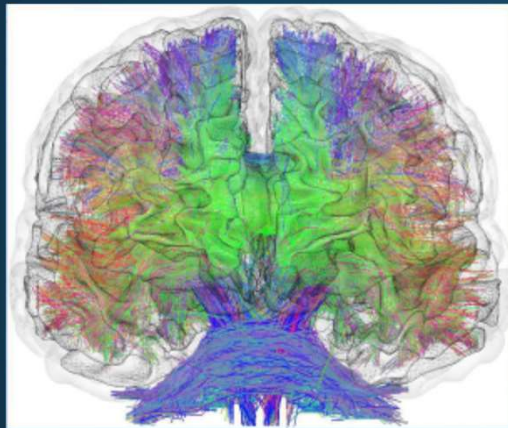


Optimal transport for comparison of short brain connectivity between individuals



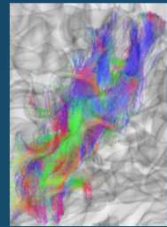
ESTABLISHMENT OF CONNECTIVITY MAPS

selection of streamlines

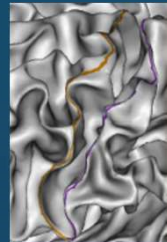


© L. Brun

Diffusion MRI

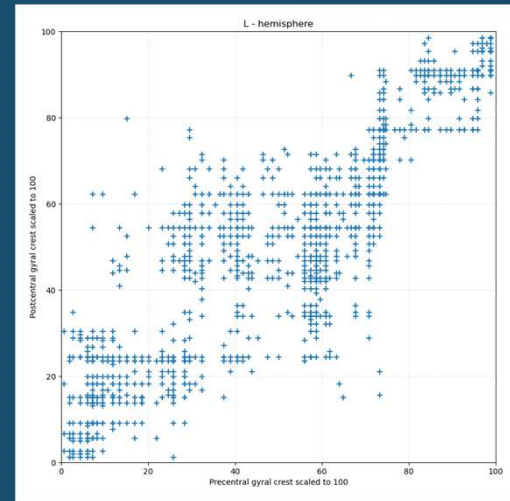
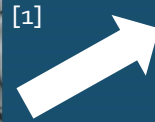


[1]

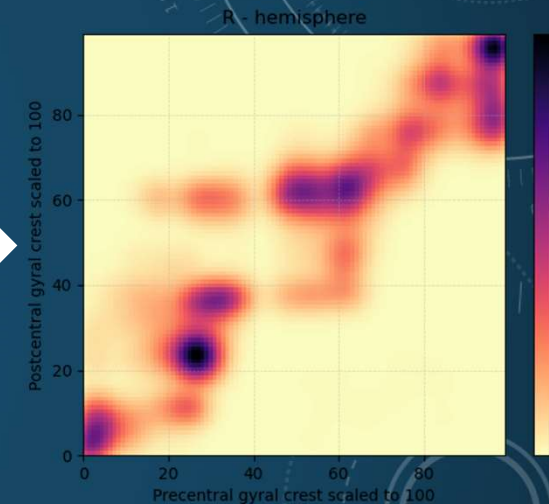


[1]

parameterisation



discrete connectivity mapping



continuous map

OPTIMAL TRANSPORT & 2-WASSERSTEIN DISTANCE

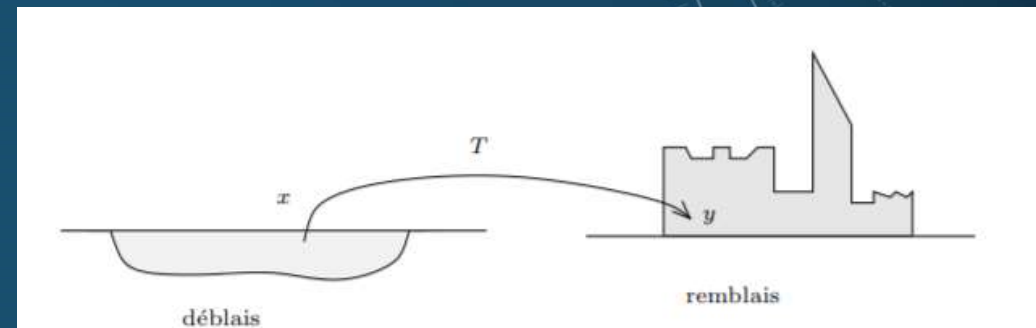
Optimal Transport matrix

$$T = \underset{\pi \in \Pi(\mu_s, \mu_t)}{\operatorname{argmin}} \langle \pi(\mu_s, \mu_t), D_2^2(\mu_s, \mu_t) \rangle$$

