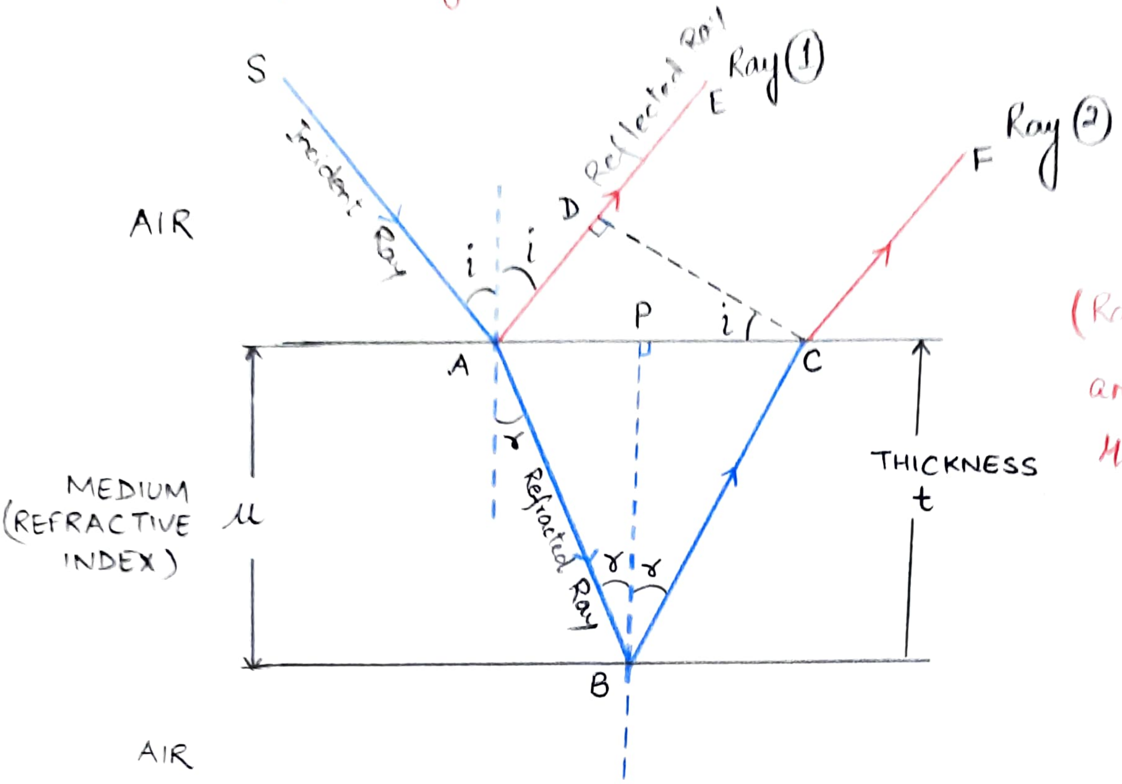
INTERFERENCE IN THIN FILM OF UNIFORM THICKNESS

Case 1 : Reflected light



(Ray 1 and Ray 2 are two successive reflected rays)

Path difference between Ray ① and Ray ② is:

