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Module - I

Interference

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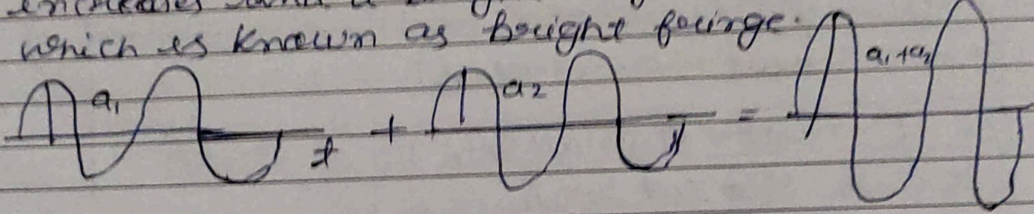
Constructive Interference

At certain points, two waves may be in phase the resultant aplitude then is equal to sum of amplitude of two waves. the intensity increases and a bright reigon is obtained which is known as bright fringe.



Distructive interference

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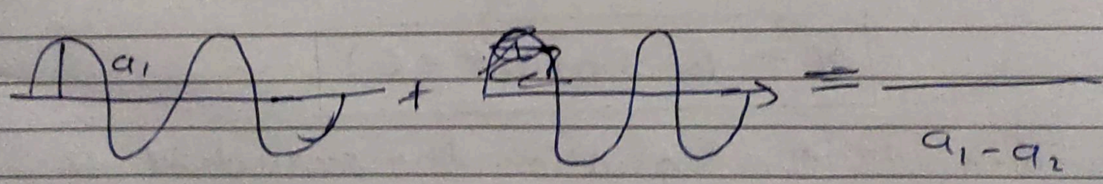

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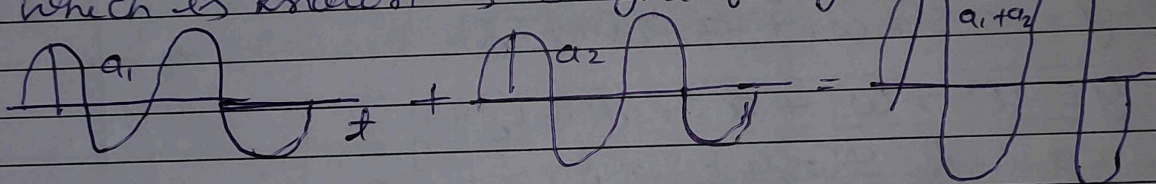

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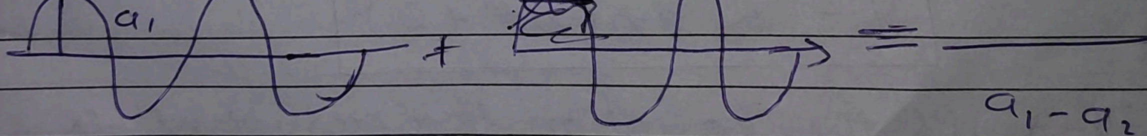

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Interference

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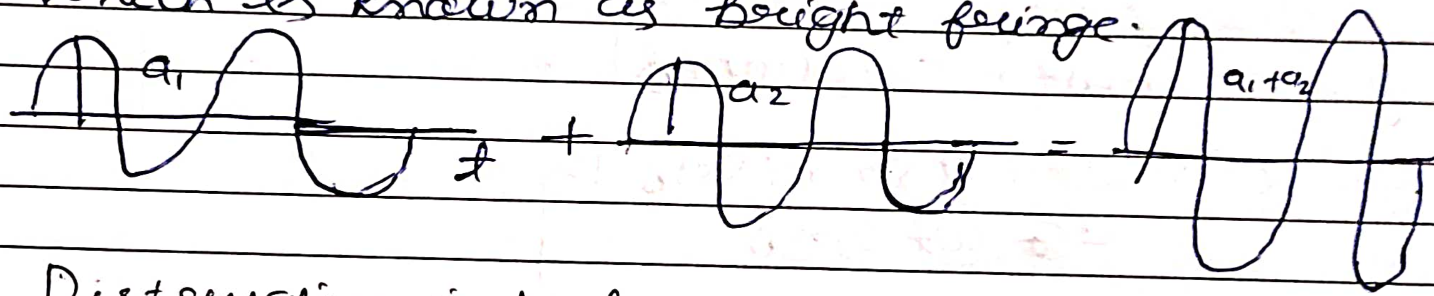

