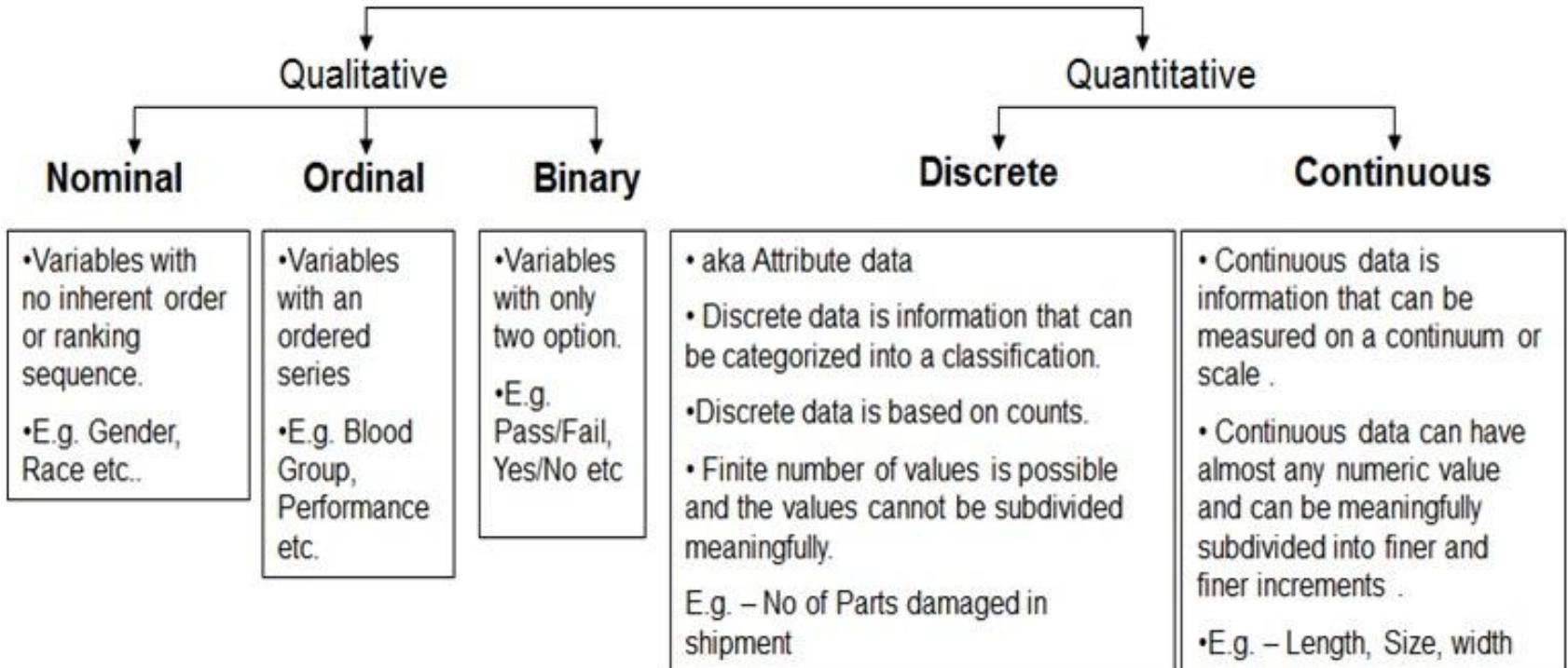


DATA PREPROCESSING & CLASSIFICATION

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DATA PREPROCESSING & CLASSIFICATION

Sr.No	Topic	Slides
1	Data Preprocessing: Measuring similarity and dissimilarity, Data Preprocessing steps, Data Cleaning; Data Integration	
2	Data Reduction: Attribute subset selection, Histograms, Clustering and Sampling,	
3	Data Transformation & Data Discretization: Normalization, Binning, Histogram Analysis and Concept hierarchy generation	80
4	Classification: Basic Concepts, Classification methods: Decision Tree Induction, Attribute Selection Measures, Tree pruning. Bayesian Classification: Naïve Bayes Classifier, Support Vector Machine	
5	Model Evaluation & Selection: Accuracy and Error measures, Holdout, Random Sampling, Cross Validation, Bootstrap; Comparing Classifier Performance using ROC Curves	
6	Combining Classifiers: Bagging, Boosting, Random Forests. <i>Extra topic: Adaboost</i>	
7	Applications	



SIMILARITY & DISIMMILARITY OF DATA

SIMILARITY & DISIMMILARITY OF DATA

What is Similarity?

