* Interference of Light :

Light : It is a form of energy which make us able to see the objects in the dark.

Interference of Light : It is the phenomena of multiple light waves interfering with one another under certain circumstances, causing the combined amplitudes of the waves to either increase or decrease.

Types of Interference :

1. Division of wavefront
2. Division of Amplitude

- Division of wavefront

(1) YDSE (2) LLyod Mirror (3) Michelson Interferometer

- Division of Amplitude

(1) Newton's Ring Experiment

(2) Thin flim

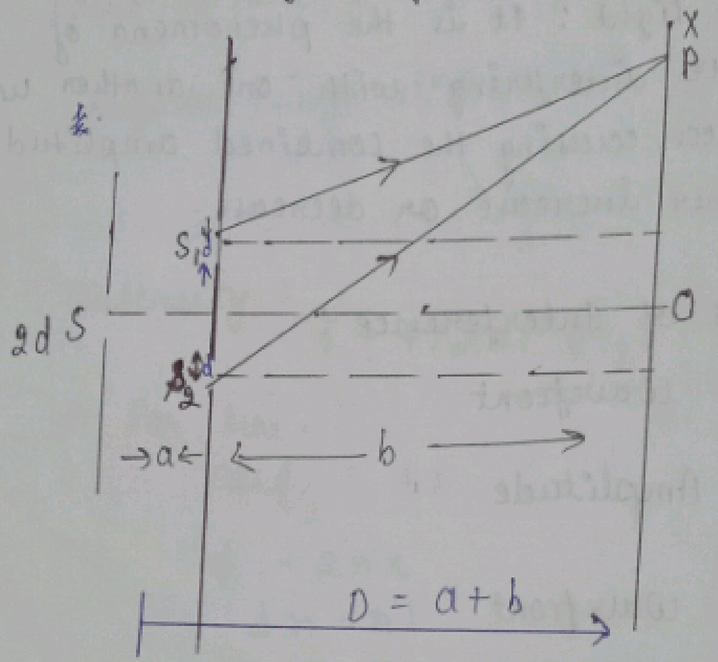
Essential Conditions for permanent/sustained Interference of Light

1. The two light sources must be coherent.
2. The interfering waves must have equal amplitude.
3. The separation b/w the two sources must be small as well as possible.

④ The two sources must be narrow.

⑤ The two coherent sources must emit monochromatic light.

Analysis of Interference:



Let a_1 and a_2 be the amplitudes of two waves from S_1 and S_2. Then the displacement due to waves at any time 't'

$$y_1 = a_1 \sin \omega t \quad -(i)$$

$$y_2 = a_2 \sin(\omega t + \delta) -(ii)$$

where δ is phase difference of two waves reaching at point P.

According to principle of superposition the resultant displacement will be

$$y = y_1 + y_2$$

$$a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta$$

$$Y = (a_1 + a_2 \cos\delta)\sin\omega t + a_2 \cos\omega t \sin\delta \quad \text{— (iii)}$$

Let $a_1 + a_2 \cos\delta = A\cos\phi$ — (4)

$a_2 \sin\delta = A \sin\phi$ — (5)

Now we have eq (3) becomes.

$$Y = A\cos\phi \sin\omega t + A\sin\phi \cos\omega t$$

$$Y = A\sin(\omega t + \phi) \text{ — (6)}$$

Hence resultant variation at P is simple harmonic of amplitude A and phase ϕ.

On squaring and adding eq (4) and (5)

$$a_1^2 + a_2^2\cos^2\delta = A^2\cos^2\phi$$

$$a_2^2\sin^2\delta = A^2\sin^2\phi$$

$$A^2\sin^2\phi + A^2\cos^2\phi = A^2$$

$$a_1^2 + a_2^2\cos^2\delta + a_2^2\sin^2\delta = A^2$$

$$\cancel{a_1^2 + 1} \quad a_1^2 + a_2^2 + 2a_1a_2\cos\delta = A^2$$

$$\boxed{A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta \text{ — (7)}}$$

The intensity at point P

$$I = A^2$$

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$\cos\delta = (2\cos^2\delta/2 - 1)$$

$$I = a_1^2 + a_2^2 + 2a_1a_2(2\cos^2\delta/2 - 1)$$

$$I = (a_1 - a_2)^2 + 4a_1a_2\cos^2\delta/2 \text{ — (8)}$$

Intensity will vary from point to point in the region of interference of two waves according to variation $\cos^2\delta/2$

Condition for Maxima:

The intensity I will be maximum at those points for which $\cos\delta$ will be maximum

$$I_{max} = (a_1+a_2)^2$$

$$\Rightarrow a_1^2 + a_2^2 + 2a_1a_2$$

path difference = even $\times \frac{\lambda}{2}$

$$\Delta = 2n\lambda/2 = n\lambda$$

$$n = 0, 1, 2, 3 \ldots$$

Condition for minima

When we put $\cos\delta = -1$

from eq (7) will get

$$I_{min} = (a_1-a_2)^2$$

Intensity distribution and conservation of energy of Interference:

The result of interference phenomenon shows the redistribution of energy.

the energy is missing at minima and appears at maxima So there is transference of energy from region to another.

We know that intensity at any point

when $a_1 = a_2 = a$

From eq (7)

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta \Rightarrow a_1^2 + a^2 + 2a^2\cos\delta$$

I_{max}, $\cos\delta = +1$

$$I_{max} = 2a^2 + 2a^2$$

$$I_{max} = 4a^2$$

$$I_{min} = 0$$

Average Intensity $\Rightarrow$ $I_{avg} = \dfrac{I_{max} + I_{min}}{2} \Rightarrow \dfrac{4a^2+0}{2}$

$\Rightarrow 2a^2$ if $a_1 = a_2 = a$

(ii) $a_1 \neq a_2$

$I_{max} = (a_1 + a_2)^2$, $I_{min}(a_1 - a_2)^2$

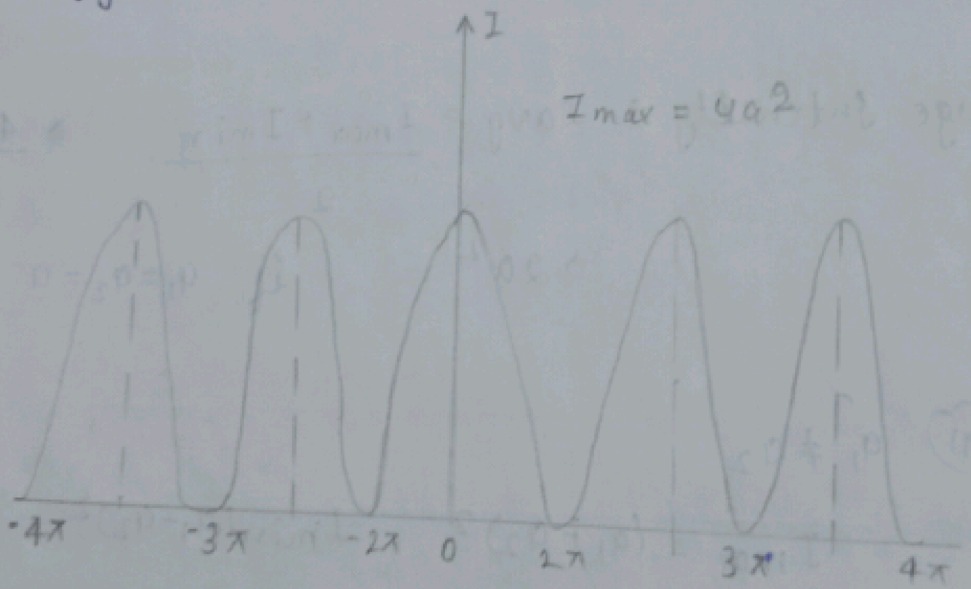
$$I_{avg} = \frac{\int_0^{2\pi} I \, d\delta}{\int_0^{2\pi} d\delta} \rightarrow \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta)\, d\delta}{\int_0^{2\pi} d\delta}$$

$$\frac{[a_1^2\delta + a_2^2\delta + 2a_1a_2 \sin\delta]_0^{2\pi}}{[\delta]_0^{2\pi}}$$

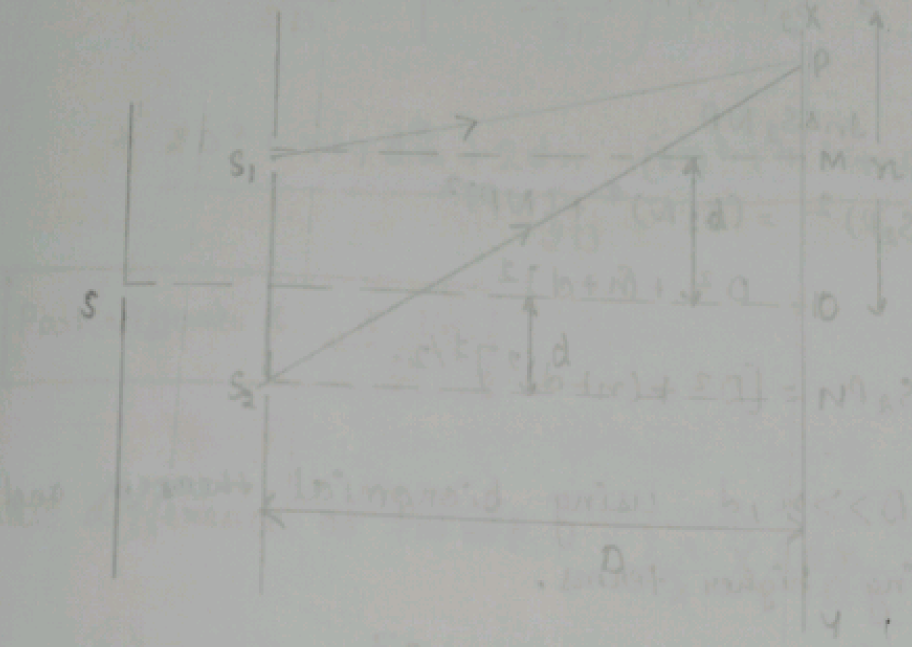
$$\frac{(a_1+a_2)^2 \cancel{2\pi}}{\cancel{2\pi}} = a_1^2 + a_2^2$$

$$I_{avg} = a_1^2 + a_2^2 = I_1 + I_2$$

Average intensity is equal to sum of the intensities of individual waves hence in interference pattern the energy simply tranfered minima to maxima. Thus the phenomenon of intensity is based on conservation of energy.



