Tarea 4: Economía Matemática:

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1. $p_t = \alpha - T - \beta U_t + g\pi_t$ (18.18)

$\pi_{t+1} - \pi_t = j(p_t - \pi_t)$ (18.19)

$U_{t+1} - U_t = -K(m - p_t)$ (18.20)

De (18.18), haciendo $t \to t+1$: $p_{t+1} = \alpha - T - \beta U_{t+1} + g\pi_{t+1}$ (18.18')

(18.18') - (18.18): $p_{t+1} - p_t = -\beta(U_{t+1} - U_t) + g(\pi_{t+1} - \pi_t)$

Reemplazando en (18.19) y (18.20): $p_{t+1} - p_t = \beta K(m - p_t) + gj(p_t - \pi_t)$ (18.21)

De (18.18): $g\pi_t = p_t - (\alpha - T) + \beta U_t$, luego:

$$p_{t+1} - p_t = \beta K(m - p_t) + gjp_t - j[p_t - (\alpha - T) + \beta U_t]$$

$(1+\beta K)p_{t+1} - [1 - j(1-g)]p_t + j\beta U_t = \beta K m + j(\alpha - T)$ (18.23)

Haciendo $t \to t+1$ en (18.23):

$(1+\beta K)p_{t+2} - [1 - j(1-g)]p_{t+1} + j\beta U_{t+1} = \beta K m + j(\alpha - T)$ (18.23')

(18.23') - (18.23)

$$(1+\beta K)p_{t+2} - [1 - j(1-g)]p_{t+1} - (1+\beta K)p_{t+1} + [1 - j(1-g)]p_t + j\beta(U_{t+1} - U_t) = 0$$

Reemplazando (18.20):

$$(1+\beta K)p_{t+2} - [1 - j(1-g)]p_{t+1} - (1+\beta K)p_{t+1} + [1 - j(1-g)] - j\beta K(m - p_{t+1}) = 0$$

$$(1+\beta K)p_{t+2} - [1 - j(1-g) + \beta K - j\beta K]p_{t+1} + [1 - j(1-g)]p_t = j\beta K m$$

$$p_{t+2} - \left[\frac{1 + gj + (1-j)(1+\beta K)}{1+\beta K}\right]p_{t+1} + \frac{1 - j(1-g)}{1+\beta K}p_t = j\beta K m$$ $\square$.

2.- Retomando (18.21):

$$\rho_{t+1}-\rho_t=\beta K(m-\rho_{t+1})+gj(\rho_t-\pi_t)$$

De (18.19): $\pi_{t+1}-\pi_t=j(\rho_t-\pi_t) \Rightarrow \rho_t=\dfrac{\pi_{t+1}-(1-j)\pi_t}{j}$ (*)

Luego $\rho_{t+1}=\dfrac{\pi_{t+2}-(1-j)\pi_{t+1}}{j}$ (**). Restando (*) de (**) se tiene

$$\rho_{t+1}-\rho_t=\frac{\pi_{t+2}-(1-j)\pi_{t+1}-[\pi_{t+1}-(1-j)\pi_t]}{j}$$

$$\rho_{t+1}-\rho_t=\frac{\pi_{t+2}-[1+(1-j)]\pi_{t+1}+(1-j)\pi_t}{j} \quad (***)$$

Reemplazando (*),(**),(***) en (18.21):

$$\frac{\pi_{t+2}-[1+(1-j)]\pi_{t+1}+(1-j)\pi_t}{j}=\beta K\left[m-\frac{\pi_{t+2}-(1-j)\pi_{t+1}}{j}\right]+gj\left[\frac{\pi_{t+1}-(1-j)\pi_t}{j}-\pi_t\right]$$

$$\pi_{t+2}-[1+(1-j)]\pi_{t+1}+(1-j)\pi_t=j\beta Km-\beta K\pi_{t+2}+\beta K(1-j)\pi_{t+1}+gj\pi_{t+1}-gj\pi_t$$

$$\pi_{t+2}(1+\beta K)-[1+gj+(1-j)(1+\beta K)]\pi_{t+1}+[1-j(1-g)]\pi_t=j\beta Km$$

$$\pi_{t+2}-\frac{[1+gj+(1-j)(1+\beta K)]\pi_{t+1}}{1+\beta K}+\frac{[1-j(1-g)]\pi_t}{1+\beta K}=\frac{j\beta Km}{1+\beta K} \quad \square$$

3. Quitar restricción de $g\leq 1$. Luego, $g>0$.

Polinomio auxiliar: $\lambda^2+a_1\lambda+a_2=0$, donde $a_1=-\dfrac{1+gj+(1-j)(1+\beta K)}{1+\beta K}$

Las raíces λ_1 y λ_2 satisfacen: $\quad a_2=\dfrac{1-j(1-g)}{1+\beta K}$.

$$\lambda_1+\lambda_2=-a_1=\frac{1+gj}{1+\beta K}+1-j>0$$

$$\lambda_1\lambda_2=a_2=\frac{1-j(1-g)}{1+\beta K}.$$

$\hookrightarrow$ Porque $0<j<1$, $g>0$ y $\beta,K>0$.

