Derivada: Sea $\phi \in D(\mathbb{R})$, $\operatorname{sop}\phi \subset [-a,a]$ con $a>0$, luego:

$$\left\langle \frac{d|x|}{dx}, \varphi \right\rangle = (-1)\left\langle f, \frac{d\phi}{dx} \right\rangle$$

$$= -\int_{\mathbb{R}} |x|\,\phi'(x)\,dx = -\int_{-a}^{a} |x|\,\phi'(x)\,dx$$

$$= \int_{-a}^{0} x\,\phi'(x) + \int_{0}^{a} x\,\phi'(x)\,dx$$

$$= \underbrace{x\cancel{\phi(x)}\Big|_{-a}^{0}}_{0} - \int_{-a}^{0} \phi(x)\,dx - \underbrace{x\phi(x)\Big|_{0}^{a}}_{0} + \int_{0}^{a} \phi(x)\,dx$$

$$= -\int_{-a}^{0} \phi(x)\,dx + \int_{0}^{a} \phi(x)\,dx$$

Como $\operatorname{sop}\phi \subset [-a,a]$ debe ocurrir que $\phi(a)=0$ y $\phi(-a)=0$

$$= \int_{-a}^{0} (-1)\,\phi(x)\,dx + \int_{0}^{a} (1)\,\phi(x)\,dx$$

$$= \int_{-a}^{0} \operatorname{sgn} x\,\phi(x)\,dx + \int_{0}^{a} \operatorname{sgn} x\,\phi(x)\,dx$$

$$= \int_{-a}^{a} \phi(x)\operatorname{sgn} x\,dx$$

Luego:

$$\left\langle \frac{d|x|}{dx}, \phi \right\rangle = \langle \operatorname{sgn} x, \phi \rangle$$ ✓