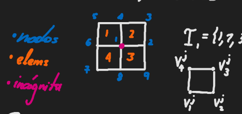
6. a) Diap. 71 ← con notación

b) $-\nabla^2 u = 2\pi^2 \sin(\pi x)\sin(\pi y)$ $\quad \Omega = (0,1)\times(0,1)$

$u = 0$, sobre $\partial\Omega$



$I_1 = \{1,2,3,4\}$

Para el elemento 4: $(0,0), (0,1/2), (1/2,0), (1/2,1/2)$

$\varphi_i(x,y) = a_i + b_i x + c_i y + d_i xy$

Para $V_1^4 = (0,0)$:

$\varphi_1(0,0) = 1 = a_1$

$\varphi_1(0,1/2) = 1 + \frac{c_1}{2} \to c_1 = -2$

$\varphi_1(1/2,0) = 1 + \frac{b_1}{2} \to b_1 = -2$

$\varphi_1(1/2,1/2) = 1 - 1 - 1 + \frac{d_1}{4} \Rightarrow d_1 = 4$

$\varphi_1(x,y) = 1 - 2x - 2y + 4xy$

$\nabla\varphi_1 = (-2+4y, -2+4x)$

Para $V_2^4 = (1/2,0)$:

$\varphi_2(0,0) = 0 = a_2$

$\varphi_2(1/2,0) = 1 = \frac{b_2}{2} \to b_2 = 2$

$\varphi_2(0,1/2) = 0 = \frac{c_2}{2} \to c_2 = 0$

$\varphi_2(1/2,1/2) = 0 = 1 + \frac{d_2}{4} \to d_2 = -4$

$\varphi_2(x,y) = 2x - 4xy$

$\nabla\varphi_2 = (2-4y, -4x)$

Para $V_3^4 = (1/2,1/2)$:

$\varphi_3(0,0) = 0 = a_3$

$\varphi_3(1/2,0) = 0 = \frac{b_3}{2} \to b_3 = 0$

$\varphi_3(0,1/2) = 0 = \frac{c_3}{2} \to c_3 = 0$

$\varphi_3(1/2,1/2) = 1 = \frac{d_3}{4} \to d_3 = 4$

$\varphi_3(x,y) = 4xy$

$\nabla\varphi_3 = (4y, 4x)$

Para $V_4^4 = (0,1/2)$:

$\varphi_4(0,0) = 0 = a_4$

$\varphi_4(1/2,0) = 0 = \frac{b_4}{2} \to b_4 = 0$

$\varphi_4(0,1/2) = 1 = \frac{c_4}{2} \to c_4 = 2$

$\varphi_4(1/2,1/2) = 1 + \frac{d_4}{4} \to d_4 = -4$

$\varphi_4(x,y) = 2y - 4xy$

$\nabla\varphi_1 = (-4y, 2-4x)$

$$K_{11} = \sum_{r\in I_1} \int_{C_r} \|\nabla\xi_1|_{C_r}\|^2 dA_{C_r}$$

$$= \int_{C_1} \|\nabla\varphi_2\|^2 dA_{C_1} + \int_{C_2} \|\nabla\varphi_1\|^2 dA_{C_2} + \int_{C_3} \|\nabla\varphi_4\|^2 dA_{C_3} + \int_{C_4} \|\nabla\varphi_3\|^2 dA_{C_4}$$

$$= \int_{C_1} [(2-4y)^2 + 16x^2]\, dA_{C_1} + \int_{C_2} [(-2+4y)^2 + (-2+4x)^2]\, dA_{C_2} + \int_{C_3} [16y^2 + (2-4x)^2]\, dA_{C_3} + \int_{C_4} [16x^2 + 16y^2]\, dA_{C_4}$$

$$= \int_0^{1/2}\int_{1/3}^{1} [(2-4y)^2 + 16x^2]\, dy\,dx + \int_{1/2}^{1}\int_{1/2}^{1} [(-2+4y)^2 + (-2+4x)^2]\, dy\,dx + \int_{1/2}^{1}\int_0^{1/2} [16y^2 + (2-4x)^2]\, dy\,dx + \int_0^{1/2}\int_0^{1/2} [16x^2 + 16y^2]\, dy\,dx.$$

$$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3}$$

$\ell_1 \approx \frac{|I_1| A_e}{4} f(v_1)$

$\approx \frac{1}{4} f(1/2, 1/2)$

$= \frac{\pi^2}{2}$

$\frac{8}{3} u_1 = \frac{\pi^2}{2} \to u_1 = \frac{3\pi^2}{16}$