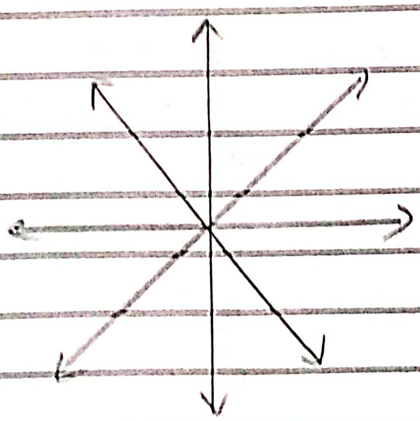
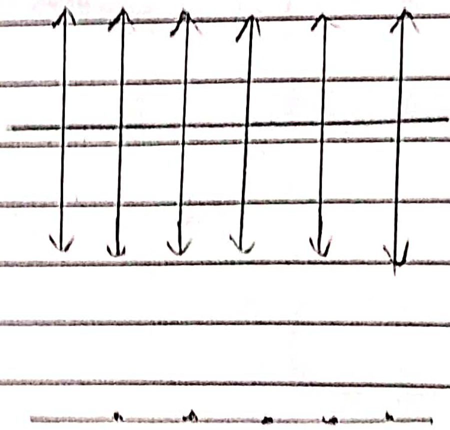
Chapter-3
Polarisation

A natural or unpolarised light, the optical vector (electric vector) has no specific orientation. When the light passes through certain crystals light tourmaline, quartz, calcite etc. the vibration of electric vector confined to a single direction in the plane perpendicular to the direction of propagation of wave. Such light is called polarised light and this concept is called polarisation.

Polarisation effects shows that light is transverse in nature.

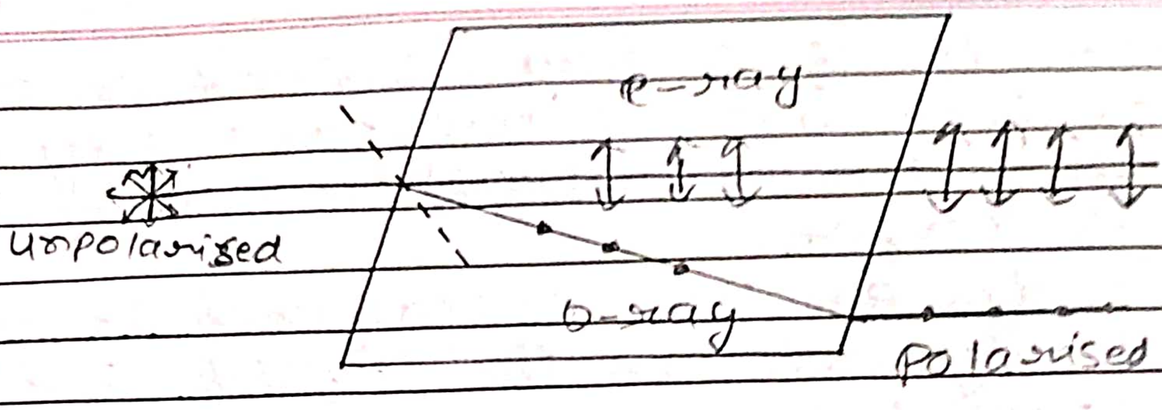


up unpolarised or natural light



polarised light

Polarisation by Double refraction



e-ray - extraordinary ray
O-ray - ordinary ray

When unpolarised light incident on Calcide Crystal ($CaCO_3$) it split into two refracted rays different in their properties. the phenomenon of causing two refracted rays by a crystal is called double refraction or birefringes. Such crystal is called birefringent.
One of the ray obeys snell's law it is called ordinary ray or O-ray the other ray does not obeys Snell's law it is called extraordinary ray or e-ray. both the rays are polarised in mutually perpendicular direction

Properties of e-ray and O-ray

1. O-ray follows Snell's law. While e-ray does not follow Snell's law.
2. both are polarised in mutualy perpendicular direction.

3. Velocity of O-ray is same in all direction but velocity of e-ray is different along different direction with in the crystal.
4. the distinction e-ray and O-ray exist only with in the crystal. outside the crystal both rays are only plane polarised wave.
5.

Refractive index of crystal for O-ray, $\mu_o = \frac{c}{V_o}$

$$\mu_o = \frac{\text{Velocity of light in air}}{\text{Velocity of O-ray in crystal}}$$

Refrative index of positive crystal for e-ray, $\mu_e = \frac{c}{(V_e)_{min}}$

$$\mu_e = \frac{\text{Velocity of light in air}}{\text{minimum Velocity of e-ray in crystal}}$$

Refractive index of -ve crystal for e-ray $\mu_e = \frac{c}{(V_e)_{max}}$

$$\mu_e = \frac{\text{Velocity of light in air}}{\text{maximum Velocity of e-ray in crystal}}$$

