Module-1

2-D of force systems.

Q What is Mechanics?

A The science which deals with the state of rest and state of motion of the body (rigid bodies) under the action of forces is called mechanics.

A knowledge of its basic concepts and principles is must for engineers engaged in the design and the construction of various types of mechanics and their structure

Classification of Mechanics:

It is divided into two types are —

Engineering Mechanics.

- STATICS (Physical science of body at rest)
- DYNAMICS. (Physical science of body in motion.)
  - KINEMATICS. (Physical science of body in motion, independent of force acting on it.)
  - KINETICS. (Physical science of motion and forces acting on it).

Force: A push or pull acting on a body is called force.

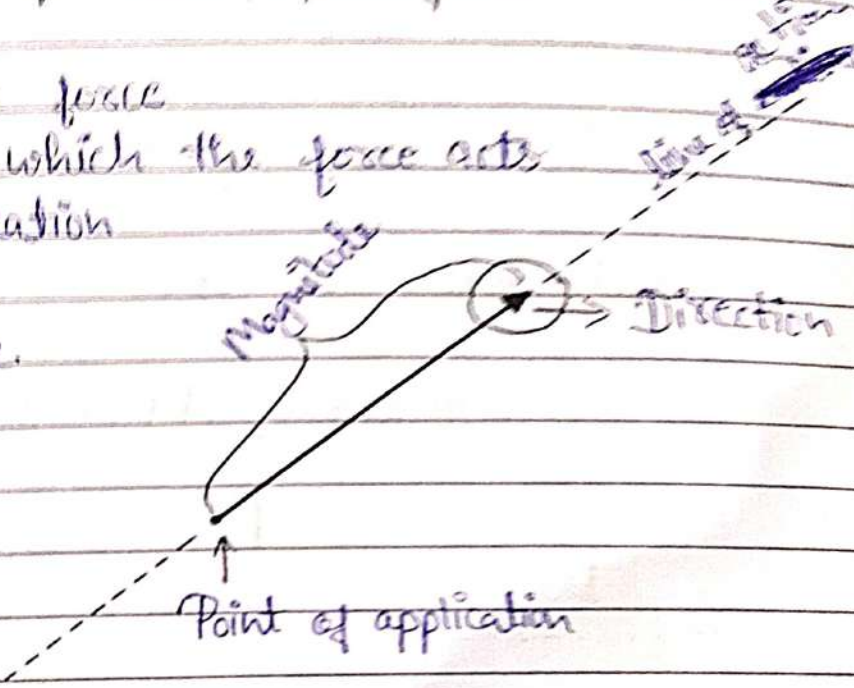
Or

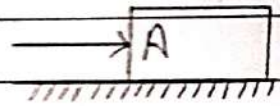
Something which tends to change the state of a body is called a force.

SI unit = N (Newton), Dimensions = $[MLT^{-2}]$

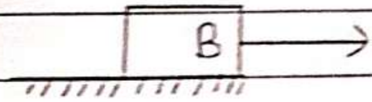
→ To defined a force completely we need—

(i) Magnitude of the force
(ii) Direction, along which the force acts
(iii) Point of Application
(iv) Line of Action
(v) Nature of force.





Push force or compressive force



Pull force or tensile force.

* Types of forces :-

(i) Applied Forces :- Applied forces are those forces which are applied externally to a body.

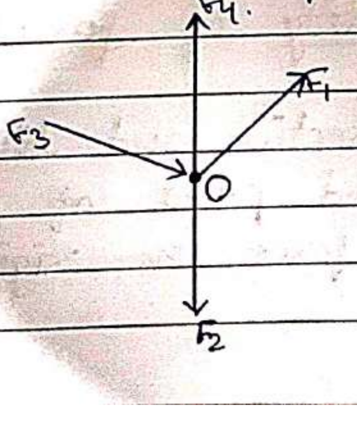
(ii) Non- Applied Force :- Non- applied forces are the reactive forces such as tension or pull forces is called forces in rod.

* Types of Rod forces.

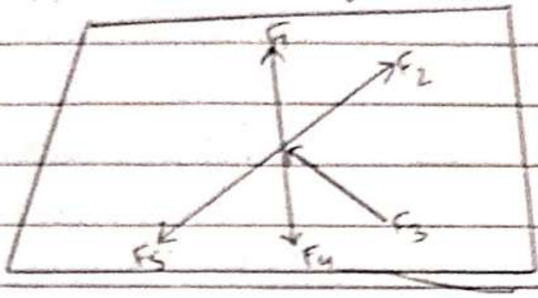
Rod forces.

- Concurrent forces (force which meet at a one point)
  - Coplanner
  - Non- Coplanner.
- Non- Concurrent forces. (force which do not meet at one point).

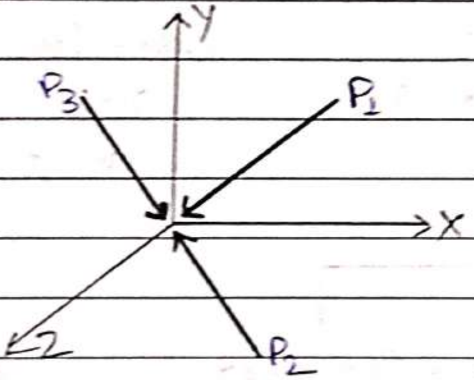
(a) Concurrent forces :- When all the forces passing through a single point then that force system is called a concurrent forces.



(i) Coplanner Concurrent forces:- The force which meet at one point and their line of action also lie on the same plane are known as coplanner concurrent forces.



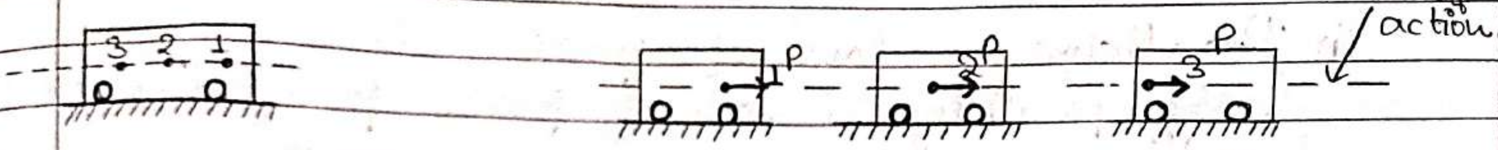
(ii) Non-Coplanner Concurrent forces:- The force which meet at one point but the line of action do not lie on the same plane are known as non-coplanner concurrent forces



Rigid Body:- A body does not applicable of application of force is called a rigid body.

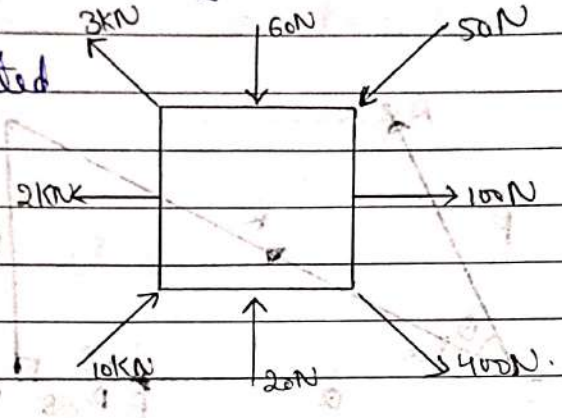
Laws of Transmissibility of forces:-

If the magnitude, direction and line of action of a force remains same and point of application change then effect of this force on rigid body will remain same



Resultant of forces:

Resultant forces:- Resultant forces is a "single force" which can produce the same effect as it is produced by the number of forces acting on the body.
It is represented by "R".

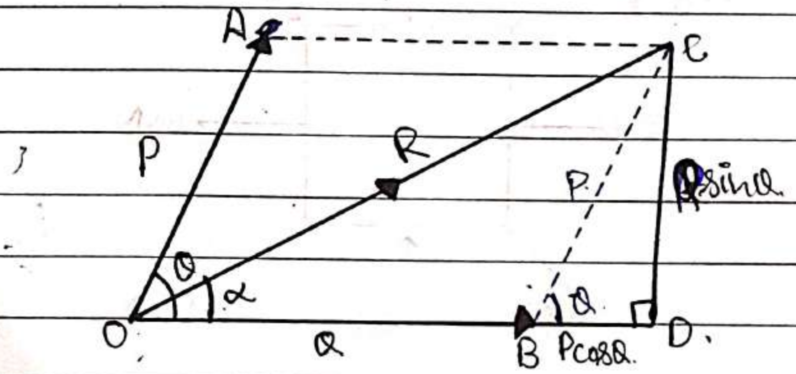


Methods to find Resultant (R):

- Analytical Method
  - Trignometric method (Law of ||gram)
  - Algebric method. (Method of Resolution).
- Graphical method.

(i) Parallelogram law of forces
(Using parallelogram law of forces for finding resultant force).

Parallelogram law of forces states that, "if two forces acting at and away from the point be represented in magnitude and direction by the two adjacent sides of parallelogram, then the diagonal of parallelogram passing through the point of intersection of the two forces represents the resultant in magnitude and direction."



In right ΔCOD.

$\sin\theta = \frac{Per}{Hyp} = \frac{CD}{P}$

$CD = P\sin\theta$

$\cos\theta = \frac{BD}{P}$

$BD = P\cos\theta$

$(OC)^2 = (OD)^2 + (CD)^2$

$(R)^2 = (P\cos\theta + Q)^2 + (P\sin\theta)^2$

$R^2 = Q^2 + P^2\cos^2\theta + 2PQ\cos\theta + P^2\sin^2\theta$

$R^2 = P^2(\sin^2\theta + \cos^2\theta) + Q^2 + 2PQ\cos\theta$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$\tan\alpha = \frac{P\sin\theta}{Q + P\cos\theta}$

$$\boxed{\alpha = \tan^{-1}\left(\frac{P\sin\theta}{Q + P\cos\theta}\right)}$$

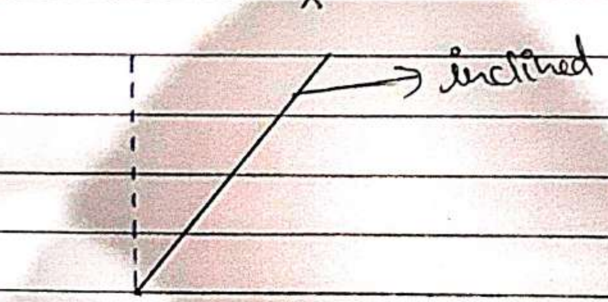
Q A block weighting $W = 10kN$ is resting on an inclined plane as shown in fig. Determine its components normal to & Parallel to the inclined plane.

Normal component = $W\cos 20° = 10\cos 20°$
$= 9.39kN.$

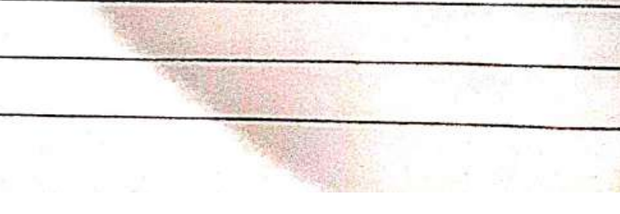
Parallel component
$= W\sin 20° = 10\sin 20°.$
$= 3.4202kN.$

(ii) Resolution of a force into a component forces -

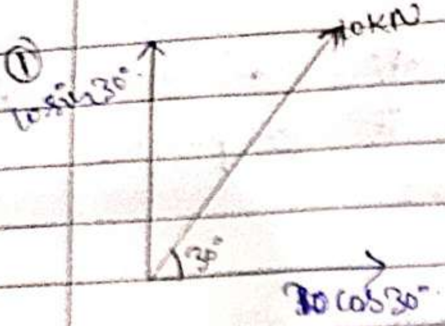
The process of spilliting up the given force into two components, without changing its effect on the body is called a resolution of force.
let us know with the help of figure.



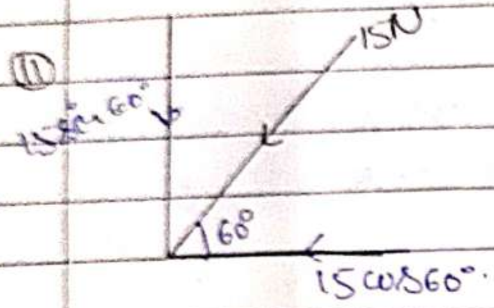
The summation of horizontal force is stands for Σfx. and vertically forces stands for Σfy



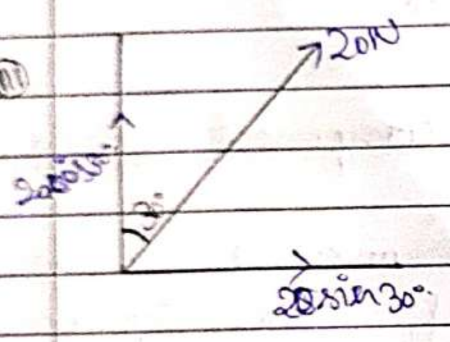
Inclined forces are divided into two parts because this force doesn't exist the summation.

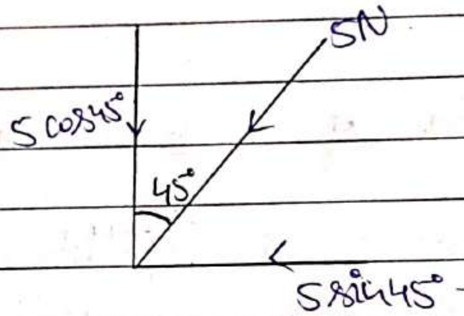
(i) 

In which line of axis the angle is given then we will written a cos. The direction is ~~ou~~ outwards

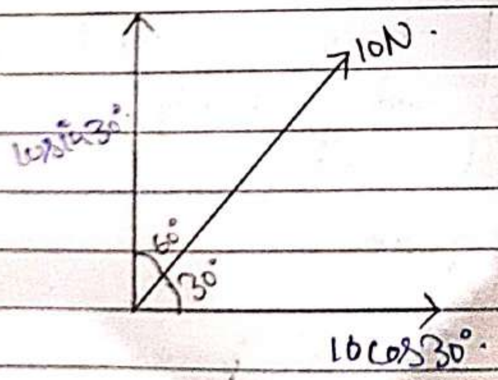
(ii) 

The direction is inwards.

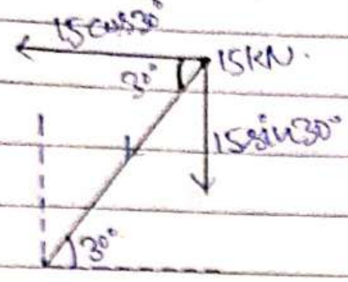
(iii) 



(iv) When the ~~given~~ figure have two angle then we can choose only one angle.



(v) If we change point of application



Using of method of resolution of forces for finding the resultant force.

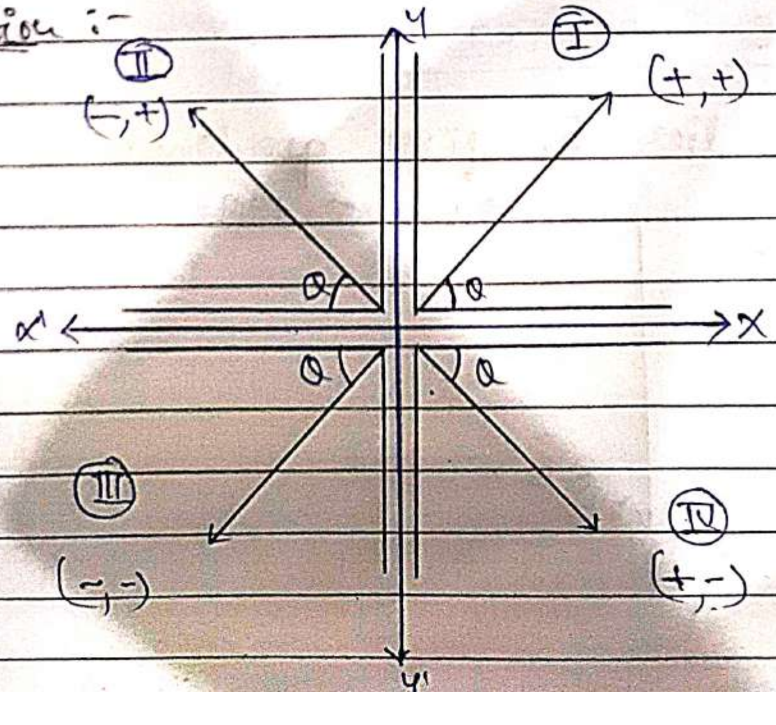
(i) ~~Direction~~ Magnitude → The resultant R of the given forces will be by the equation–

$$R = \sqrt{\Sigma fx^2 + \Sigma fy^2}$$

(ii) Direction → The resultant force will be inclined at the angle of θ, with the horizontal, such that

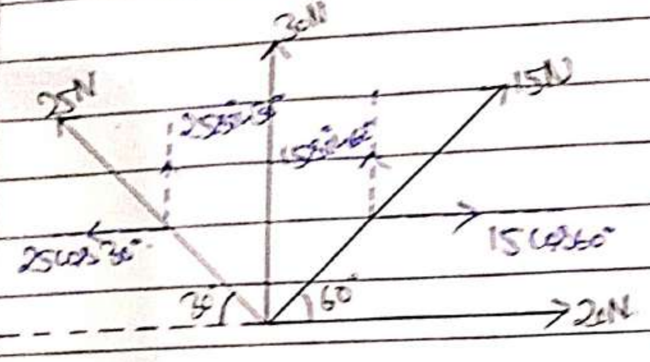
$$\tan\theta = \frac{\Sigma fy}{\Sigma fx} \Rightarrow \theta = \tan^{-1}\left|\frac{\Sigma fy}{\Sigma fx}\right|.$$

(iii) ~~Ratio~~ Position :-



Related Questions

Q



$\xrightarrow{+} \Sigma f_x = 20 + 15\cos 60° - 25\cos 30°$
$= 5.849\,N$

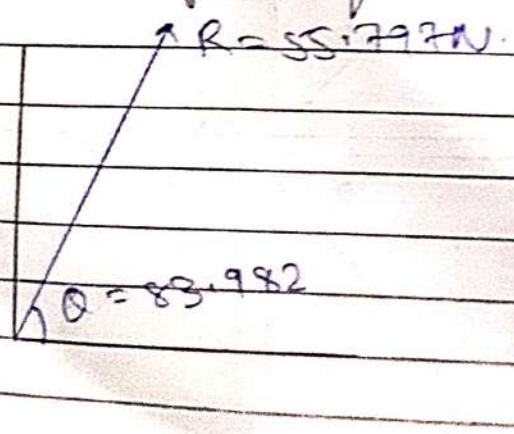
$\uparrow + \Sigma f_y = 30 + 15\sin 60° + 25\sin 30°$
$= 55.49\,N$

$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2}$
$= \sqrt{(5.849)^2 + (55.49)^2}$
$= 55.797\,N$

$\theta = \tan^{-1}\left|\frac{\Sigma f_y}{\Sigma f_x}\right| = \tan^{-1}\left|\frac{55.49}{5.849}\right| = 83.982$

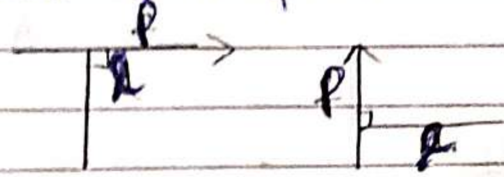
$\boxed{\theta = 83.982}$

Resultant lies in first quadrant.



Definition:- When force acts on body to produce a turning effect or rotational effect (clock or anticlockwise) is called moment of a force.

Mathematically formula-



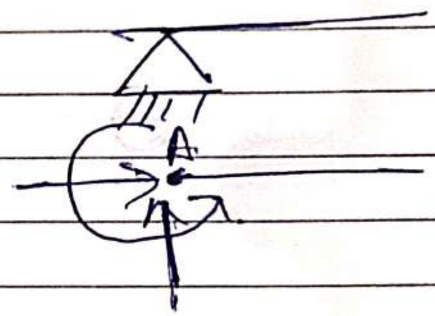
$$M = P \times l$$

where,

P = Force acting on the body.

l = Perpendicular distance btw the points about which the moment is required and the line of action of the force

Types of moments-



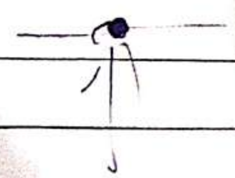
There are two types of moment-

(i) Clockwise moment

(ii) Anticlockwise moment.