$$\Delta = (AB + BC) \text{ in medium} - AD \text{ in air} \quad -①$$

$$\boxed{\Delta = \mu(AB + BC) - AD} \quad -②$$

In Δ ABP

$$\cos r = \frac{BP}{AB} = \frac{t}{AB} \quad -③$$

$$\Rightarrow \boxed{AB = \frac{t}{\cos r}} \quad -④$$

Similarly, in Δ BPC

$$\cos r = \frac{BP}{CB} = \frac{t}{BC}$$

$$\Rightarrow \boxed{BC = \frac{t}{\cos r}} \quad -⑤$$

Put eqns ④ & ⑤ in eqn ②

$$\therefore \Delta = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - AD$$

$$\boxed{\Delta = \frac{2\mu t}{\cos r} - AD} \quad -⑥$$

To find AD, consider Δ ADC

$$\sin i = \frac{AD}{AC} = \frac{AD}{AP+PC} \quad -(7)$$

$$\Rightarrow \quad \boxed{AD = (AP + PC)\sin i} \quad -⑧$$

Again, consider Δ APB and Δ PCB

$$\tan r = \frac{AP}{PB} = \frac{AP}{t}$$

$$\Rightarrow \quad AP = t\tan r \quad -⑨$$

Also, $$\tan r = \frac{PC}{PB} = \frac{PC}{t}$$

$$\Rightarrow \quad PC = t\tan r \quad -⑩$$

Put eq^ns ⑨ & ⑩ in ⑧; we get

$$AD = (t\tan r + t\tan r)\sin i$$

or

$$AD = 2t.\tan r.\sin i \quad -⑪$$

Put eq^n ⑪ in [illegible] ⑥

$$\Rightarrow \quad \boxed{\Delta = \frac{2\mu t}{\cos r} - 2t.\tan r.\sin i} \quad -⑫$$

Does not contain μ term

This term contains μ term

Hence, From Snell's law

$$\frac{\sin i}{\sin r} = \mu \quad -⑬$$

$$\Rightarrow \quad \sin i = \mu\sin r \quad -⑭$$

Put eq^n ⑭ in ⑫

$$\therefore \quad \Delta = \frac{2\mu t}{\cos r} - 2t.\tan r(\mu\sin r)$$

$$\Delta = \frac{2\mu t}{\cos r} - 2t\mu\left(\frac{\sin r}{\cos r}\right).\sin r$$

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r} \cdot \sin^2 r$$

$$\Delta = \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right] = \frac{2\mu t}{\cos r} \cdot \cos^2 r$$

$$\boxed{\Delta = 2\mu t \cos r} \quad -⑮$$

* According to Stoke's treatment, when a beam of light travels from Rarer to Denser medium it suffers a path difference of $\pm\frac{\lambda}{2}$.

No path difference change takes place when beam of light travels from Denser to Rarer medium.

Thus, applying Stoke's treatment eqⁿ ⑮ becomes

✓ Path difference $\boxed{\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}}$ –⑯

(a) Conditions for Maxima (Bright Fringe)

For Constructive interference, we know that

$$\boxed{\Delta = n\lambda} \quad -⑰$$

Comparing eqⁿˢ ⑯ & ⑰ we get;

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2\mu t \cos r = n\lambda - \frac{\lambda}{2}$$

Maxima

$$\boxed{2\mu t \cos r = (2n-1)\frac{\lambda}{2}} \quad -⑱$$ Here, n = 1,2,3 & so on

Note: If you take; $\Delta = 2\mu t \cos r - \frac{\lambda}{2}$ [from eqⁿ ⑯] and compare with eqⁿ ⑰ then

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

or $\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$ (Here, n = 0,1,2. . .)

(b) Conditions for Minima (Dark Fringe)

For Destructive Interference ;

