Taller Freddy:

sura

Sesión 1 (1) $X_t = \sqrt{B_t^3} = B_t^{3/2} \rightarrow dX_t = 3 B_t^{1/2} dB_t + \frac{1}{2}\left(\frac{3}{4} B_t^{-1/2}\right) dt$ (2) Meh.

(3) $X_t = e^{\alpha B_t^2 + \mu}$

$dX_t = \alpha e^{\alpha B_t^2+\mu}(2B_t) dB_t + \alpha\left(\alpha e^{\alpha B_t^2+\mu}(2B_t^2) + \alpha e^{\alpha B_t^2+\mu}\right) dt$

$dX_t = 2\alpha B_t e^{\alpha B_t^2+\mu} dB_t + \alpha^2 e^{\alpha B_t^2+\mu}(2B_t^2+1) dt.$

$dX_t = 2\alpha B_t X_t dB_t + \alpha^2 X_t (2B_t^2+1) dt$

(4) $X_t = \ln(B_t)$, $dX_t = \frac{dB_t}{B_t} + \frac{1}{2}\left(\frac{-dt}{B_t^2}\right) = e^{-X_t} dB_t - \frac{e^{-2X_t}}{2} dt.$

(5) $X_t = \cos(B_t)$, $dX_t = -\sin(B_t) dB_t - (1/2)\cos(B_t) dt.$

(6) $X_t = \sin(B_t)$, $dX_t = \cos(B_t) dB_t - (1/2)\sin(B_t) dt.$

(7) $X_t = \cosh(B_t)$, $dX_t = \sinh(B_t) dB_t + (1/2)\cosh(B_t) dt$

(8) $X_t = \sinh(B_t)$, $dX_t = \cosh(B_t) dB_t + (1/2)\sinh(B_t) dt.$

(9) $X_t = \sin^2(B_t) + \cos^2(B_t)$

$dX_t = [2\sin(B_t)\cos(B_t) - 2\cos(B_t)\sin(B_t)] dB_t + 1/2 (0) dt$

(10) $X_t = B_t^{n/m}.$

$dX_t = \frac{n}{m} B_t^{n/m - 1} dB_t + \frac{n}{2m}\left(\frac{n}{m} - 1\right) B_t^{n/m - 2} dt$

$X_t = \frac{n}{m} \int_0^t \frac{X_s}{B_s} dB_s + \frac{n(n-m)}{2m^2} \int_0^t \frac{X_s}{B_s^2} ds$

Sección 2: $dZ_t = dB_t$

① $X_t = e^{\alpha t Z_t}$, $Z_t = B_t$, $\alpha > 0$.

$$dX_t = \alpha Z_t e^{\alpha t Z_t} dt + \alpha t e^{\alpha t Z_t} dZ_t + \frac{1}{2} \alpha t^2 e^{\alpha t Z_t} (dZ_t)^2$$

$$dX_t = \alpha Z_t e^{\alpha t Z_t} dt + \alpha t e^{\alpha t Z_t} dZ_t + \frac{\alpha t^2}{2} e^{\alpha t Z_t} dt$$

$$dX_t = \left(\alpha Z_t e^{\alpha t Z_t} + \frac{\alpha^2 t^2}{2} e^{\alpha t Z_t}\right) dt + \alpha t e^{\alpha t Z_t} dZ_t$$

② $X_t = e^{\sigma Z_t}$, $Z_t = f(t)$, con $f(\cdot)$ determinista y $\sigma \in \mathbb{R}$.

$$dX_t = \sigma e^{\sigma Z_t} dZ_t + \frac{1}{2} \sigma^2 e^{\sigma Z_t} (dZ_t)^2 \qquad dZ_t = f'(t) dt$$

$$dX_t = \sigma e^{\sigma Z_t} f'(t) dt \qquad (dZ_t)^2 = (f'(t) dt)^2 = 0$$

③ $X_t = e^{\mu t + \sigma Z_t}$, $Z_t = E\left[\int_0^t B_s^2 ds\right] = \int_0^t E[B_s^2] ds = \int_0^t s\, ds = \frac{t^2}{2}$.

$$dZ_t = t\,dt \rightarrow (dZ_t)^2 = (t\,dt)^2 = 0$$

$$X_t = e^{\mu t + \frac{\sigma t^2}{2}}, \quad dX_t = (\mu + \sigma t) e^{\mu t + \frac{\sigma t^2}{2}} dt$$

④ $X_t = e^{B_t - Z_t}$, $Z_t = E\left[\left(\int_0^t s\, dB_s\right)^2\right] = \int_0^t E[s^2] ds = \frac{t^3}{3}$

$\Rightarrow X_t = e^{B_t - t^3/3}$, Sea $Y_t = B_t - t^3/3 = -\int_0^t s^2 ds + \int_0^t dB_s \Rightarrow dY_t = -t^2 dt + dB_t$