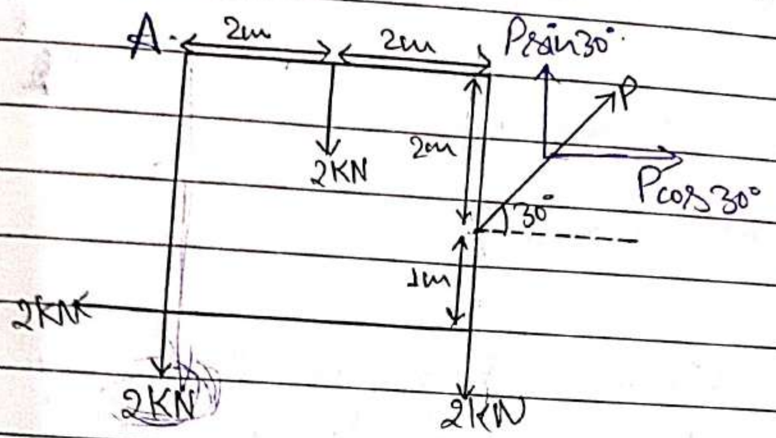
Note:(i) The general convention is to take clockwise moment as the positive and anticlockwise moment as a negative.

(ii) The moment of force at point where all forces are passing through this point is always zero.



Related Questions

Q Find the value of force P, so that moment about point A is zero as shown in figure.



Soln

Given that,

Moment of all the forces about at point A = 0.

$$\circlearrowright^{+} \sum M_A = 0.$$

$$= 2\times 2 + 2\times 4 + 2\times 3 - P\sin 30^\circ \times 4 - P\cos 30^\circ \times 2$$

$$= 18 - (4P\sin 30^\circ + 2P\cos 30^\circ) = 0$$

$$= 18 - 3.73P = 0.$$

$$+3.73P = +18$$

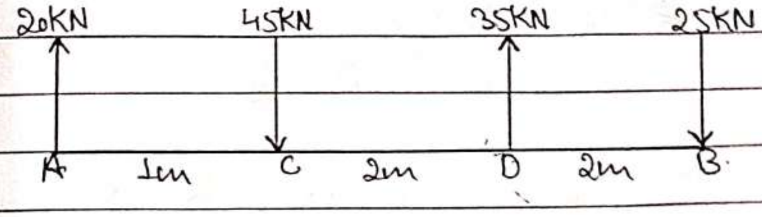
$$P = \frac{18}{3.73}$$

$$\boxed{P = 4.825\ KN}$$

Q A rigid bar AB is subjected to a system of a parallel forces as shown in figure. Reduce the given ~~resultant~~ system of forces to an equivalent.

(i) Single resultant.

(ii) Force system at D and A.



(i) $R = \sqrt{\Sigma fx^2 + \Sigma fy^2}$,

$\Sigma fx = 0.$

$\Sigma fy = 20 - 45 + 35 - 25$

$= -15$ KN.

$R = \sqrt{0 + (-15)^2}$.

$R = \sqrt{225} = 15$ KN. (Downward.

(ii) Force system at D.

$+\circlearrowright \Sigma M_D = 25 \times 2 - 45 \times 2 + 20 \times 3.$

$= 50 - 90 + 60.$

$= 20$ KN.-m.

$+\circlearrowright M_A = 45 \times 1 - 35 \times 3 + 25 \times 5$

$= 45 - 105 + 125.$

$= 170 - 105.$

$= 65$ KN-m.

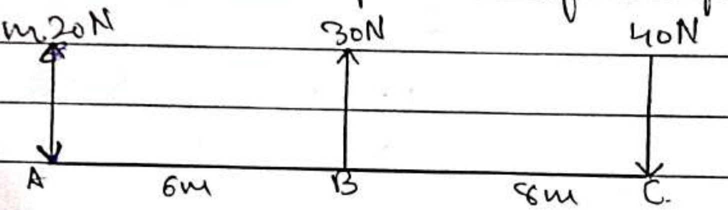
Varignon's Theorem (Law of Moments).

The algebric sum of moment of all coplanar forces is equal to moment of their resultant about same moment centre.

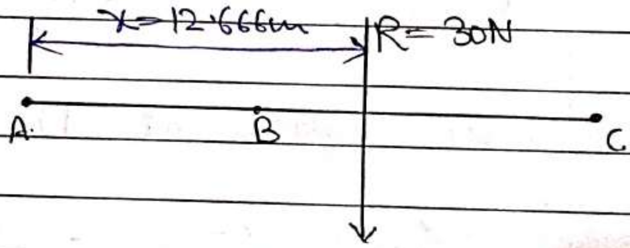
$$\Sigma M_A = R \times x.$$

Related Questions

A bar ABC carrying forces 20N at A downward, 30N at B upward and 40N at downward. Compute the resul. forces and locate its position from point A. If AB = 6m and BC = 8m.



$\xrightarrow{+} \Sigma f_x = 0.$



$+\uparrow\ \Sigma f_y = 30 - 40 - 20.$

$= -30N.$

$R = \sqrt{\Sigma f_x + \Sigma f_y^2}$

$= \sqrt{0 + 30^2}$

$\boxed{R = 30N}$ (downward).

By Varignon's theorem.

$\Sigma M_A = -30 \times 6 + 40 \times 14$

$= 380 N\text{-}m.$

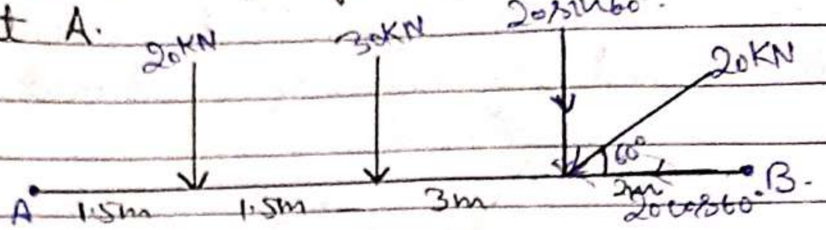
$\Sigma M_A = R \times x$

$380 = 30 \times x.$

$\frac{380}{30} = x = 12.666m$

Imp

Q Find the resultant, angle of resultant & distance of result. from point A.



$\Sigma f_x = -20\cos 60° = \boxed{-10N = \Sigma f_x}$

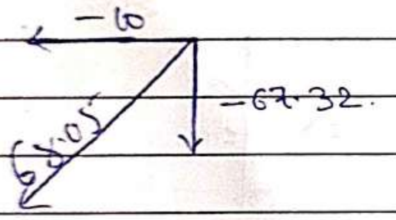
$\Sigma f_y = -20 - 30 - 20\sin 60°$

$\boxed{\Sigma f_y = -67.32N}$

$\theta = \tan^{-1}\left|\frac{\Sigma f_y}{\Sigma f_x}\right| \Rightarrow \tan^{-1}\left|\frac{-67.32}{-10}\right| = \boxed{81.55°}$

$R = \sqrt{\Sigma f_y^2 + \Sigma f_x^2} = \sqrt{4529.29 + 100} = \sqrt{4629.29}$

$= 68.05\ N.$



Resultant lies in third quadrant

By varignon's theorem

(i) $\Sigma M_A = 20 \times 1.5 + 30 \times 3 + 20\sin 60° \times 6$

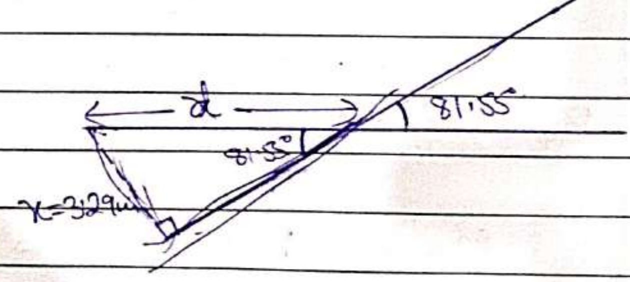
$\boxed{\Sigma M_A = 223.92\ N\text{-}m}$

$\Sigma M_A = R \times x.$

$223.92 = 68.05 \times x.$

$x = \frac{223.92}{68.05}$

$\boxed{x = 3.29m.}$



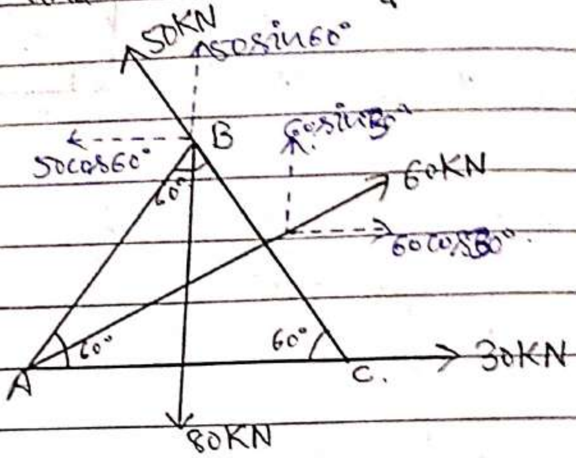
$\sin 81.55° = \frac{P}{H} = \frac{3.29}{d}$

$d = \frac{3.29}{\sin 81.55°}$

$d = \frac{3.29}{0.98}$

$\boxed{d = 3.35m}$

Imp
Q. An equilateral Δ of side 200mm is acted by four forces as shown in fig. Find resultant, angle of resultant & and location of resultant from point A.



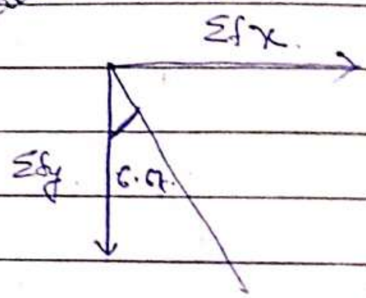
$\Sigma f_x = 30 + 60\cos 30^\circ - 50\cos 60^\circ$

$= 56.96N.$

$\Sigma f_y = -80 + 60\sin 30^\circ + 50\sin 60^\circ$

$= -6.69.$

Resultant lies in fourth quadrant.



$R = \sqrt{(56.96)^2 + (6.69)^2}$

$= \sqrt{3257.82}$

$= 57.35$

$\theta = \tan^{-1}\left|\frac{\Sigma f_y}{\Sigma f_x}\right| = \tan^{-1}\left|\frac{-6.69}{56.96}\right|$

$\theta = 6.67^\circ$

By Varignon's theorem

$\Sigma M_A = R \times x.$

$-50 \times \sqrt{3} \times 100 + 0 + 100 \times 80 = 57.35 \times d.$

$d = \frac{-8660.25 + 8000}{57.35}$

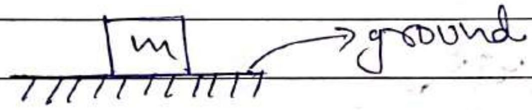
$\boxed{d = -11.51.}$

Free Body Diagrams.

Definitions :- A body is said to be free if it is free from all contact of surfaces and applications of all applied and non- applied forces on this rigid body makes it (FBD).

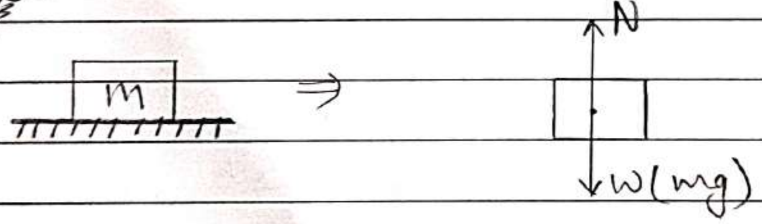
Points to remember :-

Body is assumed to be a point mass.
We will show all the forces acting on the body.
Frame of the reference must be (most of the time) ground / inertial frame.

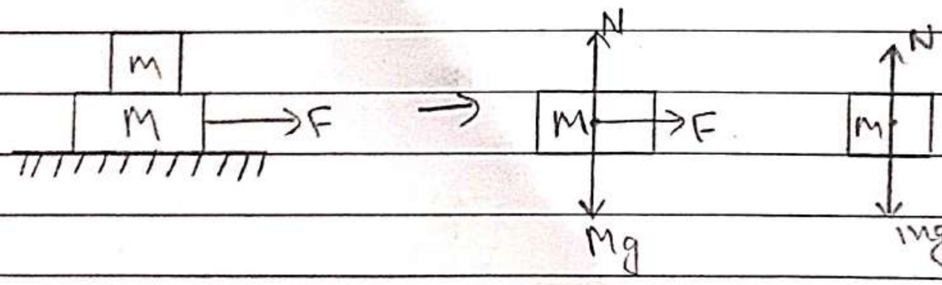


For example

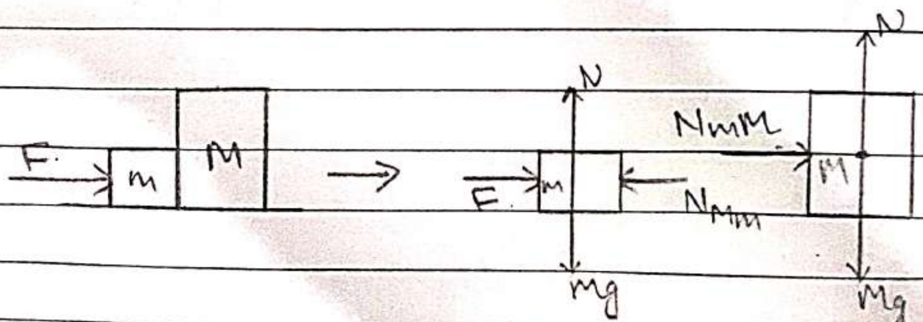
(i)



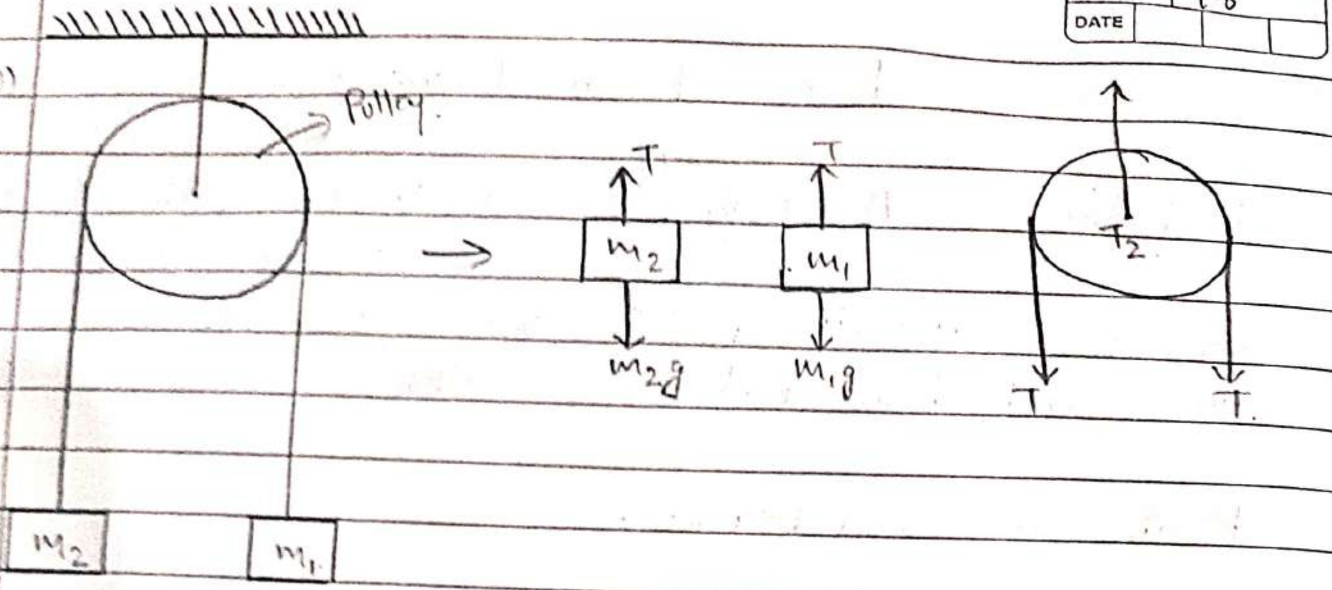
(ii)



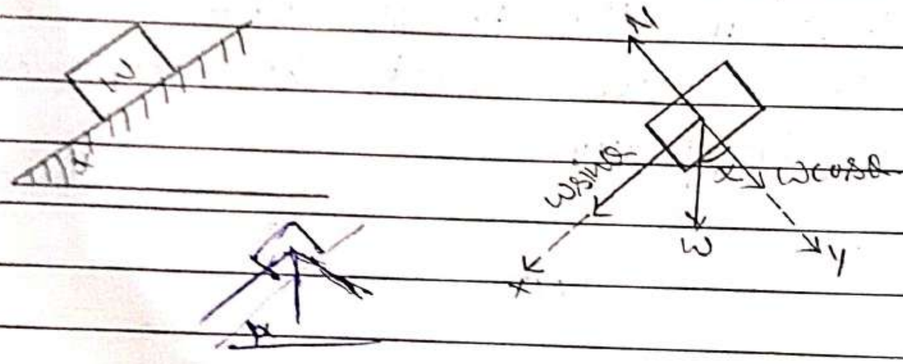
(iii)



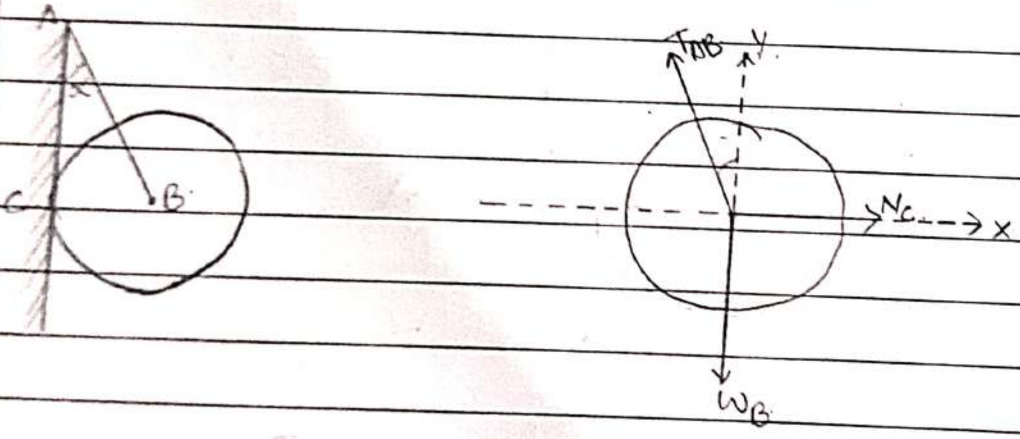
(iv)


Pulley.
m_2
m_1
T
m_2
$m_2 g$
T
m_1
$m_1 g$
T_2
T
T.

