

Master on Foundations of Data Science



# Recommender Systems

Collaborative Filtering

Santi Seguí | 2023-2024

**Task:**

**Implement a RecSys method for MovieLens dataset**

- you can used all methods we studied during the course -

**Your notebook should contain a discussion and conclusion of what you have learnt**

**Groups** of up to **2-3 people** are allowed

**How to submit:**

Upload the code at Campus Virtual  
+ share the notebook via Kaggle

**Deadline:** March 31th

# Collaborative-based methods

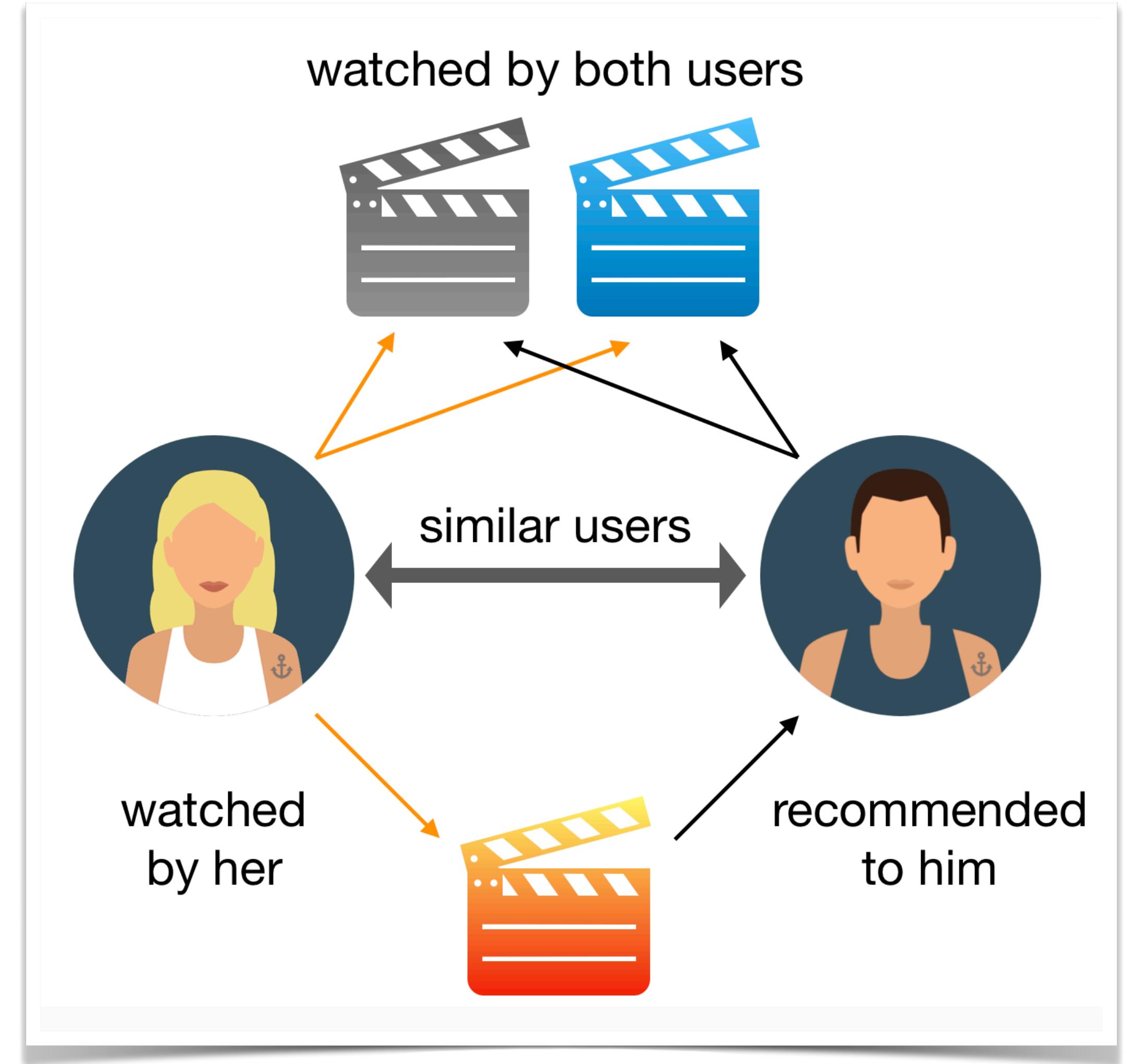
## **Conceptual goal:**

Give me recommendation based on an approach that leverages the ratings and actions of my peers and myself

## **Input:**

User ratings + community ratings

background



background

# Generalization of Supervised Classification

	Features						
	x1	x2	x3	x4	x5	x6	x7
U1	13,1	4	2,34	1	5,3	0,32	?
U2	1,1	3	2	4,5	4,5	9,9	?
U3	4	4,4	4,5	0,3	7,4	2,3	?
U4	9,3	32	3	5	3,2	7,54	?
U5	-2	3	5,3	5,3	3,5	9,9	?
U6	-6,3	46,3	6,2	5	8,3	4,5	?
U7	3,5	5	3,2	5,3	6,2	7,8	?

	Items						
	I1	I2	I3	I4	I5	I6	I7
U1	1	?	?	?	?	?	3
U2	?	3	?	4,5	4,5	?	?
U3	4	?	4,5	?	?	?	4
U4	?	?	3	5	?	?	?
U5	?	3	?	?	3,5	?	?
U6	?	?	?	5	?	4,5	3
U7	3,5	5	?	5	?	?	3

background

Using

# COLLABORATIVE FILTERING

to Weave an Information  
**TAPESTRY**

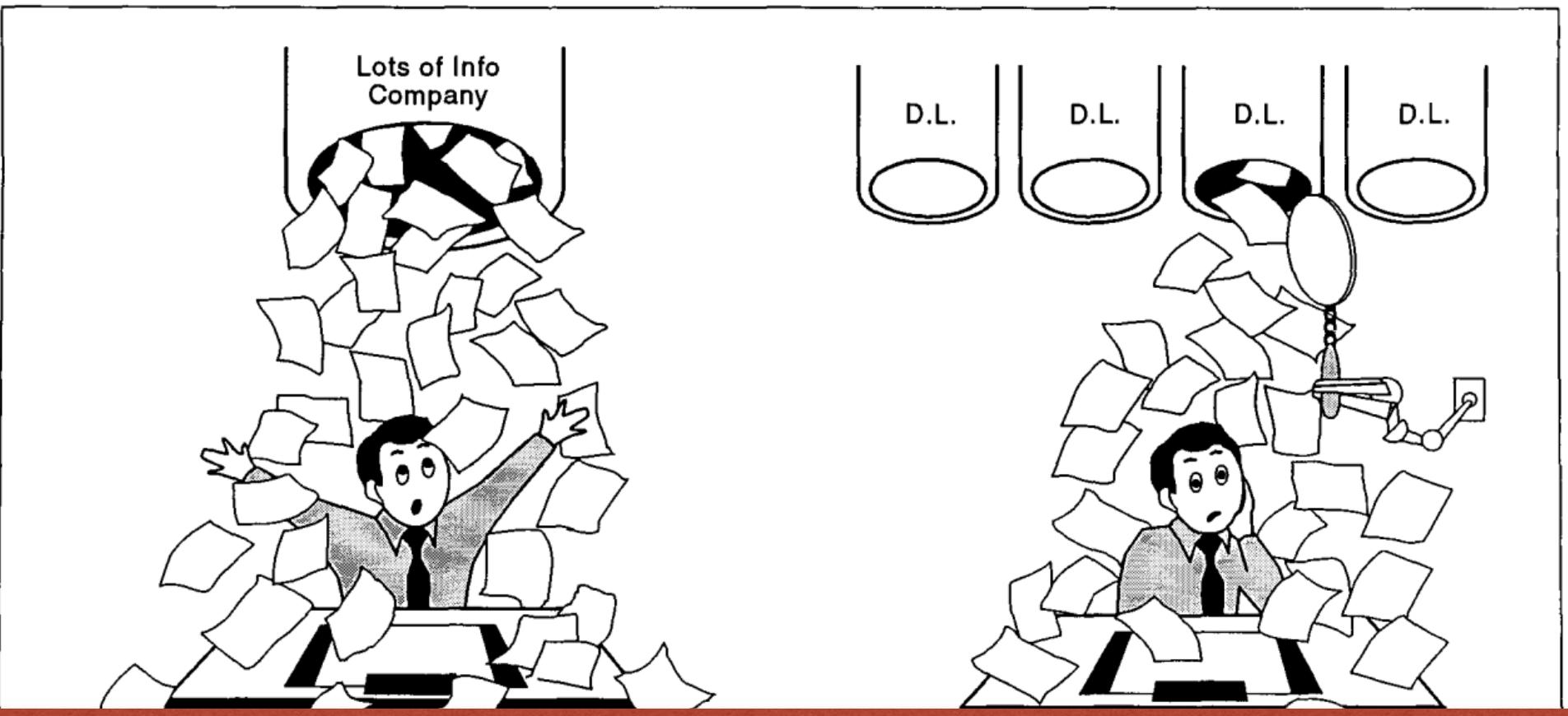
*David Goldberg, David Nichols, Brian M. Oki, and Douglas Terry*

Tapestry is an experimental mail system developed at the Xerox Palo Alto Research Center. The motivation for Tapestry comes from the increasing use of electronic mail, which is resulting in users being inundated by a huge stream of incoming documents [2, 7, 12]. One way to handle large volumes of mail is to provide mailing lists, enabling users to subscribe only to those lists of interest to them. However, as illustrated in Figure 1, the set of documents of interest to a particular user rarely map neatly to existing lists. A better solution is for a user to specify a *filter* that scans all lists, selecting interesting documents no matter what list they are in. Several mail systems support filtering based on a document's contents [3, 5, 6, 8]. A basic tenet of the Tapestry work is that more effective filtering can be done by involving humans in the filtering process.

