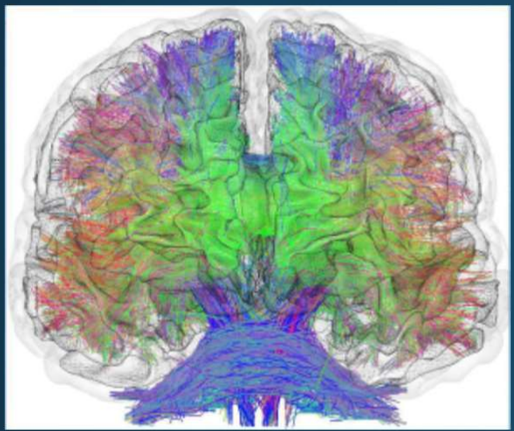
Optimal transport for comparison of short brain connectivity between individuals

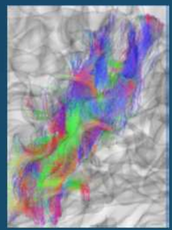
ESTABLISHMENT OF CONNECTIVITY MAPS



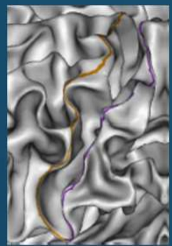
© L. Brun

Diffusion MRI

selection of streamlines

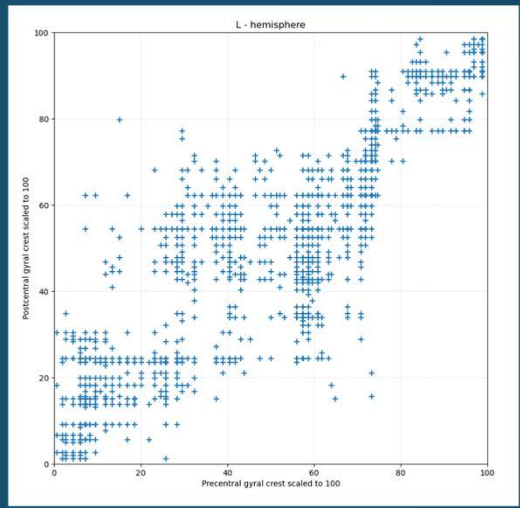


[1]

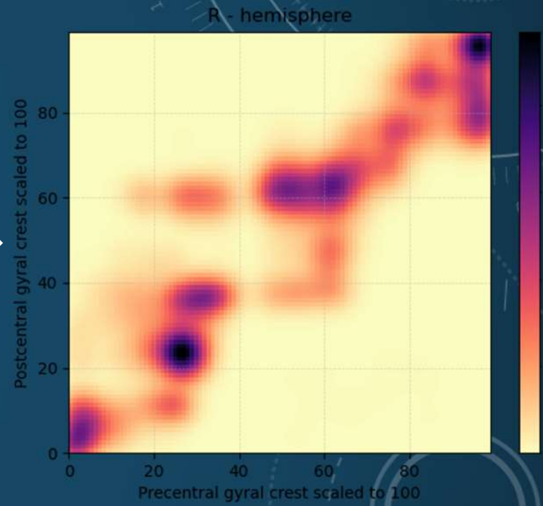


[1]

parameterisation



discrete connectivity mapping



continuous map

[1] *Etude de la connectivité structurelle des faisceaux d'association courts de la substance blanche du cerveau humain en IRM de diffusion* by Alexandre Pron

OPTIMAL TRANSPORT & 2-WASSERSTEIN DISTANCE

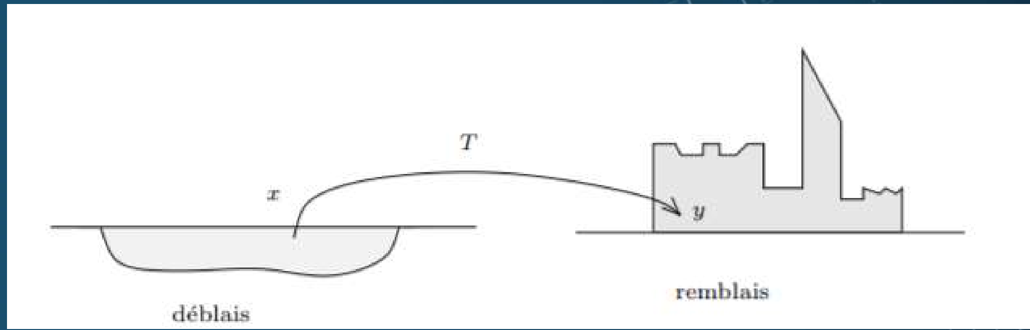
Optimal Transport matrix

$$T = \underset{\pi \in \Pi(\mu_s, \mu_t)}{argmin} < \pi(\mu_s, \mu_t), D_2^2(\mu_s, \mu_t) >$$

2-Wasserstein distance

$$W_2^2 = \min_{\pi \in \Pi(\mu_s, \mu_t)} \int_{X \times Y} ||x, y||_2^2 d\pi(x, y)$$
$$= \min_{\pi \in \Pi(\mu_s, \mu_t)} < \pi(\mu_s, \mu_t), D_2^2(\mu_s, \mu_t) >$$

such μ_s, μ_t two probability densities associated with the space X and Y



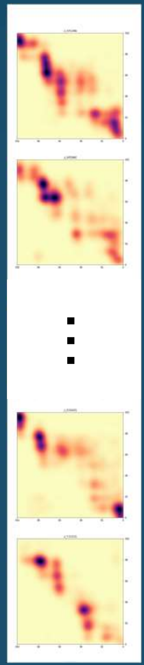
Monges problem of déblais and remblais [2]

