Module - IV Vector Calculus

* Vector :-

* Position of vector :-

$\vec{AB}$ = Position vector of B - Position vector of A

* unit vector :-

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$$

Product :-

(*) ① Dot / Scalar product :- ~~$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$~~

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

$$i\cdot i = j\cdot j = k\cdot k = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

② Cross / vector product ->

$$\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}]$$

* Vector differentiation operation (∇) (del)

$$\nabla = \hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}$$

* Gradient of a scalar function :-

let $\phi(x,y,z)$ be a scalar function

than grad $\phi = \nabla\phi = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)\phi$

Q) If $\phi = 3x^2y - y^3z^2$ find the grad ϕ at the point $(1,-2,-1)$

Sol we have $\phi = 3x^2y - y^3z^2$

~~observed~~

grad $\phi = \nabla\phi = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(3x^2y - y^3z^2)$

$$\nabla\phi = \hat{i}(6xy) + \hat{j}(3x^2 - 3y^2 z^2) + \hat{k}(-2y^3z)$$

~~$\nabla\phi =$~~ grad ϕ at $(1,-2,-1)$

$= 6(1)(-2)\hat{i} + \hat{j}(3(1)^2 - 3(-2)^2(-1)^2) + \hat{k}(-2(-2)^3(-1))$

$\Rightarrow -12\hat{i} - 9\hat{j} - 16\hat{k}$ Ans

Normal:- If $\phi(x,y,z) = c$ represent a family of surfaces for difference c. then $\nabla\phi$ is vector normal to the surface $\phi(x,y,z) = c$.

the direction derivative of $\phi(x, y, z) = c$ in the direction of vector $\vec{d}$ is equal to $\nabla\phi \cdot \hat{d}$.

Q) Find the directional derivative of $x^2 y^2 z^2$ at the point $(1, 1, -1)$ in the direction of the tangent to the curve $x = e^t$, $y = \sin 2t - 1$, $z = 1 - \cos t$ at ~~to~~ $t = 0$.

Sol we have $\phi = x^2 y^2 z^2$

$$\text{grad } \phi = \nabla\phi$$

$$= \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(x^2 y^2 z^2)$$

$$\cancel{\Rightarrow \hat{i}(2x) + \hat{j}(2y}$$

$$\Rightarrow \hat{i}(2xy^2z^2) + \hat{j}(2yx^2z^2) + \hat{k}(2zx^2y^2)$$

grad ϕ at point $(1, 1, -1)$

$$\text{grad } \phi = \hat{i}(2 \times 1 \times 1) + \hat{j}(2 \times 1 \times 1) + \hat{k}(-2 \times 1 \times 1)$$

$$= 2\hat{i} + 2\hat{j} - 2\hat{k}$$

At ~~$t = 0$~~ curve is

$x = e^0 - 1$

$y = \cancel{\sin 0 - 1} - 1$

$z = 0$

$\vec{r} = e^t \hat{i} + (\sin 2t - 1)\hat{j} + (1-\cos t)\hat{k}$

tangent vector $\left(\frac{d\vec{r}}{dt}\right) = e^t \hat{i} + 2\cos 2t\,\hat{j} + \sin t\,\hat{k}$

tangent vector $\left(\frac{d\vec{r}}{dt}\right)_{t=0} = \hat{i} + 2\hat{j}$

$\vec{d} = \hat{i} + 2\hat{j}$

$\hat{d} = \frac{\hat{i}+2\hat{j}}{\sqrt{1+2^2}} = \frac{\hat{i}+2\hat{j}}{\sqrt{5}}$

required direction derivative of x^2y^2z along the tangent vector.

$= \nabla\phi \cdot \hat{d}$

$= (2\hat{i} + 3\hat{j} - 2\hat{k}) \cdot \frac{\hat{i}+2\hat{j}}{\sqrt{5}}$

$\Rightarrow \frac{2\times 1 + 2\times 2 + 2\times 0}{\sqrt{5}}$

$= \frac{6}{\sqrt{5}}$ Ans

Q Find the unit normal vector to the surface $xy^3z^2 = 4$ at $(-1,-1,2)$.

Sol we have

$\phi = xy^3z^2 - 4$

normal vector of $\phi = \nabla\phi$

$= \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(xy^3z^2 - 4)$

$\Rightarrow \hat{i}(y^3z^2) + \hat{j}(3xy^2z^2) + \hat{k}(2xy^3z)$

normal vector of ϕ at $(-1,-1,2)$

$$\Rightarrow -4\hat{i} - 12\hat{j} + 4\hat{k}$$

unit normal vector of at $(-1,-1,2)$

$$= \frac{-4\hat{i} - 12\hat{j} + 4\hat{k}}{\sqrt{16+144+16}} = \frac{-4}{4\sqrt{11}}(\hat{i}+3\hat{j}-\hat{k})$$

$$= \frac{-1}{\sqrt{11}}(\hat{i}+3\hat{j}-\hat{k}) \text{ Ans}$$

Q) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$.

Sol Normal vector of surface

$$x^2+y^2+z^2-9=0$$

$$\nabla\phi = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(x^2+y^2+z^2-9)$$

$$\Rightarrow 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

Normal vector at $(2,-1,2) = 4\hat{i} - 2\hat{j} + 4\hat{k}$ —— ①

and normal vector of surface

$$x^2+y^2-z-3=0$$

$$= \nabla\phi = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(x^2+y^2-z-3)$$

$$= 2x\hat{i} + 2y\hat{j} - \hat{k}$$

normal vector at point $(2,-1,2)$

$$= 4\hat{i} - 2\hat{j} - \hat{k} \quad \text{——} ②$$

Let θ be angle between ① & ②

$$\Rightarrow (4\hat{i}-2\hat{j}+4\hat{k})\cdot(4\hat{i}-2\hat{j}-\hat{k})$$

$$= \sqrt{16+4+16}\ \sqrt{16+4+1}\ \cos\theta$$

$$\Rightarrow 16+4-4 = 6\sqrt{21}\cos\theta$$

$$16 = 6\sqrt{21}\cos\theta$$

$$\cos\theta = \frac{8}{3\sqrt{21}}$$

$$\theta = \cos^{-1}\frac{8}{3\sqrt{21}}$$ Ans

Q) find the directional derivative of $\frac{1}{r}$ in the direction $\vec{r}$.

Where $\vec{r} = xi + y\hat{j} + z\hat{k}$.

$$r = \sqrt{x^2+y^2+z^2}$$

$$r^2 = x^2+y^2+z^2$$

we have

$$\nabla\phi = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)\left(\frac{1}{r}\right)$$

$$\Delta\phi = \hat{i}\left(-\frac{1}{r^2}\frac{\delta r}{\delta x}\right) + \hat{j}\left(-\frac{1}{r^2}\frac{\delta r}{\delta y}\right) + k\left(-\frac{1}{r^2}\frac{\delta}{\delta z}\right)$$

$$\Delta\phi = -\frac{1}{r^2}\left(\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right)$$

$$\Delta\phi = -\frac{1}{r^2}\frac{(xi+y\hat{j}+z\hat{k})}{r}$$

$$= \frac{-\vec{r}}{r^3}$$

So directional Derivative $-\frac{\vec{r}}{r^3} \cdot \frac{\vec{r}}{|\vec{r}|}$

$\boxed{\vec{r} \cdot \vec{r} = r^2}$

$$\Rightarrow \frac{-r^2}{r^4} = -\frac{1}{r^2} \quad \underline{\text{Ans}}$$

~~Q) find the directional~~

Q) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then show that (i) $\text{grad}\, r = \frac{\vec{r}}{r}$

(ii) $\text{grad}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$

Sol (i) $\text{grad}\, r = \Delta r$

$$\Delta r = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(r)$$

$$= \frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}$$

$$\Rightarrow \frac{x\hat{i} + y\hat{j} + z\hat{k}}{r}$$

$$\Rightarrow \frac{\vec{r}}{r}$$ Hence proved

(ii) $\text{grad}\left(\frac{1}{r}\right) = \Delta \frac{1}{r}$

$$\Delta\left(\frac{1}{r}\right) = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)\left(\frac{1}{r}\right)$$

$$\Rightarrow \hat{i}\left(-\frac{1}{r^2}\frac{\delta r}{\delta x}\right) + \hat{j}\left(-\frac{1}{r^2}\frac{\delta r}{\delta y}\right) + \hat{k}\left(-\frac{1}{r^2}\frac{\delta r}{\delta z}\right)$$

$$= -\frac{x}{r^3}\hat{i} - \frac{y}{r^3}\hat{j} - \frac{z}{r^3}\hat{k}$$

$$\Rightarrow -\frac{1}{r^3}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$\Rightarrow -\frac{\vec{r}}{r^3}$$ Hence proved

* Divergence of a vector point function:-

the divergence of a vector point function $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ is defined as

$$\text{div}\vec{F} = \nabla\cdot\vec{F}$$

$$= \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)\cdot\left(F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\right)$$

$$\left[\text{div}\vec{F} = \frac{\delta F_1}{\delta x} + \frac{\delta F_2}{\delta y} + \frac{\delta F_3}{\delta z}\right]$$

(div $\vec{F}$ is a scalar function)

* Solenoidal vector function:-

A vector $\vec{V}$ is said to be solenoidal if div $\vec{V} = 0$.