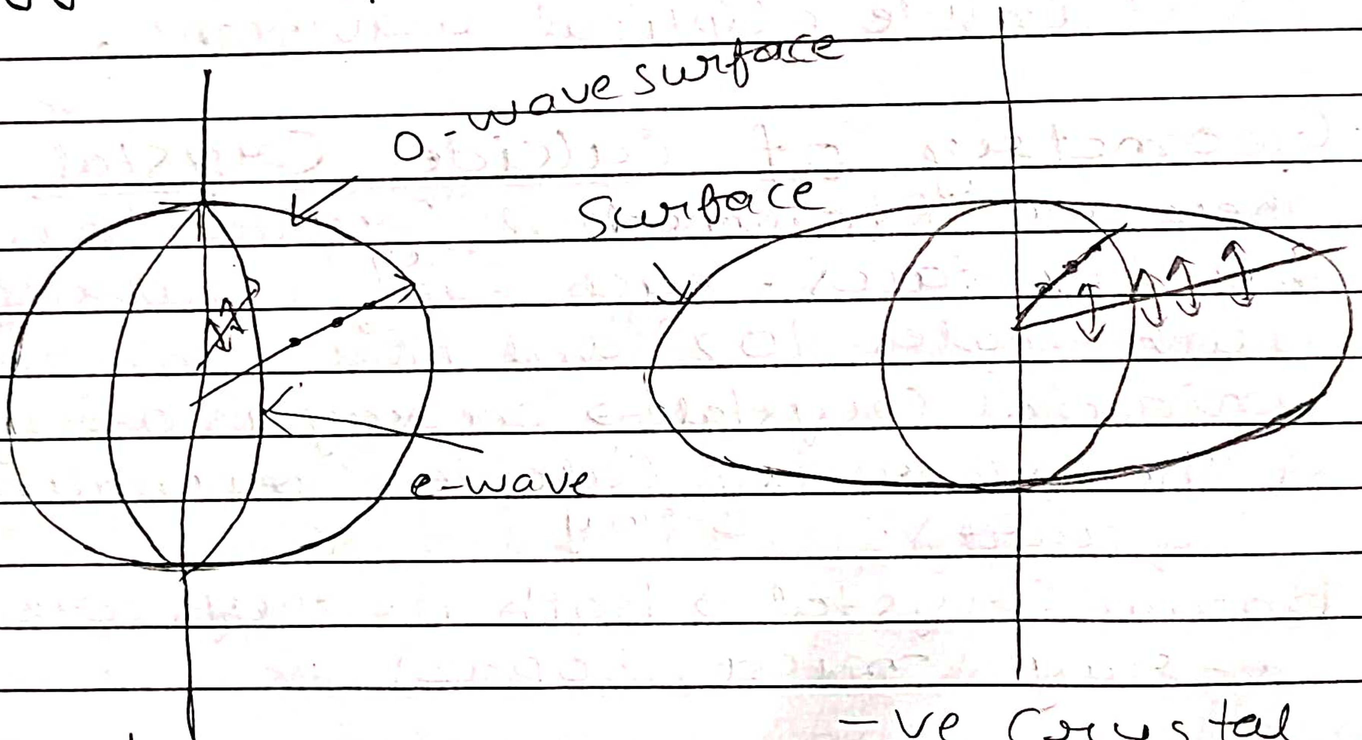
6. If 't' is thickness of crystal then Path difference between e-ray and o-ray is given by

Path difference $= \mu_e t - \mu_o t$

Phase difference $= \frac{2\pi}{\lambda}(\mu_e - \mu_o)t$

+ve → $\mu_e > \mu_o$
−ve → $\mu_e < \mu_o$

Hygen's explanation of double refraction



+ve crystal
$\mu_e > \mu_o$

−ve Crystal
$\mu_e < \mu_o$

if a point source exist with in a Crystal, the weights expand in all direction with same velocity so, the wave front is spherical.
In Case of double refracting crystal the wave surface of o-ray is

Spherical because it travels with same velocity in all direction. e-ray travels with different velocity in different direction therefore wave surface of e-ray is eliptical these two wave surfaces travel in different direction. Ultimately two refracted rays emerged out ~~of the~~ from the ~~cy~~ crystal in positive crystal eliptical wave front exist in side the spherical wave front.
negative crystal the spherical wavefront exist inside eliptical wavefront.

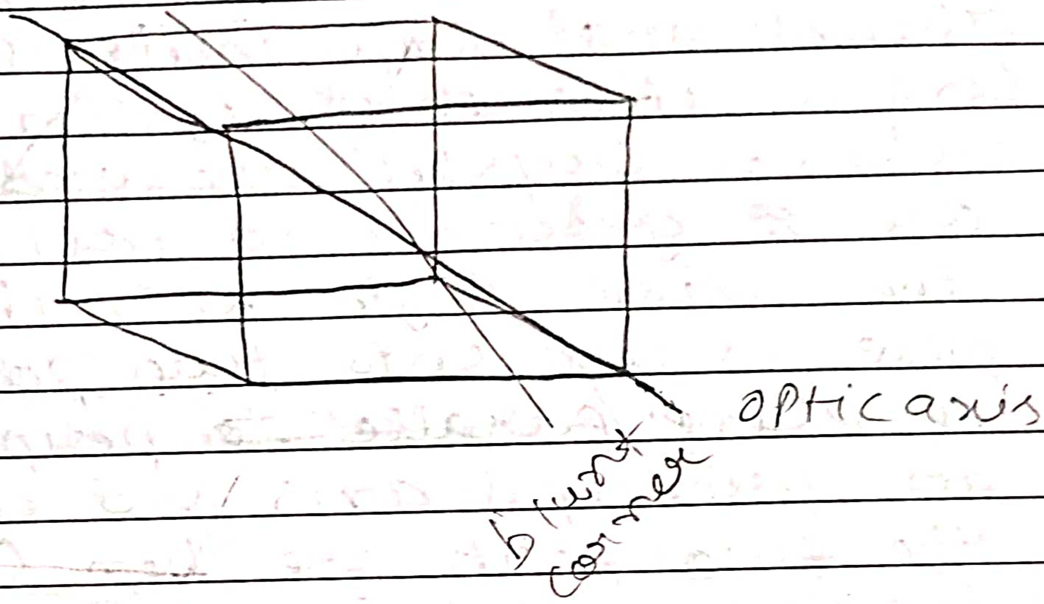
Geometry of Calcide Crystal

they are Rhombohedral crystal bounded by six faces. each ~~face~~ ray is parallelogram with angles 102° and 78°.
Uniaxial crystal → one ray is O-ray & the other is (Calcide, tourmaline, quart) e-ray.
Baxial crystal → Both the rays are e-ray (mica, topaz)

Optic axis

the two corner of crystal where all three angle are obtuse are called blunt corner. A line bisecting any of the blunt corner gives the direction of optic axis any line parallel to optic axis is also optic axis.