[2] *Optimal transport : old and new* by Villani Cédric

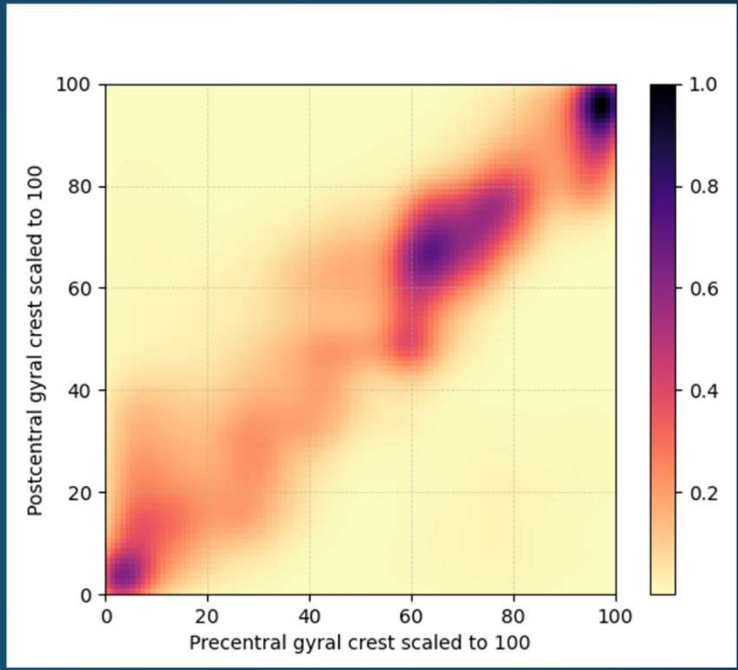
REPRESENTATIVE SUBJECT

Summation of subjects

Barycenter



Individual profile aligned



Group Profile

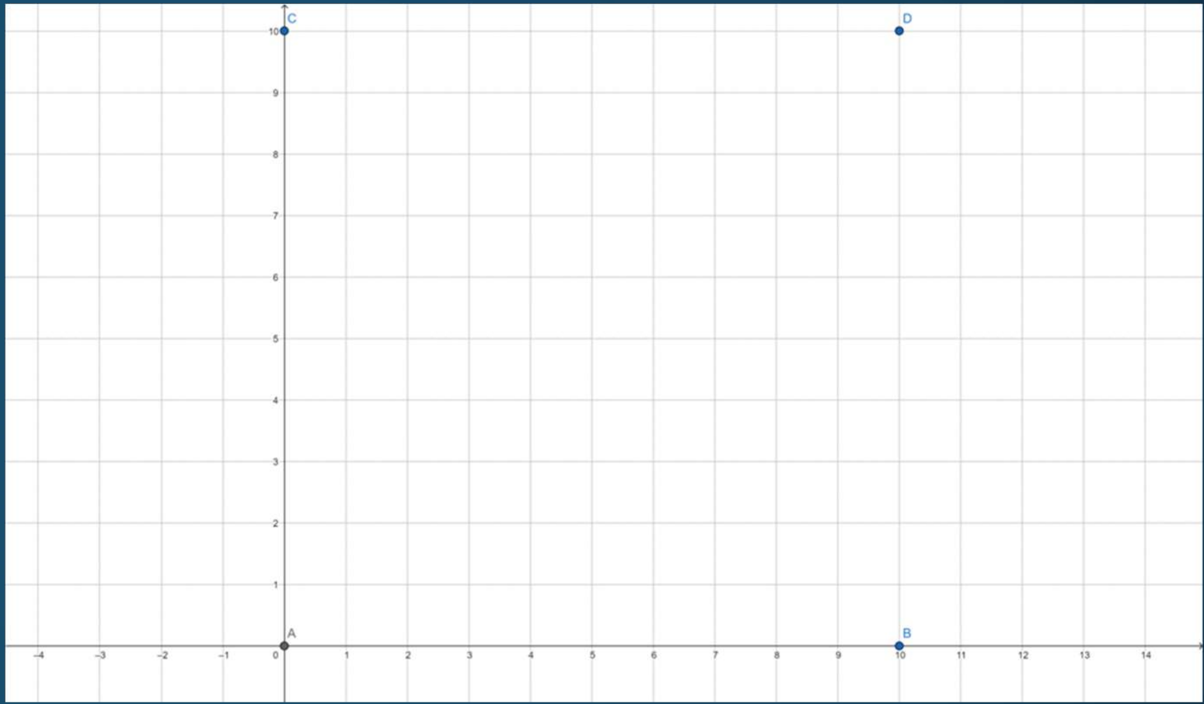
ITERATIVE BARYCENTER [3]

$$m = \underset{\mu \in (\mathbb{R}^2)^k}{argmin} \sum_{i=1}^{N} W_2^2(\mu, \nu_i) \text{ [4]}$$

[3] *Population Averaging of Neuroimaging Data Using Lp Distance-based Optimal Transport* by Q. Wang, I. Redko, and S. Takerkart

[4] *Fast Computation of Wasserstein Barycenters* by Marco Cuturi and Arnaud Doucet