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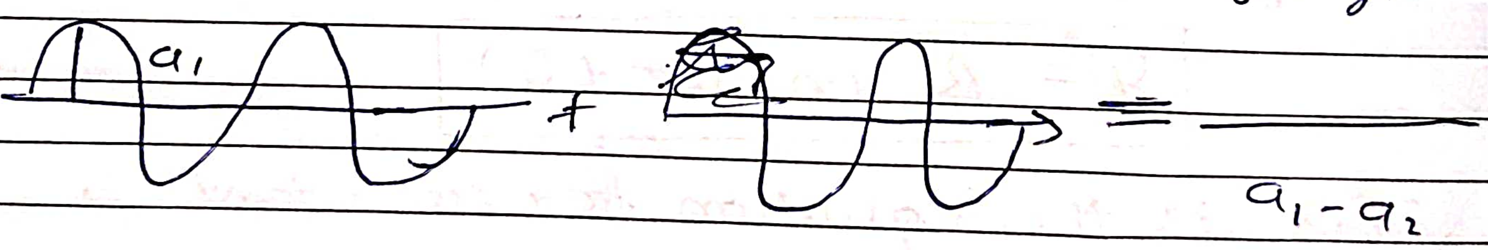
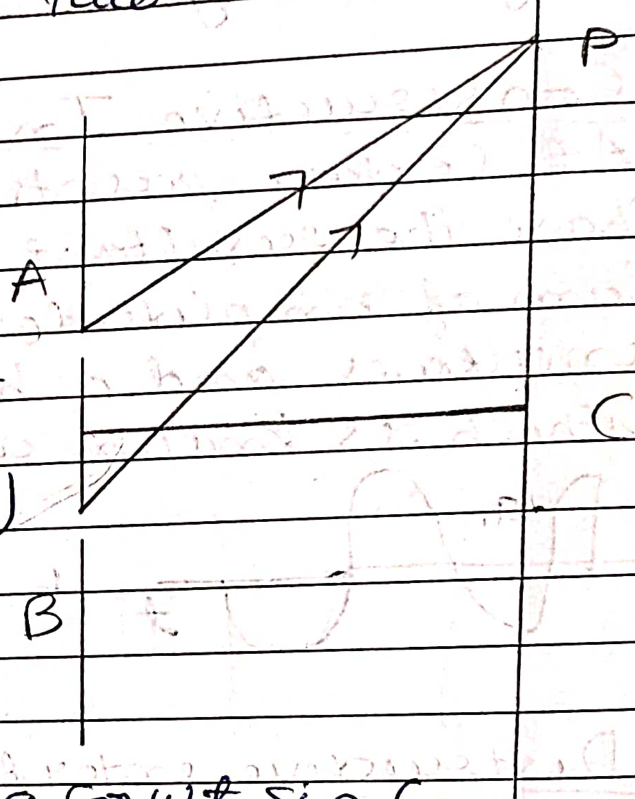


This phenomenon of redistribution of light energy due to superposition of light wave is called interference.

Analytical Treatment of Interference

Let A & B are two slits in front of mono-chromatic source S the waves from slit A and B reach at a point P on screen the amplitude and frequency of this waves are same the equation two waves can be written as



$$y_1 = a \sin \omega t$$
$$y_2 = a \sin(\omega t + \delta)$$

where $\delta \rightarrow$ Phase difference

Resultant displacement

$$y = y_1 + y_2$$
$$y = a \sin \omega t + a \sin(\omega t + \delta)$$
$$y = a \sin \omega t + a \sin \omega t \cos \delta + a \cos \omega t \sin \delta$$
$$y = a \sin \omega t [1 + \cos \delta] + a \cos \omega t \sin \delta$$

let $a(1 + \cos \delta) = R \cos \theta$ --- (1)

and $a \sin \delta = R \sin \theta$ --- (2)

Substracting in above equation

$$y = \sin \omega t \, R \cos \theta + R \sin \theta \cos \omega t$$

$$\boxed{y = R \sin(\omega t + \theta)}$$

This is the equation for resultant displacement where,

$$\tan \theta = \frac{\sin \delta}{1 + \cos \delta} \quad \text{or} \quad \theta = \tan^{-1}\left[\frac{\sin \delta}{1 + \cos \delta}\right]$$

Squaring and adding eq (1) and (2)

$$R^2(\cos^2\theta + \sin^2\theta) = a^2(1+\cos\delta)^2 + a^2\sin^2\delta$$

$$R^2 = a^2 + a^2\cos^2\delta + 2a^2\cos\delta + a^2\sin^2\delta$$

$$= 2a^2 + 2a^2\cos\delta$$

$$R^2 = 2a^2(1+\cos\delta) = 2a^2\left(1+2\cos^2\frac{\delta}{2}-1\right)$$

$$R^2 = 4a^2\cos^2\frac{\delta}{2} \quad \text{or} \quad \boxed{R = 2a\cos\frac{\delta}{2}}$$

Resultant intensity

$$I = R^2 = 4a^2\cos^2\frac{\delta}{2}$$

(i) For maximum intensity, $\cos\frac{\delta}{2} = \pm 1$

(Constructive interference)

$$\Rightarrow \frac{\delta}{2} = 0, \pi, 2\pi, 3\pi, \ldots\ldots$$

$$\Rightarrow \delta = 0, 2\pi, 4\pi, 6\pi \text{ or } \boxed{\delta = 2n\pi}$$

$n = 0, 1, 2, 3$

(ii) For minimum intensity

$$\cos\frac{\delta}{2} = 0$$

$$= \frac{\delta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots$$

or $\delta = \pi, 3\pi, 5\pi$ (Distructive interference)

