# Impact of Virial ratios on Mass Segregation and Binary Formation in Young Stellar Clusters

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**Abstract**

This study explores the dynamical evolution of young, stellar cluster containing 70 stars solely on gravitational interactions. The initial cluster configuration is generated using the fractal method proposed by Goodwin and Whitworth which creates a sub-structured distribution of stellar positions and velocities. This represents the inherent complexity of young star-forming regions. The masses of the stars are drawn from the Chabrier Initial Mass Function (IMF), which provides a realistic mass distribution of young star clusters. The N-body simulations are conducted using a $P(EC)^n$ fourth-order Hermite integrator with a global time-step to ensure accuracy in tracking stellar trajectories over time. The clusters are simulated with varying virial ratios (0.5, 1.0, 1.5, 2.0) to investigate how different initial kinetic-to-potential energy influence the cluster's evolution. The primary focus is on mass segregation and binary formation where the mass segregation is quantified using the mass Segregation Ratio $\Lambda_{MSR}$. The binary formation is analysed based on the binding energy criterion which identifies gravitationally bound star pairs in the cluster. The results show that clusters with low virial ratios undergo faster mass segregation and exhibit high rate of early binary formation. While the clusters with high virial ratios experience slower evolution with more stable binary systems over time. By comparing the outcomes for different virial ratios, this work provides insights into how initial conditions affect the long-term structural and dynamical properties of young stellar clusters, with implications for star formation and cluster survival.

## I. Introduction

### I.1. Subsection

Small open stellar clusters plays a crucial role in the formation of galaxies and dynamical evolution of stellar populations. Understanding the evolution of these stellar clusters is important as they represent the key stage in the life-cycle of stellar systems. Open clusters are young and contain a few dozen to few thousand stars and they are found within the galactic disk. Unlike globular clusters, they are less massive and disperse over time as stars drift away. But they provide valuable insights into the star formation and early stellar dynamics.

In small open clusters, star formation occurs within the dense regions of the molecular clouds. The protoclusters emerge from these dense molecular clouds due to the gravitational collapse of these dense cores[1]. The cores will form into small clump which gives rise to the birth of individual stars. Various factors effect the stability of these clusters such as Star Formation Rate, tidal disruption by galaxy ($\rho_* \geq 0.1 M_{\odot} pc^{-3}$[2]), by passing through interstellar clouds ($\rho_* \geq 1.0 M_{\odot} pc^{-3}$ [3]), by the number of enough stars in the cluster such that the evolution time $\tau_{ev} > 10^8 yrs$ [4].

The formation of stars within clusters is often not uniform. Many clusters exhibit substructure, where stars form in hierarchical or fractal patterns. This substructure is believed to be a consequence of turbulence in the molecular clouds where the stars form[5]. This leads to clumps of stars with correlated positions and velocities. The fractal method described by Goodwin and Whitworth[6] provides an effective way to model this substructure, replicating the observed distributions of young clusters in star-forming regions. This fractal method is used to initialise the cluster for the simulation.

The small clusters experience intense dynamical interactions, leading to phenomena such as mass segregation and the formation of binary star systems. The mass segregation is a phenomena where more massive stars migrate toward the cluster core while lower-mass stars migrate to the outer edge of the cluster. Escape rates in the cluster tends to increase as the clusters lose their low-mass stars. These dynamics are crucial for understanding how small clusters evolve and merge into larger systems over time. It can provide insights into whether this process is driven by primordial conditions or from dynamical interactions.

Another critical outcome of stellar interactions in clusters is the formation of binary star systems. The binary star systems are where two stars become gravitationally bound and orbit each other. Binaries play an important role in the overall dynamics of star clusters, at which they act as energy sinks and absorbing kinetic energy through close encounters with other stars. This interaction can prevent cluster collapse or lead to the ejection of stars which influences the evolution of cluster. The formation of binaries is also a crucial factor in understanding star formation itself, as a significant fraction of stars are born in binary or multiple-star systems. Due to the high stellar densities of small stellar clusters, the formation and evolution of binary star systems occur more rapidly[7]. The study shows that the mean binary fraction is high in low-density star clusters when compared to the high-density star clusters[8].

Virial ratio is a critical parameter in determining the dynamical state of a stellar cluster. It is the ratio of a system's kinetic energy to its potential energy. Systems with a virial ratio less than 1 are dynamically cold, where gravitational forces dominate, potentially leading to collapse. While systems with a ratio greater than 1 are dynamically hot, indicating a tendency for expansion. Previous studies have shown that clusters with lower virial ratios are more likely to exhibit rapid dynamical interactions which leads to faster mass segregation and vice-versa[9].