Illustration of the iterative barycenter



ROBUSTNESS OF THE BARYCENTER

Initialization support

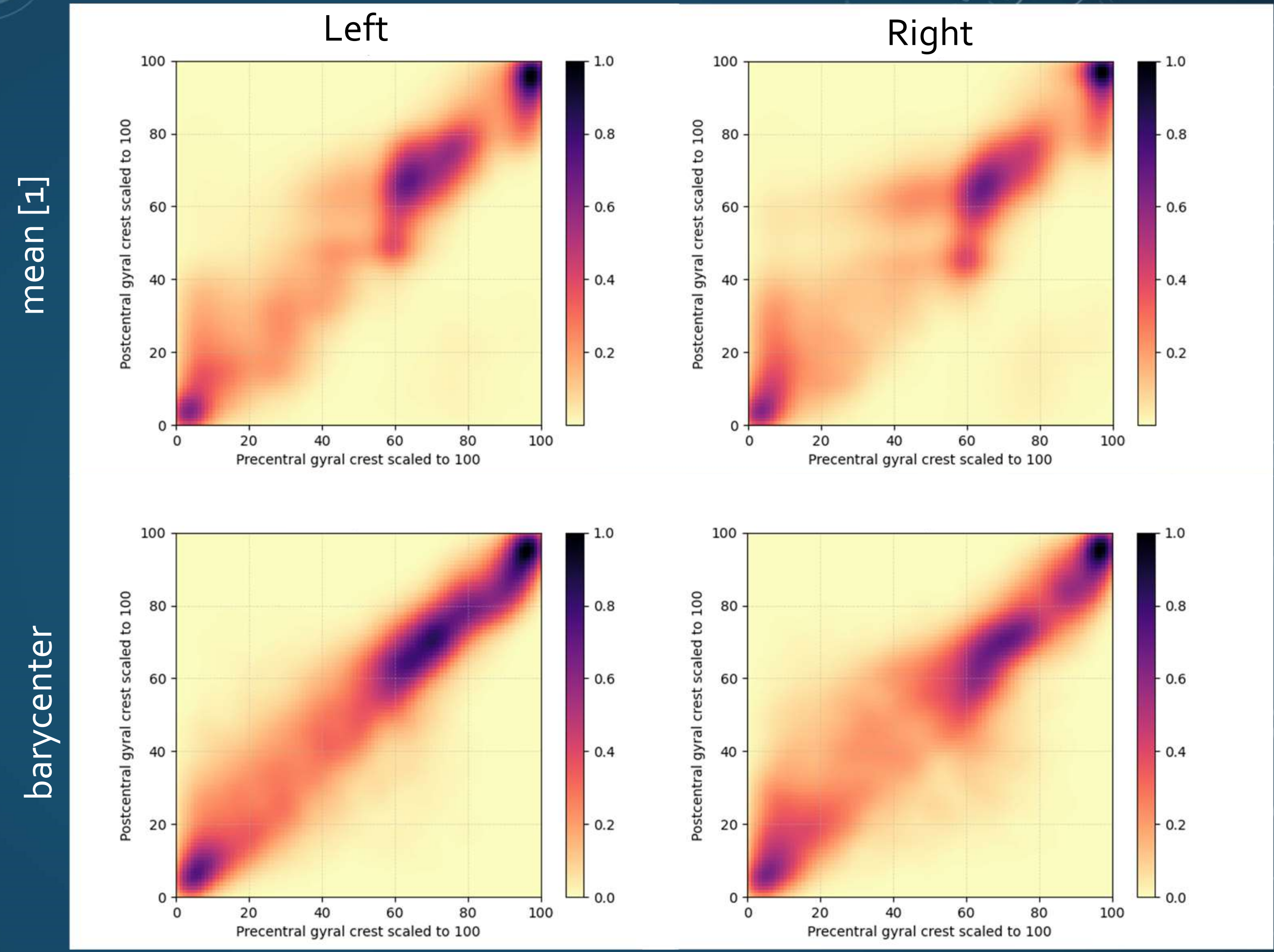
Name	Number of points	W_2 to all subjects	W_2 to all barycenters
Minimum	582	13.70	1.17
Random	768	13.69	1.10
Median	2040	13.67	0.94
Centroid	3190	13.67	0.92
Maximum	4029	13.67	0.9

Order of subjects

Experiment	W_2 to all subjects	W_2 to all barycenters
1	13.66	0.99
2	13.67	0.99
3	13.66	0.99
4	13.66	1.02
5	13.67	0.99

BARYCENTERS

[1] *Etude de la connectivité structurelle des faisceaux d'association courts de la substance blanche du cerveau humain en IRM de diffusion* by Alexandre Pron

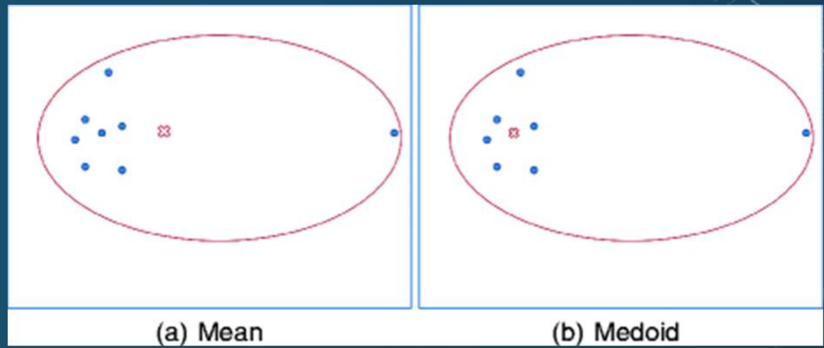


IS THERE A STRATIFICATION WITHIN OUR SUBJECTS?

K-MEDOIDS

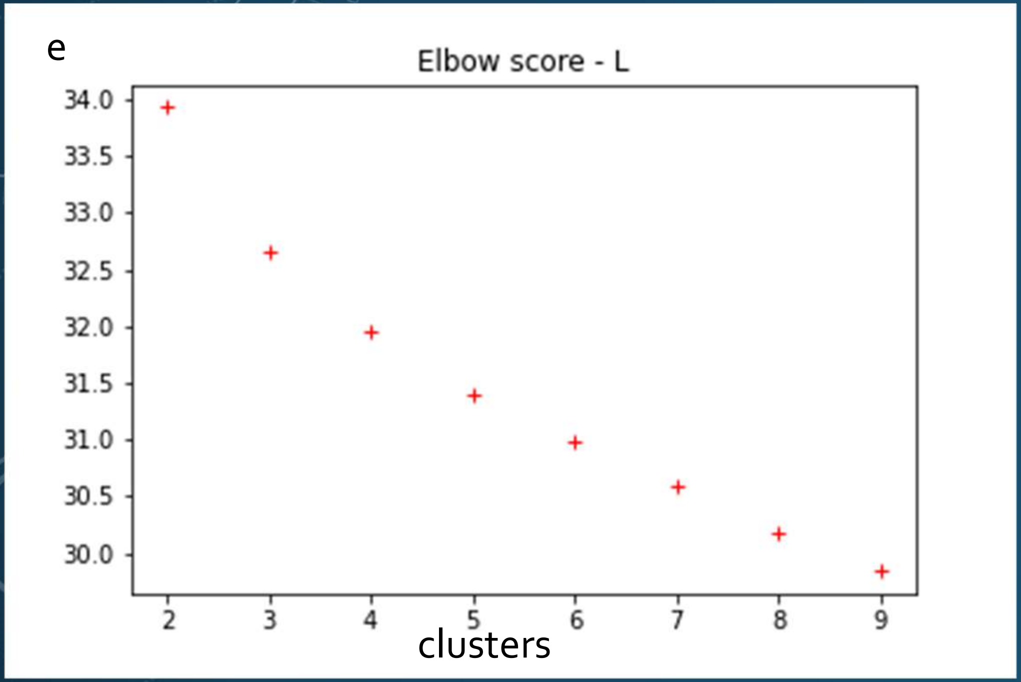
"K-Medoids Clustering" in
Encyclopedia of Machine Learning