Using collaborative filtering to weave an information tapestry

D Goldberg, D Nichols, BM Oki, D Terry  
Communications of the ACM 35 (12), 61-70

3887 1992



Collaborative filtering simply means that people collaborate to help one another perform filtering by recording their reactions to documents they read

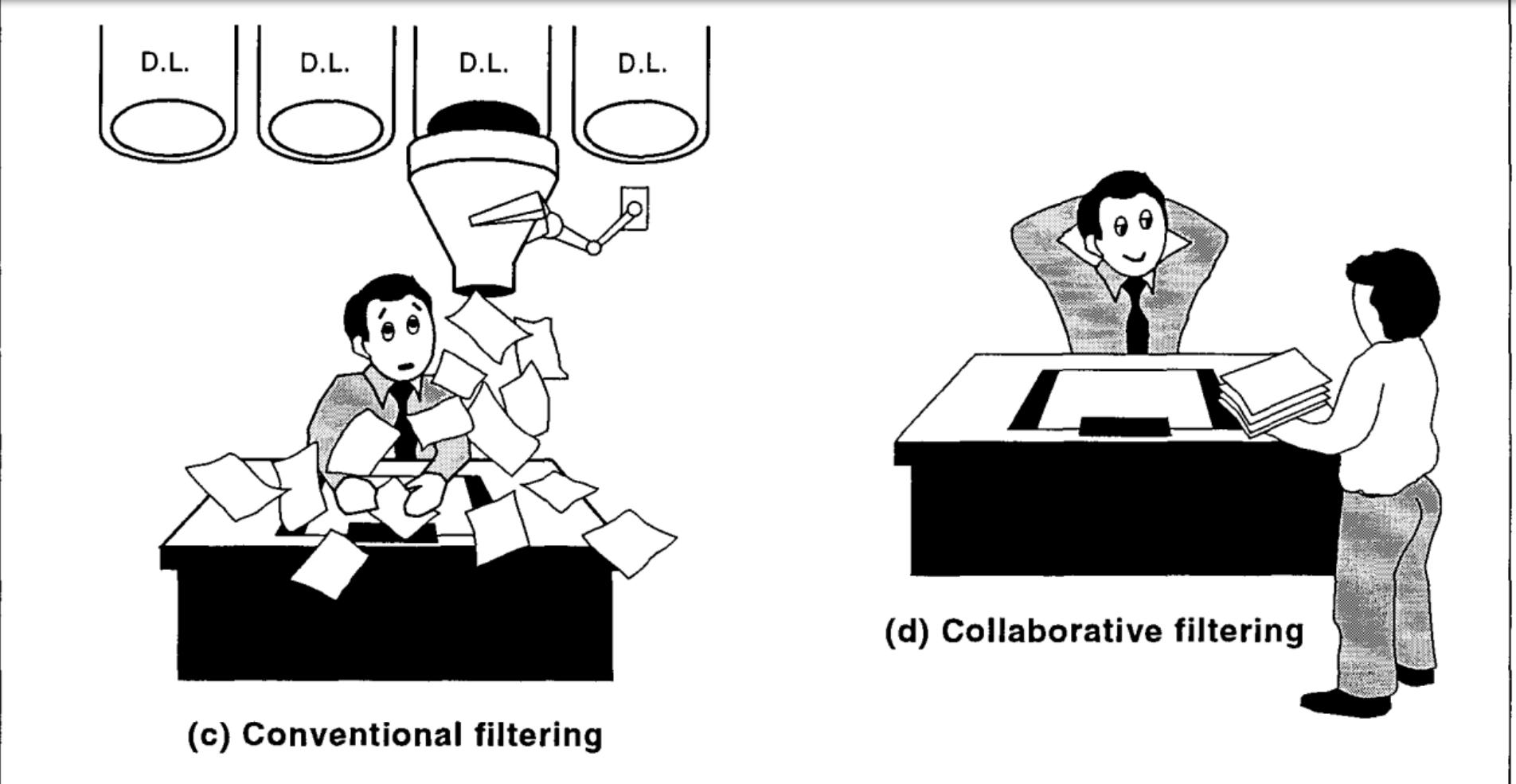


Image from : <http://dl.acm.org/citation.cfm?id=138867>

### Using collaborative filtering to weave an information tapestry

D Goldberg, D Nichols, BM Oki, D Terry  
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# Collaborative Filtering

- Collaborative filtering methods are based on collecting and **analyzing a large amount of information** on users' behaviors, activities or preferences and predicting what users will like based on their **similarity to other users**.
- Hypothesis: **Similar users tend to like similar items.**
- Requires a user community.

background

# Collaborative Filtering

- $R$  is the  $M \times N$  rating matrix where  $M$  is the number of users and  $N$  the number of items.
- Rating can be defined in a variety of ways:
  - **Continuous ratings:** from -10 to 10
  - **Interval-based ratings:** 5 stars, 3 stars
  - **Ordinal ratings:** {strongly disagree, disagree, neutral, agree and strongly agree}
  - **Binary ratings:** Like/dislike
  - **Unary ratings:** Buy

background

# Collaborative Filtering Problems:

- **Cold Start**: There needs to be enough other users already in the system to find a match.
- **Sparsity**: If there are many items to be recommended, even if there are many users, the user/ratings matrix is sparse, and it is hard to find users that have rated the same items.
- **First Rater**: Cannot recommend an item that has not been previously rated.
  - New items
  - Esoteric items
- **Popularity Bias**: Cannot recommend items to someone with unique tastes.
  - Tends to recommend popular items.

background

# Two main approaches

## 1. Memory-based methods

- **Neighbourhood-based methods** focused on computing the relationship between items or between users.
  - Two sub approaches:
    - **User based** and **Item based** Collaborative Filtering

## 2. Model-based methods

- **Latent factor methods** that explains the rating by characterizing both items and users on many factors inferred from the rating pattern

