3.

i) Es claro que:

$$W = \{(x,y,z) : 5x = 3y = z\}$$

$$W = \{(x,y,z) : x = \frac{z}{5} \wedge y = \frac{z}{3}\}$$

$$W = \{(\frac{z}{5}, \frac{z}{3}, z) : z \in \mathbb{R}\} = \{(\underbrace{1/5, 1/3, 1}_{w_1})z : z \in \mathbb{R}\}$$

$$= \text{gen}\{(1/5, 1/3, 1)\}.$$

Sea $w' \in W^{\perp}$. Con $w' = (a,b,c)$ luego:

$$\langle w', w_1 \rangle = 0 \rightarrow \langle (a,b,c), (1/5, 1/3, 1) \rangle = 0$$

$$\frac{a}{5} + \frac{b}{3} + c = 0$$

Luego:

$$\begin{aligned} a &= -\frac{5}{3}\alpha - \beta \\ b &= \alpha \\ c &= \beta \end{aligned} \longrightarrow \begin{aligned} w' &= (-\tfrac{5}{3}\alpha - \beta, \alpha, \beta) \\ &= (-5/3, 1, 0)\alpha + (-1, 0, 1)\beta \end{aligned}$$

Así:

$$W^{\perp} = \text{gen}\{(-5/3, 1, 0), (-1, 0, 1)\} \checkmark$$