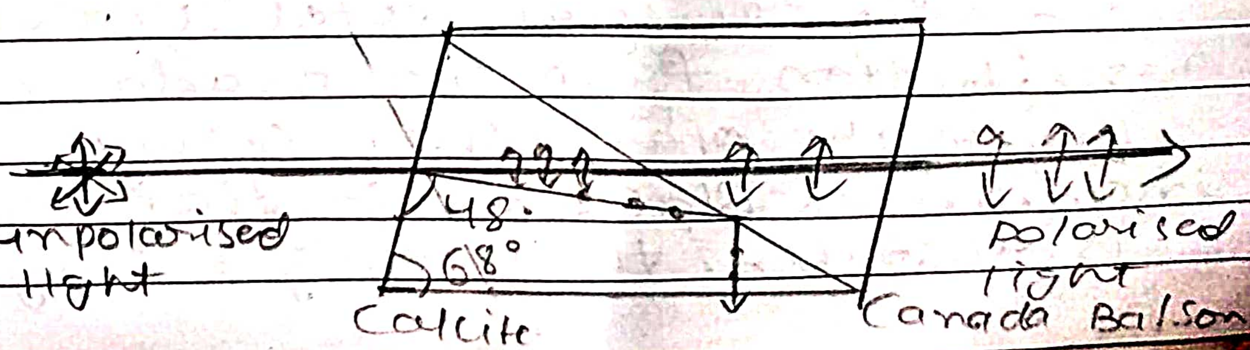
If light is incident parallel to optic axis, it will not be split into e-ray and o-ray.



* Principal Section

A plane containting optic axis and also perpendicular to the pair opposite phases is called principal section for that pair of faces, there are three principal section corresponding to three pairs.
the e-rays has vibrations parallel to principal section and o-ray has vibration perpendicular to principal section.