- It is a numerical measure of the degree to which the two objects are alike.
- Higher for pair of objects that are more alike.
- Usually non-negative and between 0 & 1.

What is Dissimilarity?

- It is a numerical measure of the degree to which the two objects are different.
- Lower for pair of objects that are more similar.
- Range 0 to *infinity*.

0 ~ No Similarity, 1 ~ Complete Similarity

Proximity refers to a similarity or dissimilarity

Data Matrix and Dissimilarity Matrix

Data matrix

- n data points with p dimensions
- Two modes

$$\xrightarrow{\hspace{1cm}} \begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

Dissimilarity matrix

- n data points, but registers only the distance
- A triangular matrix
- Single mode

$$\xrightarrow{\hspace{1cm}} \begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & \ddots & \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$

Proximity Measure for Nominal Attributes

- **Nominal Attributes:** Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
 - m : # of matches, p : total # of variables

$$d(i, j) = \frac{p - m}{p}$$

- Method 2: Use a large number of binary attributes
 - creating a new binary attribute for each of the M nominal states

Proximity Measure for Binary Attributes

- A contingency table for binary data

	1	0	sum
1	q	r	$q + r$
0	s	t	$s + t$
sum	$q + s$	$r + t$	p

- Distance measure for symmetric binary variables:

$$d(i, j) = \frac{r + s}{q + r + s + t}$$

- Distance measure for asymmetric binary variables:

$$d(i, j) = \frac{r + s}{q + r + s}$$

- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):

$$sim_{Jaccard}(i, j) = \frac{q}{q + r + s}$$

Dissimilarity between Binary Variables

Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- Gender is a symmetric attribute
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N be 0

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

Z-score

Z-score: It gives you an idea of how far from the mean a data point is

$$z = \frac{x - \mu}{\sigma}$$

1. x : raw score to be standardized,
2. μ : mean of the population,
3. σ : standard deviation
4. the distance between the raw score and the population mean in units of the standard deviation
5. negative when the raw score is below the mean, “+” when above