(**DOI:** https://doi.org/10.1007/978-0-387-30164-8_426)



ELBOW SCORE

e: the mean intra-cluster distance

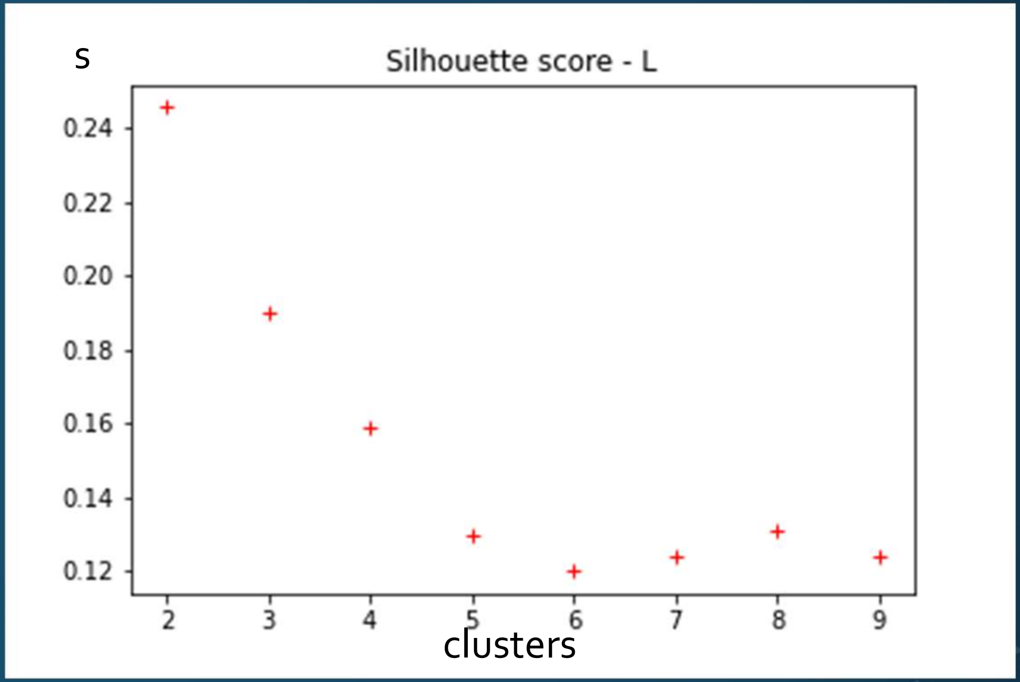


SILHOUETTE SCORE

$$s = \frac{a - e}{\max(a, e)}$$

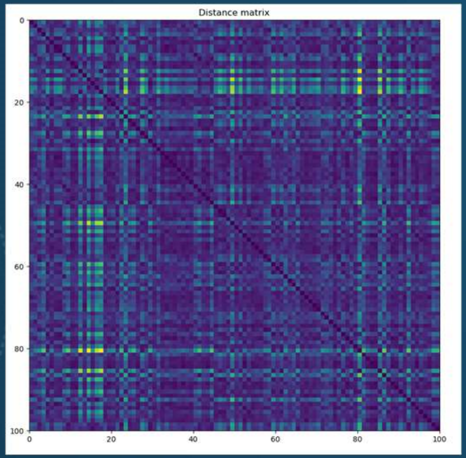
e: the mean intra-cluster distance