This study focuses on exploring these processes, mass segregation and binary formation in small young clusters containing 70 stars. By simulating clusters with varying virial ratios, we aim to investigate how initial conditions influence the spatial and dynamical evolution of the cluster. We focus on understanding whether clusters with virial ratios below or above equilibrium exhibit faster or slower rates of mass segregation and also how they influence the formation and stability of binary star systems.

## II. Methodology

### *II.1. Initial Cluster Configuration*

The initial conditions of the stellar cluster are constructed using the fractal method described by Goodwin and Whitworth[6]. This method is close to describe the sub-structured and clumpy distributions observed in the real star-forming regions. The main reason to use this fractal method is that star formation does not happen uniformly but rather in localized, clumpy regions. This is often influenced by the turbulence in the interstellar medium[10].

The initial stellar masses are drawn from the Chabrier Initial Mass Function (IMF)[11]. The Chabrier IMF is used to model the mass distribution of stars in young clusters and reflects the observed stellar population in star-forming regions. This IMF provides a log-normal distribution for stars with masses below $1M_{\odot}$ and a power-law distribution for higher-mass stars which ensures a realistic representation of both low-mass stars and more massive stars.

#### *II.1.1. Fractal Method*

The stellar cluster is designed using fractal distribution method by the Goodwin and Whitworth[6] is as follows:

1. The initial volume is defined as a cube with sides of length $N_{div} = 2$ which represents the spatial boundary of the cluster.
2. A single parent star is placed at the center of the cube. This parent star is used to generate the next generation of stars.
3. The cube is subdivided into smaller equal-volume $N_{div}^3$ sub-cubes and at the center of each sub-cube, a "child" star is placed as the first generation of children.
4. The probability that a child matures to become a parent is proportional to $N_{div}^{D-3}$, where D is the fractal dimension which controls the fractal nature of the cluster to be clumpy or uniform distribution.
5. If a child does not become a parent, it is pruned with all their parent stars.
6. A small amount of randomness or noise is added to the positions of the child stars, who become parents to avoid grid-like appearance. The noise is drawn from the Gaussian distribution with mean 0 and standard deviation of 0.05.
7. The recursive subdivision and selection process continues until the generation with little more than required number of stars is created.
8. The entire parent stars are pruned so that only the last generation of stars will remain.
9. The entire region is pruned to fit within the spherical volume, with the radius of the cluster and the stars within the spherical region is pruned randomly until the target stars are reached.
10. The root parent star is assigned velocity randomly drawn from a Gaussian distribution, simulating the independent motion of the star.
11. The child stars inherit their velocities from the same Gaussian distribution with unit variance. This preserves the substructure in velocity space, simulating the localized star formation dynamics[12].
12. Once the initial velocities are assigned, the velocities are adjusted to ensure that the cluster's center of mass (COM) has zero net velocity.
13. Then the velocities scaled to achieve the desired virial ratio, which governs the dynamical state of the cluster.

By varying the fractal dimension and virial ratio, we can simulate the clusters with different degrees of substructure and initial dynamical states. In this study, the fractal dimension of $D = 2.6$ is used to create a moderate level of clumpiness to represent the young stellar environments.

### *II.2. Numerical Simulation*

To simulate the evolution of the cluster, we employed an N-body integrator based on the fourth-order Hermite scheme with global time-step. This method is suitable for high precision simulations of gravitational systems. The Hermite scheme, a time-symmetric version is used as it provides a higher degree of accuracy by reducing scalar energy errors.

### II.2.1. Hermite $4^{th}$ Order Method

The method involves predicting positions and velocities using Taylor expansions and correcting them iteratively by calculating the higher-order terms such as acceleration and its time derivative. This $P(EC)^n$ Hermite integrator ensures that the gravitational interactions between stars are handled accurately, even in densely packed environments where close encounters occur. The evaluation and corrector steps can be repeated multiple times to further improve the accuracy of the solution. This time-symmetric nature of this integrator minimises the energy drift as seen in long-term N-body simulations[13]. The time-symmetric fourth-order Hermite scheme works as follows:

1. Predictor Step:

   The prediction step is done by using Taylor expansion of position and velocity of each star which are accurate to a fourth-order approximation in time. The predictor formulae[14] is given by

   $x_{p,j} = x_j + v_j(t - t_j) + \frac{a_j}{2}(t - t_j)^2 + \frac{\dot{a}_j}{6}(t - t_j)^3$

   $v_{p,j} = v_j + a_j(t - t_j) + \frac{\dot{a}_j}{2}(t - t_j)^2$

   where j is the index of the body. All the parameters in the formula is calculated without using interpolation since $\dot{a}_j$ is calculated form the $x_j$ and $v_j$.