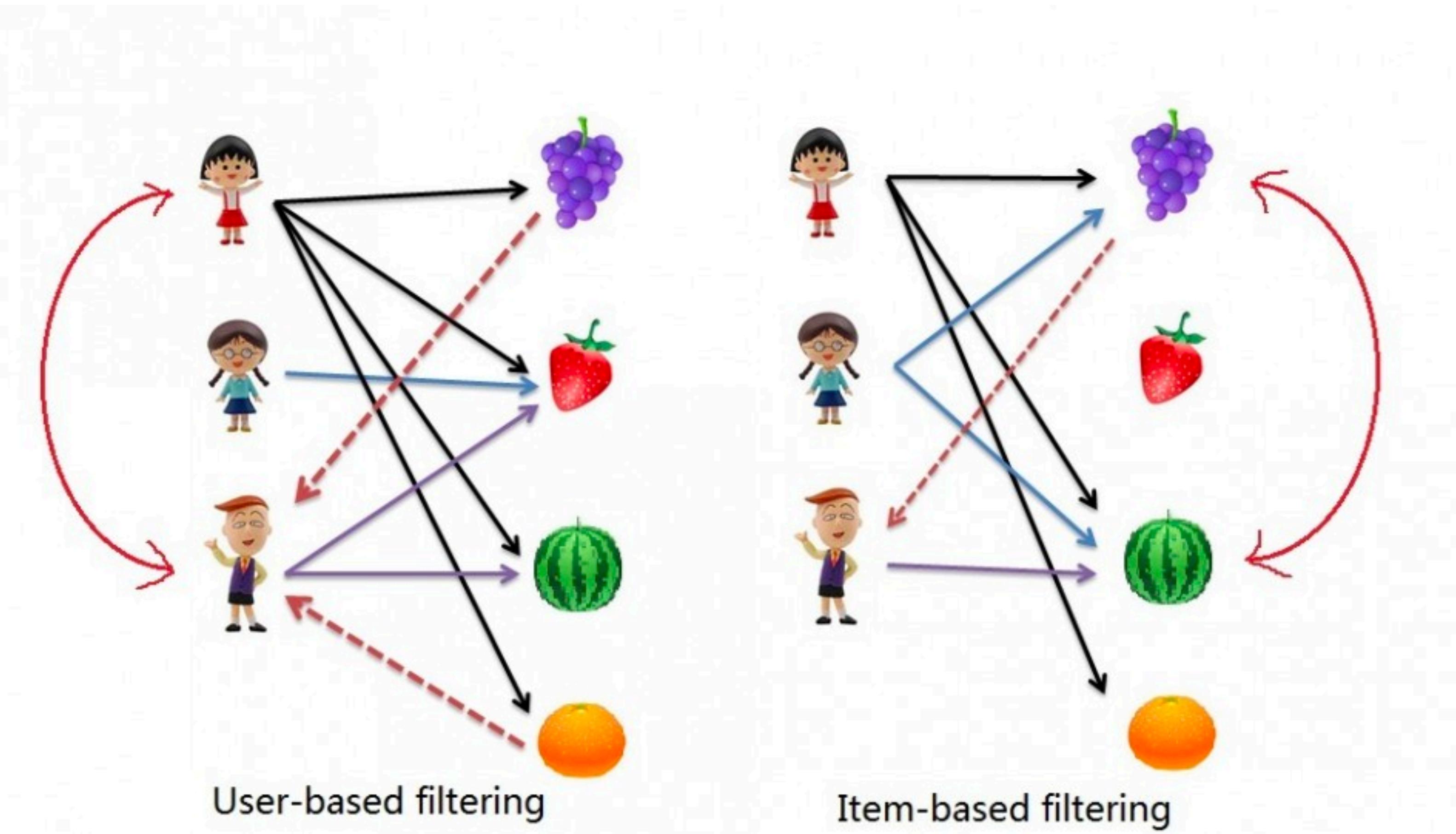
background

# 1 - Neighbourhood methods

# Neighborhood-based methods

- Neighbourhood-based methods were among the earliest algorithms developed for collaborative filtering. These methods are based on the fact that similar users display similar patterns or rating behaviors and similar items receive similar ratings.
- There are two primary types of neighbourhood -based algorithms:
  - **User-based** CF works like this: take a user  $U$  and a set of other users  $D$  whose ratings are similar to the ratings of the selected user  $U$  (user with similar preferences). And, use the ratings from those like-minded users to calculate a prediction for the selected user  $U$ .
  - In **Item-based** CF you build an item-item matrix determining relationships between pairs of items and using this matrix and data on the current user, infer the user's taste.

# Neighborhood-based methods



Let's see how we can create a **User-Based CF** for Movie recommendations.

# Example: Movie Recommender System.

## User-Based Collaborative Filtering

- Given an "active **user**" and an **item** that has not been seen by the **user**, the goal is to estimate the rating for the **item**.

	Superman	Star Wars 1	Matrix	Spiderman
Santi	3	3.5	4.5	-
Jake	3.5	4	5	5
Anne	3	-	4.5	3
Caroline	3.5	5	3.5	2

# The basic technique

- User-based nearest-neighbour collaborative filtering
  - Given an "**active user**" (e.g. *Santi*) and the items not yet seen (e.g. *Spiderman*) the goal is to estimate Santi's rating for the those (e.g. *Spiderman*) items,
  - find a set of **users** (peers) **who have rated the item** (*Spiderman*) **and has similar tastes than the active user** (*Santi*) in the past and estimate the rating for the active user, e.g. the weighted averaged with similarly between the users and the given score.
  - Repeat this estimation for all items not seen by the active user (*Santi*) and recommend the best-rated

	Superman	Star Wars 1	Matrix	Spiderman
Santi	3	3.5	4.5	-
Jake	3.5	4	5	5
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Caroline	3.5	5	3.5	2

# What is needed?

- We need to define two things:
  1. How to compute **similarities** between users
  2. How to make the **prediction** using the similar users

# How to measure **similarity** between users?

- The computation of the **similarity** between the items is one **critical step** in the CF algorithms.
- The basic idea in similarity computation between two users  $a$  and  $b$  is to first isolate the items commonly rated by both users (set  $P$ ), and then to apply a similarity computation technique to determine the similarity.

# Similarity Measures

- Euclidean Distance
- Pearson Correlation
- Person Correlation corrected
- Spearman Correlation
- Cosine Distance

# How to measure similarity between users?

- Euclidean distance

$$sim(a, b) = \sqrt{\sum_{p \in P} (r_{a,p} - r_{b,p})^2}$$

- Pearson Correlation

$$sim(a, b) = \frac{\sum_{p \in P} (r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_{p \in P} (r_{a,p} - \bar{r}_a)^2} \sqrt{\sum_{p \in P} (r_{b,p} - \bar{r}_b)^2}}$$

- Cosine distance

$$sim(a, b) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| * |\vec{b}|}$$

Where:

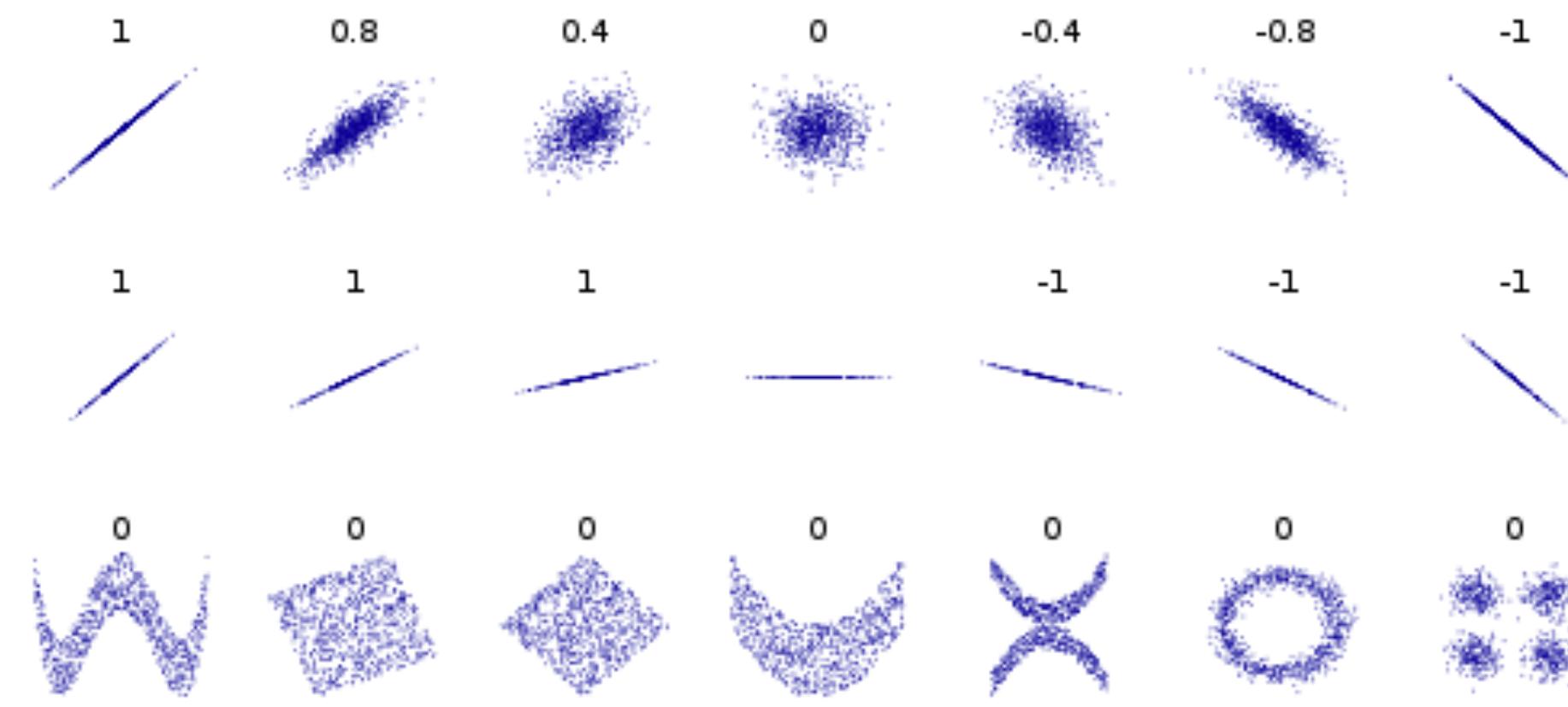
- $sim(a, b)$  is the similarity between user "a" and user "b"
- $P$  is the set of common rated movies by user "a" and "b"
- $r_{a,p}$  is the rating of movie "p" by user "a"
- $\bar{r}_a$  is the mean rating given by user "a"

# Similarity Measures: Euclidean distance

$$sim(u, v) = \sqrt{\sum_{j \in P} (r_{uj} - r_{vj})^2}$$

**CAUTION:** if the users use to rate with a different mean and standard deviation euclidean distance can give some problems

# Similarity Measures: Pearson Correlation



**Negative** Values?

Strange correlations are rare,  
and do not carries interesting information

# Hands on time!



# Which is the best similarity function?

- .... there is not a clear answer...but, there are some tips:
  - Pearson Correlation used to work better than euclidean distance since it is based more on the ranking than on the values.
  - In general, Pearson Correlation coefficient is preferable to the raw cosine because of the bias adjustment effect of mean-centering
  - Cosine distance is usually used when our data is binary/unary, i.e. “like vs. not like” or “buy vs. not buy”.