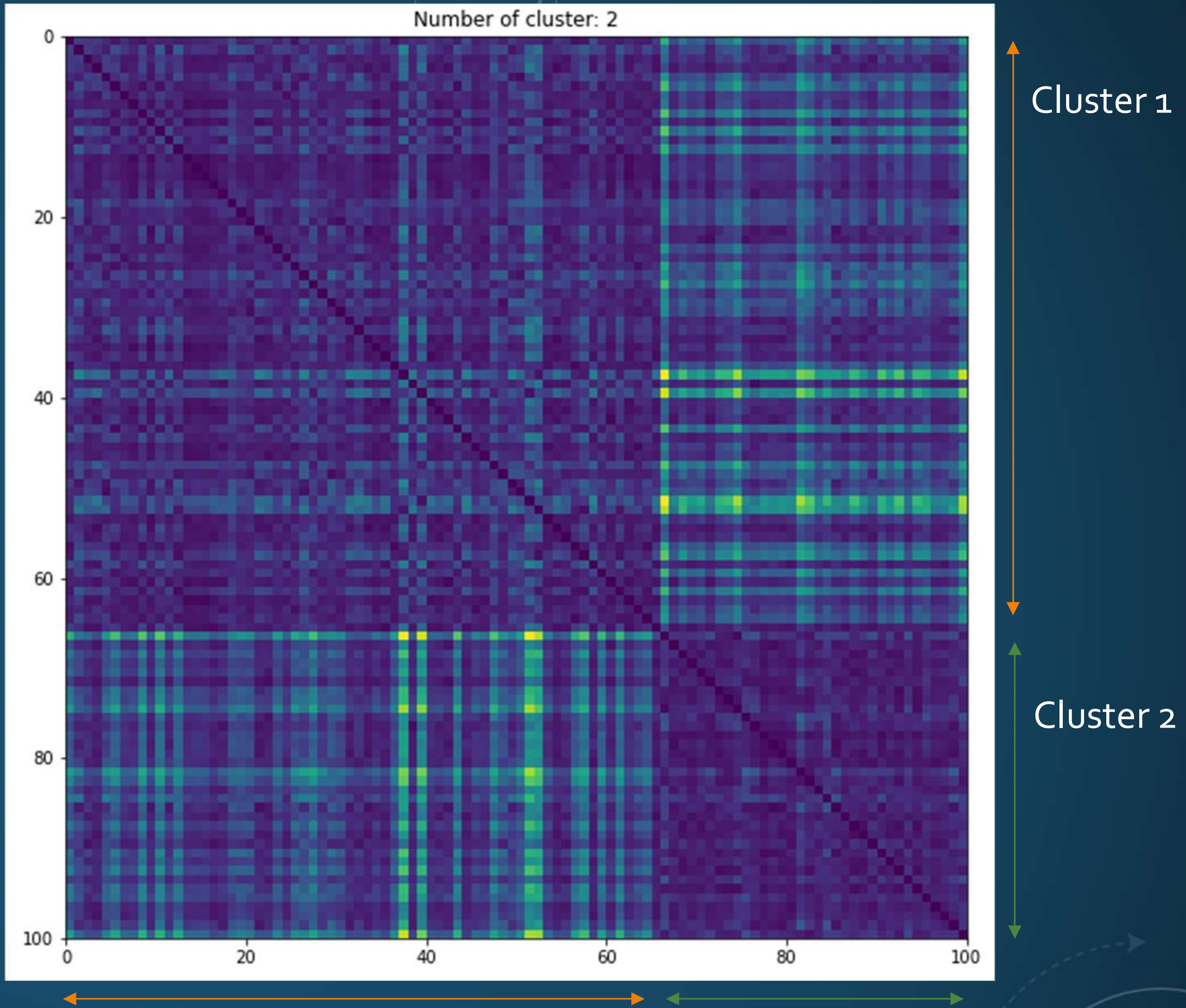
a: the mean nearest-cluster distance



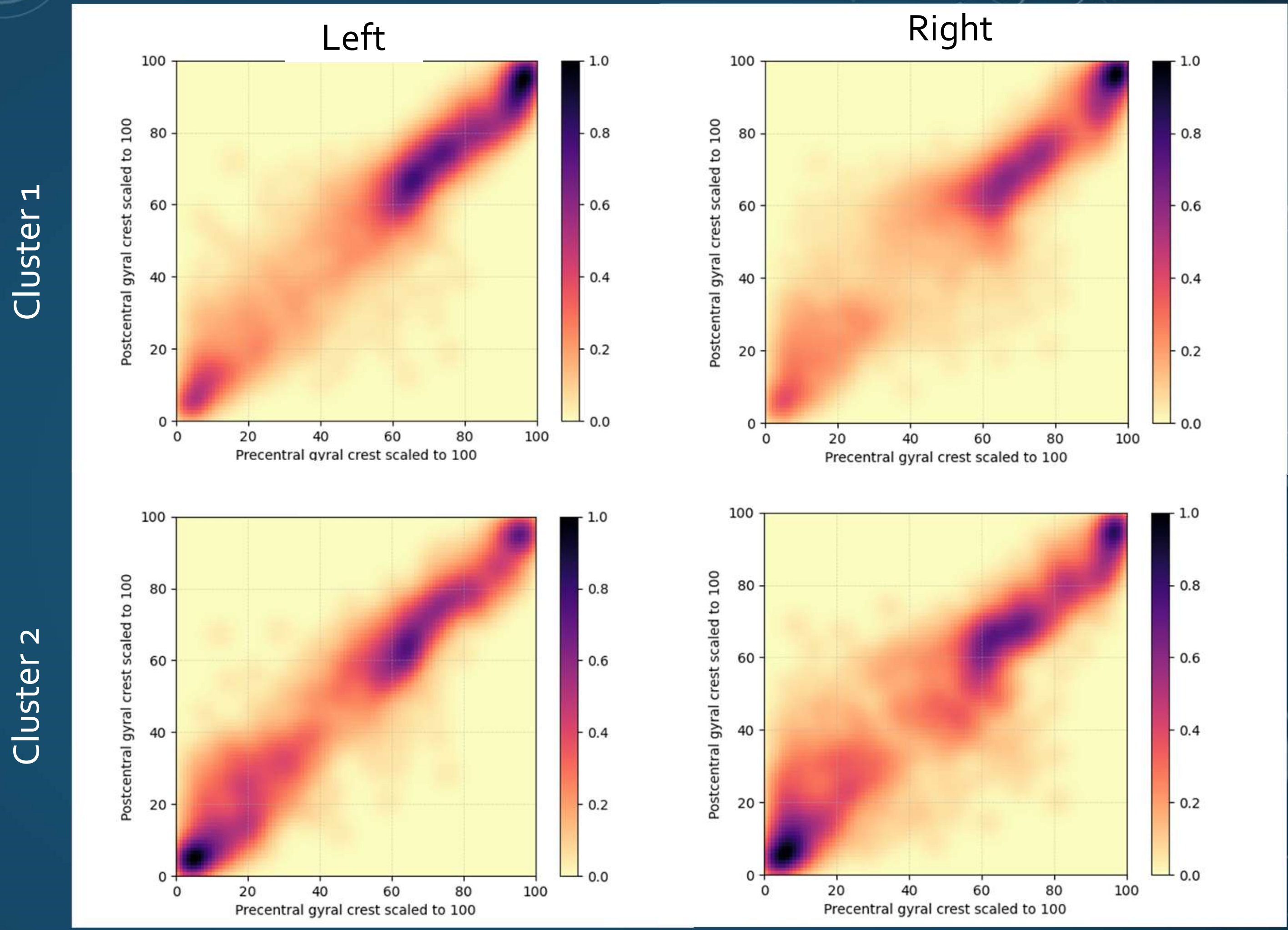
K=2

Reorganised Distance Matrix of subjects



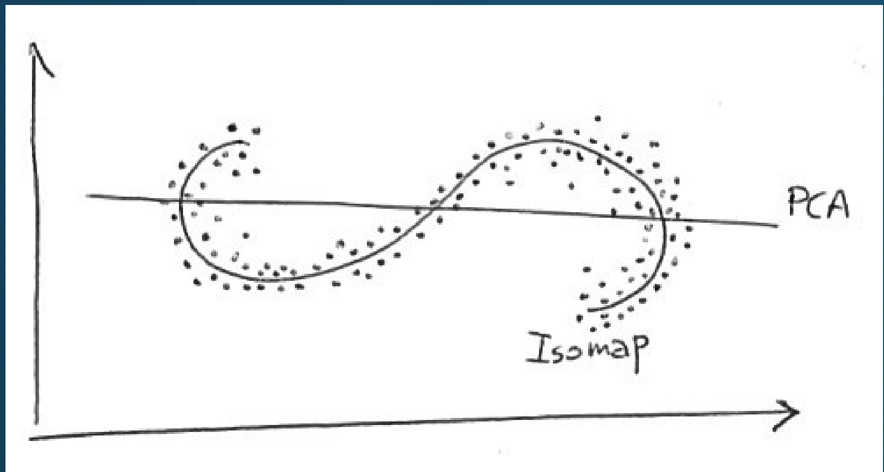


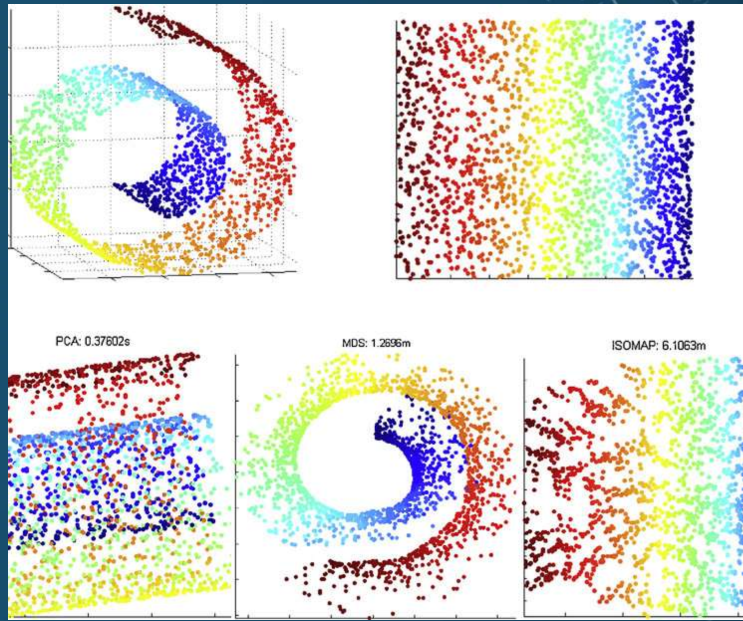
SUB-BARYCENTERS