2-Wasserstein distance

$$\begin{aligned} W_2^2 &= \min_{\pi \in \Pi(\mu_s, \mu_t)} \int_{X \times Y} \|x, y\|_2^2 d\pi(x, y) \\ &= \min_{\pi \in \Pi(\mu_s, \mu_t)} \langle \pi(\mu_s, \mu_t), D_2^2(\mu_s, \mu_t) \rangle \end{aligned}$$

such μ_s, μ_t two probability densities associated with the space X and Y



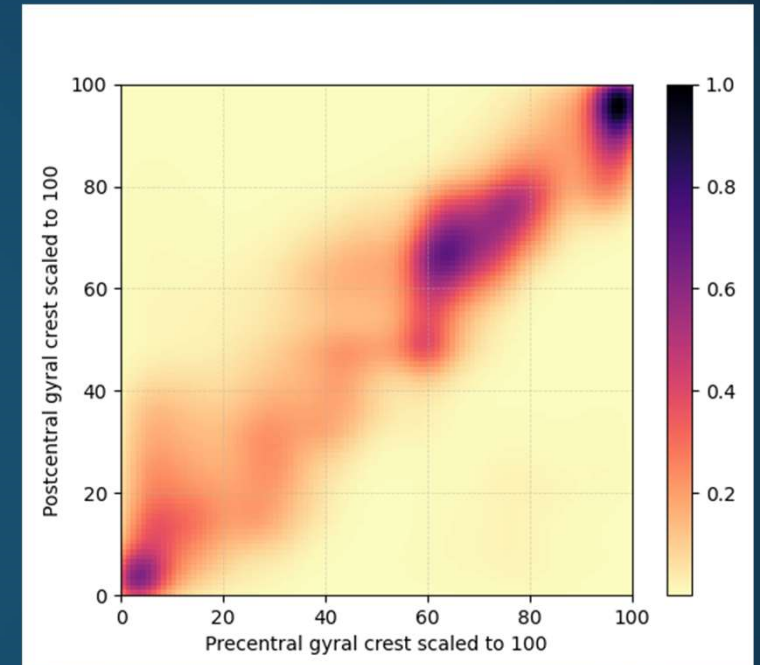
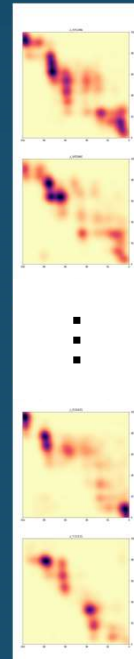
Monges problem of déblais and remblais [2]

[2] *Optimal transport : old and new* by Villani Cédric

REPRESENTATIVE SUBJECT

Summation of subjects

Barycenter



Individual profile aligned

Group Profile

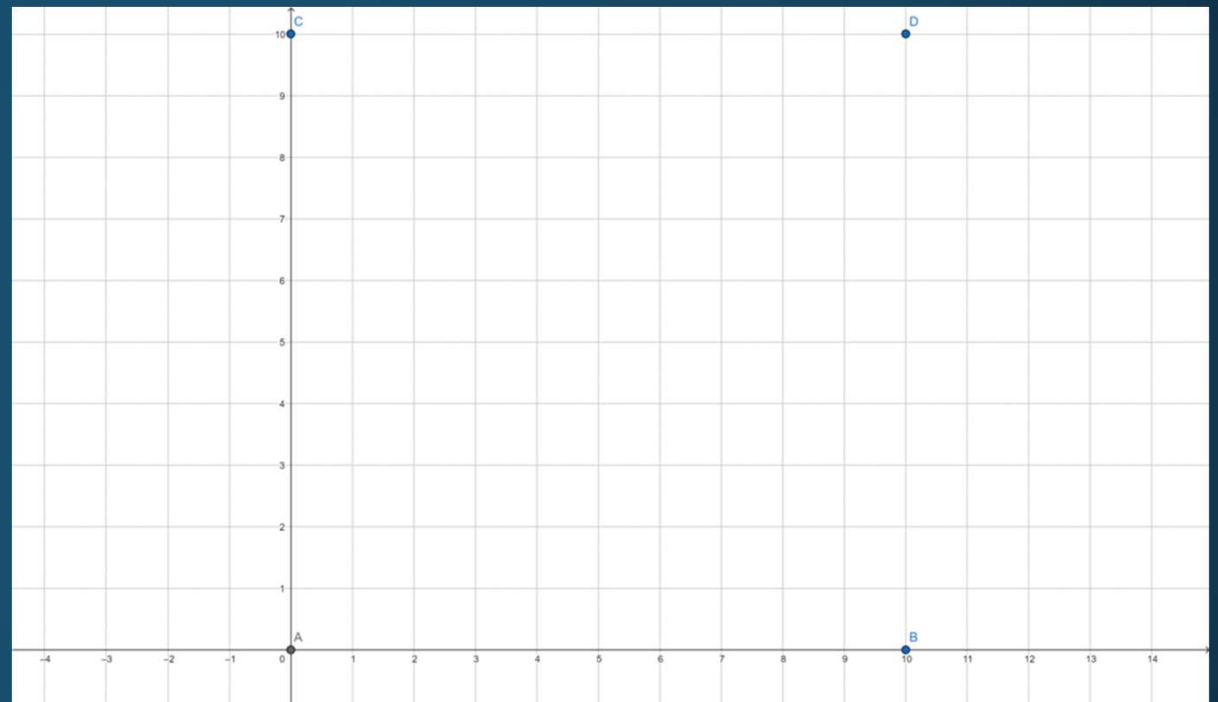
ITERATIVE BARYCENTER [3]

$$m = \underset{\mu \in (\mathbb{R}^2)^k}{\operatorname{argmin}} \sum_{i=1}^N W_2^2(\mu, v_i) \quad [4]$$

