Q-2) A soap film of refractive index 1.33 is illuminated with light of different wavelength at an angle of 45° there is complete destructive interference for λ 5890 Å find the minimum thickness of the film

Sol

$\lambda = 5890\,Å = 5890 \times 10^{-10}\,m$

$\mu = 1.33, \quad i = 45°, \quad m = 1$

$$\mu = \frac{\sin i}{\sin r}$$

$$1.33 = \frac{\sin 45}{\sin r}$$

$$\sin r = \frac{1}{\sqrt{2} \times 1.33}$$

$$\sin r = 0.5316$$

$$r = 32.11$$

$$2\mu t \cos r = m\lambda$$

$$2 \times 1.33 \times t \times \cos 32.11 = 1 \times 5890 \times 10^{-10}$$

$$t = \frac{5890 \times 10^{-10}}{2.2531}$$

$$t = 2614.17 \times 10^{-10}$$

$$t = 2.61 \times 10^{-7}\,m$$

Q-3) A thin film of soap solution is illuminated by white light at an angle of incidence $i = \sin^{-1}\left(\frac{4}{5}\right)$ in reflected light two dark consecutive overlaping fringes are observed corresponding to wavelength 6.1×10^{-7} m and 6.0×10^{-7} m the refractive index for soap solution is $\frac{4}{3}$. calculate the thickness of film.

Sol $i = \sin^{-1}\left(\frac{4}{5}\right)$, $\lambda_1 = 6.1 \times 10^{-7}$ m

$\lambda_2 = 6.0 \times 10^{-7}$ m, $\mu = \frac{4}{3} = 1.33$

$$\sin i = \frac{4}{5}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3} = \frac{3}{5}$$

$$\sin r = 0.6$$

$$\cos r = \sqrt{1 - \sin^2 r}$$

$$= \sqrt{1-(0.6)^2} = \sqrt{0.64}$$

$$= 0.8$$

$$2\mu t \cos r = m\lambda_1 \quad \text{--- (1)}$$

$$2\mu t \cos r = (m+1)\lambda_2 \quad \text{--- (2)}$$

from eq (1) and (2)

$$m\lambda_1 = m\lambda_2 + \lambda_2$$

$$\lambda_2 = m(\lambda_1 - \lambda_2)$$

$$m = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{6.0 \times \cancel{10^{-7}}}{(6.1 - 6.0) \times \cancel{10^{-7}}}$$

$$m = 60$$

putting the value of m in eq (1)

$$2\mu t \cos r = m\lambda_1$$

$$2 \times 1.33 \times t \times 0.8 = 60 \times 6.1 \times 10^{-7}$$

$$t = \frac{60 \times 6.1 \times 10^{-7}}{2 \times 1.33 \times 0.8}$$

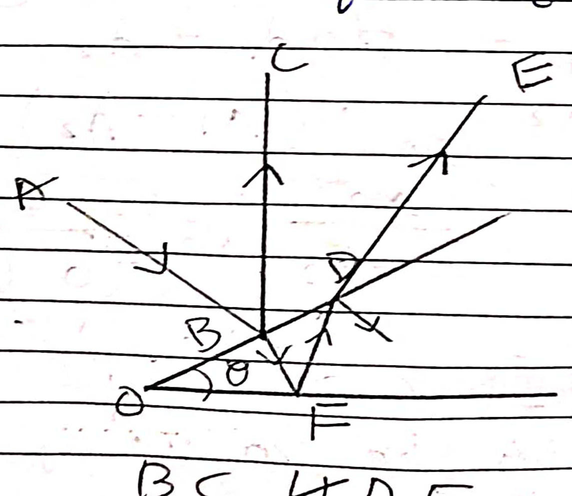
$$t = 171.99 \times 10^{-7}$$

$$t = 1.71 \times 10^{-5}\ m$$

Wedge Shaped film

A Thin film having zero thickness ~~and~~ at one end progressively increasing thickness at the other end is called wedge shaped film.

When parallel beam of monochromatic light illuminates the ~~wedge~~ film from above, the rays reflected from its two boundary surfaces will not be parallel they appear to diverge from a point near the film. the path difference between rays reflected from upper and lower surfaces (BC and DE) depends upon the thickness of the film.



BC ∥ DE

path difference b/w BC & DE

$$\boxed{\Delta = 2\mu t \cos r - \frac{\lambda}{2}}$$

For maxima

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

For minima

$$2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (n+1)\lambda$$

$$\boxed{2\mu t \cos r = n\lambda}$$

* Fringe width

Consider a wedge shaped film having normal incidence. let x_n be the distance of n^{th} dark fringe from apex (O).