$$\delta = (2n+1)\pi \text{ where, } n = 0, 1, 2, 3 \ldots$$

$\because$ Phase diff $= \frac{2\pi}{\lambda} \times$ Path diff

$\therefore$ Path diff $= \frac{\lambda}{2\pi} \times$ Phase diff

For bright fringe

$$\text{path diff} = \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$$

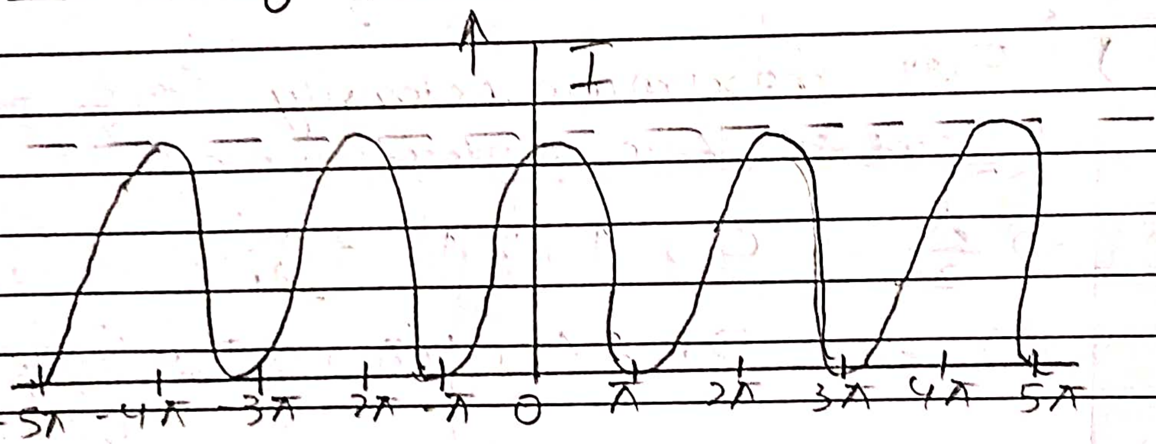
for dark fringe

$$\text{path diff} = \frac{\lambda}{2\pi} \times (2n+1)\pi$$

$$\text{path diff} = \cancel{(2n+1)} (2n+1)\frac{\lambda}{2}$$

$$n = 0, 1, 2, 3 \ldots$$

Intensity distribution Curve



Classification of Interference

(1) Division of wavefront - the wavefront originating from a source of light is divided in two parts which serve as two sources
Eg - fresenel Biprism, Laser young's double Slit

(2) Division of amplitude - the amplitude of beam of light is divided into two parts by reflection or refraction. Eg → newtons rings, michelson interferometer.

Coherent Sources - Two Sources of light are said to be Coherent when they have a Constant phase difference between them eg → young's double slit, fresnel's biprism.

Analysis of fringe (theory of interference)

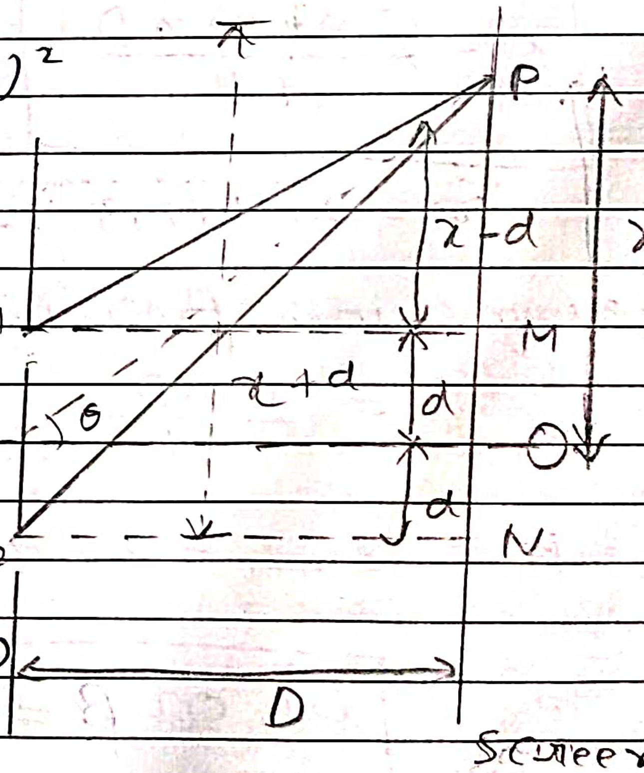
let 'S' be a narrow source of light and S_1 and S_2 be the two narrow slits separated by a distance 2d. Distance between slit & Screen is D. Let us Consider a point P on Screen at a distance x from C

from ΔS_1MP

$$(S_1P)^2 = (S_1M)^2 + (MP)^2$$
$$= D^2 + (x-d)^2 \quad \text{--- (1)}$$

from ΔS_2NP

$$(S_2P)^2 = (S_2N)^2 + (NP)^2$$
$$= D^2 + (x+d)^2 \quad \text{--- (2)}$$



Subtracting eq (2) from (1)

$$(S_2P)^2 - (S_1P)^2 = D^2 + (x+d)^2 - D^2 - (x-d)^2$$

$$(S_2P - S_1P)(S_2P + S_1P) = (x+d)^2 - (x-d)^2 = 4xd$$

$\because D >>> 2d \quad \therefore S_2P + S_1P \approx 2D$

$$\therefore (S_2P - S_1P) \cdot 2D = 4xd$$

$$S_2P - S_1P = \frac{\cancel{4}^{2} xd}{\cancel{2} D}$$

$$\boxed{S_2P - S_1P = \frac{2xd}{D}}$$

Position of bright fringe

path diff $= n\lambda$

$$\frac{2xd}{D} = n\lambda$$

$$\boxed{x = \frac{Dn\lambda}{2d}} \quad n = 0, 1, 2, \ldots$$

Position of dark frings

path diff $= (2n+1)\frac{\lambda}{2}$

$$\frac{2xd}{D} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{x = \frac{(2n+1)\lambda D}{4d}}$$