[3] *Population Averaging of Neuroimaging Data Using Lp Distance-based Optimal Transport* by Q. Wang, I. Redko, and S. Takerkart

[4] *Fast Computation of Wasserstein Barycenters* by Marco Cuturi and Arnaud Doucet

Illustration of the iterative barycenter



ROBUSTNESS OF THE BARYCENTRE

Initialization support

Name	Number of points	W_2 to all subjects	W_2 to all barycentres
Minimum	582	13.70	1.17
Random	768	13.69	1.10
Median	2040	13.67	0.94
Centroid	3190	13.67	0.92
Maximum	4029	13.67	0.9

Order of subjects

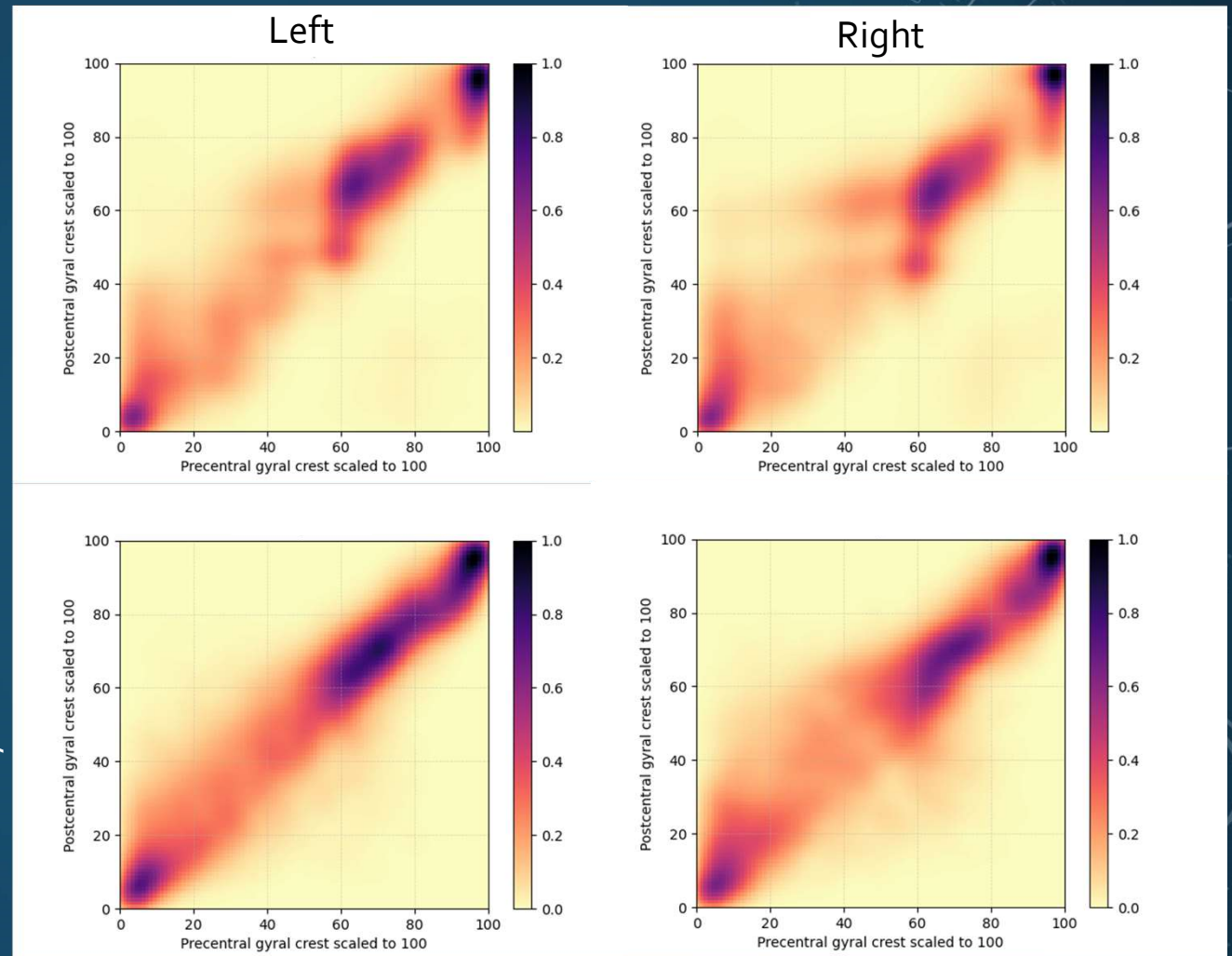
Experiment	W_2 to all subjects	W_2 to all barycentres
1	13.66	0.99
2	13.67	0.99
3	13.66	0.99
4	13.66	1.02
5	13.67	0.99

BARYCENTERS

[1] *Etude de la connectivité structurelle des faisceaux d'association courts de la substance blanche du cerveau humain en IRM de diffusion* by Alexandre Pron

mean [1]

barycenter

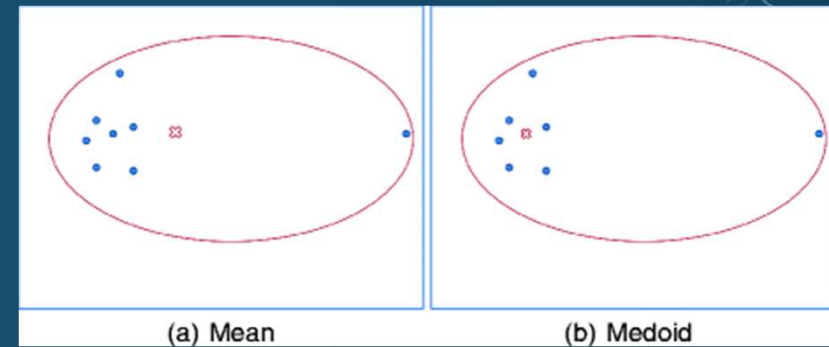


IS THERE A STRATIFICATION WITHIN OUR SUBJECTS?