Analysis of Interference fringes for division of wave in a two slit arrangment —



Let S be a narrow slit illuminated by monochromatic light of wavelength λ and S_1 and S_2 be the two coherent sources separated by distance 2d, screen X, Y is placed at distance D from S_1 and S_2.

Let O be the foot of $\perp$ draw from the midpoint of S_1 and S_2.

O is equidistant from S_1 and S_2 therefore the path difference b/w the two waves at 'O' is zero. Thus point 'O' has maximum intensity.

Now consider a point P on the screen at a distance x from O. Draw S_1M and S_2N from S_1 and S_2 on the screen.

The path difference b/w the points waves reaching at point P from $S_2 \rightarrow S_2P - S_1P$

In ΔS_2NP

$$(S_2P)^2 = (S_2N)^2 + (NP)^2$$

$$D^2 + (n+d)^2$$

$$S_2P = [D^2 + (n+d)^2]^{1/2}$$

Here; $D >> n, d$ using bionomial theorem and neglecting higher terms.

$$S_2P = D\left[1 + \frac{(n+d)^2}{2D^2}\right]$$

$$S_2P = D + \frac{(n+d)^2}{2D} \quad -1$$

Similarly In ΔS_1MP

$$(S_1P)^2 = (S_2M)^2 + (PM)^2$$

$$(S_1P)^2 = D^2 + (n-d)^2$$

$$S_1P = [D^2 + (n-d)^2]^{1/2}$$

Using binominal theorem

$$S_1P = D\left[1 + \frac{(n-d)^2}{2D^2}\right]$$

$$\left[D + \frac{(n-d)^2}{2D}\right] \quad -2$$

point

Now Path difference b/w the waves reaching at P

$$\therefore \quad D + \frac{(x+d)^2}{2D} - \left[D + \frac{(x-d)^2}{2D}\right]$$

$$\Rightarrow \frac{2D^2 + x^2 + d^2 + 2dx - (2D^2 + x^2 + d^2 - 2dx)}{2D}$$

$$\boxed{\text{Path difference : } \frac{2dx}{D}}$$

Phase difference at Point P : $\frac{2\pi}{\lambda} \times \frac{2dx}{D} \Rightarrow \frac{4\pi dx}{\lambda D}$

- For maxima

$$\text{Path difference} \Rightarrow \frac{2dx}{D} = 2n\frac{\lambda}{2}$$

$$2dx = Dn\lambda$$

$$\boxed{x = \frac{Dn\lambda}{2d}}$$

Now n^{th} bright fringes from O replace x by x_n

$$x_n = \frac{Dn\lambda}{2d}$$

Distance of n^{th} bright fringes for central maxima.

$$x_n = \frac{Dn\lambda}{2d} \quad n = 0, 1, 2, 3$$

For Minima : $(2n+1)\lambda/2$

$$\frac{2dx}{D} = \frac{(2n+1)\lambda}{2}$$

$$4dx = (2n+1)\lambda D$$

$$x = \frac{(2n+1)\lambda D}{4d}$$

n^{th} dark fringes distance of n^{th} dark fringes from central maxima

$$x_n = \frac{(2n+1)\lambda D}{4d} \qquad n = 0, 1, 2, 3 \ldots$$

Separation of Fringes (Fringe width) :

The separation b/w any two successive bright or dark fringes is known as fringe width. It is denoted by 'ω'.

Fringe width for Bright fringes —

If x_{n+1} and x_n denotes the distances of $(n+1)^{th}$ and n^{th} bright fringes from central maxima, then

$$x_{n+1} = \frac{(n+1)D\lambda}{2d}$$

$$x_n = \frac{nD\lambda}{2d}$$

$$\frac{(n+1)D\lambda}{2d} - \frac{nD\lambda}{2d}$$

$$\omega_{Bright} = \frac{D\lambda}{2d}$$

Fringe width for Dark fringes

$$x_n = \frac{(2n+1)D\lambda}{4d}$$

$$x_{n+1} = \frac{[2(n+1)+1]D\lambda}{4d} = \frac{(2n+3)D\lambda}{4d}$$

$$\omega_{Dark} = \frac{D\lambda}{2d}$$

Angular fringe width: Angular fringe width is defined as angular separation b/w two successive bright or dark fringes. It is denoted by ω_θ.

$$\omega_\theta = \theta_{n+1} - \theta_n$$

Then we get $= \frac{x_{n+1}}{D} - \frac{x_n}{D}$

$$\frac{x_{n+1} - x_n}{D} = \frac{\omega}{D}$$

$$= \frac{D\lambda}{2d} / D$$

$$\boxed{\omega_\theta = \frac{\lambda}{2d}}$$

$$\tan\theta_n = \frac{x_n}{D}$$

$$\tan\theta_{n+1} = \frac{x_{n+1}}{D}$$

If θ_n is very small

$$\tan\theta_n \approx \theta_n \approx \theta_{n+1}$$

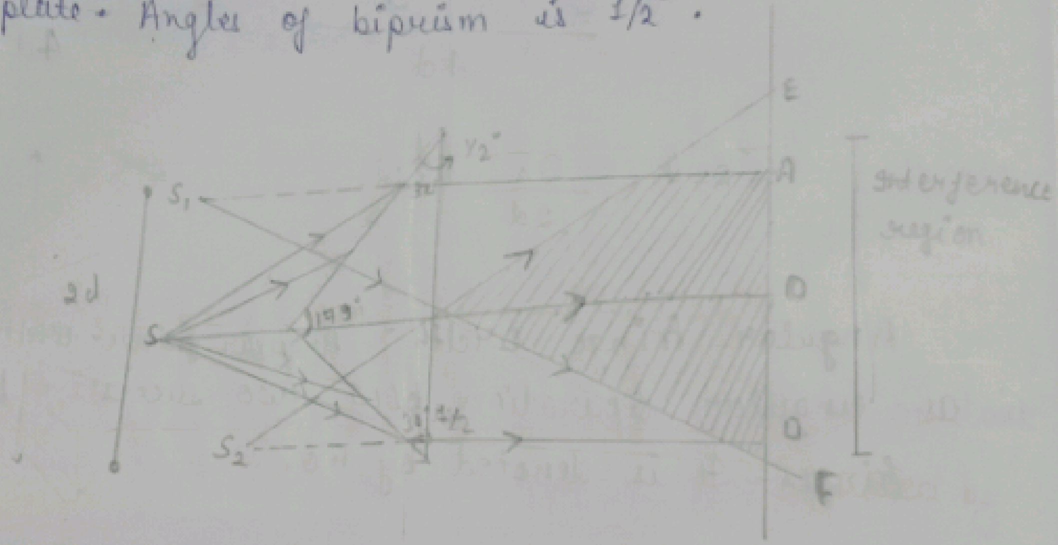
1

Fresnel's Biprism Experiment :

It is a device for producing coherent sources by division of wavefront.

Fresnel's Biprism is a combination of two prism the two prisms thin, acute angled or their base and joined.

There two faces made an obtuse angle of about 179°, so the other two angles are about 30 mins or $1/2°$. In practice biprism is prepared by grinding the sides of single optical glass plate. Angles of biprism is $1/2°$.



(a) Measurements of Fringe Width

$$\omega = \frac{\text{Distance moved}}{\text{No. of fringes passed}}$$

b) Measurement of $2d = \sqrt{d_1 d_2}$

Numericals :

(1) In a two slit interference pattern at a point we have observe the 10th order maximum for wavelength $\lambda = 7000 A^\circ$ what order will be visible here if the source of light is replaced by $\lambda = 5000 A^\circ$

$$\text{Path difference} = \frac{2n\lambda}{2}$$

$$n_1 \lambda_1 = n_2 \lambda_2$$

$$10 \times 7 \times 10^{-7} = n_2 \times 5 \times 10^{-7}$$

$$\boxed{n_2 = 14}$$

(2) The given light of wavelength $(\lambda) = 5700 A^\circ$ from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on screen 200 cm away is 2 cm. Find the double slit separation.

$$\lambda = 5700 A^\circ = 5700 \times 10^{-10} = 5700 \times 10^{-8} \text{ cm}$$

$$D = 200 \text{ cm}$$

$$10\, \omega = 2$$

$$\omega = 0.2 \text{ cm}$$

we know that

$$\omega = \frac{D\lambda}{2d}$$

$$0.2 = \frac{200 \times 5100 \times 10^{-8}}{2d}$$

$$2d = 51 \times 10^{-3}$$

$$\boxed{2d = 0.051 \text{ cm}}$$

(3) The distance b/w slit and biprism and that biprism and screen are each 50 cm. The obtuse angle of biprism is 179° and its refractive index is 1.5. If the width of fringes is 0.0135 cm. Calculate the λ of light use.

$\mu = 1.5$

angle of biprism $= 179°$

Remaining angle $\Rightarrow 180° - 179°$

$= 1°$

$\alpha = \frac{1}{2}° \Rightarrow \frac{1}{2} \times \frac{\pi}{180}$ radian

$a = 50$ cm

$b = 50$ cm

$D = (a+b) = (50+50)$

$\Rightarrow 100$ cm

$2d = 2a(\mu - 1)\alpha$

$2d = 2 \times 50 (1.5 - 1) \frac{1}{2} \times \frac{\pi}{180}$

$2d = 100(0.5) \times \frac{1}{2} \times \frac{\pi}{180}$

$2d = \frac{25 \times 3.14}{180} \Rightarrow 0.436$ cm

$\omega = \frac{D\lambda}{2d} \Rightarrow 0.0135 = \frac{100 \times \lambda}{0.436}$

$\lambda = \frac{0.005886}{100} \times 10^{10}$

$\boxed{\lambda = 588600 \text{ A}°}$

④ In a frensenal biprism experiment, the fringe width observe is 0.087 mm. What will it become if the ~~to~~ slit to biprism is reduce to 3/4 times the original distance.