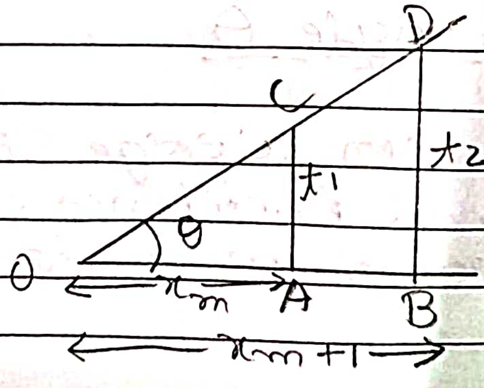
$x_{n+1} \rightarrow$ distance of $(n+1)^{th}$ dark fringe from O.

t_1 & t_2 be the thickness at point A & B

$\theta \rightarrow$ wedge angle

Normal incidence

$\Rightarrow r = 0^\circ$ or $\cos r = 1$



Condition for dark fringe

$$2\mu t \cos r = m\lambda$$

$$2\mu t = m\lambda$$

For m^{th} fringe, $2\mu t_1 = m\lambda$ --- (1)

For $(m+1)^{th}$ ", $2\mu t_2 = (m+1)\lambda$ --- (2)

In ΔAOC	In ΔBOD
$\tan\theta = \frac{t_1}{x_m}$	$\tan\theta = \frac{t_2}{x_{m+1}}$
$t_1 = x_m \tan\theta$	$t_2 = x_{m+1}\tan\theta$

~~Similarly $t_2 = x_{m+1}\tan\theta$~~

putting the value of t_1 & t_2 in eq (1) & (2)

$$2\mu x_m \tan\theta = m\lambda \longrightarrow (3)$$

$$2\mu x_{m+1}\tan\theta = (m+1)\lambda \text{ --- } (4)$$

Subtracting (3) from (4)

$$2\mu\tan\theta(x_{m+1} - x_m) = (m+1)\lambda - m\lambda$$

$$= \lambda$$

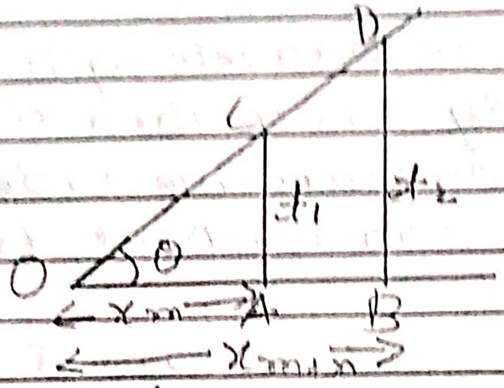
$$x_{m+1} - x_m = \frac{\lambda}{2\mu\tan\theta}$$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}} \quad \because \theta \to \text{Very Small}$$

Wedge angle θ

Consider a thin wedge shape film having normal incidence. A dark fringe of order m is found at A and

Dark fringe of order $m+n$ is found at B



$$\tan\theta = \frac{t_1}{x_m}$$

$$t_1 = x_m \tan\theta = x_m \cdot \theta$$

& $\quad 2\mu t_1 = m\lambda$

$$2\mu x_m \theta = m\lambda \quad \text{--- (1)}$$

Similarly $2\mu x_{m+n}\theta = (m+n)\lambda$ --- (2)

Subtracting (1) from (2)

$$2\mu(x_{m+n} - x_m)\theta = (m+n)\lambda - m\lambda$$
$$= n\lambda$$

$$\boxed{\theta = \frac{n\lambda}{2\mu(x_{m+n} - x_n)}}$$

Properties of interference pattern

1) fringes ~~will~~ are straight and parallel.
2) fringes are of equal thickness and equidistant.

3) fringes are localized.
4) fringes at the apex is dark.

Q-1) Light of wavelength of 6000 Å falls normally on a thin wedge shaped film of refractive index 1.4 forming fringe that 2 mm apart find the angle of wedge

Sol given:- $\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m

$\mu = 1.4$

$\beta = 2$ mm $= 2 \times 10^{-3}$ m

$$\beta = \frac{\lambda}{2\mu\theta}$$

$$\theta = \frac{\lambda}{2\mu\beta} = \frac{6000 \times 10^{-10}}{2 \times 1.4 \times 2 \times 10^{-3}}$$

$$\theta = 1.07 \times 10^{-4}$$

Q-2) Interference fringes are produce with monochromatic light on a wedge shaped film of refractive index 1.4 the angle of the wedge is 10 seconds of an arc and the distance between successive fringes is 0.5 cm. Cal. the ~~light~~ of wavelength of light used.