* Nicol Prism

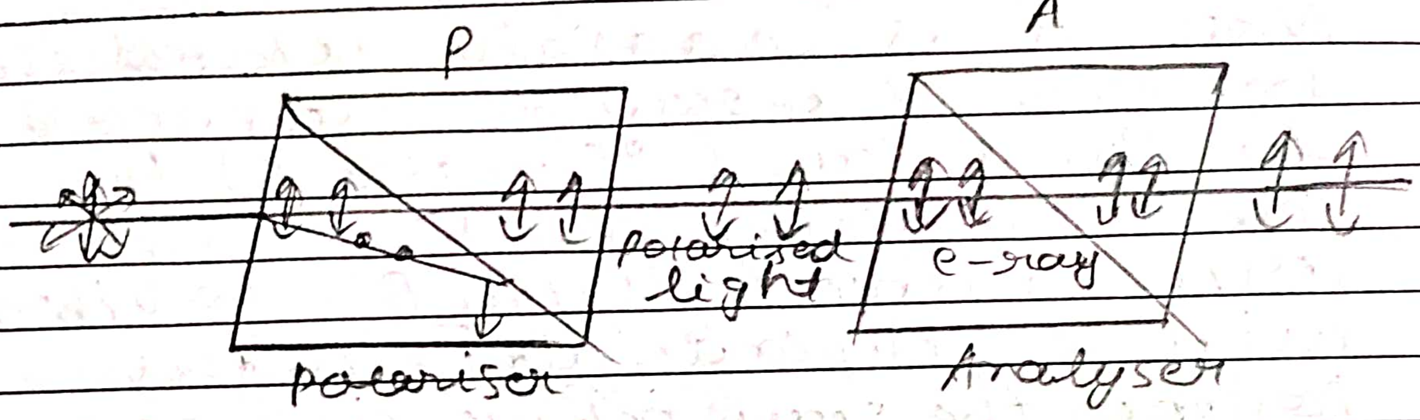


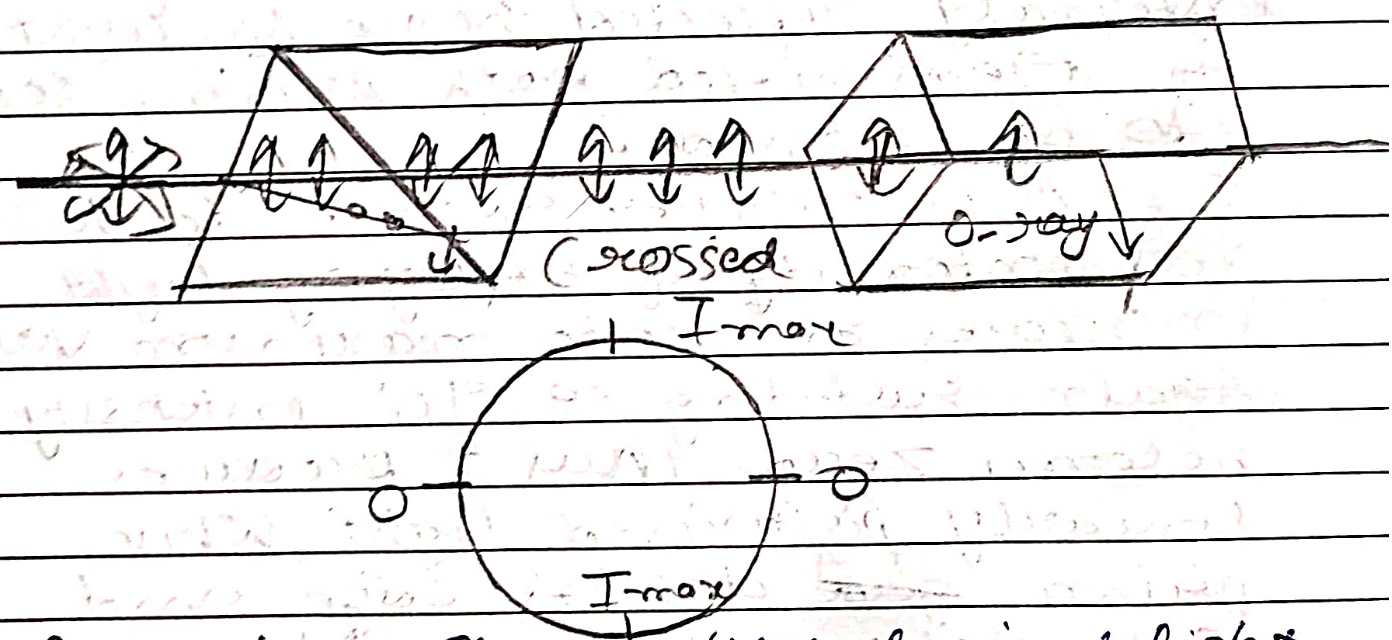
$\mu_o = 1.66$
$\mu_c = 1.55$
$\mu_e = 1.486$

Construction:- Calcide crystal of rhomohedral shape about three times long as it is thick is obtain from original crystal. the ends surfaces are ground and polished until the make Angle 68°. then this piece is cut into two parts along a plane ~~parallel to~~ perpendicular to principal axis and ee and surface. the two parts are then cemented togather with Canada Balson whose refractive index lies between refractive index of calcide for e-ray and O-ray.

Working:- Unpolarised light fall on nicol Prism at an angle of incidence of about 15° it is split into e-ray and O-ray the values of refractive indexes and the angle of incidence at canada Balson layer are such that e-ray is transmitted while O-ray suffers total internal reflection from the Canada Balson layer the ~~phase~~ face where O-ray incidence is blacked so that O-ray is absorbed. thus only e-ray comes out of crystal which is linearly polarised.

imp Nicol Prism as Polariser & Analyser





A Polariser changes unpolarised light into polarised light. Analyser is a device which is used to detect the direction of vibration of polarise light.
When two Nicol Prism are used together, one of them act as polariser they other act as analyser. if Principal Section of both prism

are parallel, the polariser allows e-ray to come out, this e-ray enters analyser. Since Principal Section is parallel therefore e-ray comes out of analyser.
Now A is gradually rotated the intensity of e-ray decreases according to Malus law ($I = a^2 \cos^2\theta$) the Principal section of A becomes perpendicular to that of P, the vibration will be perpendicular. In this position the ray behave as o-ray inside analyser it is totally internally reflected hence No light is transmitted both prism set to be crossed.
If A is further rotated to 90° the intensity of emerging light increases ~~to~~ up to maximum value. Again rotating to 90° intensity becomes zero. Thus it produces linearly polarised light while prism ~~gets~~ detects edge and P is polarised A is Analyser

* Theory of Plane, Circularly & Elliptically polarised light

1. plane Polarised light
the light having vibration along the straight line in a single plane

is called plane polarised light.

2. Circularly Polarised light.
it is Combination of two plane polarised light having same magnitude and Phase difference of 90° the resultant swips around the circle.

3. elliptically Polarised light
it is combination of two plane polarised wave having different magnitude and phase difference of 90° the resultant sweeps across an ellipse

Analytical Treatment

Case-1 both vibration are perpendicular.
let two plane polarised wave having vibration along x and y axis super impose the waves may be represented as

$$x = a \sin(\omega t + \delta) \quad \text{--- (1)}$$
$$y = b \sin \omega t \quad \text{--- (2)}$$

from eq (2)
$$\sin \omega t = \frac{y}{b}$$
$$\cos \omega t = \sqrt{1 - y^2/b^2}$$

from eq (1), $\frac{x}{a} = \sin(\omega t + \delta)$

$$\frac{x}{a} = \sin \omega t \cos \delta + \cos \omega t \sin \delta \text{ - (3)}$$

Putting values of $\sin\omega t$ & $\cos\omega t$

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1-\frac{y^2}{b^2}}(\sin\delta)$$

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sin\delta\sqrt{1-\frac{y^2}{b^2}}$$

squaring both side

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta = \sin^2\delta - \frac{y^2}{b^2}\sin^2\delta$$
$$- \frac{2xy}{ab}\cos\delta$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta} \quad (4)$$

