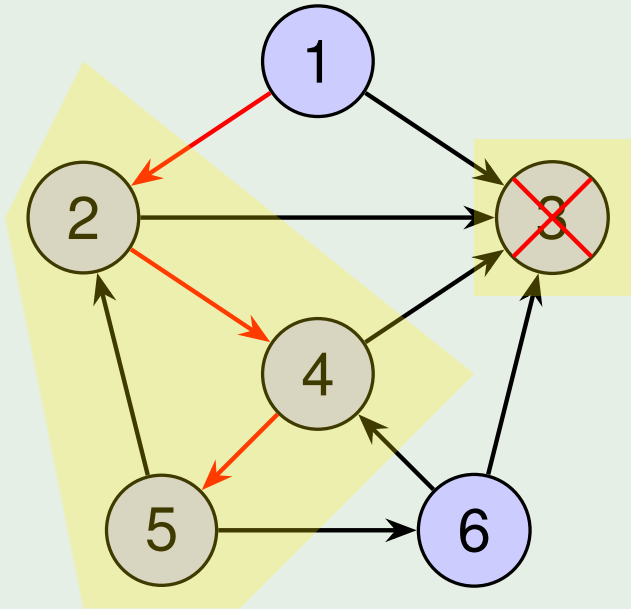
Data in memory

B and S: stack. I: array.

B	S
1	1
2	2
	4
	5

	I
1	1
2	2
3	7
4	3
5	4
6	0

Graph H



- Call Stack: $\cdots \rightarrow$DFS(1)$\rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)
- Code: `if I[w]=0 then DFS(w);`
- $w = 6$.

Demo: Gabow's strong components algorithm

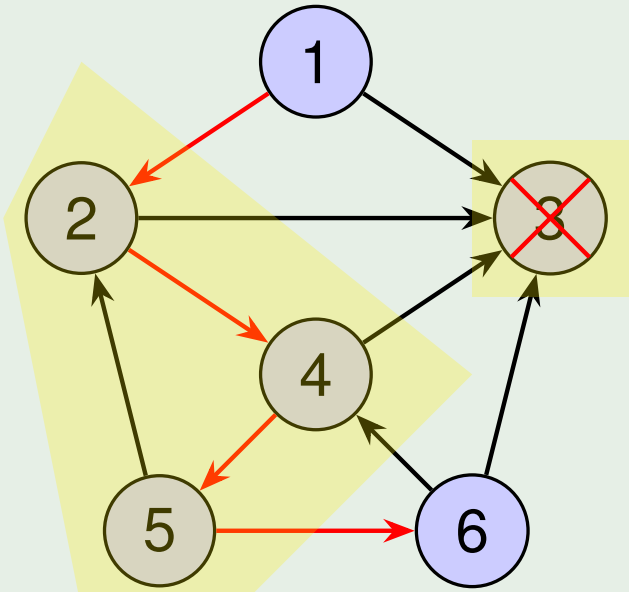
Data in memory

B and S: stack. I: array.

B	S
1	1
2	2
5	4
	5
	6

	I
1	1
2	2
3	7
4	3
5	4
6	5

Graph H



- Call Stack: $\cdots \rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)$\rightarrow$DFS(6)
- Code: `for edges(v,w)∈E do ...`
- $w = 4$.

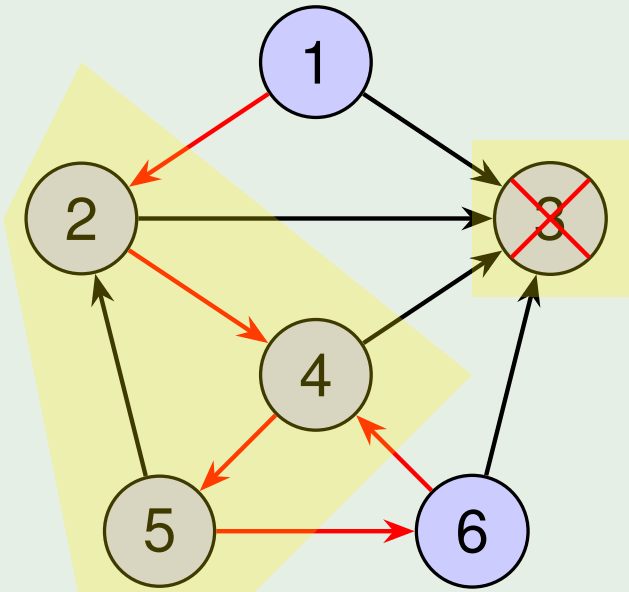
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
2	2	2	2
5	4	3	7
	5	4	3
	6	5	4
		6	5

Graph H



- Call Stack: $\cdots \rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)$\rightarrow$DFS(6)
- Code: `while I[w]<B[TOP(B)] do POP(B);`
- Now, $w = 4$, contract!

