Al ser una solución aproximada se obtiene que:

$$\frac{d^2\bar{u}_i}{dx^2} + 20 = R_i \quad \text{con} \quad \int_{x_{i-1}}^{x_i} R_i\, \phi_i^j = 0 \quad \text{con } j=1,2 \quad i=1,\dots,4$$

Luego:

$$\int_{x_{i-1}}^{x_i} \frac{d^2 u_i}{dx^2}\phi_i^j dx = -\int_{x_{i-1}}^{x_i} 20\, \phi_i^j(x)\, dx \quad (1)$$

Nótese que:

$$\phi_i^1 = \frac{x_i - x}{x_i - x_{i-1}} \qquad \frac{d\phi_i^1}{dx} = -\frac{1}{x_i - x_{i-1}}$$

$$\phi_i^1(x_{i-1}) = 1$$

$$\phi_i(x_i) = 0$$

$$\int_{x_{i-1}}^{x_i} \phi_i^1(x)\,dx = \int_{x_{i-1}}^{x_i} \frac{x_i - x}{x_i - x_{i-1}}\,dx$$

$$= \frac{1}{x_i - x_{i-1}}\left(x_i x - \frac{1}{2}x^2\right)\Big|_{x_{i-1}}^{x_i}$$

$$= \frac{x_i - x_{i-1}}{2}$$

$$\phi_i^2(x) = \frac{x - x_{i-1}}{x_i - x_{i-1}}$$

$$\phi_i^2(x_{i-1}) = 0$$

$$\phi_i^2(x_i) = 1$$

$$\frac{d\phi_i^2}{dx} = \frac{1}{x_i - x_{i-1}}$$

$$\int_{x_{i-1}}^{x_i} \phi_i^2(x)\,dx = \frac{x_i - x_{i-1}}{2}$$

Integrando por partes (1):

$$\phi_i^j(x)\frac{d\bar{u}_i}{dx}\Big|_{x_{i-1}}^{x_i} - \int_{x_{i-1}}^{x_i} \frac{d\bar{u}_i}{dx}\cdot\frac{d\phi_i^j}{dx}\,dx = -20\int_{x_{i-1}}^{x_i} \phi_i^j(x)\,dx$$