(i) if $\delta = 0$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\frac{x}{a} = y/b$$

$$y = \frac{b}{a}x$$

It is eq of Straight line

(ii) If $\delta = \frac{\pi}{2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

it is eq of ellipse

(iii) if $\delta = \frac{\pi}{2}$, $a = b$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$

$$x^2 + y^2 = a^2$$

it is eq of circle

Case-2 - both vibration are in same directio
Consider two light waves having vibration along x- axis

$$x_1 = a\sin\omega t \quad \text{--- (1)}$$

$$x_2 = b\sin(\omega t + \delta) \quad \text{--- (2)}$$

Resultant $X = x_1 + x_2$

$$X = a\sin\omega t + b\sin(\omega t + \delta)$$

$$= a\sin\omega t + b\sin\omega t\cos\delta + b\cos\omega t\sin\delta$$

$$= X = (a + b\cos\delta)\sin\omega t + b\sin\delta\cos\omega t$$

Let $a + b\cos\delta = R\cos\phi$

$$b\sin\delta = R\sin\phi$$

$$X = R\cos\phi\sin\omega t + R\sin\phi\cos\omega t$$

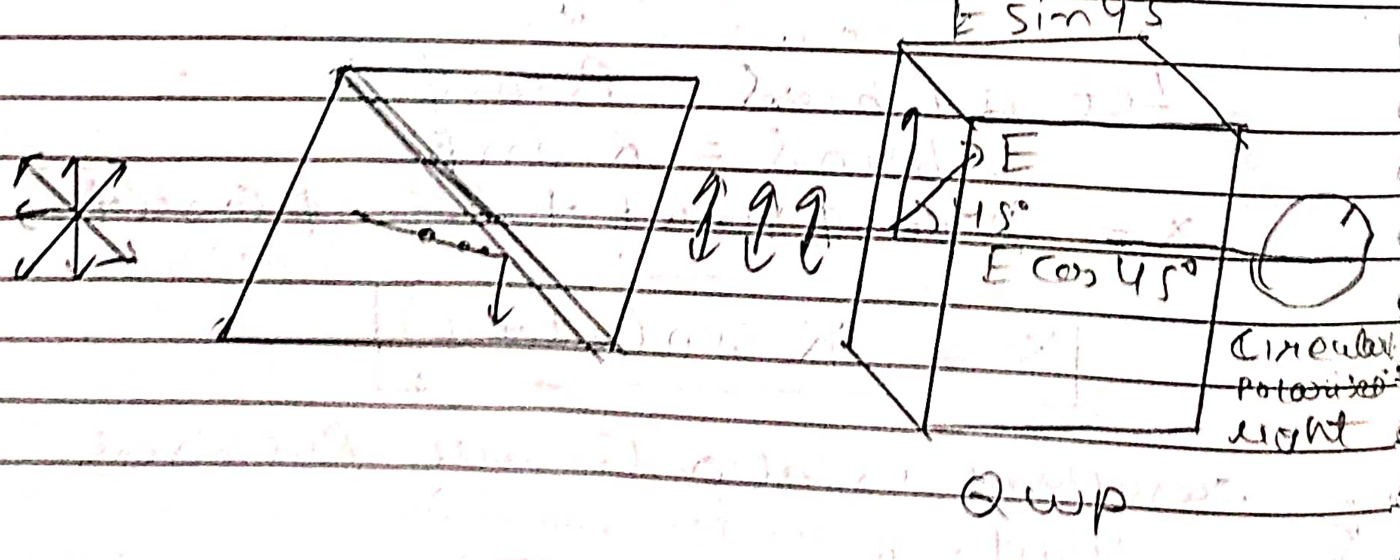
$$\boxed{X = R\sin(\omega t + \phi)}$$

resultant is also linearly Polarised light.

Production of Plane, Circularly & elliptically Polarised light

1. Plane polarised light.
when ordinary beam of light passes through nicol prism it split into o-ray and e-ray. the o-ray is totally internally reflected by e-ray passes through nicol prism this ray is plane polarised.

2. Circularly polarised light
the unpolarised light is made polarised with nicol prism and allow to pass through quarter waveplate (qwt). the angle between optic axis of quarter ~~wav~~ waveplate and vibration of light is 45°. the two components ($E\cos 45°$ and $E\sin 45$) have same magnitude. quarter waveplate introduces a phase difference of $\frac{\pi}{2}$ between them they superimpose introduce Circularly polarised light.



3. Elliptically Polarised light
unpolarised is made polarise with Nicol Prism than it is allows to pass through quarter wave plate. angle between optic axis and vibration of light $\theta \neq 45^\circ, 0^\circ, 90^\circ$. the two components will have different magnitudes. A phase difference of 90° is ~~for~~ created by quater waveplate hence they make eliptically Polarised light.