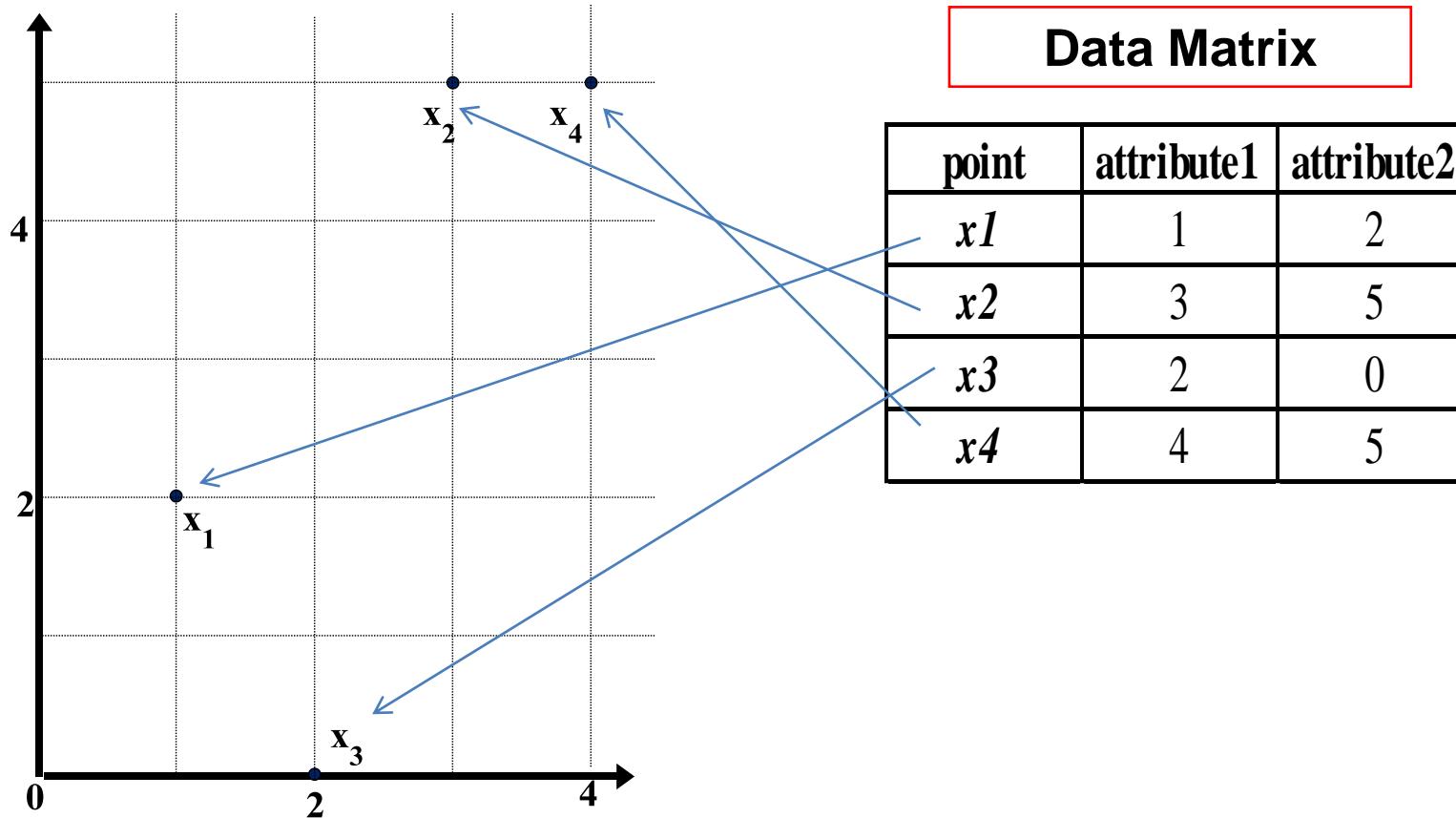
For example: let's say you have a test score of 190. The test has a mean (μ) of 150 and a standard deviation (σ) of 25. Assuming a normal distribution, your z score would be:

$$\begin{aligned} z &= (x - \mu) / \sigma \\ &= 190 - 150 / 25 = 1.6. \end{aligned}$$

The z score tells you how many standard deviations from the mean your score is. In this example, your score is 1.6 standard deviations *above* the mean.

Example:

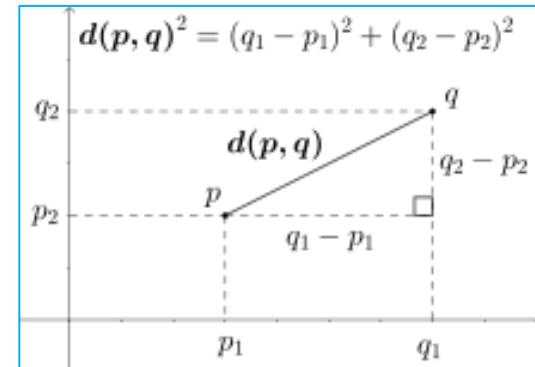
Data Matrix and Dissimilarity Matrix



Euclidean Distance: Dissimilarity Matrix

Dissimilarity Matrix: **Euclidean Distance** (is the "ordinary" straight-line distance between two points in Euclidean space.) A generalized term for the Euclidean norm is the **L² norm** or L² distance

$$\begin{aligned}d(\mathbf{p}, \mathbf{q}) = d(\mathbf{q}, \mathbf{p}) &= \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \cdots + (q_n - p_n)^2} \\&= \sqrt{\sum_{i=1}^n (q_i - p_i)^2}.\end{aligned}$$