Sol^n: $\omega = 0.087\,mm$

On reducing slit to biprism distance $\frac{3}{4}a$, 2d will reduce.

Now,

using deviation formula

$$2d = 2a(\mu-1)\alpha$$

Now if 'a' change then 2d and ω also changed

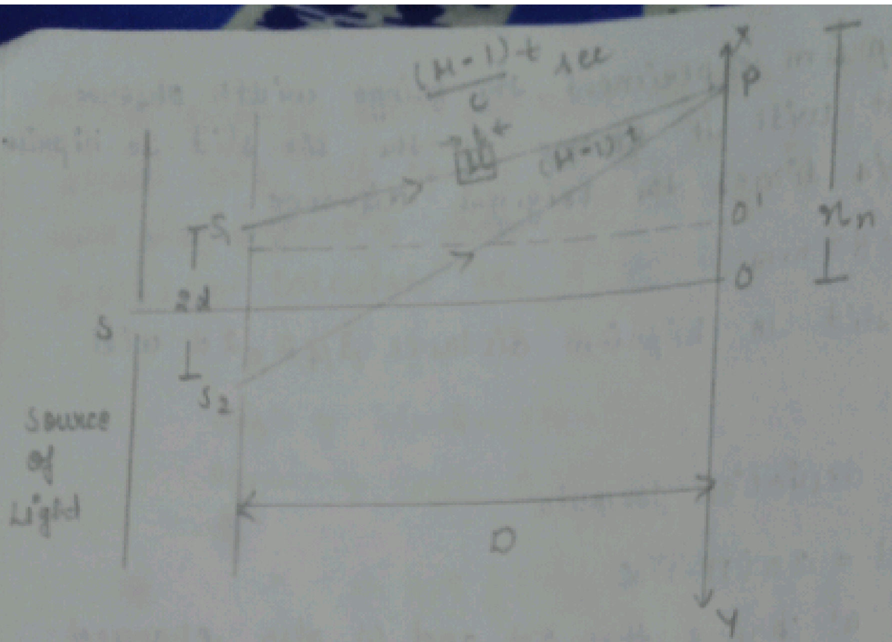
$$2d' = 2\times\frac{3}{4}a(\mu-1)\alpha$$

$$\Rightarrow \frac{3}{4}(2d) \qquad \omega' = \frac{D\lambda}{2d}$$

$$\Rightarrow \frac{D\lambda}{3/4(2d)} \qquad \Rightarrow \omega' = \frac{4}{3}\omega$$

$$\omega' = \frac{4}{3}\times 0.087\times 10^{-3}\,m$$

$$\boxed{\omega' = 0.116\times 10^{-3}\,m}$$ Ans



Displacement of Fringe :

The light reaching point P from S_1 has travel from S_1 has travel a distance 't' in sheet and $(S_1P - t)$ in the air

If 'c' and v be the velocities of ray in air and sheet respectively. Then time taken by light to reach from S_1 to P

$$\therefore \mu = \frac{c}{v}$$

$$v = \frac{c}{\mu}$$

$$= \frac{S_1P - t}{c} + \frac{t}{v}$$

$$\Rightarrow \frac{S_1P - t}{c} + \frac{\mu t}{c}$$

$$\Rightarrow \boxed{\frac{1}{c}[S_1P + (\mu-1)t]} \quad -(1)$$

Now the effective path difference at point P -

$$= S_2P - [S_1P + (\mu-1)t]$$

$$= \boxed{S_2P - S_1P - (\mu-1)t} \quad -(2)$$

we know the position of n^{th} bright fringes, then the path difference in the absence of sheet

$$\boxed{S_2P - S_1P = \frac{2d}{D} x_n} \quad -③$$

Now the effective path difference from eq ② and ③ we get

$$= \boxed{\frac{2d}{D} x_n - (\mu-1)t} \quad -④$$

condition for bright fringes at P -

Path diff = even $\times \frac{\lambda}{2}$ ⇒ $2n\lambda/2$ $= n\lambda$ $[n = 0, 1, 2 \ldots]$

$$\frac{2d}{D} x_n = (\mu-1)t = n\lambda$$

$$\boxed{x_n = \frac{D}{2d}[n\lambda + (\mu-1)t]} \quad -⑤$$

Similarly, in the absence of sheet $(t = 0)$

condition for bright fringes :-

$$\boxed{x'_n = \frac{Dn\lambda}{2d}} \quad -⑥$$

Now, the displacement of bright fringes

$x_0 = x_n - x'_n$ from eq ⑤ and ⑥

$$x_0 = \frac{D}{2d}[n\lambda + (\mu-1)t] - \frac{D}{2d} n\lambda$$

$$\boxed{x_0 = \frac{D}{2d}(\mu-1)t}$$ Bright

Similarly

$$\boxed{x_0 = \frac{D}{2d}(\mu-1)t} \text{ for Dark}$$

we know that $\omega = \frac{D\lambda}{2d}$

$$\frac{\omega}{\lambda} = \frac{D}{2d}$$

then

$$\boxed{x_0 = \frac{\omega}{\lambda}(\mu-1)t}$$

$\therefore$ n^{th} fringe width displaced then

$$x_0 = n\omega$$

$$n\omega = \frac{\omega}{\lambda}(\mu-1)t$$

$$\boxed{t = \frac{n\lambda}{(\mu-1)}}$$

* Interference in Thin film

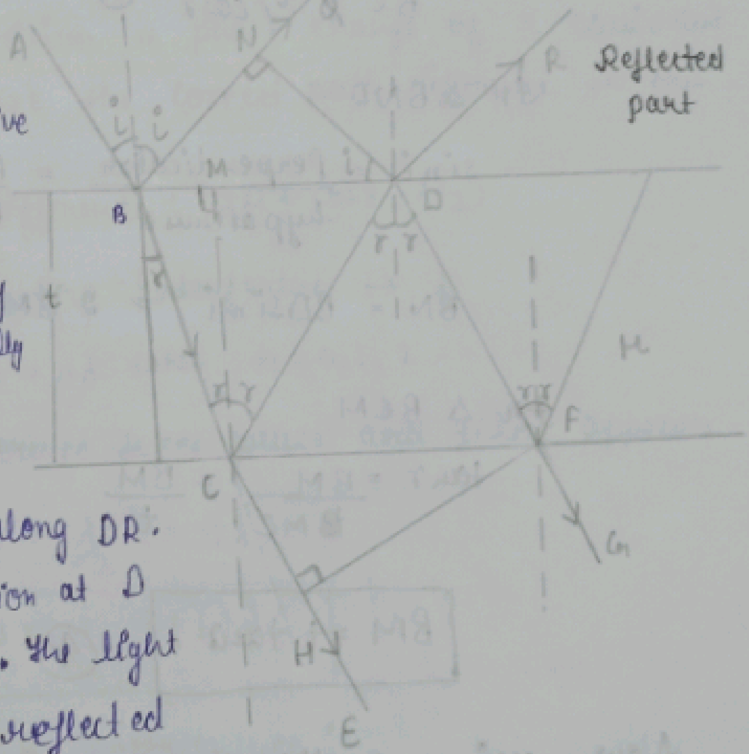
① By Reflected Light

② By transmitted Light

Thin film Interference : Q: How interference takes place in thin films? Show that the reflected and transmitted interference patterns are complimentary?

Newton and Hooks observed and develop the ~~stat~~ interference phenomenon due to multiple reflections from the space of thin transparent material. Every one is familiar with beautiful colour produced by a thin film of oil on the surface of water and also thin film of soap bubble. Young was able to explain the phenomenon on the basis of interference b/w light reflected from top and bottom surface of thin film. It has been observed in thin film takes place.

① By Reflected light ② transmitted light



Consider transparent film of thickness t and refractive index μ. A ray AB is incident on the upper surface of film is partially reflected along BQ and partially refracted along BC. At C part of it reflected along CD and partially emerge along DR. The ray CD after reflection at D finally emerge along FG. The light rays BQ and DR are reflected part & CE and FG are transmitted part.