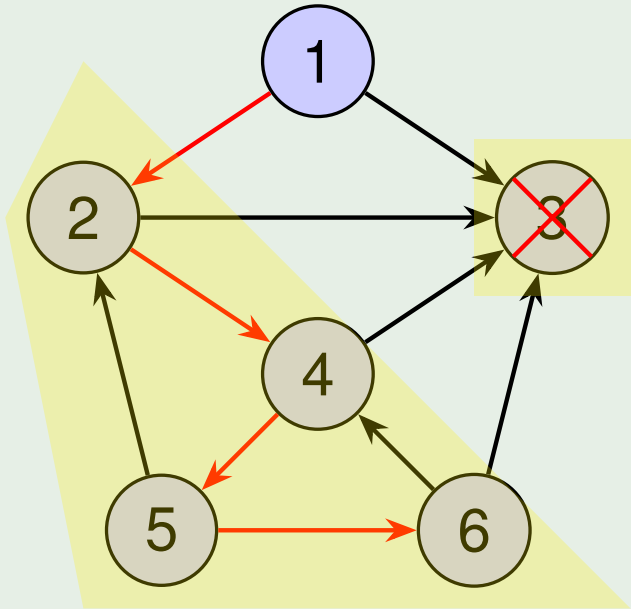
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
2	2	2	2
	4	3	7
	5	4	3
	6	5	4
		6	5

Graph H



- Call Stack: $\cdots \rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)$\rightarrow$DFS(6)
- Code: `if I[v]=B[TOP(B)] then ...`
- Go back.

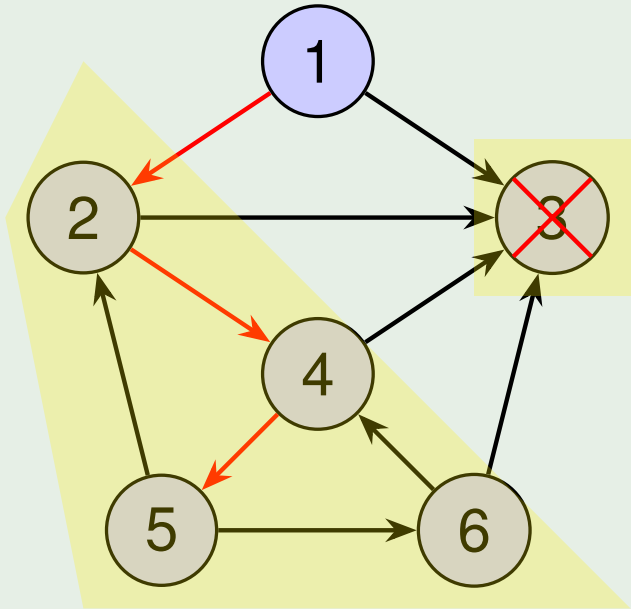
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
2	2	2	2
	4	3	7
	5	4	3
	6	5	4
		6	5

Graph H



- Call Stack: $\cdots \rightarrow$DFS(2)$\rightarrow$DFS(4)$\rightarrow$DFS(5)
- Code: `if I[v]=B[TOP(B)] then ...`
- Go back.

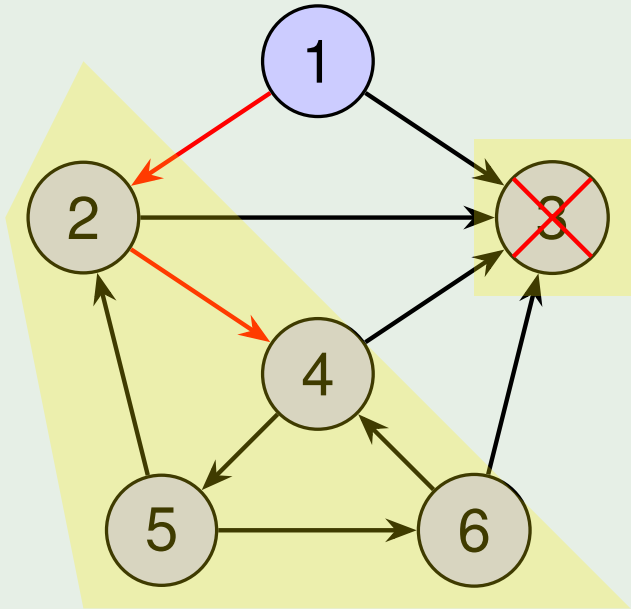
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
2	2	2	2
	4	3	7
	5	4	3
	6	5	4
		6	5

Graph H



- Call Stack: $\cdots \rightarrow$DFS(2)$\rightarrow$DFS(4)
- Code: `if I[v]=B[TOP(B)] then ...`
- Go back.

