
Summer Project Report

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Chapter 1

Background Reading Work

1.1 Introductory Newtonian Cosmology

1.1.1 Units used in cosmology

$$1 \text{ lightyear} = 9.4605 \times 10^{12} km$$

$$1 \text{ parsec} = 3.0856 \times 10^{13} km$$

$$M_{\odot} = 1.989 \times 10^{33} gm$$

$$1 \text{ year} = 3 \times 10^7 s$$

L-Luminosity

D-Distance

$$\text{Magnitude} : l = L \div 4\pi D^2$$

$$\text{Apparent Magnitude} : m = -2.5 \log l + \text{constant}$$

$$m=0 \text{ corresponds to flux } l = l_o = 2.48 \times 10^{-5} \text{ erg cm}^{-2} s^{-1}$$

$$\text{Absolute Magnitude} : M = -2.5 \log L + \text{constant} \quad (\text{m at } D=10\text{pc})$$

$$\implies m = M + 5 \log D_{pc} - 5$$

D_{pc} = Distance of the source measured in parsecs

1.1.2 Some insights

We treat the milky way as a point source for purpose of cosmology.

Galaxy \rightarrow *Group* \rightarrow *Cluster* \rightarrow *Supercluster* \rightarrow *HubbleRadius*

Cluster $\Rightarrow 1000 G \div 10^{14} M_{\odot} \Rightarrow 5 MPc$

Super - Cluster $\Rightarrow 50 MPc$ *Voids* $\Rightarrow 100 MPc$

1.1.3 Hubble's Law

Transverse motion

It is difficult to observe the Hubble expansion via measuring the transverse motion of galaxies because by calculations, in order to do so, one must have a resolution less than 2×10^{-3} arc sec. Measuring radio signals using the Very Long Baseline

Interferometry (VLBI) yields milli-arc sec resolution. So it is a promising way in measuring the same.

Radial motion

Doppler effect has been a useful tool for measuring radial motions in astronomy. The Spectrum of a galaxy shows well identified line at λ instead of lab measured λ_o . Hence we can say that the line is spectral shifted by a fraction z of the original wavelength.

$z = (\lambda - \lambda_o) \div \lambda_o$, if $z > 0$ then it is redshifted and if $z < 0$ then it is blueshifted.

When spectra from population of galaxies are considered, then they show blue-shifts only, giving rise to the following relation:

$$z = (H_o \times D) \div c$$

H_o is termed as the Hubble's constant and is estimated to be 530 km/Mpc s for $0.5 \leq h_o \leq 1$. h corresponds to the range of uncertainty and H_o changes with epochs. c/H_o corresponds to the Hubble radius at the present epoch.

When Special Theory of Relativity is considered, then the corresponding z equation becomes:

$$1 + z = \sqrt{(1 + v/c) \div (1 - v/c)}$$

1.1.4 Theoretical models

Various models are constructed explaining the large scale structure of the universe. Gravitational and electromagnetic interactions are used for long ranges while weak and strong interactions are used for others. Galaxies and IGMs are electrically neutral hence using Newtonian gravity one can view things in a simpler fashion (although at large distances and redshifts Newtonian gravity and assumptions break down)

Postulates

The Weyl Postulate

The trajectories of a special class of observers, to be identified with galaxies (points), form a bundle of non-intersecting lines in space-time so that there is a unique line passing through each point in space at any given time.

$$r = F(t, r_o)$$

$$F(t, r_o) = F(t, r_o) \Rightarrow r_o' = r_o$$

Where F is a vector function satisfying the non-intersection conditions.

The Cosmological Principle:-

At any epoch t , the universe is homogeneous and isotropic. That is, given any position in the universe and any direction in which it is viewed from that position, the large scale aspect of the universe is same for all fundamental observers.

$$v = \left. \frac{dr}{dt} \right|_{r_o} = \frac{\partial F(t, r_o)}{\partial t} \equiv G(t, r) \text{ (Say)}$$

At any epoch v can be a function of r only (unique fundamental observer).

$$v_1 = G(t, r_1) v_2 = G(t, r_2) \rightarrow \text{observer at } r=0$$

$$v_2 - v_1 = G(t, r_2) - G(t, r_1)$$

By cosmological principle, observer at $r=0$ has no speciality

$$v_2 - v_1 = G(t, r_2 - r_1)$$

$$\Rightarrow G(t, r_2) - G(t, r_1) = G(t, r_2 - r_1)$$

$$G_\mu(t, r) = \sum_\mu A_{\mu\nu} r_\nu; \lambda, \mu = 1, 2, 3$$

where $r=r_\mu \rightarrow$ triplet of cartesian

$A_{\mu\nu} = H(t)\delta_{\mu\nu} \rightarrow$ A second rank tensor depending on t only