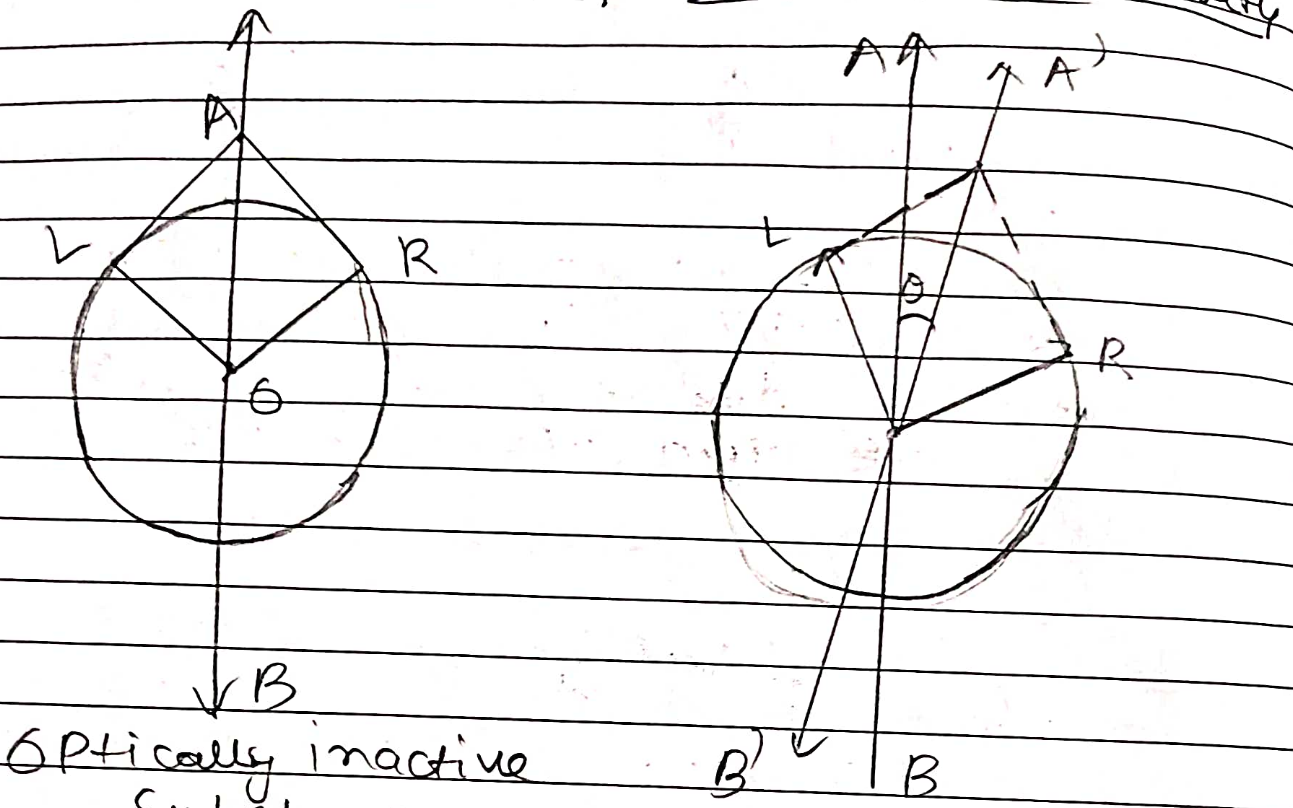
Defection of Plane, Circularly & elliptically polarised light
the light under test is pass through analyser (nicol Prism) and it is rotated to ~~360~~° through 360°
(i) if intensity varries from maximum to zero light was plane polarised.
(ii) if no-variation of intensity is found, light was circularly Polarised or unpolarised
(iii) if intensity varries from maximum to minimum but never zero than light was elliptically polarised or Partially polarised.

imp

* To differentiate between unpolarised and circularly polarised, the light is allow to pass from a QWP and then on analyser
(i) if intensity varries from maximum to zero light was circularly polarised.
(ii) if there is no change light was unpolarised.

* to differentiate between partially and elliptically polarised light we used QWP then analyser.
(i) if intensity varries from maximum to zero light was elliptically polarised.
(ii) if intensity varries from maximum to minimum but never zero light was partially polarised.

Fresnel's Theory of optical activity



Optically inactive substance

it is based on following Assumption
(i) when plane polarised light passes through a crystal along optic axis it is broken into clockwise and anticlockwise component.
(ii) Both components travel with same velocity in normal substance.
(iii) In optically active substances, velocity of two components are different and they have a relative phase difference.
(iv) In dextro rotatory $V_R > V_L$ and in levo rotatory.
(v) After emergence the two components recombine to produce plane polarise light in which vibration

is rotated through angle 'θ'

(vi) the rotation of vibration of resultant weight depends upon the Phase difference between two components.

Mathematical Analysis

let a ray of plane polarised light incident normally on an optically active Substance the equation of incident ray

$$y = 2a \sin \omega t \quad \text{-----} (1)$$

According to fresnel assumption it will be divided into two Circular bipreserss let a Phase difference of δ is introduced due to change in velocity. the equation of two Components Can be given as

$$X_1 = a \cos \omega t, \quad Y_1 = a \sin \omega t \quad \text{--} (2)$$

$$X_2 = -a \cos(\omega t + \delta), \quad Y_2 = a \sin(\omega t + \delta) \quad \text{--} (3)$$

Resultant :- $X = X_1 + X_2$

$$X = a \cos \omega t - a \cos(\omega t + \delta)$$

$$X = 2a \sin\left(\frac{2\omega t + \delta}{2}\right) \sin\frac{\delta}{2} \quad \text{----} (4)$$

$$Y = Y_1 + Y_2 = a \sin \omega t + a \sin(\omega t + \delta)$$

$$Y = 2a \sin\left(\frac{2\omega t + \delta}{2}\right) \cos\frac{\delta}{2} \quad \text{---} (5)$$

Dividing (4) by (5)

$$\boxed{\frac{X}{Y} = \tan\frac{\delta}{2}}$$

Let μ_R & μ_L be the R.I of substance for right handed & left handed components & l is the thickness of substance.

~~Path difference = $\mu_L l - \mu_R l$~~

Phase difference $(\delta) = \frac{2\pi}{\lambda}(\mu_L - \mu_R)l$

Angle of rotation $\theta = \frac{\delta}{2}$

$$\boxed{\theta = \frac{\pi}{\lambda}(\mu_L - \mu_R)l}$$

$$\theta = \frac{\pi}{\lambda}\left(\frac{c}{V_L} - \frac{c}{V_R}\right)l$$

$$\theta = \frac{\pi c}{\lambda}\left(\frac{1}{V_L} - \frac{1}{V_R}\right)l$$

$$\boxed{\theta = \frac{\pi}{T}\left(\frac{1}{V_L} - \frac{1}{V_R}\right)l} \quad \because c = n\lambda, \quad \frac{1}{T} = n = \frac{c}{\lambda}$$