# Similarity Measures

- *Significance weighting:* When two users (or items) have **very few** items (or users) **in common** the **reliability** of the similarity scores is **low**. In these cases the similarity score can be reduced with a discount factor to de-emphasize the importance of that user pair.

$$sim_c(u, v) = sim(u, v) \times \frac{\min(50, |I_u \cap I_v|)}{50}$$

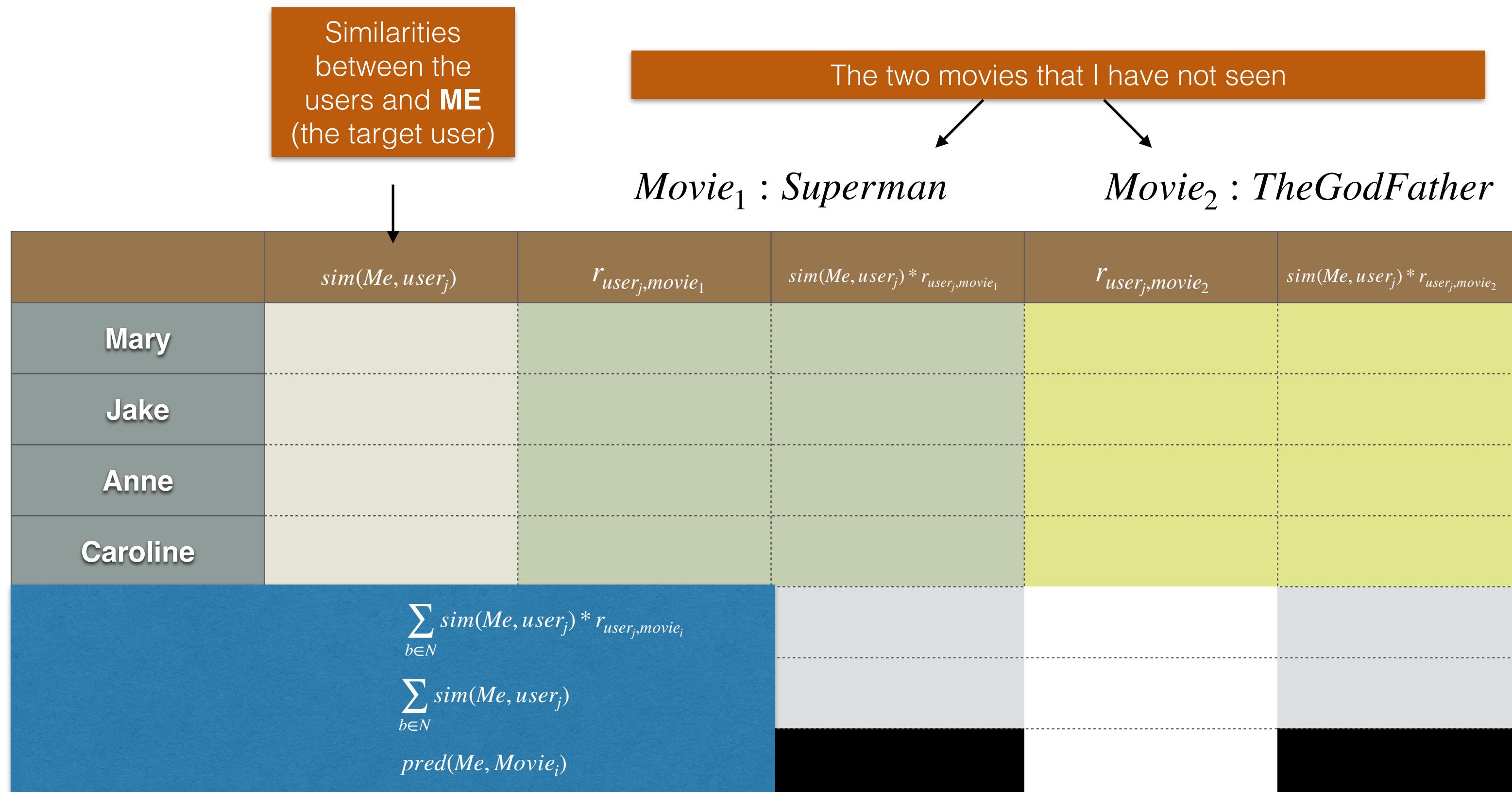
# How do we generate a prediction from the neighbour's ratings?

$$\hat{r}_{u,j} = \frac{\sum_{v \in P_u(j)} sim(u, v) \times r_{v,j}}{\sum_{v \in P_u(j)} sim(u, v)}$$

the similarity of u and v  
the rating of the user v to the movie j  
The sum of the weight used  
the set of top-k similar users

Where  $P_u(j)$  is denoted to the set of the top-k similar users to target user  $u$ ,  $sim(u, v)$  the similarity between user  $u$  and  $v$  and  $r_{u,j}$  the rating of the user  $u$  to the movie  $j$ .

# How do we generate a prediction from the neighbour's ratings?



# How do we generate a prediction from the neighbour's ratings?

Similarities between the users and **ME** (the target user)

The two movies that I have not seen

	$sim(Me, user_j)$	$r_{user_j, movie_1}$	$sim(Me, user_j) * r_{user_j, movie_1}$	$r_{user_j, movie_2}$	$sim(Me, user_j) * r_{user_j, movie_2}$
Mary	0.99	3	2.97	2.5	2.48
Jake	0.38	3	1.14	3	1.14
Anne	0.89	4.5	4.0	-	-
Caroline	0.92	3	2.77	3	2.77
	$\sum_{b \in N} sim(Me, user_j) * r_{user_j, movie_i}$		10.87		6.39
	$\sum_{b \in N} sim(Me, user_j)$		3.18		2.29
	$pred(Me, Movie_i)$		<b>3.41</b>		<b>2.79</b>

# Other prediction functions

- **Different users** may provide ratings on **different scales**. Some users rate all items highly, whereas another rate all items negatively

$$\hat{r}_{u,j} = \bar{r}_u + \frac{\sum_{v \in P_u(j)} sim(u, v) \times (r_{v,j} - \bar{r}_v)}{\sum_{v \in P_u(j)} sim(u, v)}$$

The average bias of the user u

The difference between the rank of the user v to movie j minus the average bias of user v

**Caution:** predicted scores can goes outside the range.  
However, the rank is correct.

# Other prediction functions

$$\hat{r}_{u,j} = \bar{r}_u + \sigma_u \frac{\sum_{v \in P_u(j)} sim(u, v) \times z_{v,j}}{\sum_{v \in P_u(j)} |sim(u, v)|}$$

Where  $z_{uj}$  is the standardized rating computed as follows:

$$z_{uj} = \frac{r_{uj} - \bar{r}_u}{\sigma_u} = \frac{s_{uj}}{\sigma_u}$$

and  $\sigma_u = \sqrt{\frac{\sum_{j \in I_u} (r_{uj} - \bar{r}_u)^2}{|I_u| - 1}}$

**Caution:** predicted scores can goes outside the range.  
However, the rank is correct.

# Some tricks

# Some tricks (I)

- **Top-k** most similar users to the target user in order to do the predictions.
  - Weakly correlated users might add to the error in prediction, as well as, negative correlations often do not have a predictive value.

# Some tricks (II)

**Recursive methods:** In order to avoid cold-start we can apply a recursive method for new users or for sparse data sets.



# Some tricks (III)

**Similarity amplification:**

$$sim(u, v) = Pearson(u, v)^\alpha$$

Where  $\alpha > 1$

# Some tricks (IV)

- **Clustering** applied as a “first step” for **shrinking** the candidate set of users.
- Computing the similarity between users is computationally expensive.
- Clustering is cheaper to than computing the  $MxM$  similarity matrix
- Redefine top similar users using only the subset of users in the same cluster
- Problem:  $MxN$  is an incomplete, really sparse, matrix.

