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Master's Foundation of Data Science – 2023-2025

FOUNDATIONS OF STATISTICAL LEARNING

Final Exam, 30 January 2024 (Sem. II) (Sol. of Sol. Sem)

Attempt any four questions. Each question carries 25 marks.

1. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) Calculate the SVD decomposition of A (i.e. find U, Σ, V such that $A = U\Sigma V^T$ and U, V are orthogonal matrices and Σ is diagonal).

(b) Calculate the eigenvalues and eigenvectors of A. Is A diagonalizable? Why or why not?

(c) Calculate the rank of A and the dimension of the column space (range) of A.

2. Consider the function $f(x, y) = x^2 + 2y^2 + 3xy + 4x + 5y + 6$.

(a) Calculate the gradient $\nabla f(x, y)$ and the Hessian $H(x, y)$.

(b) Is f convex? Is it strictly convex? Is it strongly convex? Justify your answer.

(c) Find the minimizer (x^*, y^*) of f using the gradient descent method with step size $\eta = 0.1$ and initial point $(x_0, y_0) = (0, 0)$. Write the update rule explicitly and compute the first two iterations.

3. Consider the function $f(x) = x^4 - 2x^2 + x$.

(a) Calculate the derivative $f'(x)$ and the second derivative $f''(x)$.

(b) Find all critical points of f. Classify each as a local minimum, local maximum, or neither.

(c) Find the global minimum of f on the interval $[-2, 2]$. Justify your answer.

4. Consider the function $f(x, y) = x^2 - y^2$.

(a) Calculate the gradient $\nabla f(x, y)$ and the Hessian $H(x, y)$.

(b) Show that $(0, 0)$ is a critical point. Is it a local minimum, local maximum, or saddle point? Justify your answer using the second derivative test.

(c) Show that $(0, 0)$ is a saddle point by showing that f increases along one direction and decreases along another direction.

5. Consider the function $f(x, y) = x^2 + y^2 + 2xy$.

(a) Calculate the gradient $\nabla f(x, y)$ and the Hessian $H(x, y)$.

(b) Is f convex? Is it strictly convex? Is it strongly convex? Justify your answer.

(c) Find the minimizer (x^*, y^*) of f using the gradient descent method with step size $\eta = 0.1$ and initial point $(x_0, y_0) = (1, 1)$. Write the update rule explicitly and compute the first two iterations.

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1. [illegible]

2. [illegible]

3. [illegible]

4. [illegible]

5. [illegible]