	$x1$	$x2$	$x3$	$x4$
$x1$	0			
$x2$	3.61	0		
$x3$	5.1	5.1	0	
$x4$	4.24	1	5.39	0

Distance on Numeric Data: Minkowski Distance

Minkowski distance: A popular distance measure

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where $i = (x_{i1}, x_{i2}, \dots, x_{ip})$ and $j = (x_{j1}, x_{j2}, \dots, x_{jp})$ are two p -dimensional data objects, and h is the order (the distance so defined is also called L- h norm)

Properties:

$d(i, j) > 0$ if $i \neq j$, and $d(i, i) = 0$ (Positive definiteness)

$d(i, j) = d(j, i)$ (Symmetry)

$d(i, j) \leq d(i, k) + d(k, j)$ (Triangle Inequality)

A distance that satisfies these properties is a **metric**

Special Cases of Minkowski Distance

- $h = 1$: Manhattan (city block, L_1 norm) distance
 - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \dots + |x_{ip} - x_{jp}|$$

- $h = 2$: (L_2 norm) Euclidean distance

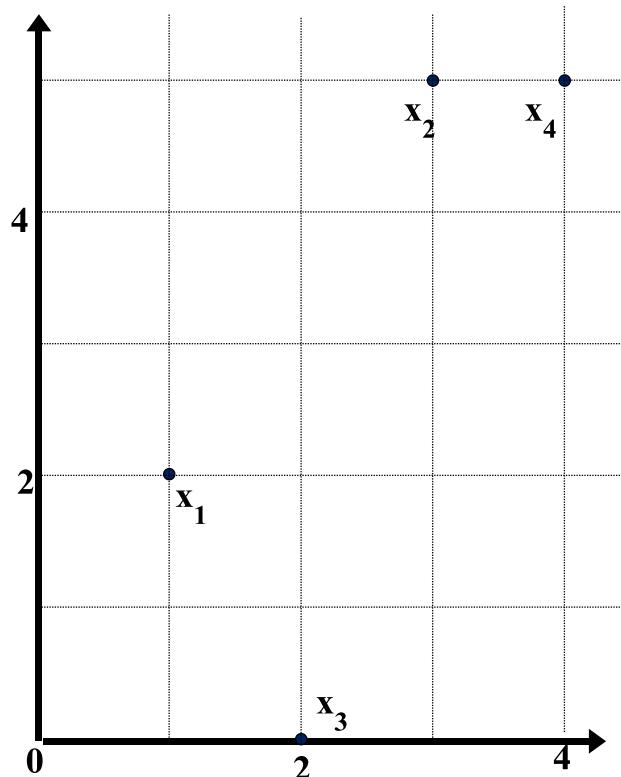
$$d(i, j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \dots + |x_{ip} - x_{jp}|^2)}$$

- $h \rightarrow \infty$. “supremum” (L_{\max} norm, L_{∞} norm) distance.
 - This is the maximum difference between any component (attribute) of the vectors

$$d(i, j) = \lim_{h \rightarrow \infty} \left(\sum_{f=1}^p |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_f^p |x_{if} - x_{jf}|$$

Example: Minkowski Distance

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



Manhattan (L_1)

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

Euclidean (L_2)

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

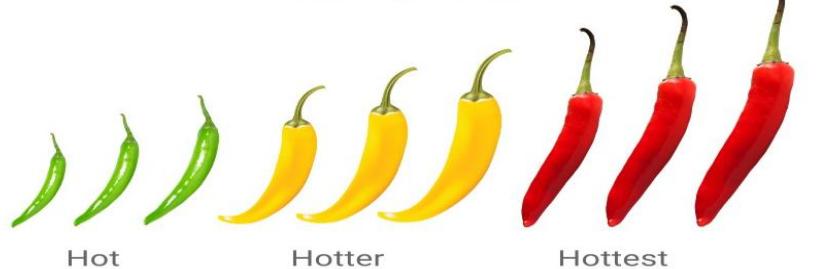
Supremum

L_∞	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	10

Swapnali Kulkarni

Dissimilarity Matrices

Ordinal Variables



- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
 - replace x_{if} by their rank $r_{if} \in \{1, \dots, M_f\}$
 - map the range of each variable onto $[0, 1]$ by replacing i -th object in the f -th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