$X_t = e^{Y_t}$

$$dX_t = e^{Y_t} dY_t + \frac{1}{2} e^{Y_t} (dY_t)^2 = e^{Y_t}(-t^2 dt + dB_t) + \frac{e^{Y_t}}{2} dt$$

$$= e^{Y_t}\left(-t^2 + \frac{1}{2}\right) dt + e^{Y_t} dB_t = e^{B_t - \frac{t^3}{3}}\left(-t^2 + \frac{1}{2}\right) dt + e^{B_t - \frac{t^3}{3}} dt.$$

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(5) $X_t = \ln(Z_t)$, con $dZ_t = \alpha Z_t dt + \mu Z_t dB_t$.

$$dX_t = \frac{dZ_t}{Z_t} + \frac{1}{2}\left(\frac{-1}{Z_t^2}\right)(dZ_t)^2 = Z_t^{-1}(\alpha Z_t dt + \mu Z_t dB_t) - \frac{Z_t^{-2}}{2}(\mu^2 Z_t^2 dt)$$

$$dX_t = \left(\alpha - \frac{\mu^2}{2}\right)dt + \mu dB_t.$$

Sección 3

(1) $$X_t = \begin{bmatrix} B_1(t)B_2(t) - B_3(t) \\ B_1^2(t)B_3^2(t) \end{bmatrix}.$$

$$dX_t = \frac{1}{2}\begin{bmatrix} 0+0+0 \\ 2B_3^2(t)+0+2B_1^2(t) \end{bmatrix}dt + \begin{bmatrix} B_2(t) & B_1(t) & -1 \\ 2B_1(t)B_3^2(t) & 0 & 2B_3(t)B_1^2(t) \end{bmatrix}\begin{bmatrix} dB_1(t) \\ dB_2(t) \\ dB_3(t) \end{bmatrix}$$

(2) $$Y_t = \begin{bmatrix} B_1^2(t)B_3^2(t) \\ B_2^2(t)B_3^2(t) \end{bmatrix}$$

$$dY_t = \frac{1}{2}\begin{bmatrix} 2B_3^2(t)+2B_1^2(t) \\ 2B_3^2(t)+2B_2^2(t) \end{bmatrix}dt + \begin{bmatrix} 2B_1(t)B_3^2(t) & 0 & 2B_1^2(t)B_3(t) \\ 0 & 2B_2(t)B_3^2(t) & 2B_2^2(t)B_3(t) \end{bmatrix}\begin{bmatrix} dB_1(t) \\ dB_2(t) \\ dB_3(t) \end{bmatrix}$$

(3) $$g(B(t)) = \begin{bmatrix} B_1(t)-B_4(t)-B_2^2(t) \\ B_2^2(t)+B_3^2(t) \\ B_4^2(t)-B_1^2(t) \end{bmatrix}$$

$$dg(B(t)) = \frac{1}{2}\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}dt + \begin{bmatrix} 1 & -2B_2(t) & 0 & -1 \\ 0 & 2B_2(t) & 2B_3(t) & 0 \\ -2B_1(t) & 0 & 0 & 2B_4(t) \end{bmatrix}\begin{bmatrix} dB_1(t) \\ dB_2(t) \\ dB_3(t) \\ dB_4(t) \end{bmatrix}$$

$$e^{B_1^2(t)+B_3^2(t)} \to 2B_1(t)e^{B_1^2(t)+B_3^2(t)} \to 2e^{B_1^2(t)+B_3^2(t)} + 2B_1(2B_1e^{B_1^2(t)+B_3^2(t)})$$

④ $$X_t = \begin{bmatrix} \cos(B_1(t)-B_2(t)) \\ \sin(B_1(t)+B_2(t)) \end{bmatrix}$$

$$dX_t = \frac{1}{2}\begin{bmatrix} -\cos(B_1(t)-B_2(t)) - \cos(B_1(t)-B_2(t)) \\ -\sin(B_1(t)+B_2(t)) - \sin(B_1(t)+B_2(t)) \end{bmatrix} dt + \begin{bmatrix} -\sin(B_1(t)-B_2(t)) & \sin(B_1(t)-B_2(t)) \\ \cos(B_1(t)+B_2(t)) & \cos(B_1(t)+B_2(t)) \end{bmatrix} \times \begin{bmatrix} dB_1(t) \\ dB_2(t) \end{bmatrix}$$

⑤ $$Z_t = \begin{bmatrix} e^{B_1(t)B_2(t)} - B_3(t) \\ e^{B_1^2(t)+B_3^2(t)} + B_2(t) \end{bmatrix}$$

$$dZ_t = \frac{1}{2}\begin{bmatrix} B_2^2(t)e^{B_1(t)B_2(t)} + B_1^2(t)e^{B_1(t)B_2(t)} \\ 2e^{B_1^2(t)+B_3^2(t)}(1+2B_1^2(t)) + 2e^{B_1^2(t)+B_3^2(t)}(1+2B_3^2(t)) \end{bmatrix} dt + \begin{bmatrix} B_2(t)e^{B_1(t)B_2(t)} & B_1(t)e^{B_1(t)B_2(t)} & -1 \\ 2B_1(t)e^{B_1^2(t)+B_3^2(t)} & 1 & 2B_3(t)e^{B_1^2(t)+B_3^2(t)} \end{bmatrix} \begin{bmatrix} dB_1(t) \\ dB_2(t) \\ dB_3(t) \end{bmatrix}$$

