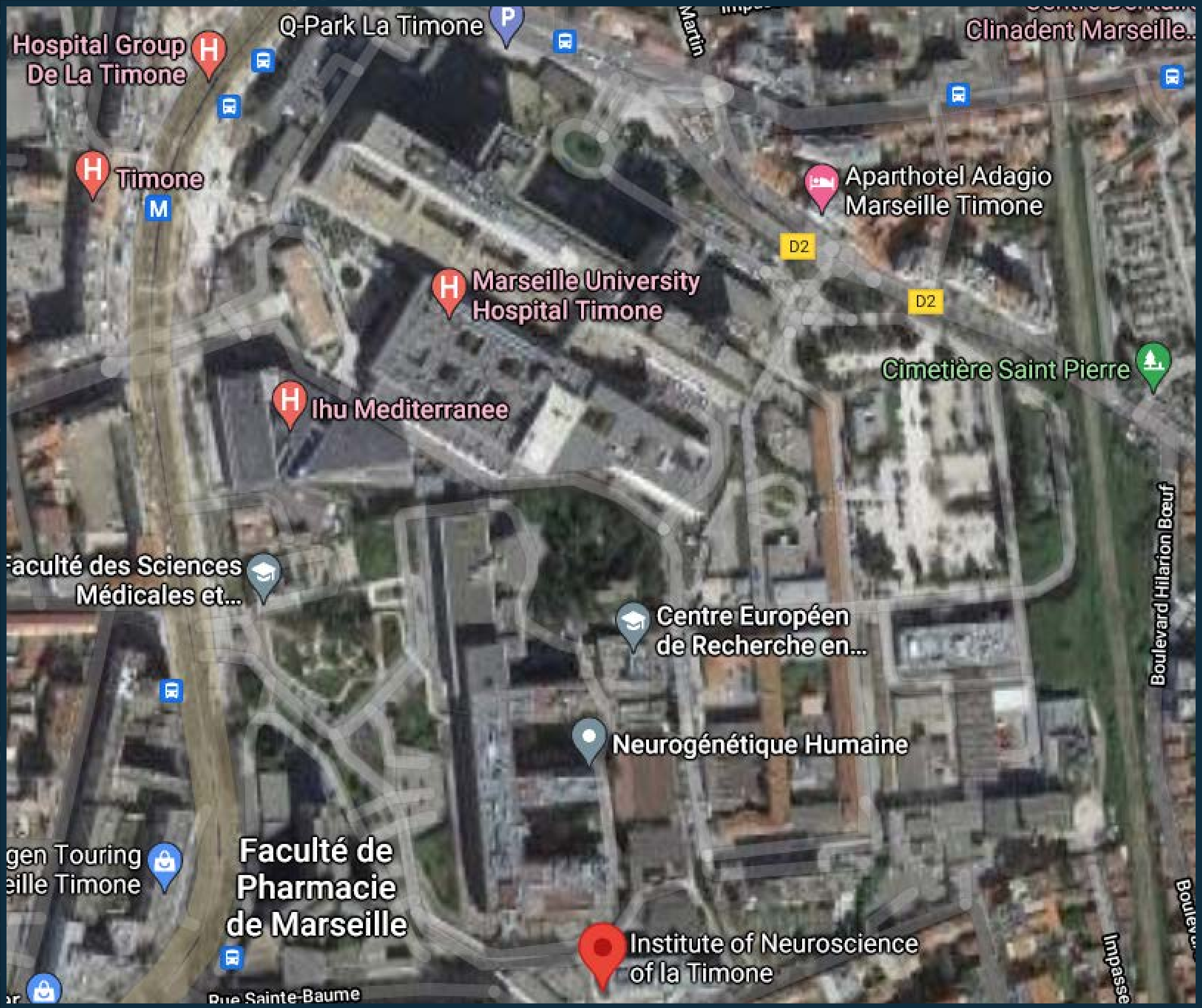
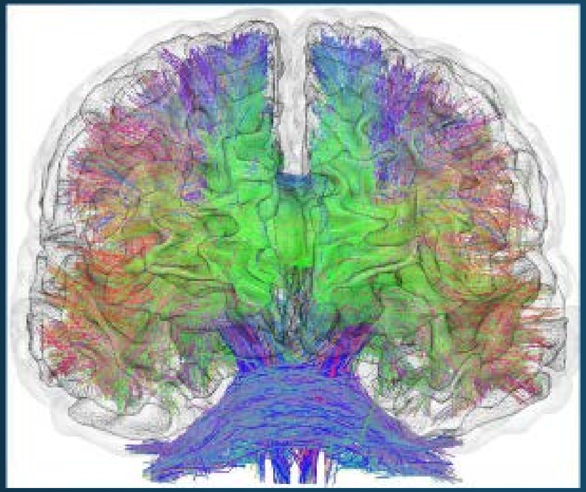
Transport optimal pour la comparaison des fibres courtes du cerveau entre individus

INSTITUT DE NEUROSCIENCES DE LA TIMONE



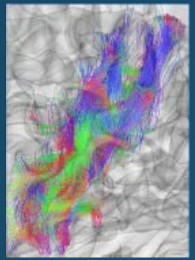
ÉTABLISSEMENT DE CARTES DE CONNECTIVITÉ

sélection des streamlines

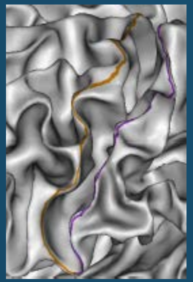


© L. Brun

IRM de diffusion

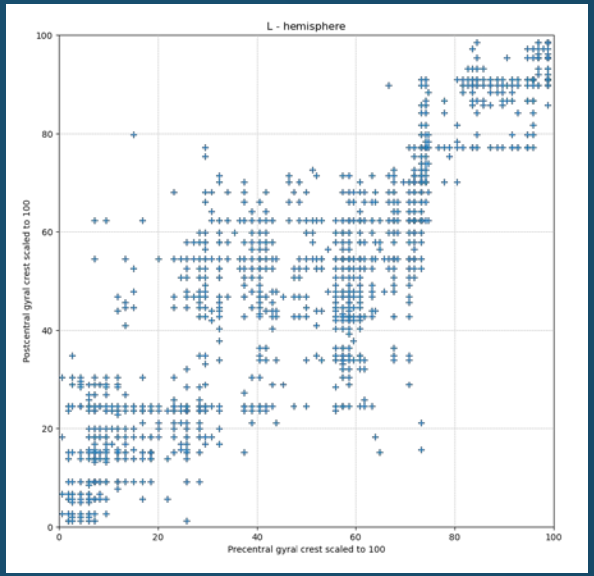


[1]

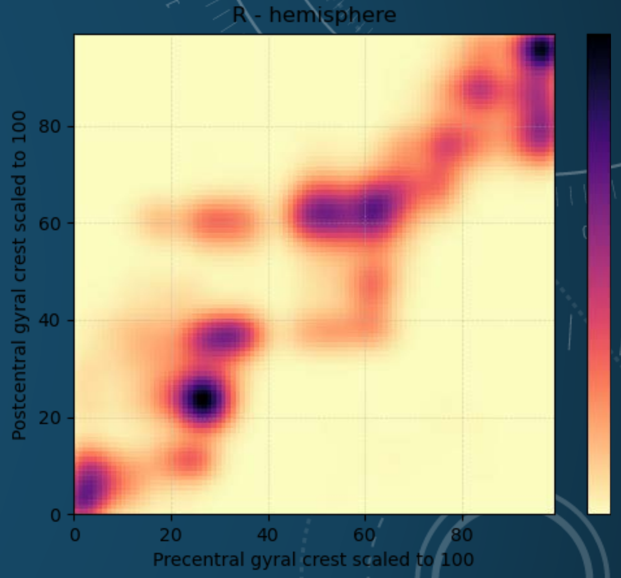


[1]

paramétrisation



Carte de connectivité discrète



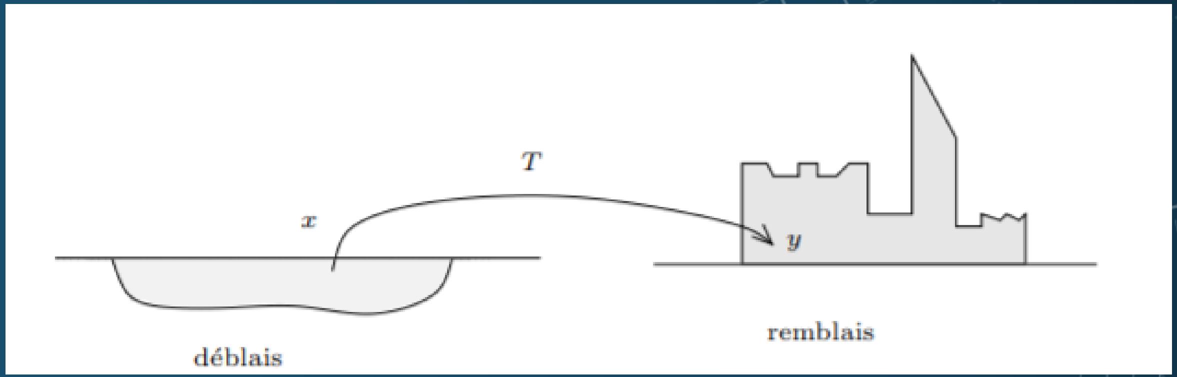
Carte continue

[1] *Etude de la connectivité structurelle des faisceaux d'association courts de la substance blanche du cerveau humain en IRM de diffusion* by Alexandre Pron