- compute the dissimilarity using methods for interval-scaled variables

Attributes of Mixed Type

- A database may contain all attribute types
 - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i, j) = \frac{\sum_{f=1}^p \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^p \delta_{ij}^{(f)}}$$

- f is binary or nominal:
 $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$, or $d_{ij}^{(f)} = 1$ otherwise
- f is numeric: use the normalized distance
- f is ordinal
 - Compute ranks r_{if} and
 - Treat r_{if} as interval-scaled

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

Cosine Similarity

- A **document** can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

Document	team	coach	hockey	baseball	soccer	penalty	score	win	loss	season
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, ...
- **Applications:** information retrieval, biologic taxonomy, gene feature mapping, ...
- **Cosine measure:** If d_1 and d_2 are two vectors (e.g., term-frequency vectors), then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| ||d_2||$$

where \bullet indicates vector dot product, $||d||$: the length of vector d

Example: Cosine Similarity

- $\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| ||d_2||$,
where \bullet indicates vector dot product, $||d||$: the length of vector d
- Ex: Find the **similarity** between documents 1 and 2.

$$d_1 = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)$$

$$d_2 = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)$$

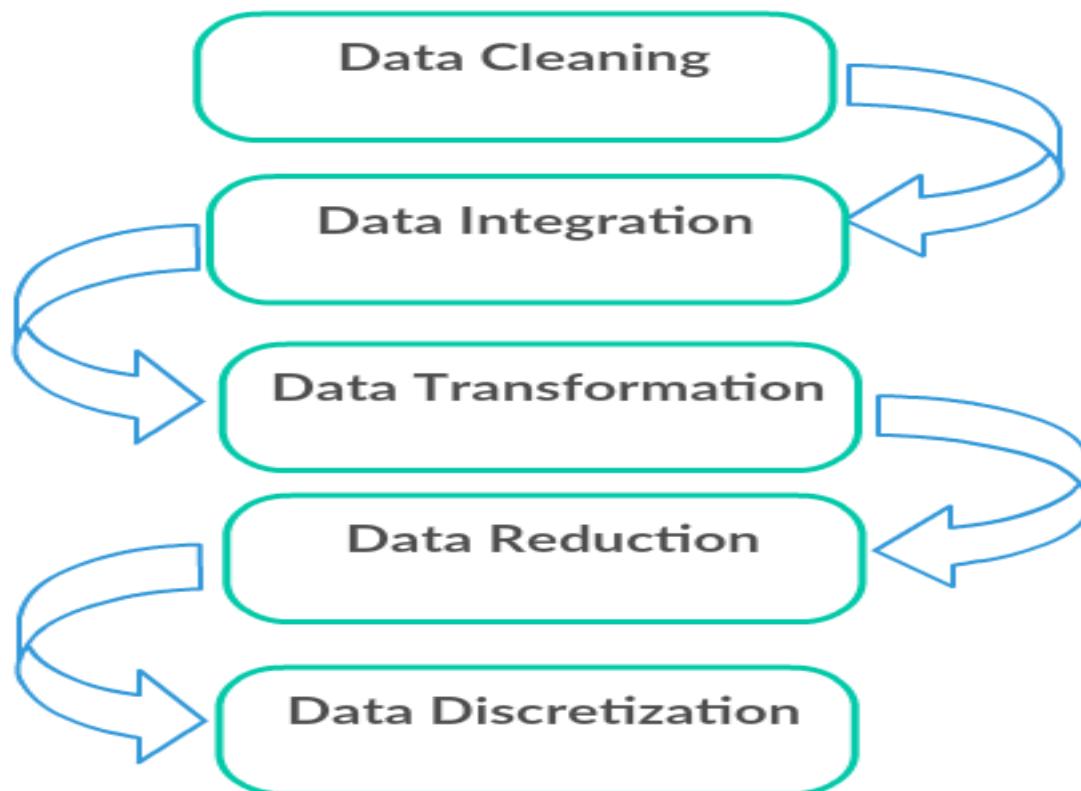
$$d_1 \bullet d_2 = 5*3+0*0+3*2+0*0+2*1+0*1+0*1+2*1+0*0+0*1 = 25$$