⑥ $$Y_t = \begin{bmatrix} \cos^3(B_1(t)+B_3(t)) \\ -\sin^3(B_1(t)+B_3(t)) \\ B_2(t) \end{bmatrix}$$

$$\cos^3(B_1(t)+B_3(t)) \to 3\cos^2(B_1(t)+B_3(t))(-\sin(B_1(t)+B_3(t)))$$
$$\to 6(\cos(B_1(t)+B_3(t)))(\sin^2(B_1(t)+B_3(t))) - 3\cos^3(B_1(t)+B_3(t))$$
$$-\sin^3(B_1(t)+B_3(t)) \to -3\sin^2(B_1(t)+B_3(t))(\cos(B_1(t)+B_3(t)))$$
$$\to -6\sin(B_1(t)+B_3(t))\cos^2(B_1(t)+B_3(t)) + 3\sin^3(B_1(t)+B_3(t))$$

$$dY_t = \frac{1}{2}\begin{bmatrix} 12\cos(B_1(t)+B_3(t))\sin^2(B_1(t)+B_3(t)) - 6\cos^3(B_1(t)+B_3(t)) \\ -12\sin(B_1(t)+B_3(t))\sin^2(B_1(t)+B_3(t)) + 6\sin^3(B_1(t)+B_3(t)) \\ 0 \end{bmatrix} dt$$
$$+ \begin{bmatrix} -3\cos^2(B_1(t)+B_3(t))\sin(B_1(t)+B_3(t)) & 0 & -3\cos^2(B_1(t)+B_3(t))\sin(B_1(t)+B_3(t)) \\ -3\sin^2(B_1(t)+B_3(t))\cos(B_1(t)+B_3(t)) & 0 & -3\sin^2(B_1(t)+B_3(t))\cos(B_1(t)+B_3(t)) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} dB_1(t) \\ dB_2(t) \\ dB_3(t) \end{bmatrix}$$

7 Dane HW5.

Sección 1:

1. $g(t, X_t) = t^3 + t + X_1(t) - X_2(t)$; $dX_t = \begin{bmatrix} 3 \\ t \end{bmatrix} dt + \begin{bmatrix} 1/2\,t & 0 \\ 0 & -1 \end{bmatrix} dB(t)$

$$dg(t, X_t) = (3t^2+1)dt + dX_1(t) - dX_2(t) + \frac{1}{2}\left(0(dX_1(t))^2 + 2(0)\,dX_1(t)\,dX_2(t) + (0)(dX_2(t))^2\right)$$

$$= (3t^2+1)dt + \left[3dt + \frac{t\,dB_1(t)}{2}\right] - \left[t\,dt - dB_2(t)\right]$$

$$= \underbrace{(3t^2 - t + 4)}_{f(t)}dt + \underbrace{\begin{bmatrix} t/2 & 1 \end{bmatrix}}_{h(t)} \begin{bmatrix} dB_1(t) \\ dB_2(t) \end{bmatrix}$$

2. $g(t, X_t) = e^t + X_1(t)X_2(t)$, $dX_1(t) = \cos t\,dt + \sin t\,dB_1(t)$

$dX_2(t) = -\sin t\,dt + \cos t\,dB_2(t)$.

$$dg(t, X_t) = e^t dt + X_2(t)\,dX_1(t) + X_1(t)\,dX_2(t) + \frac{1}{2}\left[(0)(dX_1(t))^2 + 2(1)\,dX_1(t)\,dX_2(t) + (0)(dX_2)^2\right]$$

$$dg(t, X_t) = e^t dt + X_2(t)\left[\cos t\,dt + \sin t\,dB_1(t)\right] + X_1(t)\left[-\sin t\,dt + \cos t\,dB_2(t)\right]$$
$$+ \left[\cos t\,dt + \sin t\,dB_1(t)\right]\left[-\sin t\,dt + \cos t\,dB_2(t)\right]$$

$$= \underbrace{(e^t + X_2(t)\cos t - X_1(t)\sin t)}_{f(t)}dt + \underbrace{\begin{bmatrix} X_2(t)\sin t & X_1(t)\cos t \end{bmatrix}}_{h(t)} \begin{bmatrix} dB_1(t) \\ dB_2(t) \end{bmatrix}$$

$$\alpha X_2(t)e^{\alpha X_1X_2} \xrightarrow{X_1} \alpha e^{\alpha X_1 X_2} + \alpha^2 X_2 X_1 e^{\alpha X_1 X_2}_{f(t)}$$

