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Final-Exam, MTH 320, Fall 2020

Ayman Badawi

$$\text{Score} = \frac{\qquad}{48}$$

QUESTION 1. (6 points) Let $F = (1\ 3\ 2\ 4)\ o\ (1\ 2\ 3)\ o\ (4\ 5)$

(i) Is $F \in A_5$? Explain

(ii) Find $|F|$

(iii) Find F^{-1}

QUESTION 2. (6 points) (up to isomorphic) classify all noncyclic abelian group with 36 elements, such that each has unique subgroup with 9 elements. Write down the invariant factors of each group.

QUESTION 3. (6 points) Let $F : Z_5 \oplus Z_5 \to Z_5$ such that $F(a, b) = a^{-1} + 2b$ (note that a^{-1} means inverse of a under addition mod 5 and 2b means 2 times b mod 5)

(i) Show that F is a group homomorphism.

(ii) Find $Ker(F)$

(iii) For each left cosets, say L, of $Ker(f)$, find $F(w)$ for every $w \in L$.

QUESTION 4. (6 points)
(i) We know that $(Aut(Z_{24}),\ o) \approx Z_{m_1} \oplus \cdots \oplus Z_{m_w}$, where $m_1, ..., m_w$ are the invariant factors of $Aut(Z_{24})$. Find $m_1, ..., m_w$.

(ii) Construct a subgroup, H, of $Aut(Z_{24})$ such that $|H| = 4$. Is it possible that H is cyclic? Explain.

QUESTION 5. (4 points) Give me an example of a group $(D, .)$ such that D has a normal subgroup H such that D/H is cyclic, but D is not abelian.

QUESTION 6. (4 points) (up to isomorphic) classify all abelian group with 72 elements.

QUESTION 7. (4 points) We know $U(360) \approx Z_{m_1} \oplus \cdots \oplus Z_{m_w}$, where $m_1, ..., m_w$ are the invariant factors of $U(360)$. Find $m_1, ..., m_w$. [Note $360 = 2^3 \cdot 3^2 \cdot 5$]

QUESTION 8. (4 points) Let D be a simple group such that $|D| \geq 60$. Prove that D does not have a subgroup H such that $1 < [H : D] \leq 4$ (Recall that $[H : D] = |D|/|H|$)

QUESTION 9. (4 points) Let $F : D \to L$ be a group homomorphism and H be a subgroup of $Range(F)$. Prove that $K = \{a \in D \mid F(a) \in H\}$ is a subgroup of D and $Ker(F) \subseteq K$.

QUESTION 10. (4 points) Let D be a group such that $|D| = 65$. Assume that D has a normal subgroup with 5 elements. Prove that D is cyclic.

Faculty information

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