K-MEDOIDS

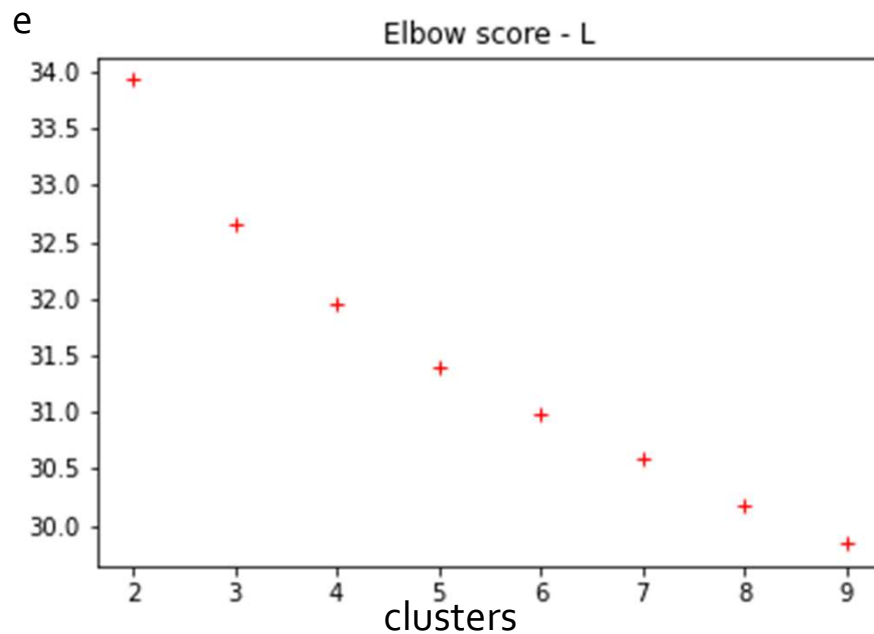
"K-Medoids Clustering" in
[Encyclopedia of Machine Learning](https://doi.org/10.1007/978-0-387-30164-8_426)

(DOI: https://doi.org/10.1007/978-0-387-30164-8_426)