Reflected Part: The difference in the path b/w BQ and DR can be calculated. Draw normal DN to BQ. The angle of incident is i and angle of refraction is r, then

Path difference $\Delta = (BC + CD)$ in film $- BN$ in air

$$= (BC + CD)\mu - BN \quad (1)$$

$$~~= (BC + BC)\mu - BN$$

$$2(BC)\mu - BN \quad (1)$$

In ΔBCM

$$\cos r = \frac{Base}{hypotenuse} \Rightarrow \frac{MC}{BC} = \frac{t}{BC}$$

$$BC = t/\cos r \quad (2)$$

In ΔBND

$$\sin i = \frac{Perpendicular}{hypotenuse} = \frac{BN}{BD}$$

$$BN = BD \sin i = 2\, BM \sin i \quad (3)$$

In ΔBCM

$$\tan r = \frac{BM}{\cancel{B}MC} = \frac{BM}{t}$$

$$\boxed{BM = t \tan r} \quad (4)$$

Now, using Snell's law

$$\frac{\sin i}{\sin r} = \mu \Rightarrow \sin i = \sin r \quad (5)$$

from eq (4) and (5) eq(3) can be written as

$$BN = 2t \tan r\, \mu \sin r$$

$$BN = 2t\, \mu \tan r \sin r \quad (6)$$

from eq ⑤ & ⑥ eq ① can be written as

$$\Delta = \frac{2\mu t}{\cos r} - 2\mu t \tan r \sin r$$

$$= \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin r}{\cos r} \sin r$$

$$= \frac{2\mu t}{\cos r}(1-\sin^2 r) = \frac{2\mu t}{\cancel{\cos r}} \times \cancel{\cos^2 r}$$

$$\boxed{\Delta = 2\mu t \cos r} \quad — ⑦$$

This eq ⑦ in case of reflected light does not represent correct path difference but only the apparent. According to stokes laws of reflection, when light is reflected from surface of optically denser medium a phase change of π equivalent to $\lambda/2$ occurs. Therefore the correct path difference in this case :

effective path difference : $(2\mu t \cos r - \lambda/2)$

Condn for constructive interference →

If path diff. $\Delta = n\lambda$ when $n = 0, 1, 2 ....$

constructive Interference takes place and film appears bright. $2\mu t \cos r - \frac{\lambda}{2} = n\lambda$

$$\boxed{2\mu t \cos r = (2n \pm 1)\lambda/2}$$

Condn for destructive interference :

If path difference is $(2n \pm 1)\frac{\lambda}{2}$ where

$n = 0, 1, 2 .....$ destructive interference

takes place and the film appears dark

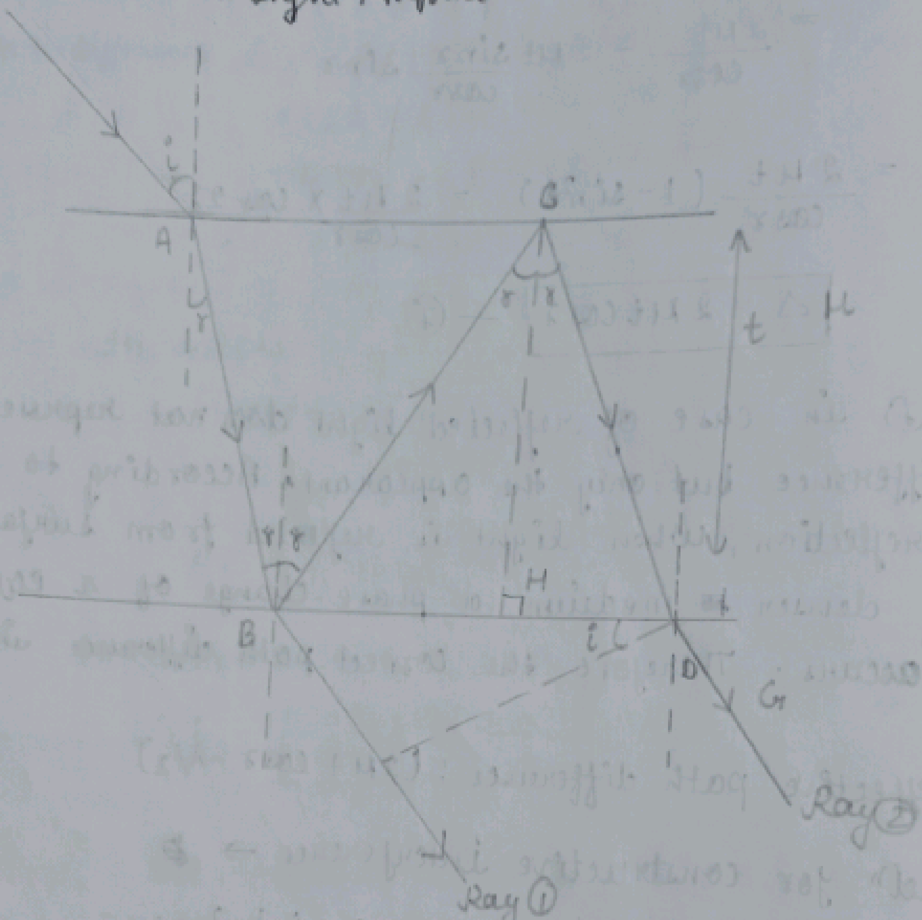
$$2\mu t \cos r - \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (n+1)\lambda$$

$\therefore n \rightarrow$ integer so $(n+1)$ will also be integer takes as 'n'

$$\boxed{2\mu t \cos r = n\lambda}$$

(b) In transmitted light / Refracted



$\Delta = (BC + CD)$ in film $-$ BE in air $-①$

$\Delta = \mu(BC+CD) - BE$ $-②$

In ΔBCH and ΔCHD

$\cos r = \frac{CH}{BC} = \frac{t}{BC} \Rightarrow BC = \frac{t}{\cos r}$ $-③$

$\cos r \Rightarrow \frac{CH}{BCD} = \frac{t}{CD} \Rightarrow CD = \frac{t}{\cos r}$ $-④$

$\Delta = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - BE$

Put eq ③ and ④ in eq ②

$\frac{2\mu t}{\cos r} - BE$ — (5)

In ΔBED

$\sin i = \frac{BE}{BD} \Rightarrow \frac{BE}{BH + HD}$ — (6)

In ΔBCH and ΔCHD

$\tan r = \frac{BH}{CH}$ $\quad BH = t \cdot \tan r$ — (7)

$\tan r = \frac{HD}{CH} \Rightarrow \frac{HD}{t} \Rightarrow HD = t \cdot \tan r$ — (8)

$\sin i = \frac{BE}{2t \tan r}$

$$\boxed{BE = 2t \cdot \tan r \cdot \sin i} \text{ — (9)}$$

Put eq (9) in (5)

$\frac{2\mu t}{\cos r} - 2t \cdot \tan r \cdot \sin i$

$2t\left(\frac{\mu}{\cos r} - \tan r \sin i\right)$ — (10)

From Snell's Law

$\frac{\sin i}{\sin r} = \mu$ — (11)

$\sin i = \mu \sin r$ — (12)

$\Delta = \frac{2\mu t}{\cos r} - \frac{2t \sin r}{\cos r} \mu \cdot \sin r$

$\Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$

$$\boxed{\Delta = 2\mu t \cos r} \text{ — (13)}$$

Applying Stoke's treatment

$$\Delta = 2\mu t \cos r \pm 0$$

$$\boxed{\Delta = 2\mu t \cos r} \quad -(14)$$

Condition for Maxima (Bright Fringe)

For constructive Interference :

$$\Delta = n\lambda \quad -(15)$$

$$\boxed{2\mu t \cos r = n\lambda} \quad -16$$

B. Fringes

Condition for Minima (D. Fringes)

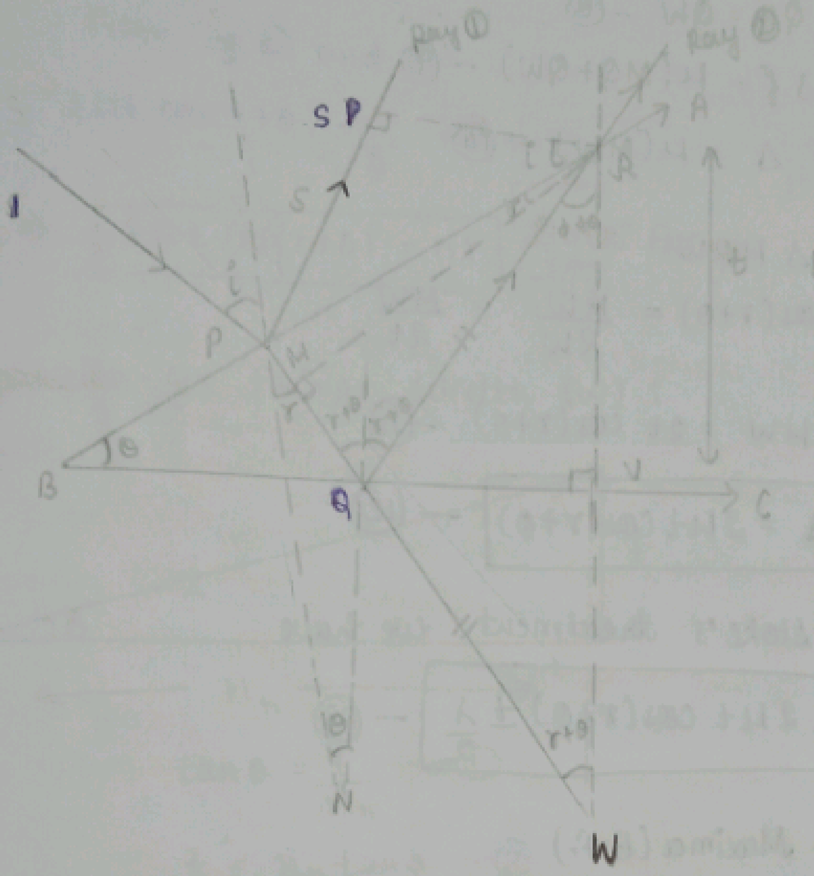
$$\Delta = (2n \pm 1)\frac{\lambda}{2} \quad -(17)$$

$$\boxed{2\mu t \cos r = (2n \pm 1)\lambda/2} \quad -(18)$$

Dark Fringe

Hence, it is quiet clear that the interference patterns due to reflected and transmitted light are complementary (opposite) to each other.

Wedge-shaped Interference :



$\Delta = (PQ + QR)$ in film $- SP$ in air — ①

$\Delta = \mu(PQ + QR) - SP$ — ②

From Snell's law

$\mu = \frac{\sin i}{\sin r}$ — ③

$\Rightarrow \frac{\frac{PS}{PR}}{\frac{PM}{PR}} \Rightarrow \frac{PS}{PM} \Rightarrow PS = \mu PM$ — ④

$\Delta = \mu(PQ + QR) - \mu PM$

$\mu(\cancel{PM} + MQ + QR) - \mu \cancel{PM}$ — ⑤

$\Rightarrow \mu[MQ+QR]$ — (7)

$\therefore QR = QW$ — (8)

$\Rightarrow \Delta \xi = \mu(MQ+QW)$ — (9)

$\Delta = \mu(MW)$ — (10)

In ΔMRW

$$\cos(r+\theta) = \frac{MW}{RW} = \frac{MW}{2t}$$

$$MW = 2t\cos(r+\theta) \quad \text{— (11)}$$

$$\boxed{\Delta = 2\mu t\cos(r+\theta)} \quad \text{— (12)}$$

Applying Stoke's treatment we have

$$\boxed{\Delta = 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2}} \quad \text{— (13)}$$

Cond^n for Maxima (B.F.)