$H(t)$ being an undetermined function of t

$v=H(t)r \rightarrow$ Velocity distance relation by Hubble

$$r=S(t)r_o \quad H_o = H(t_o) \quad \dot{S}/S = H(t)$$

1.1.5 Cosmological models

Dust- pressure less fluid

In general relativity, a dust solution is a fluid solution, a type of exact solution of the Einstein field equation, in which the gravitational field is produced entirely by the mass, momentum, and stress density of a perfect fluid that has positive mass density but vanishing pressure. The simplest model of the Universe one can think of is that of a universe filled with dust, i.e. non-relativistic pressureless matter, $p = 0$. We can think of this dust as a collection of point particles, and on the cosmological scales of approximately 10-1000 Mpc these point particles are a sufficiently good approximation for galaxies or even galaxy clusters. The cosmological principle states that there is no preferred place or direction in the Universe on large scales, so we can pick any coordinate system with respect to which we can measure the positions and velocities of these test particles. An important idealization taken in this model is that the large scale motion of galaxies has no random component in it.

According to the continuity equation :-

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$$

$$\nabla v = 3H(t) \nabla \rho = 0$$

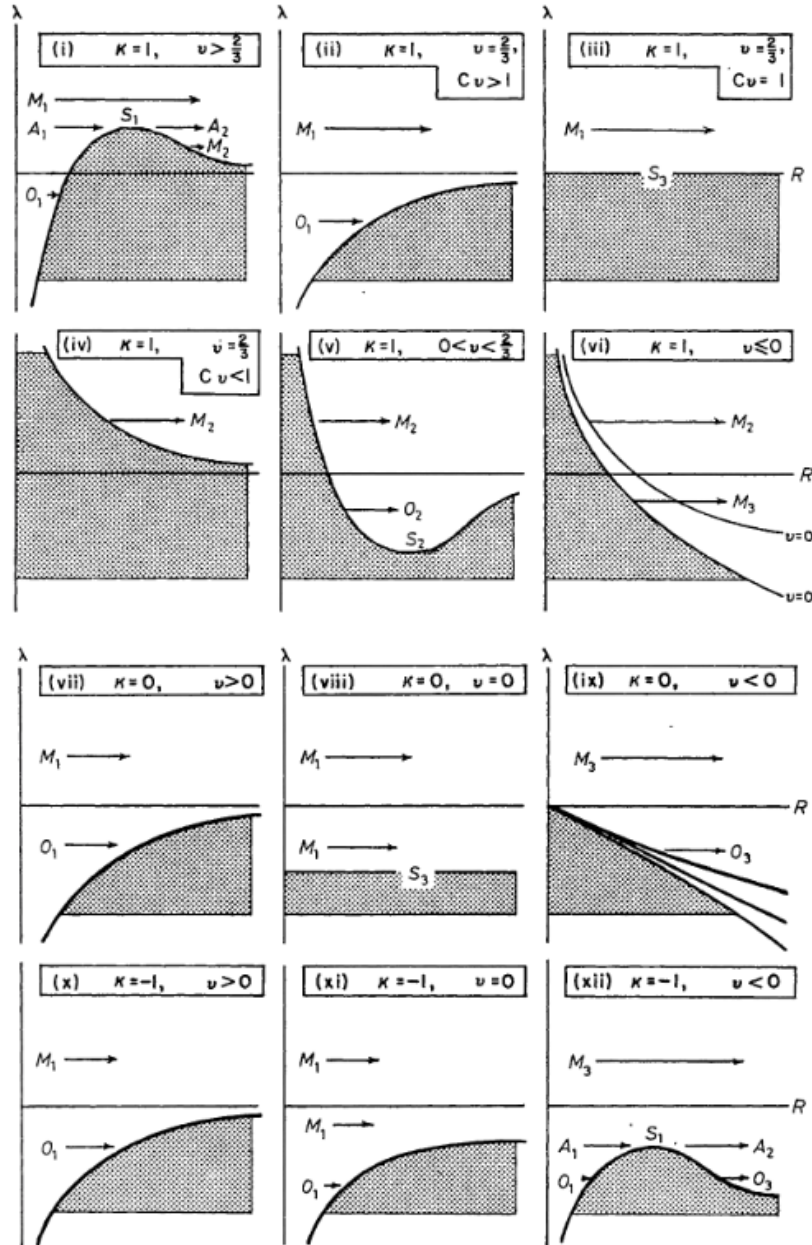
$$\therefore \frac{\partial}{\partial t} + 3\frac{\dot{S}}{S}\rho = 0 \Rightarrow \rho S^3 = \kappa = \rho_o S_o^3$$

if $p = 0$, $\dot{H}r + H^2 r = F$ (comes from Euler equation for fluid dynamics)

Furthermore got a brief idea on other models such as Einstein-de Sitter model, Milne model different classes (for $k<0$; $k=0$; $k>0$) of cosmological models.