# Some tricks (V): Long Tail Problem

- Difficult to provide good rating prediction for those item in the long tail.
- **Popular items** use to provide **less information about tastes** than non-popular items
- Usually, **popular** items provide **less profit** than non-popular.
- Usually, **non-popular** items generates more **surprise** to the user

# Impact of the long Tail

Some movies are really popular since other very unpopular. **Popular items** can sometimes **worsen the quality** of the recommendations since they tend to be **less discriminative** across different users

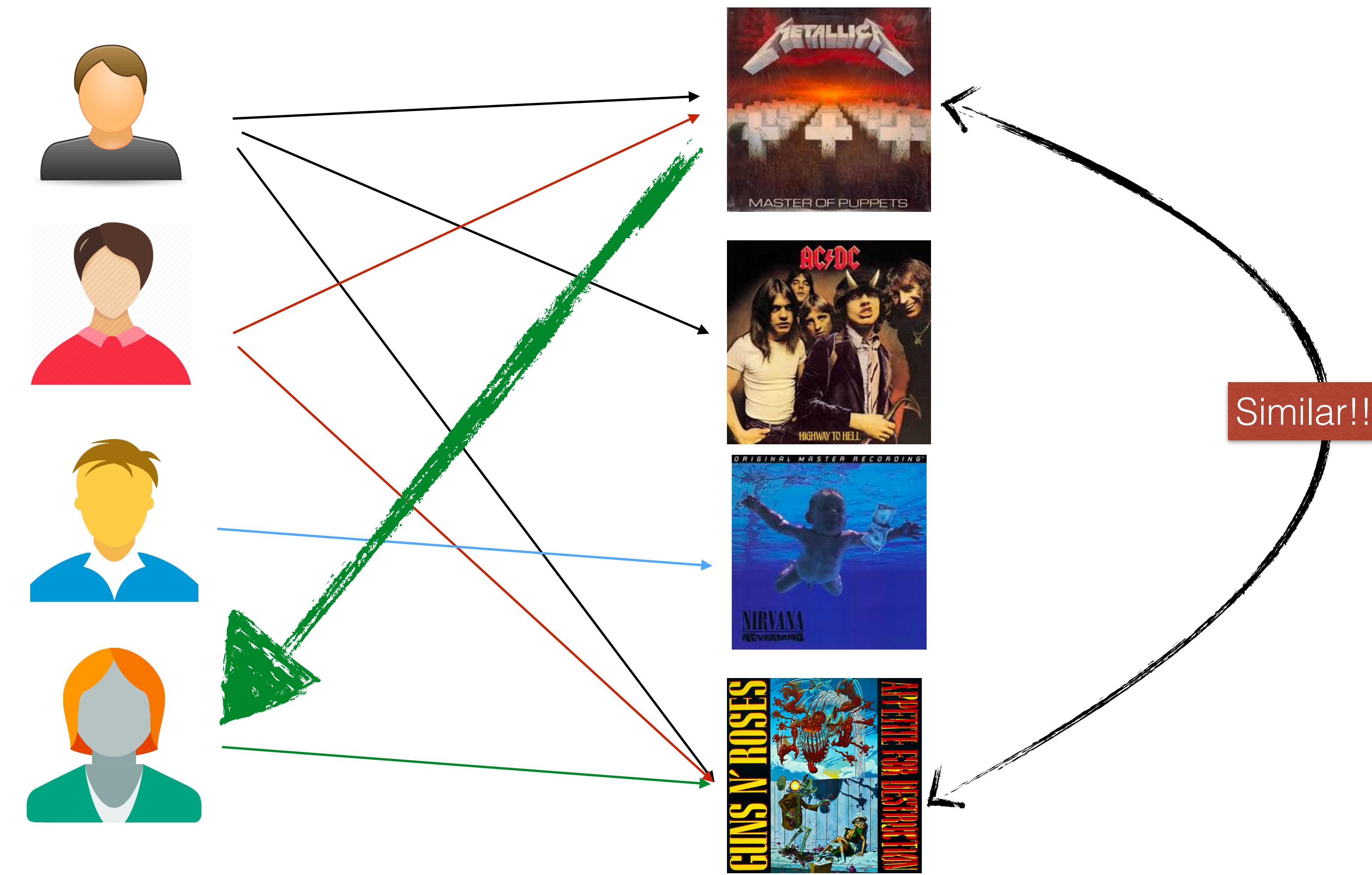
Each item  $j$  can be weighted by  $w_j$  as follows

$$w_j = \log\left(\frac{m}{m_j}\right)$$

where  $m$  is the total number of users, and  $m_j$  is the number of users who have rated the item  $j$

$$\text{Pearson}(u, v) = \frac{\sum_{k \in I_u \cap I_v} w_k (r_{uk} - \bar{r}_u)(r_{vk} - \bar{r}_v)}{\sqrt{\sum_{k \in I_u \cap I_v} w_k (r_{uk} - \bar{r}_u)^2} \sqrt{\sum_{k \in I_u \cap I_v} w_k (r_{vk} - \bar{r}_v)^2}}$$

# Item-Based Recommender



Memory-based Methods

Let's see how we can create an **Item-Based CF**  
for Movie recommendations.

# Item-based Recommenders

- Instead of relying on the user similarity, prediction can rely on **item similarities**.
- Item similarity used to be **more stable** than user-similarity. So, the update frequency of the items similarity is not as critical than user-similarity
- Item-similarities are more static, while user-similarities are more dynamic

Item-based collaborative filtering recommendation algorithms

B Sarwar, G Karypis, J Konstan, J Riedl

Proceedings of the 10th international conference on World Wide Web, 285-295

5944

2001

# Similarity Measures: What happens with item-base systems?

- Pearson Correlation

$$sim(a, b) = \frac{\sum_{p \in P} (r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_{p \in P} (r_{a,p} - \bar{r}_a)^2} \sqrt{\sum_{p \in P} (r_{b,p} - \bar{r}_b)^2}}$$

- Cosine distance

$$sim(a, b) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| * |\vec{b}|}$$

Where:

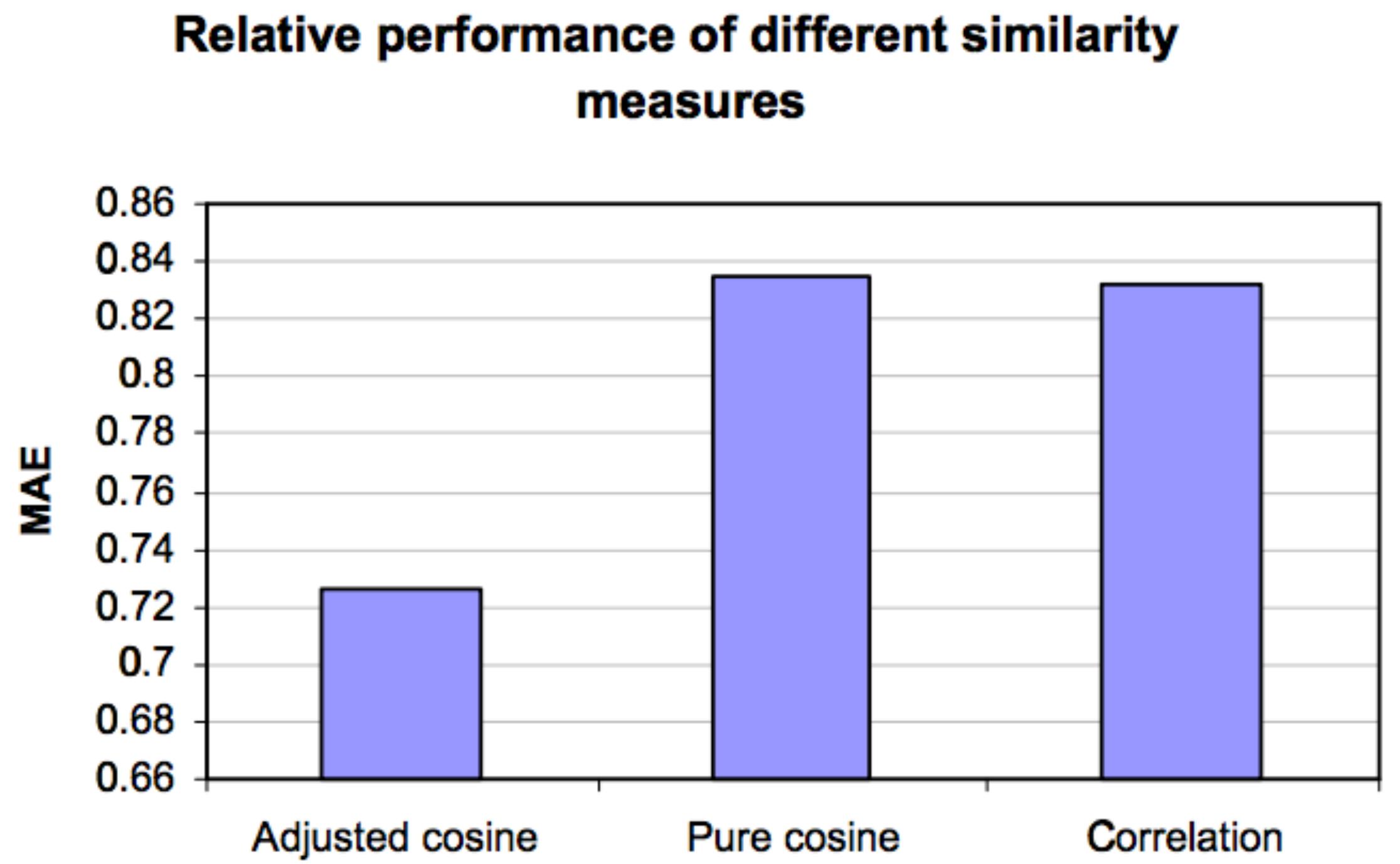
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Are these measures good?