TRANSPORT OPTIMAL & DISTANCE DE 2-WASSERSTEIN

Optimal Transport matrix

$$T = \underset{\pi \in \Pi(\mu_s, \mu_t)}{argmin} < \pi(\mu_s, \mu_t), D_2^2(\mu_s, \mu_t) >$$



Le problème du déblai et du remblai de Monges [2]

[2] *Optimal transport : old and new* by Villani Cédric

Tels que μ_s, μ_t deux densités de probabilité associées à l'espace X et Y

TRANSPORT OPTIMAL & DISTANCE DE 2-WASSERSTEIN

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Distance de 2-Wasserstein

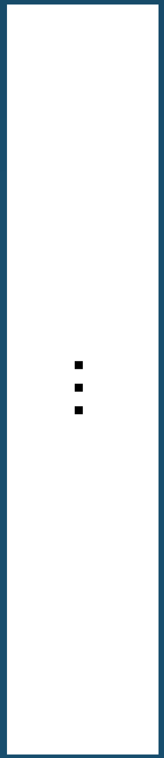
$$W_2^2 = \min_{\pi \in \Pi(\mu_s, \mu_t)} \int_{X \times Y} ||x, y||_2^2 d\pi(x, y)$$
$$= \min_{\pi \in \Pi(\mu_s, \mu_t)} < \pi(\mu_s, \mu_t), D_2^2(\mu_s, \mu_t) >$$

Tels que μ_s, μ_t deux densités de probabilité associées à l'espace X et Y

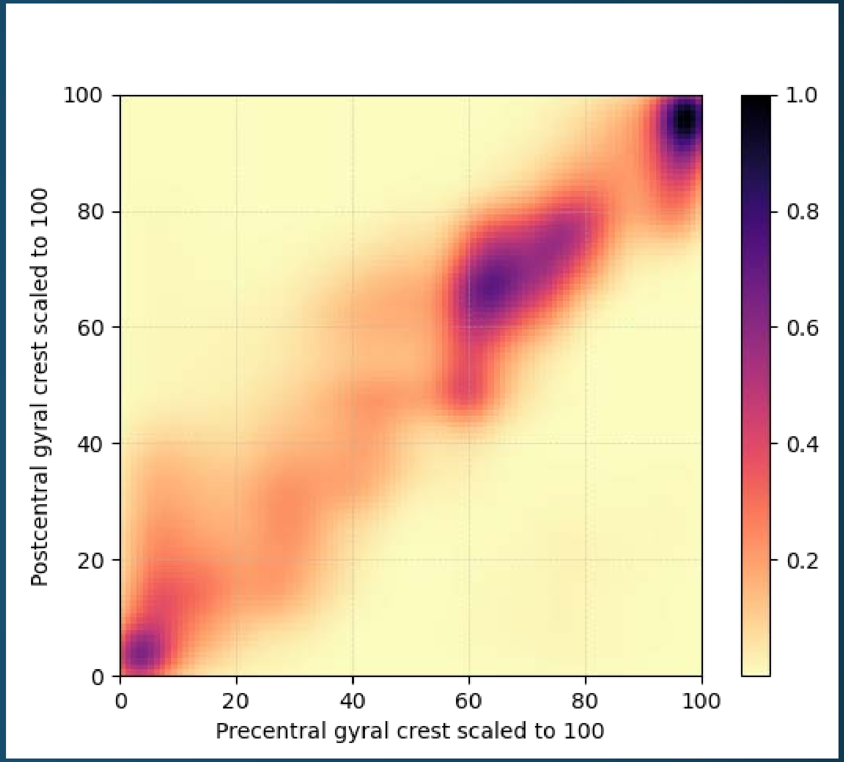
SUJET REPRÉSENTATIF

Moyenne des sujets

Barycentre



Profil individuel aligné



Profil du groupe

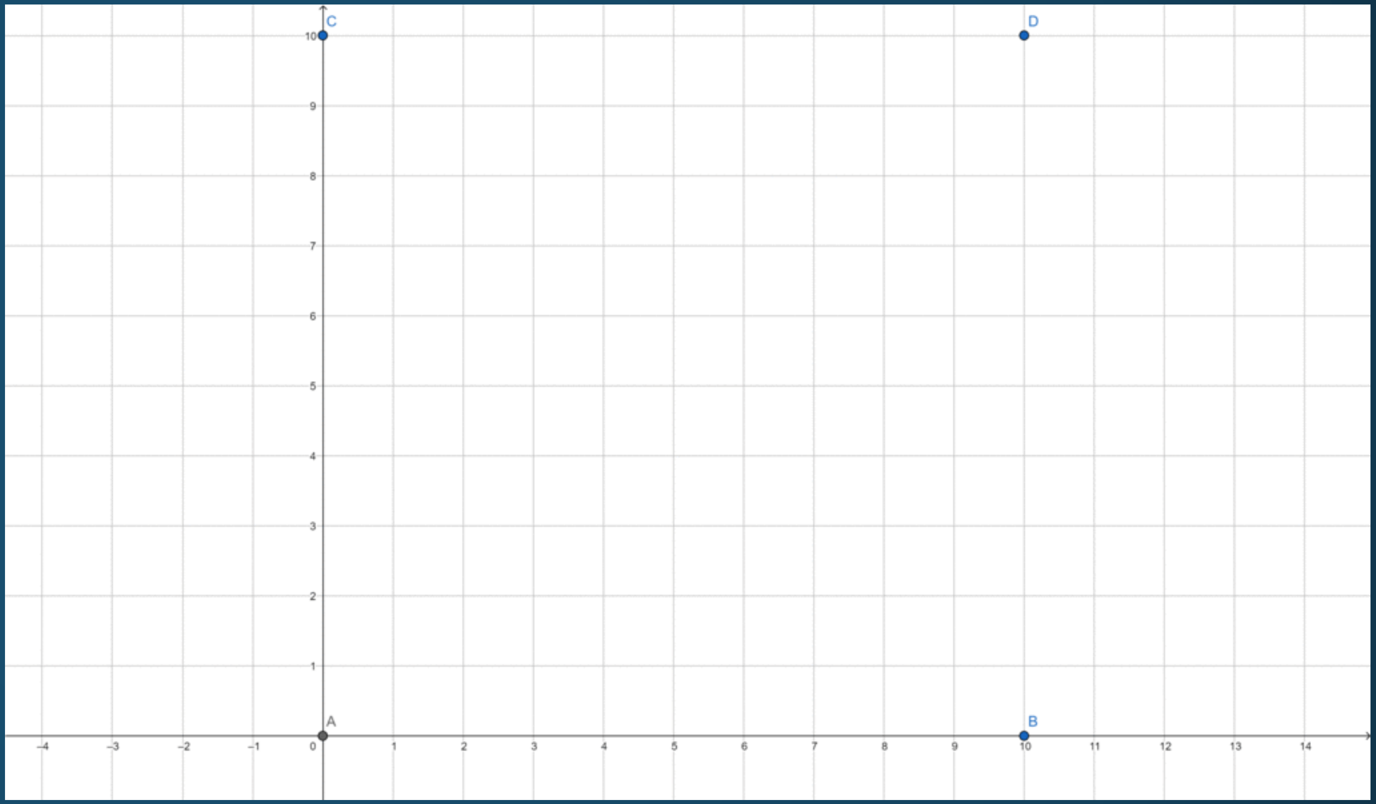
BARYCENTRE ITÉRATIF [3]

$$m = \underset{\mu \in (\mathbb{R}^2)^k}{argmin} \sum_{i=1}^{N} W_2^2(\mu, \nu_i) \text{ [4]}$$

[3] *Population Averaging of Neuroimaging Data Using Lp Distance-based Optimal Transport* by Q. Wang, I. Redko, and S. Takerkart

[4] *Fast Computation of Wasserstein Barycenters* by Marco Cuturi and Arnaud Doucet