$$\Delta = (2n+1)\frac{\lambda}{2} \quad \text{— ⑲}$$

or

$$(2n-1)\frac{\lambda}{2}$$

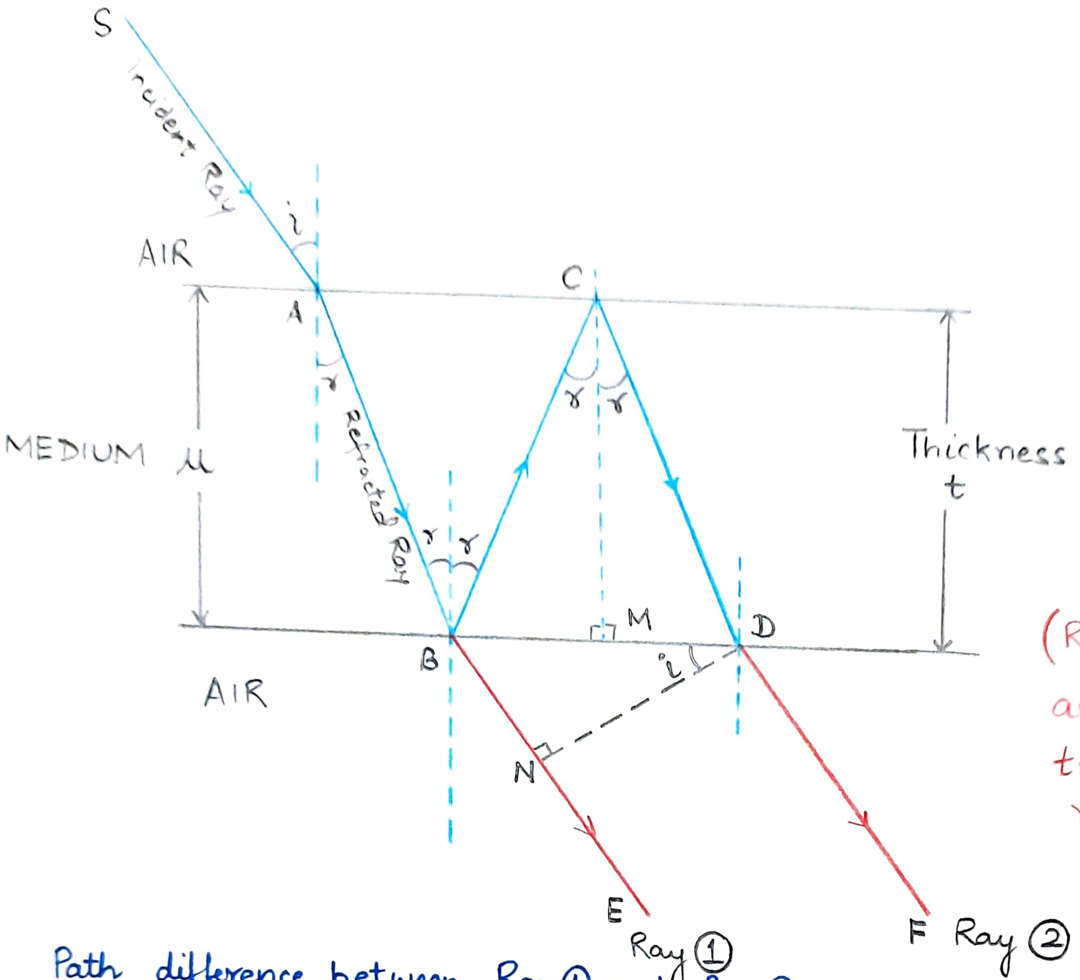
Comparing eqns ⑯ & ⑲, we get

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \quad 2\mu t \cos r = (2n+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$\Rightarrow \quad \boxed{2\mu t \cos r = n\lambda} \quad \text{— ⑳}$$ Minima

Case 2 : Refracted or Transmitted light



(Ray 1 and Ray 2 are two successive transmitted (or Refracted rays)

Path difference between Ray ① and Ray ②

$\Delta = (BC + CD)$ in medium $-$ BN in air —①

$$\boxed{\Delta = \mu(BC + CD) - BN} \quad \text{—②}$$

In ΔBCM and ΔCMD

$$\cos\gamma = \frac{CM}{BC} = \frac{t}{BC}$$

$$\Rightarrow \boxed{BC = \frac{t}{\cos\gamma}} \quad \text{—③}$$

Similarly, $\cos\gamma = \dfrac{CM}{CD} = \dfrac{t}{CD}$

$$\Rightarrow \boxed{CD = \frac{t}{\cos\gamma}} \quad \text{—④}$$

Put eqns ③ & ④ in ②

$$\Delta = \mu\left(\frac{t}{\cos\gamma} + \frac{t}{\cos\gamma}\right) - BN \quad \text{—⑤}$$