$$||d_1|| = (5^2 + 0^2 + 3^2 + 0^2 + 2^2 + 0^2 + 0^2 + 2^2 + 0^2 + 0^2)^{0.5} = (42)^{0.5} = 6.481$$

$$||d_2|| = (3^2 + 0^2 + 2^2 + 0^2 + 1^2 + 1^2 + 0^2 + 1^2 + 0^2 + 1^2)^{0.5} = (17)^{0.5} = 4.12$$

$$\cos(d_1, d_2) = 0.94$$

DATA PREPROCESSING



Data Preprocessing

- Data Preprocessing: An Overview
 - Data Quality
 - Major Tasks in Data Preprocessing
- Data Cleaning
- Data Integration
- Data Reduction
- Data Transformation and Data Discretization
- Summary



Data Quality: Why Preprocess the Data?

- Measures for data quality: A multidimensional view
 - Accuracy: correct or wrong, accurate or not
 - Completeness: not recorded, unavailable, ...
 - Consistency: some modified but some not, dangling, ...
 - Timeliness: timely update?
 - Believability: how trustable the data are correct?
 - Interpretability: how easily the data can be understood?

Major Tasks in Data Preprocessing

❖ Data cleaning

- Fill in missing values, smooth noisy data, identify or remove outliers, and resolve inconsistencies

❖ Data integration

- Integration of multiple databases, data cubes, or files

❖ Data reduction

- Dimensionality reduction
- Numerosity reduction
- Data compression

❖ Data transformation and data discretization

- Normalization
- Concept hierarchy generation

Data Cleaning

Data in the Real World Is Dirty: Lots of potentially incorrect data, e.g., instrument faulty, human or computer error, transmission error

- **incomplete**: lacking attribute values, lacking certain attributes of interest, or containing only aggregate data
 - e.g., *Occupation=“ ”* (missing data)
- **noisy**: containing noise, errors, or outliers
 - e.g., *Salary=“-10”* (an error)
- **inconsistent**: containing discrepancies in codes or names, e.g.,
 - *Age=“42”, Birthday=“03/07/2010”*
 - Was rating “1, 2, 3”, now rating “A, B, C”
 - discrepancy between duplicate records
- **Intentional** (e.g., *disguised missing data*)
 - Jan. 1 as everyone’s birthday?

Incomplete (Missing) Data

- **Data is not always available**
 - E.g., many tuples have no recorded value for several attributes, such as customer income in sales data
- **Missing data may be due to**
 - equipment malfunction
 - inconsistent with other recorded data and thus deleted
 - data not entered due to misunderstanding
 - certain data may not be considered important at the time of entry
 - not register history or changes of the data
- **Missing data may need to be inferred**

How to Handle Missing Data?

- **Ignore the tuple:** usually done when class label is missing (when doing classification)—not effective when the % of missing values per attribute varies considerably
- **Fill in the missing value manually:** tedious + infeasible?
- **Fill in it automatically with**
 - a global constant : e.g., “unknown”, a new class?!
 - the attribute mean
 - the attribute mean for all samples belonging to the same class: smarter
 - the most probable value: inference-based such as Bayesian formula or decision tree

Noisy Data

- **Noise:** random error or variance in a measured variable
- **Incorrect attribute values** may be due to
 - faulty data collection instruments
 - data entry problems
 - data transmission problems
 - technology limitation
 - inconsistency in naming convention
- **Other data problems** which require data cleaning
 - duplicate records
 - incomplete data
 - inconsistent data

How to Handle Noisy Data?