Illustration du barycentre itératif



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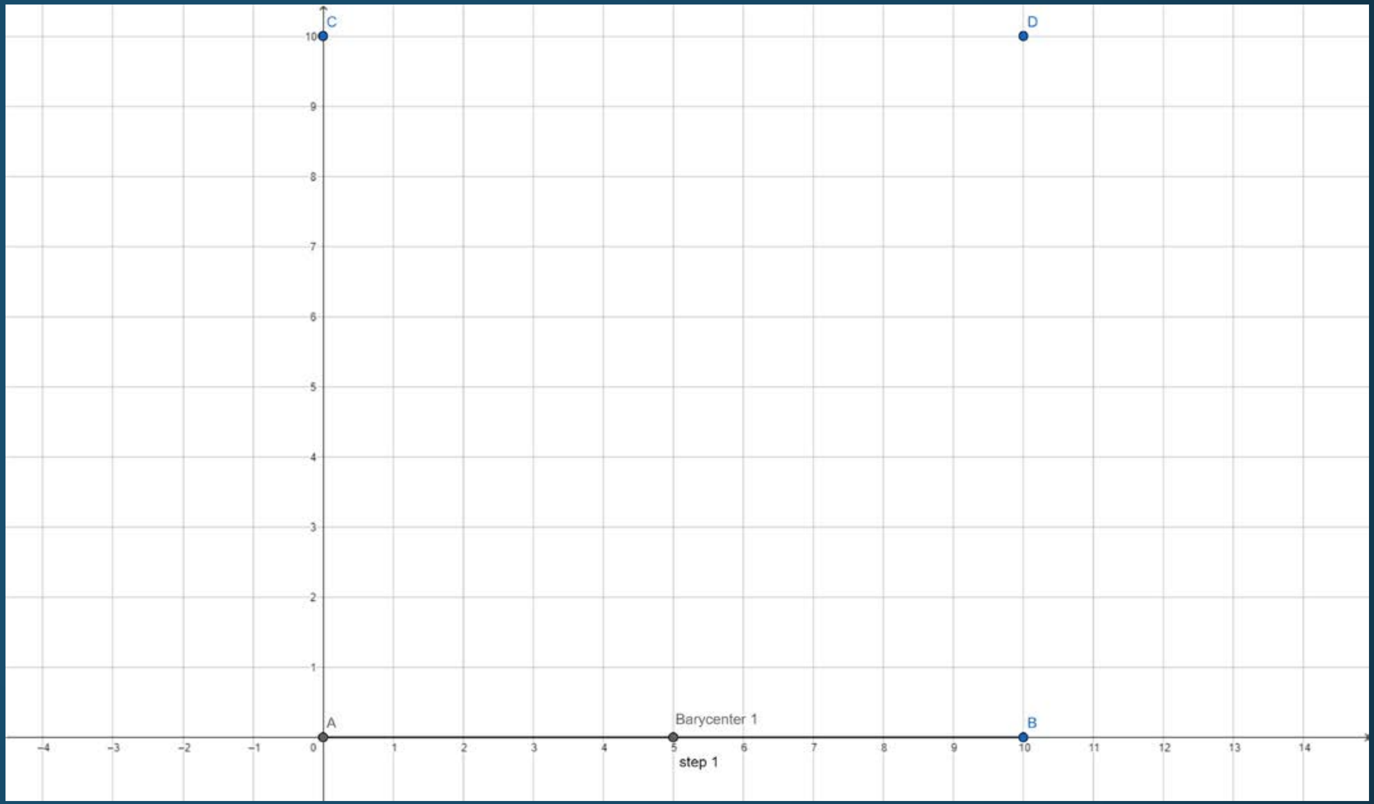

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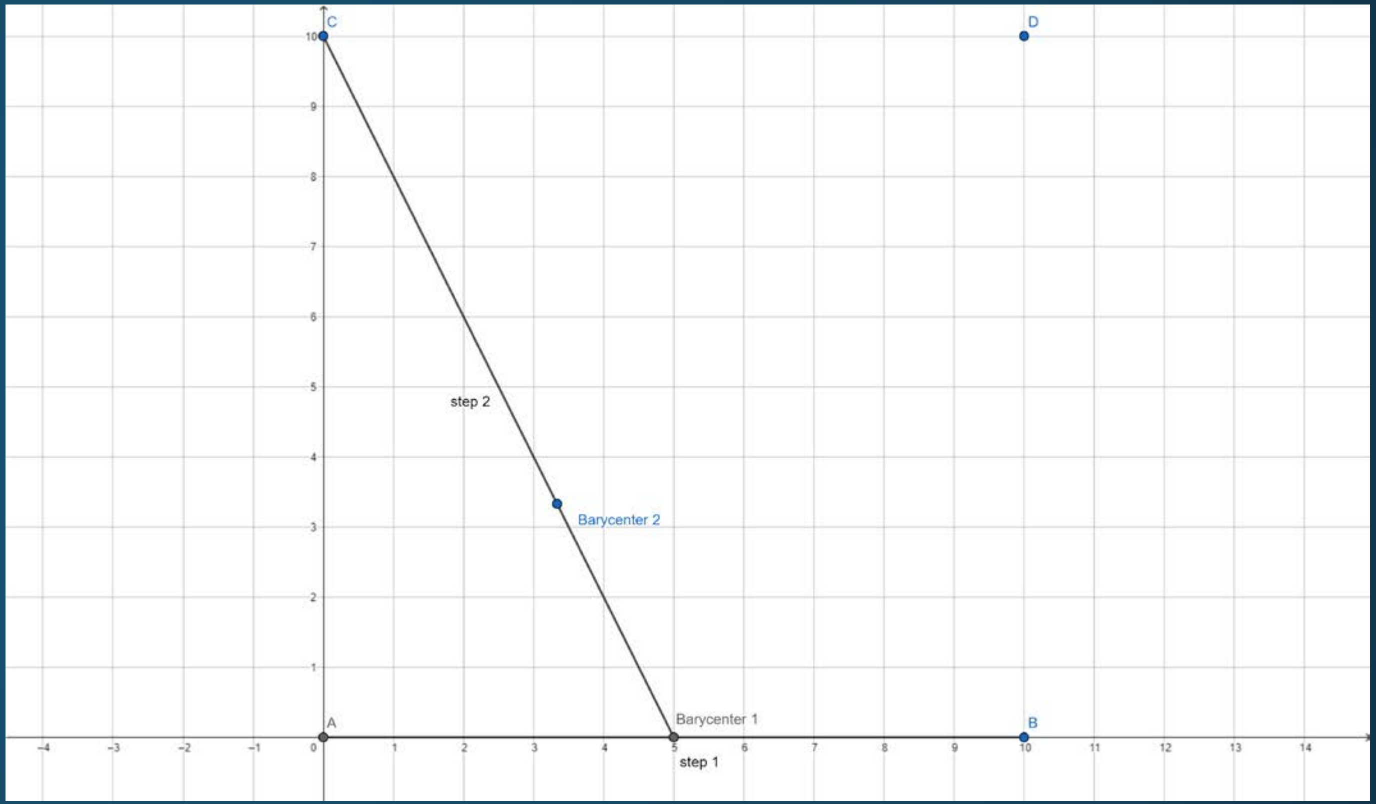

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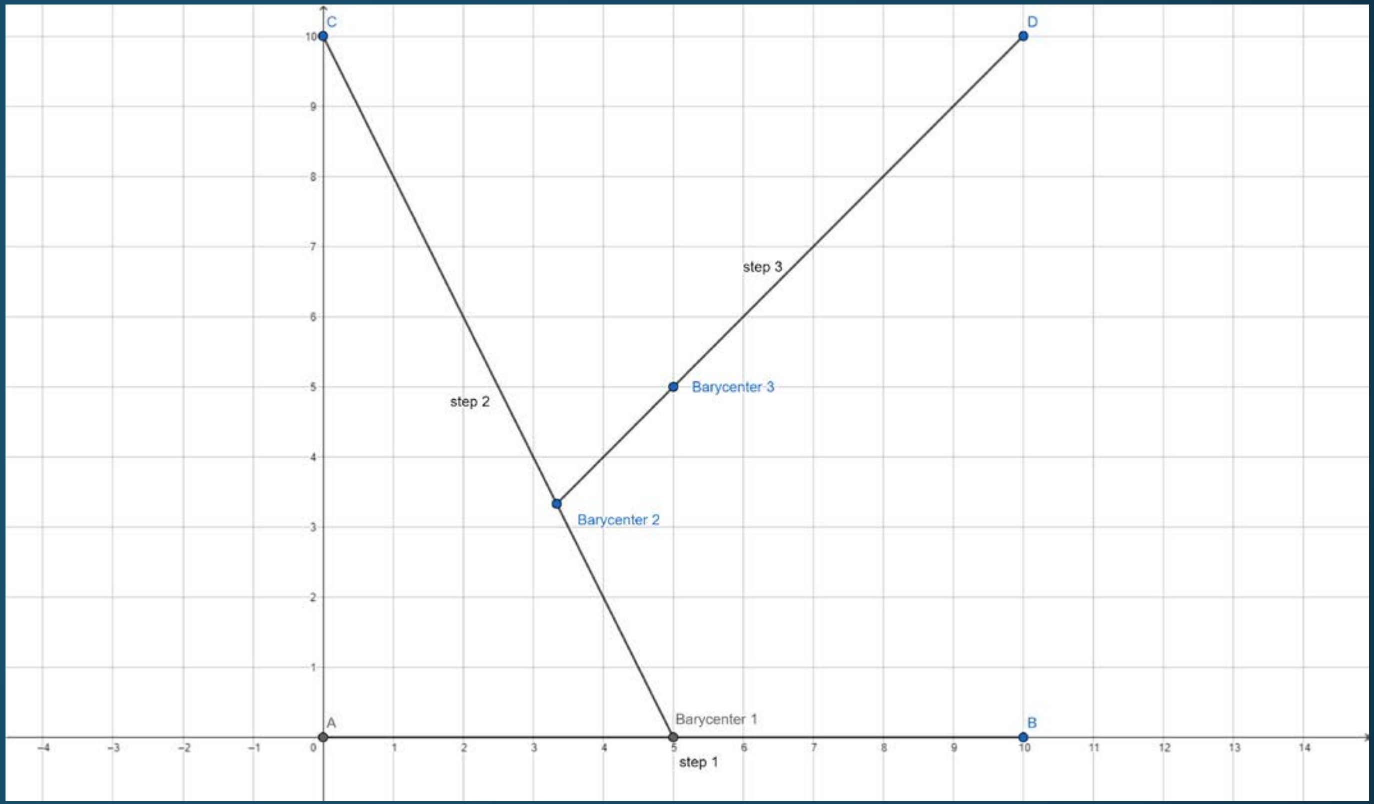