Sol $\beta = 0.5$ cm , $\mu = 1.4$

$= 5 \times 10^{-3}$ m

$$\theta = 10'' = \frac{10 \times \pi}{60 \times 60 \times 180} \text{ read}$$

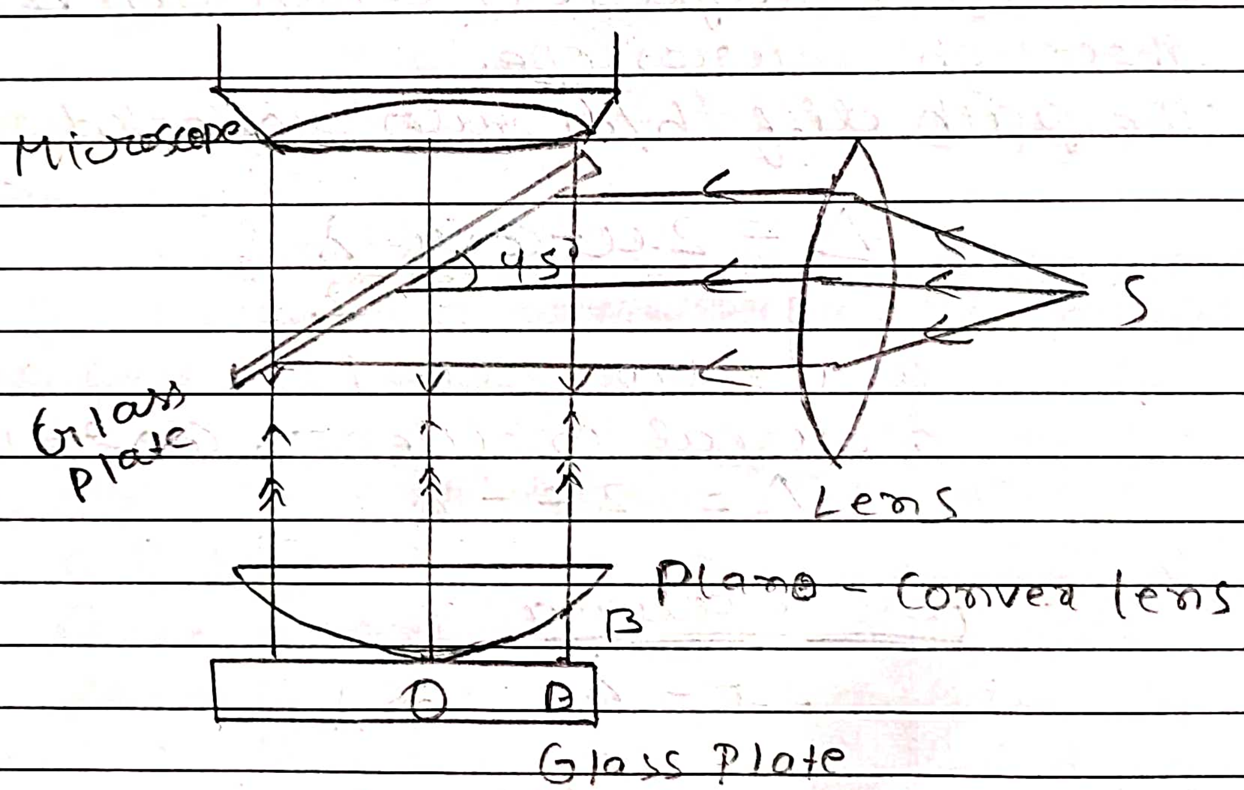
$$\beta = \frac{\lambda}{2\mu\theta}$$

$$\lambda = \beta \times 2\mu\theta$$

$$\lambda = \frac{5 \times 2 \times 1.4 \times 10 \times 3.14 \times 10^{-3}}{60 \times 60 \times 180}$$

$$\lambda = 6.78 \times 10^{-8}$$

* Newton's Ring



newton's ring are formed when a plano-convex lens placed on a plane glass sheet is illuminated from the top with monochromatic light. A thin air film is formed between the lens and plate whose thickness is zero at the point of contact.

monochromatic light from extended source is made parallel by a lens. It is incident on glass plate inclined at 45° degree, and Is reflected normaly downward on the plano-convex lens. A small part of light is reflected from the upper surface of thin film (B) (Point B). the remainder is reflected from the lower surface of film (Point D). these two rays are coherent. introduce interference pattern which can be seen through microscope.

The path diff b/w two reflected rays is

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$$

air film, $\mu = 1$

normal incident, $\cos r = 1$

$$\Delta = 2t - \frac{\lambda}{2}$$

for maxima

$$2t - \frac{\lambda}{2} = n\lambda$$

$$2t = (2n+1)\frac{\lambda}{2}$$

for minima

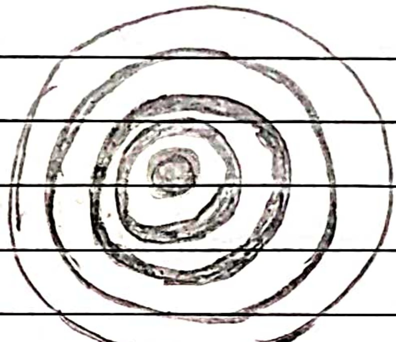
$$2t - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{2t = m\lambda}$$

where $m = n+1$

Circular fringes

the focus of all points corresponding to ~~eg~~ specific thickness of air film falls on a circle whose ~~point of~~ centre is at point of contact O, since each maxim and minima is focus of constant film thickness therefore fringes are circular they are also called fringes in ~~eg~~ equal thickness.



Central dark spot

path diff $\Delta = 2t - \frac{\lambda}{2}$

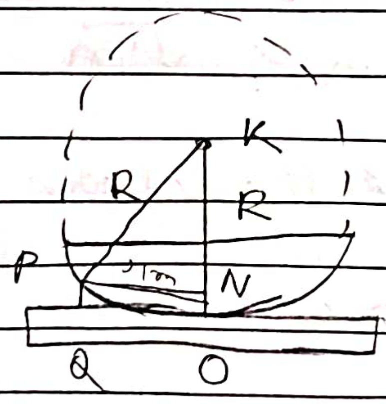
at the centre $t = 0$

Path diff $= \frac{\lambda}{2}$

$\Rightarrow$ which is a condition for dark fringe therefore central spot is dark.

Radii of dark fringes

let ~~a~~ R be the radius of curvature of the plano lens let a dark fringe be located at Q where thickness of film is t. if the order of this fringe or ring is ~~small n~~ m. then OQ is equal to r_m ($OQ = r_m$)



ΔKPN, $KP^2 = PN^2 + KN^2$

$$R^2 = r_m^2 + (R-t)^2$$

$$r_m^2 = R^2 - (R-t)^2$$

$$r_m^2 = R^2 - R^2 - t^2 + 2Rt$$

$$r_m^2 = 2Rt - t^2$$

$\because R >> t$

$\therefore 2Rt >> t^2$

$\Rightarrow t^2$ is neglected

$$r_m^2 = 2Rt \quad --(1)$$

$$\boxed{r_m = \sqrt{2Rt}}$$

$$\boxed{D_m = 2\sqrt{2Rt}}$$

For dark ring

$$2t = m\lambda$$

Putting in eq (1)

$$\boxed{\begin{aligned} r_m &= \sqrt{m\lambda R} \\ D_m &= 2\sqrt{m\lambda R} \end{aligned}}$$

$\because \lambda$ & R are constant

$$D_m \propto \sqrt{m}$$

$\Rightarrow$ the diameter of the dark fring is directly proptional to square root of natural numbers

For bright ring

$$2t = (2n+1)\frac{\lambda}{2}$$

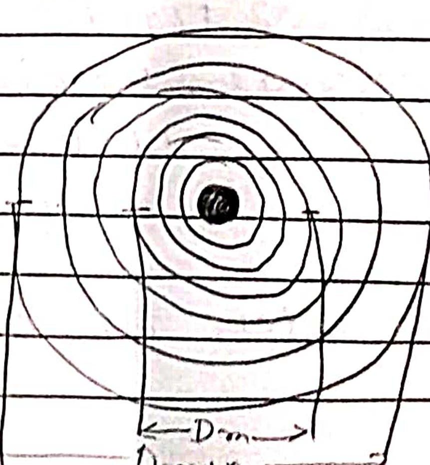
putting in eq (1)

$$\boxed{\begin{aligned} r_n &= \sqrt{\frac{(2n+1)\lambda R}{2}} \\ D_n &= 2\sqrt{\frac{(2n+1)\lambda R}{2}} \end{aligned}}$$

⇒ Application

1) Determination of wavelength

Airfilm is illuminated by monochroma light the microscope is adjusted till circular rings come into focus. Now turning the screw, microscope is move towards one side. tangential of ~~fifth~~ 5th, 10th, 15th ...... Dark rings are noted. the microscope is move to the other side and again the position of 5th, 10th, 15th, ... ~~that~~ Dark rings are noted. the difference b/w these two ridings gives the diameter.



Diameter of dark fring

$$D_m = 2\sqrt{m\lambda R}$$

$$D_m^2 = 4m\lambda R$$

$$D_{m+p}^2 = 4(m+p)\lambda R$$

$$D_{m+p}^2 - D_m^2 = 4\lambda R(m+p-m)$$

$$D_{m+p}^2 - D_m^2 = 4P\lambda R$$

$$\boxed{\lambda = \frac{D_{m+p}^2 - D_m^2}{4PR}}$$

2) Determination of refractive index of liquid

we know that

$$\left[D_{m+p}^2 - D_m\right]_{air} = 4P\lambda R \quad \text{--- (1)}$$

Now A liquid whose refractive index is to be determine, is filled between the lens and plate. now the condition for dark fring becomes

Condition for dark ring is

$$2\mu t \cos r = m\lambda$$

Normal incidence, $\cos r = 1$

$$2\mu t = m\lambda$$

$$2t = \frac{m\lambda}{\mu}$$

diameter of dark ring

$$D_m = 2\sqrt{2Rt}$$

$$\boxed{[D_m]_{liq} = 2\sqrt{\frac{m\lambda R}{\mu}}}$$

$$[D_m]^2_{liq} = \frac{4m\lambda R}{\mu}$$

$$[D_{m+p}]^2_{liq} = \frac{4(m+p)\lambda R}{\mu}$$

$$[D_{m+p}^2 - D_m^2]_{liq} = \frac{4p\lambda R}{\mu} \quad \text{--- (2)}$$

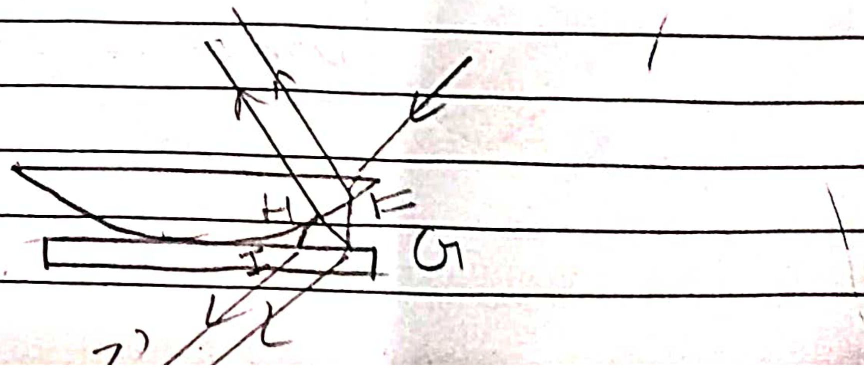
Dividing eq (1) by (2)

$$\boxed{\frac{[D_{m+p}^2 - D_m^2]_{air}}{[D_{m+p}^2 - D_m^2]_{liq}} = \mu}$$

the diameter of the ring decreases when liquid is filled between lens and plate.

Newton's Ring by transmitted light

A monochromatic light being is transmitted along G, it partialy reflected at G and H and finaly transmit along I_2". the ray" - Suffers reflection two times from denser medium before coming out



Path diff = $2\mu t \cos r - \frac{\lambda}{2} + \frac{\lambda}{2}$

$$\Delta = 2\mu t \cos r$$

normal incidence, $\cos r = 1$

$$\boxed{\Delta = 2\mu t}$$

for bright ring

$$2\mu t = n\lambda$$

for dark ring

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

<u>Diameter of rings</u>

we know that $r_n^2 = 2Rt$

For ~~dark ring~~ bright ring

$$2\mu t = n\lambda$$

$$2t = \frac{n\lambda}{\mu}$$

$$r_n^2 = \frac{n\lambda R}{\mu}$$

$$r_n = \sqrt{\frac{n\lambda R}{\mu}}$$

$$\boxed{D_n = 2\sqrt{\frac{n\lambda R}{\mu}}}$$

for air film $\mu = 1$

$$\boxed{D_n = 2\sqrt{n\lambda R}}$$

for dark ring

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

$$2\mu = \frac{(2n+1)\lambda}{2\mu}$$

$$r_n = \sqrt{\frac{(2n+1)\lambda R}{2\mu}}$$

$$\boxed{D_n = 2\sqrt{\frac{(2n+1)\lambda R}{2\mu}}}$$

For air

$$\boxed{D_n = 2\sqrt{\frac{(2n+1)\lambda R}{\mu}}}$$

⇒ the ring pattern observe in reflected light is exactly complementary to those seen in transmitted light.

Q) effect of placing mirror in place of glassplate.

Ans) the ring would disappear and uniform brightness is observed because transmitted rays will also be reflected from mirror. two complementary & rings pattern will overlap.

Q) Effect of increasing distance between lens and plate

Ans) the ring come closer to each

other until the can we longer be separately observed.

Q) Effect of including liquid between lens and plate.

Ans) diameter of the ring decreases because $D_m = 2\sqrt{m\lambda R/\mu}$.

Q) Effect of small radius of curvature.

Ans) Since $D_m = 2\sqrt{m\lambda R}$ therefore R is ~~Cat~~ small Dm also small.

Q) Effect of placing plano-concave lens.

Ans) fringes are circular but order of the ring is reverse.

Numericals

Q-1) Newton's ring are observed in reflected light of wavelength 5900 Å the diameter of the 10th dark ring is 0.5 cm. find the Radius of Curvature of lens and thickness of airfilm.

Sol
$$\lambda = 5900\text{Å} = 5900 \times 10^{-8}\text{ cm}$$
$$m = 10 \quad , \quad D_m = 0.5\text{ cm}$$

$$D_m = 2\sqrt{m\lambda R}$$
$$D_m^2 = 4m\lambda R$$
$$R = \frac{D_m^2}{4m\lambda} = \frac{0.25}{4 \times 10 \times 5900 \times 10^{-8}}$$
$$= 1.059 \times 10^{-2}\text{ cm}$$

Condition for dark

$$2t = m\lambda$$

$$t = \frac{10 \times 5900 \times 10^{-8}}{2}$$

$$= 2950 \times 10^{-7} \text{ cm}$$

Q-2) A Newton's ring experiment the diameter of 15th ring was found to be 0.590 cm and that of 5th ring is 0.336 cm if the radius of plano-convex lens is 100 cm Calculate the wave length of light used

Sol

$$D_{15} = 0.590 \text{ cm}$$

$$D_5 = 0.336 \text{ cm}$$

$$R = 100 \text{ cm}$$

$$m+p = 15 \quad , m = 5$$

$$P = 10$$

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4PR}$$

$$\lambda = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= \frac{0.2352}{4000}$$

$$= 5880 \times 10^{-5} \text{ cm}$$

Q-3) A ~~~~ ~~~~ Newton's Ring experiment the diameter of 4th and 12th dark rings are 0.4 and 0.7 cm respectively find the diameter of 20th dark ring.

Sol $D_4 = 0.4$ cm

$D_{12} = 0.7$ cm

$D_{12}^2 - D_4^2 = 4 \times 8 \lambda R = 32 \lambda R$ --- ①

$D_{20}^2 - D_{12}^2 = 4 \times 8 \lambda R = 32 \lambda R$ --- ②

$D_{20}^2 - D_{12}^2 = D_{12}^2 - D_4^2$

$D_{20}^2 = 2 D_{12}^2 - D_4^2$

$D_{20}^2 = 2 \times 0.49 - 0.16$

$= 0.98 - 0.16$

$= 0.82$

$D_{20} = \sqrt{0.82}$

$D_{20} = 0.905$ cm

Q-4) Newton's Rings are formed in reflected ~~~~ light of ~~~~ wavelength 6000 Å with a liquid between plane and curved surface if the diameter of 5th bright ring is 3.1 mm and the radius of curvature is 1m Calculate the refractive index of liquid.

Sol $\lambda = 6000 \times 10^{-8}$ cm , $n = 5$

$D_n = 3.1$ mm $= 0.31$ cm

$R = 1m = 100 cm$

$$D_n = 2\sqrt{\frac{(2n+1)\lambda R}{2\mu}}$$

$$D_n^2 = \frac{4(2n+1)\lambda R}{2\mu}$$

$$\mu = \frac{(2n+1)\lambda R}{D_n^2}$$

$$\mu = \frac{2(10+1) \times 6000 \times 10^{-8} \times 100}{0.31}$$

$$\mu = \frac{11 \times 6000 \times 100 \times 10^{-8}}{0.31}$$

$$\mu = 0.21$$

Dif