2. Corrector Step:

   After the prediction step, the forces are recalculated using the predicted positions and velocities which is then used to correct the initial prediction using higher-order terms. The corrector formulae[14] is given by

   $x_{c,i}(t_i + \Delta t_i) = x_{p,i} + \frac{a_{0,i}^{(2)}}{24}\Delta t_i^4 + \frac{a_{0,i}^{(3)}}{120}\Delta t_i^5$

   $v_{c,i}(t_i + \Delta t_i) = v_{p,i} + \frac{a_{0,i}^{(2)}}{6}\Delta t_i^3 + \frac{a_{0,i}^{(3)}}{24}\Delta t_i^4$

   where

   $a_{0,i}^{(2)} = \frac{-6(a_{0,i}-a_{1,i})-\Delta t_i(4\dot{a}_{0,i}+2\dot{a}_{1,i})}{\Delta t_i^2}$

   $a_{0,i}^{(3)} = \frac{12(a_{0,i}-a_{1,i})+6\Delta t_i(\dot{a}_{0,i}+\dot{a}_{1,i})}{\Delta t_i^3}$

   where $a_{1,i}$ and $\dot{a}_{1.i}$ are the acceleration and its time derivative at time t.

Generally, In N-body integration methods, even small errors in the energy calculation can accumulate over-time results in artificial expansion or contraction of the system. The time-symmetric version[14] is used to reduce the long-term drift in the total energy of the system. In our simulations, we used global time step for simplicity, which avoids complications related to individual time steps. This approach ensures more uniformity and stability in the integration process. This method is useful for systems where the majority of stars follow regular orbits, as it can iterate nearly circular orbits with high precision. In our case, the cluster contains a sub-structured distribution of stars at which the Hermite scheme efficiently handles the gravitational interactions while minimising the integration error.

### II.2.2. Assumptions and Limitations

For the simulations, we used a random sample from the Chabrier IMF to assign masses to the 70 stars in the cluster, ranging from sub-solar masses to a few solar masses($< 100M_{\odot}$). We also assume the system evolves purely under gravitational interactions, with no contribution from gas, dust or other dissipative forces. This simplifies the computational model but limits its applicability in gas-rich environments where the additional forces alter the dynamics. The individual stars are treated as point masses, with no internal evolution or mass loss over time. The simulations are conducted using a global time step scheme with the fourth-order Hermite integrator. The maximum global time step change is set to one year since the global time step will change over time based on the calculations between the stars. This setup provided a robust framework for running the simulations efficiently. This allowed multiple runs with different virial ratios while ensuring sufficient accuracy to capture the dynamical evolution of the cluster. For an Open cluster, the evolution time is of order $\tau_{ev} \approx 10^2\tau_{relax}$[1] where the relaxation time is given by

$\tau_{relax} \approx \frac{0.1N}{lnN}\tau_{cross}$ [15]

where N is the number of the stars in the cluster and $\tau_{cross}$ is the dynamical crossing time. The dynamical crossing time $\tau_{cross}$ is the time it takes for the star to cross the boundary of the cluster. Since, our simulations are purely dynamical, this is an essential parameter to calculate the ending time of the simulation. The simulations data is shown in the tableII.2.2 given below.

| Virial ratio | Total Mass | Evolution time |
|---|---|---|
| 0.5 | 24.15 $M\odot$ | 88 Myrs |
| 1.0 | 24.02 $M\odot$ | 125.4 Myrs |
| 1.5 | 24.44 $M\odot$ | 152.5 Myrs |
| 2.0 | 23.93 $M\odot$ | 177.6 Myrs |

## III. Results

The evolution of the cluster for different virial ratios reveals significant changes inn their spatial distribution overtime. This is identified in the variation of INDICATE index[16] across different virial ratios. The INDICATE method is used to measure the dynamical state of the cluster by analyzing the spatial and kinematic distributions of the stars. It measures the local clustering by assessing the number of neighbouring stars within a critical radius. We implemented this INDICATE method where for each star, the distance to its $5^{th}$ nearest neighbour is calculated using a k-dimensional tree (k-d tree) algorithm. This value is compared to the mean neighbour distance from the control distribution. The INDICATE index is derived by measuring the proportion of stars whose nearest neighbour distance is smaller than the mean control distance. This index quantifies the relative local density around each star.