$n = 0, 1, 2, 3 \ldots\ldots$

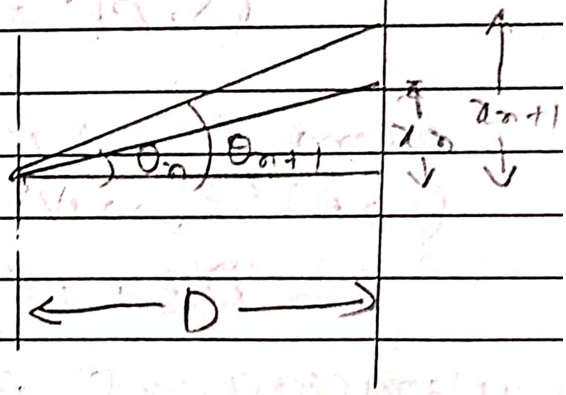
Angular fringe width

$$\omega_\theta = \theta_{n+1} - \theta_n$$

$$= \frac{x_{n+1}}{D} - \frac{x_n}{D}$$

$$= \frac{D(n+1)\lambda}{D \cdot 2d} - \frac{Dn\lambda}{D \cdot 2d}$$

$$\boxed{\omega_\theta = \frac{\lambda}{2d}}$$



fring width β or ω (Bright)

$$x_n = \frac{n\lambda D}{2d}, \quad x_{n+1} = (n+1)\frac{\lambda D}{2d}$$

$$\beta = x_{n+1} - x_n = \frac{(n+1)\lambda D}{2d} - \frac{n\lambda D}{2d}$$

$$\boxed{\omega \text{ or } \beta = \frac{\lambda D}{2d}}$$

Dark $\quad x_n = \frac{(2n+1)\lambda D}{4d}, \quad x_{n+1} = [2(n+1)+1]\frac{\lambda D}{4d}$

$$w_0 \text{ or } \beta = x_{n+1} - x_n$$

$$= (2n+2+1)\frac{\lambda D}{4d} - (2n+1)\frac{\lambda D}{4d}$$

$$= \frac{2\lambda D}{4d}$$

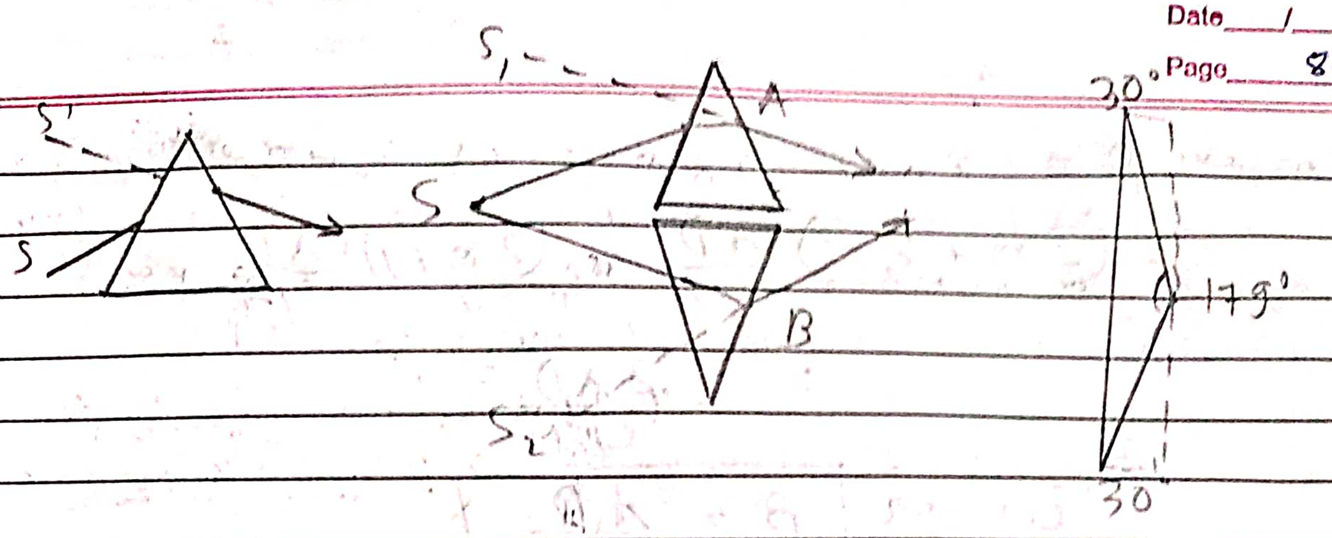
$$w \text{ or } \boxed{\beta = \frac{\lambda D}{2d}}$$

Fresnel Biprism

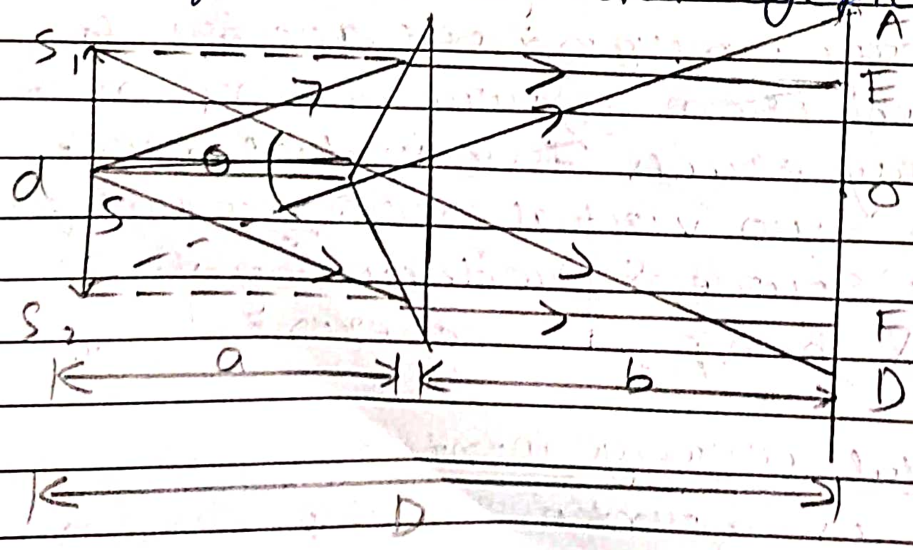
Construction- Fresnel used a biprism to show interference phenomenon the biprism consist of two prism of very small refracting angles joined base to base In practice a thin glass plate is taken and one of its focus is ground and polished will a prism is formed with as obtuse angle of about 179° & two side angles of 30' or 1/2°

