2. Sea $(V, \langle \cdot, \cdot \rangle)$ e.p.i., sean $u, v \in V$

a)
$$\langle u+v, u+v\rangle = \langle u, u+v\rangle + \langle v, u+v\rangle$$
$$= \langle u,u\rangle + 2\langle u,v\rangle + \langle v,v\rangle$$
$$\langle u-v, u-v\rangle = \langle u, u-v\rangle - \langle v, u-v\rangle$$
$$= \langle u,u\rangle - 2\langle u,v\rangle + \langle v,v\rangle$$
$$\langle u+v, u+v\rangle + \langle u-v, u-v\rangle = 2\langle u,u\rangle + 2\langle v,v\rangle$$
$$\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$$

b) $(\Rightarrow)$

$\rightarrow$ Como $\langle u,v\rangle = 0$
$$\|u+v\|^2 = \|u\|^2 + \|v\|^2$$

$(\Leftarrow)$ $\|u+v\|^2 = \|u^2\| + \|v\|^2$
$$\langle u+v, u+v\rangle = \langle u,u\rangle + \langle v,v\rangle$$
$$\cancel{\langle u,u\rangle} + 2\langle u,v\rangle + \cancel{\langle v,v\rangle} = \cancel{\langle u,u\rangle} + \cancel{\langle v,v\rangle}$$
$$\langle u,v\rangle = 0.$$

c) Sale de las pruebas del a).

d) Sea $x, y \in V \subset \mathbb{C}$

$$\langle x+iy, x+iy\rangle = \langle x, x+iy\rangle + \langle iy, x+iy\rangle$$
$$= \overline{\langle x+iy, x\rangle} - i\langle y, x+iy\rangle$$
$$= \overline{\langle x,x\rangle} + \overline{\langle iy,x\rangle} - i\,\overline{\langle x+iy, y\rangle}$$
$$= \overline{\langle x,x\rangle} + i\,\overline{\langle y,x\rangle} - i\left[\overline{\langle x,y\rangle} + \overline{\langle iy,y\rangle}\right]$$
$$= \overline{\langle x,x\rangle} + i\,\overline{\langle y,x\rangle} - i\left[\overline{\langle x,y\rangle} + i\,\overline{\langle y,y\rangle}\right]$$
$$= \langle x,x\rangle + i\langle x,y\rangle - i\,\overline{\langle x,y\rangle} + \langle y,y\rangle$$
$$= \langle x,x\rangle + i\langle x,y\rangle - i\langle y,x\rangle + \langle y,y\rangle$$

$$\langle x-iy, x-iy\rangle = \langle x, x-iy\rangle - \langle iy, x-iy\rangle$$
$$= \overline{\langle x-iy, x\rangle} + i\langle y, x-iy\rangle$$
$$= \overline{\langle x,x\rangle} - \overline{\langle iy,x\rangle} + i\,\overline{\langle x-iy, y\rangle}$$
$$= \overline{\langle x,x\rangle} - i\,\overline{\langle y,x\rangle} + i\left[\overline{\langle x,y\rangle} - \overline{\langle iy,y\rangle}\right]$$
$$= \overline{\langle x,x\rangle} - i\,\overline{\langle y,x\rangle} + i\left[\overline{\langle x,y\rangle} - i\,\overline{\langle y,y\rangle}\right]$$
$$= \langle x,x\rangle - i\langle x,y\rangle + i\,\overline{\langle x,y\rangle} + \langle y,y\rangle$$
$$= \langle x,x\rangle - i\langle x,y\rangle + i\langle y,x\rangle + \langle y,y\rangle$$

$$\|x+yi\|^2 - \|x-iy\|^2 = \cancel{\langle x,x\rangle} + i\langle x,y\rangle - i\langle y,x\rangle + \cancel{\langle y,y\rangle} - \cancel{\langle x,x\rangle} + i\langle x,y\rangle - i\langle y,x\rangle - \cancel{\langle y,y\rangle}$$

$$\|x+iy\|^2 - \|x-iy\|^2 = 2i\langle x,y\rangle - 2i\langle y,x\rangle \rightarrow i\|x+iy\|^2 - i\|x-iy\| = -2\langle x,y\rangle + 2\langle y,x\rangle \quad (1)$$

$$\langle x+y, x+y\rangle = \langle x, x+y\rangle + \langle y, x+y\rangle$$
$$= \overline{\langle x+y, x\rangle} + \overline{\langle x+y, y\rangle}$$
$$= \overline{\langle x,x\rangle} + \overline{\langle y,x\rangle} + \overline{\langle x,y\rangle} + \overline{\langle y,y\rangle}$$
$$= \langle x,x\rangle + \langle x,y\rangle + \langle y,x\rangle + \langle y,y\rangle$$

$$\|x-y\|^2 = \langle x-y, x-y\rangle = \langle x, x-y\rangle - \langle y, x-y\rangle$$
$$= \overline{\langle x-y, x\rangle} - \overline{\langle x-y, y\rangle}$$
$$= \langle x,x\rangle - \overline{\langle y,x\rangle} - \overline{\langle x,y\rangle} + \langle y,y\rangle$$
$$= \langle x,x\rangle - \langle x,y\rangle - \langle y,x\rangle + \langle y,y\rangle$$
$$\|x+y\|^2 - \|x-y\|^2 = 2\langle x,y\rangle + 2\langle y,x\rangle \quad (2)$$

(1) + (2)

$$4\langle y,x\rangle = i\|x+iy\|^2 - i\|x-iy\| + \|x+y\|^2 - \|x-y\|^2$$

$$\langle y,x\rangle = \frac{\|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2}{4} \quad \square$$