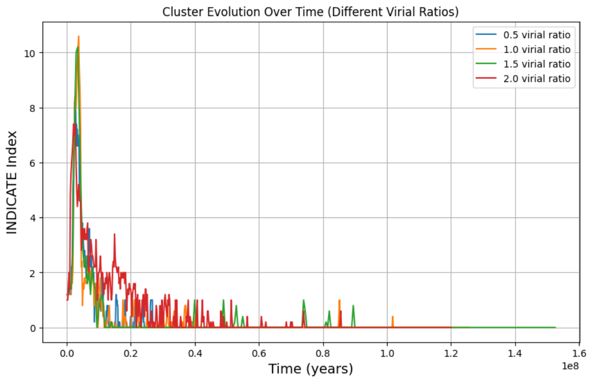


**Figure 1:** INDICATE index during the cluster's evolution at different virial ratios

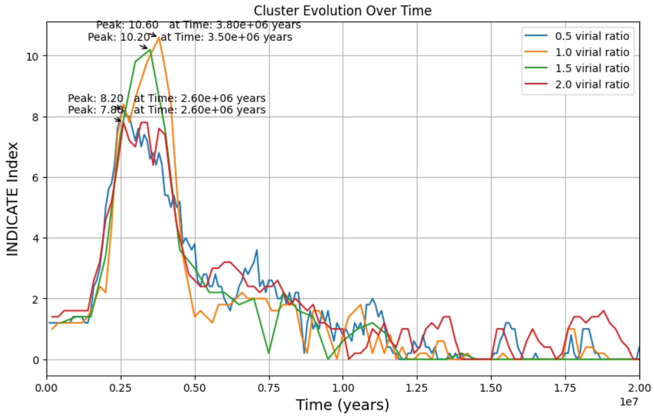


**Figure 2:** INDICATE index during the cluster's evolution till 20Myears at different virial ratios

From figure1, the INDICATE index for all the virial ratios settles towards lower values. This means the cluster have reached a relatively stable state with minimal dynamical evolution. The cluster with high virial ratio 2.0 show more variability in the INDICATE index, suggest that the systems take longer to settle compared to those with lower virial ratios.

The graph from the figure2 shows the time evolution of the INDICATE index for each virial ratio trimmed to the span of approximately 20 million years. The initial peak in the INDICATE index indicate periods of strong local clustering followed by a rapid decline. This suggests that the stellar systems undergo rapid early-dynamical interactions, indicating the formation of local sub-clusters. This is due to the initial conditions, where the stars tend to clump together before the cluster starts expanding.

*III.0.1. Mass Segregation Analysis*

In this study, the mass segregation is analysed by computing the Mass Segregation Ratio ($\Lambda_{MSR}$), which compares the spatial distribution of the most massive stars to a randomly selected sample from the overall population of stars. The $\Lambda_{MSR}$ is calculated by determining the mean minimum spanning tree (MST) method from the scipy package. The MST length of the top 10% massive stars is calculated and compared it to the MST length of random stars of the same size sample. This method allows for quantifying the degree of mass segregation, where a $\Lambda_{MSR} > 1$ indicates that the massive stars are more spatially concentrated, while a value of 1 suggests no segregation[9][17]. This method ensures that the measurement of mass segregation is robust across varying cluster conditions and provides valuable insights into the dynamics of star clusters at different stages of their evolution.

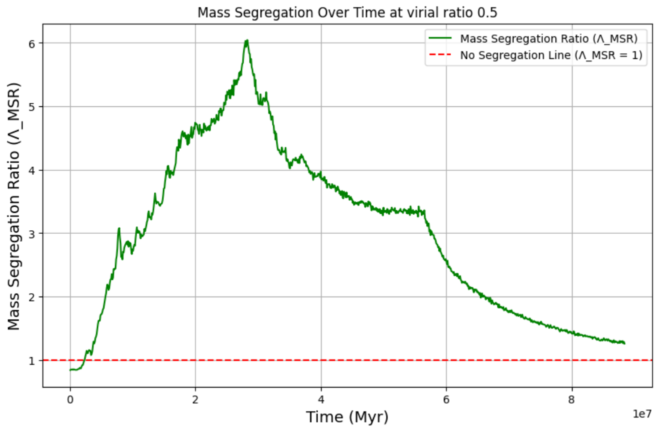


**Figure 3:** Mass segregation of cluster at virial ratio 0.5

The mass segregation of the cluster at virial ratio 0.5 is plotted in the figure3, where $\Lambda_{MSR}$ steadily increases peaking around 6. This indicates a significant level of mass segregation, before slowly declining over time. This suggests a dynamic state of mass segregation at which eventually stabilizes around $\Lambda_{MSR}$ of 1.5 by the end of simulation.