ELBOW SCORE

e: the mean intra-cluster distance

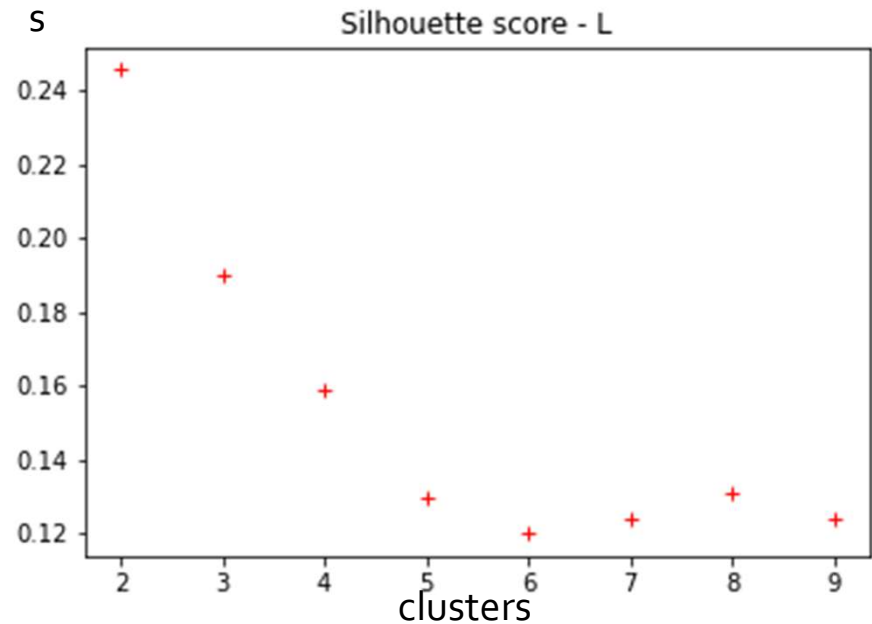


SILHOUETTE SCORE

$$s = \frac{a - e}{\max(a, e)}$$

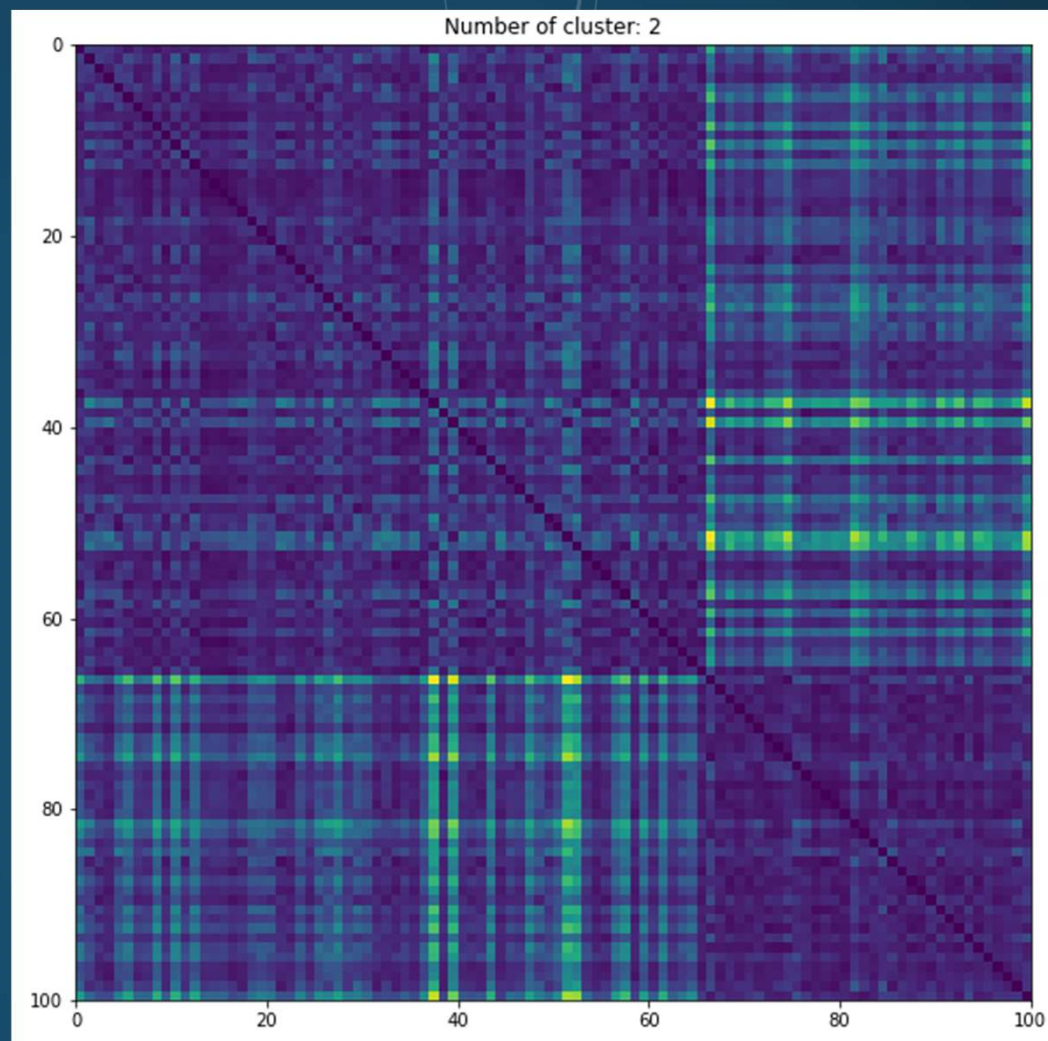
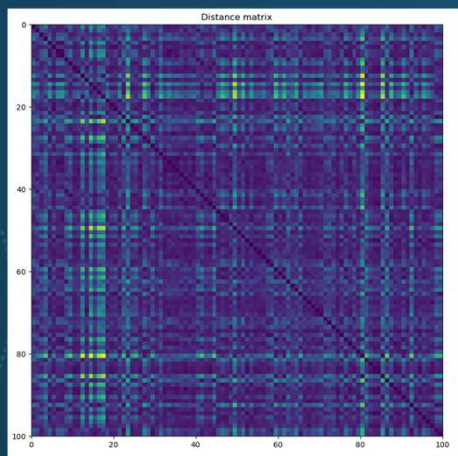
e: the mean intra-cluster distance

a: the mean nearest-cluster distance



$K=2$

Reorganised Distance Matrix of subjects

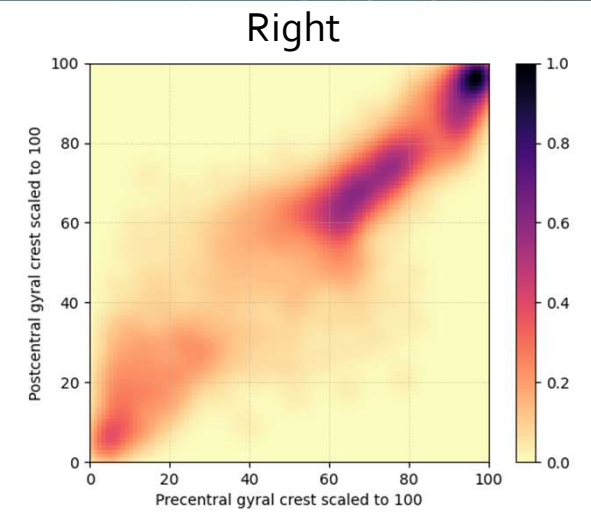
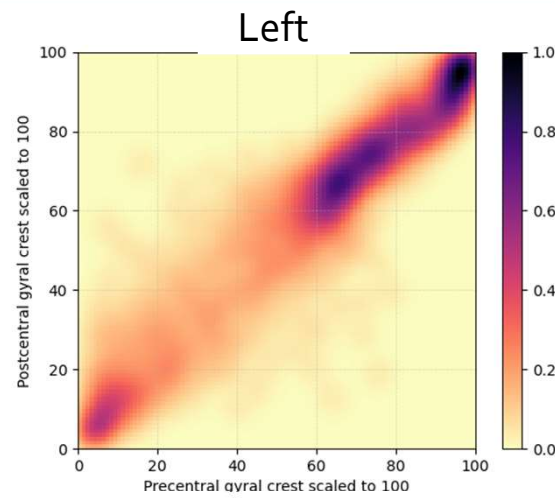


