

Para $j=1$

$$-\frac{d\bar{u}_i}{dx}(x_{i-1}) + \frac{1}{x_i - x_{i-1}} \int_{x_{i-1}}^{x_i} \frac{d\bar{u}_i}{dx} dx = -10(x_i - x_{i-1})$$

$$-\frac{d\bar{u}_i}{dx}(x_{i-1}) + \frac{1}{x_i - x_{i-1}} \bar{u}_i \Big|_{x_{i-1}}^{x_i} = -10(x_i - x_{i-1})$$

$$-\frac{d\bar{u}_i}{dx}(x_{i-1}) + \frac{u_i - u_{i-1}}{x_i - x_{i-1}} = -10(x_i - x_{i-1})$$

Para $j=2$:

$$\frac{d\bar{u}_i}{dx}(x_i) - \frac{1}{x_i - x_{i-1}} \int_{x_{i-1}}^{x_i} \frac{d\bar{u}_i}{dx} dx = -10(x_i - x_{i-1})$$

$$\frac{d\bar{u}_i}{dx}(x_i) - \frac{u_i - u_{i-1}}{x_i - x_{i-1}} = -10(x_i - x_{i-1})$$

Luego, como $x_i - x_{i-1} = 4$, escribiéndolo matricialmente:

$$\frac{1}{4} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix} = \begin{bmatrix} -\bar{u}_i'(x_{i-1}) + 40 \\ \bar{u}_i'(x_i) + 40 \end{bmatrix}$$

Ensamblando:

$$\begin{bmatrix} 1/4 & -1/4 & 0 & 0 & 0 \\ -1/4 & 0 & -1/4 & 0 & 0 \\ 0 & -1/4 & 1/2 & -1/4 & 0 \\ 0 & 0 & -1/4 & 1/2 & -1/4 \\ 0 & 0 & 0 & -1/4 & 1/4 \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} -\bar{u}_1'(x_0) + 40 \\ 80 \\ 80 \\ 80 \\ -\bar{u}_4'(x_4) + 40 \end{bmatrix}$$

Luego las ecuaciones son: