Final Year Project Report

Formalising Alternative Models of Computation in Isabelle
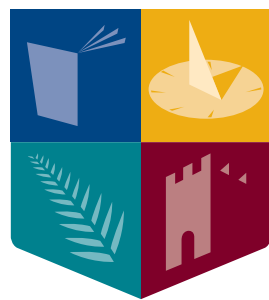
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5 Credits

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ABSTRACT

The goal of this project was to formalise and implement alternative models of computation inside the proof assistant Isabelle, and then to make use of this to assist in providing definitions on and proving various results about these models. In particular it focuses on Cellular Automata, in both one and two-dimensional variants, and with differing topologies.

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INTRODUCTION

Cellular Automata (CA) are a very simple model of computation. There are many variations and extensions of them, and some like Conway's Game of Life and Rule 110 are known to be Turing Complete. Isabelle is a proof assistant that can be used for anything from mathematical proofs to formal verification of software properties.

1.1 Topic addressed in this project

This project looks at formalising models of Cellular Automaton in Isabelle to help deal with the complexity that comes with trying to mathematically prove results about them. In addition to providing a definition of six different variants of CA, this project provides definitions of important properties they may have, and proves some lemmas and theorems necessary to work with them.

1.2 Motivation

It is very difficult to work with high level concepts while still being rigorous, and theoretical Computer Science is a very abstract and mathematical discipline that requires exactly that. as it strikes a balance between simplicity and complexity. They are

1.3 Approach

Isabelle was chosen as the language to implement these models in.

1.4 Metrics

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