③ $dX(t) = M(t)dt + H(t)dB_t$, $M(t) = \begin{bmatrix} t \\ 1/2 \end{bmatrix}$ $H(t) = e^t \mathbb{I}_2$

a) $g(t, X_t) = e^{\alpha(X_1(t)+X_2(t))}$

$$dg(t,X_t) = \left\{ \begin{bmatrix} \alpha e^{\alpha(X_1(t)+X_2(t))} & \alpha e^{\alpha(X_1(t)+X_2(t))} \end{bmatrix} \begin{bmatrix} t \\ 1/2 \end{bmatrix} + \frac{1}{2} tr\left(e^t \mathbb{I}_2 \begin{bmatrix} \alpha^2 e^{\alpha(X_1(t)+X_2(t))} & \alpha^2 e^{\alpha(X_1(t)+X_2(t))} \\ \alpha^2 e^{\alpha(X_1(t)+X_2(t))} & \alpha^2 e^{\alpha(X_1(t)+X_2(t))} \end{bmatrix} e^t \mathbb{I}_2 \right) \right\} dt$$

$$+ \begin{bmatrix} \alpha e^{\alpha(X_1(t)+X_2(t))} & \alpha e^{\alpha(X_1(t)+X_2(t))} \end{bmatrix} e^t \mathbb{I}_2 \, dB_t$$

$$dg(t,X_t) = \left(\alpha e^{\alpha(X_1(t)+X_2(t))}(t+1/2) + \frac{1}{2} tr\left[\alpha^2 e^{\alpha(X_1(t)+X_2(t))+2t} \mathbb{I}_2 \right] \right) dt$$

$$+ \alpha e^{\alpha(X_1(t)+X_2(t))+t} \begin{bmatrix} 1 & 1 \end{bmatrix} dB_t$$

$$dg(t,X_t) = \left(\alpha Y_t (t+1/2) + \alpha^2 Y_t e^{2t} \right) dt + \alpha Y_t \begin{bmatrix} 1 & 1 \end{bmatrix} dB_t \quad Y_t = e^{\alpha(X_1(t)+X_2(t))}$$

b) $g(t, X_t) = e^{\alpha X_1(t) X_2(t)}$

$$dg(t,X_t) = \left(\begin{bmatrix} \alpha X_2(t) e^{\alpha X_1(t)X_2(t)} & \alpha X_1(t) e^{\alpha X_1(t)X_2(t)} \end{bmatrix} \begin{bmatrix} t \\ 1/2 \end{bmatrix} \right.$$

$$\left. + \frac{1}{2} tr\left(e^t \mathbb{I}_2 \begin{bmatrix} \alpha^2 X_2^2(t) e^{\alpha X_1(t)X_2(t)} & \alpha e^{\alpha X_1(t)X_2(t)}(1+\alpha X_1(t)X_2(t)) \\ \alpha e^{\alpha X_1(t)X_2(t)}(1+\alpha X_1(t)X_2(t)) & \alpha^2 X_1^2(t) e^{\alpha X_1(t)X_2(t)} \end{bmatrix} e^t \mathbb{I}_2 \right) \right) dt +$$

$$+ \begin{bmatrix} \alpha X_2(t) e^{\alpha X_1(t)X_2(t)} & \alpha X_1(t) e^{\alpha X_1(t)X_2(t)} \end{bmatrix} e^t \mathbb{I}_2 \, dB_t$$

$$dg(t,X_t) = \left[\alpha e^{\alpha X_1(t)}(X_2(t)t + X_1(t)/2) + \alpha^2 e^{2t+\alpha X_1(t)X_2(t)} (X_2^2(t) + X_1^2(t)) \right] dt$$

$$+ \alpha e^{t+\alpha X_1(t)X_2(t)} \begin{bmatrix} X_2(t) & X_1(t) \end{bmatrix} dB_t$$

c) $g(t, X_t) = e^{t^2 \sin(X_1(t))\cos(X_2(t))}$

$$dg(t, X_t) = \Big\{ \left(2t \sin X_1(t) \cos X_2(t)\right) e^{t^2 \sin X_1(t) \cos(X_2(t))}$$

$$+ \left[\left(t^2 \cos X_1(t) \cos X_2(t)\right) e^{t^2 \sin X_1(t) \cos X_2(t)} \quad - t^2 \sin X_1(t) \sin X_2(t) e^{t^2 \sin X_1(t) \cos X_2(t)} \right] \begin{bmatrix} t \\ 1/2 \end{bmatrix}$$

$$+ \frac{1}{2} tr\left(e^t \mathbb{I}_2 \begin{bmatrix} a & b \\ b & c \end{bmatrix} e^t \mathbb{I}_2 \right) \Big\} dt + t e^{t + t^2 \sin X_1(t) \cos X_2(t)} \left[\cos X_1(t) \cos X_2(t) \quad - \sin X_1(t) \sin X_2(t) \right] dB_t$$

$$a = -t^2 \sin X_1(t) \cos X_2(t) e^{t^2 \sin X_1(t) \cos X_2(t)} + t^4 \cos^2 X_1(t) \cos^2 X_2(t) e^{t^2 \sin X_1(t) \cos X_2(t)}$$

$$b = -t^2 \cos X_1(t) \sin X_2(t) e^{t^2 \sin X_1(t) \cos X_2(t)} - t^4 \cos X_1(t) \cos X_2(t) \sin X_1(t) \sin X_2(t) e^{t^2 \sin X_1(t) \cos X_2(t)}$$

$$c = -t^2 \sin X_1(t) \cos X_2(t) e^{t^2 \sin X_1(t) \cos X_2(t)} + t^4 \sin^2 X_1(t) \sin^2 X_2(t) e^{t^2 \sin X_1(t) \cos X_2(t)}$$

$$dg(t, X_t) = t e^{t^2 \sin X_1(t) \cos X_2(t)} \left[2 \sin X_1(t) \cos X_2(t) + t^2 \cos X_1(t) \cos X_2(t) - \frac{t}{2} \sin X_1(t) \sin X_2(t) \right] dt$$

$$+ \frac{1}{2} \left[-t^2 e^{t^2 \sin X_1(t) \cos X_2(t) + 2t} \left(2 \sin X_1(t) \cos X_2(t) - t^2 \left(\cos^2 X_1(t) \cos^2 X_2(t) + \sin^2 X_1(t) \sin^2 X_2(t) \right) \right) \right] dt.$$

$$+ t^2 e^{t + t^2 \sin X_1(t) \cos X_2(t)} \left[\cos X_1(t) \cos X_2(t) \quad - \sin X_1(t) \sin X_2(t) \right] dB_t .$$

$$dg(t, X_t) = t e^{t^2 \sin X_1(t) \cos X_2(t)} \Big[2 \sin X_1(t) \cos X_2(t) + t^2 \cos X_1(t) \cos X_2(t) - \frac{t}{2} \sin X_1(t) \sin X_2(t)$$

$$- \frac{t e^{2t}}{2} \left(2 \sin X_1(t) \cos X_2(t) - t^2 \left(\cos^2 X_1(t) \cos^2 X_2(t) + \sin^2 X_1(t) \sin^2 X_2(t) \right) \right) \Big] dt.$$

$$+ t^2 e^{t + t^2 \sin X_1(t) \cos X_2(t)} \left[\cos X_1(t) \cos X_2(t) \quad - \sin X_1(t) \sin X_2(t) \right] dB_t .$$

$$X_t = X_0 + \int_0^t M\,dt \qquad f(t)=M$$

(4) Malo.

(5) $g(t,X_t) = e^{\alpha X_t}$, $dX_t = M\,dt$, $M = \mathbb{I}_2$, $X_0 = 0_{1\times 1}$

$$dg(t,X_t) = \alpha e^{\alpha X_t}\,dX_t + \cancelto{0}{\frac{1}{2}(\alpha^2 e^{\alpha X_t})(dX_t)^2}$$

$$dg(t,X_t) = \alpha e^{\alpha X_t}(M\,dt) = \alpha e^{\alpha X_t}\,dt.$$

(6) $g(t,X_t) = \ln(t) + X_1(t)X_2(t)X_3(t)$; $dX_t = \begin{bmatrix} 1/t \\ t \\ t^2 \end{bmatrix} dt + \begin{bmatrix} t/2 & t & t^2 \\ 0 & 1 & t \\ 1/t & 0 & e^t \end{bmatrix} dB_t$

$$dg(t,X_t) = \left\{ \frac{1}{t} + \begin{bmatrix} X_2(t)X_3(t) & X_1(t)X_3(t) & X_1(t)X_2(t) \end{bmatrix} \begin{bmatrix} 1/t \\ t \\ t^2 \end{bmatrix} + \frac{1}{2} tr\left(\begin{bmatrix} t/2 & 0 & 1/t \\ t & 1 & 0 \\ t^2 & t & e^t \end{bmatrix} \begin{bmatrix} 0 & X_3(t) & X_2(t) \\ X_3(t) & 0 & X_1(t) \\ X_2(t) & X_1(t) & 0 \end{bmatrix} \begin{bmatrix} t/2 & t & t^2 \\ 0 & 1 & t \\ 1/t & 0 & e^t \end{bmatrix} \right) \right\} dt$$

$$+ \begin{bmatrix} X_2(t)X_3(t) & X_1(t)X_3(t) & X_1(t)X_2(t) \end{bmatrix} \begin{bmatrix} t/2 & t & t^2 \\ 0 & 1 & t \\ 1/t & 0 & e^t \end{bmatrix} dB_t$$

$$dg(t,X_t) = \left\{ \frac{1}{t} + \frac{X_2(t)X_3(t)}{t} + tX_1(t)X_3(t) + t^2X_1(t)X_2(t) \right.$$