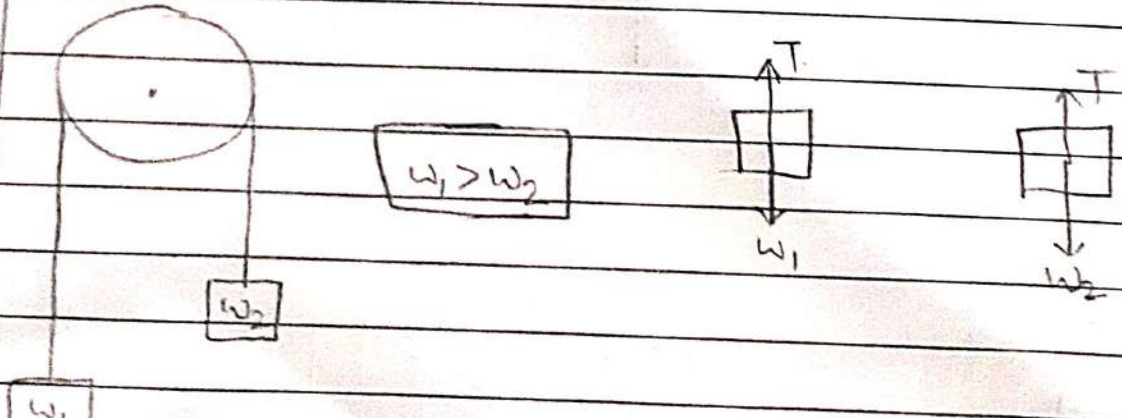
(v)


W
α
N
Wsinα
Wcosα
α
W
x
y

(vi)

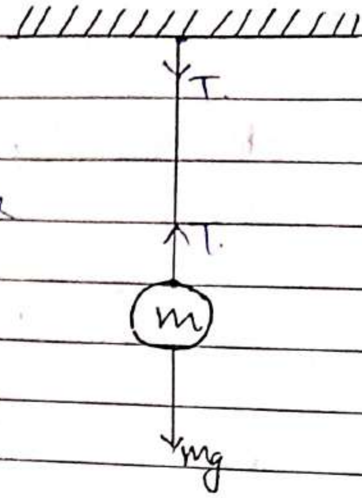

A
α
C
B
T_{AB}
y
N_C
x
W_B

(vii)


W_2
W_1
$W_1 > W_2$
T
W_1
T
W_2

Tension (T) :-

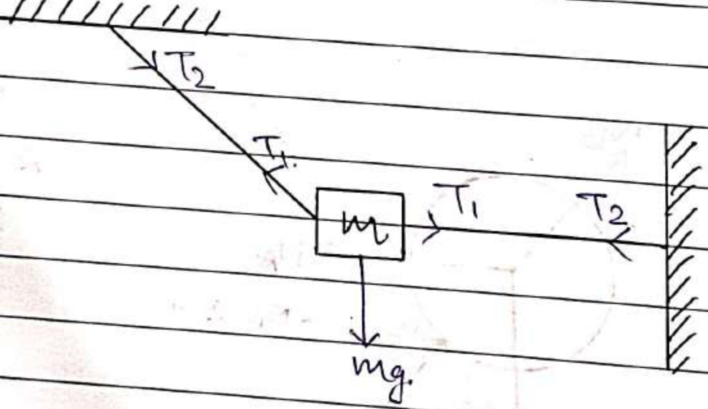
(Rod, Cable, Rope, String, Chain)



- Tension always acts away from the point

- if string changes then tension changes
- Tension remains same if string remains same
- ~~if string changes then tension changes.~~

Example



Equilibrium of Forces (for concurrent forces)

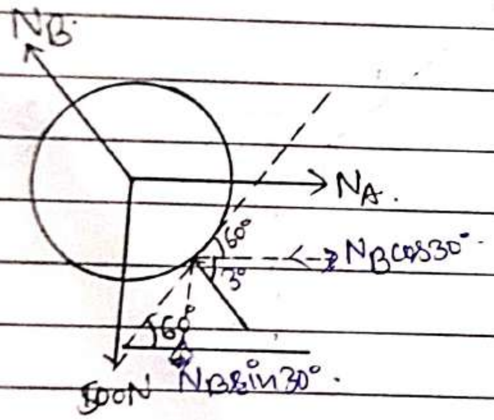
For equilibrium condition

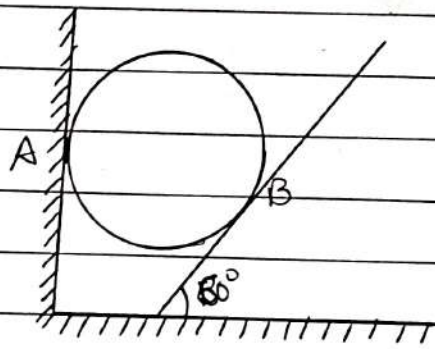
if $R = 0$.
$\Sigma F_x = 0$
$\Sigma F_y = 0$ for concurrent forces.

Example

(1) Find the reaction at the point of a contact A and B for the sphere of weight 500N shown below.

Sol^n FBD





$+\uparrow \Sigma F_y = -500 + N_B \sin 30°$

$500 = N_B \sin 30°$

$N_B = \frac{500}{\sin 30} = 500 \times 2 = 1000N.$

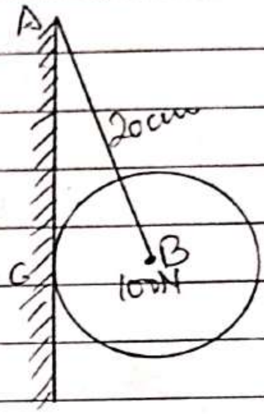
$\xrightarrow{+} \Sigma F_x = N_A + N_B \cos 30°$

$= N_A - 1000 \cos 30°$

$= N_A - 866.02$

$N_A = 866.02 N$

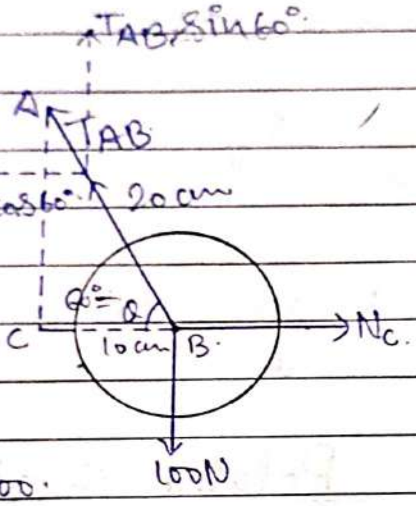
Q-2 A circular roller of weight 100N and radius 10cm hang by a tie rod AB 20cm and rest against a smooth vertical wall at C. Determine the force F in the tie rod and the reaction at the point of contact C



Soln FBD for roller

$\cos\theta = \frac{B}{H} = \frac{10}{20}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$.



$+\uparrow \Sigma f_y = T_{AB}\sin 60° - 100.$

$T_{AB}\sin 60° = 100.$

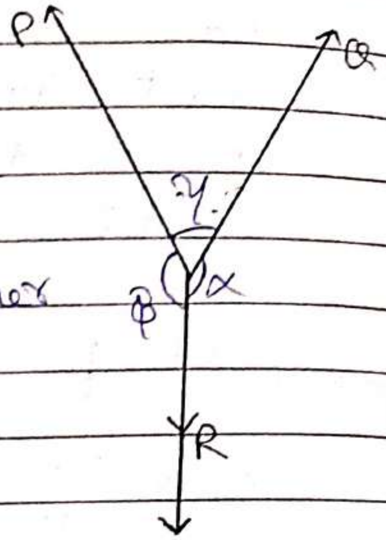
$T_{AB} = \frac{100}{\sin 60°} = 115.47.$

$\xrightarrow{+} \Sigma f_x = N_C - T_{AB}\cos 60°.$

$N_C - 115.47\cos 60°.$

$\boxed{N_C = 57.735\,N.}$

Lami's Theorem



It states "If three coplanar forces acting at a point be in equilibrium then each force is proportional to the sine of the angle between the other two." Mathematically expressed that –

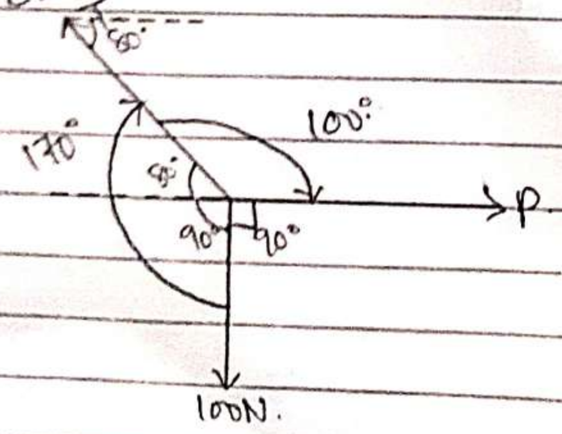
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}.$$

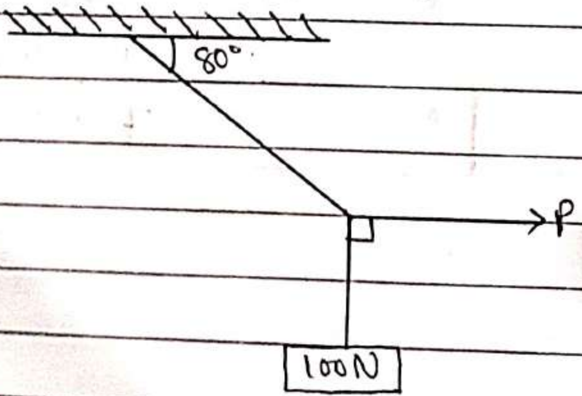
where, P, Q and R are three forces and α, β, γ are the angles as shown in fig.(a).

Related Questions

A horizontal forces P is a shown in figure keep the weight of 100N in the equilibrium. Find the magnitude of force P and tension of the string.

FBD.





By lami's theorem

$$\frac{P}{\sin 170^\circ} = \frac{100}{\sin 100^\circ} = \frac{T}{\sin 90^\circ}.$$

$\Rightarrow \frac{P}{\sin 170^\circ} = \frac{100}{\sin \cancel{130}^\circ}$ $\Rightarrow \frac{T}{\sin 90^\circ} = \frac{100}{\sin 100^\circ}$

$\Rightarrow P \sin 100^\circ = \frac{100 \sin 170^\circ}{\cancel{\sin 170^\circ}}$ $T \sin 100^\circ = \sin 90^\circ \times 100$

$T = \frac{100}{\sin 100^\circ}$

$P \sin 100^\circ = \cancel{575.87}\ 17.36$

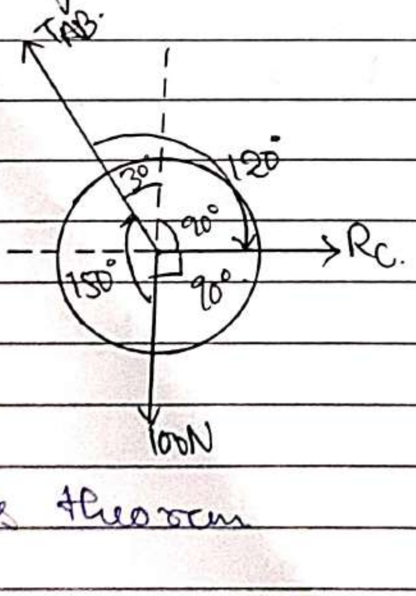
$P = \frac{\cancel{575.87}\ 17.36}{\sin 100^\circ}$

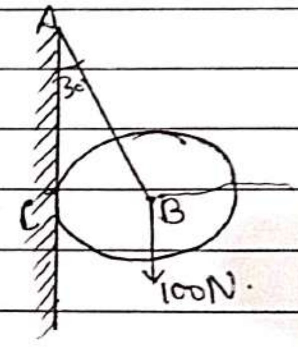
$T = 101.54\,N$

$P = \cancel{584.76\,N}\ 17.63\,N$

Q-2 A cylinder of weight 100N is connected by chord AB and a vertical wall at point C. find the tension at chord and reaction at C for equilibrium of cylinder.

Sol^n FBD for cylinder.





By lami's theorem

$\Rightarrow \frac{T_{AB}}{\sin 90^\circ} = \frac{100}{\sin 120^\circ} = \frac{R_C}{\sin 150^\circ}$

$\Rightarrow \frac{T_{AB}}{\sin 90^\circ} = \frac{100}{\sin 120^\circ} \Rightarrow T_{AB} \sin 120^\circ = \sin 90 \times 100$

$T_{AB} = \frac{100}{\sin 120^\circ}$

$T_{AB} = 115.47\,N.$

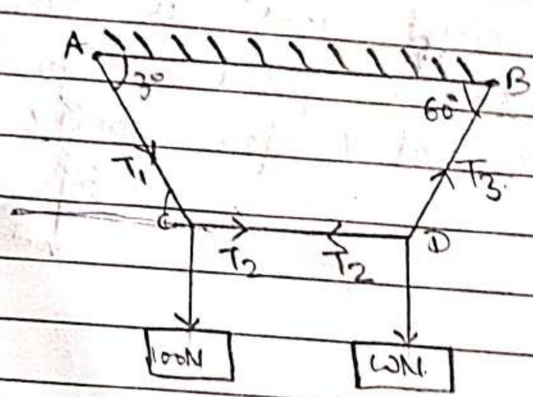
$$\Rightarrow \frac{R_E}{\sin 150^\circ} = \frac{100}{\sin 120^\circ}.$$

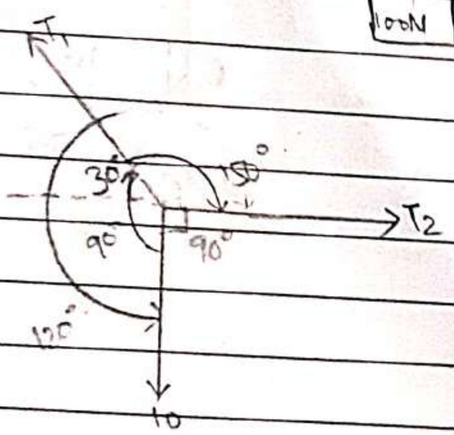
$$R_C \sin 120^\circ = \sin 150^\circ \times 100$$

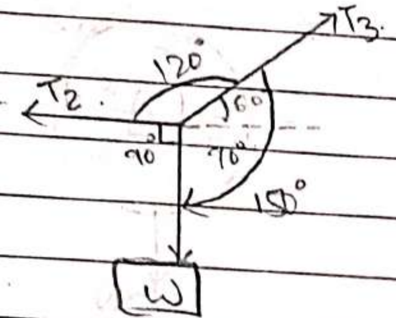
$$R_C = \frac{50}{\sin 120^\circ}.$$

$$R_C = 57.73N.$$

Q-3 A chord ACDB connected at Point A & B having a weight 10N & WN at C & D respectively. Find W, such that CD remains horizontal.







$$\frac{T_2}{\sin 120^\circ} = \frac{10}{\sin 150^\circ}$$

$$T_2 \sin 150^\circ = \sin 120^\circ \times 10.$$

$$T_2 = \frac{8.66}{\sin 150^\circ} = 17.32\ N$$

$$\boxed{T_2 = 17.32N}$$

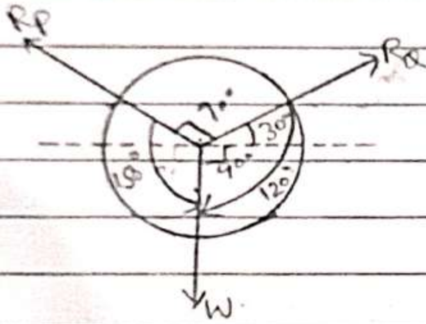
$$\frac{T_2}{\sin 150^\circ} = \frac{W}{\sin 120^\circ}$$

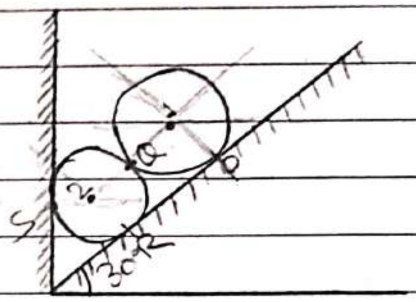
$$17.32 \times \sin 120^\circ = W \times \sin 150^\circ$$

$$\frac{14.99}{\sin 150^\circ} = W$$

$$\boxed{W = 29.98N}$$

Q.4 Two identical cylinder each of weight 100N is rested as shown. Find the reaction P, Q, R & S for equi.





$$\frac{R_P}{\sin 120^\circ} = \frac{R_Q}{\sin 150^\circ} = \frac{W}{\sin 90^\circ}$$

$$\Rightarrow \frac{R_P}{\sin 120^\circ} = \frac{100}{\sin 90^\circ} \Rightarrow R_P \sin 90^\circ = 100 \times \sin 120^\circ$$

$$R_P = 86.60N.$$

$$\frac{R_Q}{\sin 150^\circ} = \frac{100}{\sin 90^\circ} \Rightarrow R_Q \sin 90^\circ = 100 \times \sin 150^\circ$$

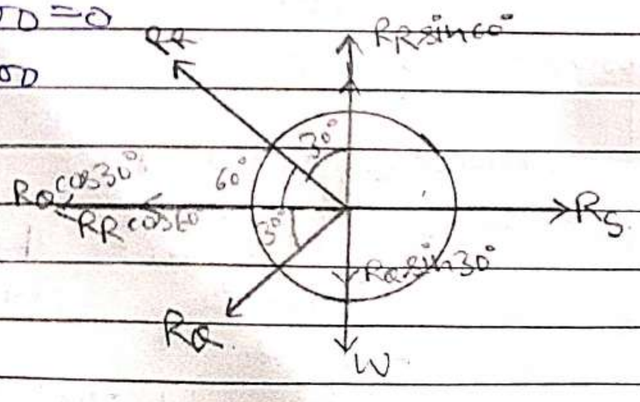
$$R_Q = 50N.$$

$$\Sigma f_y = R_R \sin 60^\circ - R_Q \sin 30^\circ - 100 = 0$$

$$= R_R \sin 60^\circ - 50 \sin 30^\circ - 100$$

$$+R_R \sin 60^\circ = +125N$$

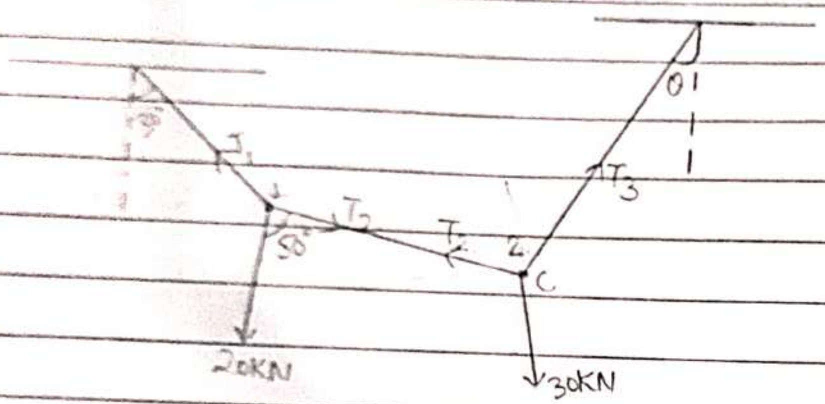
$$R_R = \frac{125}{\sin 60^\circ} = 144.63.$$



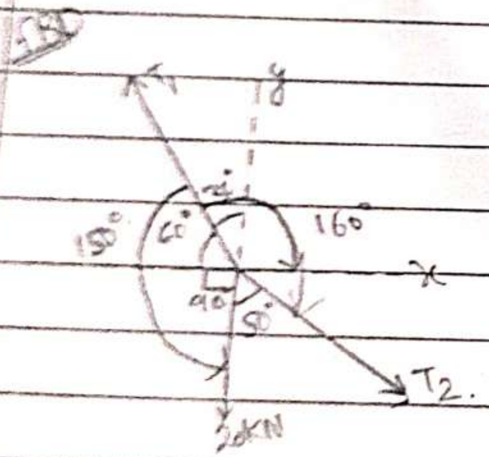
$\Sigma fx = -R_R\cos 60^\circ - R_Q\cos 30^\circ + R_S$
$= -144.63\cos 60^\circ - 50\cos 30^\circ + R_S$
$= -72.31 - 43.30 + R_S$
$= -115.61 + R_S$

$\boxed{R_S = 115.61\,N}$

Q-5 Find the tension in all segments of wire and also find the angle θ



FBD



$\frac{T_2}{\sin 150^\circ} = \frac{20KN}{\sin 160^\circ}$

$\cancel{T_2 \sin 160^\circ = 20 \times \sin 150^\circ}$

$T_2 = \frac{10}{\sin 160^\circ}$

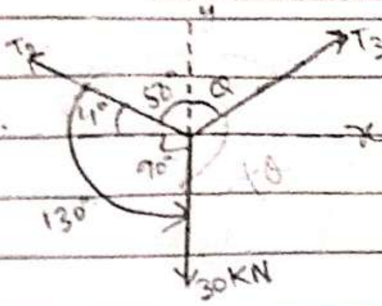
$\boxed{T_2 = 29.23}$

$\frac{T_1}{\sin 50^\circ} = \frac{20KN}{\sin 160^\circ} = \frac{T_2}{\sin 150^\circ}$

$T_1 \sin 160^\circ = 20 \times \sin 150^\circ$

$T_1 = \frac{15.32}{\sin 160^\circ} = \cancel{49.23KN}$
$44.79KN.$

$\boxed{T_1 = 44.79KN}$



$\Rightarrow \frac{T_2}{\sin(90(90-Q))} = \frac{T_3}{\sin 130°} = \frac{30°}{\sin(Q+50°)}$.

$\Rightarrow \frac{T_2}{\sin(+Q)} = \frac{T_3}{\sin 130°} = \frac{30}{\sin(Q+50°)}$.

$\Rightarrow \frac{T_2}{\sin Q} = \frac{T_3}{0.76} = \frac{30°}{\sin(50+Q)}$

$\Rightarrow \frac{T_2}{\sin Q} = \frac{T_3}{0.76}$. $\Rightarrow \frac{29.23}{\sin Q} = \frac{T_3}{0.76}$.

$T_3 = \frac{22.21}{\sin Q}$

$\Rightarrow \frac{T_3}{0.76} = \frac{30}{\sin(50+Q)}$.

$\Rightarrow T_3 = \frac{0.76 \times 30°}{\sin(50+Q)}$.

$\Rightarrow \frac{22.21}{\sin Q} = \frac{22.8}{\sin(50+Q)}$.

$\Rightarrow \frac{\sin(50+Q)}{\sin Q} = \frac{22.8}{22.21}$.

$\Rightarrow \frac{\sin 50 \cos Q + \cos 50° \sin Q}{\sin Q} = \frac{22.8}{22.21}$.

$\Rightarrow \sin 50° \cot Q + \cos 50° = 1.02$

$\cot Q = \frac{1.02 - \cos 50°}{\sin 50°}$.