HOW CAN WE STUDY THE VARIABILITY OF OUR SUBJECTS?

Isomap:
- Principal Component Analysis
- MultiDimensional Scaling

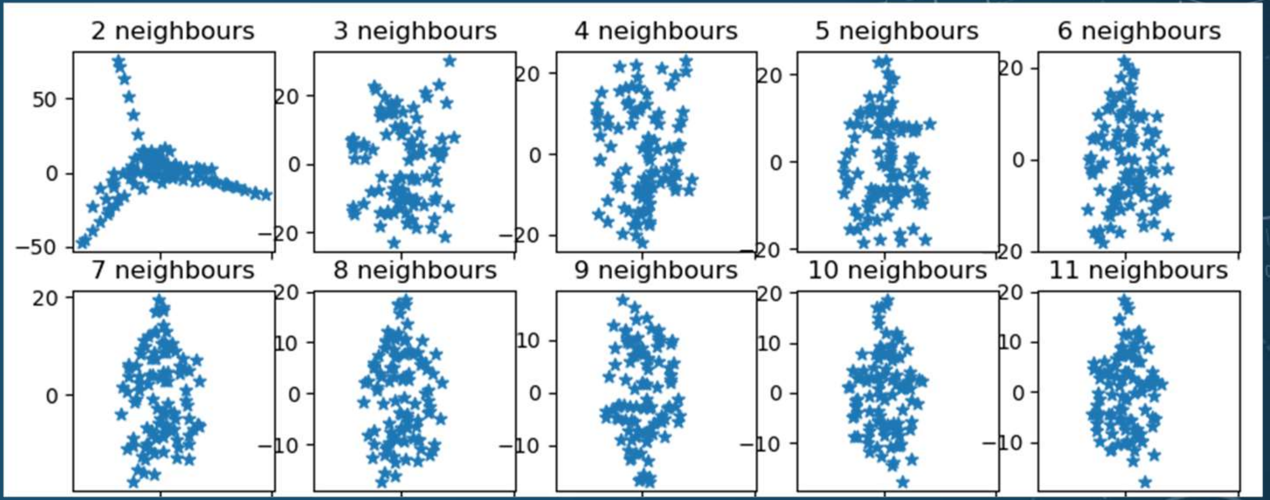




[α] *Multivariate methods for the analysis of complex and big data in forensic sciences. Application to age estimation in living persons*