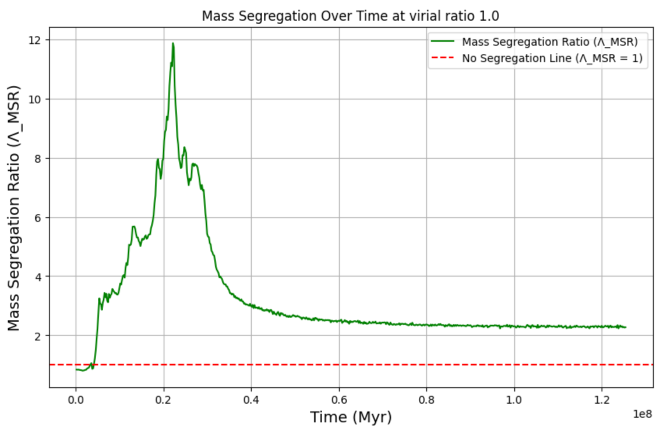


**Figure 4:** Mass segregation of cluster at virial ratio 1.0

At a virial ratio of 1.0, the $\Lambda_{MSR}$ also exhibits a strong peak, reaching a maximum value near 12. The decline is more rapid and the system stabilizes faster, converging to a mass segregation ratio near 2 as shown in the figure4.

For the virial ratio of 1.5, the mass segregation is less

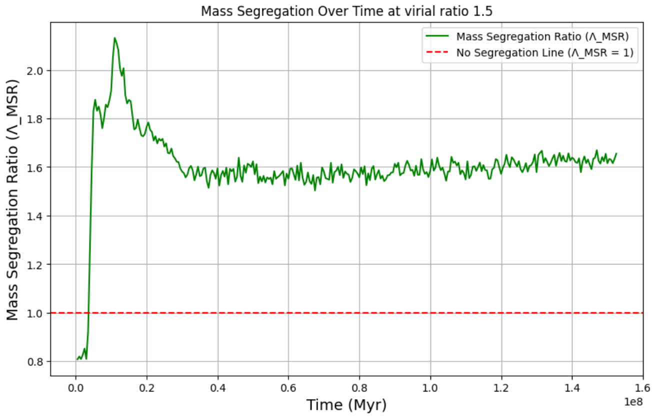


**Figure 5:** Mass segregation of cluster at virial ratio 1.5

clear, with a peak value just above 2. The plot shows a consistent behaviour over time, with the $\Lambda_{MSR}$ fluctuating between 1.5 and 2.0 after the initial peak as shown in the figure5.

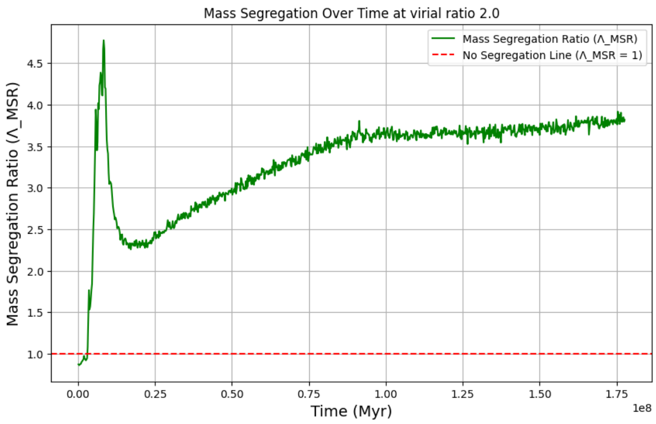


**Figure 6:** Mass segregation of cluster at virial ratio 2.0

The mass segregation at a virial ratio of 2.0 shows an even more subtle peak which is just over 4 before stabilizing at 3 over time. This suggests that the minimal mass segregation compared to lower virial ratios as shown in the figure6.

### *III.1. Binary Formation*

The formation of binary systems through dynamical interactions acts as crucial indicators of the internal cluster dynamics. They play an important role in the redistribution of kinetic energy within the system. The cumulative number of binaries formed over time for the clusters with virial ratio 0.5, 1.0, 1.5, 2.0 are presented in the figures78910 respectively. The method used to identify binary systems is based on the binding energy criterion[18], for detecting gravitationally bound systems. The binding energy is calculated for each pair of stars in the cluster during the evolution time. If the binding energy is negative, the stars form a binary which indicates a gravitationally bound system. The stars are tracked over time and identified as binaries if they remain bound in the subsequent snapshots of time.