ROBUSTESSE DU BARYCENTRE

Support d'initialisation

Nom	Nombre de points	W_2 à tous les sujets	W_2 aux barycentres
Minimum	582	13.70	1.17
Random	768	13.69	1.10
Médian	2040	13.67	0.94
Centroïde	3190	13.67	0.92
Maximum	4029	13.67	0.9

Ordre des sujets

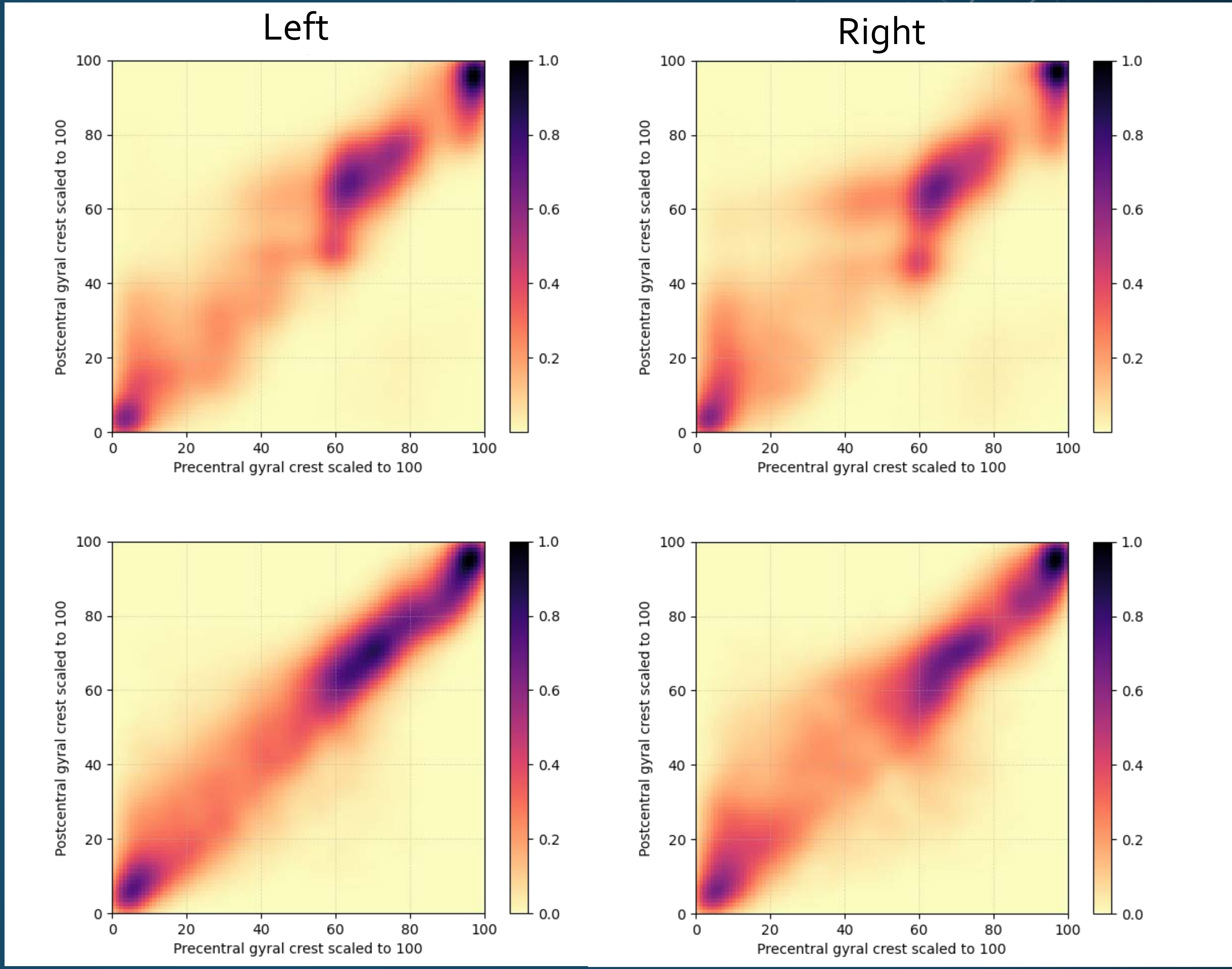
Expérience	W_2 à tous les sujets	W_2 aux barycentres
1	13.66	0.99
2	13.67	0.99
3	13.66	0.99
4	13.66	1.02
5	13.67	0.99

BARYCENTRES

[1] *Etude de la connectivité structurelle des faisceaux d'association courts de la substance blanche du cerveau humain en IRM de diffusion* by Alexandre Pron

moyenne [1]

barycentre

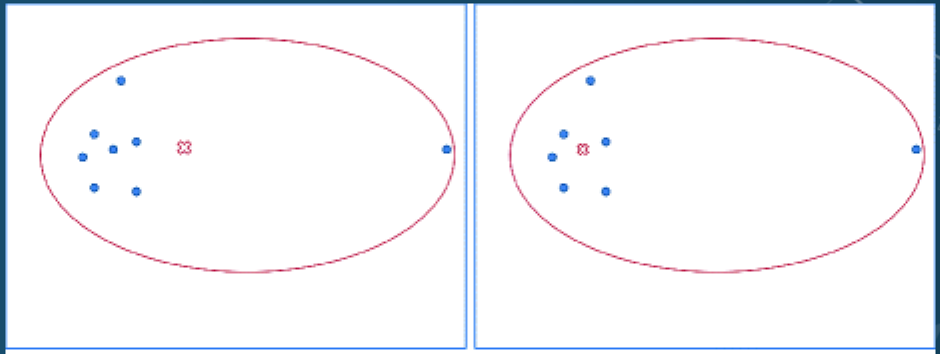


EXISTE-T-IL UNE STRATIFICATION AU SEIN DE NOS SUJETS ?

K-MEDOIDS

"K-Medoids Clustering" in
Encyclopedia of Machine Learning

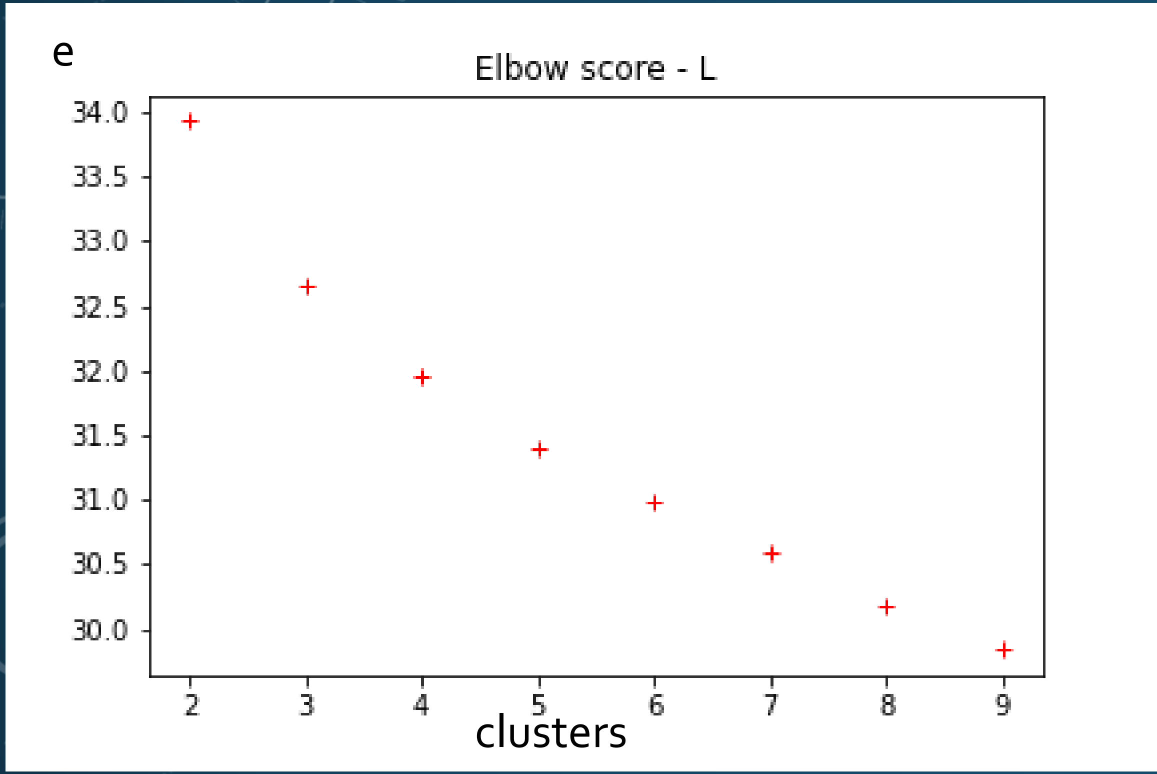
(**DOI:** https://doi.org/10.1007/978-0-387-30164-8_426)