Cosmological constant

Einstein introduced a λ term corresponding to the radial force of repulsion between two masses that varies in proportion to the distance between them. Further derivations using this constant yields results like a universe with motions without matter and Einstein's matter without motion. Hence there needed to be made some modifications for λ cosmology, emerging concepts of a steady state model, perfect cosmological principle and matter, creation fields.



(Graph taken from "Classification of uniform cosmological models" Harrison, E.R.)

Chapter 2

Understanding the Voigt profile

The equivalent width of an absorption line is the measure of the area of the line on a plot of intensity versus wavelength. It is defined by an integration of the difference in the intensities divided by the original intensity. where I is the observed spectral intensity, I_c the interpolation of the absorption-free continuum over the absorption feature and $\tau(\lambda)$ the optical depth. Furthermore, for redshifted absorption lines the wavelength is multiplied by a factor of $(1+z)$.

Column density is a measure of the amount of intervening matter between an observer and the object being observed integrated over a Gaussian distribution centred over the line of sight.

$$\tau(\nu) = N \frac{1}{\sqrt{\pi}b} \int_{-\infty}^{\infty} \sigma(\nu)' e^{-\frac{\nu-\nu_0}{b^2} dv}$$

The Voigt profile is an important model in molecular spectroscopy. The velocity distribution of atoms in IGM is described by an overall Gaussian curve which turns out Lorentzian at the wings. This convolution function of the two curves and the absorption feature itself is called a Voigt profile. Here b is the doppler parameter which is linearly dependent on the FWHM (Full Width at Half Maximum) of the distribution. The Voigt profile is defined by the function $H(a,u)$ whose relation is given below:

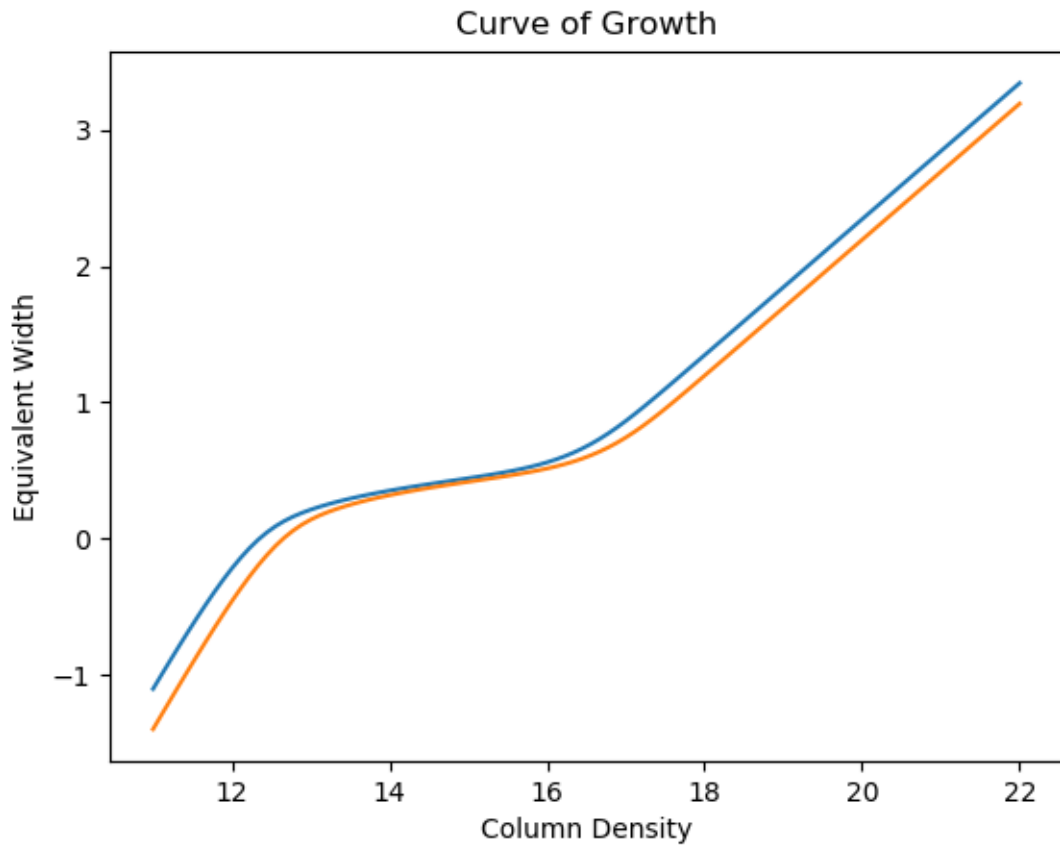
$$H(a, u) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(u-y)^2 + a^2} dy$$

As seen, the Voigt profile is a convolution of a Gaussian and a Lorentzian. At the core, this profile is Gaussian and the wings are Lorentzian. Hence, the variations of the optical depth at the core will be governed by the Gaussian distribution and the probability of finding the atoms at the wings will be of Lorentzian nature. The transition between these two natures are quite sharp and it is quite apparent in the profile as it corresponds to more than twice the FWHM. Hence the damping wings are apparent only for strong absorption lines.