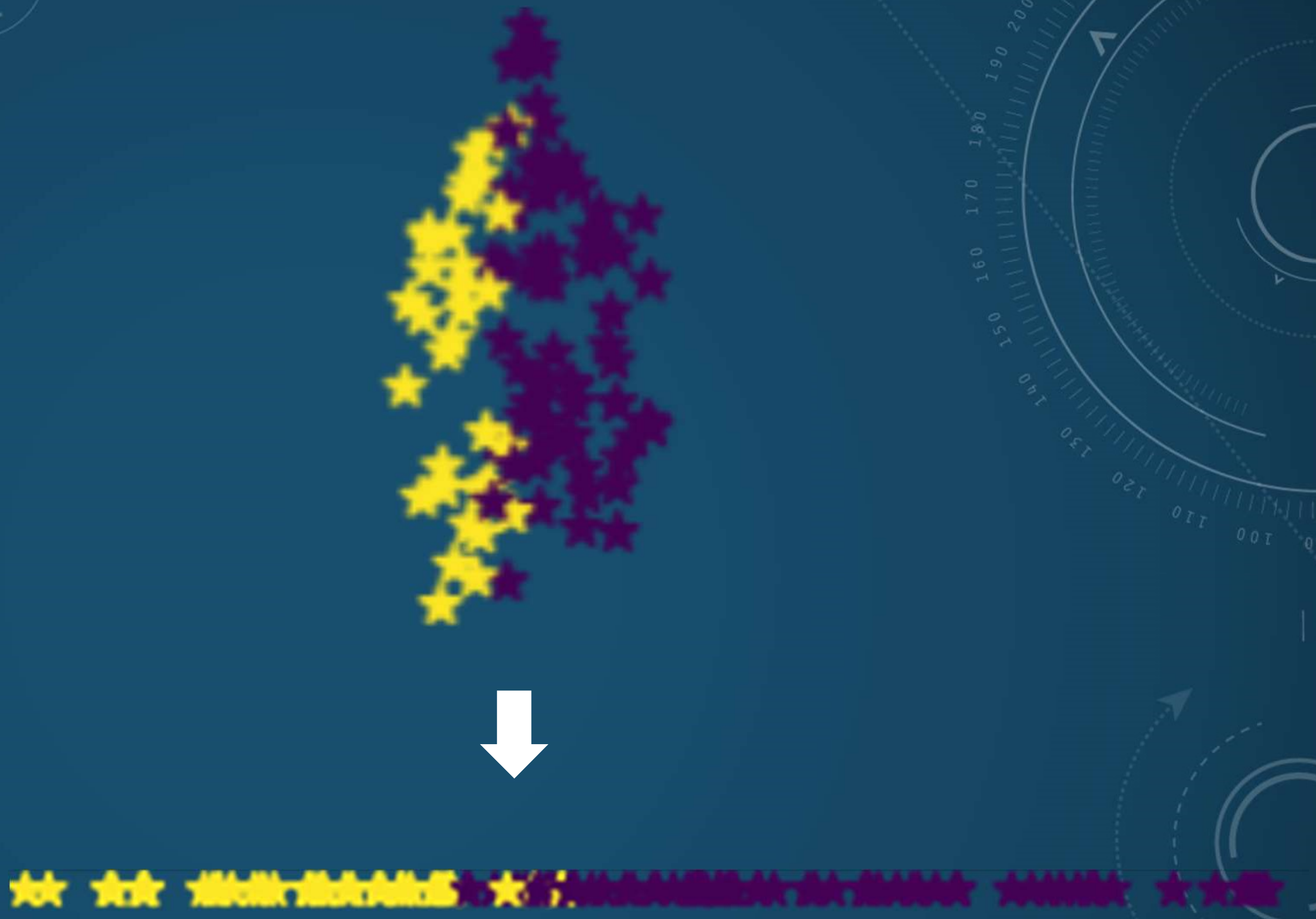
NUMBER OF NEIGHBOURS [7]

[7] *Selection of the Optimal Parameter Value for the Isomap Algorithm* by Samko, A. D. Marshall, and P. L. Rosin

