

General

$$\text{Variance: } \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{N-1}$$

$$\text{Standard Deviation: } \sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}}$$

$$\text{Pearson: } \rho = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{\frac{1}{N} \sum (x_i - \bar{x})(y_i - \bar{y})}{\sigma_X \sigma_Y}, -1 < \rho < 1$$

$$\text{Spearman: } 1 - \frac{6 \sum d^2}{n(n^2 - 1)}, -1 < \rho < 1$$

$$\text{Point Biserial: } r_{pb} = \frac{M_1 - M_0}{\sigma_y} \sqrt{p_1 p_2}$$

$$\text{ANOVA: } F = \frac{\text{var}_b}{\text{var}_w} = \frac{\text{MSB}}{\text{MSW}}, df_b = \text{classes} - 1, df_w = m - \text{classes}$$

$$\text{Chi-squared: } \chi^2 = \sum \frac{(O - E)^2}{E}, E = \frac{O_A \times O_B}{O_{\text{total}}}$$

$$\text{Min-Max Scaling: } X' = \frac{X - \min(X)}{\max(X) - \min(X)}$$

$$\text{Standard Scaling: } \mu = 0, \sigma = 1; Z = \frac{X - \mu}{\sigma}$$

Linear Regression:

$$\text{Cost: } J(w) = \frac{1}{2m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})^2 = \frac{1}{2m} \sum_{i=1}^m (w^T x^{(i)} - y^{(i)})^2$$

$$\text{Derivative: } \frac{\partial J(w)}{\partial w} = \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) \cdot x_n^{(i)}$$

$$\text{Update: } w_n := w_n - \alpha \frac{\partial J(w)}{\partial w} = w_n - \alpha \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) \cdot x_n^{(i)}$$

$$\text{Normal equation: } J(w) = \frac{1}{2m} (w^T X^T X w - 2w^T X^T y + y^T y)$$

$$\text{Derivative for normal equation: } \frac{\partial J(w)}{\partial w} = \frac{1}{m} (X^T X w - X^T y)$$

$$\text{Parameters: } w = (X^T X)^{-1} X^T y$$

$$\text{OLS } (w_0): w_1 = \frac{\sum_{i=1}^m (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^m (x_i - \bar{x})^2}$$

$$\text{OLS } (w_1): w_0 = \bar{y} - w_1 \bar{x}$$

Metrics:

$$\text{RMSE: } \sqrt{\frac{1}{m} \sum_{i=1}^m (y^{(i)} - \hat{y}^{(i)})^2}, 0 \rightarrow \infty$$

$$R^2 = 1 - \frac{\text{SS}_{\text{res}}}{\text{SS}_{\text{tot}}}$$

$$\text{SS}_{\text{tot}} = \sum_{i=1}^m (y^{(i)} - \bar{y})^2$$

$$\text{SS}_{\text{reg}} = \sum_{i=1}^m (\hat{y}^{(i)} - \bar{y})^2$$

$$\text{SS}_{\text{res}} = \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

$$\text{Adjusted } R^2: R^2_{\text{adj}} = 1 - \frac{(1 - R^2)(m - 1)}{m - n - 1}$$

$$\text{Precision: } \frac{\text{TP}}{\text{TP} + \text{FP}}$$

$$\text{Recall: } \frac{\text{TP} + \text{FN}}{\text{Precision} \times \text{Recall}}$$

$$\text{F1: } 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

Logistic Regression

$$h_w(x) = g(w^T x), \text{ where } g(w^T x) = \frac{1}{1 + e^{-(w^T x)}}$$

$$\text{Cost: } J(w) = -\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(h_w(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_w(x^{(i)}))$$

$$\frac{\partial h_w(x)}{\partial w_j} = h_w(x) (1 - h_w(x)) x_j$$

$$\text{Derivative: } \frac{\partial}{\partial w_j} J(w) = \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

$$\text{Update: } w_j := w_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

$$\text{Odds: } \frac{p}{1-p}, \text{ logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$\text{Coefficients: logit} = w^T x, \text{ odds} = e^{w^T x}$$

Ridge (Linear Regression)

$$\text{Cost: } J(w) = \frac{1}{2m} \left[\sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n w_j^2 \right]$$

$$\text{Derivative: } \frac{\partial J(w)}{\partial w} = \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) + \frac{\lambda}{m} w_j$$

$$\text{Update: } w_0 := w_0 - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})$$

$$\text{Update: } w_j := w_j \left(1 - \alpha \frac{\lambda}{m} \right) - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_j$$

$$\text{Normal Equation: } w = \left(X^T X + \lambda \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \right)^{-1} X^T y, I \in \mathbb{R}^{n-1 \times n-1}$$

LASSO (Linear Regression)

$$\text{Cost: } J(w) = \frac{1}{2m} \left[\sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n |w_j| \right]$$

$$\text{Derivative: } \frac{\partial J(w)}{\partial w} = \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) + \frac{\lambda}{2m} \text{sign}(w_j)$$

$$\text{Update: } w_0 := w_0 - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})$$

$$\text{Update: } w_j := w_j - \alpha \frac{\lambda}{m} \cdot \text{sign}(w_j) - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_j$$