Cluster 1

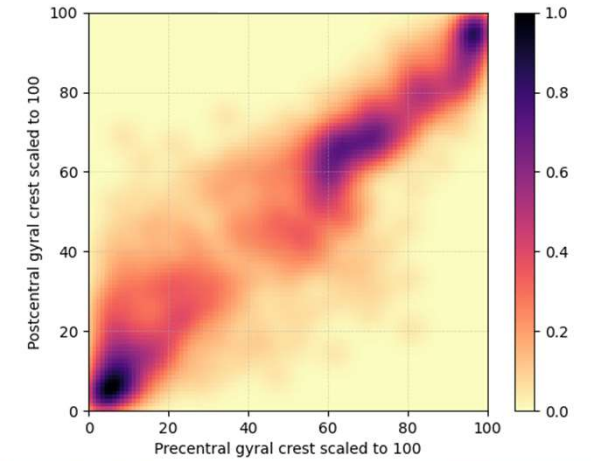
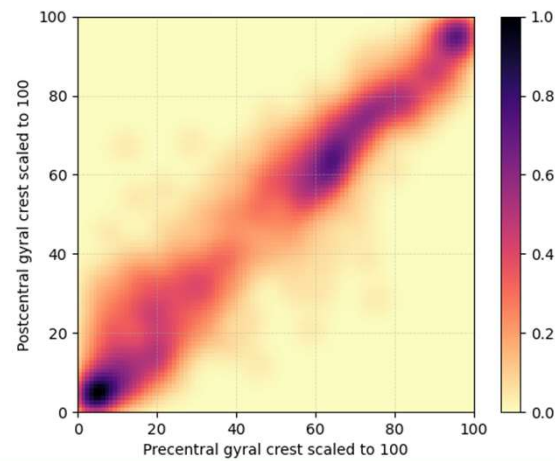
Cluster 2

SUB- BARYCENTERS

Cluster 1



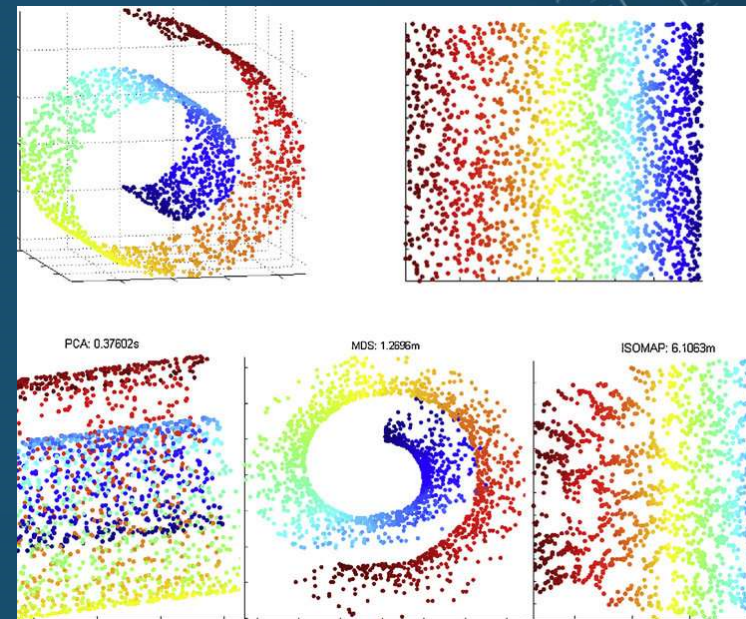
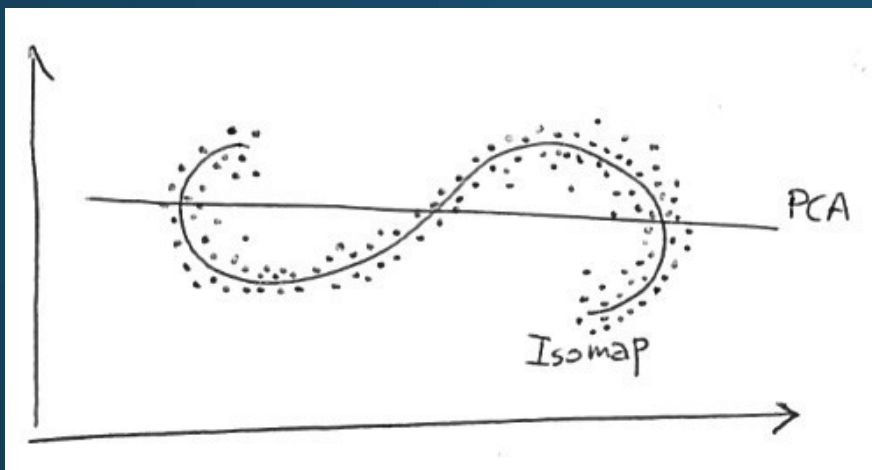
Cluster 2



HOW CAN WE STUDY THE VARIABILITY OF OUR SUBJECTS?