(a) Mean (b) Medoid

SCORE ELBOW

e: la distance intra-cluster moyenne



SCORE SILHOUETTE

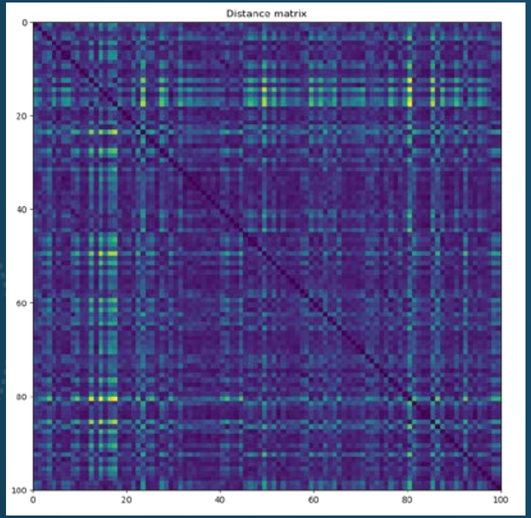
$$s = \frac{a - e}{\max(a, e)}$$

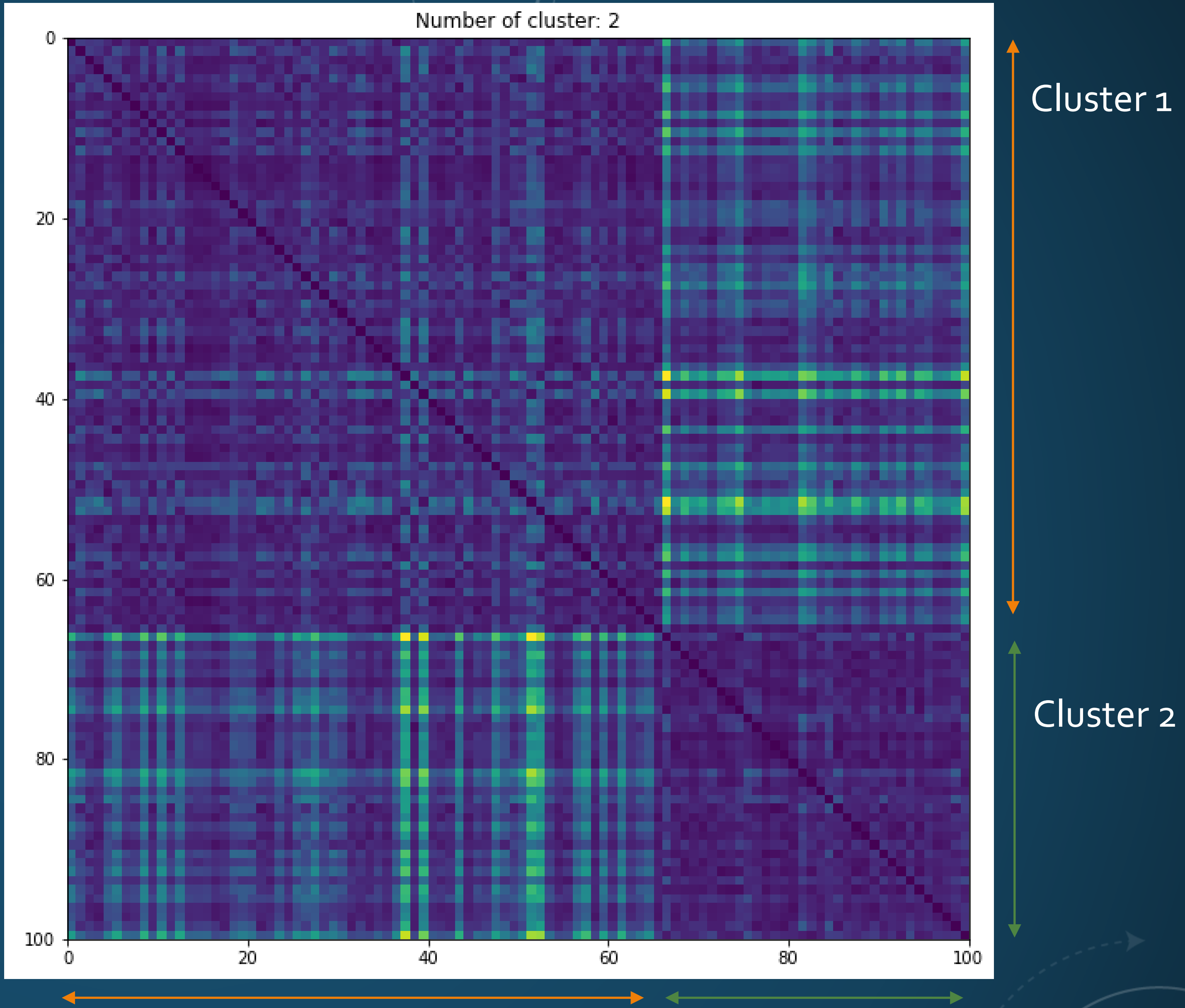
e: la distance intra-cluster moyenne

a: la distance moyenne entre les clusters les plus proches

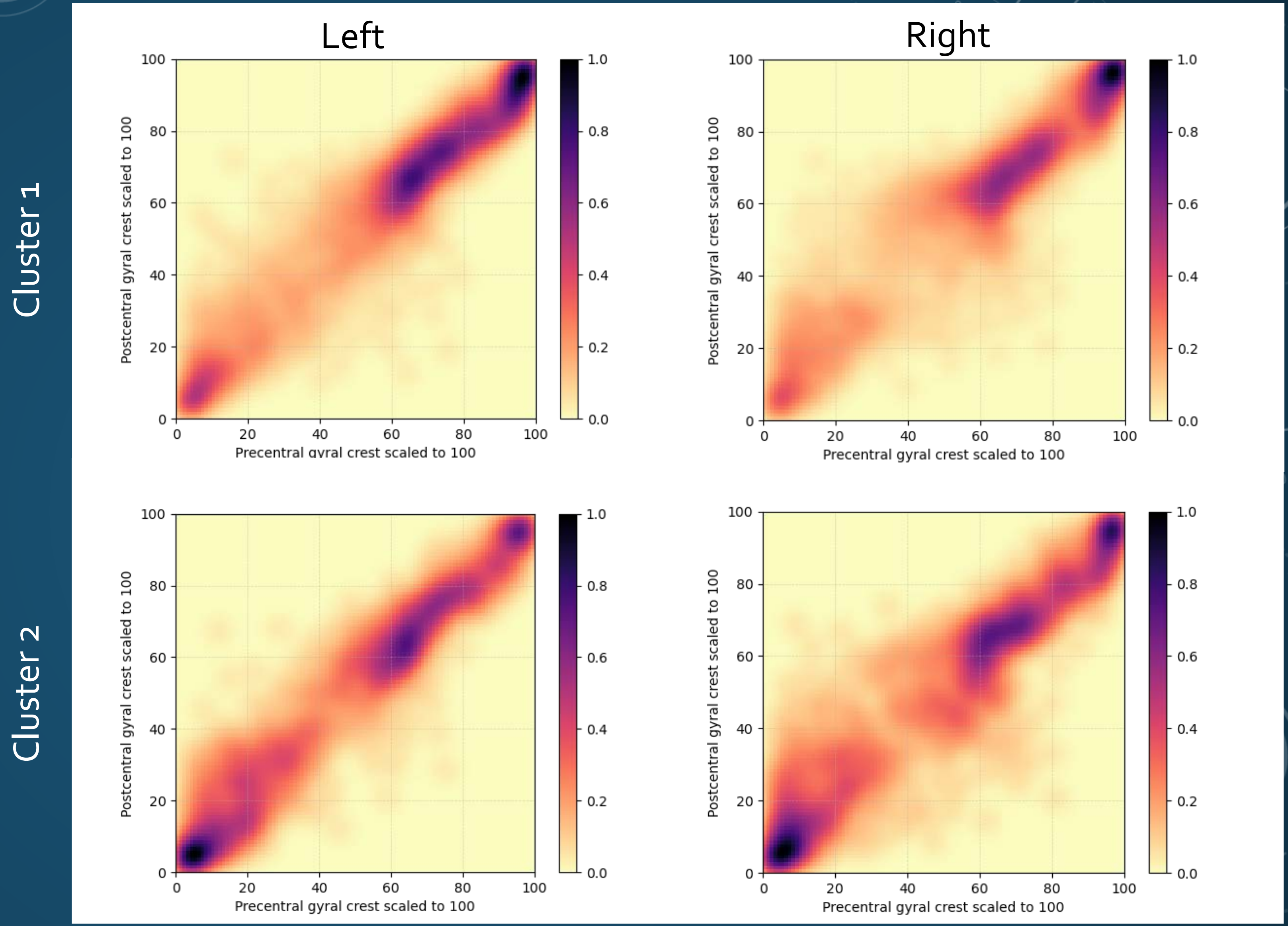
K=2

Matrice de distance réorganisée des sujets