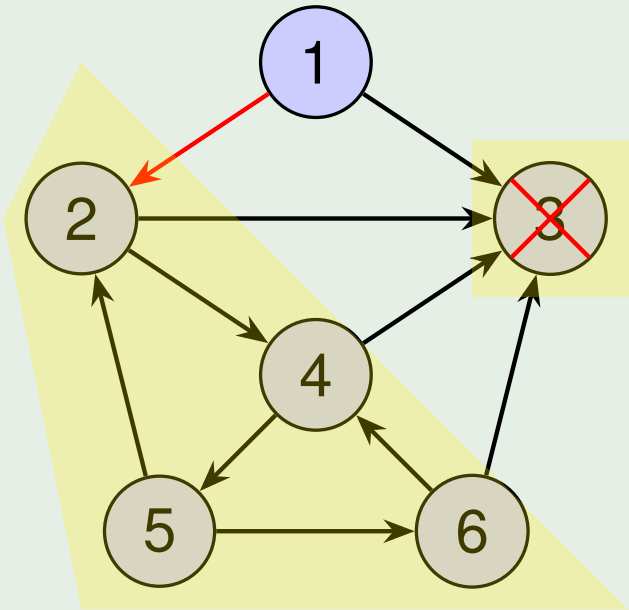
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
1	1	1	1
2	2	2	2
	4	3	7
	5	4	3
	6	5	4
		6	5

Graph H



- Call Stack: STRONG()→DFS(1)→DFS(2)
- Code: `if I[v]=B[TOP(B)] then ...`
- Go back. But this time, Condition in last line is satisfied!

Demo: Gabow's strong components algorithm

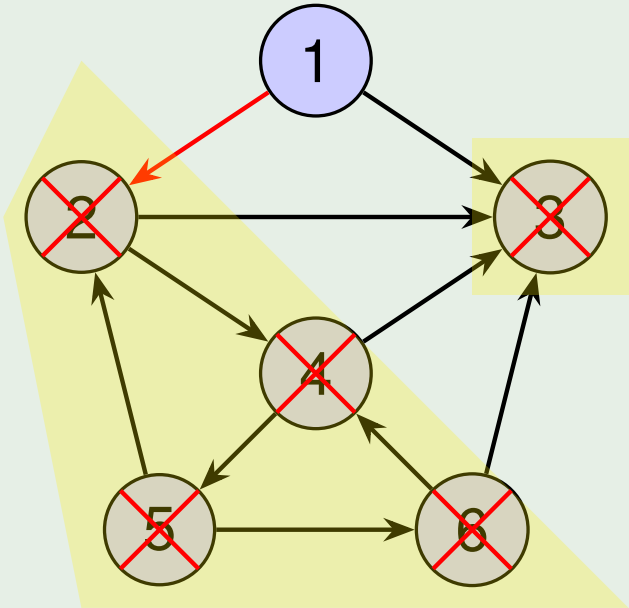
Data in memory

B and S: stack. I: array.

B	S
1	1

	I
1	1
2	8
3	7
4	8
5	8
6	8

Graph H



- Call Stack: STRONG()→DFS(1)→DFS(2)
- Code: `while I[v]≤TOP(S) do I[POP(S)]=c;`
- Pop 2 from B, while 2, 4, 5, 6 in S are also popped.

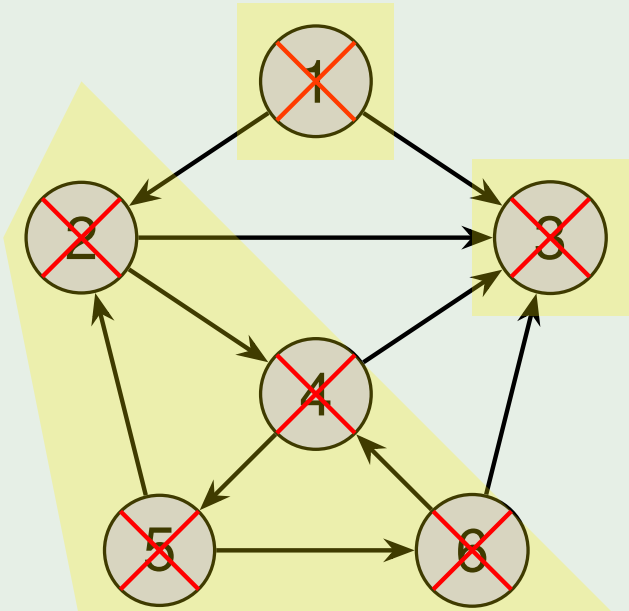
Demo: Gabow's strong components algorithm

Data in memory

B and S: stack. I: array.

B	S		I
		1	9
		2	8
		3	7
		4	8
		5	8
		6	8

Graph H



- Call Stack: STRONG()→DFS(1)
- Code: `while I[v]≤TOP(S) do I[POP(S)]=c;`
- Pop the last one both in B and in S. *Finished!!*

Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

For Further Reading I


A. Author.
Handbook of Everything.
Some Press, 1990.


S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.