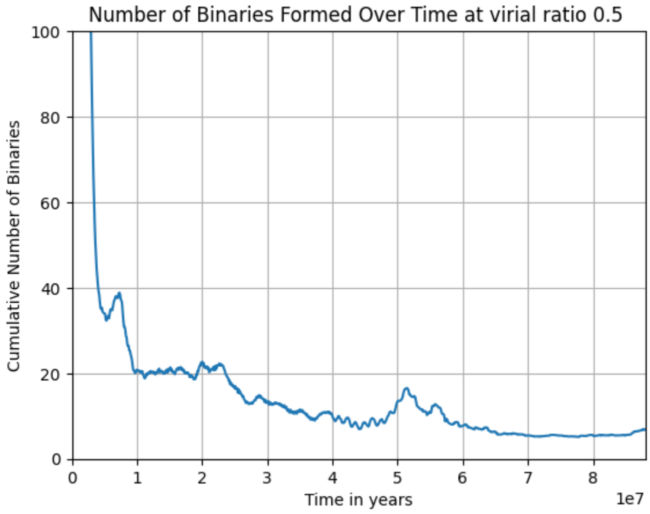


**Figure 7:** Cumulative binaries formed over time at virial ratio 0.5

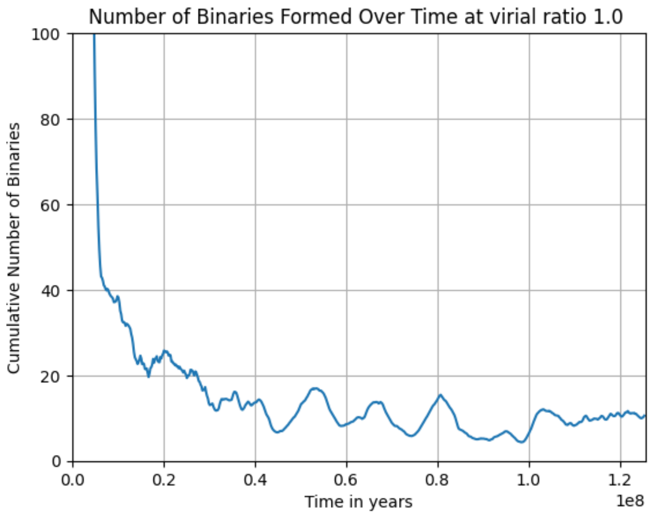


**Figure 8:** Cumulative binaries formed over time at virial ratio 1.0

As observed in the figure7, the initial phase of the simulation exhibits a rapid formation of binary systems, followed by a sharp decrease within the first million years. This is the characteristic of the clusters with high gravitational binding, where the strong gravitational interactions facilitate binary formation but also lead to quick disruption. In such dynamically cold systems, the early dominance of close encounters results in frequent binary encounters, causing a steep decline in the cumulative binary count. After this initial period of binary destruction, the number of binaries stabilizes as the cluster relaxes. This equilibrium state is notable in dense centrally concentrated clusters, where binaries are more likely to form and evolve under the influence of multiple stellar interactions.

For the virial ratio 1.0, the figure8 explains the evolution of binaries in the cluster which is in virial equilibrium at the initial stage. The cumulative binary formation curve exhibits a similar rise and decline as seen in the case of virial ratio 0.5 but with more gradual decrease. The cluster's balanced initial energy state results in fewer disruptive close encounters which allows more gradual binary dissolution over time. In this case, the binary formation rate does not reach steady point quickly. The oscillations observed in the cumulative binary count suggests that the binaries are being continuously formed and destroyed over a longer period as the system slowly approaches equilibrium.

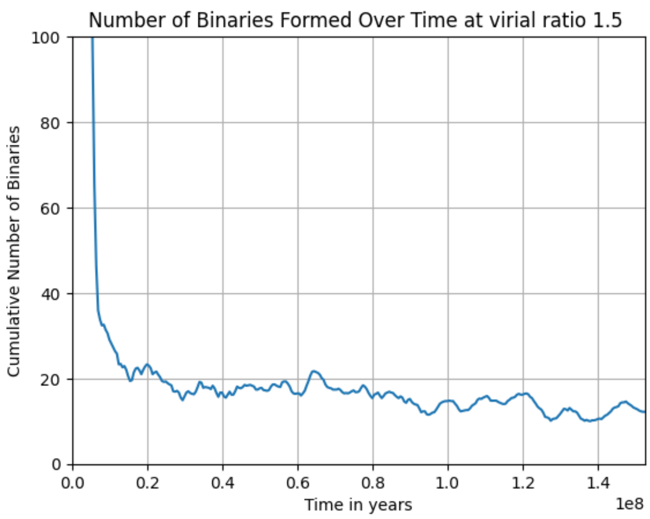


**Figure 9:** Cumulative binaries formed over time at virial ratio 1.5

As shown in the figure9, the binary formation and dissolution is different for the clusters with virial ratio of 1.5. The higher virial ratio indicates a system with excess kinetic energy at start which leads to fewer initial close encounters and slower binary formation. As the stars are dynamically hot systems which are less likely to come into close with one another, the decline in the number of binaries is explainable which reduced the interaction rate. But, this case of higher virial ratio leads to more stable binary systems over time. The stable and slower decrease in the cumulative binary count suggest that the clusters with high virial ratios can sustain their binary populations for longer period.

