Master in Foundations of Data Science — 2020-2021

NUMERICAL LINEAR ALGEBRA

Final exam, 15 January 2021 from 15:00h till 18:00h. Exercises should be delivered in separated pages. All answers should be suitably justified.

1. Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 2 & -1 \\ -2 & -1 & 9 \end{pmatrix}.$$

(1) Compute the PLU factorization of A given by the GEPP algorithm.
(2) Compute the Cholesky factorization of A.
(3) Explain how you would solve the equation $A\,x = b$ for the vector $b = (1, 0, -1)^T$ using each of these factorizations, and compute the solution vector x using one of them.

2. A Schur normal form for the matrix in Exercise 1 is $A = U\,T\,U^*$ with

$$U = \begin{pmatrix} 0.24 & -0.90 & 0.37 \\ 0.16 & 0.41 & 0.90 \\ -0.96 & -0.16 & 0.24 \end{pmatrix} \quad \text{and} \quad T = \begin{pmatrix} 9.66 & 0 & 0 \\ 0 & 0.19 & 0 \\ 0 & 0 & 2.14 \end{pmatrix} \tag{0.1}$$

up to two decimal digits.

(1) Compute the eigenvalues and eigenvectors of A from this normal form.
(2) Determine the rate of convergence of the power method applied to the matrix A.

3. Let A be the 3×3 matrix in Exercise 1.

(1) Compute a singular value decomposition (SVD) of A using the data obtained in Exercise 2(1).
(2) Using this SVD, compute the condition number of A with respect to the 2-norm.
(3) Compute the best rank 1 approximation to A with respect to the 2-norm, and determine its distance to A. Repeat this task replacing the 2-norm by the Frobenius norm.

4. Given a symmetric matrix $B \in \mathbb{R}^{n \times n}$, explain a procedure to obtain a Schur normal form of it in a computationally efficient way. Give details about the algorithms for computing the involved factorizations.