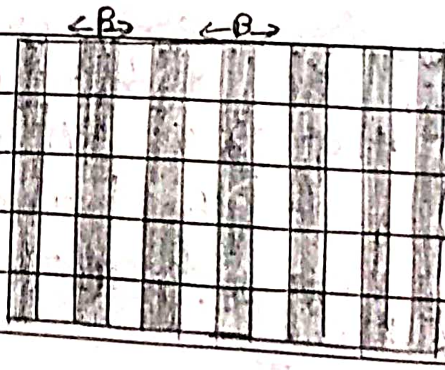
Working- When a light ray is incident on an ordinary prism the ray is bent. As a result, the ray emerging out appears to have emmerted from a source A biprism in the same way erects two virtual source S_1 & S_2 these two virtual source are images of the same source S produced by the refraction & hence are coherent.

Experimental arrangement -
The biprism is mounted suitably on an optical bench. A monochromatic light



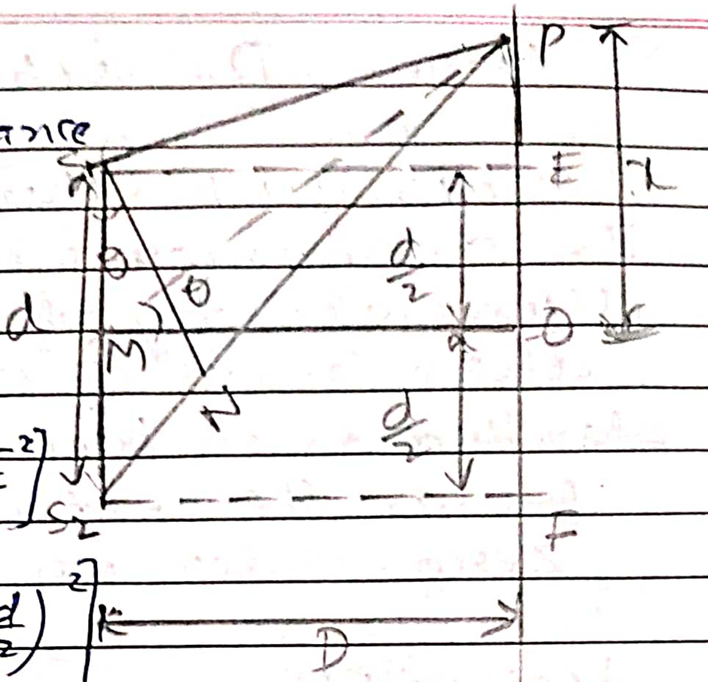
Source such as sodium lamp illuminates a slit S the biprism is placed in such a way that its refracting edge is parallel to the length of slit S. A single cylindrical wavefront impinges on both prisms. the top portion of wave-front is refracted downward & appears to have emmerted from virtual source S_1. The lower segment is refracted upward and appears to have emmerted from virtual source S_2. the two virtual sources are coherent & hence light waves are in a position to interference in the region beyond the biprism & interference fringes are seen to observe frings a micrometer eyepiece is used.





Theory –

Let point P be at a distance x from axis then



$PE = x - \frac{d}{2}$ & $PF = x + \frac{d}{2}$

so, $(S_2P)^2 - (S_1P)^2$

$$= [S_2F^2 + PF^2] - [S_1E^2 + PE^2]$$

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$= 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

we can approximate $S_2P \cong S_1P \cong D$

So path difference $S_2P - S_1P = \frac{2xd}{2D} = \frac{xd}{D}$

The Central bright fringe of max intensity occurs at O. On both sides of O alternate bright & dark frings are produce the width of dark or brighter frings is given as the mth order frings occurs when

$$x_m = \frac{m\lambda D}{d}$$

& m+1th order fringe occurs when

$$x_{m+1} = \frac{(m+1)\lambda D}{d}$$

frings width, $\beta = x_{m+1} - x_m$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

Here $D = a+b$

Frings with white light
In the biprism experiment if slit S is illuminated by white light, the interference pattern consist of a contract white fringe blanked on it both sides by a few coloured frings the central white fringe is the zero-order fringe.

