Question 1a

Two models were developed to find the maximum number of banana that will arrive at the market. They include;

1) **A brute force model**: This model suggests a straight travel from the farm to the market using the available number of camels.
2) **A path model**: This model suggests stop points along the journey from the farm to the market. At each stop points, number of camels will be reduced to cut down on the cost of traveling for the brute force model.

The mathematical basis for each model is outlined below:

Brute-force Model: Straight journey from farm to Market.

Mathematical basis;

$$N_{max} = x - C \qquad (\boldsymbol{1})$$

N $_{\mathbf{max}}$ = maximum amount of Banana to arrive at Market.

x = amount of banana from farm.

C = amount of banana required to feed available camels during Journey to Market.

#calculating ***x*** *: maximum amount of banana from farm*

$$x = B \quad (\mathbf{2})$$

B = amount of banana that can be transported from Market by available Camels.

$$B = z * n$$

z = maximum number of available camels

n = amount of banana 1 camel can transport (equal to 1000)

$$B_{max} = z * n = 1000 * z$$

*#Substituting **B** into equation (2)*

$$x = 1000 * z \qquad (\mathbf{3})$$

*# calculating **C**: minimum amount of banana required to feed available camels during Journey to Market:*

$$C = k * y * z \qquad (\mathbf{4})$$

k = minimum number of bananas eaten by 1 camel per Km.

y = minimum distance between farm and market per Km.

z = maximum number of available camels

Substituting equation (3) and (4) into equation (1)

$$N = 1000z - k * y * z$$

$$\boldsymbol{N = z(1000 - k * y) \quad \#mathematical\ model}$$

Minimum condition for this model;

1) The cost **C** must be lesser than **x** for a particular number of **z.**

Mathematically,

$$C < x$$

$$k * y * z < 1000z$$

To check suitability of this model,

*#smallest cost for a particular **z***

$$C_{min} = k_{min} * y_{min} * z = 1 * 1000 * z = \mathbf{1000z}$$

*#biggest amount of banana for a particular **z***

$$x_{max} = 1000 * z = \mathbf{1000z}$$

Therefore,

$$C_{min} = x_{max}$$

This relation violates the minimum condition for the model, hence the models won't work for the constraints of this problem.

Path Model: Stop at certain points in the journey

x = number of bananas from farm

x_n = number of bananas at point n

y = distance to farm (km)

z = number of camels from farm

z_n = number of camels at point n

k = number of banana eaten at each Km of the journey

m = maximum capacity of 1 camel = 1000 banana

G = maximum number of banana available camels can carry = m * z = 1000z

N = maximum number of banana that can arrive at market

C_n = cost of journey at point n during the journey.

Mathematical Model;

$$\text{Farm} \xrightarrow{D_1} \text{P1} \xrightarrow{D_2} \text{P2} \xrightarrow{D_3} \text{Pn} \overset{D_n}{\ldots\ldots\ldots\ldots} \text{Market } (\text{P}_{\text{final}})$$

As the journey progresses, $\mathbf{x_n}$ bananas is reduced by cost $\mathbf{C_n}$. Mathematically represented as;

$$C_n = k * z_n * D_n$$

To reduce the cost of the journey, camels have to be dropped at points $\mathbf{P_n}$, which are points that $\mathbf{C_n}$ equals to **m**, the maximum capacity of 1 camel; **1000**.

Represented mathematically as;

$$k * z_n * D_n = 1000$$

$$D_n = \frac{1000}{z_n * k} \qquad (1)$$

The distance D_n moved by points P_n is represented by equation (1). The total distance (**movedKm**) moved from the farm is sum total of D_1, D_2, D_3 to D_n.

Mathematically,

$$movedKm = D_1 + D_2 + D_3 + ---- + D_n \qquad (2)$$

$$movedKm = \left(\frac{1000}{z_1 * k}\right) + \left(\frac{1000}{z_2 * k}\right) + \left(\frac{1000}{z_3 * k}\right) + \text{----------} + \left(\frac{1000}{z_n * k}\right)$$

To calculate the maximum output **N**, for every stop point P_n, 1000 bananas will be subtracted from **x**, as long as **movedKm** is less than **y**.

$$\boldsymbol{x_n = x - 1000n} \quad movedKm < y$$

$$where\ \boldsymbol{n}\ is\ number\ of\ stop\ point$$

When **movedKm** is greater than **y**, excess Km (**excKm**) is calculated and the cost (C_{excess}) of the excess Km is compensated.

$$excKm = movedKm - y \qquad movedKm > y$$

$$C_{excess} = k * z_n * excKm$$

$$N = x_n + C_{excess}$$

The only exception is if x_n is 1000 bananas left at some point **Pn**, at that point the cost of remaining km left to complete the journey is subtracted from 1000 bananas.

Mathematically,

$$N = x_n - C_n \qquad (**)$$

$$x_n = 1000$$

$$C_n = k * z_n * r$$

$$r = y - movedKm \quad (remaining\ part\ of\ the\ journey)$$

$$z_n = 1 \quad (x_n = 1000, only\ one\ camel\ is\ needed)$$

$$C_n = k * r$$

#subtituting $\boldsymbol{x_n}$ *and* $\boldsymbol{C_n}$ *into equation ***

$$N = 1000 - (k * r)$$

To explain the drop of camels better for instance at P_1, when 1000 bananas is gone into transportation cost, the remaining banana **x -1000** can easily be shared among $\boldsymbol{z_1 - 1}$ camels, and giving them a maximum capacity again.

Certain Conditions for this solution;

1) Since **z** can only be a whole number, hence **x** will always be multiple of 1000.

   Mathematically;

$$x = G$$

$$x = 1000z$$

2) The amount of bananas from farm, **x** is always equal to the capacity of the camels **G** from farm, and vice versa. Therefore, **x** has an upper bound determined by maximum number of camels.

$$x = 1000 * \ z$$

$$x = 1000 * 10 = 10{,}000$$

Question 1b

The model chosen for the Question 1a solution is the **Path Model**. The maximum output for the path model occurs when, **x = G**, which can be represented by:

$$x = G = 1000z$$

$$z = \frac{x}{1000}$$