Q) If $u = x^2+y^2+z^2$ and $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ then find the $\operatorname{div}(u\vec{r})$.

Sol we have

$$\operatorname{div}(u\vec{r}) = \nabla\cdot(u\vec{r})$$

$$= \left(\hat{i}\frac{\delta}{\delta x}+\hat{j}\frac{\delta}{\delta y}+\hat{k}\frac{\delta}{\delta z}\right)\cdot\left((x^2+y^2+z^2)(x\hat{i}+y\hat{j}+z\hat{k})\right)$$

$$\Rightarrow \left(\hat{i}\frac{\delta}{\delta x}+\hat{j}\frac{\delta}{\delta y}+\hat{k}\frac{\delta}{\delta z}\right)\left(x(x^2+y^2+z^2)\hat{i}+y(x^2+y^2+z^2)\hat{j} + z(x^2+y^2+z^2)\hat{k}\right)$$

$$\Rightarrow \frac{\delta}{\delta x}(x^3+xy^2+xz^2)+\frac{\delta}{\delta y}(yx^2+y^3+yz^2) + \frac{\delta}{\delta z}(zx^2+zy^2+z^3)$$

$$\Rightarrow (3x^2+y^2+z^2)+(x^2+3y^2+z^2)+(x^2+y^2+3z^2)$$

$$\Rightarrow 5(x^2+y^2+z^2)$$

$$= \cancel{5u}$$

$$= 5u$$

Q) Find the value of n for which the vector $r^n\vec{r}$ is solenoidal where $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$.

Sol $\operatorname{div}\vec{V} = \operatorname{div}(r^n\vec{r})$

$$\nabla (r^n \vec{r}) = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z} \right) \left(r^n x\hat{i} + r^n y\hat{j} + r^n z\hat{k} \right)$$

$$\Rightarrow \frac{\delta}{\delta x}(r^n x) + \frac{\delta}{\delta y}(r^n y) + \frac{\delta}{\delta z}(r^n z)$$

$$\Rightarrow r^n + n r^{n-1}\frac{\delta r}{\delta x}\cdot x + r^n + n r^{n-1}\frac{\delta r}{\delta y}\cdot y$$

$$+ r^n + n r^{n-1}\frac{\delta r}{\delta z}\cdot z$$

$$\Rightarrow 3r^n + n r^{n-1}\left[\frac{x}{r}\cdot x + \frac{y}{r}\cdot y + \frac{z}{r}\cdot z \right]$$

$$= 3r^n + \frac{n r^{n-1}}{r}(x^2+y^2+z^2)$$

$$\text{div}\,\vec{v} = 3r^n + \frac{n r^{n-1}}{r} r^2$$

$$= 3r^n + n r^n = (3+n) r^n$$

for $\vec{v} = r^n \vec{r}$ is a solenoidal $\text{div}\,\vec{v} = 0$

$$= (3+n) r^n = 0$$

$$\Rightarrow 3 + n = 0$$

$$n = -3 \quad \text{Ans}$$

Q) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ $= \sqrt{x^2+y^2+z^2}$ then show that

① $\text{div}\left(\frac{\vec{r}}{r^2}\right) = \frac{1}{r^2}$

② $\text{div}\left(\frac{\vec{r}}{|\vec{r}|^3}\right) = 0$

③ div(grad r^n) $= n(n+1)\, r^{n-2}$ Hence Show that $\nabla^2\left(\frac{1}{r}\right) = 0$

Sol ①

L.H.S

$\text{div}\left(\frac{\vec{r}}{r^2}\right)$

$\text{div}(\vec{r}\cdot r) = \Delta(r\cdot\vec{r})$

$\Rightarrow \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)\left(r x\hat{i} + r y\hat{j} + r z\hat{k}\right)$

$\Rightarrow r + \frac{\delta r}{\delta x}\cdot x + r + \frac{\delta r}{\delta y}\cdot y + r + \frac{\delta r}{\delta z}\cdot z$

$\Rightarrow 3r + \frac{x}{r}\cdot x + \frac{y}{r}\cdot y + \frac{z}{r}\cdot z$

$\Rightarrow \frac{3r + 1}{r}\left(x^2 + y^2 + z^2\right)$

$\Rightarrow 3r + \frac{r^2}{r}$

$= 3r +$

Q) Find the directional derivative of divergence of $\vec{V} = x^4\hat{i} + y^4\hat{j} + z^4\hat{k}$ at the point $(1, 2, 2)$ in the direction of order normal of the sphere $x^2+y^2+z^2=9$.

Sol $\text{Div}\,\vec{V} = \nabla \cdot \vec{V} = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)$

$(x^4\hat{i} + y^4\hat{j} + z^4\hat{k})$

$$= \frac{\delta x^4}{\delta x} + \frac{\delta y^4}{\delta y} + \frac{\delta z^4}{\delta z}$$

$$\Rightarrow 4x^3 + 4y^3 + 4z^3$$

Directional derivative of ~~div~~ $\vec{V}$ =

$$\text{div}\,\vec{v} = \Delta(4x^3+4y^3+4z^3) = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)(4x^3+4y^3+4z^3)$$

$$= 12x^2\hat{i} + 12y^2\hat{j} + 12z^2\hat{k}$$

Directional Derivative at $(1, 2, 2)$

$$= 12\hat{i} + 48\hat{j} + 48\hat{k}$$

* Curl :- Curl of a vector point function $\vec{V}$ is defined as

$$\text{curl } \vec{V} = \nabla \times \vec{V}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ v_1 & v_2 & v_3 \end{vmatrix}$$

If curl $\vec{V} = 0$ then $\vec{V}$ is irrational

Q) If

Q) find the divergences and curl of the $\vec{V} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$ at $(2,-1,1)$.

Sol $\text{div}\,\vec{V} = \nabla\cdot\vec{V} = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right)$

$\left(xyz\hat{i} + 3x^2y\hat{j} +\right.$

$\text{div}\,\vec{V} = -1\times1 + 3(2)^2 + 2\times2(1) - (-1)^2 = 14$

and curl $\vec{V} = \nabla\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ xyz & 3x^2y & xz^2 - y^2z \end{vmatrix}$

$= \hat{i}\left[\frac{\delta}{\delta y}(xz^2 - y^2z) - \frac{\delta}{\delta z}(3x^2y)\right] - \hat{j}\left[\frac{\delta}{\delta x}(xz^2 - y^2z) - \frac{\delta}{\delta z}xyz\right]$

$\text{curl}\,\vec{V} = i[-2yz] - j[z^2 - xy] + \hat{k}[6xy - xz]$

at $(2,-1,1)$

$\text{curl}\,\vec{V} = i[-2\times-1\times1] - \hat{j}[1^2 - 2\times-1] + \hat{k}[6\times2\times-1 - 2\times1]$

$= 2\hat{i} - 3\hat{j} - 14\hat{k}$ Ans

Q) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ & $r^2 = x^2 + y^2 + z^2$ then show that the vector $r^n \vec{r}$ is irrotational.

Sol we have

$$\text{curl}(r^n \vec{r}) = \nabla \times r^n \vec{r}$$

$$\Rightarrow \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right) \times (r^n x\hat{i} + r^n y\hat{j} + r^n z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ r^n x & r^n y & r^n z \end{vmatrix}$$

$$\Rightarrow \hat{i}\left[\frac{\delta}{\delta y} r^n z - \frac{\delta}{\delta z} r^n y\right] - \hat{j}\left[\frac{\delta}{\delta x} r^n z - \frac{\delta}{\delta z} r^n x\right]$$

$$+ \hat{k}\left[\frac{\delta}{\delta x} r^n y - \frac{\delta}{\delta y} r^n x\right]$$

$$\Rightarrow \hat{i} \quad 0$$

$r^n \vec{r}$ is irrotational

Q) find the divergent and curl of
$\vec{V} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$

Q) Prove that for every vector field $\vec{v}$

$$\boxed{\operatorname{div}(\operatorname{Curl} \vec{v}) = 0}$$

Sol We have

$$\operatorname{Curl} \vec{v} = \nabla \times \vec{v}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$\Rightarrow i\left[\frac{\delta}{\delta y} v_3 - \frac{\delta}{\delta z} v_2\right] + \hat{j}\left[\frac{\delta}{\delta x} v_3 - \frac{\delta}{\delta z} v_1\right]$$

$$+ \hat{k}\left[\frac{\delta}{\delta x} v_2 - \frac{\delta}{\delta y} v_1\right]$$

$\text{div}(\text{curl } \vec{v})$

$$= \left(i\frac{\delta}{\delta x} + j\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right) \cdot \left[i\left(\frac{\delta v_3}{\delta y} - \frac{\delta v_2}{\delta z}\right) - \hat{j}\left(\frac{\delta v_3}{\delta x} - \frac{\delta v_1}{\delta z}\right) + \hat{k}\left(\frac{\delta v_2}{\delta x} - \frac{\delta v_1}{\delta y}\right) \right]$$

$$\Rightarrow \cancel{\frac{\delta^2 v_3}{\delta x \delta y}} - \cancel{\frac{\delta^2 v_2}{\delta x \delta z}} - \cancel{\frac{\delta^2 v_3}{\delta y \delta x}} + \cancel{\frac{\delta^2 v_1}{\delta y \delta z}} + \cancel{\frac{\delta^2 v_2}{\delta z \delta x}} - \cancel{\frac{\delta^2 v_1}{\delta z \delta y}}$$

$\Rightarrow 0$ proved

* **Line Integral** :- Let $\vec{F}(x, y, z)$ be a vector function and AB is given curve then Line integral

$$= \int_C \vec{F} \cdot d\vec{r}$$

Work :- If $\vec{F}$ represent the variable force acting on a particle along are AB then the total work done

$$= \int_A^B \vec{F} \cdot d\vec{r}$$

Q) If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in xy-plane from (0,0) to (1,4) along a curve $y = 4x^2$. Find the work done.