Application
① Determination of wavelength of light
The slit is illuminated by light from monochromatic source the biprism is moved along the optical bench till on looking through it, the two equally bright vertical slits are seen then on moving eyepiece the frings appear in eyepiece.
(i) Determination of β.
When frings are observed. the cross wire is set on centre of one of bright fringe the position of eyepiece is read say x_0 the micrometer screw of the eyepiece is moved slowly & the no. of bright frings N that pass across the cross wire is counted & position of cross wire is again read say x_N.

$$\text{Fringe width } \beta = \frac{x_N - x_0}{N}$$

(ii) Determination of 'd'
A convex lens of short focal length is placed at the slit & eyepiece. the lens

is moved back & forth near biprism till a sharp pair of images of slit is obtained the distance of images is measured ~~say~~ say d_1. If u is distance of slit & V that of eyepiece from lens then magnification $\frac{V}{u} = \frac{d_1}{d}$ — ①

Now the lens is moved to a position nearest to eyepiece & same procedure is repeated & note distance d_2 again $\frac{V}{u} = \frac{d_2}{d}$ — ②

from eq ① and ② $d = \sqrt{d_1 d_2}$

Using values of β, d & D we can calculate wavelength λ using the relation

$$\boxed{\lambda = \frac{\beta d}{D}}$$

② Determination of thickness of sheet

suppose S_1 & S_2 are the virtual coherent monochromatic source we obtain central bright fringe optical path $S_1O = S_2O$ Now let a transparent plate G of thickness t & refractive index μ be introduced in the path of one of beams. the central fringe shifts to P from O. the light waves from S_1 to P travel partly in air ($S_1P - t$) and partly in sheet (L) the optical path

$$\Delta S_1P = (S_1P - t) + \mu t = S_1P + (\mu - 1)t$$

$$\Delta S_2P = S_2P$$

Path diff $\Delta S_1P - \Delta S_2P = 0$

$$\Rightarrow \Delta S_1P = \Delta S_2P$$

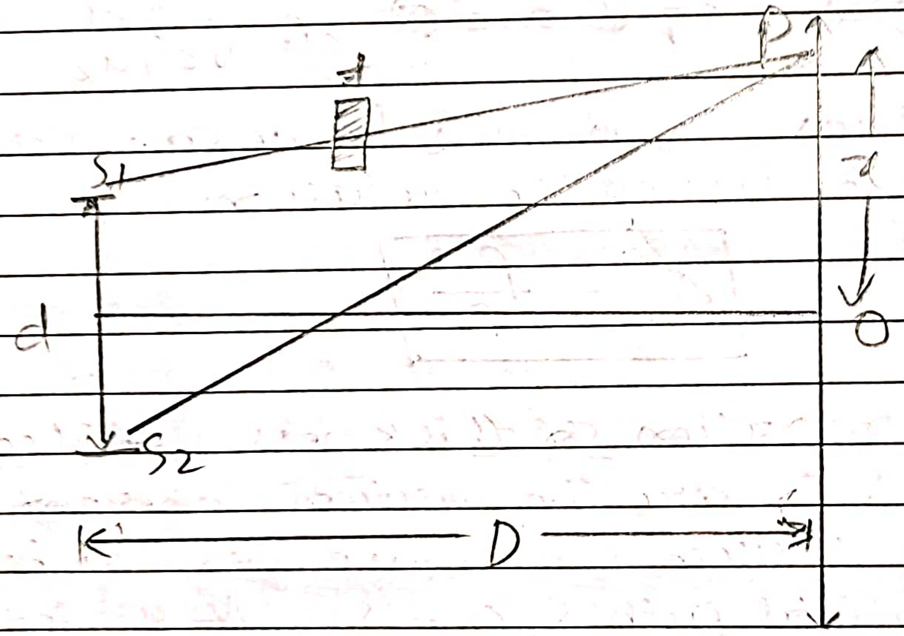
$\Rightarrow$ $S_1P - t + \mu t = S_2P$

$S_2P - S_1P = (\mu - 1) t$

$(\mu - 1) t = \frac{xd}{D}$

hence thickness $\boxed{t = \frac{xd}{D(\mu - 1)}}$

$\left[\because S_2P - S_1P = \frac{xd}{D}\right]$



determination of 2d by deviation method

$2d = 2a(\mu - 1)\alpha$

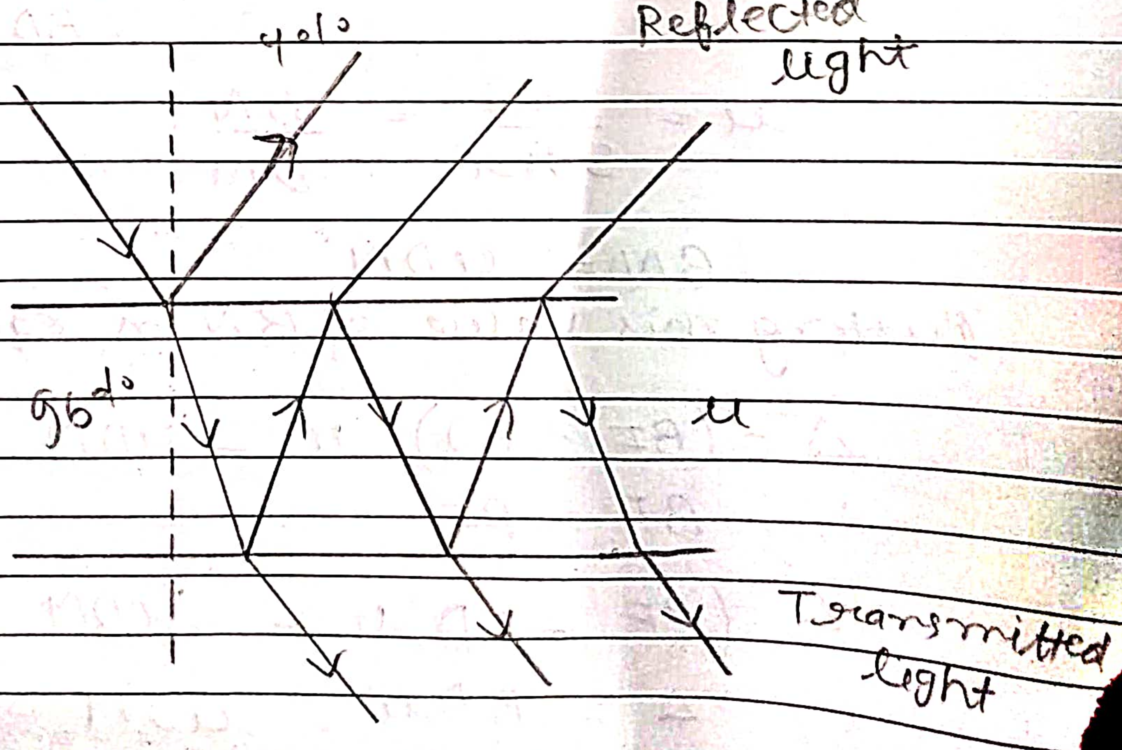
$a \rightarrow$ distance between source & biprism