For CI

$$\Delta = n\lambda \quad \text{— (14)}$$

From eq^n (13) & (14)

$$\Rightarrow 2\mu t\cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \boxed{2\mu t\cos(r+\theta) = (2n-1)\frac{\lambda}{2}} \quad \text{— (15)}$$

$\forall\ n = 1, 2, 3 \ldots$

Cond^n for Minima (D.F)

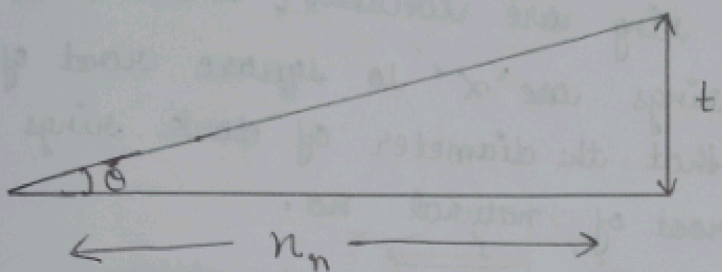
$$\Delta = (2n \pm 1)\frac{\lambda}{2} \quad -(16)$$

From eq (13) and (16)

$$2\mu t \cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t \cos(r+\theta) = n\lambda} \quad \text{Dark Fringes} \quad -(17)$$

Expression for Fringe width (ω) :



$$\tan\theta = \frac{t}{x_n} \quad -(1)$$

$$t = x_n \tan\theta \quad -(2)$$

$$2\mu t \cos(r+\theta) = n\lambda \quad -(3)$$

Put eq^n (2) in (3)

$$= 2\mu x_n \tan\theta \cdot \cos(r+\theta) = n\lambda \quad -(4)$$

At normal incidence $i = r = 0$

$$\Rightarrow 2\mu x_n \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta} = n\lambda \quad -(5)$$

$$\Rightarrow 2\mu x_n \sin\theta = n\lambda \quad -(6)$$

For $(n+1)^{th}$ order

$$2\mu x_{n+1} \sin\theta = (n+1)\lambda \quad -(7)$$

Subtract eq (6) from (7)

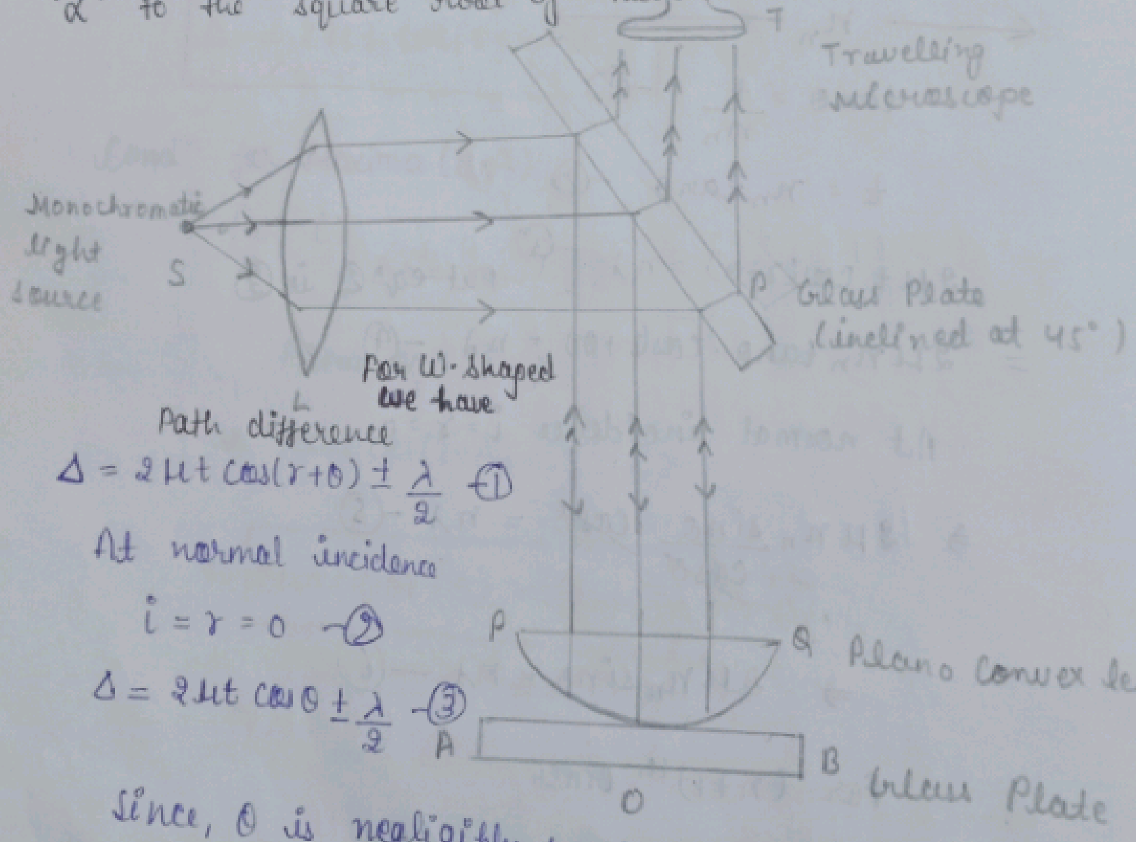
$= 2\mu(x_{n+1} - x_n)\sin\theta = \lambda$ — (8)

$2\mu w \sin\theta = \lambda$ — (9)

$$\boxed{w = \frac{\lambda}{2\mu \sin\theta}} \quad \text{— (10)}$$

Newton's Ring: (Application of Wedge-shaped Interference)

(1) Describe the construction and working of Newton's ring experiment. Why the centre of Newton's ring generally appear dark? Why Newton's ring are circular? Show that the diameter of bright rings are 'α' to square root of odd natural no. Show that the diameter of dark rings are 'α' to the square root of natural no.



For W-shaped we have

Path difference

$\Delta = 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2}$ — (1)

At normal incidence

$i = r = 0$ — (2)

$\Delta = 2\mu t\cos\theta \pm \frac{\lambda}{2}$ — (3)

Since, θ is negligibly small hence

Newton's Ring

$$\boxed{\Delta = 2\mu t \pm \frac{\lambda}{2}} \quad \text{— (4)}$$

At the point of contact O, $t=0$

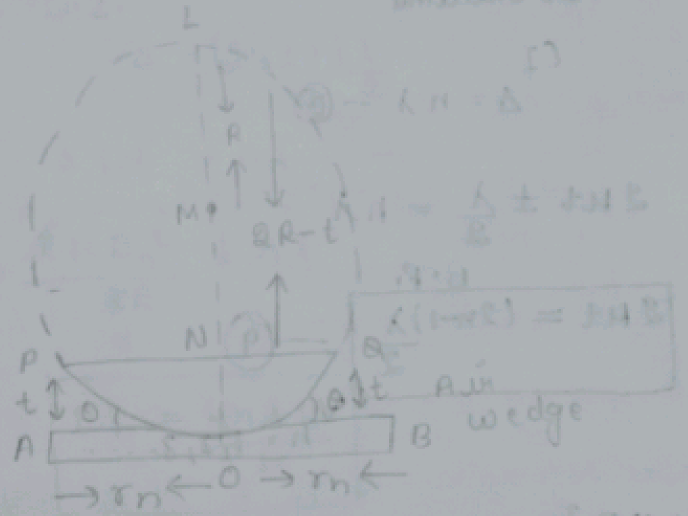
Hence $\boxed{\Delta = \pm \frac{\lambda}{2}}$ — (5)

* CI $\boxed{\Delta = n\lambda}$ — (6)

* DI $\Delta = (2n+1)\frac{\lambda}{2}$ — (7)

from eq (5) & (7) we get a minima
Hence, the centre generally appears dark.

Diameters of Rings:



from the property of chord (PQ).
we have

$$PN \times NQ = ON \times NL \quad -(1)$$

$$r_n \times r_n = t(2R-t) \quad -(2)$$

$$r_n^2 = 2Rt - t^2 \quad -(3)$$

Since t is very small hence
t^2 becomes neglible

$$r_n^2 = 2Rt$$

$$t = \frac{r_n^2}{2R}$$

If diameter of nth ring is D_n

$$D_n = 2r_n \quad \text{—} (5)$$

$$t = \frac{\left(\frac{D_n}{2}\right)^2}{2R}$$

$$\boxed{t = \frac{D_n^2}{8R}} \quad \text{—} (6)$$

For Bright Ring : since,

$$\Delta = 2\mu t \pm \frac{\lambda}{2} \quad \text{—} (7)$$

For Maxima

CI

$$\Delta = n\lambda \quad \text{—} (8)$$

$$2\mu t \pm \frac{\lambda}{2} = n\lambda$$

B.F.

$$\boxed{2\mu t = (2n-1)\frac{\lambda}{2}} \quad \text{—} (9)$$

$\forall \; n = 1, 2, 3 \ldots$

$$\Rightarrow 2\mu \times \frac{D_n^2}{8R} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{D_n^2 = \frac{2(2n-1)\lambda R}{\mu}} \quad \text{—} (10)$$

For Air Wedge

$$\mu = 1$$

$$D_n^2 = \frac{2(2n-1)\lambda R}{\mu} \quad \text{—} (11)$$

$$\boxed{D_n = \sqrt{2(2n-1)\lambda R}} \quad \text{--- (12)}$$

$$\boxed{D_n \propto \sqrt{(2n-1)}} \quad \text{--- (13)}$$

$$D_1 : D_2 : D_3 \ldots$$
$$\sqrt{1} : \sqrt{3} : \sqrt{5} \ldots \quad \text{--- (14)}$$

Hence diameter is directly $\propto$ to square root of odd natural no.