Como $g>0 \rightarrow 1-g<0 \rightarrow 1-j(1-g)>0 \rightarrow \dfrac{1-j(1-g)}{1+\beta K}>0$.

Luego, $\lambda_1,\lambda_2>0$, ya que $\lambda_1+\lambda_2>0$ y $\lambda_1\lambda_2>0$.

Claramente, $(1-\lambda_1)(1-\lambda_2) = 1-(\lambda_1+\lambda_2)+\lambda_1\lambda_2 = \frac{j\beta K}{1+\beta K} > 0$. (*)

Caso 1: $\lambda_1, \lambda_2 \in \mathbb{R}$ y $\lambda_1 \neq \lambda_2$.

i) Si $\lambda_1 = 1$ ó $\lambda_2 = 1 \Rightarrow (1-\lambda_1)(1-\lambda_2) = 0 \rightarrow$ Contradicción con (*)

ii) Si $0 < \lambda_1 < 1 < \lambda_2 \Rightarrow (1-\lambda_1)(1-\lambda_2) < 0 \rightarrow$ Contradicción con (*)

iii) Si $0 < \lambda_2 < 1 < \lambda_1 \Rightarrow (1-\lambda_1)(1-\lambda_2) < 0 \rightarrow$ Contradicción

iv) Si $0 < \lambda_1 < \lambda_2 < 1$ ó $0 < \lambda_1 < \lambda_2 < 1 \Rightarrow (1-\lambda_1)(1-\lambda_2) > 0$ ✓ $\rightarrow$ Este caso claramente implica p_t convergente.

v) Si $\lambda_1 > \lambda_2 > 1$ ó $\lambda_1 > \lambda_2 > 1 \Rightarrow (1-\lambda_1)(1-\lambda_2) > 0$ ✓ $\rightarrow$ Casos de divergencia

En el caso v) es claro que $\lambda_1\lambda_2 > 1$, luego

$$\lambda_1\lambda_2 = \frac{1-j(1-g)}{1+\beta K} > 1 \Rightarrow -j(1-g) > \beta K \Rightarrow 1-g < -\frac{\beta K}{j} \Rightarrow \boxed{g > 1+\frac{\beta K}{j}}$$

Luego, se tiene divergencia cuando $g > 1+\frac{\beta K}{j}$

Caso 2: $\lambda_1 = \lambda_2 \in \mathbb{R} \Rightarrow$ i) $\lambda_1 = \lambda_2 = 1$

$\Rightarrow (1-\lambda_1)(1-\lambda_2) = 0 \rightarrow$ Contradicción con (*).

ii) $0 < \lambda_1 = \lambda_2 < 1 \Rightarrow (1-\lambda_1)(1-\lambda_2) > 0$ ✓ $\rightarrow$ Caso convergente.

iii) $\lambda_1 = \lambda_2 > 1 \Rightarrow (1-\lambda_1)(1-\lambda_2) > 0$ ✓ $\rightarrow$ Caso divergencia

$\Rightarrow \lambda_1\lambda_2 > 1 \Rightarrow g > \frac{1+\beta K}{j}$

Caso 3: $\lambda_1, \lambda_2 \in \mathbb{C}$ conjugados.

Nos interesa el módulo de λ_1 y λ_2 para analizar convergencia.

Se sabe que el módulo es $r = \sqrt{a_2}$

$$r = \sqrt{\frac{1-j(1-g)}{1+\beta K}} > 1 \Rightarrow \boxed{g > 1+\frac{\beta K}{j}}$$

En cualquier caso, p_t es divergente si $g > 1+\frac{\beta K}{j}$.

4. a) $p_t = \alpha - T - \beta U_t + g\pi_t$ (1) $\rightarrow p_{t+1} = \alpha - T - \beta U_{t+1} + g\pi_{t+1}$ (4)

$U_{t+1} - U_t = -K(m - p_t)$ (2) $\rightarrow \dfrac{p_t - (\alpha - T) + \beta U_t}{g} = \pi_t$ (Δ)

$\pi_{t+1} - \pi_t = j(p_t - \pi_t)$ (3)

(4)-(1): $p_{t+1} - p_t = -\beta(U_{t+1} - U_t) + g(\pi_{t+1} - \pi_t)$ (5)

(2) y (3) en (5): $p_{t+1} - p_t = \beta K(m - p_t) + gj(p_t - \pi_t)$ (6)

(Δ) en (6): $p_{t+1} - p_t = \beta K(m - p_t) + gj\left[p_t + \dfrac{-p_t + (\alpha - T) - \beta U_t}{g}\right]$

$p_{t+1} + (-1 - gj + j + \beta K)p_t = \beta K m + j(\alpha - T) - j\beta U_t$ (7)