$$\boxed{\Delta = \frac{2\mu t}{\cos\gamma} - BN} \quad \text{—⑥}$$

For finding BN, consider ΔBND

$$\sin i = \frac{BN}{BD} = \frac{BN}{BM + MD}$$

$$\Rightarrow \boxed{BN = (BM + MD)\sin i} \quad -⑦$$

Again, In ΔBCM and ΔCMD

$$\tan r = \frac{BM}{CM} = \frac{BM}{t}$$

$$\Rightarrow \boxed{BM = t.\tan r} \quad -⑧$$

Similarly, $\tan r = \frac{MD}{CM} = \frac{MD}{t}$

$$\Rightarrow \boxed{MD = t\cdot\tan r} \quad -⑨$$

Put eqns ⑦, ⑧ & ⑨ in ⑥ ; we get

$$\boxed{\Delta = \frac{2\mu t}{\cos r} - 2t.\tan r.\sin i} \quad -⑩$$

Using Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \sin i = \mu \sin r \quad -⑪$$

Put eqn ⑪ in ⑩

$$\Delta = \frac{2\mu t}{\cos r} - 2t.\tan r.(\mu \sin r)$$

$$= \frac{2\mu t}{\cos r} - 2\mu t\frac{\sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}\left(1 - \sin^2 r\right)$$

$$\therefore \boxed{\Delta = 2\mu t \cos r} \quad -⑫$$

Applying Stoke's treatment, no phase or path change will occur hence,

$$\Delta = 2\mu t \cos r \pm 0$$

Path difference $\boxed{\Delta = 2\mu t \cos r}$ $-⑬$

(a) Condition for Maxima (Bright Fringe)

For Constructive Interference, we know that

$$\boxed{\Delta = n\lambda} \qquad -\text{⑭}$$

Comparing eqⁿˢ ⑬ & ⑭

$$\boxed{2\mu t\ \cos r = n\lambda} \qquad \text{Maxima} \quad -\text{⑮}$$

(b) Condition for Minima (Dark Fringe)

For Destructive Interference, we know that

$$\boxed{\Delta = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad (2n-1)\frac{\lambda}{2}} \qquad -\text{⑯}$$