# Adjusted Cosine Similarity

- Computing similarity using basic cosine measure in item-based case has one important drawback: **The differences in rating scale between different users are not taken into account.**
- The Adjusted Cosine Similarity offsets this drawback by subtracting the corresponding user average from each co-rated pair:

$$sim(i, j) = \frac{\sum_{u \in U} (R_{u,i} - \bar{R}_u)(R_{u,j} - \bar{R}_u)}{\sqrt{\sum_{u \in U} (R_{u,i} - \bar{R}_u)^2} \sqrt{\sum_{u \in U} (R_{u,j} - \bar{R}_u)^2}}$$



**Figure 4: Impact of the similarity computation measure on item-based collaborative filtering algorithm.**

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# Item-Based: How do we generate a prediction?

$$\hat{r}_{u,j} = \bar{r}_u + \frac{\sum_{v \in P_u(j)} sim(u, v) \times (r_{v,j} - \bar{r}_v)}{\sum_{v \in P_u(j)} sim(u, v)}$$

Why not another equation?

# User-Based vs. Item-Based

- $m = \#users ; n = \#items$
- Normally, the number of users is much bigger than the number of items.

Computational time:

$O(m^2 n)$

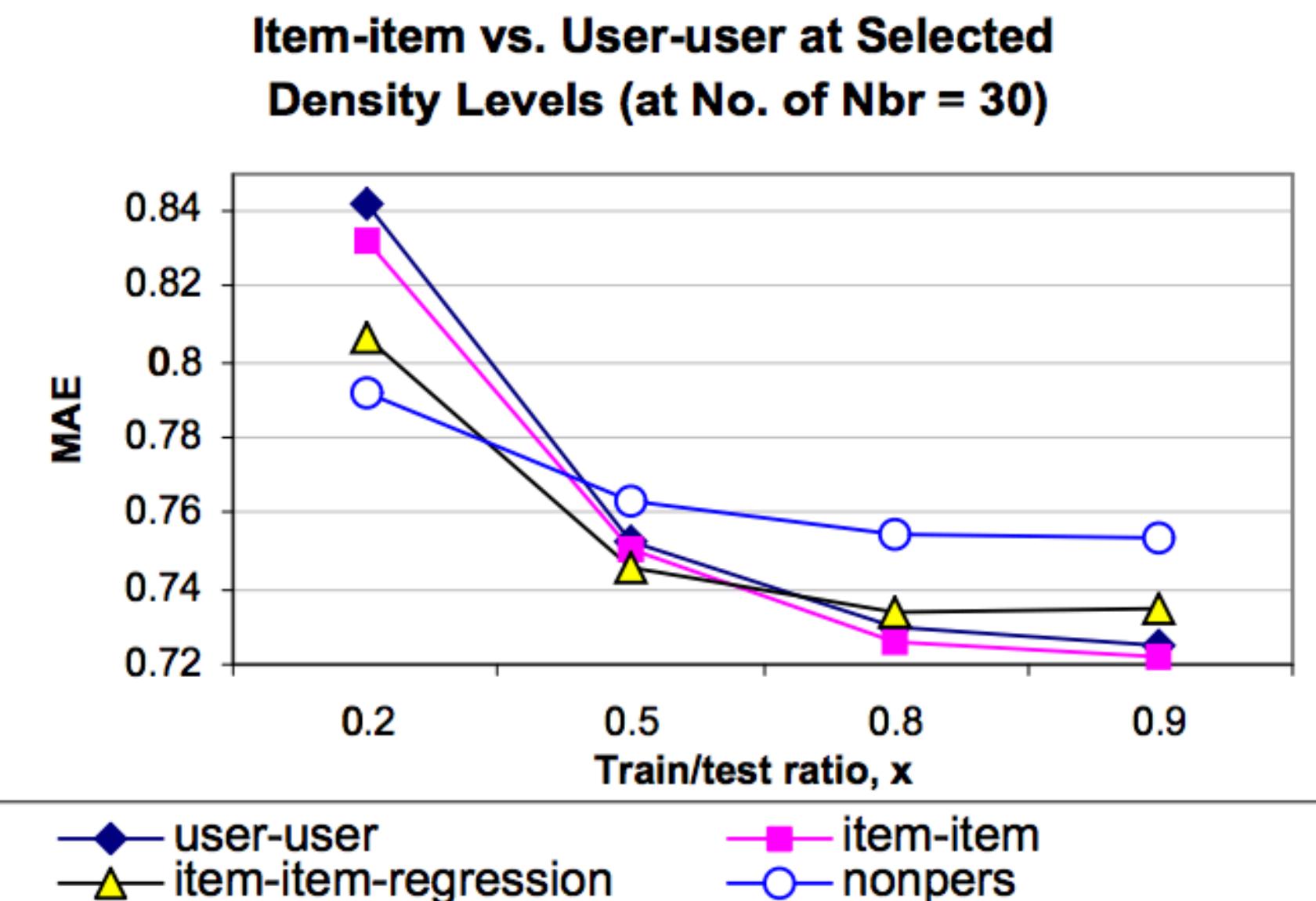
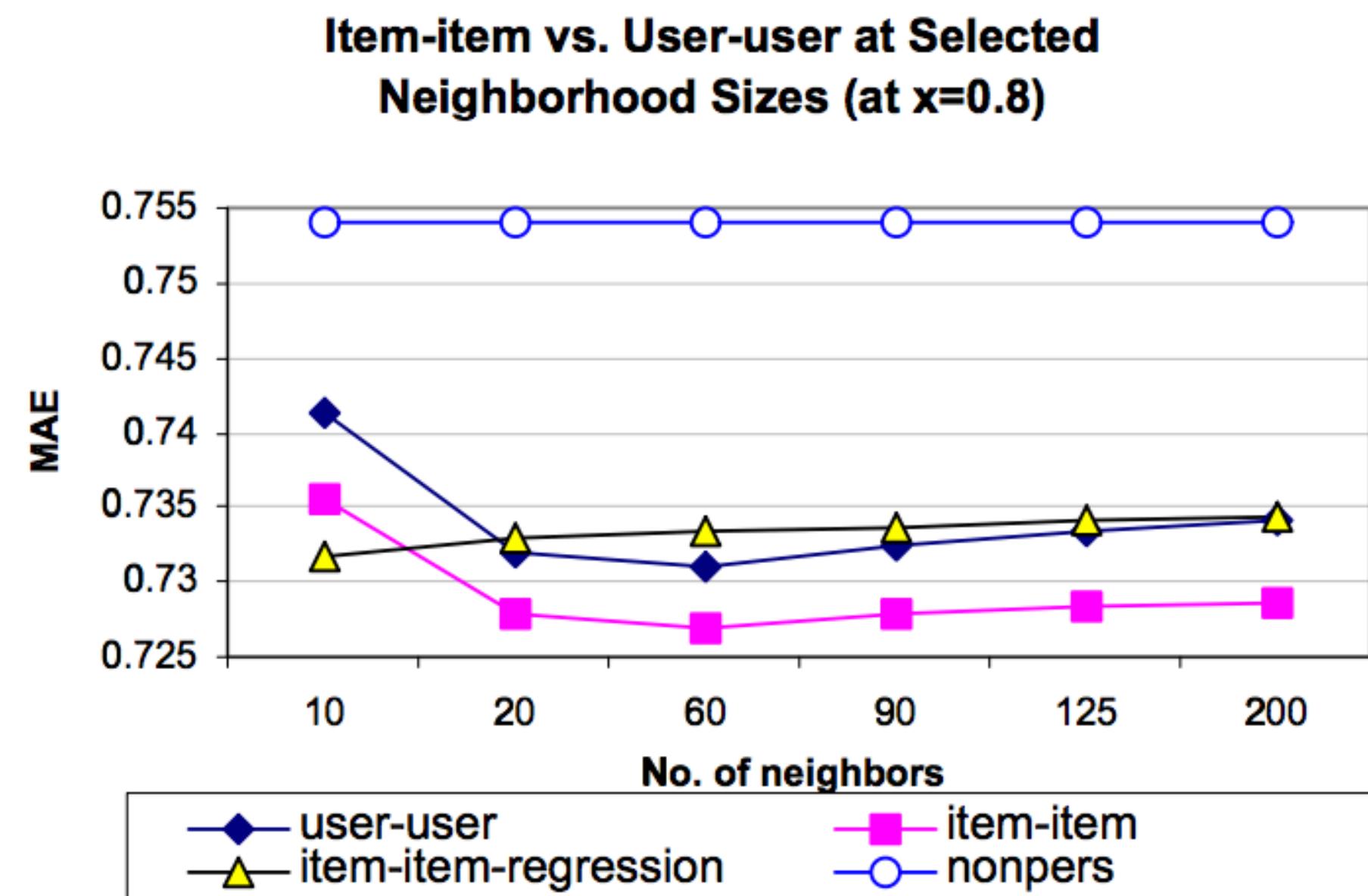
$O(n^2 m)$