$$\left. + \frac{1}{2} tr\left(\begin{bmatrix} \frac{X_2(t)}{t} & \frac{tX_3(t)}{2} + \frac{X_1(t)}{t} & \frac{tX_2(t)}{2} \\ X_3(t) & tX_3(t) & tX_2(t) + X_1(t) \\ tX_3(t) + e^t X_2(t) & t^2X_3(t) + e^t X_1(t) & t^2X_2(t) + tX_1(t) \end{bmatrix} \begin{bmatrix} t/2 & t & t^2 \\ 0 & 1 & t \\ 1/t & 0 & e^t \end{bmatrix} \right) \right\} dt$$

$$+\left[t x_2(t)x_3(t)+\frac{x_1(t)x_2(t)}{t} \quad t x_1(t)x_3(t)+x_1(t)x_3(t) \quad t^2 x_2(t)x_3(t)+t x_1(t)x_3(t)+e^t x_2(t)x_1(t)\right] dB_t$$

$$dg(t,X_t)=\left\{\frac{1}{t}+\frac{x_2(t)x_3(t)}{t}+t^2 x_1(t)x_2(t)\right.$$

$$+\frac{1}{2}\,tr\left(\begin{bmatrix} x_2(t) & \frac{t x_3(t)}{2}+x_2(t)+\frac{x_1(t)}{t} & \frac{t^2 x_3(t)}{2}+t x_2(t)\left(\frac{e^t}{2}+1\right)+x_1(t) \\ \frac{t x_3(t)}{2}+x_2(t)+\frac{x_1(t)}{t} & 2t x_3(t) & 2t^2 x_3(t)+e^t(t x_2(t)+x_1(t)) \\ \frac{t^2 x_3(t)}{2}+t x_2(t)\left(\frac{e^t}{2}+1\right)+x_1(t) & 2t^2 x_3(t)+e^t(t x_2(t)+x_1(t)) & \Delta \end{bmatrix}\right)\Bigg\} dt$$

$$+\left[\frac{t x_2(t)x_3(t)}{2}+\frac{x_1(t)x_2(t)}{t},\ t x_1(t)x_2(t)+x_1(t)x_3(t),\ t^2 x_2(t)x_3(t)+t x_1(t)x_3(t)+e^t x_1(t)x_2(t)\right] dB_t.$$

donde $\Delta = 2t^3 x_3(t)+2t^2 x_2(t)+2te^t x_1(t)$

$$dg(t,X_t)=\left[\frac{1}{t}+\frac{x_2(t)x_3(t)}{t}+t^2 x_1(t)x_2(t)+\frac{1}{2}\left[x_2(t)+2t x_3(t)+2t^3 x_3(t)+2t^2 x_2(t)+2te^t x_1(t)\right]\right] dt$$

$$+\left[\frac{t x_2(t)x_3(t)}{2}+\frac{x_1(t)x_2(t)}{t},\ t x_1(t)x_2(t)+x_1(t)x_3(t),\ t^2 x_2(t)x_3(t)+t x_1(t)x_3(t)+e^t x_1(t)x_2(t)\right] dB_t$$

$$⑦\ g(t,X_t)=e^{tX_1(t)}+e^{-tX_2(t)},\quad dX_t=f(t)dt+h(t)dB_t,\quad f(t)=(e^{-t},e^{t})^T$$

$$h(t)=\begin{bmatrix}1 & 0 & 1\\ B_1(t) & -B_2(t) & B_3(t)\end{bmatrix},\quad B_t=(B_1(t),B_2(t),B_3(t))^T$$

$$dg(t,X_t)=\Big\{X_1(t)e^{t}-X_2(t)e^{-tX_2(t)}+\begin{bmatrix}te^{tX_1(t)} & -te^{-tX_2(t)}\end{bmatrix}\begin{bmatrix}e^{-t}\\ e^{t}\end{bmatrix}$$

$$+\frac{1}{2}tr\left(\begin{bmatrix}1 & B_1(t)\\ 0 & -B_2(t)\\ 1 & B_3(t)\end{bmatrix}\begin{bmatrix}t^2e^{tX_1(t)} & 0\\ 0 & t^2e^{-tX_2(t)}\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ B_1(t) & -B_2(t) & B_3(t)\end{bmatrix}\right)\Big\}dt$$

$$+\begin{bmatrix}te^{tX_1(t)} & -te^{-tX_2(t)}\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ B_1(t) & -B_2(t) & B_3(t)\end{bmatrix}dB_t$$

$$dg(t,X_t)=\Big\{X_1(t)e^{tX_1(t)}-X_2(t)e^{-tX_2(t)}+te^{tX_1(t)-t}-te^{-tX_2(t)+t}$$

$$+\frac{1}{2}tr\left(\begin{bmatrix}t^2e^{tX_1(t)} & t^2B_1(t)e^{-tX_2(t)}\\ 0 & -t^2B_2(t)e^{-tX_2(t)}\\ t^2e^{tX_1(t)} & t^2B_3(t)e^{-tX_2(t)}\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ B_1(t) & -B_2(t) & B_3(t)\end{bmatrix}\right)\Big\}dt$$