Chapter 3

Curve of Growth

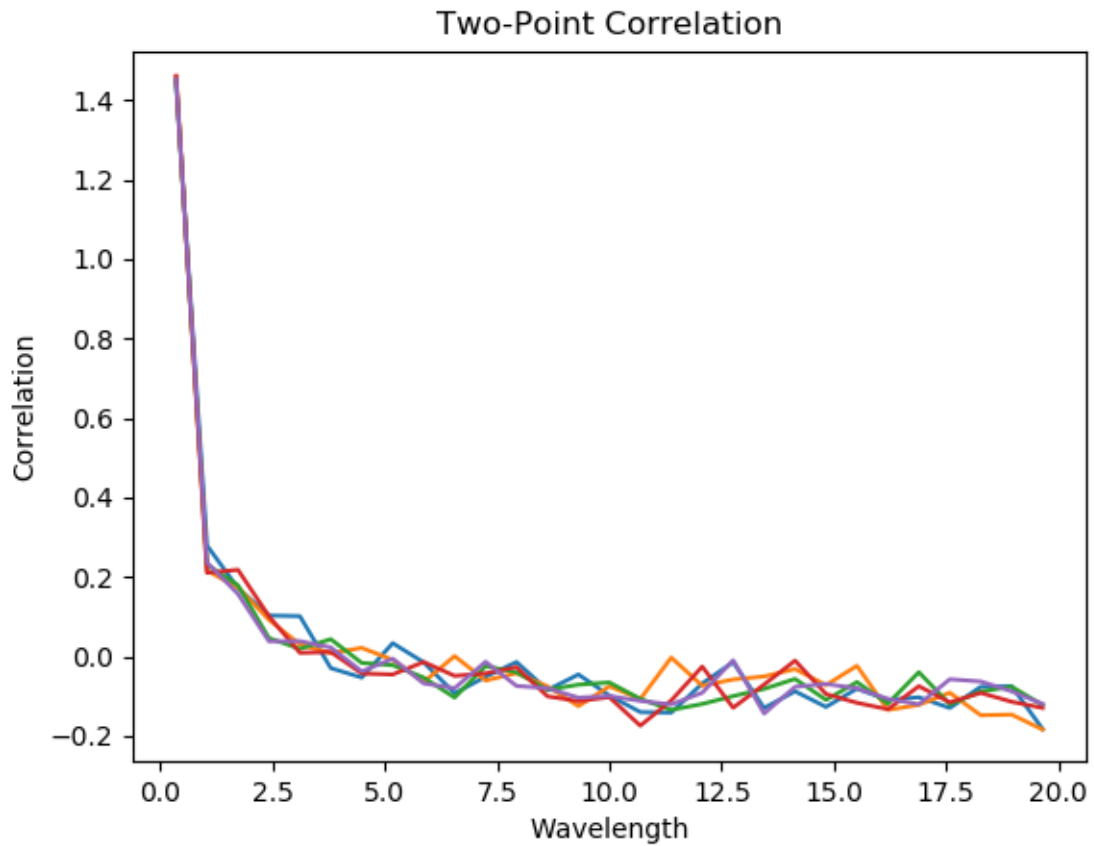
The curve of Growth is the relation between the equivalent width of a line and the column density for different values of the Doppler parameter. A plot was done for the curve of growth for Mg(2) lines using the survey data. The resultant graph after calculating the profile for different ranges is given below:



Chapter 4

Correlation analysis of Hydrogen and Magnesium (2) lines

A two point correlation analysis was first done for Hydrogen lines as a practice and then the same code is being implemented for corresponding Mg(2) lines taken from SDSS. The code compared each data with a set of corresponding random values generated from a uniform distribution whose mean was taken from a mean generated by a Poisson random variable. The graph is given below:



The wavelength can be converted to the corresponding number of absorbers across a particular line of sight. The correlation proves that there is an obvious clustering of the absorbers till a particular scale and after this particular point, the Poisson and random error dominates resulting in a random 'noise-like' distribution.

Bibliography

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