From the figure10, we can see a fast decline in the formation of binaries in the first million years. It is stabilized for a longer period which suggests that the most stable binary systems are formed at higher virial ratios. The binary systems can be sustained for longer periods due to the excess kinetic energy in the cluster.

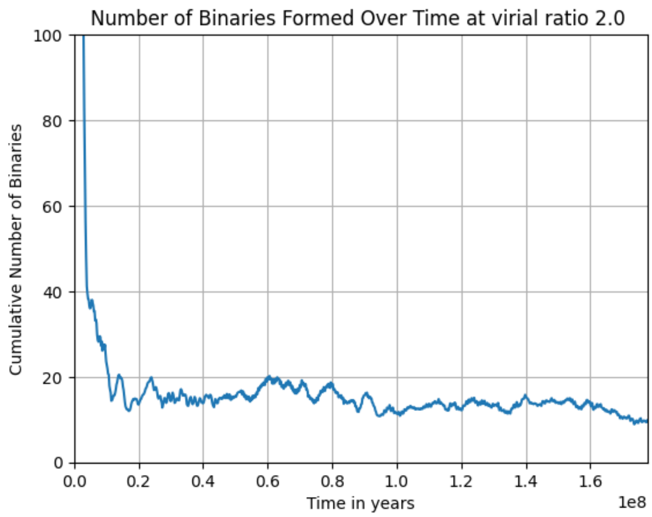


**Figure 10:** Cumulative binaries formed over time at virial ratio 2.0

## IV. Discussion and Conclusion

The results of this study highlight how the initial virial ratio influence the dynamical evolution of the star clusters in relation to mass segregation and binary formation. These findings validate the earlier studies[6], which demonstrated that the clusters with low virial ratios collapse rapidly, leading to burst of dynamical interactions, early binary formation and disruption.

The results show that with lower virial ratios exhibit stronger and earlier mass segregation with $\Lambda_{MSR}$ peaking in the earlier stages of simulation. This is consistent with the case where the more massive stars sink into the cluster core which validates the previous study[9]. The higher virial ratios show weaker mass segregation and slower evolution towards a steady state, as observed in the previous studies[17].

The formation and destruction of binary systems offers more insight into the role of initial conditions in the long-term evolution of the star clusters. The rapid formation and disruption of the binaries in low virial ratio clusters is consistent with the previous findings[18][19]. As cluster relaxes over time, the binary count stabilizes. This suggests that fewer binary systems are being formed or destroyed as the cluster reaches equilibrium.

## V. Future Work

In the future studies, it would be interesting to explore other factors such as initial mass function (IMF) and the fractal dimension of the cluster, impact these dynamical processes in conjunction with virial ratio. The role of stellar evolution is neglected in this purely dynamical study could provide further insights into the long-term survival and dynamical evolution of binary systems in young star clusters.

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### A. Spatial Distribution of clusters at different virial ratios

To visualise the clumping of stars in the clusters, the spatial distribution of the clusters at the peak of the INDICATE index are generated as shown in the figures11121314. The shaded region is the boundary of the cluster with radius 0.5pc and the stars inside the shaded region. We can see the clustering of the stars at different virial ratios of 0.5, 1.0, 1.5, 2.0 in the figures11121314.

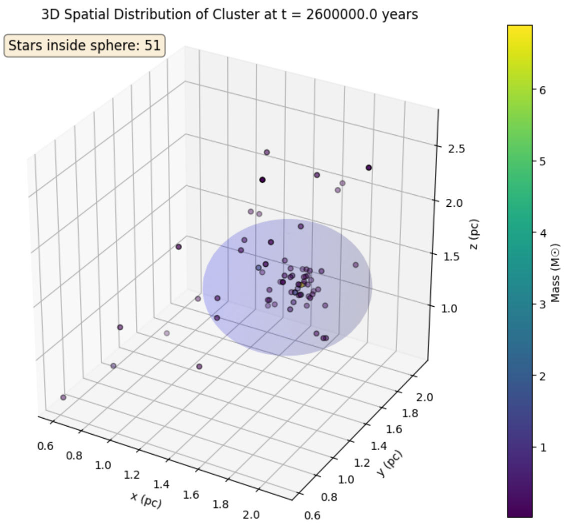


**Figure 11:** Spatial distribution of cluster at virial ratio 0.5 and at t = 2.6Myrs in 3D

As show in the figure11, the stars are clumped in localized regions at the peak of the INDICATE index, demonstrating strong local clustering. The gravitational interactions dominate, causing the stars to congregate into smaller and dense regions.