- Binning
 - first sort data and partition into (equal-frequency) bins
 - then one can smooth by bin means, smooth by bin median, smooth by bin boundaries, etc.
- Regression
 - smooth by fitting the data into regression functions
- Clustering
 - detect and remove outliers
- Combined computer and human inspection
 - detect suspicious values and check by human (e.g., deal with possible outliers)

Data Cleaning as a Process

- Data discrepancy detection
 - Use metadata (e.g., domain, range, dependency, distribution)
 - Check field overloading
 - Check uniqueness rule, consecutive rule and null rule
 - Use commercial tools
 - Data scrubbing: use simple domain knowledge (e.g., postal code, spell-check) to detect errors and make corrections
 - Data auditing: by analyzing data to discover rules and relationship to detect violators (e.g., correlation and clustering to find outliers)
- Data migration and integration
 - Data migration tools: allow transformations to be specified
 - ETL (Extraction/Transformation>Loading) tools: allow users to specify transformations through a graphical user interface
- Integration of the two processes
 - Iterative and interactive (e.g., Potter's Wheels)

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Data Integration

- **Data integration:**
 - Combines data from multiple sources into a coherent store
- Schema integration: e.g., A.cust-id \equiv B.cust-#
 - Integrate metadata from different sources
- **Entity identification problem:**
 - Identify real world entities from multiple data sources, e.g., Bill Clinton = William Clinton
- Detecting and resolving data value conflicts
 - For the same real world entity, attribute values from different sources are different
 - Possible reasons: different representations, different scales, e.g., metric vs. British units

Handling Redundancy in Data Integration

- Redundant data occur often when integration of multiple databases
 - *Object identification:* The same attribute or object may have different names in different databases
 - *Derivable data:* One attribute may be a “derived” attribute in another table, e.g., annual revenue
- Redundant attributes may be able to be detected by *correlation analysis* and *covariance analysis*
- Careful integration of the data from multiple sources may help reduce/avoid redundancies and inconsistencies and improve mining speed and quality

Correlation Analysis (Nominal Data)

- **X² (chi-square) test**

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

- The larger the X² value, the more likely the variables are related
- The cells that contribute the most to the X² value are those whose actual count is very different from the expected count
- Correlation does not imply causality
 - # of hospitals and # of car-theft in a city are correlated
 - Both are causally linked to the third variable: population

Chi-Square Calculation: An Example

	Play chess	Not play chess	Sum (row)
Like science fiction	250(90)	200(360)	450
Not like science fiction	50(210)	1000(840)	1050
Sum(col.)	300	1200	1500

- χ^2 (chi-square) calculation (numbers in parenthesis are expected counts calculated based on the data distribution in the two categories)

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840} = 507.93$$

- It shows that like_science_fiction and play_chess are correlated in the group

Correlation Analysis (Numeric Data)