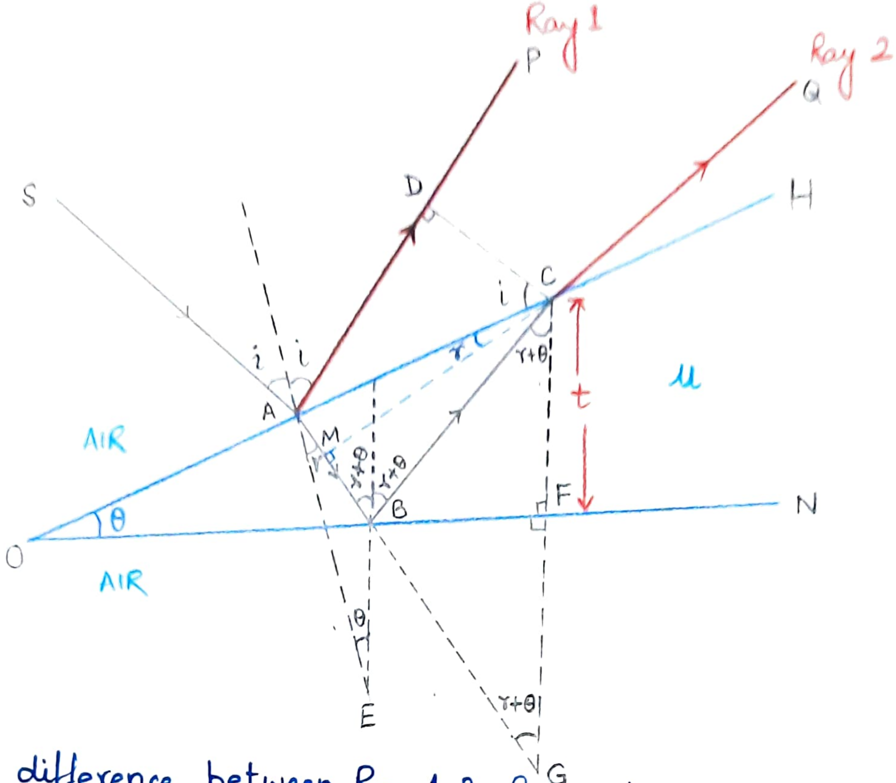
Comparing eqⁿˢ ⑬ & ⑯ we get

$$\boxed{\begin{array}{c} 2\mu t\ \cos r = (2n+1)\dfrac{\lambda}{2} \\ \text{or} \\ 2\mu t\ \cos r = (2n-1)\dfrac{\lambda}{2} \end{array}} \quad \begin{array}{l} \text{Here } n = 0, 1, 2, \ldots. \\ -\text{⑰} \\ \text{Here } n = 1, 2, 3, \ldots. \end{array}$$

"Hence, it is quite clear that the interference pattern due to reflected and transmitted light are complementary to each other."

Interference in Thin Film of Non-uniform thickness

WEDGE-SHAPED INTERFERENCE



Path difference between Ray 1 & Ray 2 is

$$\Delta = (AB + BC) \text{ in medium} - AD \text{ in air} \quad -①$$

$$\boxed{\Delta = \mu(AB + BC) - AD} \quad -②$$

From Snell's law, we have

$$\frac{\sin i}{\sin r} = \mu \quad -③$$

In ΔADC

$$\sin i = \frac{AD}{AC} \quad -④$$

In ΔAMC

$$\sin r = \frac{AM}{AC} \quad -⑤$$

Put eqns ④ & ⑤ in ③

$$\frac{\left(\frac{AD}{\cancel{AC}}\right)}{\left(\frac{AM}{\cancel{AC}}\right)} = \mu \Rightarrow \boxed{AD = \mu AM} \quad -⑥$$

Put eqn ⑥ in ②

$$\Delta = \mu(AB+BC) - \mu AM \quad -⑦$$

Since,

$$AB = AM + MB$$

$$\therefore \quad \Delta = \mu(\cancel{AM} + MB + BC) - \mu \cancel{AM}$$

$$\Rightarrow \boxed{\Delta = \mu(MB+BC)} \quad -⑧$$

Also, ∵ ΔBCF is congruent to ΔBGF

$$\therefore \boxed{BC = BG} \quad -⑨$$

$$\& \quad \boxed{CF = FG = t} \quad -⑩$$

Put eqn ⑨ in ⑧

$$\Delta = \mu(MB + BG)$$

$$\boxed{\Delta = \mu(MG)} \quad -⑪$$

Consider ΔMGC

$$\cos(r+\theta) = \frac{MG}{CG} = \frac{MG}{2t} \qquad (\text{from eq}^n\ 10)$$

$$\Rightarrow \boxed{MG = 2t\cos(r+\theta)} \quad -⑫$$

Put eqn ⑫ in ⑪

$$\boxed{\Delta = 2\mu t\cos(r+\theta)} \quad -⑬$$

Applying Stoke's treatment; Total path difference will be

✓ $$\boxed{\Delta = 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2}} \quad -⑭$$

(a) Condition for Maxima

For Constructive Interference, $\boxed{\Delta = n\lambda}$ –⑮

Comparing eqns ⑭ & ⑮

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \boxed{2\mu t\cos(r+\theta) = (2n-1)\frac{\lambda}{2}} \quad -⑯$$

where $n = 1, 2, 3 \ldots$

(b) Condition for Minima

For Destructive Interference; $\Delta = (2n\pm1)\frac{\lambda}{2}$ –⑰

Comparing eqns ⑭ & ⑰ we get

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t\cos(r+\theta) = n\lambda} \quad -⑱$$