Memory Requirements:

$O(m^2)$

$O(n^2)$

# User-Based vs. Item-Based



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# User-Based vs. Item-Based

- **Pros User-based**
  - Tend to provide higher diversity (more serendipity)
- **Pros Item-based**
  - Better results (in terms of RMSE)
  - More stable to changes

# User-Based vs. Item-Based

	User-Based	Item-Based
Scalability		
Explanation		
Novelty		
Coverage		
Cold start		
Performance		

# User-Based vs. Item-Based

	User-Based	Item-Based
Scalability	Bad when #users is huge	Bad when #items is huge
Explanation	Bad	Good
Novelty	Bad	Good
Coverage	Bad	Good
Cold start	Bad for new users	Bad for new items
Performance	Need to get many users history	Only need to get current users's history

# Item-Based Nearest Neighbor - Regression

- We can replace the (normalized) similarity coefficient  $AdjustedCosine(j, i)$  with an unknown parameter.

$w_{ji}^{item}$  to model the rating prediction of a user  $u$  for target item  $i$ .

$$\hat{r}_{ui} = \sum_{j \in Q_i(u)} w_{ij}^{item} \cdot r_{uj}$$

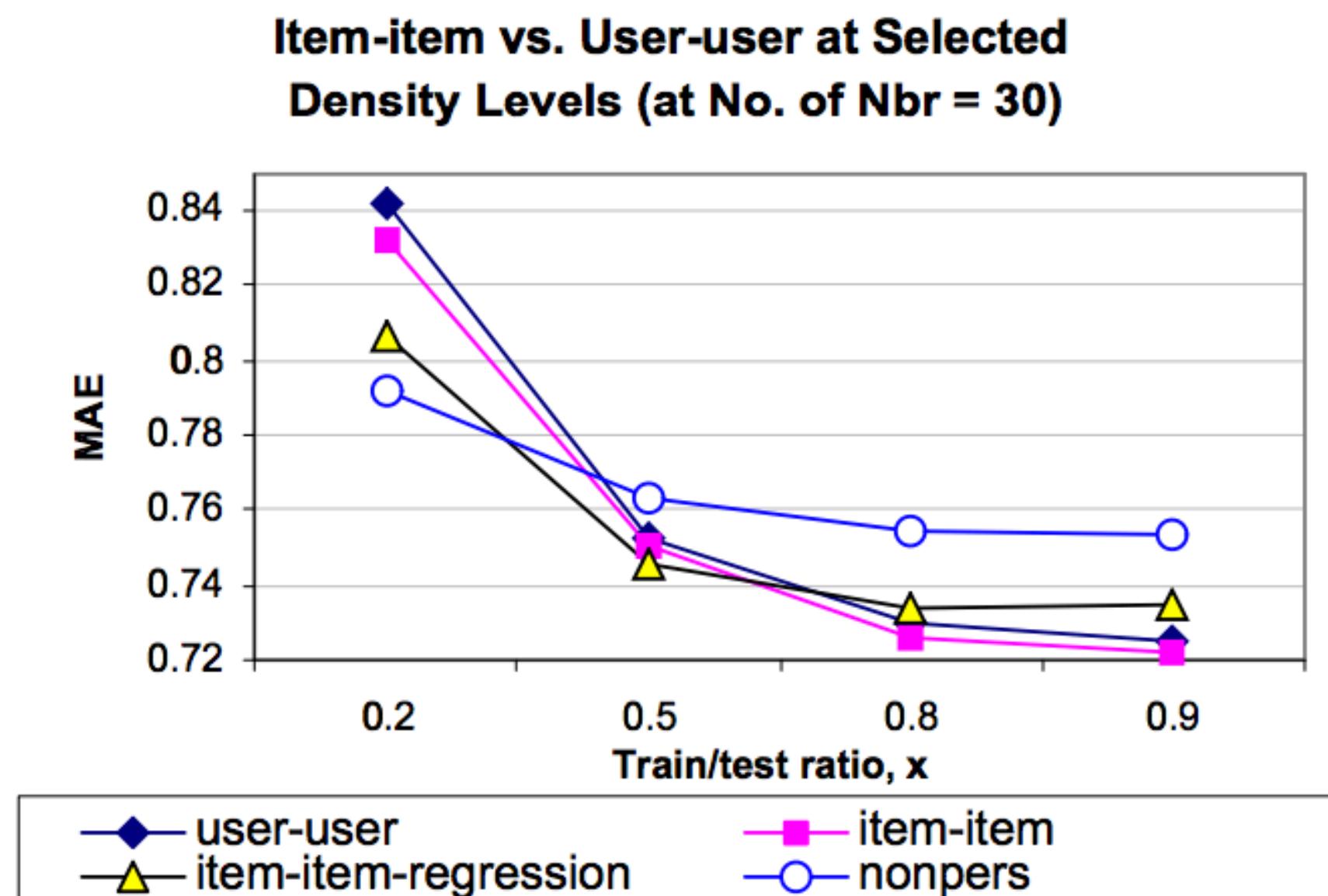
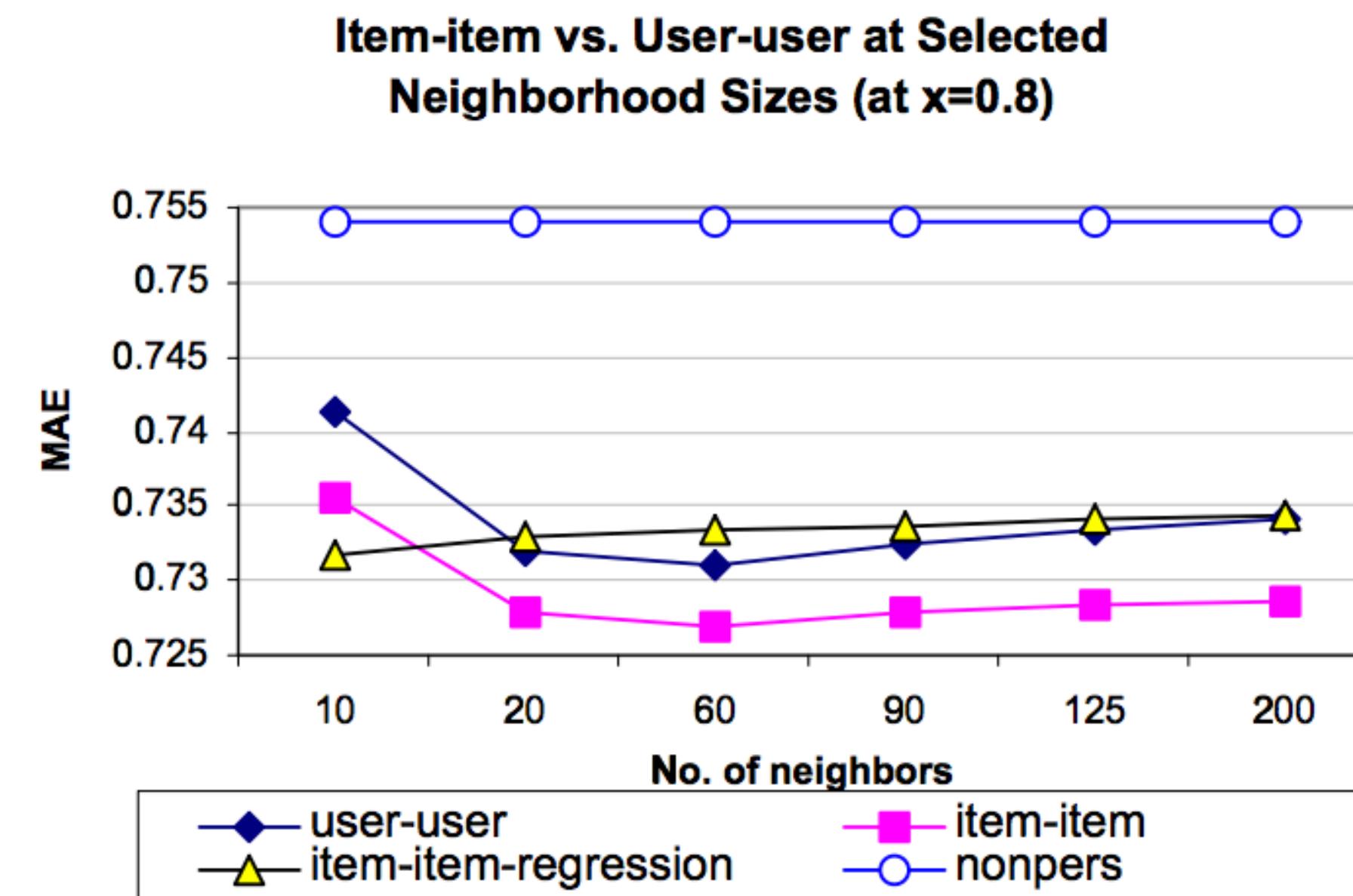
$Q_i(u)$  is the most similar item to  $i$  that have been seen by user  $u$

The nearest items in  $Q_i(u)$  can be determined using the adjusted cosine.