$\cot Q = \frac{1.02 - 0.64}{0.76} = \frac{0.38}{0.76} = 0.5$

$\cot Q = 0.5 \Rightarrow \frac{1}{\tan Q} = \frac{1}{0.5}$ 0.5

$\tan Q = 1.97$

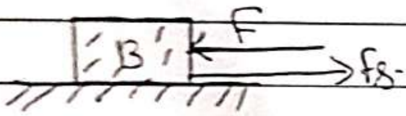
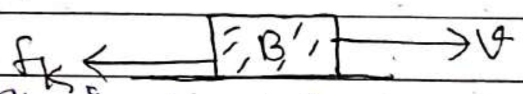
$\boxed{Q = 63.08°}$

FRICTION

<u>Friction:-</u> Friction is a contact force. It always oppose the relative motion between two bodies or tendency of relative motion. This friction is called a friction force.
friction btw two dry surfaces is called "<u>coloumb's friction</u>"

Types of friction

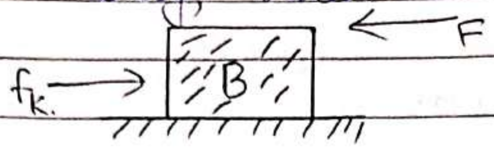
→Rest.

Static friction	Kinetic friction
When there is no relative motion, no force is applied  It is denoted by f_s.	When there is relative motion and there is the formula.  It is denoted by f_K.

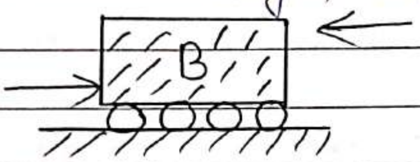
(i) <u>Kinetic friction:-</u> It is the friction experienced by a body when it is in motion is known as kinetic friction.
There are two types of friction
(i) Sliding Friction
(ii) Rolling Friction

Mathematically expression → $\boxed{f_K = \mu_K N.}$
where, μ_K = coefficient of kinetic friction
N = Normal Reaction

a) <u>Sliding</u> <u>Friction</u> :- It is the friction, experienced by the body, when it slide over another.



b) <u>Rolling</u> <u>friction</u> :- It is the friction, experienced, by the body, when it roll over another.

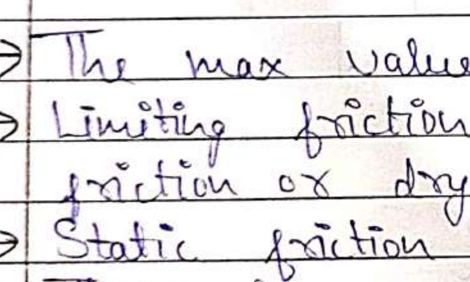


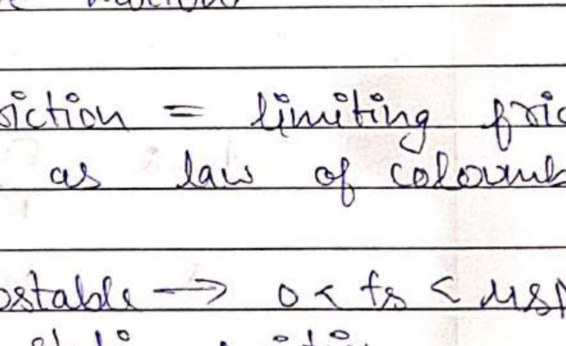
Note: Rolling friction is less then sliding friction

(ii) <u>Static</u> <u>friction</u> :- It is the friction acts on the body when it is at relative rest and has tendency of relative motion.

→ The max value of static friction = limiting friction
→ Limiting friction also known as law of coloumb's friction or dry friction.
→ Static friction is self adjustable → $0 < f_s \leq \mu_s N$.
→ There is no formula for static friction

<u>Limiting</u> ~~Force~~ <u>friction</u> <u>force</u> :- The max. friction force that can be developed at the contact surface, when the body is just on the point of moving is called limiting force of friction





Laws of Coloumb's Friction :-

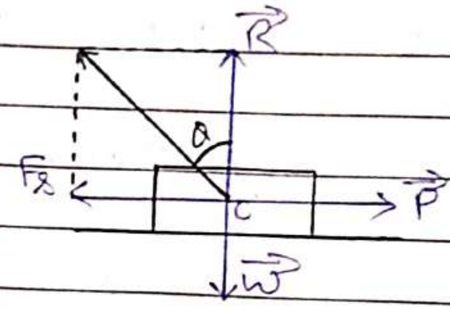
1) Force of friction is always opposite to direction of (motion impendent)
↳ (just begin to start).

2) Friction force depends upon roughness or smoothness of the surface (does not depend on contact area of surface)

3) The ratio of force of friction to normal reaction is a constant is known as "coefficient of friction"

Angle of friction :-

The angle between normal reaction and resultant force is called angle of friction.



$$\tan\theta = \frac{F_s}{R}$$

$$\theta = \tan^{-1}(F_s/R). \quad \{F_s = \mu_s R\}$$

$$\theta = \tan^{-1}\left(\frac{\mu_s \cancel{R}}{\cancel{R}}\right)$$

$$\boxed{\theta = \tan^{-1}(\mu_s)}$$

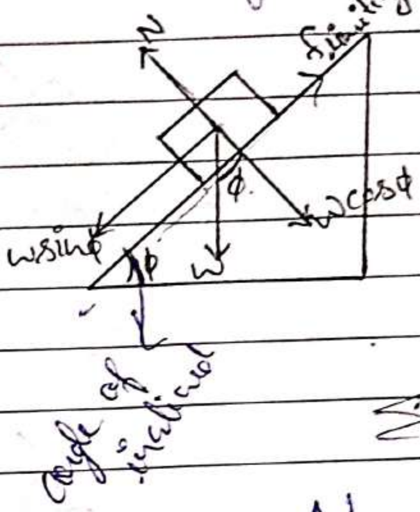
Coefficient of friction :-

The ratio of limiting friction F_{lim} and normal reaction (R) is called coefficient of friction

$$\boxed{\mu = F_{lim}/R}$$

Angle of Repose:-

With increase in angle of inclined surface, the max angle at which body starts sliding down is called angle of repose.



$N = w\cos\alpha$

$\mu N = w\sin\alpha$

$\frac{w\sin\alpha}{w\cos\alpha} = \frac{\mu \cancel{N}}{\cancel{N}}$

$\boxed{\tan\alpha = \mu}$

$\boxed{\tan\alpha = \tan\theta}$

$\Downarrow$

$\alpha = \theta$

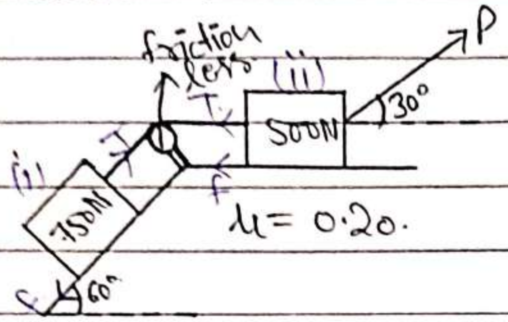
ΣF_y

$N - w\cos\phi$

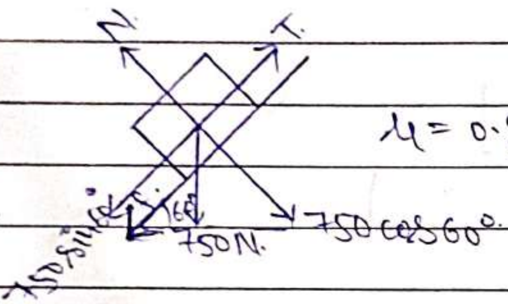
$N = w\cos\phi$

Related Questions

Q Find P to cause a motion to impend.



(i)



$\mu = 0.20N$.

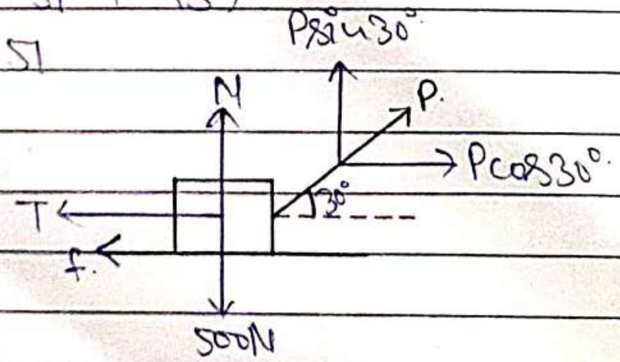
$\Sigma f_y = N - 750\cos 60°$
$N = 750\cos 60°$.

first of all we find that

$f = \mu N. = 0.20 \times 750\cos 60°$.
$f = 75N$.

$\xrightarrow{+} \Sigma f_x = T - 750\sin 60° - f_k$
$T = +(750\sin 60° + f_k)$
$T = (649.51 + 75)$
$T = 724.51$

(ii)



$+\uparrow \Sigma f_y = N + P\sin 30° - 500$.
$N = 500 - P\sin 30°$. —— ①.

$\Rightarrow \Sigma f_x = P\cos 30^\circ - T - f_k$

$P\cos 30^\circ = T + f_k$

$P\cos 30^\circ = 724.51 + \mu_k N$

$P\cos 30^\circ = 724.51 + 0.20(500 - P\sin 30^\circ)$

$P\cos 30^\circ = 724.51 + 0.20 \times 500 - 0.20 P\sin 30^\circ$

$P\cos 30^\circ + 0.20 P\sin 30^\circ = 724.51 + 100$

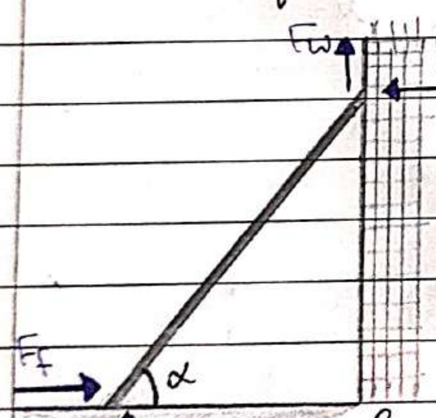
$P(\cos 30^\circ + 0.20\sin 30^\circ) = 824.51$

$P(0.86 + 0.1) = 824.51$

$$P = \frac{824.51}{0.96}$$

$$\boxed{P = 858.86\text{N}}$$

Ladder friction :- Consider a ladder AB resting on the rough ground and leaning against a wall, as shown in fig. 77.

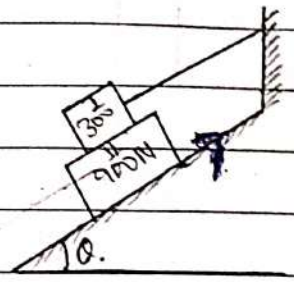


As per the upper end of the ladder tends to slip downwards, therefore, the direction of the friction force btw ladder and the wall (F_w) will be upwards. Similarly, as the lower end of the ladder tends to slip away from the wall therefore, the direction of the force of friction btw ladder and the floor (F_f) will be towards the wall.

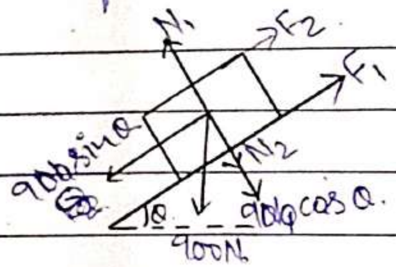
Since the system is in eqv., therefore the algebric sum of Σf_x and Σf_y of the forces must be equal to zero

Note:- The normal reaction at the floor (R_f) will act perpendicular to the floor. Similarly, normal reaction of the wall (R_w) will also act $\perp$ to the wall.

Q-2 What is the value of θ for downward motion of 900N block if $\boxed{\mu = 1/3}$



Sol^n FBD of 900N Block



$+\uparrow \sum F_y = 0$

$= N_1 - N_2 - 900\cos\theta =$ —— (I).

$N_1 = 900\cos\theta + N_2$.

$\Rightarrow \sum F_x = \mu N_1 + \mu N_2 - 900\sin\theta = 0$.

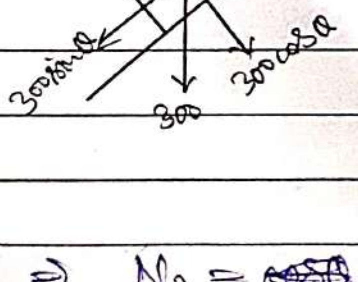
$\frac{1}{3}(N_1 + N_2) = 900\sin\theta$. —— (II).

$\cancel{900\cos\theta + N_2 + N_2 = 2700\sin\theta}$

$\cancel{2N_2}$

$\cancel{\frac{1}{3}(900\cos\theta + N_2 + N_2) - 900\sin\theta = 0}$

$\cancel{300\cos\theta + \frac{2}{3}N_2 = 2700\sin\theta}$



$+\uparrow \sum F_y = 0$.

$N_2 - 300\cos\theta \Rightarrow N_2 = 300\cos\theta$. —— (III).

Putting the eqn (III) in eqn (I) we get.

$N_1 = 900\cos\theta + 300\cos\theta$.

$\boxed{N_1 = 1200\cos\theta}$

DATE

∴ Hence, $N_1 = 1200\cos\theta$
$N_2 = 300\cos\theta$

$$\Rightarrow \frac{1}{3}(N_1 + N_2) = 900\sin\theta$$

$$\frac{1}{3}(1500\cos\theta) = 900\sin\theta$$

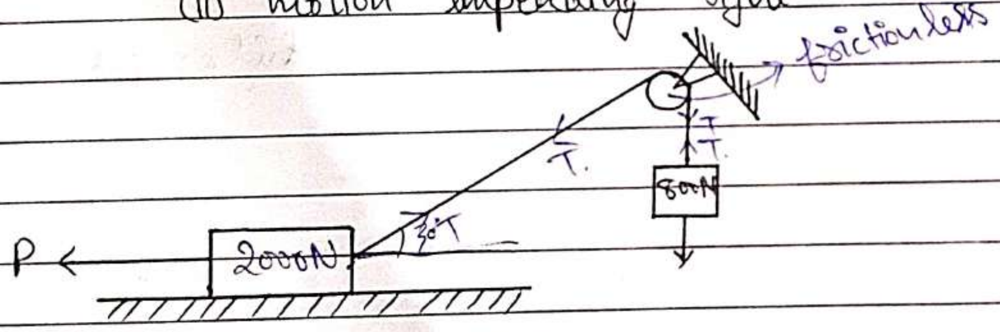
$$1500\cos\theta = 2700\sin\theta$$

$$\tan\theta = \frac{15\cancel{00}}{27\cancel{00}}$$

$$\theta = \tan^{-1}\left(\frac{15}{27}\right)$$

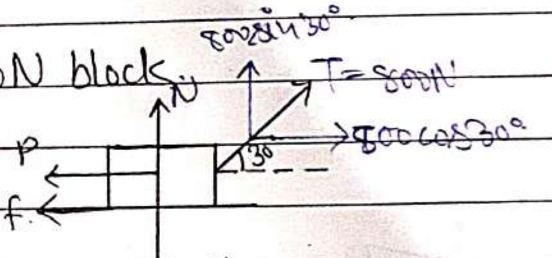
$$\boxed{\theta = 29.05^\circ}$$

Q-3 Find P (i) motion impending left
(ii) motion impending right.



$\mu = 0.35$.

(i) FBD of 2000N block.



$+\uparrow \Sigma f_y = 0$

$= N + 800\sin30^\circ - 2000 = 0$

$N = 2000 - 800\sin30^\circ$

$N = 1600N$

$\xrightarrow{+} \Sigma f_x = 0$

$\Rightarrow 800\cos30^\circ - 0.35N - P$

$P = 800\cos30^\circ - 0.35\times1600$

$\boxed{P = 132.82N}$

Belt Friction

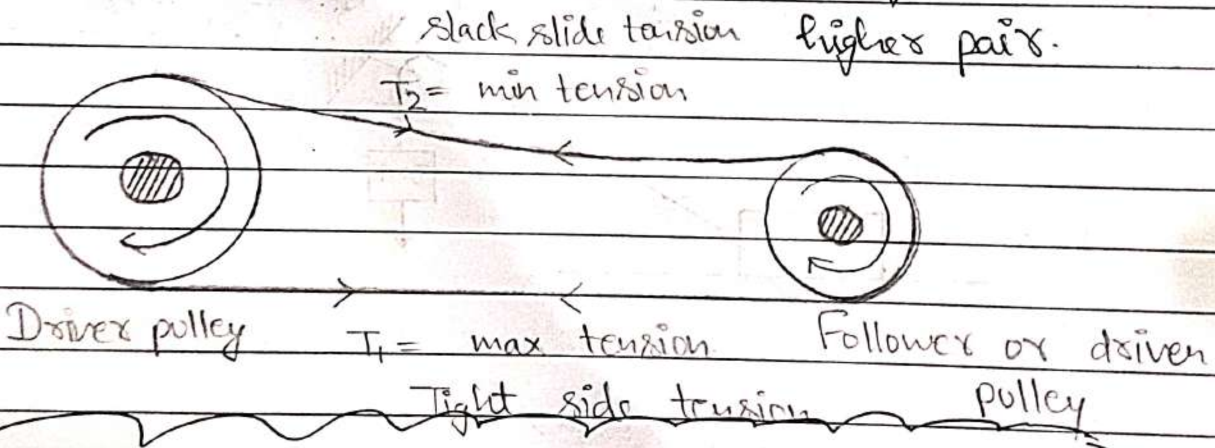
Belt drive —

- A belt drive is a mechanism in which power is transmitted by the movement of a continuous flexible belt.
- It is used to transmit rotational motion from the mechanical element to another.

pulley

- A belt is looped strip of flexible material which is used to mechanically link two or more rotating shaft.
- A belt drive is found in almost every modern machine.

Multipoint contact → wrapping pair
↓
higher pair.



To come slack side → Belt & Pulley contact ↑
friction grip ↑
slip ↓

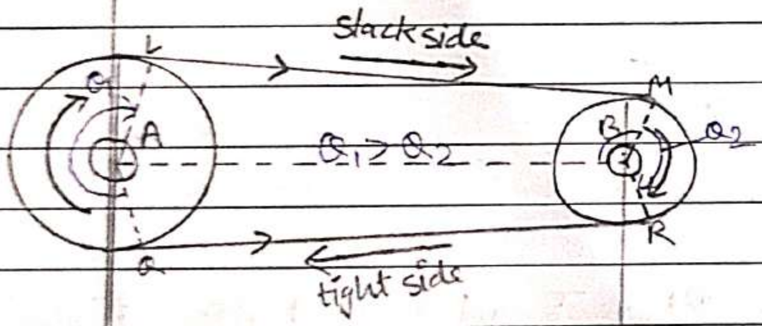
Drawback of slack side → Uneven extension or contraction on belt side is known as creep

Definition of Belt drive –

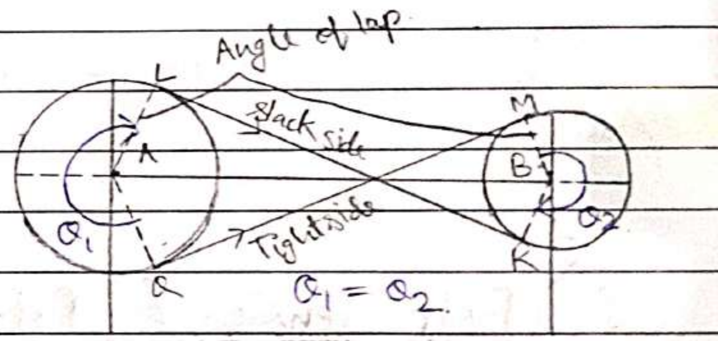
Belt drive has two rotating shafts connected to each other using a belt. The belt that drive or the one that is connected to the power source is called the driving system and the other one which is driven or to which rotational motion is transmitted is called driven system. Similarly, the pulley which transmit rotation is called driving pulley and the pulley to which rotation is transmitted is called driven pulley.

Types of Belts.

Open Belt Drive



Cross Belt Drive



Open Belt Drive	Cross Belt Drive
• Two pulley rotate in the same direction	• Two pulleys rotate in opposite direction
• Length of the belt is smaller.	• Length of belt is larger.
• Angle of lap is different for driver and driven pulley. $\therefore \theta_1 \neq \theta_2$	• Angle of lap is same for the driver and driven pulley $\therefore \theta_1 = \theta_2$.