NEWTON's RING - Construction & Working

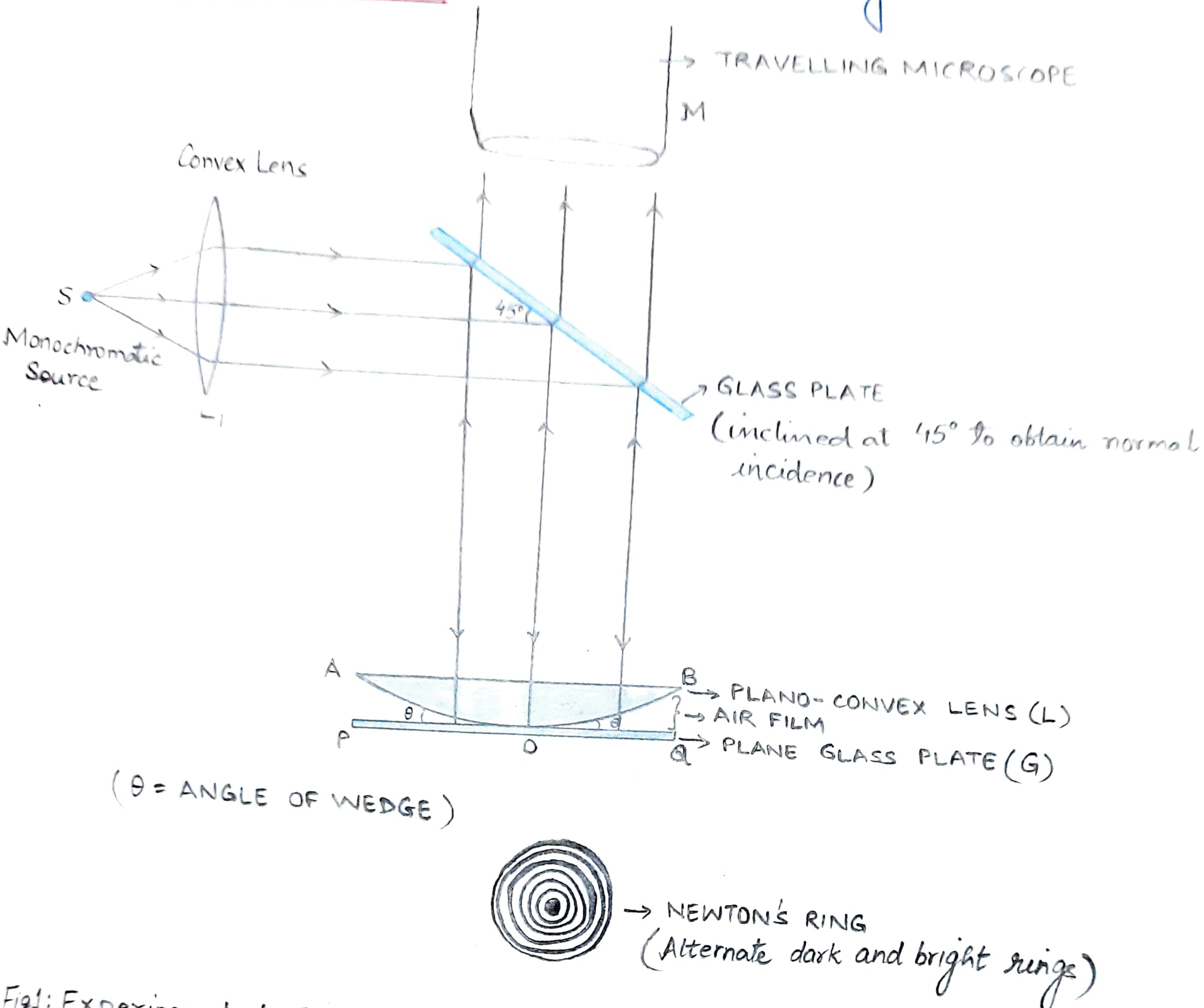
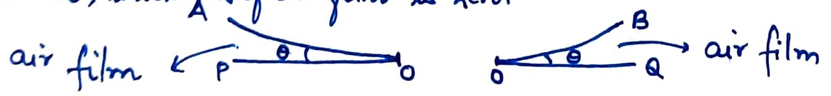


Fig 1: Experimental Set-up

When a plano-convex lens L is placed on a glass plate G, then air film of gradually increasing thickness is formed between the two surfaces. At the point of contact O, thickness of air film is Zero.