Sol: work done $= \int_c \vec{F} \cdot d\vec{r}$

$$= \int_{(0,0)}^{(1,4)} (2x^2y\hat{i} + 3xy\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$\Rightarrow \int_{(0,0)}^{(1,4)} 2x^2y\,dx + 3xy\,dy$$

$$= \int_0^1 2x^2\,4x^2\,dx + 3x(4x^2)\,8x\,dx$$

$$= \int_0^1 (8x^4 + 96x^4)\,dx \quad \left[\because \begin{array}{l} y = 4x^2 \\ dy = 8x\,dx \end{array}\right]$$

$$= \int_0^1 104x^4\,dx$$

$$\Rightarrow 104\left[\frac{x^5}{5}\right]_0^1$$

$$\Rightarrow 104 \times \frac{1}{5}$$

$$\Rightarrow \frac{104}{5} \text{ (Ans)}$$

Q) A vector field is given by
$\vec{F} = (2y+3)\hat{i} + xz\hat{j} + (yz-x)\hat{k}$

~~Sol~~ Evaluate $\int_C \vec{F}\cdot d\vec{r}$ along the path
C is $x = 2t$, $y = t$, $z = t^3$ from $t=0$ to $t=1$.

Sol

$$\int_C \vec{F}\cdot d\vec{r} = \int_C [(2y+3)\hat{i} + xz\hat{j} + (yz-x)\hat{k}]\cdot[dx\hat{i} + dy\hat{j} + dz\hat{k}]$$

$$\Rightarrow \int_C (2y+3)\,dx + xz\,dy + (yz-x)\,dz$$

we have $x = 2t \Rightarrow dx = 2dt$
$y = t \Rightarrow dy = dt$
$z = t^3 \Rightarrow dz = 3t^2 dt$ & limit of t, from $t=0$ to $t=1$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = \int_0^1 (2t+3)\,2\,dt + (2t\,t^3)\,dt + (t\,t^3-2t)\,3t^2\,dt$$

$$\Rightarrow \int_0^1 [4t + 6 + 2t^4 + 3t^6 - 6t^3]\,dt$$

$$= \left[\frac{4t^2}{2} + 6t + \frac{2t^5}{5} + \frac{3t^7}{7} - 6\frac{t^4}{4}\right]_0^1$$

$\Rightarrow \frac{4}{2} + 6 + \frac{2}{5} + \frac{3}{7} - \frac{6}{4}$

$\Rightarrow 2 + 6 + \frac{2}{5} + \frac{3}{7} - \frac{3}{2}$

$\Rightarrow 8 + \frac{2}{5} + \frac{3}{7} - \frac{3}{2}$

$\Rightarrow \frac{560 + 28 + 30 - 105}{70}$

$\Rightarrow \frac{513}{70}$ Ans

use green's theorem to evaluate $\oint_C$

$\int_C (x^2 + xy)\,dx + (x^2 + y^2)\,dy$ where c is the square formed by line $x = \pm 1$, $y = \pm 1$

Sol we ~~have~~ know that green's theorm

$$\int_C \phi\,dx + \psi\,dy = \iint_R \left(\frac{\delta \psi}{\delta x} - \frac{\delta \phi}{\delta y}\right) dx\,dy$$

$$\int_C (x^2 + xy)\,dx + (x^2 + y^2)\,dy = \iint_R (2x - x)\,dx\,dy$$

$$\Rightarrow \int_{-1}^{1}\int_{-1}^{1} x\,dx\,dy$$

$$\Rightarrow \int_{-1}^{1} x\,dx \int_{-1}^{1} dy = \int_{-1}^{1} x\,dx\,[y]_{-1}^{1}$$

$$= \int_{-1}^{1} 2x\,dx = 2\left[\frac{x^2}{2}\right]_{-1}^{+1} = (1)^2 - (-1)^2 = 0$$

Ans

---

we have $\int_C x^2 y\,dx + x^2\,dy$

$$= \int_{\text{along A, ie } y=0} x^2 y\,dx + x^2\,dy + \int_{\text{on AB, i.e } x=1} x^2 y\,dx + x^2\,dy$$

$$+ \int_{\text{along BO, } y=x} x^2 y\,dx + x^2\,dy$$

$$= 0 + \int_0^1 1^2 dy + \int_1^0 x^2 \cdot x\,dx + x^2\,dx$$

$$= \int_0^1 dy + \int_1^0 (x^3 + x^2)\,dx$$

$$= [y]_0^1 + \left[\frac{x^4}{4} + \frac{x^3}{3}\right]_1^0$$

$$\Rightarrow 1 + \left[-\frac{1}{4} - \frac{1}{3}\right]$$

$$\Rightarrow \frac{5}{12} \text{ Ans}$$

Q verify Green's theorm for $\int_C (xy + y^2)\,dx + x^2\,dy$ where C is bounded by $y = x$ & $y = x^2$

Sol $\int_C (xy + y^2)\,dx + x^2\,dy$

$$= \iint_R [2x - (x + 2y)]\,dx\,dy$$

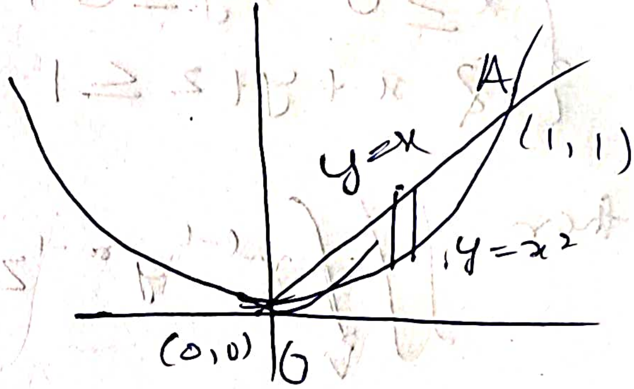
$$= \iint (x - 2y)\,dx\,dy$$

$$= \int_0^1 dx \int_{x^2}^{x} (x - 2y)\,dy$$

$$= \int_0^1 dx \left[xy - \frac{2y^2}{2}\right]_{x^2}^{x}$$

$$= \int_0^1 (x^2 - x^2) - (x^3 - x^4)\, dx$$

$$= \int_0^1 x^4 - x^3\, dx$$



$$= \left[\frac{x^5}{5} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{5} - \frac{1}{4} = -\frac{1}{20} \text{ Ans}$$

We have $\int_C (xy + y^2)\, dx + x^2 dy$

$$= \int_{C_1} (xy + y^2)\, dx + x^2 dy + \int_{C_2} (xy + y^2) dx + x^2 dy$$

$$= \int_0^1 (x x^2 + x^4)\, dx + x^2 (2x\, dx) + \int (x^2 + x^2)\, dx + x^2 dx$$

$$= \int_0^1 (x^3 + x^4 + 2x^3)\, dx + \int_1^0 3x^2 dx$$

$$\Rightarrow \left[\frac{5x^4}{4} + \frac{x^5}{5}\right]_0^1 + \left[\frac{3x^3}{3}\right]_1^0$$

$$\Rightarrow \left(\frac{3}{4} + \frac{1}{5}\right) + (0 - 1)$$

$$= \frac{15 + 4 - 20}{20} = -\frac{1}{20} \text{ Ans}$$

* Drichilet's theorem:-

$$\left\{ \begin{array}{l} x \geq 0, \; y \geq 0, \; z \geq 0 \\ \& \; x + y + z \leq 1 \end{array} \right\}$$

then $\iiint_V x^{l-1} y^{m-1} z^{n-1} \, dx \, dy \, dz$

$\Rightarrow \dfrac{\Gamma l \, \Gamma m \, \Gamma n}{\Gamma l+m+n+1}$

Q) Find the mass of an octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and the density at an point being $kxyz$.