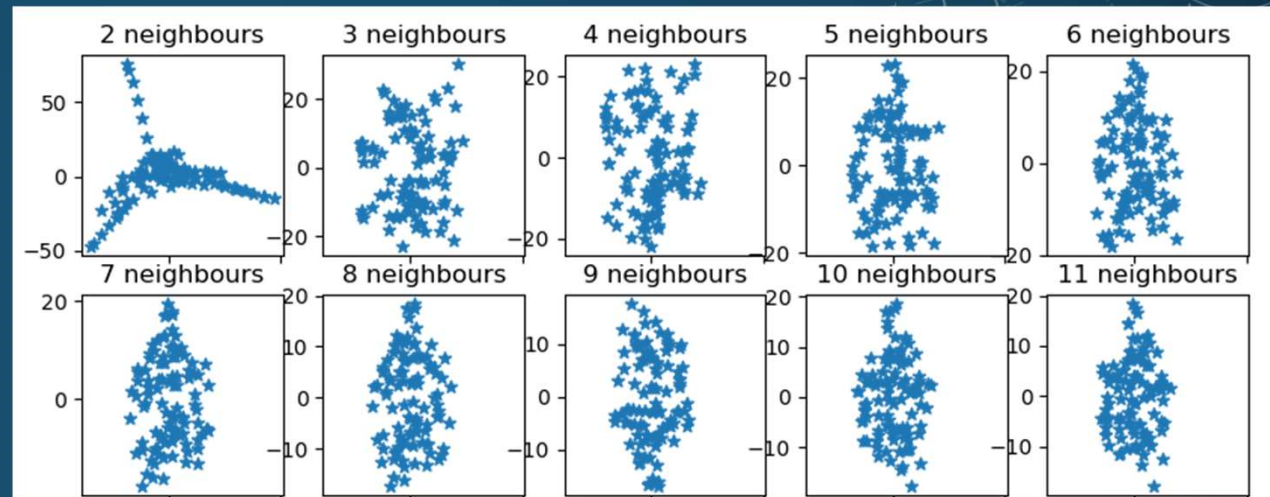
Isomap:

- Principal Component Analysis
- MultiDimensional Scaling



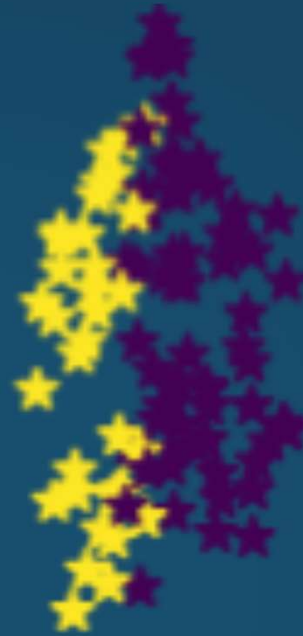
NUMBER OF NEIGHBOURS [7]

[7] *Selection of the Optimal Parameter Value for the Isomap Algorithm* by Samko, A. D. Marshall, and P. L. Rosin