$\mu \rightarrow$ refractive index

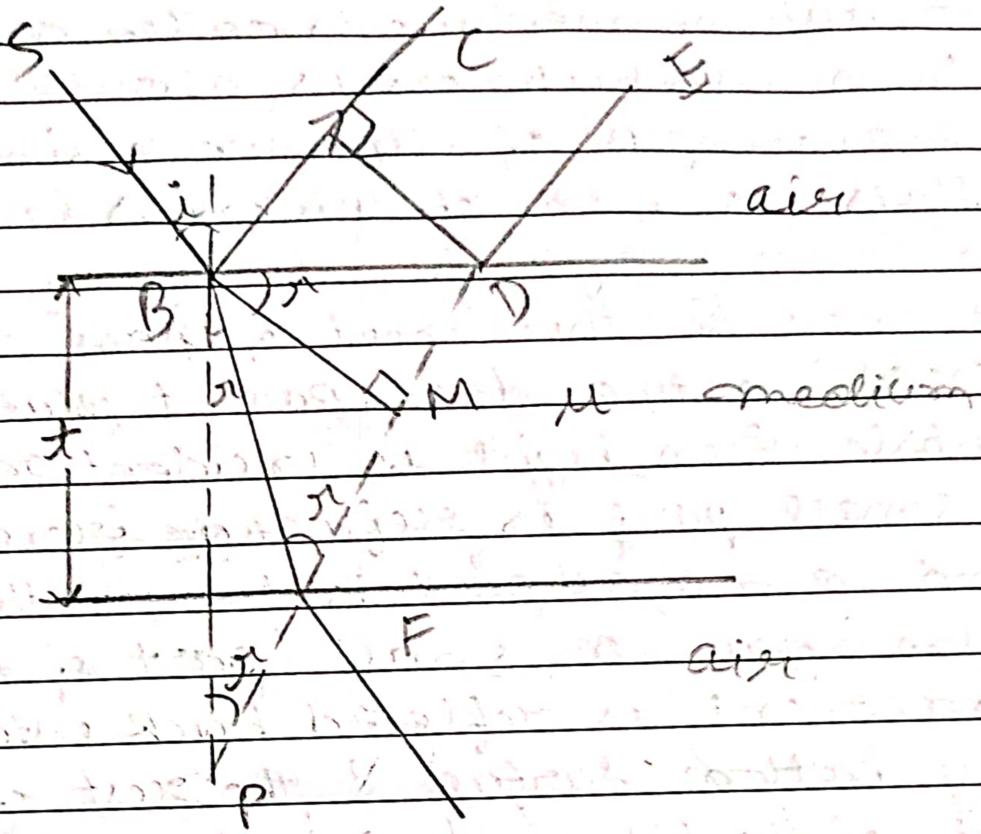
$\alpha \rightarrow$ refracting angle

Thin film

An optical medium is called a thin film when its thickness is about the order of wavelength of light in visible reigon thus, a film of thickness in range 0.5um to 10um may be considered as a thin film. It may be thin sheet of glass, mica, air film enclosed a two transparent plates or a soap bubble. When light is incident on such a film a small part is reflected from top surface and a major part is transmitted into the film again or small part of transmitted component is reflected back into the thin film by bottom surface & the rest of it emerges out of the film. A small portion of the light thus gets reflected partialy several times in succesion with in the film the reflected and refracted components travel along different portions & subsequently overlap to produce interference.



Interference due to reflected light



Path diff b/w DE & BC

$\Delta = (BF + FD)\mu - BN \times i$ --- (1)

In ΔBND, $\sin i = \frac{BN}{BD}$

In ΔBMD, $\sin r = \frac{DM}{BD}$

$$\mu = \frac{\sin i}{\sin r} = \frac{BN}{DM}$$

$BN = \mu DM$

Putting this value of BN in eq (1)

$\Delta = (BF + FD)\mu - \mu DM$ --- (2)