For Dark Ring :

For minima

D.I.

$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$2\mu t = n\lambda$$

$$\Rightarrow \frac{2\mu D_n^2}{8R} = n\lambda$$

$$\boxed{D_n^2 = \frac{4n\lambda R}{\mu}}$$

For air film $\mu = 1$.

$$D_n^2 = 4n\lambda R$$

$$\boxed{D_n = \sqrt{4n\lambda R}}$$

$$\boxed{D_n \propto \sqrt{n}}$$

$$D_1 : D_2 : D_3 \ldots$$
$$\sqrt{1} : \sqrt{2} : \sqrt{3} \ldots$$

Applications of Newton's Ring:

* To find the wavelength of monochromatic light use.

Diameter for n^{th} dark ring

$$D_n^2 = \frac{4n\lambda R}{\mu} \quad -(1)$$

For Air film

$$D_n^2 = 4n\lambda R \quad -(2)$$

For $(n+p)^{th}$ ring   Subtracting eq (2) from (3)

$$D_{n+p}^2 = 4(n+p)\lambda R \quad -(3)$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

where p is any integer
R = Radius of curvature of plano-convex lens.

* To find the refractive index 'μ' of a liquid.

For Dark ring; Diameter is given by

$$(D_n^2)_{med} = \frac{4n\lambda R}{\mu} \quad -(1)$$

If air film is used

$$\mu = 1$$

$$(D_n^2)_{air} = 4n\lambda R \quad -(2)$$

$$\Rightarrow \frac{(D_n^2)_{med}}{(D_n^2)_{air}} = \frac{\frac{4n\lambda R}{\mu}}{4n\lambda R}$$

$$\frac{(D_n^2)_{air}}{(D^2_n)_{med}} = \frac{4n\lambda R}{\left(\frac{4n\lambda R}{\mu}\right)} = \mu$$

Q.) The distance b/w two slits is 0.1mm. The width of slit formed on screen is 5 mm. If the distance b/w the slit and screen is 1m. Find the wavelength of light used.

Given: $d = 0.1\,mm \Rightarrow 1\times10^{-4}$

$\omega = 5\,mm \Rightarrow 5\times10^{-3}$ $\qquad \lambda = 5000\,A^\circ$

$D = 1\,m$

$$\beta = \frac{\lambda D}{d} \Rightarrow 5\times10^{-3} = \frac{\lambda\times1}{1\times10^{-4}}$$

$$\Rightarrow 5\times10^{-3}\times1\times10^{-4} = \lambda$$

$$\Rightarrow 5\times10^{-4+(-3)} = \lambda$$

$$5\times10^{-7} = \lambda$$

$\boxed{5000\,A^\circ = \lambda}$ Ans

Q.) A biprism of angle 1° and refractive index 1.5 is at a distance of 40cm from the slit. Find the fringe width at 60cm from the biprism for wavelength 5893 A°

$D = 100\,cm$ $\qquad \beta = \frac{\lambda D}{2d}$

$2d = \{ 2a(\mu-1)\alpha$

$d = 0.6908 \sim 0.7$

$\beta = 0.0844\times10^{-3}\,m$

Ans $= 0.0844\times10^{-3}\,m$

Q: Interference fringes are produced by the biprism on the focal plane of reading microscope which is 1 m away from the slit. A lens is placed b/w the biprism and microscope to give two images of the slit in two positions. If the images of the slit are 4.05 mm and 2.90 mm at the two positions with wavelength of light $\lambda = 5893\ \mathring{A}$, Find the distance b/w consecutive interference bands.

Formula used: $2d = \sqrt{d_1 d_2}$

$\beta = \frac{D\lambda}{2d}$ $D = 1\ m$

Ans: 0.172 mm

$\Rightarrow 2d = \sqrt{4.05 \times 2.90}$

$2d = \sqrt{3.426} \times 10^{-3}\ m$ $\beta = \frac{1 \times 5893 \times 10^{-10}}{3.426 \times 10^{-3}}$

$\boxed{\beta = 0.172\ mm}$ Ans

ight of wavelength 589.3 A°

Q: White light is reflected from an oil film, of thickness 0.01 mm and refractive index 1.4 at angle of 45° to the vertical. If the reflected light falls on the slit of a spectrometer. Find the no. of dark bands seen b/w wavelength 4000 A° and 5000 A°.

~~$2\mu t \cos\theta$~~ = ~~$n\lambda$~~.

Ans: $n_1 = 60$ (approx)

$n_2 = 48$ (approx)

$n_1 - n_2 = 12$ Ay

Given: $t = 0.01\,mm$

$\mu = 1.4$

$i = 45°$

$\lambda_1 = 4000\,A°$

$\lambda_2 = 5000\,A°$

$2\mu t \cos r = n\lambda$

$\lambda_1 = 4000\,A^\circ$

$2\mu t \cos r = n_1 \lambda_1$

$\sin r = \frac{1}{\sqrt{2} \times 1.4}$

$n_1 = \frac{2\mu t \cos r}{\lambda_1} \Rightarrow \frac{2 \times 1.4 \times 0.01 \cos r}{4000\,A^\circ}$

$n_2 = \frac{2\mu t \cos r}{\lambda_2} \Rightarrow \frac{2 \times 1.4 \times 0.01 \cos r}{5000}$

$n_1 = 60 \quad , n_2 = 48$

$\boxed{n_1 - n_2 = 12}$ Ans

Q: A thin film ~~of~~ is illuminated by white light at an angle of incidence $i = \sin^{-1}(\frac{4}{5})$. In reflected light, two dark consecutive overlapping fringes are observed corresponding to wavelength 6.1×10^{-7} m and 6.0×10^{-7} m. The refractive index of the film is $\frac{4}{3}$. Find the thickness of the film.

Given: $\lambda_1 = 6.1 \times 10^{-7}\,m$

$\lambda_2 = 6.0 \times 10^{-7}\,m$

$\mu = \frac{4}{3}$

$2\mu t \cos r = n\lambda_1$ — (1)

$2\mu t \cos r = (n+1)\lambda_2$ — (2)

Equating these two :

$n\lambda_1 = (n+1)\lambda_2$

$n\lambda_1 = n\lambda_2 + \lambda_2$

$n(\lambda_1 - \lambda_2) = n\lambda_2$

$n = \left(\frac{\lambda_2}{\lambda_1 - \lambda_2}\right) \Rightarrow 60$

Putting the value of n in eq (1)

$2\mu t \cos r = n\lambda_1$

$2 \times \frac{4}{3} \times t \times \frac{4}{5} = 60 \times 6.1 \times 10^{-7}$

$\frac{8}{3} \times \frac{4}{5} t = 60 \times 6.1 \times 10^{-7}$

$\frac{32}{15} t = 366 \times 10^{-7}$

$32t = 366 \times 10^{-7} \times 15$

$t = \frac{366 \times 10^{-7} \times 15}{32}$

$\boxed{\text{Ans: } 1.716 \times 10^{-2} \text{ mm}}$

Q: Interference fringes are produced from a wedge shaped film of μ 1.5. If the distance b/w two successive fringes is 0.02 mm. Find the angle of film. If λ of light used is 5.5×10^{-5} cm.

Formula used : $\omega = \frac{\lambda}{2\mu\theta}$

$\mu = 1.5$

$\lambda = 5.5 \times 10^{-5}$ cm

$\omega = 0.02$ mm

Ans $= 0.009166$ rad

$\Rightarrow 0.525^\circ$

0.02×10^{-1} cm

$$0.02 \times 10^{-1} = \frac{5.5 \times 10^{-5}}{2 \times 1.5 \times \theta}$$

$$0.02 \times 10^{-1} \times 3 \times \theta = 5.5 \times 10^{-5}$$

$$\theta = \frac{5.5 \times 10^{-5}}{0.06 \times 10^{-1}}$$

$$\theta = \frac{5.5 \times 10^{-5+1}}{0.06} \Rightarrow \frac{5.5 \times 10^{-4}}{0.06}$$

$$\theta = 0.009166 \text{ rad}$$

$$\theta = 0.009166 \times \frac{\pi}{180} \Rightarrow 0.525^{\circ}$$

$$\boxed{\theta = 0.525^{\circ}}$$

Q: A Newton's Ring Experiment the diameter of 15^{th} ring is 0... ring is 0.336 cm. If the radi...

A source emits light of wavelength $\lambda_1 = 6 \times 10^{-7}$ m and $\lambda_2 = 4.8 \times 10^{-7}$ m to obtain Newton's Ring in reflected light. If n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring λ_2 provided the radius of curvature of plano-convex lens is 0.96 m. Find the diameter $(n+1)^{th}$ dark ring due to λ_2.

Formula used:- $D^2_n = 4n\lambda_1 R$

$D^2_{n+1} = 4(n+1)\lambda_2 R$

Equating the value :

$$D^2_n = 4n\lambda_1 R$$

$$D^2_{n+1} = 4n\lambda_2 + 4\lambda_2 R \Rightarrow$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} \Rightarrow \frac{4.8 \times 10^{-7}}{6 \times 10^{-7} - 4.8 \times 10^{-7}}$$

$$\Rightarrow \frac{4.8 \times 10^{-7}}{10^{-7}(6 - 4.8)} \Rightarrow 4$$

$$D^2_{(n+1)} = 4(4+1)\lambda_2 \times 0.96$$

Ans: 3.04×10^{-3} m

Q: A Newton's Ring, ... with a liquid pl... The diameter of 7th ... curvature is 1 m.