Length for open belt drive and closed belt drive

(−) Open Belt Drive

$$\theta = 180^\circ - 2\alpha$$

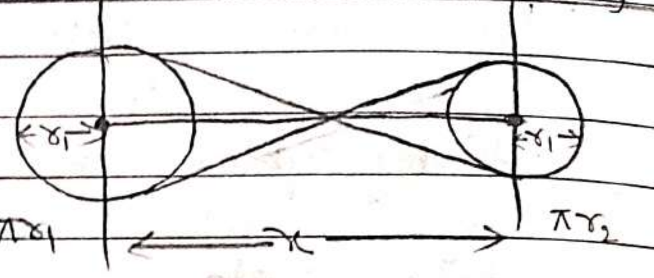
$$\sin\alpha = \frac{r_1 - r_2}{x}$$

$$\alpha = \sin^{-1}\left(\frac{r_1 - r_2}{x}\right)$$ Angle of lap

$$L_{open} = \pi r_1 + \pi r_2 + 2x + \frac{(r_1 - r_2)^2}{x}$$

θ = radian

Cross belt Drive (+)



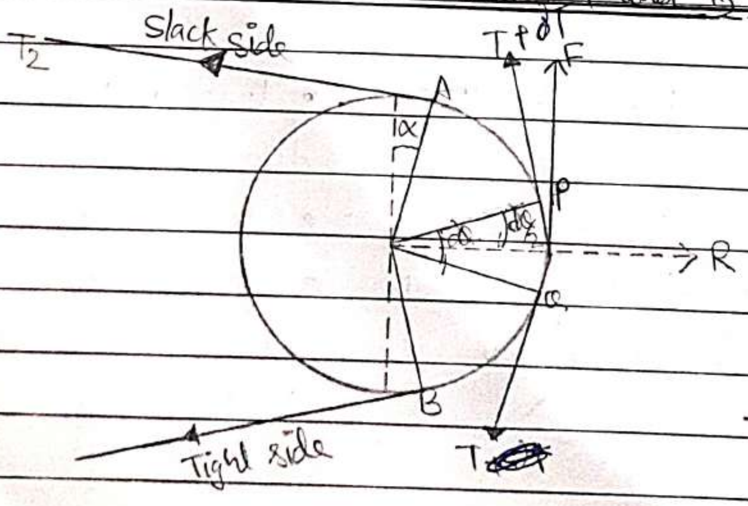
$$\theta = 180 + 2\alpha$$

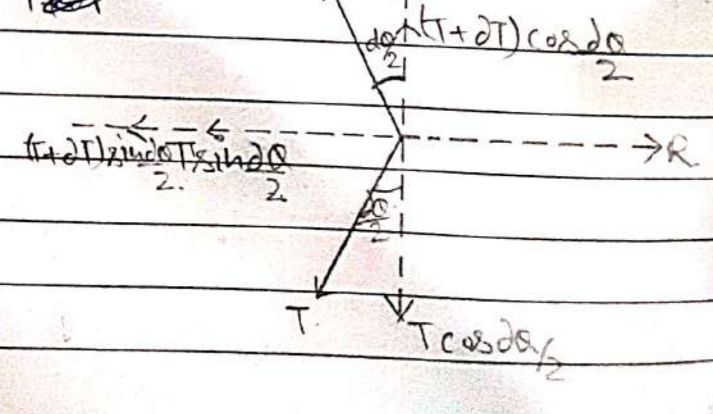
$$\alpha = \sin^{-1}\left(\frac{r_1 + r_2}{x}\right)$$

$$L_{closed} = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

θ = radian

Belt tension & Relation Btw T_1 and T_2 & Ratio of Tension





T_1 = Tension in the belt of the tight side
T_2 = Tension in the belt on the slack side
θ = Angle of contact or lap in the radians i.e, angles which subtended by the Arc AB, along which the belt touches the pulley, at the centre

Now consider a small portion of the belt PQ, subtending angle $\partial\theta$ at the centre of the pulley. The belt PQ in equilibrium under the following forces:-

1) Tension T in the belt at P
2) Tension $(T+\partial T)$ in the belt at Q
3) Normal Reaction R and
4) Friction force, $F = \mu R$.

Resolving all the forces horizontally and equating the same -

$\Rightarrow \Sigma f x = 0$

$$\Rightarrow R - (T+\partial T)\sin\frac{\partial\theta}{2} - T\sin\frac{\partial\theta}{2} = 0 \quad \text{---} \text{ (1)}.$$

Since $\partial\theta$ is very small, therefore substituting $\sin\frac{\partial\theta}{2} = \frac{\partial\theta}{2}$ in eqn (1)

$$R = (T+\partial T)\frac{\partial\theta}{2} + T\frac{\partial\theta}{2}$$

$$R = \frac{T\partial\theta}{2} + \frac{\partial T\partial\theta}{2} + \frac{T\partial\theta}{2} \quad \left\{ \text{neglecting } \frac{\partial T\partial\theta}{2} \right\}.$$

$$R = \frac{\cancel{2}T\partial\theta}{\cancel{2}}$$

$$\boxed{R = T\partial\theta} \quad \text{---} \text{ (II)}.$$

Now, resolving the force vertically and equating

$+\uparrow \sum f_y = 0$

$$\Rightarrow F + (T+\partial T)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} = 0 \quad \text{—— (III)}$$

$$F = \partial T\cos\frac{d\theta}{2} + (T+\partial T)\cos\frac{d\theta}{2}$$

$$F = \cancel{-T\cos\frac{d\theta}{2}} + \cancel{T\cos\frac{d\theta}{2}} + \partial T\cos\frac{d\theta}{2}$$

Since the angle is very small, therefore substituting $\cos\frac{d\theta}{2} = 1$ in eqn — (III).

$$\mu R = \partial T$$

$$\boxed{R = \frac{\partial T}{\mu}} \quad \text{—— (IV)}$$

∴ Equating the value of R from eqn (II) and (IV), we g

$$T\partial\theta = \frac{\partial T}{\mu}$$

$$\mu\, d\theta = \frac{\partial T}{T}$$

Integrating both sides, we get from A to B

$$\int_{T_1}^{T_2} \frac{\partial T}{T} = \int_0^{\theta} \mu\, d\theta$$

$$\log_e\left(\frac{T_2}{T_1}\right) = \mu\theta$$

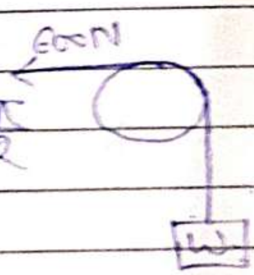
$$\boxed{\left(\frac{T_2}{T_1}\right) = e^{\mu\theta}} \quad \text{—— (V)}$$

The equation (I) may also be expressed in term of corresponding logarithm to the base 10 i.e.,

$$\boxed{2.303 \log\left(\frac{T_1}{T_2}\right) = \mu\theta} \text{ ——— (VI)}$$

Related Questions

Q A rope is making $1\frac{1}{4}$ turns around the stationary horizontal drum. to support a weight W, take coff. of friction $\mu = 0.3$ find range of weight can be the supported by ~~exactly~~ exerting 600N force and other end of the rope.



Soln Given that,

Total wrap angle $(\theta) = \frac{5}{4} \times 2\pi = \frac{5\pi}{2}$

$\mu = 0.3$.

Case I When (W) is going downward.

$T_2 = W \quad T_1 = 600N$

$$\frac{T_2}{T_1} = e^{0.3 \times 5\pi/2}$$

$$W = 600 \times 10.54 = \boxed{6324. = W}$$

Case (II) When (W) is going upward.

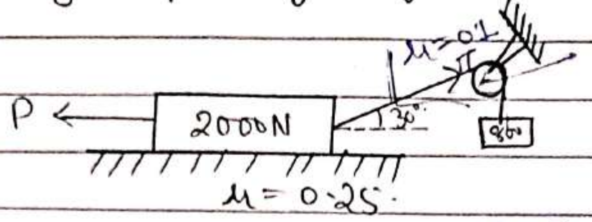
$T_2 = 600, \ T_1 = W.$

$$\frac{600}{W} = e^{0.3 \times 5\pi/2}$$

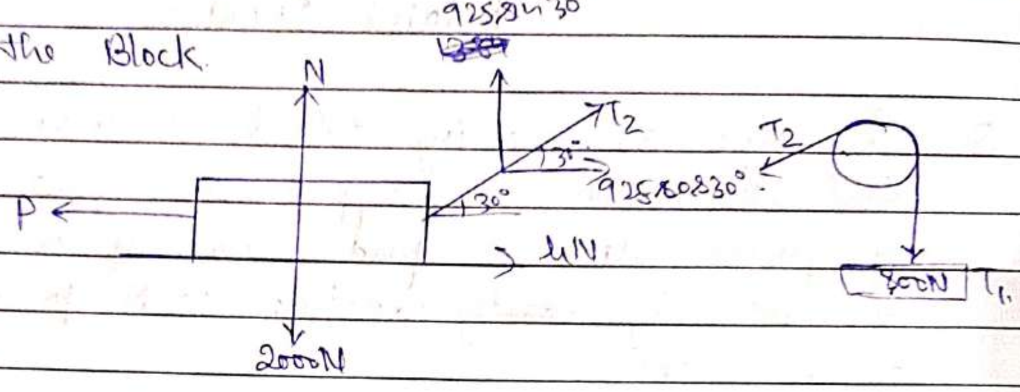
$$W = \frac{600}{10.59.}$$

$$\boxed{W = 56.65 \text{ N}}$$

Q-) Find P if motion is impending left (ii) if motion is ~~improving~~ impending right.



FBD of the Block



Case → When T_2 is going upward

$T_1 = 800N.$

$$\frac{T_2}{T_1} = e^{0.1 \times \pi/3}$$

$T_2 = 800 \times 1.1\cancel{5}$

$T_2 = \cancel{1384 N}.\ 925N.$

$+\uparrow \Sigma y = 925 \sin 30° + N - 2000.$

$N = 2000 - 925 \sin 30°$

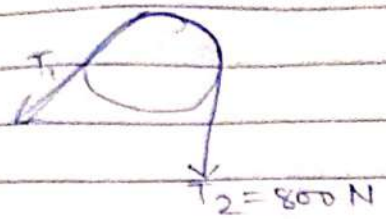
$N = 1537.5\ N$

$\xrightarrow{+} \Sigma f x = 0.25N + 925 \cos 30° - P.$

$P = 0.25 \times 1537.5 + 925 \cos 30°.$

$P = 1185.44\ N.$

Case-II



$$\frac{T_2}{T_1} = e^{0.1 \times \pi/3}$$

$$\frac{800}{T_1} = 1.15$$

$$T_1 = 800/1.15$$

$$T_1 = 695.65$$

$N = 2000 - 695.65 \sin 30°$

$N = 1652.17 N$

$P = -0.25 \times 1652.17 + 695.65 \cos 30°$

$P = 189.40 N$

Q-3 Determine max weight that can be lowered by a person who can exert 300N pull at other end. A angle of $2\frac{1}{2}$ turns, $\mu = 0.3$. What is the range of weight

Soln Total angle of wrap $(\theta) = 5/2 \times 2\pi = 5\pi$

$$\frac{T_2}{T_1} = e^{\mu\theta} \Rightarrow \frac{T_2}{300} = e^{0.3 \times 5\pi}$$

$$T_2 = 300 \times 21.20$$

$$T_2 = 6360 N$$

Q-4 The resultant of two forces one of which is double to another having 260N. If the direction of larger force is reversed resultant is reduced to 180N. find the forces.

$$(260)^2 = \sqrt{(F)^2 + (2F)^2 \times 2F \times 2F \times \cos\theta}$$

$$67600 = 5F^2 + 4F^2 \cos\theta \quad \text{--- (I)}$$

$$180 = \sqrt{(F)^2 + (-2F)^2 - 2F \times 2F \cos\theta}$$

$$(180)^2 = 5F^2 - 4F^2 \cos\theta$$

$$32400 = 5F^2 - 4F^2 \cos\theta \quad \text{--- (II)}$$

Second Module

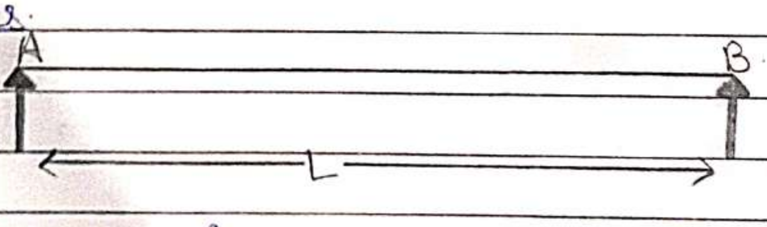
BEAM

Beam:- It is a type of structures, used in construction and engineering to provide a safe and secure or efficient load path that efficiently distributes the weight throughout the foundation of building

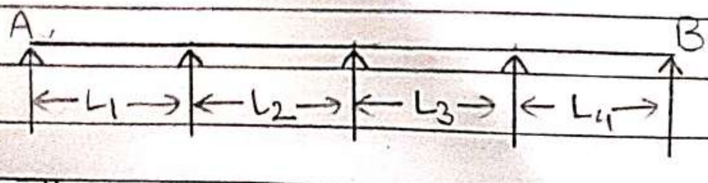
The most common types of Beam structures are.

(i) Simply supported Beam
(ii) Continuous Beam
(iii) Overhanging Beam
(iv) Cantilever Beam
(v) Fixed Beam

(i) **Simply Supported Beams:-** A beam which is freely supported on the walls or the columns at its both the ends is called as simply supported beams.

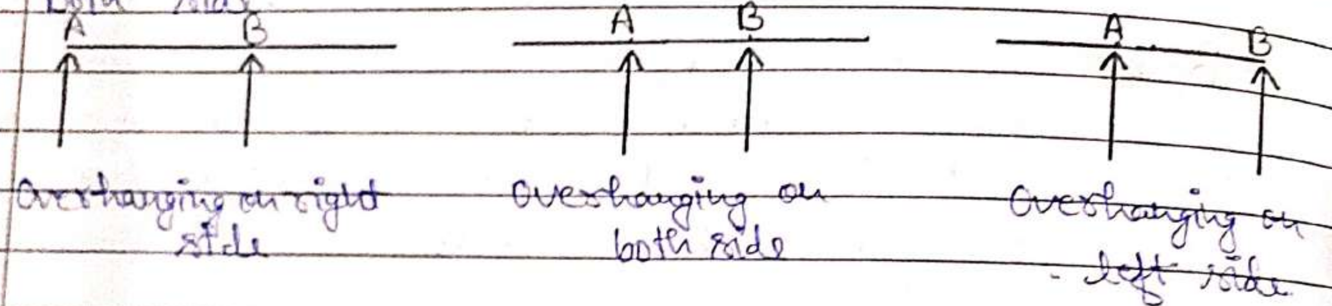


(ii) **Continuous Beam:-** A beam which is supported on more than two supports (i.e. at least three support) is called continuous beam.

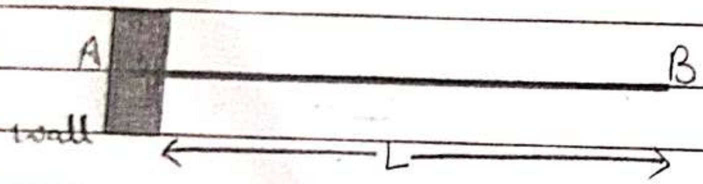


three span continuous Beam

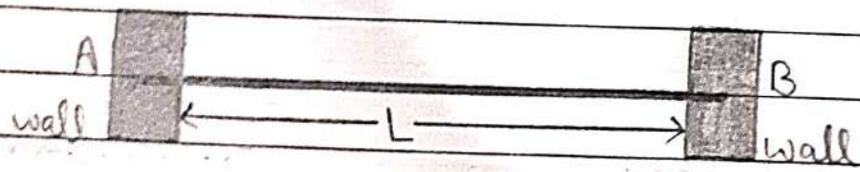
(iii) Overhanging Beam :- If the ends portion of the beam extends beyonds the support, it is called as an overhanging beam.
A Beam may be overhanging on one side or on both side



(iv) Cantilever Beam :- A beam fixed at one end and free at the other is called as a cantilever beam.



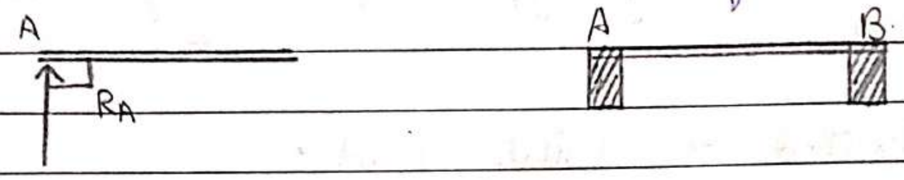
(v) Fixed Beam :- A beam whose both ends are rigidly fixed in walls is called a fixed beam or constrained beam



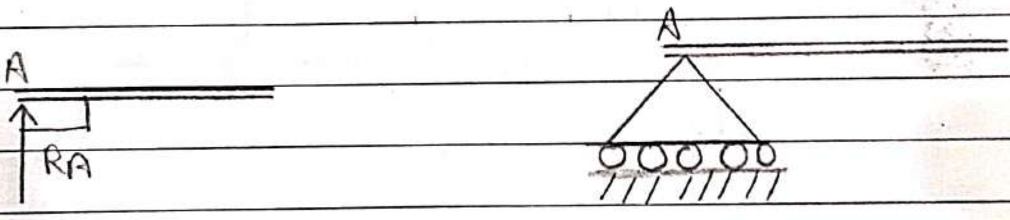
Types of Supports of Beam :-

There are four types of supports for beam are the important from the subject points of view:-

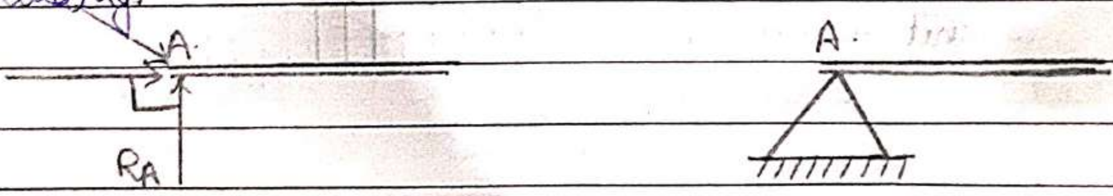
(i) **Simply Supported Beam :-** In such a case the reactio is always vertical.



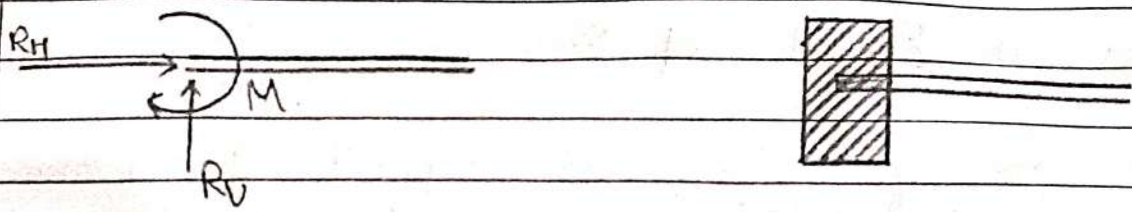
(ii) **Roller Supported Beam :-** In such a case, the end of a beam is supported on rollers and the reaction on such end is always normal to support.



(iii) **Hinge Supported Beam :-** In such a case, the reaction o such end may be horizontal, vertical or inclined depending upon the type of th loading.



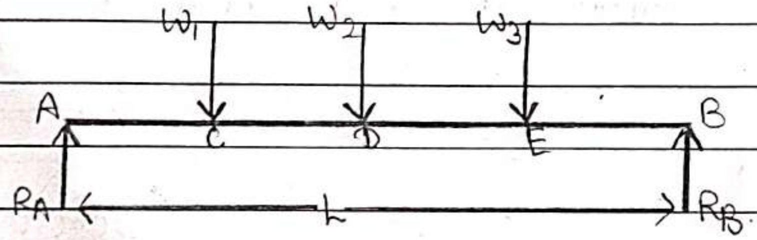
(iv) **Fixed Supported Beam :-** In such a case, magnitude of a moment is taken into consideratio while calculating the reaction ; and moment does not involve any load. therefore it has no vertical or horizontal component.



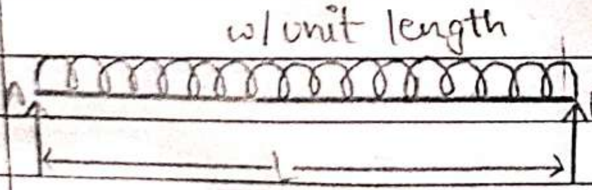
Types of Loading :-

There are many types of loading in subjet point of views -

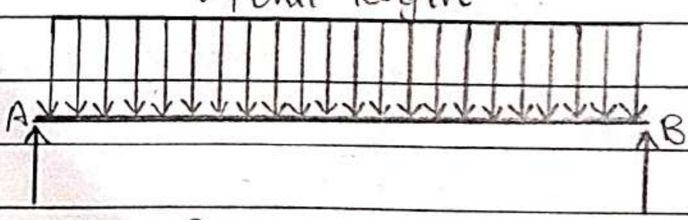
(i) Concentrated or Point load :- A load acting at a point on the beam is known as concentrated load or point load. Generally a load distributed over small area is taken as concentrated load.



