$$\boxed{\beta^2 = \left(\frac{\Delta x}{\Delta y}\right)^2}$$

**Discretized Stream Function**

$$\omega = -\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) \Rightarrow w_{i,j} = \left(\frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{\Delta x^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta y^2}\right)$$

$$\Rightarrow \Delta x^2 \omega_{i,j} = \psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j} + \beta^2\psi_{i,j+1} - 2\beta^2\psi_{i,j} + \beta^2\psi_{i,j-1}$$

$$\Rightarrow \boldsymbol{\psi_{i,j} = \frac{1}{2+2\beta^2} \times \left(\psi_{i+1,j} + \psi_{i-1,j} + \beta^2\psi_{i,j+1} + \beta^2\psi_{i,j-1} + \Delta x^2\omega_{i,j}\right)}$$

**Discretized u, v, Vorticity**

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \nu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right)$$

$$\Rightarrow \overbrace{\frac{1}{\nu}\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}\right)}^{A} = \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}$$

$$\Rightarrow \overbrace{\frac{1}{\nu}\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}\right)}^{A} = \frac{\omega_{i+1,j} - 2\omega_{i,j} + \omega_{i-1,j}}{\Delta x^2} + \frac{\omega_{i,j+1} - 2\omega_{i,j} + \omega_{i-1,j}}{\Delta y^2}$$

$$\Rightarrow \boldsymbol{\omega_{i,j} = \frac{1}{2+2\beta^2} \times \left(\omega_{i+1,j} + \omega_{i-1,j} + \beta^2\omega_{i,j+1} + \beta^2\omega_{i,j-1} - \Delta x^2 A\right)}$$

$$\begin{cases} A = \dfrac{1}{\nu}\left(u\dfrac{\partial \omega}{\partial x} + v\dfrac{\partial \omega}{\partial y}\right) = \dfrac{1}{\nu}\left(u_{i,j}\dfrac{\omega_{i+1,j} - \omega_{i-1,j}}{2\Delta x} + v_{i,j}\dfrac{\omega_{i,j+1} - \omega_{i,j-1}}{2\Delta y}\right) \\ u = \dfrac{\partial \psi}{\partial y} \Rightarrow u_{i,j} = \dfrac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta y} \\ v = -\dfrac{\partial \psi}{\partial x} \Rightarrow v_{i,j} = -\dfrac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta x} \end{cases}$$

**Discretized Boundary Condition Of Vorticity**

$$\omega_{1,j} = -2\frac{\psi_{2,j} - \Delta x . v_{1,j}}{\Delta x^2} \xrightarrow{v_{1,j}=0} \omega_{1,j} = -2\frac{\psi_{2,j}}{\Delta x^2}$$

$$\omega_{i_{max},j} = -2\frac{\psi_{i_{max}-1,j} + \Delta x . v_{i_{max},j}}{\Delta x^2} \xrightarrow{v_{i_{max},j}=0} \omega_{i_{max},j} = -2\frac{\psi_{i_{max}-1,j}}{\Delta x^2}$$

$$\omega_{i,1} = -2\frac{\psi_{i,2} - \Delta y . u_{i,1}}{\Delta y^2} \xrightarrow{u_{i,1}=-U} \omega_{i,1} = -2\frac{\psi_{i,2} + \Delta y . U}{\Delta y^2}$$

$$\omega_{i,j_{max}} = -2\frac{\psi_{i,j_{max}-1} + \Delta y . u_{i,j_{max}}}{\Delta y^2} \xrightarrow{u_{i,j_{max}}=U} \omega_{i,j_{max}} = -2\frac{\psi_{i,j_{max}-1} + \Delta y . U}{\Delta y^2}$$