In optically inactive substance $V_L = V_R \Rightarrow \theta = 0$

very imp

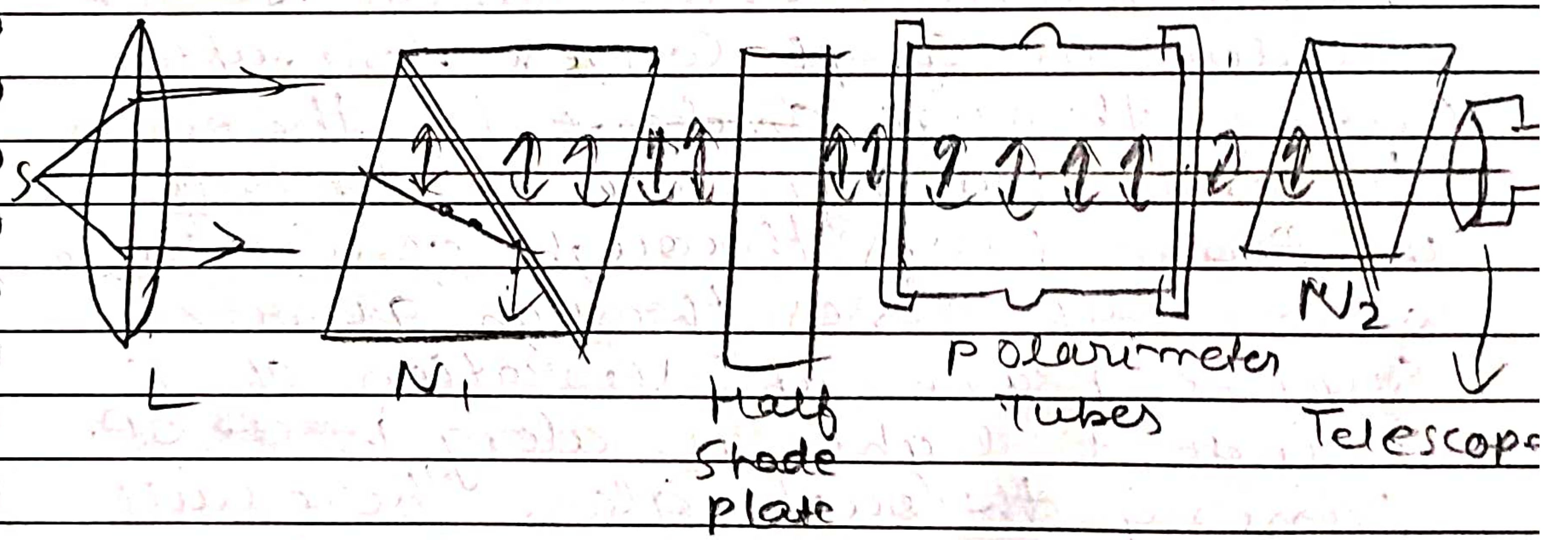
* Specific Rotation

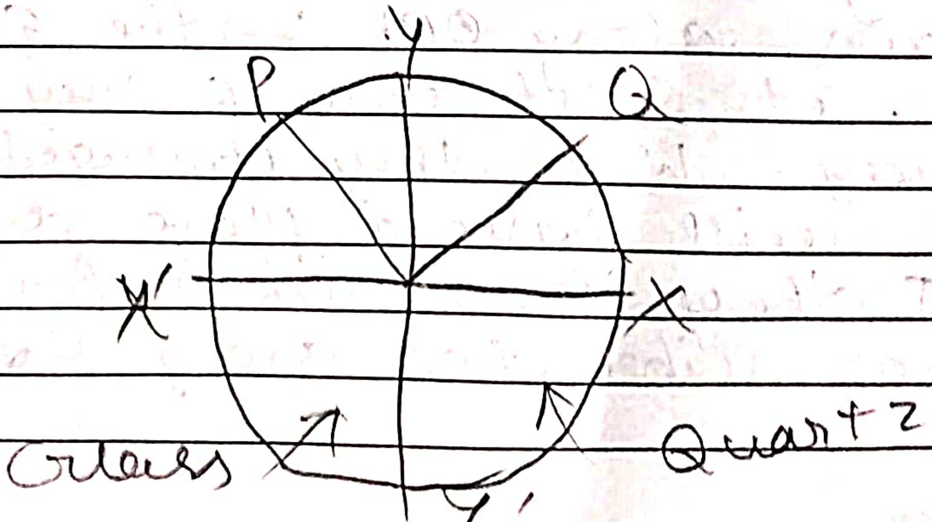
the specific rotation is defined as rotation produced by 10 cm long column of a liquid containing 1 gm of optically active substance in 1 cc of solution the symbol of specific rotation is $[S]_{\lambda}^{T}$ Temperature and wavelength are constant:

$$\boxed{[S]_{\lambda}^{T} = \frac{10\theta}{lc}}$$

where length is in centimeter and concentration in gm/cc

* Laurent's Half Shade Polarimeter





polarimeter is an instrument which is used to measure the optical rotation produced by the optically active substances.

Experimental arrangement

it consist of two nicol prism N_1 and N_2. define polariser where is plate called half shade plate it consist of two semi-circular plates one made up of quartz. the polarimeter tube have large diameter at the centre it is a glasstube containing solution and attens for used to observed intensity. ~~ato~~

working.

light from monochromatic source incident on ~~Compt~~ convex lens after passing through ~~andone~~ N_1 the beam becomes plane polarised. the half of beam passes through glass and the other half passes through quartz. Suppose plane of vibration of incident light is along ~~is at~~ OP. Passing through glass there will remain along OP. on the quartz half, it split into e-ray and o-ray. When they emerged out there will have a phase difference of π they combined to form a linear vibration along OQ.