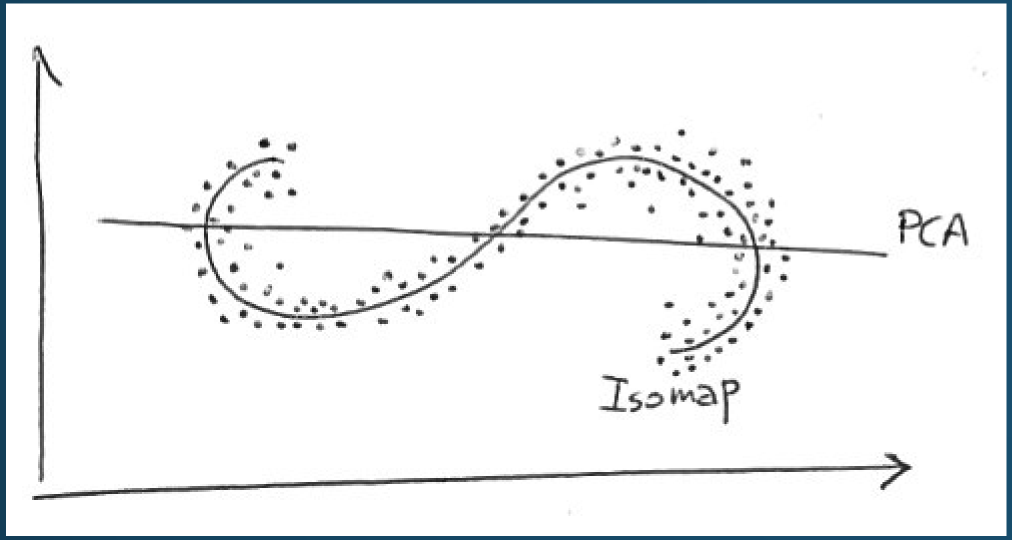
SOUS-BARYCENTRES

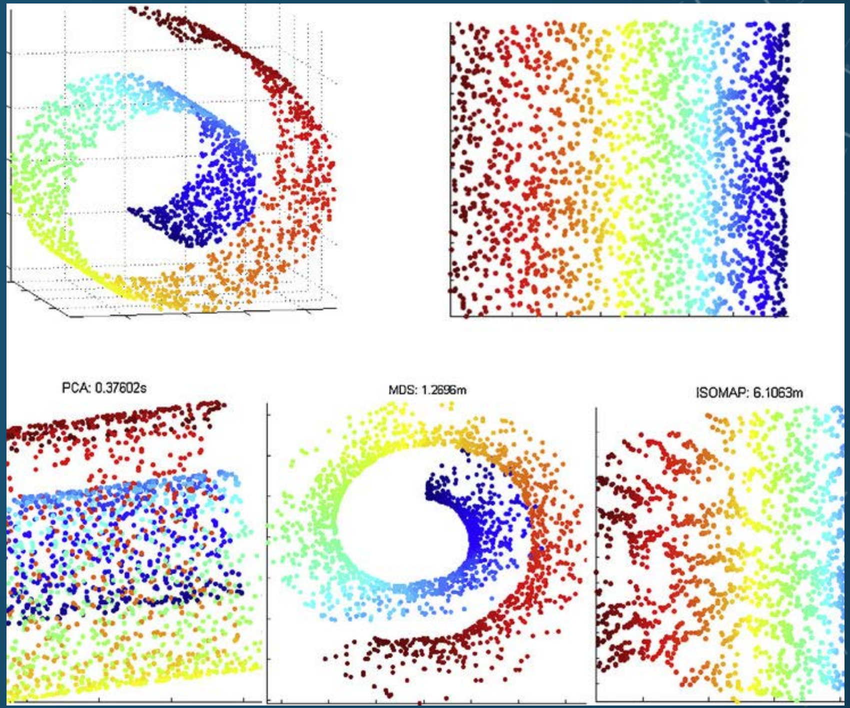


COMMENT ÉTUDIER LA VARIABILITÉ DE NOS SUJETS ?

Isomap:

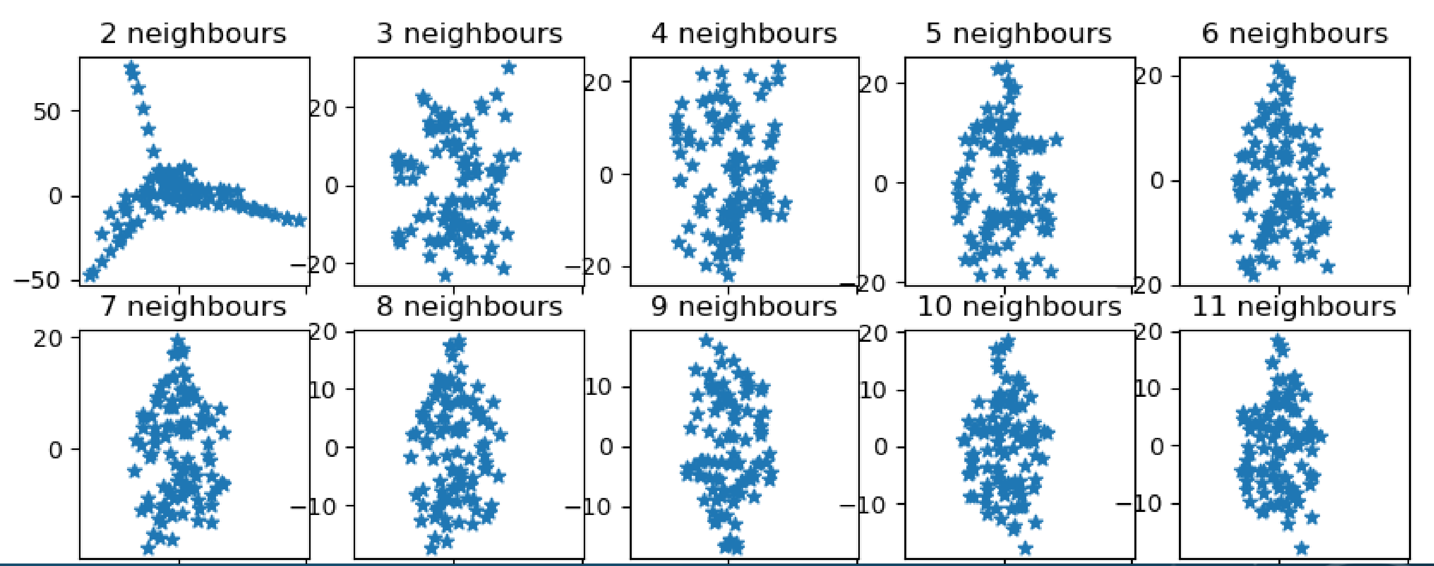
- Principal Component Analysis
- MultiDimensional Scaling





[α] *Multivariate methods for the analysis of complex and big data in forensic sciences. Application to age estimation in living persons*

