18 $$u(x)=\begin{cases} x^2, & 0\le x<1/2 \\ -x^2+2x-1/2, & 1/2\le x\le 3/2 \\ (2-x)^2, & 3/2<x\le 2 \end{cases}$$

$$\left.\begin{aligned}
&\lim_{x\to 1/2^-} u(x) = \lim_{x\to 1/2^-} x^2 = \frac{1}{4} \\
&\lim_{x\to 1/2^+} u(x) = \lim_{x\to 1/2^+} (-x^2+2x-1/2) = \frac{1}{4} \\
&\lim_{x\to 3/2^-} u(x) = \lim_{x\to 3/2^-} (-x^2+2x-1/2) = \frac{1}{4} \\
&\lim_{x\to 3/2^+} u(x) = \lim_{x\to 3/2^+} (2-x)^2 = \frac{1}{4}
\end{aligned}\right\}$$

Luego, u es continua y por (*) $u\in \mathbb{L}^1(\Omega)$.

$$u'(x)=\begin{cases} 2x, & 0\le x<1/2 \\ -2x+2, & 1/2\le x\le 3/2 \\ -2(2-x), & 3/2<x\le 2 \end{cases}$$

$$\left.\begin{aligned}
&\lim_{x\to 1/2^-} u'(x) = \lim_{x\to 1/2^-} 2x = 1 \\
&\lim_{x\to 1/2^+} u'(x) = \lim_{x\to 1/2^+} (-2x+2) = 1 \\
&\lim_{x\to 3/2^-} u'(x) = \lim_{x\to 3/2^-} (-2x+2) = -1 \\
&\lim_{x\to 3/2^+} u'(x) = \lim_{x\to 3/2} (-2(2-x)) = -1
\end{aligned}\right\}$$

Luego $u'\in \mathbb{L}^2(\Omega)$

$$u''(x)=\begin{cases} 2, & 0\le x<1/2 \\ -2, & 1/2\le x\le 3/2 \\ 2, & 3/2<x\le 2 \end{cases}$$

Claramente $[u''(x)]^2$ es continua.

Luego $u''\in \mathbb{L}^2(\Omega)$.

$$u''(x) = 2-4H(x-1/2)+4H(x-3/2)$$

$$\Rightarrow u'''(x) = -4\delta(x-1/2)+4\delta(x-3/2)$$

Que claramente $u'''\notin \mathbb{L}^2(\Omega)$

$\therefore\ u\in \mathcal{H}^2(\Omega)$

Ahora es claro que $\partial\Omega=\{0,2\}$

y $u(0)=0=u(2)$

Luego $u|_{\partial\Omega}=0$

$\Longrightarrow u\in \mathcal{H}_0^2(\Omega)$.