

$$15. \Omega = (0,1)$$

$$\varphi \in \mathcal{D}(\Omega)$$

$$\langle T, \varphi \rangle = \sum_{n=1}^{\infty} \varphi^{(n)}\left(\frac{1}{n}\right)$$

Lineal: Sean $a, b \in \mathbb{R}$, $\varphi, \psi \in \mathcal{D}(\Omega)$

$$\begin{aligned} \langle T, a\varphi + b\psi \rangle &= \sum_{n=1}^{\infty} \left. \frac{d^n}{dx^n} (a\varphi(x) + b\psi(x)) \right|_{x=1/n} \\ &= \sum_{n=1}^{\infty} [a\varphi^{(n)}(1/n) + b\psi^{(n)}(1/n)] \\ &= a \sum_{n=1}^{\infty} \varphi^{(n)}(1/n) + b \sum_{n=1}^{\infty} \psi^{(n)}(1/n) \\ &= a \langle T, \varphi \rangle + b \langle T, \psi \rangle \end{aligned}$$

Continuo: Sea $K \subset \Omega$ compacto
Sea $\varphi \in \mathcal{D}_K(\Omega)$

$$\begin{aligned} |\langle T, \varphi \rangle| &= \left| \sum_{n=0}^{\infty} \varphi^{(n)}(1/n) \right| \\ &\leq \sum_{n=0}^{\infty} |\varphi^{(n)}(1/n)| \\ &\leq \sum_{n=0}^{\infty} \sup_{x \in K} |\varphi^{(n)}(x)| \end{aligned}$$

Entonces, T es continuo con $C_m = 1$

y $m = \infty$.

Entonces, T es distribución de orden infinito