When a beam of monochromatic light is incident normally on a combination of plano-convex lens L and glass plate G (i.e air film), an alternate dark and bright circular fringes are formed. These circular rings are formed because of the interference between the reflected rays from the top and bottom surfaces of the air film. These rings are called Newton's ring.

Newton's ring are circular because the air film has a circular symmetry. The thickness of the air film corresponding to each fringe is same throughout the circle.

Q. 'Centre of Newton's Ring generally appears dark.' Why?

Path difference in Wedge shaped film is

$$\boxed{\Delta = 2\mu t \cos(r+\theta) \pm \frac{\lambda}{2}} \quad -①$$

In Newton's Ring, at normal incidence $i = r = 0$

Also, for air film $\mu = 1$

$$\therefore \quad \Delta = 2 \cdot 1 \cdot t \cos(0+\theta) \pm \frac{\lambda}{2}$$

Since wedge angle θ is very small

hence $\cos\theta = 1$

$$\therefore \quad \boxed{\Delta = 2t \pm \frac{\lambda}{2}} \quad -②$$

At the centre (i.e. pt. of contact o)

$$t = 0$$

$$\therefore \quad \boxed{\Delta = \pm\frac{\lambda}{2}} \quad -③$$

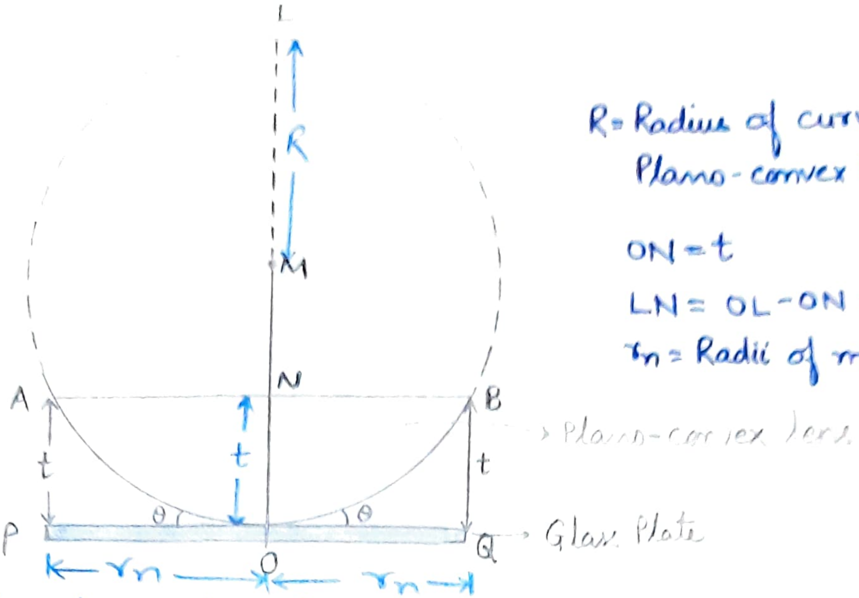
As we know that for minima; condition is

$$\Delta = (2n \pm 1)\frac{\lambda}{2} \quad -④$$

For $n = 0$; $\Delta = \pm\frac{\lambda}{2}$ [similar to eq^n ③]

Hence, centre of Newton's Ring is a minima or Dark spot.