(ii) Uniformly Distributed Load :- A load which is spread up uniformly on the beam, i.e, each unit length is loaded on the same extent. It is written as U.D.L.

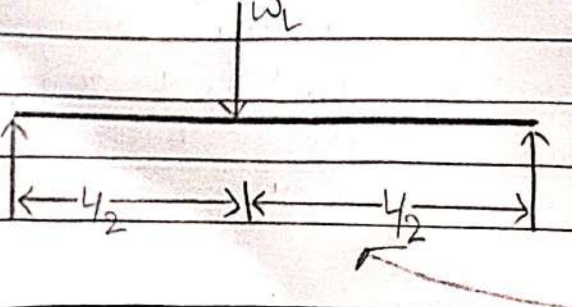


Coil shape U.D.L



Rectangular shape

U.D.L converted into point load. and use area of □ = base x ~~length~~ height



Area of □ = b × h
= wL.

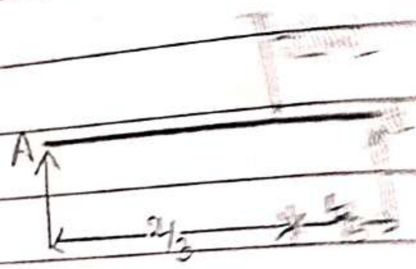
b/2 ⇒ L/2.

(iii) Uniformly Vary...

intensity of load ... change is uniform ...

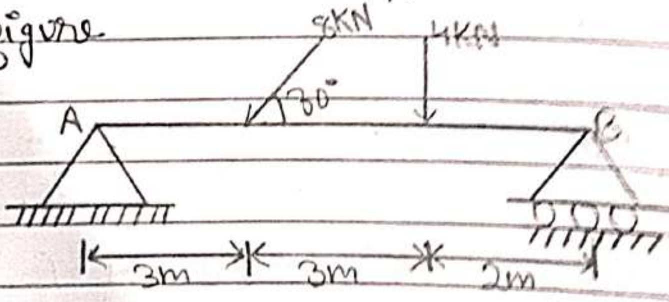
A and B for the beam

Trapezoidal Shape UVL



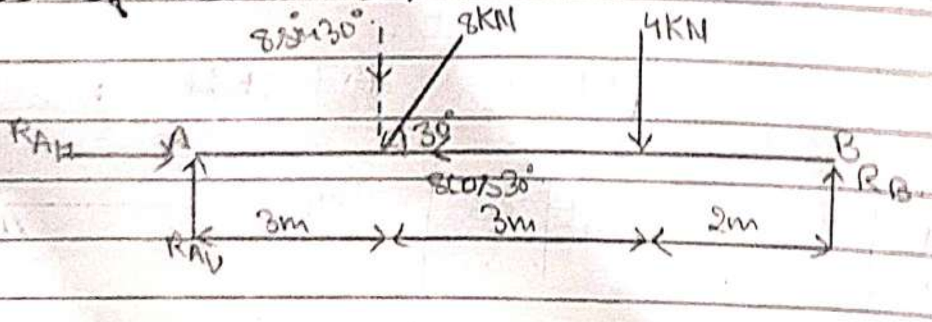
Related Questions

Q-1 Find the reaction at support A and B ... shown in figure



FBD of beam AB ⇒

Soln



$\xrightarrow{+} \Sigma f_x = 0$

$= R_{AH} - 8\cos 30°$

$\Rightarrow R_{AH} = 8\cos 30° \Rightarrow \boxed{R_{AH} = 6.928\,KN}$

$+\uparrow \Sigma f_y = 0$

$\Rightarrow R_{Av} + R_B - 4 - 8\sin 30^\circ = 0$

$\Rightarrow R_{Av} + R_B - 4 - 4$

$\Rightarrow R_{Av} + R_B - 8 = 0$ —— ①

Now, Moments at point A, clockwise

$\Sigma M_A = 0$

$\Rightarrow 8\sin 30^\circ \times 3 + 4 \times 6 + R_B \times 8 = 0$

$12 + 24 - R_B 8 = 0.$

$\cancel{-} R_B \times 8 = \cancel{-}36$

$R_B = 36/8 = 4.5\,KN$

$\boxed{R_B = 4.5\,KN}$

Put the value R_B in eqn ①, we get

$\Rightarrow R_{Av} + 4.5 - 8 = 0.$

$R_{Av} = 8 - 4.5$

$\boxed{R_{Av} = 3.5\,KN}$

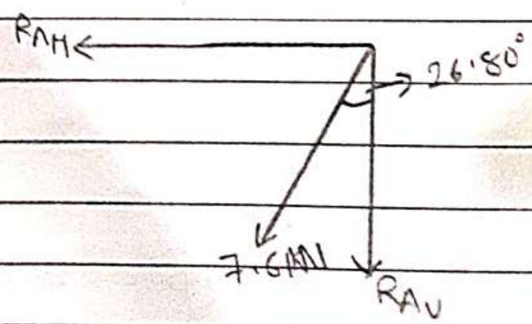
Let's find resultant and degree of A

$R_A = \sqrt{R_{AH}^2 + R_{Av}^2} \Rightarrow \sqrt{(6.928)^2 + (3.5)^2}$

$\boxed{R_A = 7.761\,KN}$

$\tan\theta = \left|\frac{R_{Av}}{R_{AH}}\right| \Rightarrow \theta = \tan^{-1}\left|\frac{3.5}{6.928}\right|$

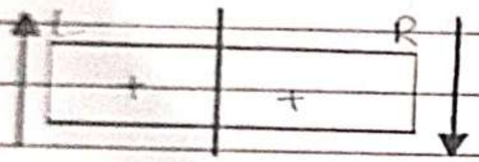
$\boxed{\theta = 26.80^\circ}$



SHEAR FORCE & BENDING MOMENT

a) **Shear force :-** Shear force at any cross-section of the beam is the algebric sum of all the vertical forces on the beam acting on the right or left side of the section.

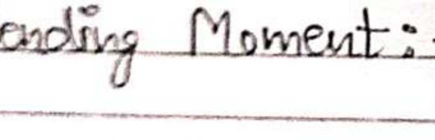
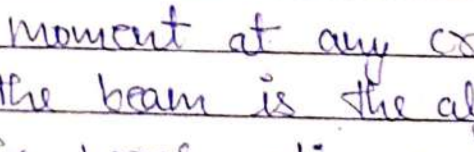
Sign Convention for Shear Force.

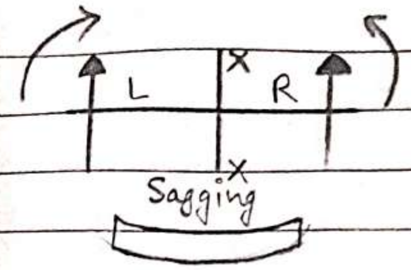
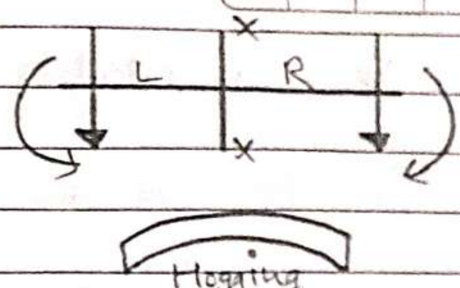
	
An upward force to the left of the section and downward force to the right of the section is taken as positive	Downward force to the left of the section and upward force to the right of section is taken as negative.

b) **Bending Moment :-** Bending moment at any cross section of the beam is the algebric sum of the moments of all the forces acting on the right or left side of the section.

Sign Convention for Bending Moment.

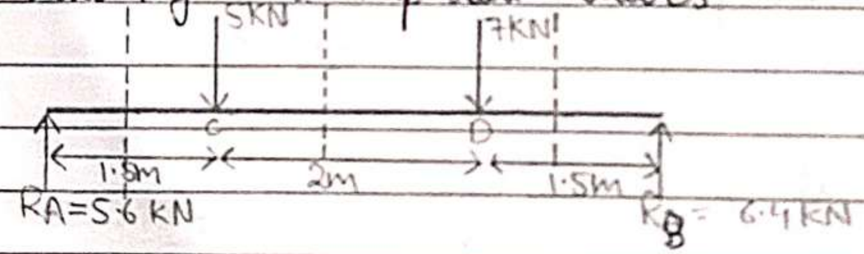
(i)

	
Clockwise moment of the left and anticlockwise to right of the section is taken as +ve.	Anticlowise of the left and clockwise moment to the right of the section is taken as -ve.

(ii)		
		
	Upward forces to any side of section produces positive B.M. The portion of the beam deflects in the sagging pattern due to these forces.	Downward forces to any side of section produces negative B.M. The portion of the beam deflects in the hogging manner due to these forces.

Related Questions

Q-1 A simply supported beam of span 5m carries two point loads of 5KN and 7KN as shown in fig. Draw S.F.D and B.M.D showing the important values.

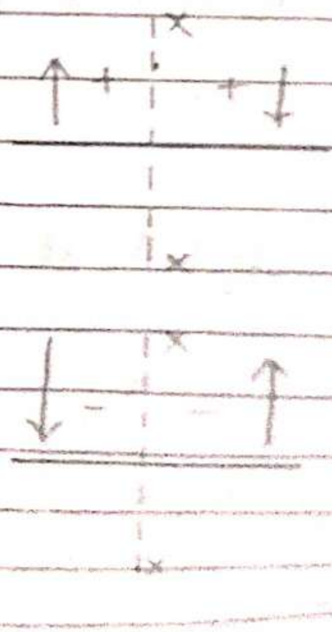


Soln. $\circlearrowright \sum M_A = 0$

$\Rightarrow 5 \times 1.5 + 7 \times 3.5 - R_B \times 5 = 0$

$\Rightarrow 32 = 5R_B$

$R_B = 32/5 = 6.4\ KN$



$\uparrow \sum f_y = 0$

$\Rightarrow R_A - 5 - 7 + 6.4 = 0$

$R_A - 12 + 6.4 = 0$

$R_A = 5.6\ KN$

Now we calculate S.F calculations.

i) S.F btw A and C = 5.6 KN for taking left side of the section
or $(5+7)-6.4 = 12-6.4 = 5.6$ KN.

(ii) S.F btw C and D →
Σ of left side of the section $= (5.6-5) = 0.6$ KN.
Σ of right side of the section $= (7-6.4) = 0.6$ KN.

(iii) SF btw D and B →
Σ of left side of the section $= (5.6-5-7) = -6.4$ KN
Σ of right side of the section $= -6.4$ KN.

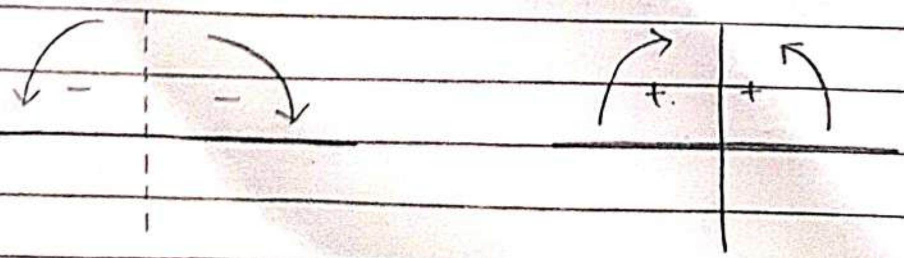
Similarly we calculate B.M calculations

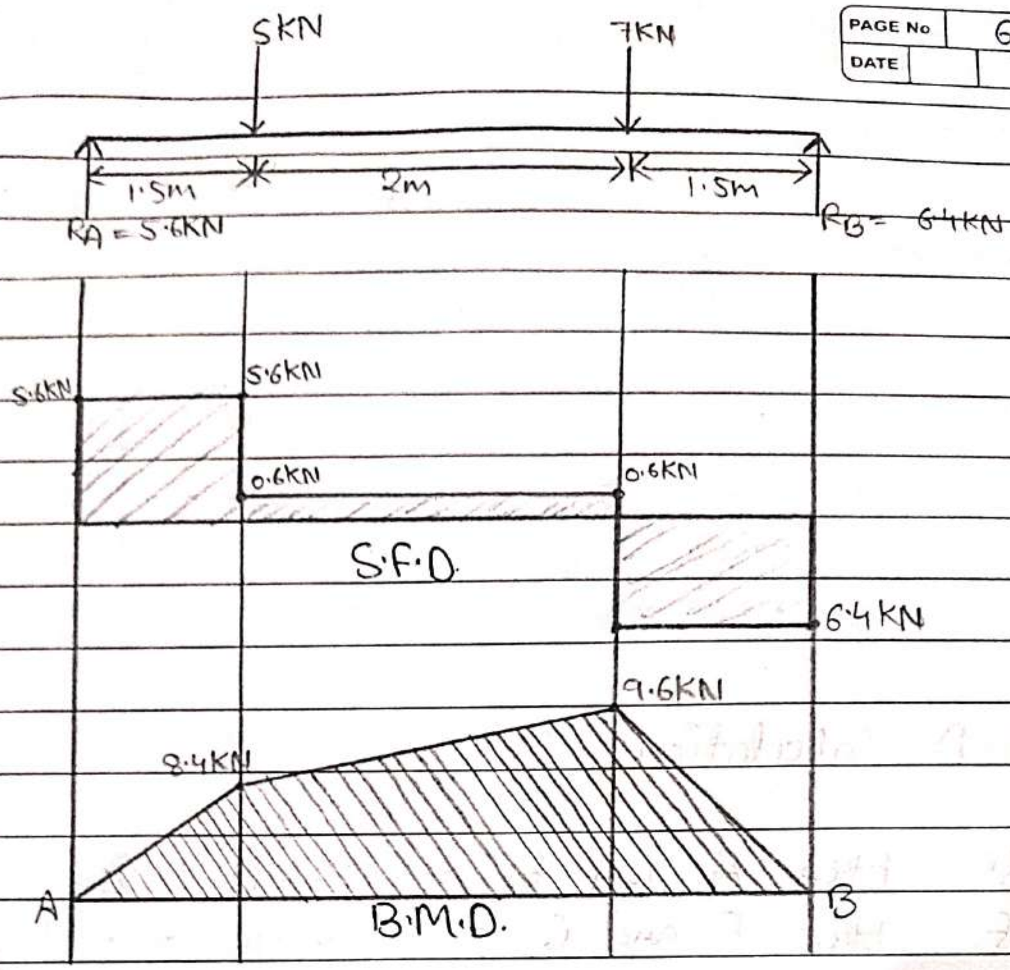
i) B.M at point A → $= 0$.
OR $-5\times1.5 - 7\times3.5 + 6.4\times5 = 0$.

(ii) B.M at point B $= 0$.
OR $= 5.6\times5 - 5\times3.5 - 7\times1.5 = 0$.

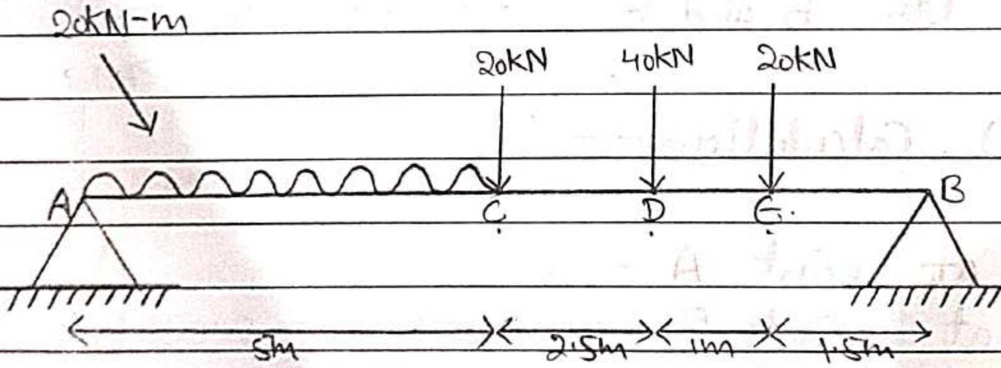
(iii) B.M at point C $= 5.6 \times 1.5 = 8.4$ KN-m
OR $= -7\times2 + 6.4\times3.5$
$= 8.4$ KN

(iv) B.M at point D $= (6.4\times1.5) = 9.6$ KN-m
OR $5.6\times3.5 - 5\times2 = 9.6$ KN-m

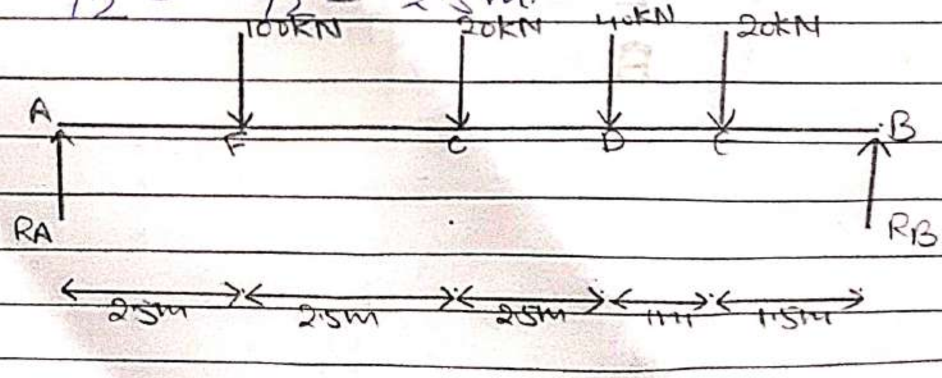




Imp
Q-2 Draw the SFD and BMD showing the important valu



Soln Total load btw AC is 20×5 = 100kN and their cent id is B/2 = 5/2 = 2.5m.



$\circlearrowright \Sigma M_A = 0$

$\Rightarrow 100 \times 2.5 + 20 \times 5 + 40 \times 7.5 + 20 \times 8.5 - R_B \times 10.$

$= 820 = 10 R_B$

$\boxed{R_B = 82 kN.}$

$\uparrow \Sigma F_y = 0$

$= R_A - 100 - 20 - 40 - 20 + R_B = 0$

$= R_A - 180 + 82 = 0$

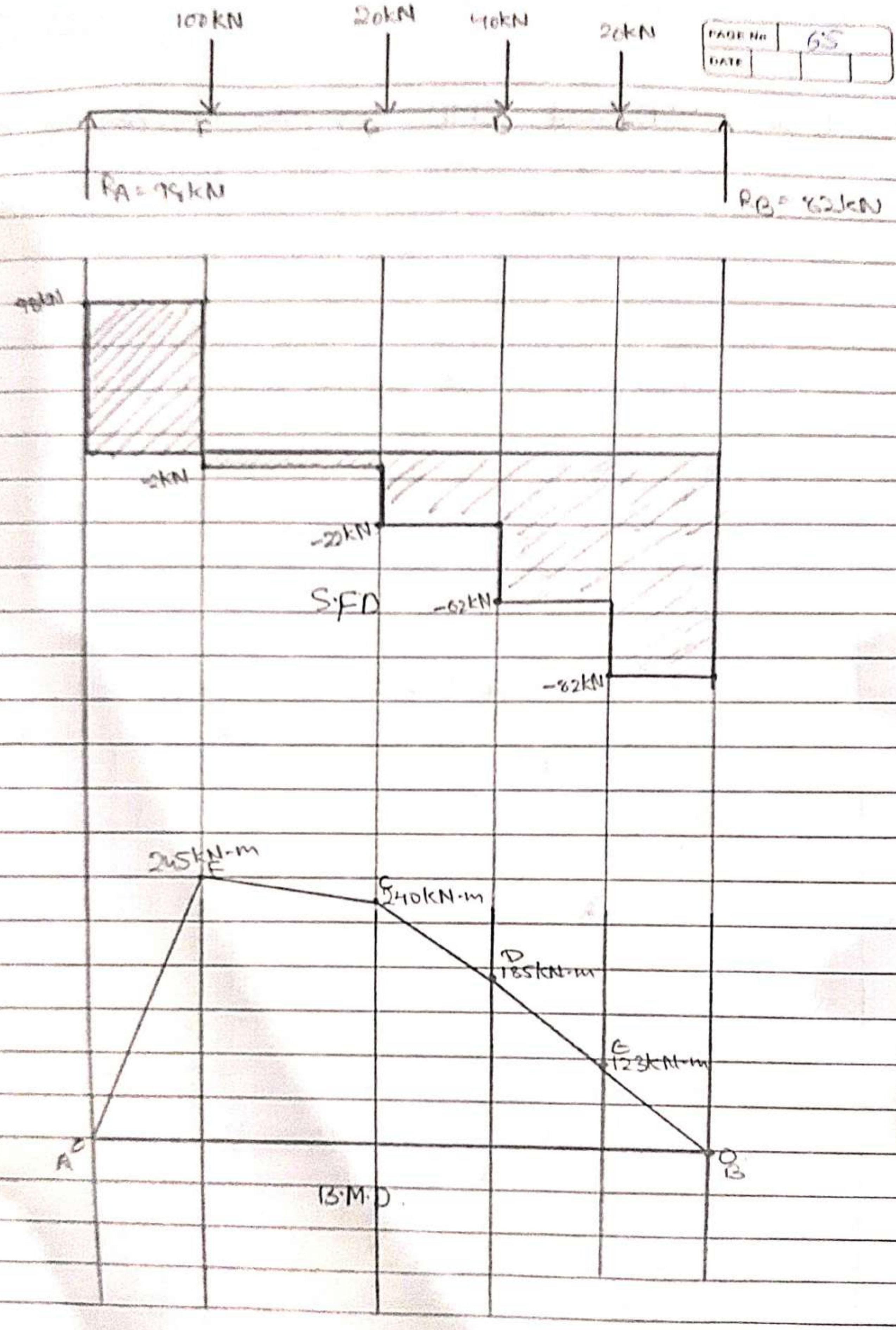
$\boxed{R_A = 98 kN.}$

SFD Calculations.