$$+\begin{bmatrix}te^{tX_1(t)}-tB_1(t)e^{-tX_2(t)} & tB_2(t)e^{-tX_2(t)} & te^{tX_1(t)}-tB_3(t)e^{-tX_2(t)}\end{bmatrix}dB_t.$$

$$dg(t,X_t)=\Big\{e^{tX_1(t)}(X_1(t)+te^{-t})-e^{-tX_2(t)}(X_2(t)+te^{t})$$

$$+\frac{1}{2}tr\left(\begin{bmatrix}t^2e^{tX_1(t)}+t^2B_1^2(t)e^{-tX_2(t)} & -t^2B_1(t)B_2(t)e^{-tX_2(t)} & t^2e^{tX_1(t)}+t^2B_1(t)B_3(t)e^{-tX_2(t)}\\ -t^2B_1(t)B_2(t)e^{-tX_2(t)} & t^2B_2^2(t)e^{-tX_2(t)} & -t^2B_2(t)B_3(t)e^{-tX_2(t)}\\ t^2e^{tX_1(t)}+t^2B_1(t)B_3(t)e^{-tX_2(t)} & -t^2B_2(t)B_3(t)e^{-tX_2(t)} & t^2e^{tX_1(t)}+t^2B_3^2(t)e^{-tX_2(t)}\end{bmatrix}\right)\Big\}dt.$$

$$+\begin{bmatrix}te^{tX_1(t)}-tB_1(t)e^{-tX_2(t)} & tB_2(t)e^{-tX_2(t)} & te^{tX_1(t)}-tB_3(t)e^{-tX_2(t)}\end{bmatrix}dB_t.$$

$$\nabla_x^T g = \left[-2\cos(\varphi t + X_1(t))\sin(\varphi t + X_1(t)) \quad 2\sin(\varphi t + X_2(t))\cos(\varphi t + X_2(t))\right]$$
$$= \left[-\sin(2\varphi t + 2X_1(t)) \quad \sin(2\varphi t + 2X_2(t))\right]$$

$$dg(t, X_t) = \left[e^{tX_1(t)}(X_1(t) + te^{-t}) - e^{-tX_2(t)}(X_2(t) + te^{t}) + t^2 e^{tX_1(t)} + t^2 e^{-tX_2(t)}\left(B_1^2(t) + B_2^2(t) + B_3^2(t)\right)\right] dt$$
$$+ \left[te^{tX_1(t)} - tB_1(t)e^{-tX_2(t)} \quad tB_2(t)e^{-tX_2(t)} \quad te^{tX_1(t)} - tB_3(t)e^{-tX_2(t)}\right] dB_t.$$

⑧. $g(t, X_t) = \cos^2(\varphi t + X_1(t)) + \sin^2(\varphi t + X_2(t))$, $dX_t = f(t)dt + h(t)dB_t$

$$f(t) = (\cos(\varphi t), \sin(\varphi t))^T \quad h(t) = \begin{bmatrix} \cos B_1(t) & 0 & \sin(B_3(t)) \\ \sin B_2(t) & 0 & \cos B_1(t) \end{bmatrix}$$

$$dg(t, X_t) = \Big\{\varphi \cdot 2\cos(\varphi t + X_1(t))(-\sin(\varphi t + X_1(t))) + \varphi \cdot 2\sin(\varphi t + X_2(t))\cos(\varphi t + X_2(t))$$
$$+ \left[-2\cos(\varphi t + X_1(t))\sin(\varphi t + X_1(t)) \quad 2\sin(\varphi t + X_2(t))\cos(\varphi t + X_2(t))\right]\begin{bmatrix} \cos(\varphi t) \\ \sin(\varphi t) \end{bmatrix}$$
$$+ \frac{1}{2} tr\left(\begin{bmatrix} \cos B_1(t) & \sin B_2(t) \\ 0 & 0 \\ \sin B_3(t) & \cos B_1(t) \end{bmatrix}\begin{bmatrix} -2\cos(2\varphi t + 2X_1(t)) & 0 \\ 0 & 2\cos(2\varphi t + 2X_2(t)) \end{bmatrix}\begin{bmatrix} \cos B_1(t) & 0 & \sin B_3(t) \\ \sin B_2(t) & 0 & \cos B_1(t) \end{bmatrix}\right)\Big\} dt$$
$$+ \left[-\sin(2\varphi t + 2X_1(t)) \quad \sin(2\varphi t + 2X_2(t))\right]\begin{bmatrix} \cos B_1(t) & 0 & \sin B_3(t) \\ \sin B_2(t) & 0 & \cos B_1(t) \end{bmatrix} dB_t.$$