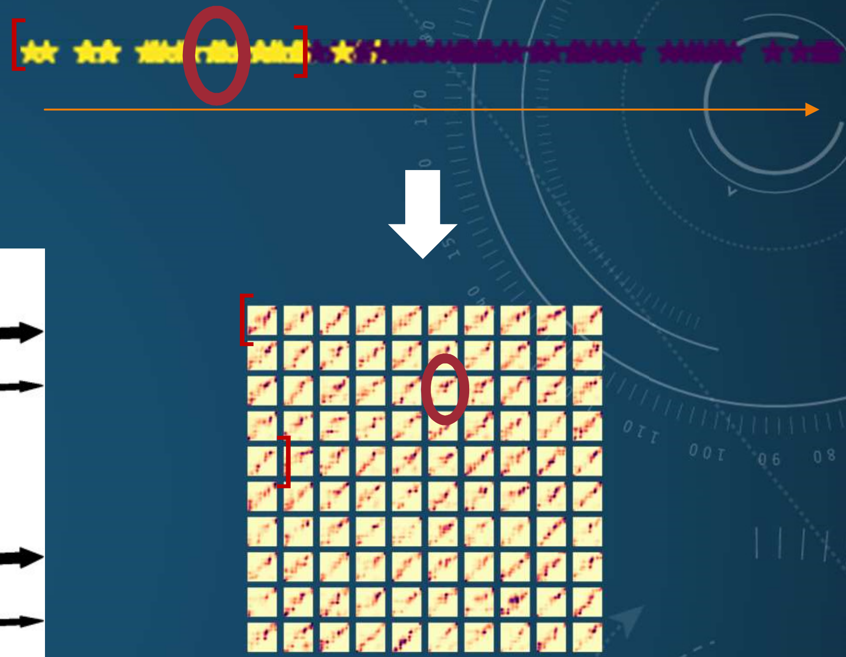


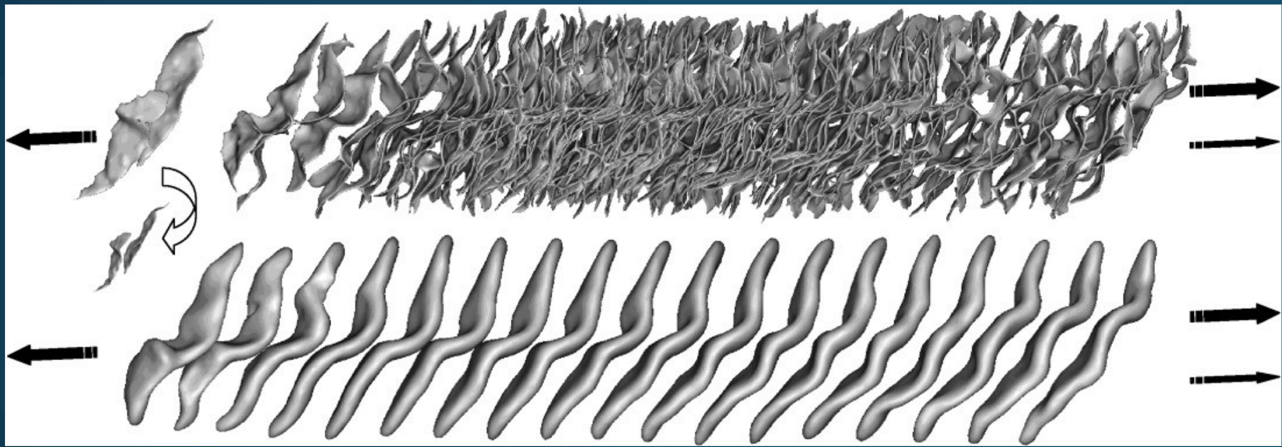
ISOMAP

7 neighbours



SLIDING BARYCENTER

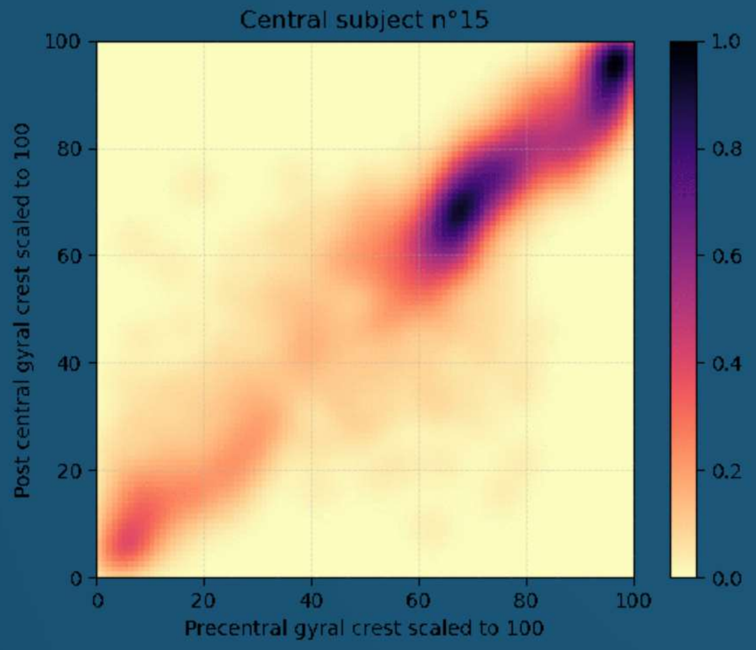


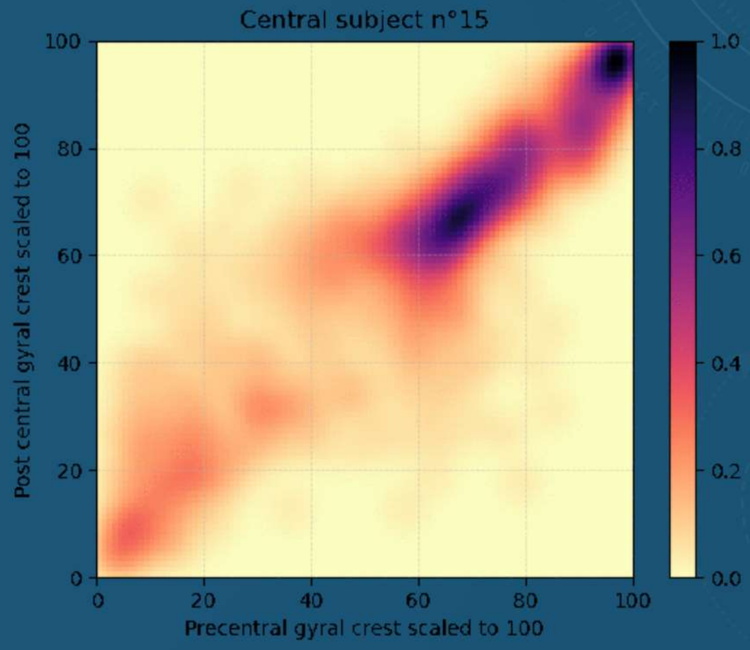


Hand Knob position moving [8]

[8] The effect of handedness on the shape of the central sulcus by Zhong Yi Sun et al

SLIDING BARYCENTER





CONCLUSION & FUTURE WORKS

Representative subject

- The barycenter determined by the 2-Wasserstein metric provides a representative suject but it is a smoother solution than alexendre's.

Clustering

- It would seem that clustering it is not relevant.

Isomap

- The axis of variation that we have highlighted would correspond to a variation of the position of the central zone and a shift in density towards the ventral zone.

Determine the variability parameter with certainty

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[4] Marco Cuturi and Arnaud Doucet. « Fast Computation of Wasserstein Barycenters ». In: Proceedings of the 31st International Conference on Machine Learning. Ed. by Eric P. Xing and Tony Jebara. Vol. 32. Proceedings of Machine Learning Research 2. Bejing, China: PMLR, 22–24 Jun 2014, pp. 685–693. url: http: //proceedings.mlr.press/v32/cuturi14.html (cit. on p. 5).

[5] Leonard Kaufman and Peter J. Rousseeuw. Clustering by means of medoids. Ed. by In: Dodge Y and editor. Amsterdam: 1987 (cit. on p. 6).

[6] Chao Shao and Haitao Hu. « Extension of ISOMAP for Imperfect Manifolds ». In: J. Comput. 7.7 (2012), pp. 1780–1785. doi: 10.4304/jcp.7.7.1780-1785. url: http://www.jcomputers.us/index.php?m=content%5C&c=index%5C&a=show% 5C&catid=121%5C&id=2301 (cit. on p. 7).

[7] . Samko, A. D. Marshall, and P. L. Rosin. « Selection of the Optimal Parameter Value for the Isomap Algorithm ». In: Pattern Recogn. Lett. 27.9 (July 2006), pp. 968–979. issn: 0167-8655. doi: 10.1016/j.patrec.2005.11.017. url: https: //doi.org/10.1016/j.patrec.2005.11.017 (cit. on p. 7).

[8] Zhong Yi Sun et al. « The effect of handedness on the shape of the central sulcus ». In: NeuroImage 60.1 (2012), pp. 332–339. issn: 1053-8119. doi: https://doi.org/ 10.1016/j.neuroimage.2011.12.050. url: https://www.sciencedirect.com/ science/article/pii/S1053811911014522 (cit. on p. 13).