When principal of N_2 is parallel to OQ, quartz half will appear bright. When principal plane is parallel to OP glass ~~haf~~ half will bright and quartz half will be dark. When it parallel to ~~yy'~~ yy' both will appear equally bright.

Biquartz Polarimeter

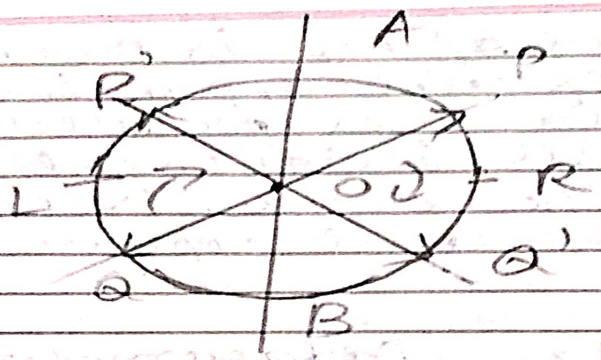
the optical arrangement in biquartz polarimeter is almost the same as lorentz polarimeter however there are two differences

(i) In place of monochromatic source a white source is used.

(ii) In place of half shade plate, A biquart plate is used.

Biquartz plate

In this plate both semicircles are made up of quartz one is left handed and the other is right handed quartz the vibration of incident light is along POQ. then in bright fringes plate it gets rotated to OQ' and In left handed plate it rotated to OP'. one half ~~of~~ appears red and the other half appears blue. ~~When~~ the Principle plane of N_2 is parallel to AB then both ~~halfs~~ halves will appear grey this position is called ~~tint~~ tint of passage



Bi quartz Plate

Q1) the specific rotation of quartz at 5086 Å is 29.73 degree/mm Cal. the difference in refractive indexes

Sol $\lambda = 5086\ \text{Å} = 5086 \times 10^{-7}\ mm$

$\frac{\theta}{l} = 29.73$ degree/mm

$\mu_L - \mu_R = ?$

$\frac{\theta}{L} = \frac{\pi}{\lambda}(\mu_L - \mu_R)$

$(\mu_L - \mu_R) = \frac{5086 \times 10^{-7}}{3.14} \times 29.73$

$= 4.81 \times 10^{-3}$

Q2) the indices of refraction of quartz for right handed left handed circularly polarise light of wavelength 6500 Å travelling in a direction of optic axis have the following values $\mu_R = 1.53914$
$\mu_L = 1.53920$

Cal. the rotation of plane of polarisation of light in decrease produced by plate 0.2 mm thick

Sol $\mu_R = 1.53914$, $\mu_L = 1.53920$

$L = 0.2\,mm = 0.02\,cm$

$$\theta = \frac{\pi}{\lambda}(\mu_L - \mu_R)L$$

$$= \frac{\pi(1.53920 - 1.53914)}{6500 \times 10^{-8}} \times 0.02 \text{ radi}$$

$$\Rightarrow \frac{\pi \times 0.00006 \times 0.02}{6500 \times 10^{-8}} \times \frac{180}{\pi}$$

$\theta \Rightarrow 3.323$ degree.

Q) A sugar solution in tube of 20cm produces optical rotation of 13° the solution is when diluted to 1/3 of its previous concentration find the optical rotation produced by 30 cm long tube containing the diluted solution.

Sol $l_1 = 20\,cm$, $\theta_1 = 13°$, $C_1 = C$

$l_2 = 30\,cm$, $\theta_2 = ?$ $C_2 = C/3$

$$S = \frac{10\theta_1}{l_1 C_1} = \frac{10\theta_2}{l_2 C_2}$$

$$\theta_2 = \frac{\theta_1}{l_1 C_1} \times l_2 C_2$$

$$\theta_2 = \frac{13 \times 36^{10} \times \cancel{c}}{20 \times \cancel{3} \times \cancel{c}}$$

$$\theta = 6.5^\circ$$

Q) ~~of~~ 5% solution ~~obtain~~ of Cane-Sugar placed in a tube of length 40cm causes optical rotation of 20° how much length of 10% solution of the same substance will produce a rotation of 35°.

Sol ~~A~~ $l_1 = 40cm$, $\theta_1 = 20^\circ$

$C_1 = \frac{5}{100}$ $\theta_2 = 35^\circ$

$\Rightarrow \frac{1}{20}$ | $C_2 = \frac{10}{100} = \frac{1}{10}$

$$S = \frac{100\theta_1}{l_1 C_1} = \frac{100\theta_2}{l_2 C_2}$$

$$l_2 = \frac{l_1 C_1 \theta_2}{\theta_1 C_2} = \frac{40 \times 1 \times 35 \times 10}{20 \times 20 \times 1}$$

$$l_2 = \frac{1400 \times 10}{400}$$

$$\Rightarrow \frac{140}{4} = \cancel{35}$$

$$l_2 = 35 \text{ cm}$$

Q) 80 gm of ~~cas~~ impure sugar is resolved in 1L of water the solution gives ~~and~~ an optical rotation of 9.9° when placed in a tube of length 20cm if the specific rotation of pure sugar is 66°/Decimeter(gm/cc)$^{-1}$ find the percentage of purity of sugar sample

Sol $l = 20cm$, $\theta = 9.9°$

$S = 66° (dm)^{-1} (gm/cc)^{-1}$

$$S = \frac{10\theta}{lC}$$

$$C = \frac{10 \times 9.9}{20 \times 66} = \cancel{98}$$

$$C = 0.075 \ gm/cc$$

$$C = 75 \ gm/litre$$

impure $C = 80 gm/lit$

$$\% \text{ purity} = \frac{75}{80} \times 100$$

$$\rightarrow 93.75\%$$