$\circ\ V = 2.68 \times 10^8$ m/s

Module - II

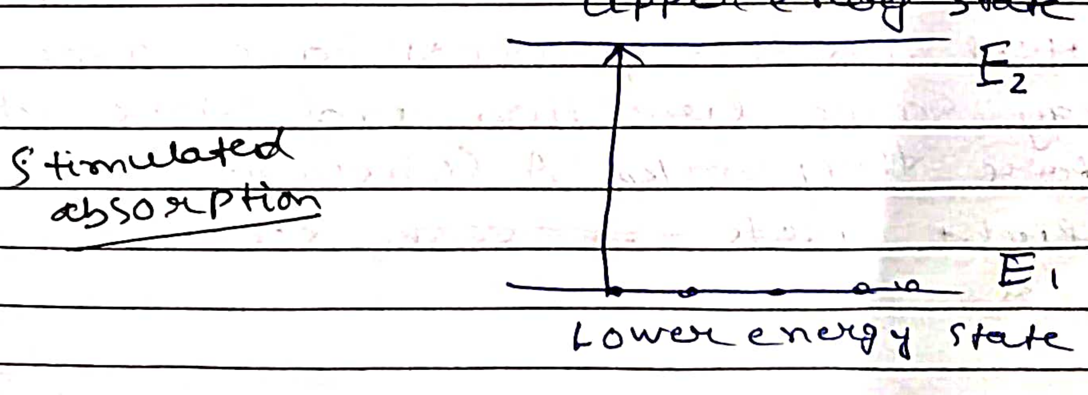
Laser

laser Stands for light amplification by stimulated emmison of radiation.

property of laser

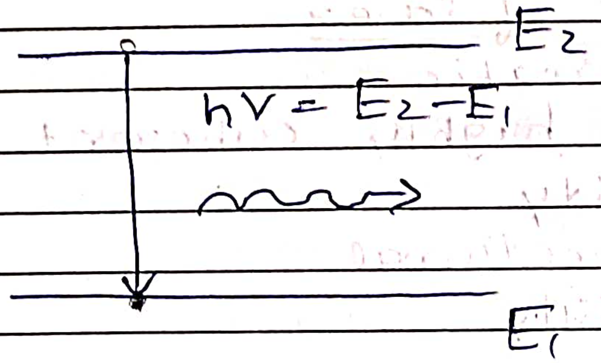
① Monochromatic
② ~~Coherent~~ highly coherent beam.
③ Intensity
④ Unidirectional
⑤ Focussily

① **Stimulated Absorption** :- In normal condition most of the atoms exist in ground State. if we provide energy to them, atoms will get excited and jump to upper energy level this transition is called Stimulated absorption.



Date / /
Page

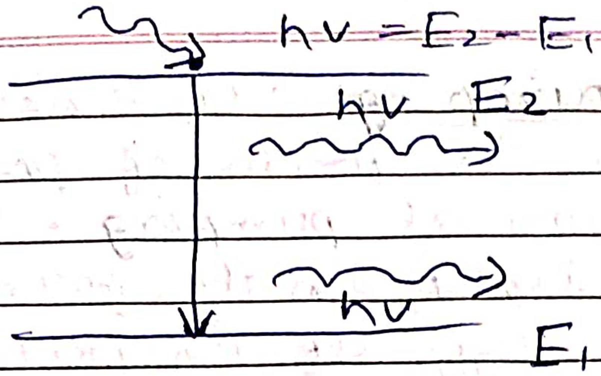
② Spontaneous emission.
the atoms can exist in excited state only for 10^{-8} seconds. after that they return to lower energy state by emitting a photon of energy $h\nu = E_2 - E_1$.
this Transition is called Spontaneous emission. the photons emitted by spontaneous emission process have random direction and random phase they make in coherent light beam



Spontaneous emission

③ Stimulated emission
let an atom exist in excited state. and in coming photon of energy ~~hv~~ = $h\nu = E_2 - E_1$ forces this atom to return lower energy state. two photons of energy $h\nu$ are emitted this is called : Stimulated emission. the photons emitted in this process have same energy same direction and same phase therefore they make A coherent beam of light ~~make as lase~~ known as laser.

Stimulated emission



Population Inversion

- In ordinary condition number of atoms N_1 in lower energy state is greater than number of atom ~~and~~ N_2 ~~those~~ in upper state. therefore probability of stimulated emission is very low. If by some technique the number of atom in excited state is made much larger than lower state, probability of stimulated emission increases.

"the situation in which number of atom in upper state is greater than number of atom in lower state is called population inversion.

$$\boxed{N_2 > N_1}$$

* Pumping

the method use to increase number of atom in excited state is called pumping. several techniques are used to provide energy to the ground state atom. Commonly used methods are

Date___/___/___
Page______

(1) Optical pumping → if energy is provided in the form of photon pumping is called optical pumping. the light source gives light in the form of pulses. the energy of excitation photon is greater than energy of emitted photon
eg → ~~Reo~~ Ruby laser.

(2) Electric discharge pumping
In this method accelerated electrons produce by electric discharge give their energy to the atom so that atoms get excited to upper ~~limit~~ level.
e.g → helium neon laser

(3) Chemical pumping :- In this process chemical reaction is used to excite the atoms.
eg → Carbondioxide laser.

(4) X- Ray Pumping → X-ray Photons are use for pumping process.
eg → dye laser

Meta Stable State :-
this energy state is a ~~known~~ long lived energy state in which atom can stay for 10^{-3} sec. the atoms do not return to lower state imidiately thus the probability of spontaneous emission is quite negligible ~~active~~

Date___/___/___

Page________

Active System

A material in which population inversion is achieve is called active system of laser. it contains callection of atom molecules or ions which is responsible for laser action it can be a liquid solid or gas.

eg → i) In ruby laser Chromium work as active system.

ii) In helium Neon laser, Neon works as active system

* Condition necessary to achieve laser action.

i) there must be population inversion.

ii) there must exist a metastable state. in which atom can stay for 10^{-3} sec

iii) we must use a Resonator (cavity).

→ there must be a pair of mirror at the ends of active system. One mirror is fully silvered and the other is partially silvered. the pair of mirrors is called resonator or cavity.

Relation between einstein ~~position~~ Cofficien

let N_1 be the no of atom in lower energy state E_1 and N_2 ~~th~~ be the no. of atom in upper energy state E_2. let a radiation of frequency beam incident such that $h\nu = E_2 - E_1$

$u(\nu) \rightarrow$ energy density of radiation

① The no. of absorption per unit volume per unit time $\propto N_1 u(\nu)$

$$= B_{12} N_1 u(\nu)$$

$B_{12} \rightarrow$ Einstein coefficient for absorption

② the no. of spontaneous emission per unit volume per unit time $\propto N_2$

$$= A_{21} N_2$$

$A_{21} \rightarrow$ Einstein coefficient for spontaneous emission.

③ the no. of ~~time~~ stimulated emission per unit time per unit volume $\propto N_2 u(\nu)$

$$= B_{21} N_2 u(\nu)$$

$B_{21} \rightarrow$ Einstein coefficient for stimulated emission.

In thermal equilibrium

rate of absorption = rate of emission

$$B_{12} N_1 u(\nu) = A_{21} N_2 + B_{21} N_2 u(\nu)$$

$$u(\nu) \left[B_{12} N_1 - B_{21} N_2 \right] = A_{21} N_2$$

Date / /
Page

$$u(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} = \frac{A_{21} N_2}{N_2\left[B_{12} \frac{N_1}{N_2} - B_{21}\right]}$$

$$u(\nu) = \frac{A_{21}}{B_{12}\left[\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}\right]} \quad \text{--- (4)}$$

According to Boltzmann distribution la

$$N = N_0 e^{-E/KT}$$

$N_0 \rightarrow$ Total no. of atoms.
$K \rightarrow$ Boltzmann Constant

$$N_1 = N_0 e^{-E_1/KT}$$
$$N_2 = N_0 e^{-E_2/KT}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{-E_1/KT}}{N_0 e^{-E_2/KT}} = e^{(E_2-E_1)/KT}$$

$$\frac{N_1}{N_2} = e^{h\nu/KT} \quad \text{--- (5)}$$

Putting eq (5) in (4)

$$u(\nu) = \frac{A_{21}}{B_{12}\left[e^{h\nu/KT} - \frac{B_{21}}{B_{12}}\right]} \quad \text{--- (6)}$$

According to planck's law of radiation

$$u(\nu) = \frac{8\pi h \nu^3}{c^3\left[e^{h\nu/KT} - 1\right]} \quad \text{--- (7)}$$

Date / /
Page

Compairing (6) and (7)

$$\frac{B_{21}}{B_{12}} = 1 \Rightarrow B_{12} = B_{21}$$

$$\boxed{\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3}}$$

Ruby laser (3-level laser)

Construction -1:- Active System

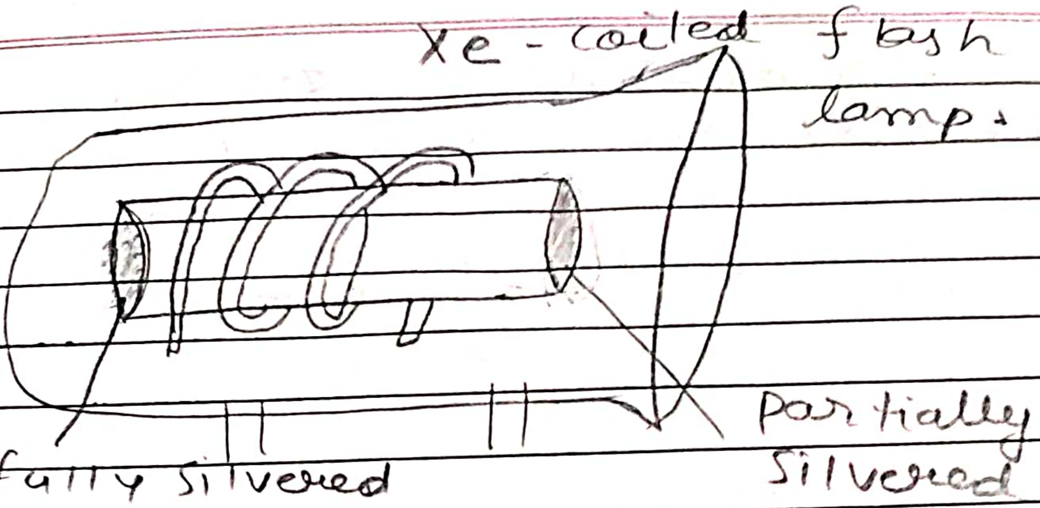
It consist a pink Ruby rod of cylindrical Shape. Ruby is a crystal of Almunium oxide (Al_2O_3) doped with 0.05 percent Chromium oxide (Cr_2O_3)

Construction 2:- ~~of~~ Pumping

Optical pumping used A ~~a~~ Coild lamp field with ~~X~~enon is bounded over the rod it gaves light of wavelength 5500 Å in the form of flash.