L2 (Logistic Regression)

$$\text{Cost: } J(w) = \left[-\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(h_w(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_w(x^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^n w_j^2$$

$$\text{Derivative: } \frac{\partial J(w)}{\partial w} = \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) + \frac{\lambda}{m} w_j$$

$$\text{Update: } w_0 := w_0 - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})$$

$$\text{Update: } w_j := w_j \left(1 - \alpha \frac{\lambda}{m} \right) - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_j$$

L1 (Logistic Regression)

$$\text{Cost: } J(w) = \left[-\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(h_w(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_w(x^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^n |w_j|$$

$$\text{Derivative: } \frac{\partial J(w)}{\partial w} = \frac{1}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) + \frac{\lambda}{2m} \text{sign}(w_j)$$

$$\text{Update: } w_0 := w_0 - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})$$

$$\text{Update: } w_j := w_j - \alpha \frac{\lambda}{m} \cdot \text{sign}(w_j) - \frac{\alpha}{m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)}) x_j$$

Decision Trees

$$\text{Entropy: } H(p_1) = -\sum_{i=1}^n p_i \log_2(p_i)$$

$$\text{Gini: } \sum_k p_k (1 - p_k) = 1 - \sum_k p_k^2$$

$$\text{Information gain: } H(p_1^{\text{root}}) - (w^{\text{left}} H(p_1^{\text{left}}) + w^{\text{right}} H(p_1^{\text{right}}))$$

SVMs

$$\text{Soft: } \min_w C \sum_{i=1}^m \max(0, 1 - y^{(i)} (w^T x^{(i)})) + \frac{1}{2} \sum_{j=1}^n w_j^2, C = \frac{1}{\lambda}$$

$$\text{Hard: } \min_w \frac{1}{2} \|w\|^2$$

$$\text{Kernel: } \min_w C \sum_{i=1}^m \max(0, 1 - y^{(i)} (w^T f^{(i)})) + \frac{1}{2} \sum_{j=1}^m w_j^2$$

$$f = K(x, l^{(i)})$$

Anomaly Detection

$$\text{Gaussian Distribution: } p(x_j; \mu_j, \sigma_j^2) = \frac{1}{\sqrt{2\pi} \sigma_j} \exp\left(-\frac{(x_j - \mu_j)^2}{2\sigma_j^2}\right)$$

$$\text{Density Estimation: } p(x) = \prod_{j=1}^n p(x_j; \mu_j, \sigma_j^2) < \varepsilon$$

$$\text{Multivariate: } p(x_1; \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu)\right)$$

$$\Sigma = \frac{1}{m} \sum_{i=1}^m (x^{(i)} - \mu)(x^{(i)} - \mu)^T, \mu = \frac{1}{m} \sum_{i=1}^m x^{(i)}$$

Feature Selection / Dimensionality Reduction

$$\text{Cramer's V: } \sqrt{\frac{\chi^2}{n(k-1)}}$$

$$\text{Correlation Ratio: } \eta^2 = \frac{\text{SS}_{\text{between}}}{\text{SS}_{\text{total}}}, \rightarrow \eta = \sqrt{\eta^2}, 0 < \eta < 1$$

$$\text{Covariance Matrix: } \Sigma = \frac{1}{m} \sum_{i=1}^m (x^{(i)})(x^{(i)})^T, \text{ find } \lambda$$

TSNE:

$$p_{j|i} = \frac{\exp\left(-\frac{\|x^{(i)} - x^{(j)}\|^2}{2\sigma_i^2}\right)}{\sum_{k \neq i} \exp\left(-\frac{\|x^{(i)} - x^{(k)}\|^2}{2\sigma_i^2}\right)}, p_{ij} = \frac{p_{j|i} + p_{i|j}}{2m}$$

$$\text{Perplexity: } \mathcal{P} = 2^{\mathcal{H}}, \mathcal{H} = -\sum_{j \neq i} p_{j|i} \log_2(p_{j|i})$$

$$q_{ij} = \frac{w_{ij}}{Z}, w_{ij} = k(\|y_i - y_j\|), Z = \sum_{k \neq l} w_{kl}$$

$$\text{Similarity kernel in SNE: } k(d) = \exp(-d^2)$$

$$\text{Similarity kernel in tSNE: } k(d) = \frac{1}{1 + d^2}$$

$$q_{ij} = \frac{(1 + \|y_i - y_j\|^2)^{-1}}{\sum_{k \neq l} (1 + \|y_i - y_j\|^2)^{-1}}$$

Cost function (Kullback-Leibler Divergence) :

$$J = \sum_i \text{KL}(p_{ij} | q_{ij}) = \sum_{i \neq j} p_{ij} \log\left(\frac{p_{ij}}{q_{ij}}\right)$$

$$= \sum_{i \neq j} p_{ij} \log(p_{ij}) - \sum_{i \neq j} p_{ij} \log(q_{ij}) + \log \sum_{i \neq j} (w_{ij})$$

Update rule:

$$\frac{\partial}{\partial y_i} J = 4 \sum_{j \neq i} (p_{ij} - q_{ij}) q_{ij} (y_i - y_j)$$