(i) SF btw A and F = 98 kN. (left section).
(ii) SF btw F and C = (98 − 100) = −2 kN. (left section
(iii) SF btw C and D = (98 − 100 − 20) = −22 kN (left section)
(iv) SF btw D and E = (20 − 82) = −62 kN (right section).
(v) SF btw B and E = −82 kN (right section).

BMD Calculations —

(i) BM at point A = 0
BM at point F = 98 × 2.5 = 245 kN-m
BM at point C = 98 × 5 − 100 × 2.5 = 240 kN-m.
" " " D = 82 × 2.5 − 20 × 1 = 185 kN-m
" " " E = 82 × 1.5 = 123 kN-m
" " " B = 0.


PAGE No 65
DATE
100kN
20kN
40kN
20kN
F
G
D
E
RA = 98kN
RB = 82kN
98kN
-2kN
-22kN
SFD
-62kN
-82kN
245kN-m
F
G
240kN-m
D
185kN-m
E
123kN-m
A
0
B
B.M.D

Third Module.

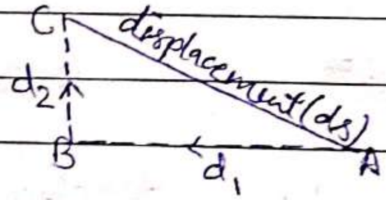
Kinematics and Kinetics

Kinematics:- When equation of motion are used to find velocity, acceleration & time without causes (forces) is known as kinematics.
for v, a & t,

$$v = u + at \quad \text{——(i)}$$
$$v^2 = u^2 + 2as \quad \text{——(ii)}$$
$$s = at + \frac{1}{2}at^2 \quad \text{——(iii)}$$

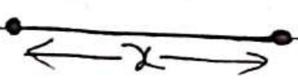
Displacement:- The shortest distance covered from the particle from the reference point.

Total distance $= d_1 + d_2$
displacement, ds
It is the vector quantity



Velocity:- The rate of change of position of a body w.r.t time is called velocity

$v = \frac{dx}{dt}$, Unit = m/s, or km/hr.



Acceleration:- The rate of change velocity w.r.t time is called acceleration

$a = \frac{dv}{dt}$, Unit — m/s^2 or km/hr^2

Related Questions

Q The motion of the particle is given by
$s = t^3 - 3t^2 + 2t + 5$.
find, (i) velocity & acceleration after 4sec.
(ii) max. & min velocity & corresponding displacement.
(iii) Time at which velocity is 0.

(i) $\frac{ds}{dt} = 3t^2 - 6t + 2$, at $t=4$, $\frac{ds}{dt} = 26m/sec$ (velocity)

$\frac{d^2s}{dt^2} = \frac{dv}{dt} = 6t - 6$ at $t=4$, $\frac{d^2s}{dt^2} = 18m/sec^2$ (acceleration)

Second derivative,

$$\frac{d^2v}{dt^2} = 6 \text{ [min]}.$$

$\frac{dv}{dt} = 0$, $= 6t - 6$ as $t = 1sec$ velocity is min

$$\frac{ds}{dt} = 3 - 6 + 2 = -1m/sec$$

Corresponding displacement

$s = (1)^3 - 3(1)^2 + 2(1) + 5$

$\boxed{s = 5m}$

(iii) $3t^2 - 6t + 2 = 0$

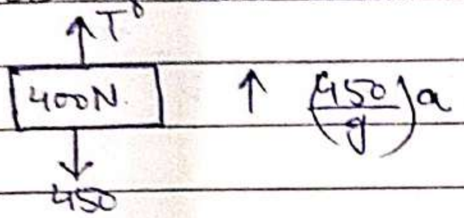
$t = 0.422$ & 1.57 sec.

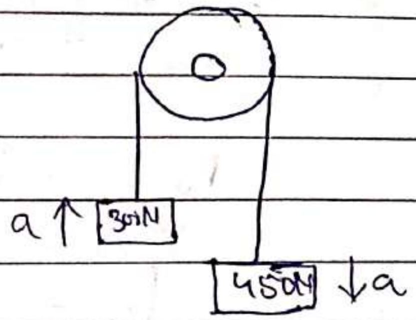
Q-2 $v = t^3 - t^2 - 3t + 2$.
if particle is at a distance 10m after 2sec.
find (i) acc. $\left(\frac{dv}{dt}\right)$ & displacement (s) after 4sec

Related Questions

Q Find acceleration of each block and tension in block

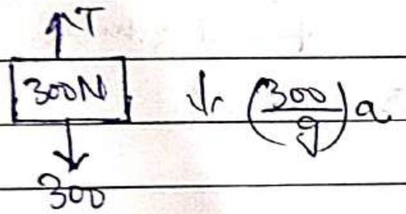
FBD of 400N block





$$\Sigma f_y = T + \left(\frac{450}{g}\right)a - 450 = 0 \quad \text{— (I)}.$$

FBD of 300.



$$\Sigma f_y = T - \left(\frac{300}{g}\right)a - 300 = 0 \quad \text{— (II)}$$

Solve Eqn (I) and Eqn (II), we get

$$a = 9.962 \text{ m/sec}^2, \quad T = 360\text{N}.$$

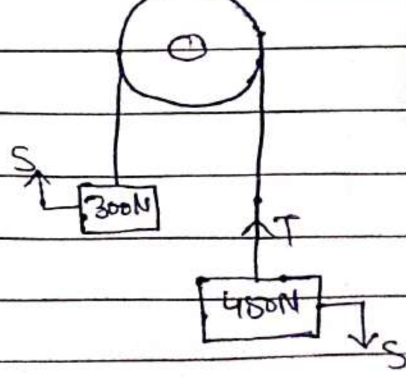
Q How much distance block will move its increasing velocity from 2 to 4sec.

Soln $u = 2, \quad v = 4$

Total work done $= +450 \times S - 300 \times S$

$= S \times (450 - 300)$

$= 150S$

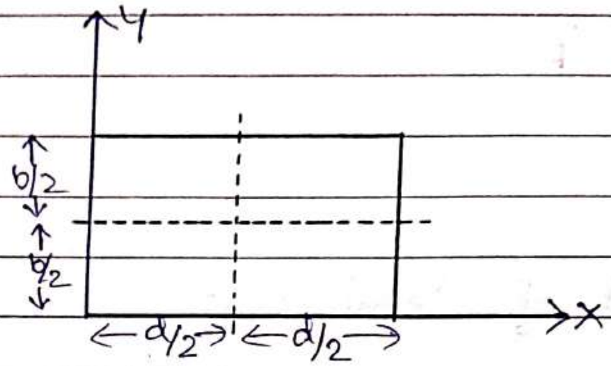


2nd Module (Part-2).

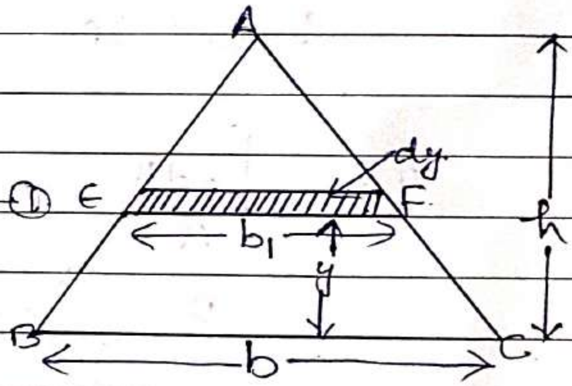
Centroid and Moment of Inertia.

Centroid:- It is the point on plane area where total area of fig. concentrated.
It always lies on the axis of symmetry if it is exist.

Eg (i) Rectangle:-



(ii) Triangle:-



$\therefore$ Area of element $= dA = b_1 dy$

$dA = \left(1-\frac{y}{h}\right) b\,dy$ —— (I)

Area of $\Delta = \frac{1}{2} \times b \times h$ —— (II)

From eqn (I) and (II)

$$\frac{b_1}{b} = \left(\frac{h-y}{h}\right)$$

$$b_1 = \left(\frac{h-y}{h}\right) b = \left(1-\frac{y}{h}\right) b$$

$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\int y\,dA}{A}$$

$$\int_0^h y\left(1-\frac{y}{h}\right) b\,dy \Rightarrow \int_0^h \left(y - \frac{y^2}{h}\right) b\,dy$$

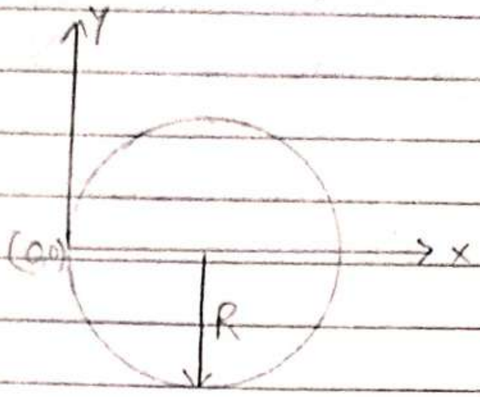
$$\Rightarrow b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h = \frac{bh^2}{6}$$

$$\bar{y} = \frac{\cancel{b}h^{\cancel{2}}}{6_3} \times \frac{2}{\cancel{bh}} = h/3$$

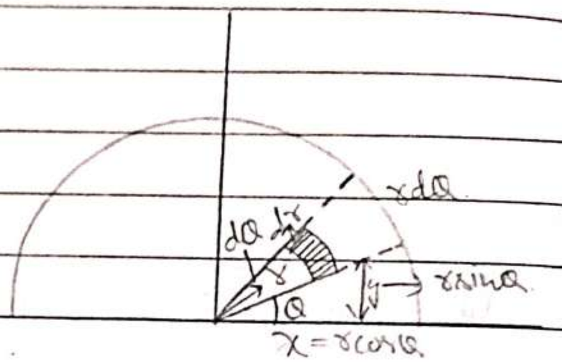
$\therefore (\bar{x}, \bar{y}) = (b/2, h/3)$

(iii) Circle –

$(\bar{x}, \bar{y}) = (R, 0)$



(iv) Centroid of Semicircle.



Area of semicircle $= \frac{\pi r^2}{2}$

Area $(dA) = r d\theta \cdot dr$.

$$\bar{y} = \frac{\int y\,dA}{A} = \int_0^{\pi}\int_0^{R} (r^2 \sin\theta)\,d\theta\,dr / A$$

$$= \frac{R^3}{3}\int_0^{\pi} \sin\theta\,d\theta = \frac{2R^3}{3/A}$$

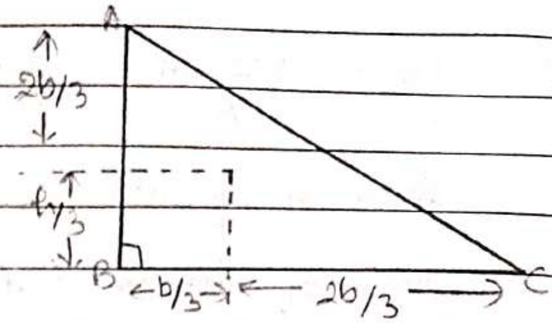
$$\bar{y} = \frac{\frac{2R^3}{3}}{\frac{\pi r^2}{2}} = \frac{2R^{\cancel{3}}}{3} \times \frac{2}{\pi \cancel{R^2}} = \frac{4R}{3\pi}$$

Hence, $(\bar{x}, \bar{y}) = \left(R, \frac{4R}{3\pi}\right)$

(vi) Centroid of right angled triangle.

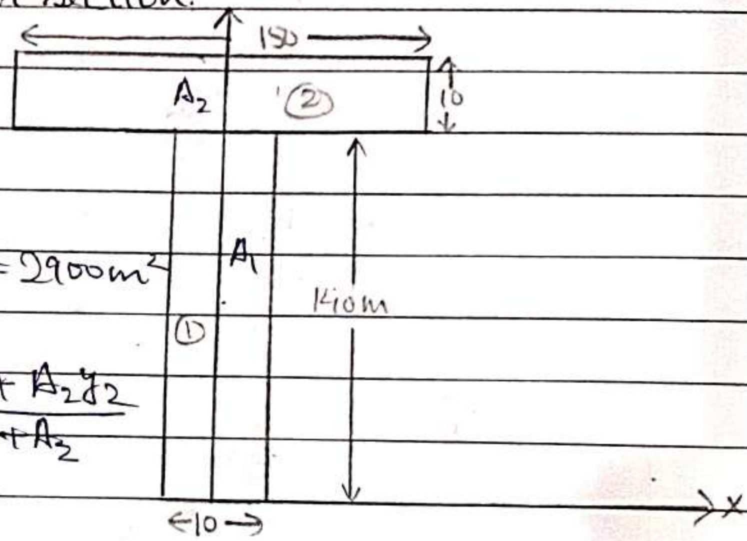
Area $= \frac{1}{2} \times b \times h$

$(\bar{x}, \bar{y}) = (b/3, h/3)$.



Related Questions

Q Find the centroid of given section.



$A_1 = 140 \times 10 = 1400 m^2$

$A_2 = 150 \times 10 = 1500 m^2$

$A = A_1 + A_2 = 1500 + 1400 = 2900 m^2$

$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$, $\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

Centroid of 1st ~~[illegible]~~ □

$(\bar{x}_1, \bar{y}_1) = (0, 70)$

Centroid of 2nd □

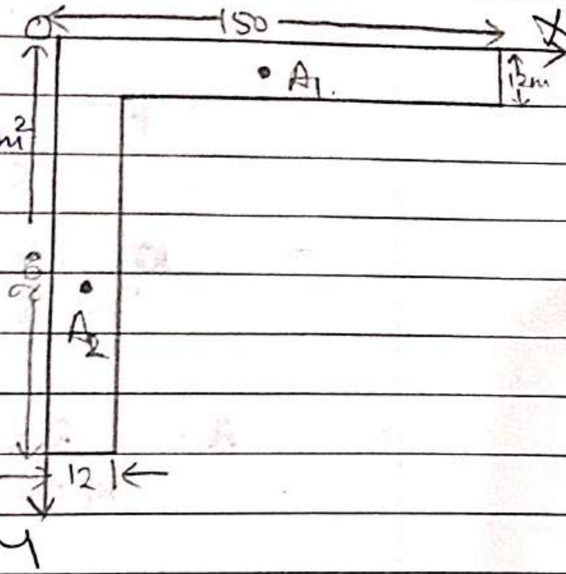
$(\bar{x}_2, \bar{y}_2) = (0, 145)$

$\bar{x} = \frac{1400 \times 0 + 1500 \times 0}{2900} = 0$.

$\bar{y} = \frac{1400 \times 70 + 1500 \times 145}{2900} = 108.79$.

Hence; Centroid of given figure $(0, 108.79)$

Q Find the centroid of given section.



$A_1 = 150 \times 12 = 1800 m^2$

$A_2 = \cancel{20}(200-12) \times 12 = 188 \times 12 = 2256 m^2$

$A = A_1 + A_2 = 1800 + 2256 = 4056 m^2$

Centroid of 1st ~~triangle~~.

$(\bar{x}_1, \bar{y}_1) = (75, 6)$.

$(\bar{x}_2, \bar{y}_2) = \left(6, 12 + \frac{1}{2}(188)\right)$.

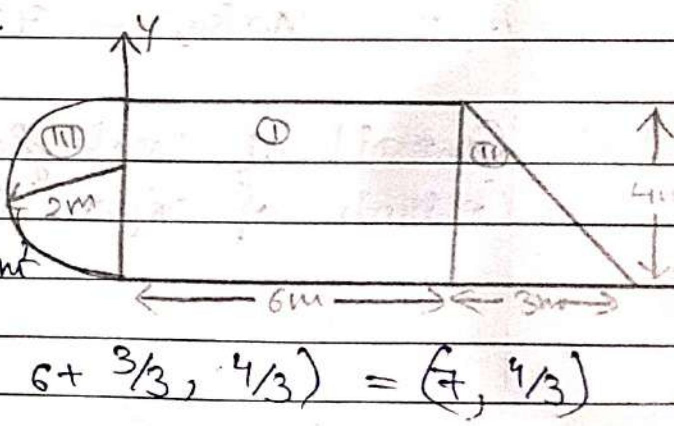
$= (6, \cancel{24}\,12 + 94) = (6, 106)$.

$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{1800 \times 75 + 2256 \times 6}{4056} = 36.62 m^2$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{1800 \times 6 + 2256 \times 106}{4056} = 61.62 m^2$

∴ Hence the centroid of section is $(36.62, 61.62)$

Q Find the centroid of the section



$A_{II} = \frac{1}{2} \times 3 \times 4 = 6 m^2$

$A_1 = 6 \times 4 = 24 m^2$

$A_3 = \frac{\pi r^2}{2} = \frac{\pi \times 4}{2} = 2\pi = 6.28 m^2$

$(x_1, y_1) = (3, 2)$, $(x_2, y_2) = (6 + 3/3, 4/3) = (7, 4/3)$

$(x_3, y_3) = \left(-\frac{4 \times 2}{3\pi}, 2\right) \Rightarrow \left(-\frac{8}{3\pi}, 2\right) = (-0.848, 2)$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1+A_2+A_3}$$

$$= \frac{6\times 7 + 24\times 3 + 6.28\times(-0.848)}{6+24+6.28}$$

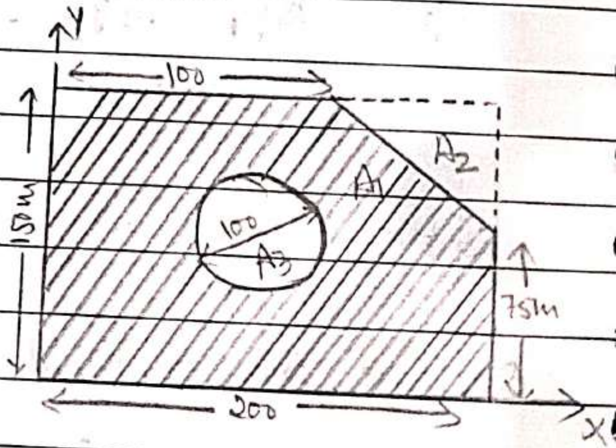
$$= \frac{108.674}{36.28} = 2.995\,m$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1+A_2+A_3} = \frac{24\times 2 + 6\times 4/3 + 6.283\times 2}{36.283}$$

$$= \frac{68.57}{36.283} = 1.889\,m.$$

Hence, the centroid of section is $(\bar{x},\bar{y}) = (2.99, 1.88)$

Q Find centroid of centre of circle such that it will be centroid of remaining shaded area.