Construction 3:- Resonator

Both ends of the rod are made optically flat and parallel. One ~~it~~ end ~~perfectly~~ is fully silvered and they other is partially Silvered.



working

1. When flash of light from ~~X~~enon lamp falls upon ruby rod, chromium ions (Cr^{+++}) absorb photon of wavelength 5500 Å and move to excited state E_2. (Transition 1)

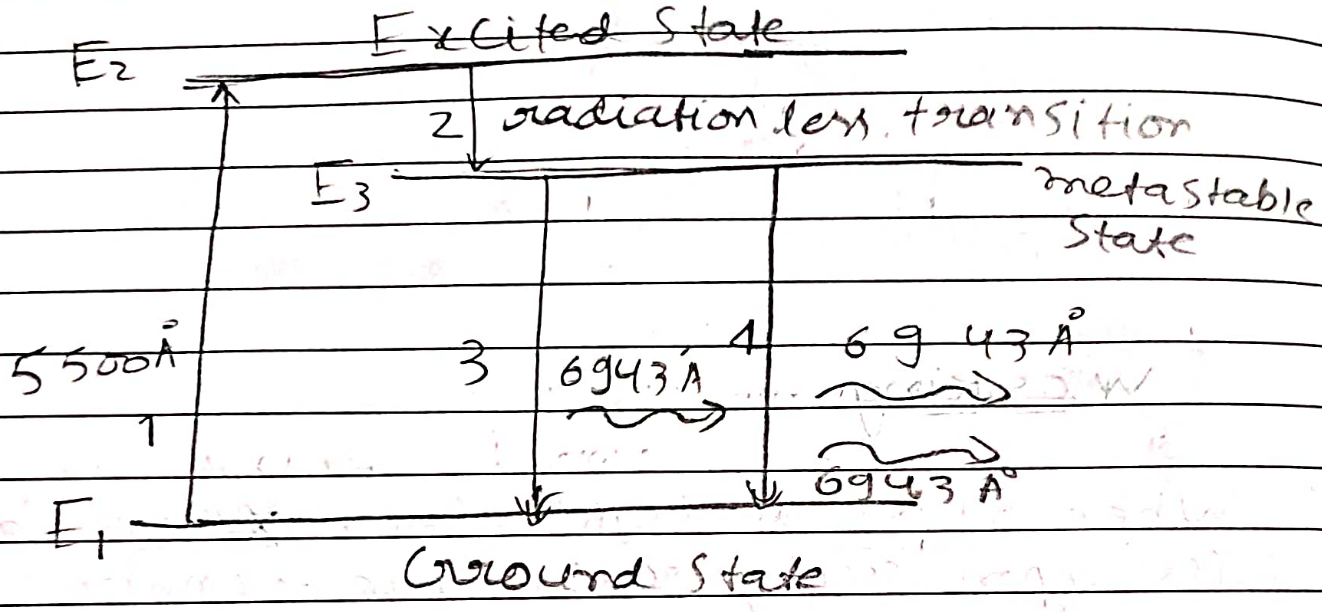
2. the excited ion gives there part of energy to the crystal and moves to metastable state E_3 (Transition 2).

3. One ~~a~~ ion returns to ground state by spontaneous emission emitting a photon of wavelength 6943 Å (Transition 3)

4. this photon travels throug the rod and is reflected baking forth until its stimulate an ion in E_3 to stimulated emission. (Transition - 4). Hence two photons are emitted

5. Process 4 is repeated again and again

When heat becomes intense, laser comes out the partially on silvered end.



Drawback
the laser light emitted is in the form of pulses.

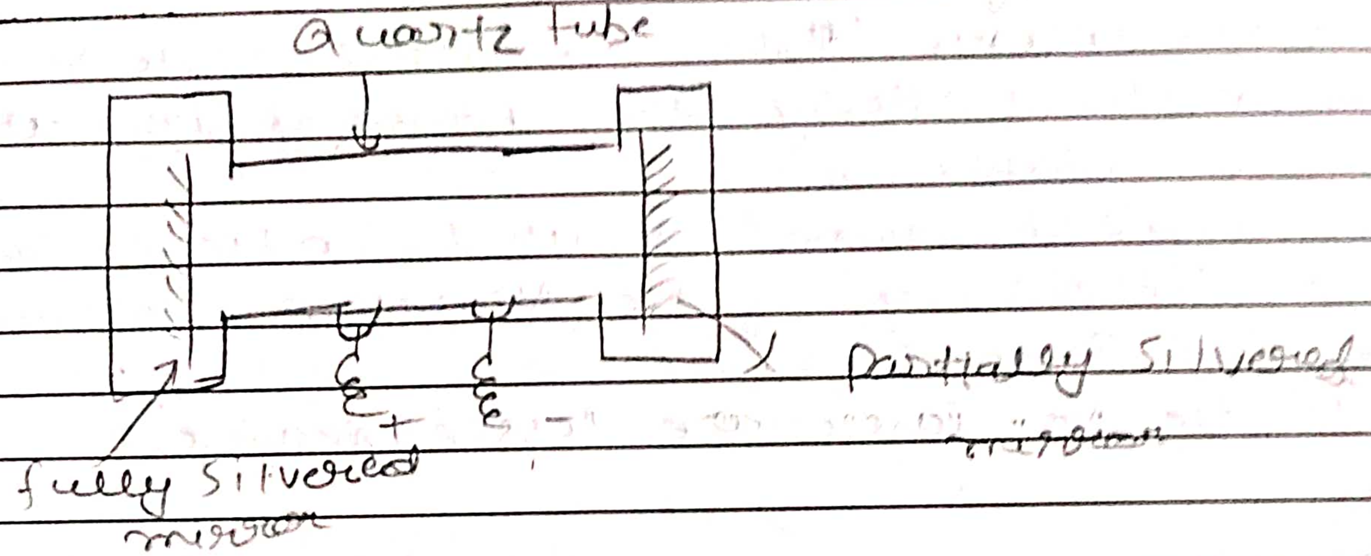
* Helium-Neon laser (4- level laser)

Construction 1:- Active System
it consist a narrow discharge tube of ~~Chords~~ quartz filled with Helium and Neon in the ratio 7:1 at low pressure.

Costruction 2:- ~~Inelastic ato~~ Pumping
Inelastic atom atomic is used two eléctrode are connected to high frequency alternating current.

Date / /
Page

Construction 3 :- Resonator
Two plane parallel mirrors are attached with a tube. distance between mirrors is integral multiple of laser wavelength



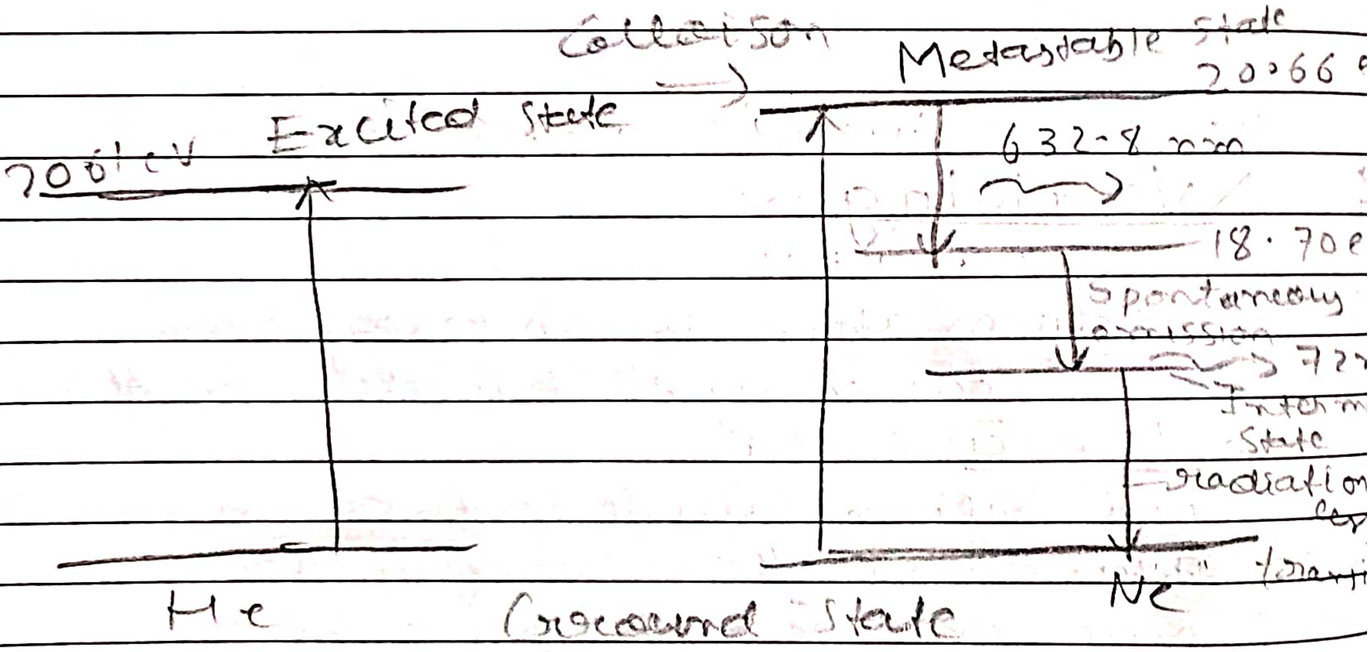
Working

1. the helium atoms absorb energy from electrons produced by electric discharge ~~At~~ An move to excited State.
2. Excited helium collide with ground state neon and transfer there energy to neon. So that neon moves to metastable state.
3. When population inversion is achieve, laser action takes place between metastable State and excited State At $\boxed{18.70\,eV}$
4. Another transition due to Spontaneous emission from excited to intermidiate state gives in coherent photon of wavelength 72 ~~nm~~ Nm
5. ~~the same~~

5. the remaining exciting energy is lost in collision with tube's walls and neon returns to ground state.

Advantage :-

1. Continuous laser is emitted in He-Ne laser while in Ruby laser pulses of light are emitted.
2. Coolent is not required in He-Ne laser
3. efficiency of He-Ne is greater than Ruby laser.
4. He-Ne laser are less expensive.



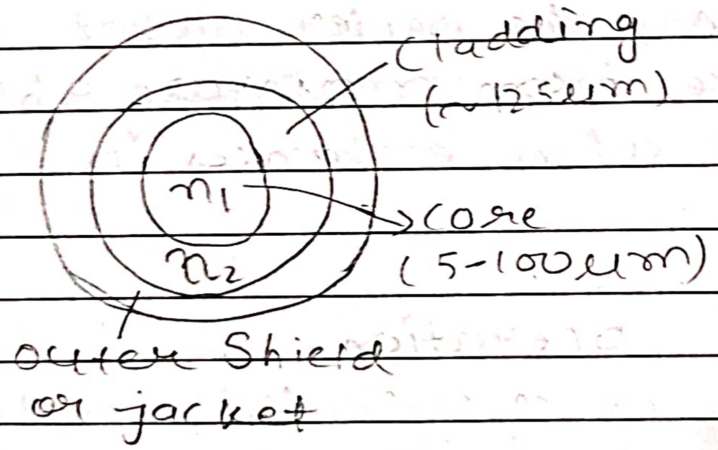
Fiber optics

fundamental ideas about optical fibre

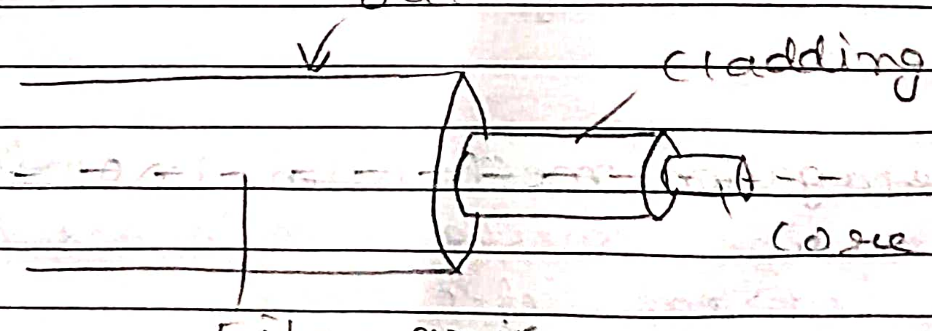
* it is a hair thin flexible transparent medium of wire shape usually made by glass or plastic. Data can be send using optical fibre cables through light way. this cable can carry use amount of information in the form of light wave from one place to other with negligible loss and width bandwidth.

Basic Structure

(Cross-sectional view)



Side view



λ

Date / /
Page

Fibre structure has three ~~instruction~~ sections

i) **Core** :-

i) it is the inner most section made by pure glass or plastic. it has diameter from 5 - 100 um

ii) **Cladding** :- core is surrounded by another layer of glass with slightly lower refractive index then core. it is called cladding and has ~~ado~~ a diameter about 125 um.

iii) **outer shield or jacket** :- it is made by plastic or polymer will diameter 250 um. the outer jacket protects the fibre from moisture absorp crushing and also enhances its tensile strength.

Principle of operation

An optical fibre is a cylindrical wave guide that propagates information coded in the form of light along its length by the process of total internal reflection.

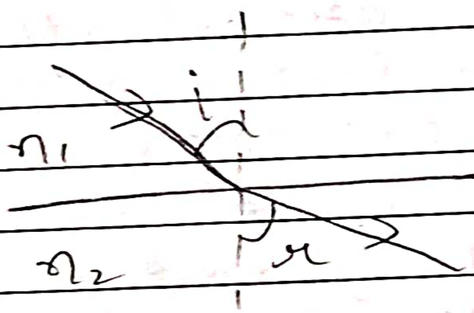
Propagation mechanism in optical fibre

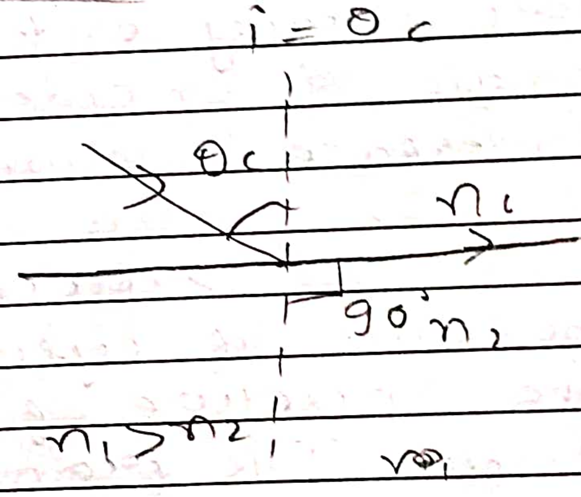
Propagation mechanism is based on total internal reflection.

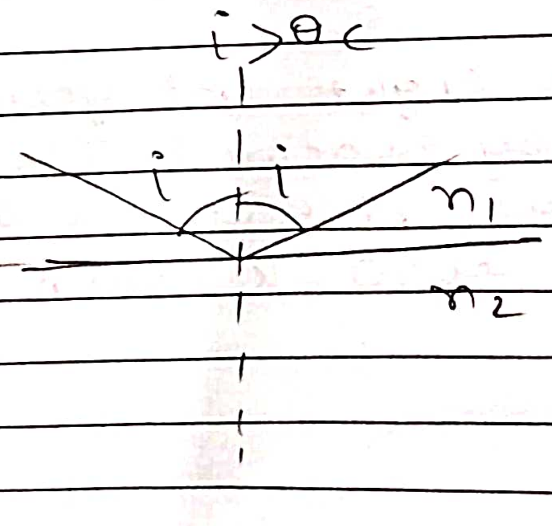
i) when light passes from denser to rarer medium it bends away from the normal if 'i' increases are also r also increases because $n = \frac{\sin i}{\sin r}$ is constant.

ii) At a particular angle of incidence θ_c the ~~reflected~~ refracted ray makes 90° with the normal. θ_c is called critical angle.

iii) When angle of incidence is further increased the refracted ray turns back into the same medium this is called total internal reflection.
This effect is used in optical fibre to confine the light.

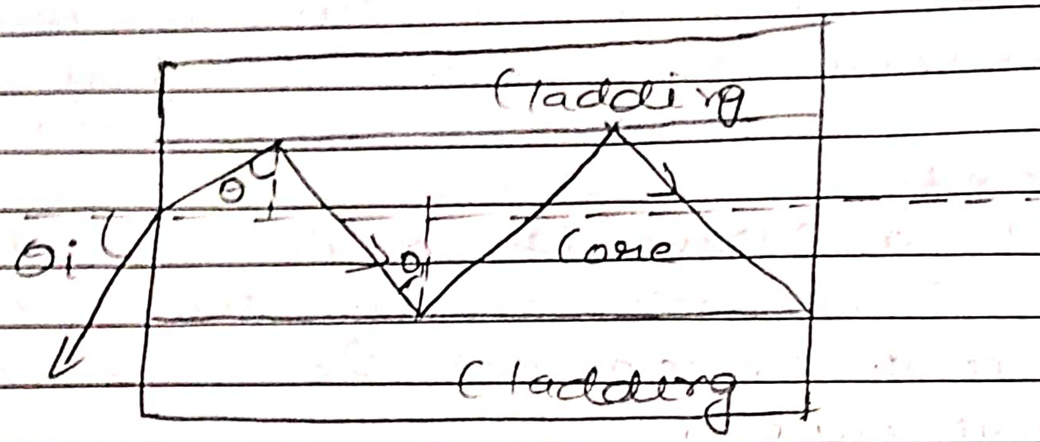






Date / /
Page

Communication in optical fibres



the effect of TIR is used to communicate via optical fibre. the refractive index of a core is slightly greater than refractive index of cladding so that core is denser than cladding.