$\lambda = 5893$ Å

$d = 0.4$ cm

$n = 7$

$R = 1$ m

$\Rightarrow$

$D^2_{n+1} =$

Q:- A Newton's Ring, sodium lamp of wavelength 5893 Å is used with a liquid placed b/w the flat and curved surface. The diameter of 7th bright fringe is 0·4 cm and Radius of curvature is 1 m. Find μ of the liquid :

Ans : 0·96

$\lambda = 5893\ \text{Å}$

$d = 0{\cdot}4\,\text{cm}$

$n = 7$

$R = 1\,\text{m}$

$$D_n^2 = \frac{4n\lambda R}{\mu} \qquad \Rightarrow D_{(7)}^2 = \frac{4 \times 7 \times 5893 \times 1\,\text{m}}{\mu}$$

$$\Rightarrow (0{\cdot}4)^2 = \frac{28 \times 5893 \times 10^{-10} \times 1}{\mu}$$

$$\mu = \frac{28 \times 5893 \times 10^{-10}}{(0{\cdot}4)^2 \times 10^{-4}}$$

$$\boxed{\mu = 0{\cdot}96} \text{ Ans}$$

(2)

$$D_{n+1}^2 = 4(n+1)\lambda_2 R$$

$$\Rightarrow D_5^2 = 20 \times 4{\cdot}8 \times 10^{-7} \times 0{\cdot}96$$

$$D_5 = \sqrt{20 \times 4{\cdot}8 \times 10^{-7} \times 0{\cdot}96}$$

$$\boxed{D_5 = 3{\cdot}04 \times 10^{-3}\ \text{m}} \text{ Ans}$$

07/11/022

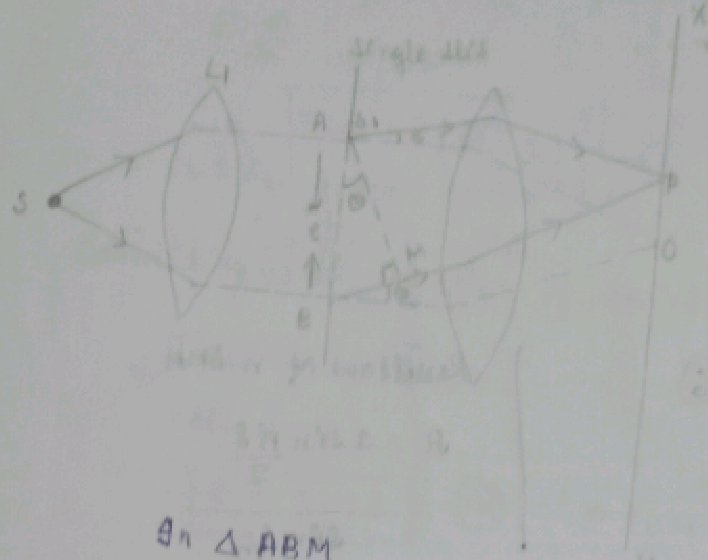
Diffraction:

It is the phenomenon of bending of light around the corners of an obstacle.

Classification of Diffraction

Fresnel	Fraunhoffer
* Either the source/screen or both are at finite distance from the obstacle of the diffracting element.	* Either the source/screen or both are at ∞ distance from the obstacle or diffracting element.
* Incident wavefront are spherical or cylindrical.	* Incident wavefronts are planar.
* Linear distances are important	* Angular inclination are important.
* Centre may be bright or dark.	* Centre is always bright.

Fraunhoffer Diffraction due to Single slit:



In Δ ABM

$$\sin\theta = \frac{BM}{AB} = \frac{BM}{e}$$

$$\boxed{BM = e\sin\theta} \quad \text{— (1)}$$

Path difference b/w two diffracted rays

$$\boxed{\Delta = BM = e\sin\theta} \quad \text{— (2)}$$

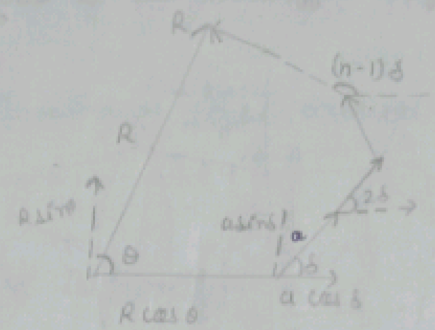
Phase difference $\delta = \frac{2\pi}{\lambda}\cdot\Delta$

$$\boxed{\delta = \frac{2\pi}{\lambda}(e\sin\theta)} \quad \text{— (3)}$$

Let us assume that the slit AB be divided into n equal parts. Hence phase difference due to each path will be :

$$\delta = \frac{1}{n}\left(\frac{2\pi}{\lambda}e\sin\theta\right) \quad \text{— (4)}$$

$$R = \frac{a \sin \frac{n\delta}{2}}{\sin \delta/2} \quad \text{—⑤}$$



Put eq ④ in ⑤

$$R = \frac{a \sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin\theta}{n\lambda}\right)} \quad \text{—⑥}$$

Resultant of n-SHM

$$R = \frac{a \sin \frac{n\delta}{2}}{\sin \delta/2}$$

Let $\alpha = \frac{\pi e \sin\theta}{\lambda}$ —⑦

$$R = \frac{a \sin\alpha}{\sin \frac{\alpha}{n}} \quad \text{—⑧}$$

Since 'n' is infinitely large hence $\frac{1}{n}$ is infinitely small $\therefore \sin\left(\frac{\alpha}{n}\right) \approx \frac{\alpha}{n}$

$$R = \frac{a \sin\alpha}{\frac{\alpha}{n}} \quad \text{—⑨}$$

$$R = na\left(\frac{\sin\alpha}{\alpha}\right) \quad \text{—⑩}$$

Let $na = A$

$$\boxed{R = A\frac{\sin\alpha}{\alpha}} \quad \text{—⑪}$$

Intensity I at point P is given by

$$I = R^2$$

$$\boxed{I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2} \quad - (12)$$

Condition and direction of Minima:

$$I_{min} = 0 \quad - (13)$$

$$\sin\alpha = 0$$

$$\boxed{\alpha = \pm m\pi} \quad - (14) \quad (m = 1, 2, 3 \ldots)$$

Condition and direction of Maxima:

(i) For Central Maxima: (I_0)

For I_{max}; $\alpha = 0$

$$I_{cm} = A^2\left(\frac{\sin 0}{0}\right)^2$$

$$\boxed{I_{cm} = I_0 = A^2} \quad - (15)$$

(ii) For Secondary Maxima:

$$\left.\frac{dI}{d\alpha}\right|_{max} = 0 \qquad \Rightarrow \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

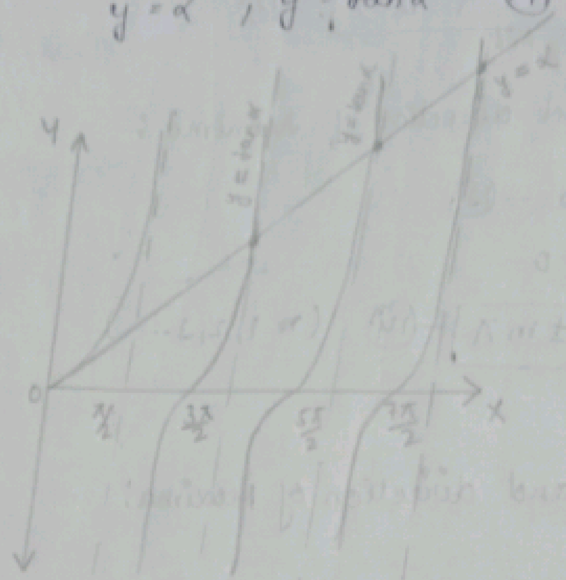
$$A^2 \times 2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0 \quad - (17)$$

$\Rightarrow \alpha \cos\alpha - \sin\alpha = 0$

$$\boxed{\alpha = \tan\alpha} \quad \text{— (18)}$$

let

$$y = \alpha \;,\; y = \tan\alpha \quad \text{— (19)}$$



$$\alpha = \pm\frac{3\pi}{2},\ \pm\frac{5\pi}{2},\ \pm\frac{7\pi}{2} \ldots \quad \text{— (20)}$$

for $\alpha = \pm\frac{3\pi}{2}$

$$I_1 = \frac{A^2 \sin^2\left(\frac{3\pi}{2}\right)}{\left(\frac{3\pi}{2}\right)^2}$$

$$\boxed{I_1 = \frac{4}{9\pi^2} A^2} \quad \text{— (21)}$$

for $\alpha = \frac{5\pi}{2}$

$$I_2 = \frac{A^2 \sin^2\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)^2} \quad \text{— (22)}$$

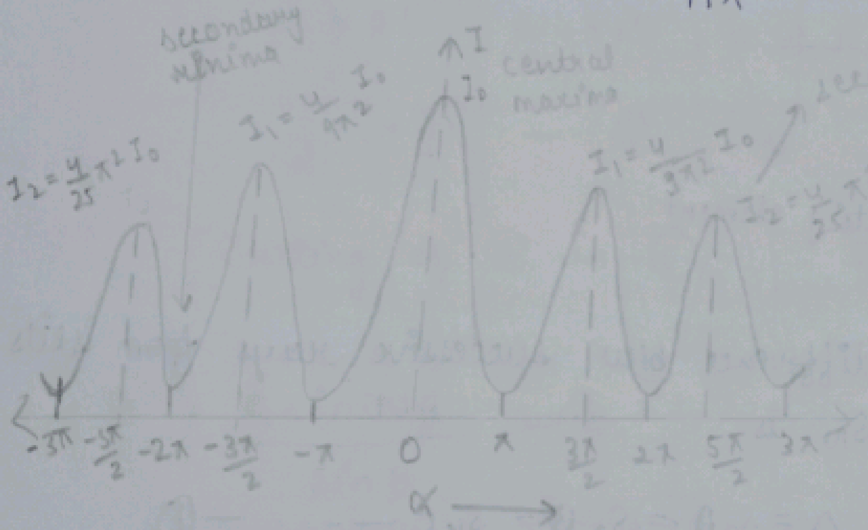
$$I_2 = \frac{4}{25}\pi A^2$$

Similarly, $I_3 = \frac{4}{49\pi^2} A^2$ — (23)