$$dg(t, X_t) = \Big\{-\varphi\sin(2\varphi t + 2X_1(t)) + \varphi\sin(2\varphi t + 2X_2(t)) - \sin(2\varphi t + 2X_1(t))\cos(\varphi t) + \sin(2\varphi t + 2X_2(t))\sin(\varphi t)$$
$$+ \frac{1}{2} tr\left(\begin{bmatrix} -2\cos B_1(t)\cos(2\varphi t + 2X_1(t)) & 2\sin B_2(t)\cos(2\varphi t + 2X_2(t)) \\ 0 & 0 \\ -2\sin B_3(t)\cos(2\varphi t + 2X_1(t)) & 2\cos B_1(t)\cos(2\varphi t + 2X_2(t)) \end{bmatrix}\begin{bmatrix} \cos B_1(t) & 0 & \sin B_3(t) \\ \sin B_2(t) & 0 & \cos B_1(t) \end{bmatrix}\right)\Big\} dt.$$
$$+ \left[-\sin(2\varphi t + 2X_1(t))\cos B_1(t) + \sin(2\varphi t + 2X_2(t))\sin B_2(t) \quad 0 \quad -\sin(2\varphi t + 2X_1(t))\sin B_3(t) + \sin(2\varphi t + 2X_2(t))\cos B_1(t)\right] dB_t$$

$$dg(t, X_t) = \Big\{-\sin(2\varphi t + 2X_1(t))(\varphi + \cos(\varphi t)) + \sin(2\varphi t + 2X_2(t))(\varphi + \sin(\varphi t)) +$$

$+$

$$+\frac{1}{2}tr\left\{\begin{bmatrix} -2\cos^2 B_1(t)\cos(2\varphi t+2X_1(t))+2\sin^2 B_1(t)\cos(2\varphi t+2X_1(t)), & 0, & -2\cos B_1(t)\sin B_3(t)\cos(2\varphi t+2X_1(t))+2\sin B_1(t)\cos B_1(t)\cos(2\varphi t+2X_1(t)) \\ 0, & 0, & 0 \\ -2\cos B_1(t)\sin B_1(t)\cos(2\varphi t+2X_1(t))+2\sin B_1(t)\cos B_1(t)\cos(2\varphi t+2X_1(t)), & 0, & -2\sin^2 B_3(t)\cos(2\varphi t+2X_1(t))+2\cos^2 B_1(t)\cos(2\varphi t+2X_1(t)) \end{bmatrix}\right\}dt$$

$$+\left[-\sin(2\varphi t+2X_1(t))\cos B_1(t)+\sin(2\varphi t+2X_1(t))\sin B_1(t),\ 0,\ -\sin(2\varphi t+2X_1(t))\sin B_3(t)+\sin(2\varphi t+2X_1(t))\cos B_1(t)\right]dB_t$$

$$dg(t,X_t)=\{-\sin(\varphi t+2X_1(t))(\varphi+\cos(\varphi t))+\sin(2\varphi t+2X_1(t))(\varphi+\sin(\varphi t))-\cos^2 B_1(t)\cos(2\varphi t+2X_1(t))+\sin^2 B_1(t)\cos(2\varphi t+2X_1(t))$$

$$-\sin^2 B_3(t)\cos(2\varphi t+2X_1(t))+\cos^2 B_1(t)\cos(2\varphi t+2X_1(t))\}dt.$$

$$+\left[-\sin(2\varphi t+2X_1(t))\cos B_1(t)+\sin(2\varphi t+2X_1(t))\sin B_1(t)\ \ 0\ -\sin(2\varphi t+2X_1(t))\sin B_3(t)+\sin(2\varphi t+2X_1(t))\cos B_1(t)\right]dB_t$$

Taller EDE: Freddy:

EDE Lineal Homogénea:

1) $dX_t=\sigma X_t dB_t,\ \sigma\in\mathbb{R}.$

$$\frac{dX_t}{X_t}=\sigma dB_t \longrightarrow \int_0^t\frac{dX_s}{X_s}=\sigma\int_0^t dB_s=\sigma B_t.\quad (1)$$

$$\text{Sea } g(X_t)=\ln(X_t)\xrightarrow{\text{Itô}} dg(X_t)=\frac{dX_t}{X_t}-\frac{1}{2X_t^2}(dX_t)^2=\frac{dX_t}{X_t}-\frac{1}{2X_t^2}(\sigma X_t dB_t)^2$$

$$dg(X_t)=\frac{dX_t}{X_t}-\frac{1}{2}\sigma^2 dt\xrightarrow{\int}\ln\left(\frac{X_t}{X_0}\right)=\int_0^t\frac{dX_s}{X_s}-\frac{1}{2}\int_0^t\sigma^2 ds\quad (2)$$

$$(1)\text{ en }(2):\ \ln\left(\frac{X_t}{X_0}\right)=\sigma B_t-\frac{1}{2}\sigma^2 t\longrightarrow X_t=X_0\exp\left(\sigma B_t-\frac{\sigma^2 t}{2}\right)$$