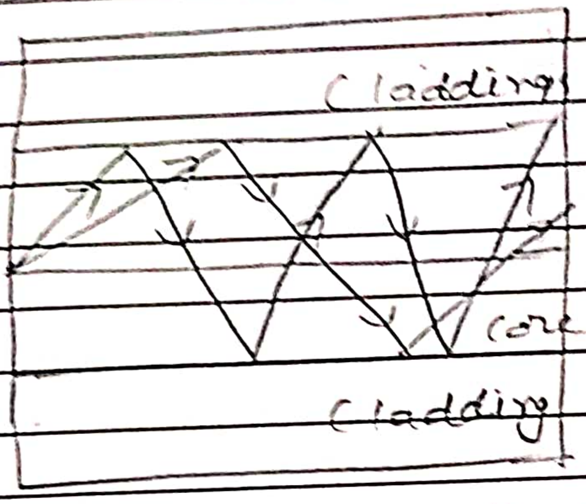
for a ray entering the fiber, if angle of incidence at core cladding interface is greater than critical angle $\theta > \theta_c$, the light gets total internal reflection into the core. because of cylindrical symmetry in the structure of fibre this ray will suffer TIR from lower interface also.

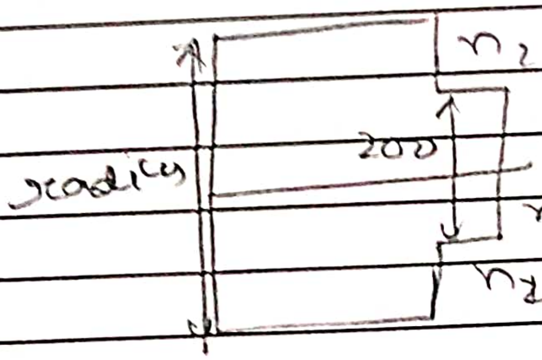
therefore the light get guided through the core by repeated TIR. thus an optical fibre act as a light guide and is also known as wave guide.

Types of optical fiber

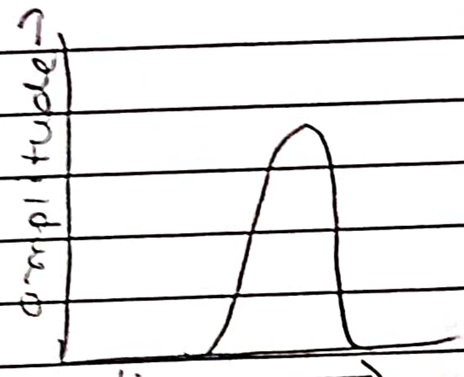
On the basis of refractive index and refractive index modes the fibers can be divided into three types.

Step:-I index multimode fiber

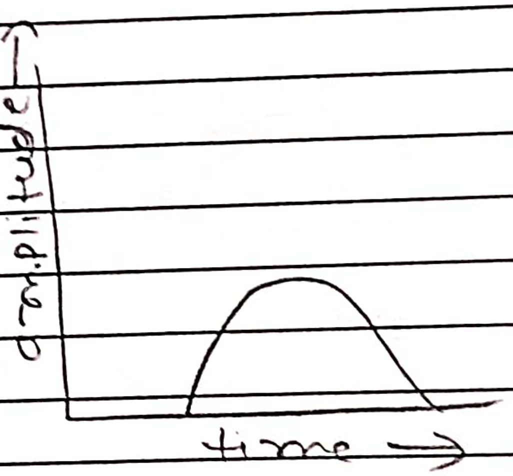




pulse propagation



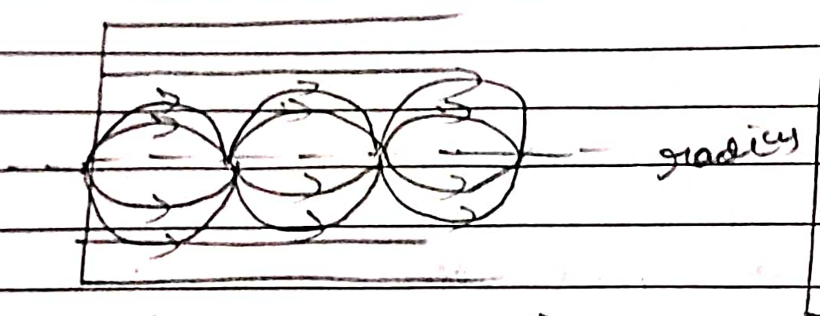
Input pulse



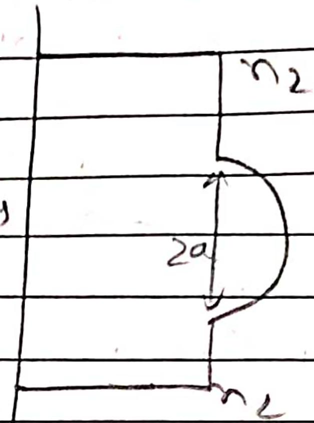
output pulse

Date / /
Page

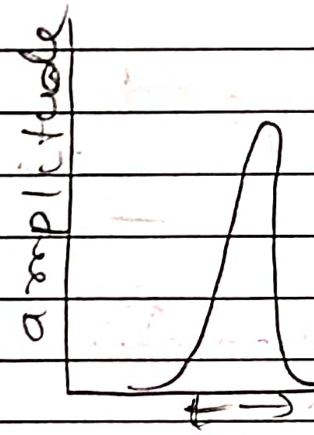
Graded index multi mode fibre



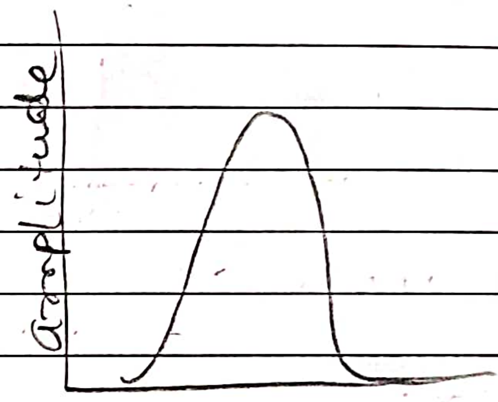
Pulse propagation



R.I profile



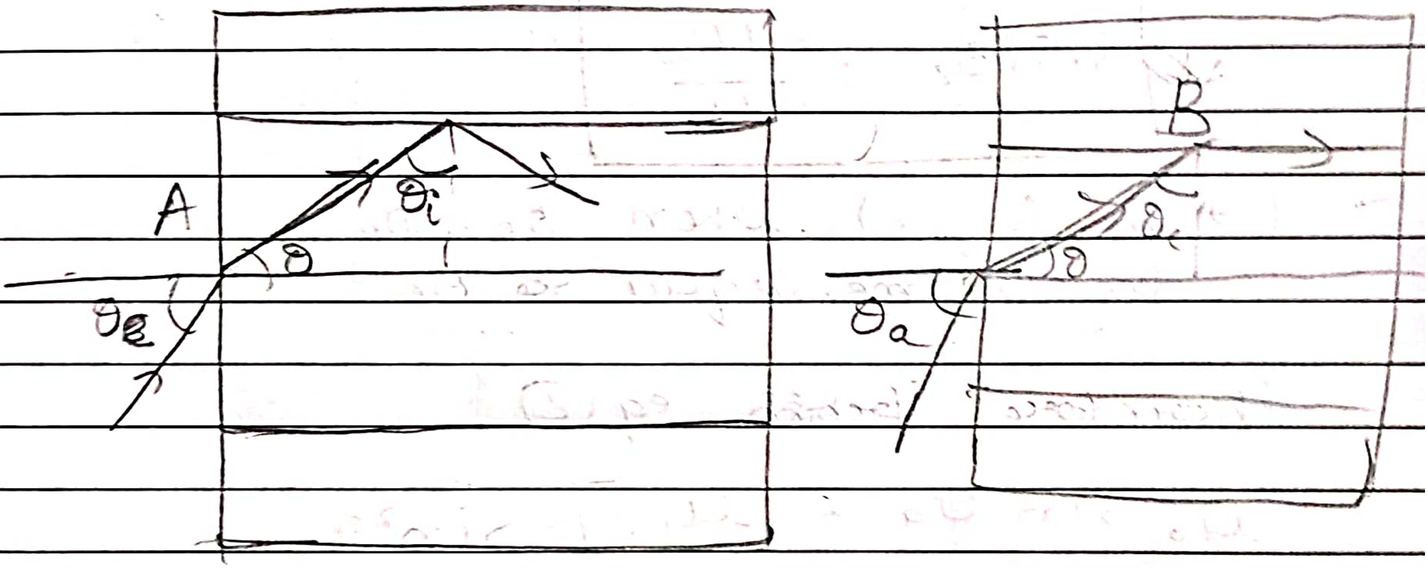
Input pulse



Output pulse

- In this fibre the R.I of core decreases gradualy from maximum value at the core to the minimum costant value at the core clading interface. R.I of Grading is constant.
- A ray entering the fibre continuously bent towards the ~~a~~ excess of ~~bit~~ fibre.
- the path of ray is smooth sinusiodal
- all the rays take same time to reach the receiving end. there fore modal dispersion is greatly minimized in ~~gra~~ GRIN.