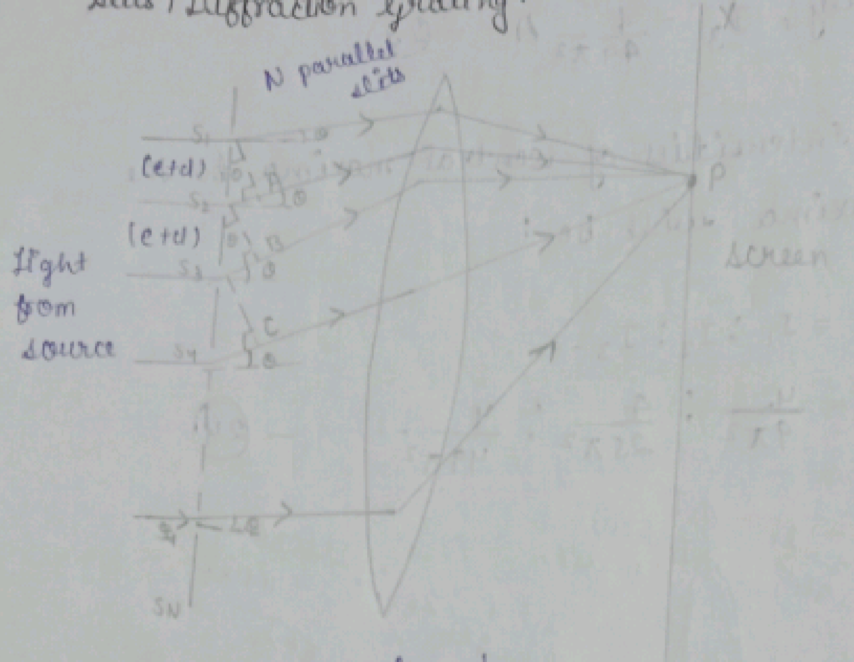
The ratio of intensities of central maxima to the secondary maxima will be :

$$I_0 = I_1 : I_2 : I_3 \ldots\ldots$$

$$1 = \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \ldots \quad -(24)$$



Fraunhoffer diffraction due to N- parallel slits / Multiple slits / Diffraction grating.



$(e+d)$ = grating element

The path difference b/w successive rays from slits S_1, S_2, S_3 ---- S_N is

$$\Delta = S_2A = S_3B = S_4C \text{ ------ } (1)$$

In ΔS_1S_2A, ΔS_2S_3B, ΔS_3S_4C

$$\sin\theta = \frac{S_2A}{S_1S_2} = \frac{S_3B}{S_2S_3} = \frac{S_4C}{S_3S_4}$$

$$\Rightarrow \frac{S_2A}{(e+d)} = \frac{S_3B}{(e+d)} = \frac{S_4C}{(e+d)} \text{ ...... } (2)$$

$$\boxed{\Delta = (e+d)\sin\theta} \text{ — } (3)$$

Phase difference $\delta = \frac{2\pi}{\lambda} \cdot \Delta$ — (4)

$$\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta \quad \text{— (5)}$$

$\Rightarrow 2\beta$

From the resultant of n simple harmonic motion

$$R = \frac{a \sin \frac{n\delta}{2}}{\sin \frac{\delta}{2}} \quad \text{— (6)}$$

At point P

$R \to R'$, $n \to N$, $a \to R$

$$\frac{\delta}{2} = \beta \quad \text{— (7)}$$

Putting eq (7) in (6)

$$R' = \frac{R \sin N\beta}{\sin\beta} \quad \text{— (8)}$$

Here, $R = A\left(\frac{\sin\alpha}{\alpha}\right)$ — (9)

Resultant Amp. at pt P.

$$R' = \frac{A\left(\frac{\sin\alpha}{\alpha}\right)\sin N\beta}{\sin\beta} \quad \text{— (10)}$$

Intensity I at pt. P is

$$I = R'^2$$

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2 \quad \text{— (11)}$$

Condition and direction of minima:

$$I_{min} = 0 \quad -(12)$$

$$\sin N\beta = 0 \quad -(13)$$

$$N\beta = \pm m\pi \quad (m = 1, 2, 3 \ldots \text{ except } 0, N, 2N \ldots) \quad -(14)$$

$$N\left[\frac{\pi}{\lambda}(e+d)\sin\theta\right] = \pm m\pi$$

Minima

$$\boxed{N(e+d)\sin\theta = \pm m\pi} \quad -(15)$$

Condition and direction of Maxima:

(i) central Maxima

For I_{max},

$$\sin\beta = 0 \quad -(16)$$

$$\beta = \pm n\pi \quad (n = 0, 1, 2 \ldots)$$

At $\beta = \pm n\pi$; $\frac{\sin 0}{\sin 0}$ = undeterminant $-(17)$

Apply L' Hospital Rule

$$\frac{\sin N\beta}{\sin\beta} = \lim_{\beta = \pm n\pi} \frac{d}{d\beta}\left[\frac{\sin N\beta}{\sin\beta}\right] \quad -(18)$$

$$\Rightarrow \left.\frac{N\cos N\beta}{\cos\beta}\right|_{\beta = \pm n\pi} = N \quad -(19)$$

Put eq (19) in (11)

I at central Max ($I_{CM} = I_0$)

Intensity at central max. ($I_{cm} = I_0$)

$$\boxed{I_{cm} = I_0 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 N^2} \quad -(20)$$

(ii) Secondary Maxima:

$$\left.\frac{dI}{d\beta}\right|_{max} = 0$$

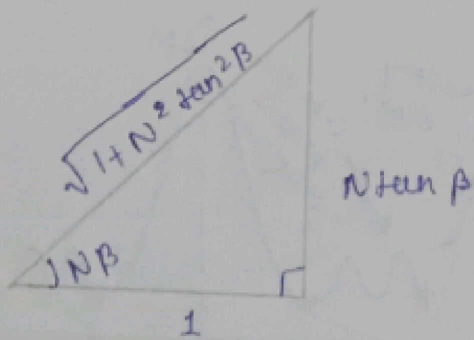
$$\frac{dI}{d\beta} = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 2\left(\frac{\sin N\beta}{\sin\beta}\right)\left[\frac{\sin\beta \cdot N\cos N\beta - \sin N\beta \cos\beta}{\sin^2\beta}\right] = 0$$

$-(21)$

$$\frac{dI}{d\beta} = \sin\beta \cdot N\cos N\beta - \sin N\beta \cdot \cos\beta = 0$$

$$\sin\beta \cdot N\cos N\beta = \sin N\beta \cdot \cos\beta$$

$$\boxed{\frac{N\tan\beta}{1} = \tan N\beta} \quad -(22)$$



$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}} \quad -(23)$$

Put eq (23) in (11)

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left[\frac{N^2\tan^2\beta}{1+N^2\tan^2\beta}\right]\frac{1}{\sin^2\beta}$$

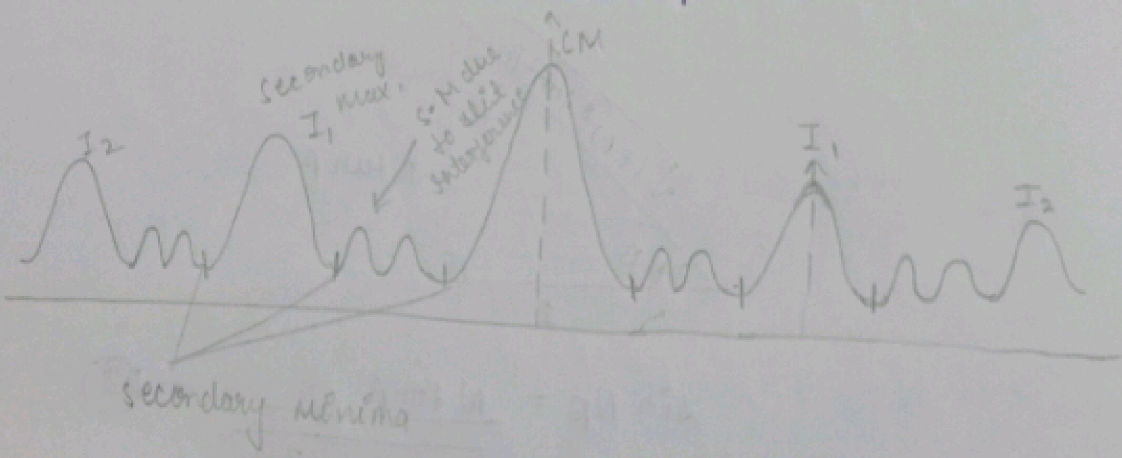
$$= A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left[\frac{N^2\frac{\sin^2\beta}{\cos^2\beta}}{1+N^2\frac{\sin^2\beta}{\cos^2\beta}}\right]\frac{1}{\sin^2\beta}$$

$$\Rightarrow A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left[\frac{N^2\frac{\cancel{\sin^2\beta}}{\cancel{\cos^2\beta}}}{\frac{\cos^2\beta+N^2\sin^2\beta}{\cancel{\cos^2\beta}}}\right]\times\frac{1}{\cancel{\sin^2\beta}}$$

$$I_{SM} \Rightarrow A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left[\frac{N^2}{(1+N^2-1)\sin^2\beta}\right] \quad -(24)$$

Ratio of intensity of 2nd maxima to CM is

$$\frac{I_{SM}}{I_{CM}} = \frac{1}{1+(N^2-1)\sin^2\beta} \quad -(25)$$



Put β in (17)

$$\boxed{(e+d)\sin\theta = \pm n\lambda} \quad \text{— (17) A}$$

Grating Equation.