5. (a) Find the half range Fourier sine series of $f(x)$ defined over the range $0 < x < 4$ as :

$$f(x) = \begin{cases} x \;, & 0 < x < 2 \\ 4 - x, & 2 < x < 4 \end{cases}$$

(b) Obtain Fourier series of the function :

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

(c) Find the half range cosine series expansion of $f(x) = x - x^2$ in $0 < x < 1$.

6. (a) Solve :

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x}$$

(b) Solve :

$$(D^2 - D D' - 2D)\, z = \cos(3x + 4y)$$

(c) Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6e^{-3x}$.

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3. (a) Solve :

$$\frac{dy^2}{dx^2} - 2\frac{dy}{dx} + y = x e^x \cos x$$

(b) Apply method of variation of parameters to solve :

$$\frac{dy^2}{dx^2} + y = \sec x$$

(c) Solve the simultaneous differential equations :

$$\frac{dx}{dt} = 3x + 8y$$

$$\frac{dy}{dt} = -x - 3y$$

with $x(0) = 6$ and $y(0) = -2$.

4. (a) Prove that :

$$P_n(x) = \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$$

(b) Prove that :

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

(c) Prove that :

$$n P_n(x) = x P_n'(x) - P_{n-1}'(x)$$

(b) Find the power series solution of :

$$\left(1-x^2\right)\frac{dy^2}{dx^2}-2x\frac{dy}{dx}+2y=0$$

about $x=0$.

(c) Find the Fourier series of the function :

$$f(x)=\frac{1}{4}(\pi-x)^2$$

in the interval $0 \leq x \leq 2\pi$. Hence obtain the relation :

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\dots\dots=\frac{\pi^2}{6}$$

(d) A string is stretched and fastened to two points ℓ apart. Motion is started by displacing the string in the from $y=a\sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at a time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by :

$$y(x,t)=a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi c t}{\ell}\right)$$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each question. 5×8=40

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(c) Evaluate :

$$\int_{-1}^{1} x^3 P_3(x)\,dx$$

(d) Show that :

$$\frac{d}{dx}\left[J_0(x)\right] = -J_1(x)$$

(e) Write Dirichlet's conditions for a Fourier series.

(f) If $f(x) = 1$ is expanded in Fourier sine series in $(0, \pi)$ then find the value of b_n.

(g) Form the partial differential equation by eliminating the arbitrary constants a and b from $z = ax + by$.

(h) Classify the partial differential equation :

$$2\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x\,\partial y} + 3\frac{\partial^2 u}{\partial y^2} = 0$$

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Solve the differential equation :

$$x\frac{d^2y}{dx^2} + \left(4x^2 - 1\right)\frac{dy}{dx} + 4x^3 y = 2x^3$$

by changing the independent variable.

B. Tech. Examination 2022-23

(Even Semester)

DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : $8\times1=8$

(a) Find the order and degree of the differential equation :

$$\frac{d^2y}{dx^2}-\sqrt{1+\frac{dy}{dx}}=0$$

(b) Reduce the differential equation :

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log x$$

to a differential equation with constant coefficients.

[P. T. O.