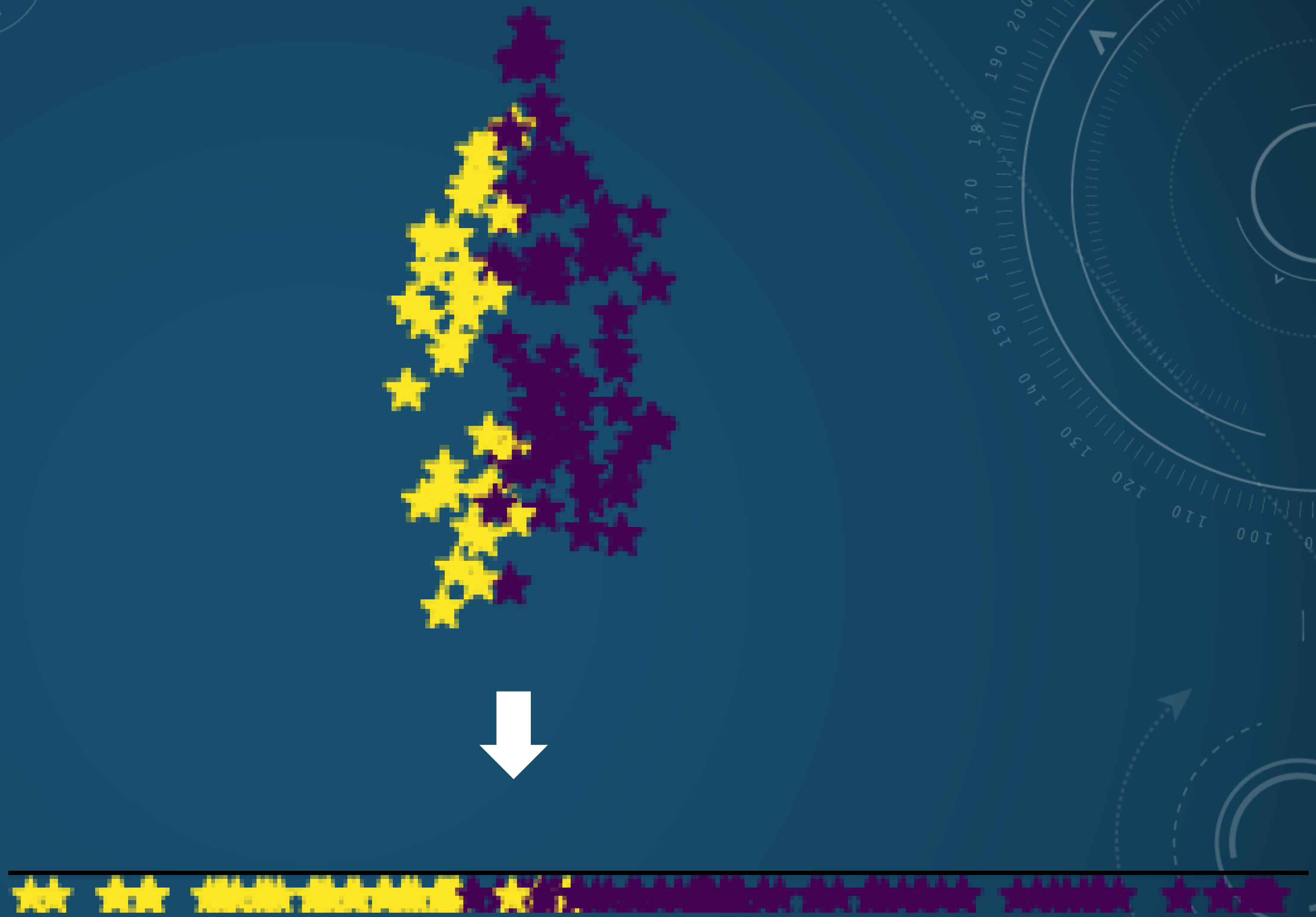
NOMBRE DE VOISINS [7]



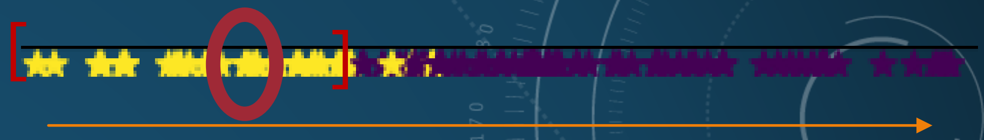
[7] *Selection of the Optimal Parameter Value for the Isomap Algorithm* by Samko, A. D. Marshall, and P. L. Rosin