# Item-Based Nearest Neighbor - Regression

$$\begin{aligned} \text{Minimize } J_t &= \sum_{u \in U_t} (r_{ut} - \hat{r}_{ut})^2 \\ &= \sum_{u \in U_t} \left( r_{ut} - \sum_{j \in Q_t(u)} w_{jt}^{item} \cdot r_{uj} \right)^2 \end{aligned}$$

# User-Based vs. Item-Based



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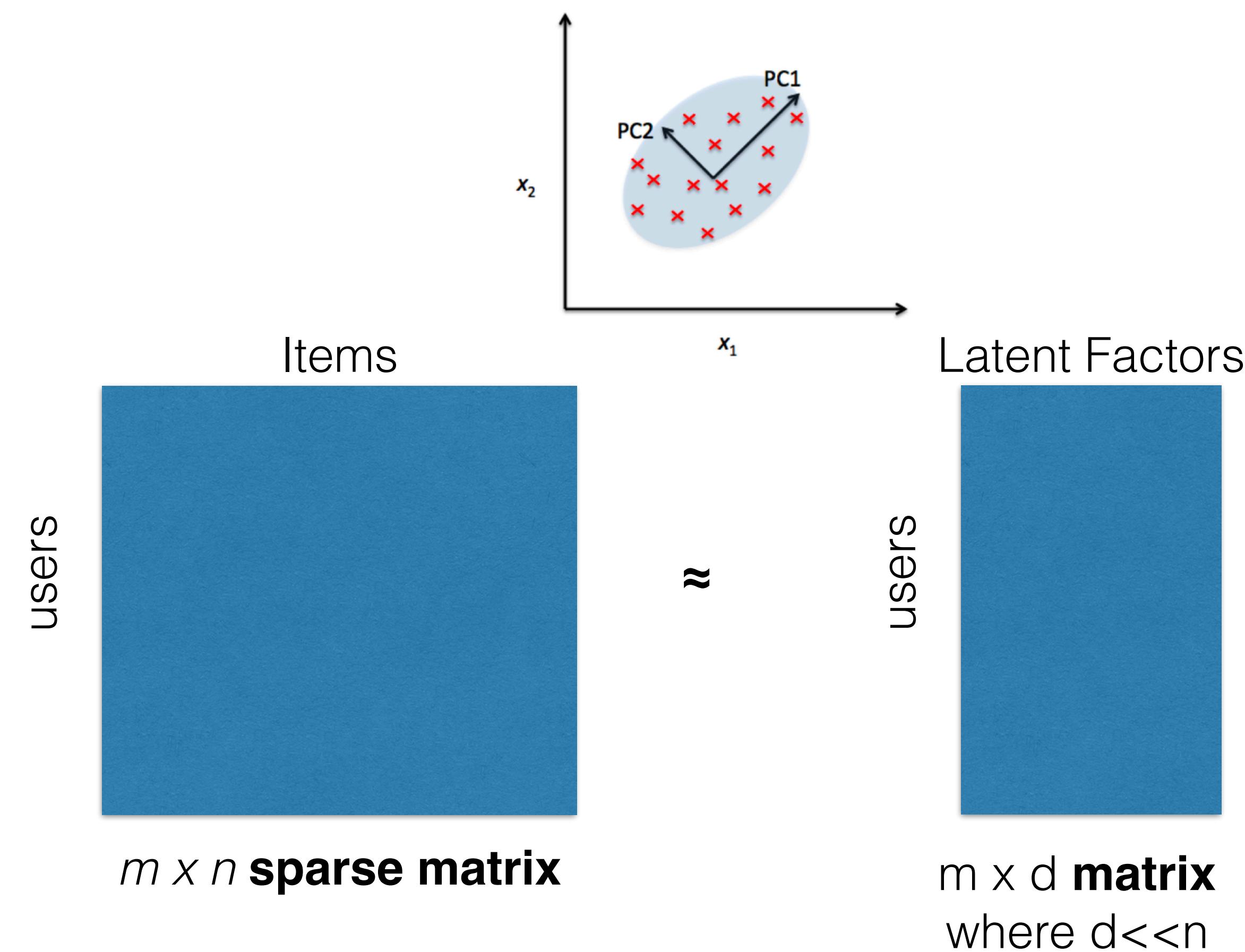
# Dimensionality Reduction

- Pairwise similarities are hard to robustly be computed in sparse matrices.
- Dimensionality reduction can be used to **improve** neighborhood-based methods both in terms of **quality** and in terms of **efficiency**
- A reduced representation of the data can be created in terms of either row-wise latent factors or in terms of column-wise latent factors.

# Dimensionality Reduction and Neighborhood Methods

- Dimensionality reduction can **improve** neighborhood methods in terms of **accuracy** and also in terms of **efficiency**.
- Similarities are hard to be computed in huge dimensional sparse rating matrices.
  - Latent factor models

# Dimensionality Reduction



# Dimensionality Reduction

- The low-dimensional representation can be computed using **PCA** or **SVD-Like** methods.
- After the  $d$ -dimensional representation of each user is estimated, the similarity between users can be computed
- Cosine or dot product on the reduced vectors can be used in order to compute the similarity
- More robust since the feature vector is fully specified
- More efficient

# Dimensionality Reduction

- How to **obtain** the **d-dimensional representation** on the sparse matrix?
- **SVD Method.** Steps:
  - Augment the  $m \times n$  incomplete rating matrix  $R \rightarrow R_f$ 
    - Mean-user rating or mean-item rating for each row/column
  - Let's define the similarity matrix  $S$  as  $S = R_f^T R_f$ .  $S$  is a positive semi-definite of size  $n \times n$
  - Determine the dominant basis vectors of  $R_f$  by computing the **diagonalization** of the similarity matrix  $S$ .
    - $S = P \Lambda P^T$ , where  $P$  is an  $n \times n$  matrix, whose columns contain the orthonormal eigenvectors of  $S$ .  $\Lambda$  is a diagonal matrix containing the non-negative eigenvalues of  $S$  along its diagonal.
  - Let denote  $P_d$  the  $n \times d$  matrix only containing the columns of  $P$  with the largest eigenvalues
  - The low representation of  $R$  is obtained by the multiplication of  $R_f P_d$

# Dimensionality Reduction

- How to **obtain** the **d-dimensional representation** on the sparse matrix?
- **PCA Method.** Steps:
  - Augment the  $m \times n$  incomplete rating matrix  $R \rightarrow R_f$ 
    - Mean-user rating or mean-item rating for each row/column
  - Lets define the similarity matrix  $S$  as **the Covariance Matrix of  $R_f$**
  - Determine the dominant basis vectors of  $R_f$  by computing the diagonalization of the similarity matrix  $S$ .
    - $S = P\Lambda P^T$ , where  $P$  is an  $n \times n$  matrix, whose columns contain the orthonormal eigenvectors of  $S$ .  $\Lambda$  is a diagonal matrix containing the non-negative eigenvalues of  $S$  along its diagonal.
  - Let denote  $P_d$  the  $n \times d$  matrix only containing the columns of  $P$  with the largest eigenvalues
  - The low representation of  $R$  is obtained by the multiplication of  $R_f P_d$

# Dimensionality Reduction

- Challenges:
  - Missing Values
    - Need a way to fill it
    - Several alternatives, including clever averages and predictions
  - Computational Complexity
  - Lack of transparency/explainability

## **Task: Implement a RecSys method for MovieLens dataset**

- you can used all methods we studied during the course -

**Your notebook should contain a discussion and conclusion of what you have learnt**

**Groups** of up to **2-3 people** are allowed

### **How to submit:**

Upload the code at Campus Virtual  
+ share the notebook via Kaggle

**Deadline:** March 31th

Link: <https://www.kaggle.com/competitions/recsysmasterfds-2024>