Diameter of Newton's Ring



R = Radius of curvature of Plano-convex lens

ON = t

LN = OL − ON = 2R − t

r_n = Radii of nth ring

From the property of chord (AB)

$$AN \times NB = ON \times NL \quad -①$$

$$r_n \times r_n = t \times (2R - t) \quad -②$$

$$r_n^2 = 2Rt - t^2 \quad -③$$

Since t is very small, hence t^2 is negligible

$$\therefore \quad r_n^2 = 2Rt$$

$$\Rightarrow \boxed{t = \frac{r_n^2}{2R}} \quad -④$$

If D_n is the diameter of Ring, then

$$D_n = 2r_n \quad -⑤$$

$$\Rightarrow \quad t = \frac{D_n^2}{4 \cdot 2R}$$

$$\Rightarrow \boxed{t = \frac{D_n^2}{8R}} \quad -⑥$$

(a) Diameter of nth Bright Ring

For Wedge film, $\Delta = 2\mu t \cos(r+\theta) \pm \frac{\lambda}{2}$

In Newton's Ring, at normal incidence $i = r = 0$ and wedge angle θ is very small

$$\boxed{\Delta = 2\mu t \pm \frac{\lambda}{2}} \quad -⑦$$

For maxima, $\Delta = n\lambda$ – ⑧

from eqⁿ ⑦ & ⑧ we get

$$2\mu t + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \boxed{2\mu t = (2n-1)\frac{\lambda}{2}}$$ Condition for Maxima – ⑨

Put eqⁿ ⑥ in ⑨

$$\cancel{2}\mu \frac{D_n^2}{4\cancel{8}R} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{D_n^2 = \frac{2(2n-1)\lambda R}{\mu}}$$ – ⑩

For Air film, $\mu = 1$ then

$$D_n^2 = 2(2n-1)\lambda R$$

$$\Rightarrow \boxed{\begin{array}{l} D_n = \sqrt{2(2n-1)\lambda R} \\ \text{or} \\ D_n \propto \sqrt{(2n-1)} \end{array}}$$ – ⑪ – ⑫ where $n = 1, 2, 3 \ldots\ldots$

$\Rightarrow D_1 : D_2 : D_3 : D_4 \ldots\ldots = \sqrt{1} : \sqrt{3} : \sqrt{5} : \sqrt{7} \ldots\ldots$

Hence, diameter of bright rings are proportional to the square root of odd natural numbers.

(b) Diameter of nth Dark Ring

For minima, $\Delta = (2n \pm 1)\frac{\lambda}{2}$ – ⑬

Comparing eqⁿˢ ⑦ & ⑬

$$2\mu t \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\boxed{2\mu t = n\lambda}$$ – ⑭

Put eqⁿ ⑥ in ⑭

$$\cancel{2}\mu \frac{D_n^2}{4\cancel{8}R} = n\lambda$$

$$\boxed{D_n^2 = \frac{4n\lambda R}{\mu}}$$ – ⑮

For Air film, $\mu = 1$ then

$$D_n^2 = 4n\lambda R$$

$$\boxed{\begin{array}{l} D_n = \sqrt{4n\lambda R} \quad (16) \\ \Rightarrow D_n \propto \sqrt{n} \quad (17) \end{array}}$$

$$D_1 : D_2 : D_3 : D_4 \cdots = \sqrt{1} : \sqrt{2} : \sqrt{3} : \sqrt{4} \cdots$$

Hence, diameter of dark rings are proportional to the square root of natural numbers.

APPLICATIONS OF NEWTON'S RING

① To determine Wavelength of Monochromatic light (λ)

Diameter of nth dark ring $= D_n^2 = 4n\lambda R$ — ①

Diameter of (n+p)th dark ring $= D_{n+p}^2 = 4(n+p)\lambda R$ — ②

Subtract eqⁿ ① from ②

$$D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\Rightarrow \boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}} \quad \text{— ③}$$

Here, p = any integer

R = Radius of curvature

② To determine Refractive Index of a liquid

For Air film, $\mu = 1$; Diameter of nth dark ring is:

$$(D_n^2)_{air} = 4n\lambda R \quad \text{— ①}$$

For any medium of refractive index, μ

diameter of nth dark ring is:

$$(D_n^2)_{med.} = \frac{4n\lambda R}{\mu} \quad \text{— ②}$$

From eqⁿˢ ① & ② we get

$$\boxed{\mu = \frac{(D_n^2)_{air}}{(D_n^2)_{med.}}} \quad \text{— ③}$$