Area $= A_1 - A_2 - A_3$

$A_1 = 150\times 200 = 30000\,m^2$

$A_2 = \frac{1}{2}\times 100\times 75 = 3750\,m^2$

$A_3 = \pi\times(50)^2 = 7853.98\,m^2$

Centroid of rectangle, $(x_1, y_1) = (100, 75)$

Centroid of right $\angle\Delta = \left(\frac{2\times 100+100}{3}, \frac{2}{3}\times 75 + 75\right)$

$= (166.66, 125)$

Centroid of circle, $(x_3 = \bar{x}, y_3 = \bar{y})$

$$\bar{x} = \frac{x_1 A_1 - x_2 A_2 - x_3 A_3}{A_1 - A_2 - A_3}$$

$$= \frac{100 \times 30000 - 166.66 \times 3750 - \bar{x}(7853.98)}{18396.02}$$

$$18396.02\bar{x} = 3000000 - 624975 - \bar{x}(7853.98)$$

$$(18396.02 + 7853.98)\bar{x} = 2375025$$

$$26250 \, \cancel{10542.04} \, \bar{x} = 2375025$$

$$\bar{x} = \frac{2375025}{\cancel{10542.04}\,26250} = 90.48$$

$$\bar{y} = \frac{y_1 A_1 - y_2 A_2 - \bar{y}(A_3)}{A_1 - A_2 - A_3}$$

$$\bar{y} = \frac{75 \times 30000 - 125 \times 3750 - \bar{y}(7853.98)}{18396.02}$$

$$18396.02\bar{y} = 2250000 - 468750 - \bar{y}(7853.98)$$

$$(18396.02 + 7853.98)\bar{y} = 1781250$$

$$\bar{y} = \frac{1781250}{26250}$$

$$\bar{y} = 67.85$$

Hence, the centroid of shaded region is (90.48, 67.8

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Chapter-4

Trusses

It is the structural member made up of slender members pin-connected at ends and is capable of taking nodes at joint and it is also called pin connected frame

- Assumptions of truss:-

(i) Self weight of members is negligible.
(ii) The cross section of the member is uniform.
(iii) The ends of the member are pin connected.
(iv) The load acts only at joints.

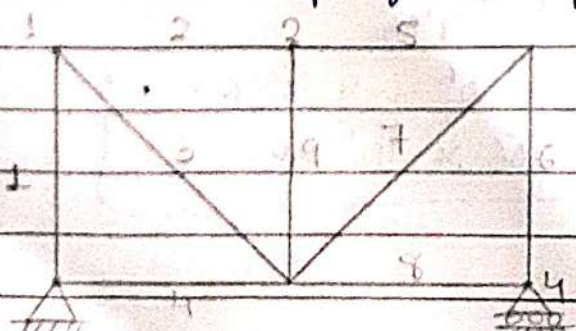
- Types of frame -

(i) Perfect frame:-

$$M = 2j - 3$$

where, M = no. of members.
j = no. of joints.

If the no. of member just sufficient to resist the load without undergoing appreciable deformation in shape is known as perfect shape.



$9 = 2 \times 6 - 2$

$9 = 9.$

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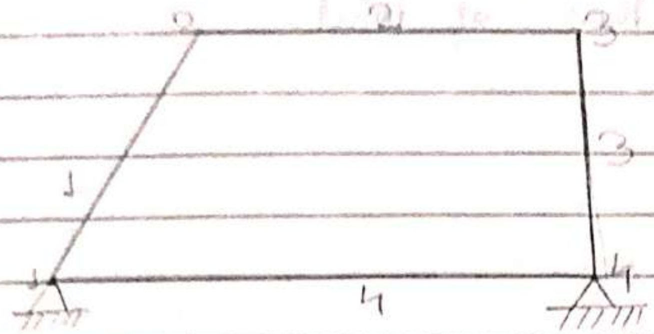
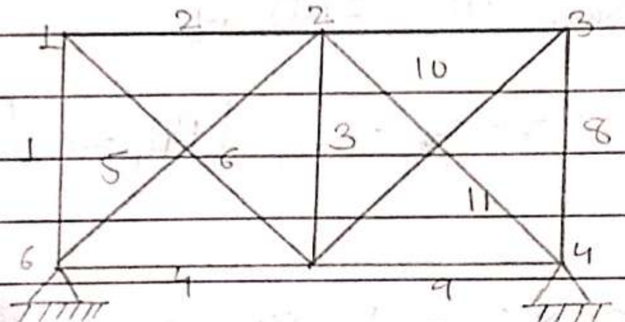
(ii) Deficient Truss:- If the no. of members in a truss is less than the required for the perfect truss is known as deficient truss.



$4 = 2\times4-3$
$4 < 5$

(iii) Redundant truss:- If the no of members more than in a truss than for required for perfect truss.



$M = 2j-3$
$11 = 2\times6-3$
$11 > 9.$

* Types of Method :-

(I) Method of joints

(II) Method of section

(III) Graphical Method (X).

→ Natures of forces in a member.

(I) Tensile →←   (II) Compression ←→

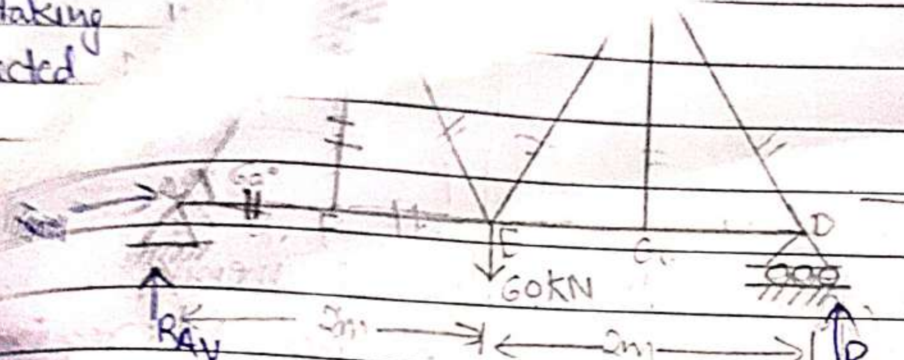
Chapter-4

Truss

the truss show

It is the struc'
members pin-
of taking
connected

• Ass



Draw the FBD having 2 unknown member, we have
Now, we draw the FBD of

$\overset{+}{\rightarrow} \Sigma f_x = 0$, $\boxed{R_{A_H} = 0}$

Σf_x
Σf_y
ΣM

$\circlearrowright^{+} M_A = 0$

$= 60 \times 2 - R_D \times 4 = 0$

$= 120 - 4R_D = 0$

$\cancel{+4}R_D = +\cancel{120}\ 30$

$\boxed{R_D = 30KN}$

$\uparrow \Sigma f_y = 0$

$= R_{A_V} + R_D - 60 = 0$

$R_{A_V} + 30KN - 60 = 0$

$\boxed{R_{A_V} = 30 KN}$

∴ Hence, it is symmetrical truss, now we solve left to right.

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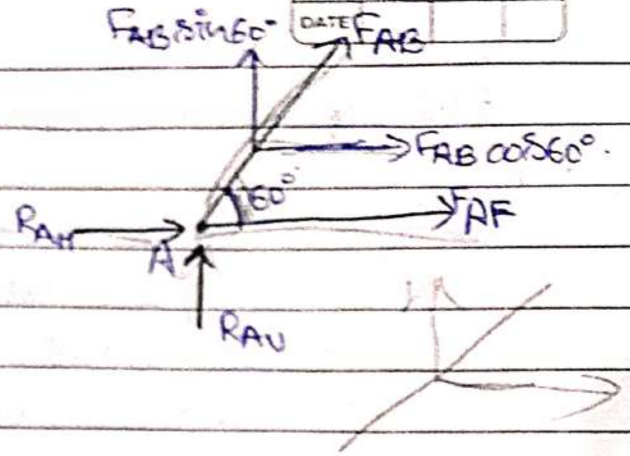
- FBD of A.



$+\uparrow \Sigma F_y = 0$

$= F_{AB}\sin 60° + R_{AV} = 0$

$F_{AB}\sin 60° = -30$

$F_{AB} = \dfrac{-30}{\sin 60°}$

$\boxed{F_{AB} = -34.64\ KN}$

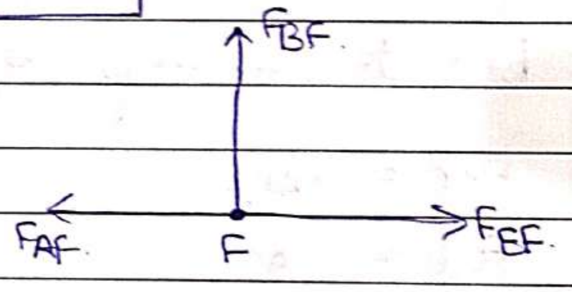
$\overset{+}{\rightarrow}\Sigma F_x = 0$

$= F_{AF} + F_{AB}\cos 60° = 0$

$= F_{AF} - 34.64 \cdot \cos 60° = 0$

$\boxed{F_{AF} = 17.32KN}$

- FBD of F:-



$\overset{+}{\rightarrow} F_x = 0$

$F_{EF} - F_{AF} = 0$

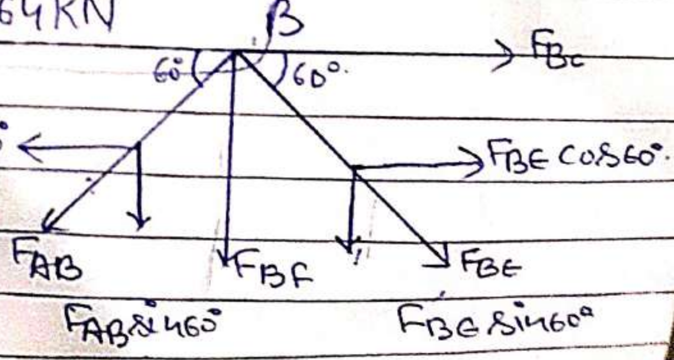
$F_{EF} - 17.32KN = 0$

$\boxed{F_{EF} = 17.32KN}$

$+\uparrow \Sigma F_y = 0 \Rightarrow \boxed{F_{BF} = 0}$

FBD of it B.

$BC = -34.64KN$



$+\uparrow F_y = \cancel{F_{BC} + F_{BC}\cos 60°}$

$= -F_{BF} + F_{AB}\sin 60° + F_{BE}\sin 60°$

$= 0 + (-34.64)\sin 60° + F_{BE}\sin 60° = 0$

$(+29.99) + F_{BE}\sin 60°$

$+F_{BE} = \dfrac{+29.99}{\sin 60°}$

$\boxed{F_{BE} = 34.62KN}$

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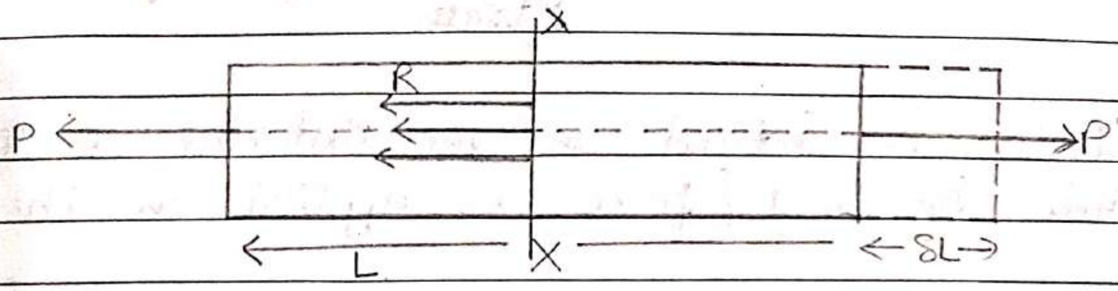
Module-4:-

Stress and Strain ⇒.



Stress:- The internal resistive force to deformation per unit cross sectional area is called a stress, intensity of stress or unit stress. It is denoted by σ

$$\boxed{\sigma = \frac{R}{A}}$$

Mathematically, stress may be defined as the force per unit area.

S.I unit of stress = $\frac{\text{S.I unit of force}}{\text{S.I unit of Area}} = \frac{N}{m^2} = Nm^{-2}$

- Nm^{-2} is called pascal and is denoted by P.a.

$$1Pa = 1N/m^2$$

- other unit of ~~pascal~~ stress are kilo pascal (KPa) Mega Pascal (MPa), Giga Pascal (GPa) etc.

Strain:- It is defined as the ratio between change in length and the original length. It is denoted by (e).

Strain = $\frac{\text{Change in length}}{\text{Original length}}$ $\boxed{e = \frac{\delta L}{L}}$

e = strain, δL = Change in length of the body

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Some important points regarding stress and strain

1.) Stress is induced in the material of the body while the load / force is applied on the body.

2.) The stress calculated by using the equation $\boxed{\sigma = \frac{P}{A}}$ is also called direct stress, longitudnal stress, axial stress or normal stress

3.) The strain calculated by using the equation $\boxed{e = \frac{\delta L}{L}}$ is also called direct strain, longitudinal strain, axial strain or linear strain or primary strain.

Stress - तनाव
Strain - [illegible]

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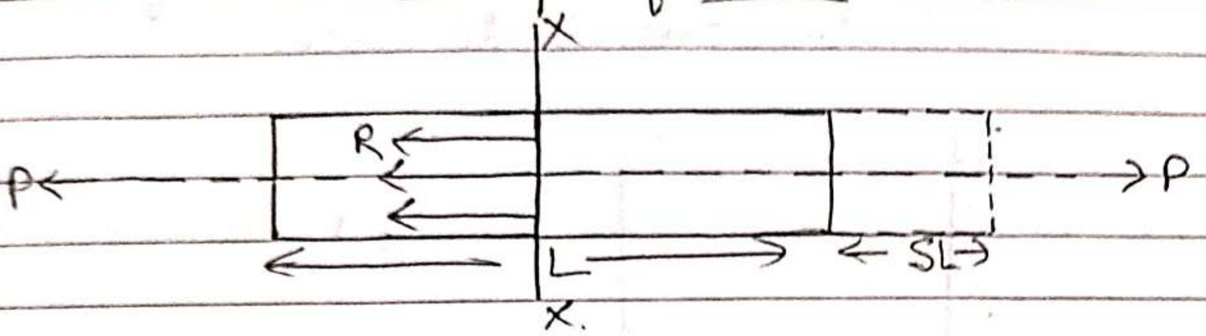
Concept of Stress



The internal resistive force to deformation per unit cross-sectional area is called stress intensity of stress or unit stress. It is denoted by σ.

$$\boxed{\sigma = \frac{R}{A}}$$

In the equillibrium condition, the internal resis R equal to P (external load) i.e., $R = P$.

$$\boxed{\sigma = \frac{P}{A}}$$

σ = Stress
P = Load acting on the body
A = Area cross section of the body

Mathematically, stress may be defined as the force per unit area

$$\boxed{\sigma = \frac{P}{A}}$$

SI unit of stress = $\frac{\text{SI unit of force}}{\text{SI unit of area}}$

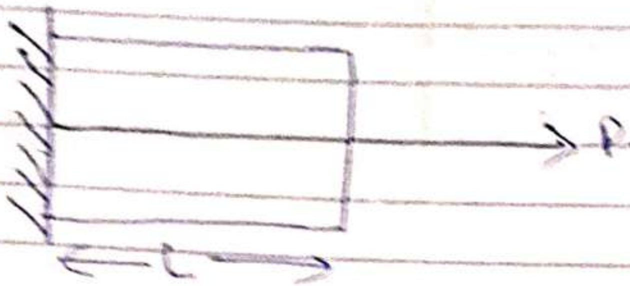
$= \frac{N}{m^2} = N/m^2$.

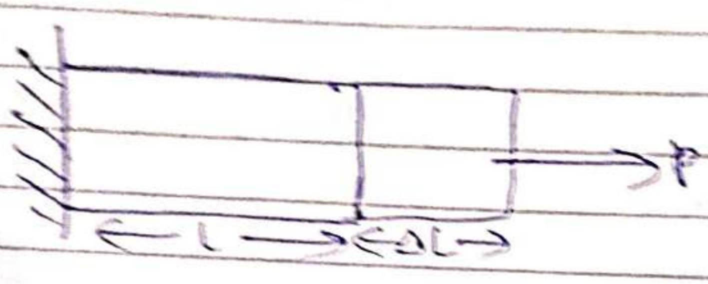
N/m^2 is called Pascal denoted by Pa

Therefore SI unit of stress is Pascal

$$\boxed{1 Pa = 1 N/m^2}$$

Strain:- It is the ratio of change in dimension to original dimension





Strain $= \frac{\Delta L}{L}$ or $\frac{\Delta b}{b}$

$$\text{Strain} = \frac{\text{change in dimension}}{\text{Original dimension}}$$

There unit is unitless. It is denoted by ϵ

$\frac{\text{stress}}{\text{strain}}$

Elastic constant:- It depends upon the property of material which having homogeneous or Inisotrophic

(i) Young modulus or modulus of rigidity (E)

$$E = \frac{\text{Normal or axail stress}}{\text{Normal or axial strain}} = \frac{\sigma}{\varepsilon}$$

(ii) Young modulos or modulus of rigidity =

$$G = \frac{\text{Shear stress}}{\text{Shear strain}} \left(\frac{\tau}{\gamma}\right).$$

(iii) Bulk modulus:- (K.)

$$K = \frac{\text{Volumetric stress } (\sigma V)}{\text{Volumetric strain } (\varepsilon V)}.$$

(iv) Poisson ratio (μ).

unitless

$$\mu = -\frac{(\text{laterial strain})}{\text{longitudinal strain}}$$

for all metal, $1/3 \leq \mu \leq 1/2$

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Factor of safety (FOS):-

$$FOS = \frac{\text{Maximum yield or ultimate strength of material}}{\text{Working stress } (\sigma w) \text{ or Allowable stress}}$$

$$FOS = \frac{S_Y \text{ or } S_u}{\sigma w}$$

$$\boxed{\sigma w = \frac{S_Y \text{ or } S_0}{FoS}}$$

Strain Energy:- It is defined as the energy absorption capacity of the material of a machine component when its ~~it~~ strained

Proof Resilance:- It is defined as the maximum energy absorption capacity of material of machine component at elastic limit is called proof resilance.

$\omega = \frac{1}{2} P \cdot \delta$ $\quad P = \text{load}$

$P.R = \frac{1}{2} P \cdot \delta$ $\quad \delta = \text{deformation } (\Delta l, \Delta b, \Delta t)$

Toughness:- Energy absorption capacity of a material of machine component at fracture limit is called toughness.

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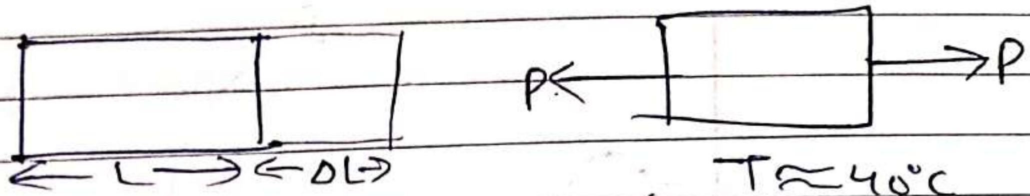
Thermal Stress :- The stress generated in the object by virtue of temp gradient when it is restricted for extension / contraction then this type of stress is called thermal stress



$T \simeq 40^\circ C$

$$(\Delta l)_{Thermal} = \alpha \Delta T L$$

where α = Thermal coefficient

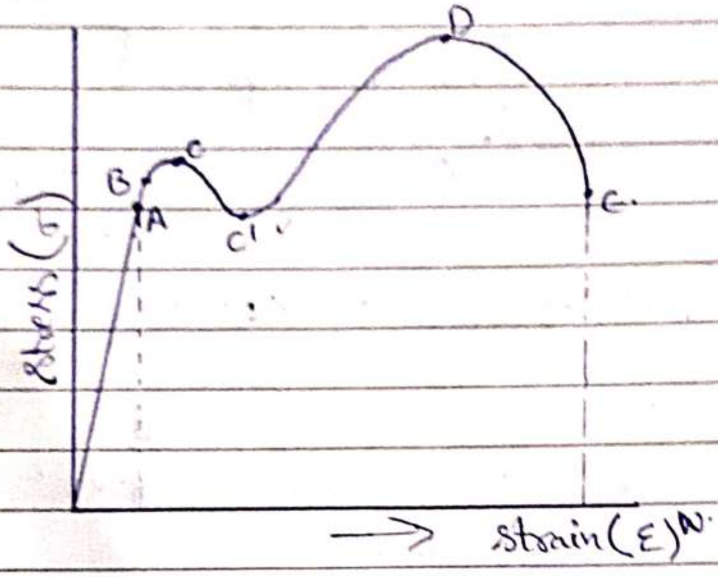
ΔT = Temp gradient

$T_2 - T_1$

L = length of the object

Stress Strain Diagram of ductile material (Mild Steel)

→ Draw with the help of Universal testing machine (UTM).
→ Gradually applied load.



O → Origin
A → Limit of proportionality
B → Limit of Elasticity
C → Upper yield point
C' → Lower end point.
D → Ultimate stress point
E → Fracture of breaking point

OM = Elastic region
ON = Plastic region

Limits of proportionality:- It is the limiting value of stress up to which stress ∝ to strain.

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Limits of Elasticity:- It is the limiting ~~va~~ value of the stress up to which the if the material is streached and released strain disappear completly and material region is original length

Upper yield point:- This is the stress at which the load starts reducing and strain increases. This phenomenon of material is calle yielding of material

Lower yield point:- At this stage, stress remains same and strain increases for sometime

Ultimate stress point:- This is the maximum stress that material can resist. At this stage, cross-sectional area at particular sect start reducing very fast. This is called neck form

Fracture point or Breaking:- The stress at which finally speciman failed or break is called fracture