* Acceptance Angle:- the rays transmitted by TIR must have angle of incident greater than critical angle at the core cliding interface.
" the light ray should incident on the core with in a maximum external angle θ_a to have the condition ($\theta_i > \theta_c$) for propagating down the core. this angle θ_a is known as Acceptance angle ~~for~~ for the fibre. it depends upon refractive index of core and cliding.



$$\frac{\sin\theta_e}{\sin\theta} = \frac{\mu_1}{\mu_0} \qquad \theta + \theta_i = 90$$
$$\theta_i = 90 - \theta$$

Using Snell's law for point A, (fig 1)

$$\frac{\sin\theta_e}{\sin\theta} = \frac{\mu_1}{\mu_0}$$

$$\mu_0 \sin\theta_e = \mu_1 \sin\theta \quad \text{---} \quad (1)$$

$\because \theta + \theta_i = 90$

$\theta = 90 - \theta_i$

$\mu_0 \sin\theta_e = \mu_1 \sin(90 - \theta_i)$

$\mu_0 \sin\theta_e = \mu_1 \cos\theta_1$

$\mu_0 \sin\theta_e = \mu_1 \sqrt{1 - \sin^2\theta_1}$ ---- (2)

using Snells law for point B (fig 2)

$$\frac{\sin\theta_c}{\sin 90'} = \frac{\mu_2}{\mu_1}$$

$$\boxed{\sin\theta_2 = \frac{\mu_2}{\mu_1}}$$

From fig(2), when $\theta_e = \theta_a$

θ_i becomes equal to θ_c

therefore from eq (2)

$\mu_0 \sin\theta_a = \mu_1 \sqrt{1 - \sin^2\theta_c}$

for Air $\mu_0 = 1$

$$\sin\theta_a = \mu_1 \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}}$$

$$\boxed{\sin\theta_a = \sqrt{\mu_1^2 - \mu_2^2}}$$

for another medium

$$\sin\theta_a = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{}$$

Date / /
Page

Numerical Aperture

Numerical Aperture is a number which defines the light gathering capacity of a fibre when light is emitted from source only a fraction $(NA)^2$ of the total amount of light can be collected by the fibre.

$$\boxed{NA = \sin\theta_a = \frac{\sqrt{\mu_1^2-\mu_2^2}}{\mu_0}}$$

In air $\mu = 1$

$$\boxed{NA = \sin\theta_a = \sqrt{\mu_1^2-\mu_2^2}}$$

Relative Refractive index difference

$$\Delta = \frac{\mu_1-\mu_2}{\mu_1}$$

$$\Delta = \frac{\mu_1-\mu_2}{\mu_1} \times \frac{(\mu_1+\mu_2)}{(\mu_1+\mu_2)}$$

$$\Delta = \frac{\mu_1^2-\mu_2^2}{\mu_1 \cdot 2\mu_1}$$

$\because$ $\mu_1-\mu_2$ is very small

$\therefore$ $\mu_1+\mu_2 \simeq 2\mu_1$

$$\Delta = \frac{\mu_1^2 - \mu_2^2}{2\mu_1^2}$$

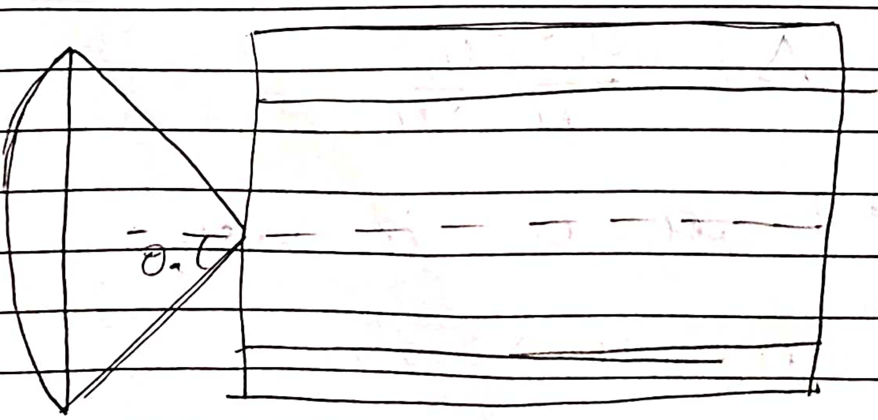
$$\mu_1^2 - \mu_2^2 = 2\Delta\mu_1^2$$

$$\sqrt{\mu_1^2 - \mu_2^2} = \mu_1\sqrt{2\Delta}$$

$$\boxed{NA = \mu_1\sqrt{2\Delta}}$$

*Acceptance Cone

the rays incident at angle greater than θ_a would be ~~refeacting~~ refracted inclding planing only the Race falling at angle 0 to θ_a would propagate to the fibre.
the ~~Core~~ Cone formed with acceptance angle θ_a as vertex angle is called acceptance cone. All the Rays falling with in the Cone can propagate ~~to the f~~ through the fibre.



Date ___/___/___
Page ______

* Numbers of modes & Cut-off parameter (V-no)

the various structural parameters of fibre can be combined into a normalised parameter known as V number or normalised frequency off cut-off.

$$V = \frac{2\pi a}{\lambda_0}(NA)$$

$$V = \frac{2\pi a}{\lambda_0}\sqrt{\mu_1^2 - \mu_2^2}$$

If fibre is in a medium of R.I. μ_0,

$$V = \frac{2\pi a}{\lambda_0}\mu_0\sqrt{\mu_1^2 - \mu_2^2}$$

For MMF

No of mode $= \frac{V^2}{2}$

For GRIN

No of mode $= \frac{V^2}{4}$

For SMF

No of mode $= 1$

For SMF, $V < 2.405$
For MMF, $V \geq 2.405$

Out of all, only those modes will be propagated for which cut off frequency is less than V number.

* Attenuation

The reduction in amplitude intensity of fibre or power of a signal as it is propagated in optical fibre is called

→ Attenuation.

Attenuation loss is measured in decibel. A decibel is express as base 10 log logarithm of ratio of ~~in~~ output power to input power.

~~$dB = 10 \log_{10} \frac{P_{oint}}{P_{in}}$~~

$$\boxed{dB = 10 \log_{10} \frac{P_{out}}{P_{in}}}$$

Attenuation coefficient is expressed in dB/Km.

$$\boxed{\alpha = \frac{-10}{L} \log_{10} \frac{P_{out}}{P_{in}} \ dB/Km}$$

Date___/___/___

Page________

-ve sign indicates loss

* Signal losses in optical fibre

1. **Absorption Loss** :- the absorption of light by the core and cladding is the main source of Attenuation. light is absorbed by material itself or impurities and imperfection. When glass molecule intract with electron neutron gama rays, x-rays etc. Structure of glass molecule changes and absorption loss occurs.

2. **Rayleigh scattering loss** :

glass has microscopic inhomogeneties such as bubble impurity particle etc. light is scattered by these object fibre with large N.A exibit greater scattering loss. this loss is more ~~to~~ prominent in the wavelength range 500 to 1550 nm.

3. **Waveguide scattering loss** :- irregulatories in geometry of optical fibre is the main source of wave guide scattering loss. this losses can be minimised by putting third layer of pure silica around thin cladding. refractive index of

Date ___/___/___
Page ______

of third layer is higher than cladding but lower than core.

4.) bending loss:- it is of two types:

i) micro bending ⇒ this loss occurs when core surface has small variation in shape. show the incident angle changes the on core cladding interface and light ~~refle~~ refracts in cladding.

ii) Macro bending:⇒ this loss occurs due to rapping the fibre or pulling the fibre cable around corner. larger core radius and smaller bending radius increases bending loss

Dispersion

the information which is to be transmitted is coded in the form of light pulse then these pulses are fed into the fibre. the pulse brodance in time as it passes through fibre.

"the phenomenon of broadening or spreading of pulse ~~is~~ is known as dispersion.". it occurs because the rays are send at the same time but after travelling through fibre, they reach output end at different times. the types of dispersion are as follows:-

Date ___/___/___

Page ______

i) Intermodal dispersion or modal dispersion :-

it occurs in multimode fibres. Since each path of light has a different length therefore some of light reaches receiving end sooner then the others this leaves to pulse spreading. if broading is large, the adajacent pulses will overlap at the output end and may not be resolved

(ii) Intramodal dispersion or Chromatic dispersion :-

Since optical source do not emit just a single frequency but a band of frequency. each wavelength travel with different speed in the core. this causes broading of pulse in each single mode it is further divided into two categories:

~~a) material dispers~~

a) material dispersion :- pulse of light usualy contains several wavelengths the speed of different wavelengths are different in core causing the spreading of light pulse.

b) wave guide dispersion :- wave guide dispersion it is cause because some light travels in the cliding also. velocity of light is higher in cliding than core (since $\mu_1 > \mu_2$). wave guide

Date
Page

dispersion depends on the difference of n_1 and n_2 and core diameter"

Advantage of fibre optics

i) optical fibre have lower transmission losses. they more data canbe send over long distances.
ii) light beam has a very high information carrying capacity due to high frequency therefore transmission at higher data rate is possible.
iii) light weight and small ~~Hair~~ Hair thin size dimension.
iv) optical fibre can with stand environmental mental results better and have a long life then copper wire.
v) optical fibre cable are cheaper.
vi) Bandwith of optical fibre is higher.

Disadvantage of fibre optics

i) working with fibres workers may face hizards of glass shards.
ii) it requires a new set of skills for insterlence and maintaince.