En (7) $\Rightarrow t \rightarrow t+1$: $p_{t+2} + (-1 - gj + j + \beta K)p_{t+1} = \beta K m + j(\alpha - T) - j\beta U_{t+1}$ (8)

(8)-(7); reemplazo (2)

$p_{t+2} + (-2 - gj + j + \beta K)p_{t+1} - (-1 - gj + j - j\beta K)p_t = -j\beta(-K(m - p_t))$

$p_{t+2} + [-2 - j(g-1) + \beta K]p_{t+1} + [1 + j(g-1) - \beta K(1-j)]p_t = j\beta K m$

b) Se sabe que la particular es $\bar{p} = \dfrac{c}{1 + a_1 + a_2}$

$\bar{p} = \dfrac{j\beta K m}{1 - 2 - j(g-1) + \beta K + 1 + j(g-1) - \beta K(1-j)} = \dfrac{j\beta K m}{j\beta K} = m \Longrightarrow$ Misma particular

c) $j = g = 1 \Rightarrow a_1 = \beta K - 2$, $a_2 = 1$ $\Big\}$ $\Delta = a_1^2 - 4a_2 = (\beta K - 2)^2 - 4$

$= \beta^2 K^2 - 4\beta K + 4 - 4$

$= \beta K(\beta K - 4)$

Caso 1: $\Delta > 0$

$\Longrightarrow \beta K(\beta K - 4) > 0$

Como $\beta K > 0 \Rightarrow \beta K - 4 > 0$

$\boxed{\beta K > 4}$

Caso 2:

$\Delta = 0 \Longrightarrow \boxed{\beta K = 4}$

Caso 3: $\Delta < 0$

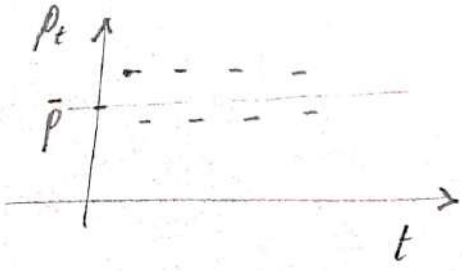
$\boxed{\beta K < 4}$

d) $j = g = 1$

$\boxed{\beta K = 3} \Longrightarrow \lambda_1, \lambda_2 \in \mathbb{C}$; conjugadas.

$\left.\begin{array}{l} \hookrightarrow a_1 = 1 \\ a_2 = 1 \end{array}\right\} r = \sqrt{a_2} = 1 \Rightarrow$ Comportamiento oscilatorio con aplitud constante.

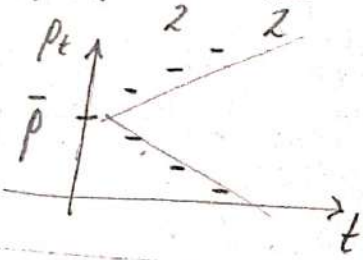
$$\theta = Arcos\left(\frac{-a_1}{2\sqrt{a_2}}\right) = Arcos\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$$



Luego, $\rho_t = \bar{\rho} + \left[A_1 \cos\left(\frac{2\pi}{3}t\right) + A_2 \cos\left(\frac{2\pi}{3}t\right)\right]$

$\boxed{\beta K = 4}$ $\lambda_1 = \lambda_2 \in \mathbb{R}$

$\left.\begin{array}{l} \hookrightarrow a_1 = 2 \\ a_2 = 1 \end{array}\right\} \lambda_1 = \lambda_2 = \frac{-a_1}{2} = \frac{-2}{2} = -1 \Longrightarrow \rho_t = A_1(-1)^t + A_2 t(-1)^t + \bar{\rho}$



$\rho_t = (-1)^t\left[A_1 + A_2 t\right] + \bar{\rho}$

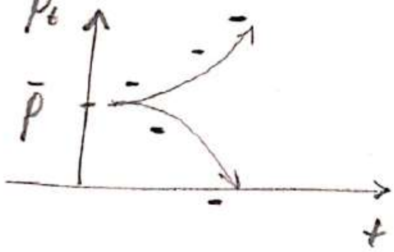
$\hookrightarrow$ Divergente

$\boxed{\beta K = 5}$: $\lambda_1, \lambda_2 \in \mathbb{R}$ y $\lambda_1 \neq \lambda_2$

$\begin{array}{l} \hookrightarrow a_1 = 3 \\ a_2 = 1 \end{array}$ $\lambda_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2} = \frac{-3 \pm \sqrt{5}}{2} \rightarrow$ $\lambda_1 = \frac{-3+\sqrt{5}}{2} \rightarrow |\lambda_1| < 1$ ✓

$\lambda_2 = \frac{-3-\sqrt{5}}{2} \rightarrow |\lambda_2| > 1$

$\rho_t = \bar{\rho} + A_1\left(\frac{-3+\sqrt{5}}{2}\right)^t + A_2\left(\frac{-3-\sqrt{5}}{2}\right)^t$



Solución Divergente.