$\because BF = PF$

$\Delta = (PF + FD)\mu - \mu DM$

$\Delta = PD\mu - \mu DM$

$$\Delta = \mu(PD - DM) \quad \text{--- (3)}$$

In ΔPBM, $\cos r = \frac{PM}{PB} = \frac{PM}{2t}$

$$\Rightarrow PM = 2t\cos r$$

putting in eq (3)

$$\boxed{\Delta = 2\mu t\cos r}$$

If a ray is reflected from denser medium it suffers a phase change of π or path diff of $\frac{\lambda}{2}$ including this we get,

$$\boxed{\Delta = 2\mu t\left\{\cos r - \frac{\lambda}{2}\right\}}$$

<u>For maxima</u>

$$2\mu t\cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2}$$

<u>For minima</u>

$$2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

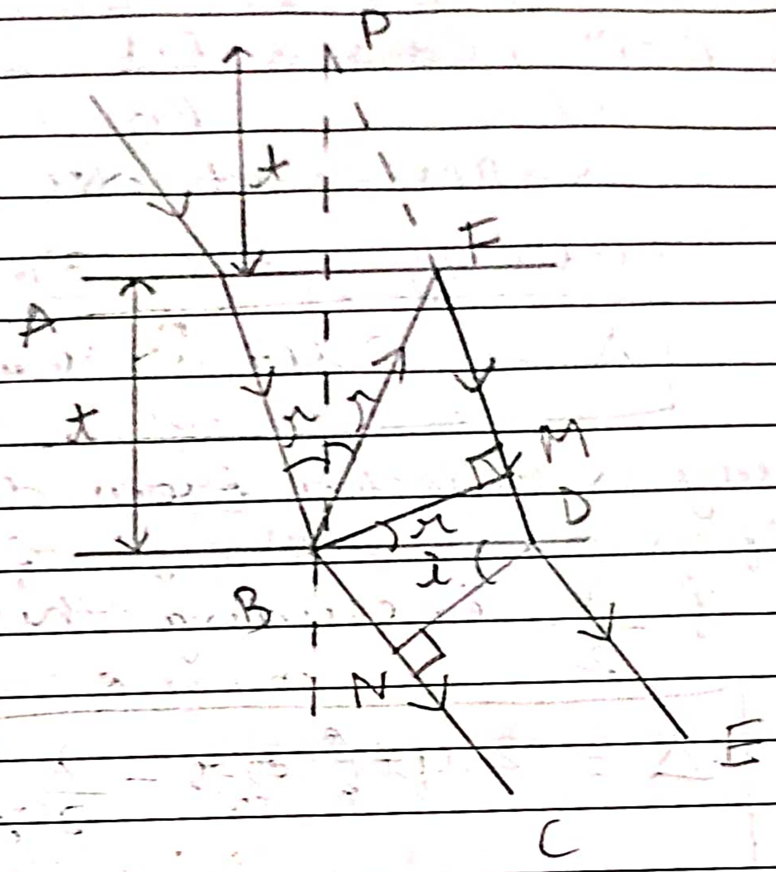
$$2\mu t\cos r = (n+1)\lambda$$

$$\boxed{2\mu t\cos r = m\lambda}$$

$$m = n+1$$

$$m = 1, 2, 3, \ldots\ldots$$

* Interference due to transmitted light



Path diff b/w BC & DE

$$\Delta = (BF + FD)\mu - BN \times 1 \quad \text{--- (1)}$$

In ΔBND, $\sin r = \frac{DM}{BD}$

$$\mu = \frac{\sin i}{\sin r} = \frac{BN}{DM}$$

$$BN = \mu DM$$

putting in eq (1)

$$\Delta = (BF + FD)\mu - \mu DM$$

$$\because PF = BF$$

$$\Delta = (PF + FD)\mu - \mu DM$$

$$= PD\mu - \mu DM$$

$$= \mu(PD - DM)$$

$= \mu PM$ ---- ②

In ΔPBM, $\cos r = \frac{PM}{PB} = \frac{PM}{2t}$

$PM = 2t \cos r$

putting in eq ②

$$\boxed{\Delta = 2\mu t \cos r}$$

For maxima, $2\mu t \cos r = n\lambda$

For minima, $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

* the interference pattern due to transmitted light is ~~complentary~~ complementary to the pattern due to reflected light.

Q-1) A parallel beam of light of wavelength 5890 Å is incident on a thin glass plate of refractive index 1.5 such that angle of refraction is 60° Calculate the smallest thickness of the plate which will appeared dark by reflection.

Sol $\lambda = 5890$ Å

$\mu = 1.5$, $r = 60°$

$2\mu t \cos r = m\lambda$

for minimum thickness, $m = 1$

$2 \times 1.5 \times t \times \cos 60 = 1 \times 5890 \times 10^{-10}$

$t = \frac{5890 \times 10^{-10}}{1.5}$

$t = 3.93 \times 10^{-7}$ m

Q-2) A soap film of refractive index 1.33 is illuminated with light of different wavelength at an angle of 45° there is complete distructive interference for λ 5890 Å find the minimum thickness of the film

Sol

$\lambda = 5890\,Å = 5890 \times 10^{-10}\,m$

$\mu = 1.33$, $i = 45°$, $m = 1$

$$\mu = \frac{\sin i}{\sin r}$$

$$1.33 = \frac{\sin 45}{\sin r}$$

$$\sin r = \frac{1}{\sqrt{2} \times 1.33}$$

$$\sin r = 0.5316$$

$$r = 32.11$$

$$2\mu t \cos r = m\lambda$$

$$2 \times 1.33 \times t \times \cos 32.11 = 1 \times 5890 \times 10^{-10}$$

$$t = \frac{5890 \times 10^{-10}}{2.2531}$$

$$t = 2614.17 \times 10^{-10}$$

$$t = 2.61 \times 10^{-7}\,m$$

Q-3) A thin film of soap solution is illuminated by white light at an angle of incidence $i = \sin^{-1}\left(\frac{4}{5}\right)$ in reflected