Kalman Filter for Observer-ARMA Model with Parameter Estimation

Juan Sebastián Cárdenas-Rodríguez
David Plazas Escudero
Mathematical Engineering, Universidad EAFIT

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Preliminaries

Notation

- Observed data vector: $y_t^* := (y_0^T, y_1^T, \ldots, y_t^T)^T$.
- $z_{t|t-1} := E(z_t|y_{t-1}^*)$ and $y_{t|t-1} := E(y_t|y_{t-1}^*)$.
- $\Delta y_t := y_t - y_{t|t-1}$ and $\Delta z_t := z_t - z_{t|t-1}$.
- $\Sigma_{t|t} := \mathrm{Cov}(\Delta z_t|y_t^*)$.

Preliminaries

Observer-ARMA model

The general scheme for the observer-ARMA model is given by

$$\begin{aligned}
&z_{t+1} = \sum_{s=0}^{l_1} a_s z_{t-s} + \sum_{s=0}^{l_2} b_s \xi_{t-s} \\
&y_t = h_t z_t + \zeta_t \\
&z_0 \in \mathbb{R},\ \ t = 0, 1, \ldots
\end{aligned} \tag{1}$$

Preliminaries

Kalman Filter

The standard KF Procedure [Kalman 1960] has two main hypothesis:

H1

$$E(\xi_t) = E(\zeta_t) = 0, \quad t = 0, 1, \ldots$$

.

$$E(\xi_t \xi_t^T) = Q_t, \quad E(\zeta_t \zeta_t^T) = R_t$$

$$E(\xi_t \xi_s^T) = E(\zeta_t \zeta_s^T) = 0, \quad \forall s \neq t$$

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$$E(z_t z_s^T) = 0, \quad \forall t \neq s$$

H2

$$\xi_t \sim \mathcal{N}(0, Q_t), \quad \zeta_t \sim \mathcal{N}(0, R_t), \quad \forall t = 0, 1, \dots$$

Preliminaries

Kalman Filter

The KF procedure is now presented:

1. **Initialization:**

$$z_{0|0} = z_0 \in \mathbb{R}^n,\ \Sigma_{0|0} = 0$$

2. **Prediction:**

$$\begin{aligned} z_{t|t-1} &= F_t z_{t-1|t-1} \\ \Sigma_{t|t-1} &= F_{t-1} \Sigma_{t-1|t-1} F_{t-1}^T + Q_{t-1} \end{aligned}$$

3. **Correction:**

$$\begin{aligned} z_{t|t} &= z_{t|t-1} + M_t^{\text{opt}} \Delta y_t \\ \Sigma_{t|t} &= \Sigma_{t|t-1} - M_t^{\text{opt}} H_t \Sigma_{t|t-1} \end{aligned}$$

where $M_t^{\text{opt}} = \Sigma_{t|t-1} H_t^T \left(H_t \Sigma_{t|t-1} H_t^T + R_t \right)^{-1}$.

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a given vector function.

$$\begin{cases} x_{t+1} = f(x_t) + \xi_t \\ y_t = H_t x_t + \zeta_t \\ x_0 \in \mathbb{R}^n, \; t = 0, 1, \dots \end{cases}$$

Preliminaries

Extended Kalman Filter

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a given vector function.

$$\begin{cases} x_{t+1} = f(x_t) + \xi_t \\ y_t = H_t x_t + \zeta_t \\ x_0 \in \mathbb{R}^n, \; t = 0, 1, \ldots \end{cases} \implies \begin{cases} x_{t+1} = \tilde{F}_t x_t + \xi_t \\ y_t = H_t x_t + \zeta_t \\ x_0 \in \mathbb{R}^n, \; t = 0, 1, \ldots \end{cases} \tag{2}$$

Where $\tilde{F}_t$ is:

$$\tilde{F}_t := \left. \frac{\partial f(x)}{\partial x} \right|_{x = x_t^{\mathrm{ref}}}$$

References I

Kalman, Rudolph Emil (1960). “A New Approach to Linear Filtering and Prediction Problems”. In: *Journal of basic Engineering* 82.1, pp. 35–45.

Thank you

Student Contact

Juan Sebastián Cárdenas-Rodríguez

jscardenar@eafit.edu.co

Student Contact

David Plazas Escudero

dplazas@eafit.edu.co