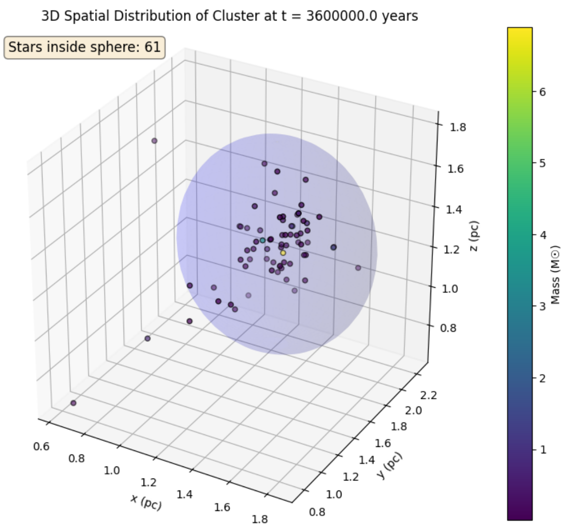


**Figure 12:** Spatial distribution of cluster at virial ratio 1.0 and at t = 3.6Myrs in 3D

For the virial ratio of 1.0, we can see in the figure12 that the clustering is still present but more diffused when compared to 0.5 virial ratio case. The balance between the gravitational potential energy and kinetic energy results in more evenly distributed cluster with some traces of sub-clustering.

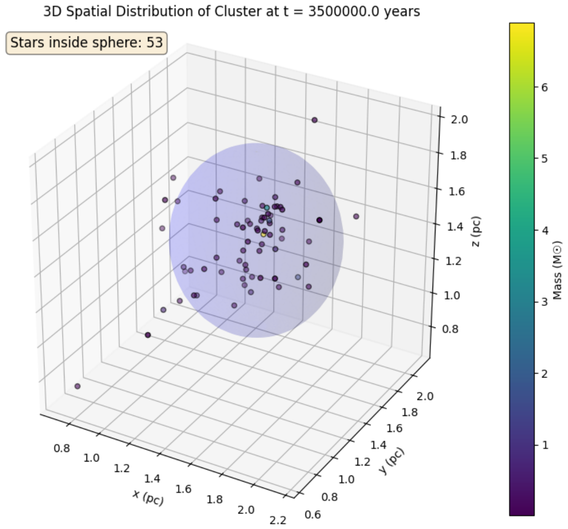


**Figure 13:** Spatial distribution of cluster at virial ratio 1.5 and at t = 3.5Myrs in 3D

In the case of virial ratio 1.5, it is less substructured as shown in the figure13. The system's higher kinetic energy makes the cluster to expand more rapidly. This leading to fewer tightly bound sub-clusters, though some local clustering is still observable.

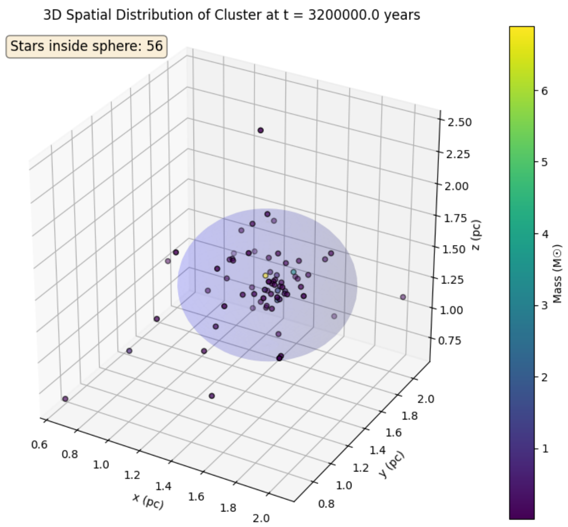


**Figure 14:** Spatial distribution of cluster at virial ratio 2.0 and at t = 3.2Myrs in 3D

The cluster with a virial ratio of 2.0 shows the most dispersed spatial distribution as shown in the figure14. The high kinetic energy causes the system to expand more rapidly, dispersing any initial clumping of stars. The stars are less gravitationally bound, resulting in more homogeneous and spread-out configuration.

The clusters with low virial ratios experience their peak clustering earlier in the evolution. This suggests that the gravitational interactions dominate which leads to rapid sub-cluster formation and dissipation. while in the clusters with high virial ratios, particularly in the 2.0 case, it shows a delayed peak and exhibit more gradual dynamical evolution. This aligns with the theoretical expectations, where virial ratios defines the relative balance between gravitational and kinetic energies. Lower virial ratios imply that gravitational forces dominate, leading to more rapid contraction and clustering. The higher virial ratios imply that the kinetic energy is dominant, which allows the cluster to expand more freely and results in slower relaxation.