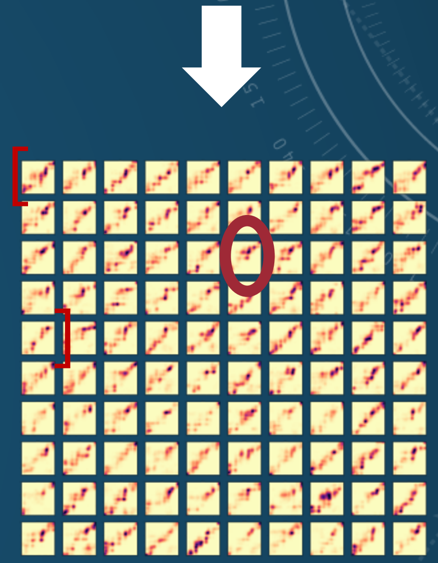
ISOMAP

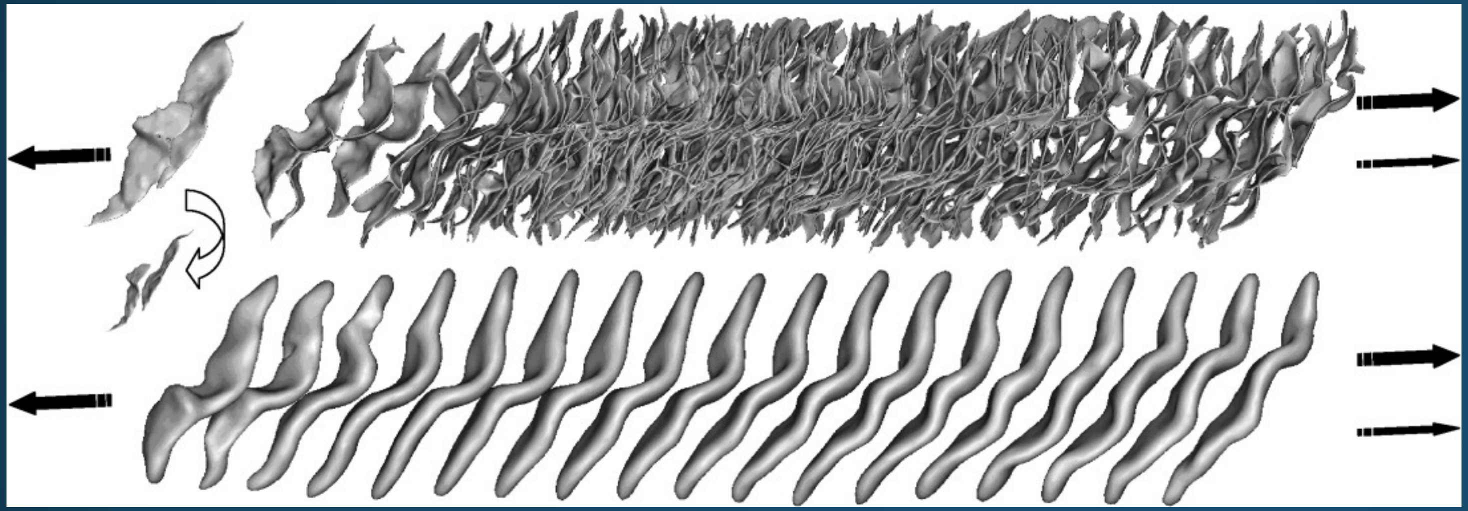
7 voisins



BARYCENTRE GLISSANT



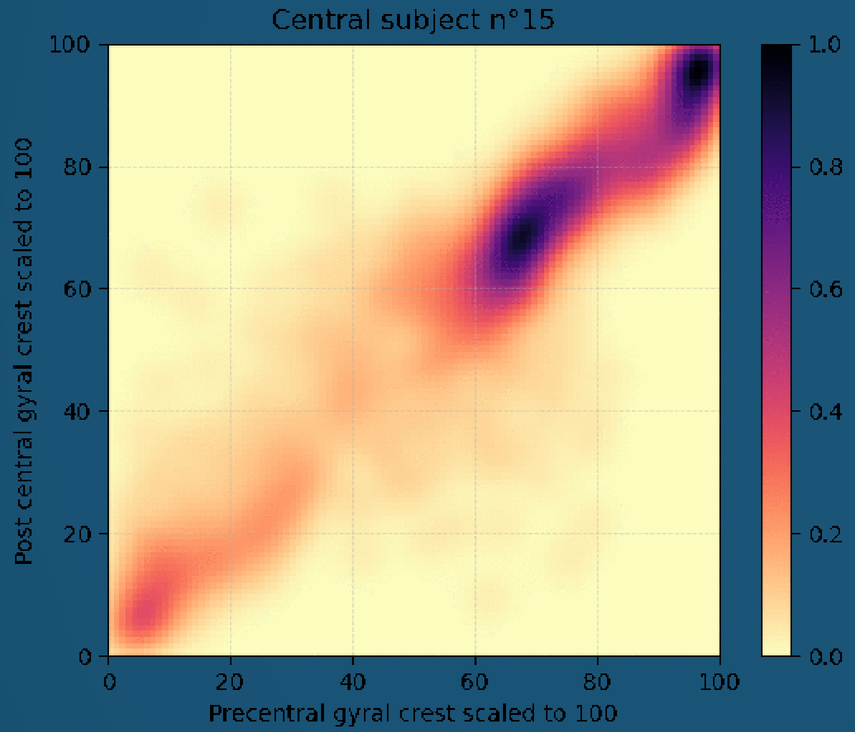


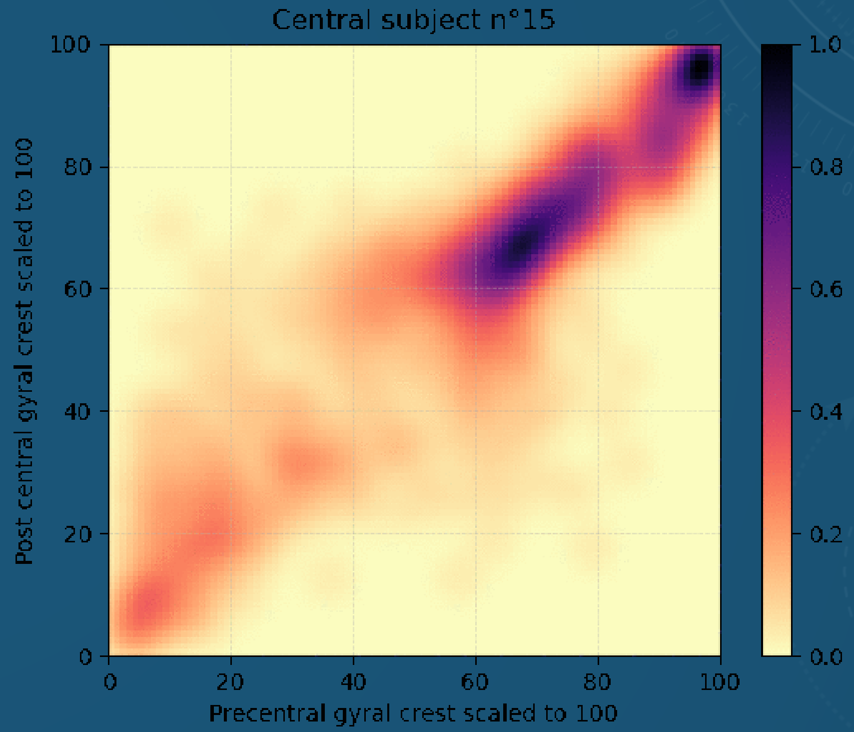


Hand Knob position moving [8]

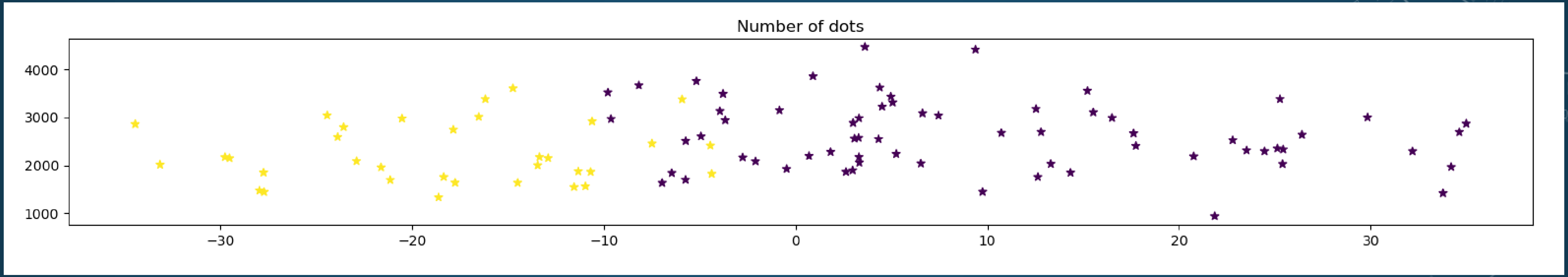
[8] The effect of handedness on the shape of the central sulcus by Zhong Yi Sun et al

BARYCENTRE GLISSANT

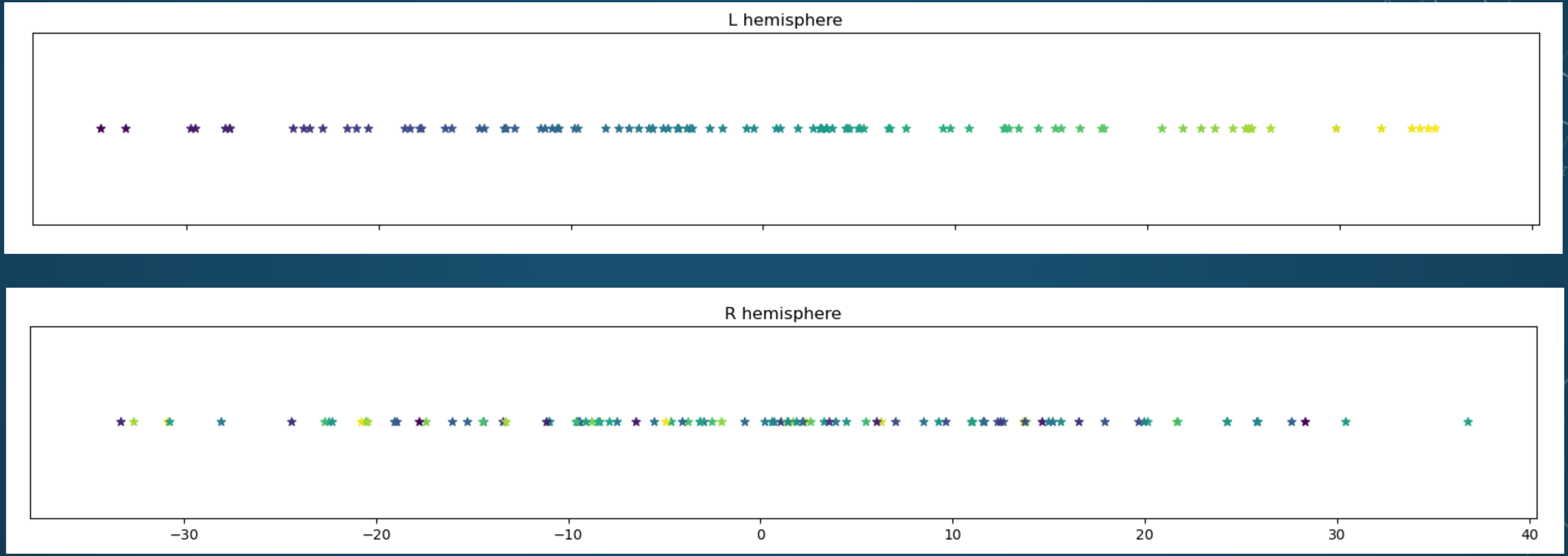




INFLUENCE DU NOMBRE DE POINTS SUR L'AXE DE VARIATION



COMPARAISON ENTRE LES DEUX HÉMISPHÈRES



CONCLUSION ET TRAVAUX FUTURS

Sujet représentatif

- Le barycentre déterminé par la métrique de 2-Wasserstein fournit un sujet représentatif mais c'est une solution plus lisse que celle d'Alexandre.

Clustering

- Il semblerait que le clustering ne soit pas pertinent.

Isomap

- L'axe de variation que nous avons mis en évidence correspondrait à la variation de la position de la tâche centrale et à un déplacement de la densité vers la zone ventrale.

Déterminer le paramètre de variabilité de manière certaine

REFERENCES

[1] Alexandre Pron. « Etude de la connectivité structurelle des faisceaux d'association courts de la substance blanche du cerveau humain en IRM de diffusion ». 2019AIXM0391. PhD thesis. 2019. url: http://www.theses.fr/2019AIXM0391/document (cit. on p. 4).

[2] Villani Cédric. Optimal transport : old and new / Cédric Villani. eng. Grundlehren der mathematischen Wissenschaften. Berlin: Springer, right 2009. isbn: 978-3-540- 71049-3.

[3] Q. Wang, I. Redko, and S. Takerkart. « Population Averaging of Neuroimaging Data Using Lp Distance-based Optimal Transport ». In: 2018 International Workshop on Pattern Recognition in Neuroimaging (PRNI). 2018, pp. 1–4. doi: 10.1109/PRNI. 2018.8423953.

[4] Marco Cuturi and Arnaud Doucet. « Fast Computation of Wasserstein Barycenters ». In: Proceedings of the 31st International Conference on Machine Learning. Ed. by Eric P. Xing and Tony Jebara. Vol. 32. Proceedings of Machine Learning Research 2. Bejing, China: PMLR, 22–24 Jun 2014, pp. 685–693. url: http: //proceedings.mlr.press/v32/cuturi14.html (cit. on p. 5).

[5] Leonard Kaufman and Peter J. Rousseeuw. Clustering by means of medoids. Ed. by In: Dodge Y and editor. Amsterdam: 1987 (cit. on p. 6).

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[7] . Samko, A. D. Marshall, and P. L. Rosin. « Selection of the Optimal Parameter Value for the Isomap Algorithm ». In: Pattern Recogn. Lett. 27.9 (July 2006), pp. 968–979. issn: 0167-8655. doi: 10.1016/j.patrec.2005.11.017. url: https: //doi.org/10.1016/j.patrec.2005.11.017 (cit. on p. 7).

[8] Zhong Yi Sun et al. « The effect of handedness on the shape of the central sulcus ». In: NeuroImage 60.1 (2012), pp. 332–339. issn: 1053-8119. doi: https://doi.org/ 10.1016/j.neuroimage.2011.12.050. url: https://www.sciencedirect.com/ science/article/pii/S1053811911014522 (cit. on p. 13).