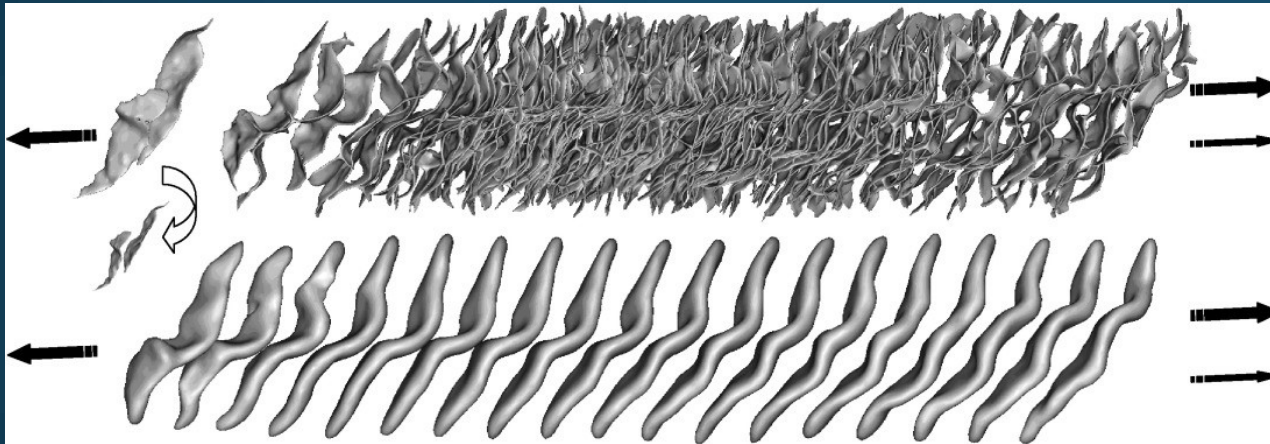


ISOMAP

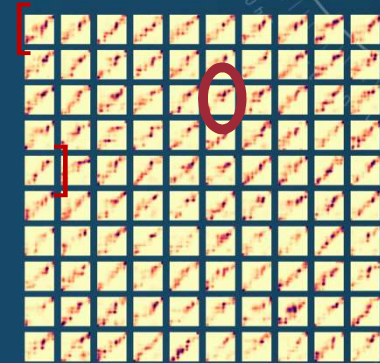
7 neighbours



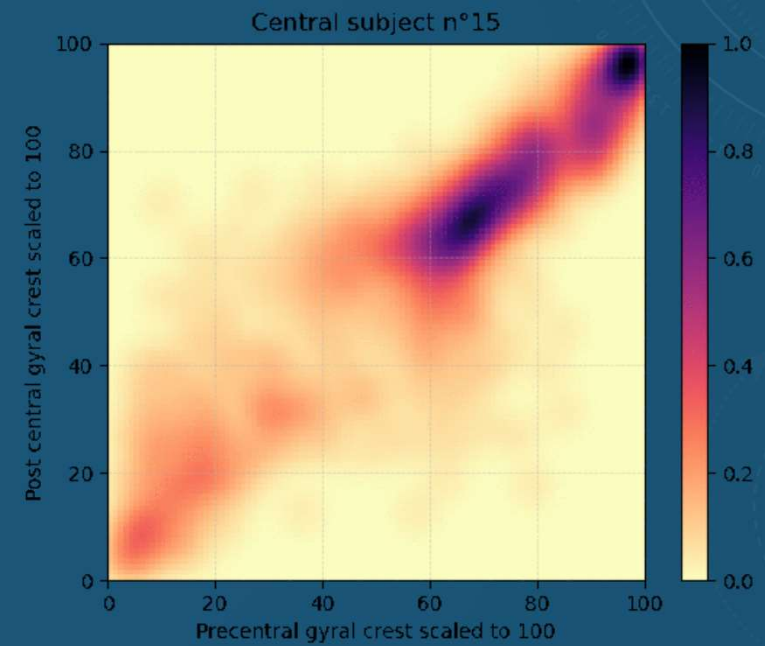
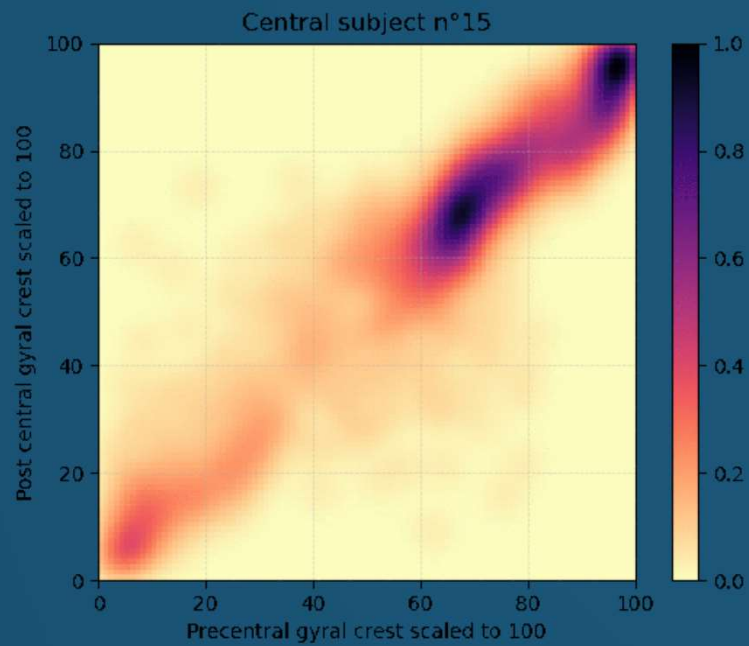
SLIDING BARYCENTER



Hand Knob position moving [8]



SLIDING BARYCENTER



CONCLUSION & FUTURE WORKS

Representative subject

- The barycenter determined by the 2-Wasserstein metric provides a representative subject but it is a smoother solution than alexandre's.

Clustering

- It would seem that clustering it is not relevant.

Isomap

- The axis of variation that we have highlighted would correspond to the position of the central zone.

Determine the variability parameter with certainty

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- [6] Chao Shao and Haitao Hu. « Extension of ISOMAP for Imperfect Manifolds ». In: J. Comput. 7.7 (2012), pp. 1780–1785. doi: 10.4304/jcp.7.7.1780-1785. url: <http://www.jcomputers.us/index.php?m=content%5C&c=index%5C&a=show%5C&catid=121%5C&id=2301> (cit. on p. 7).
- [7] . Samko, A. D. Marshall, and P. L. Rosin. « Selection of the Optimal Parameter Value for the Isomap Algorithm ». In: Pattern Recogn. Lett. 27.9 (July 2006), pp. 968–979. issn: 0167-8655. doi: 10.1016/j.patrec.2005.11.017. url: <https://doi.org/10.1016/j.patrec.2005.11.017> (cit. on p. 7).
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