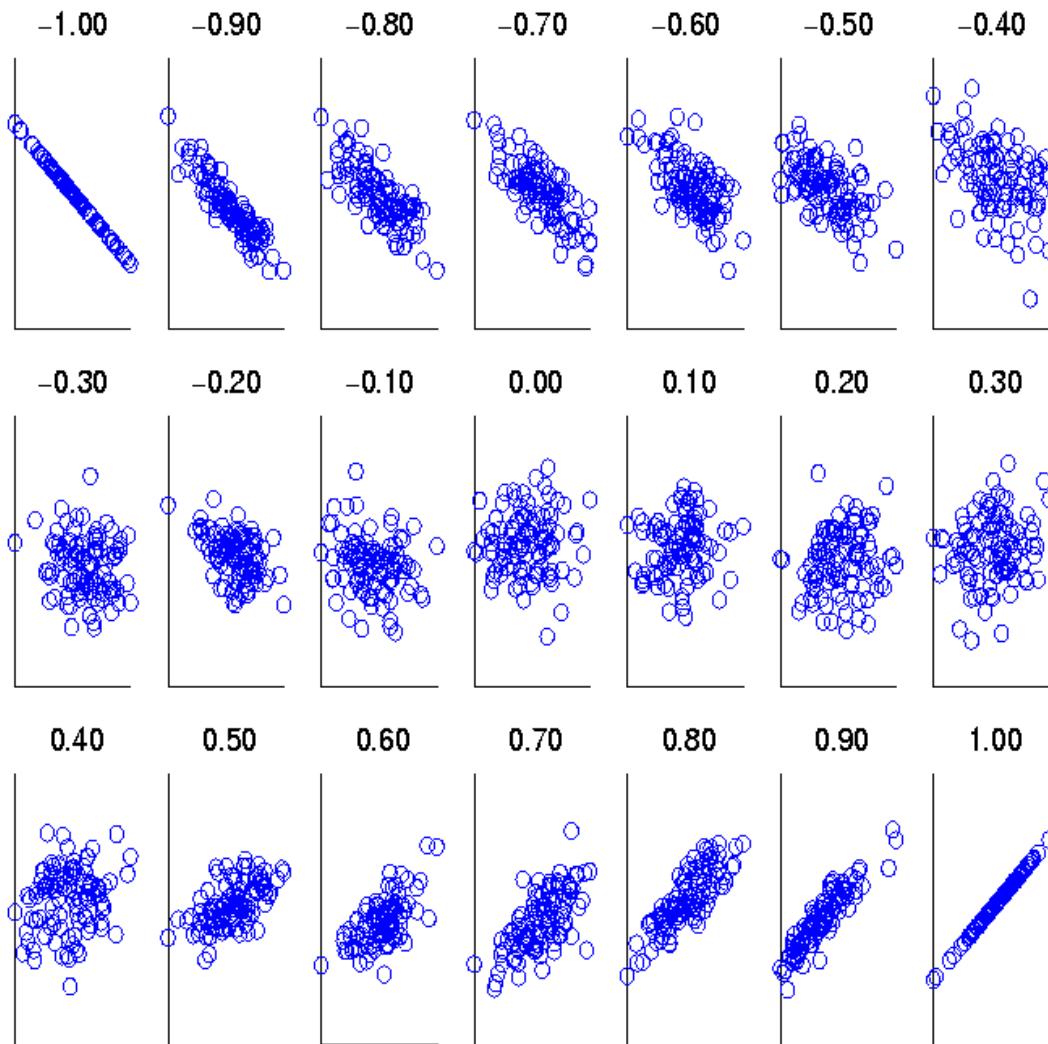
- Correlation coefficient (also called Pearson's product moment coefficient)

$$r_{A,B} = \frac{\sum_{i=1}^n (a_i - \bar{A})(b_i - \bar{B})}{(n-1)\sigma_A\sigma_B} = \frac{\sum_{i=1}^n (a_i b_i) - n\bar{A}\bar{B}}{(n-1)\sigma_A\sigma_B}$$

where n is the number of tuples, \bar{A} and \bar{B} are the respective means of A and B, σ_A and σ_B are the respective standard deviation of A and B, and $\Sigma(a_i b_i)$ is the sum of the AB cross-product.

- If $r_{A,B} > 0$, A and B are positively correlated (A's values increase as B's). The higher, the stronger correlation.
- $r_{A,B} = 0$: independent; $r_{AB} < 0$: negatively correlated

Visually Evaluating Correlation



Scatter plots showing the similarity from –1 to 1.

Correlation (viewed as linear relationship)

- Correlation measures the linear relationship between objects
- To compute correlation, we standardize data objects, A and B, and then take their dot product

$$a'_k = (a_k - \text{mean}(A)) / \text{std}(A)$$

$$b'_k = (b_k - \text{mean}(B)) / \text{std}(B)$$

$$\text{correlation}(A, B) = A' \bullet B'$$

Covariance (Numeric Data)

- Covariance is similar to correlation

$$Cov(A, B) = E((A - \bar{A})(B - \bar{B})) = \frac{\sum_{i=1}^n (a_i - \bar{A})(b_i - \bar{B})}{n}$$

Correlation coefficient: $r_{A,B} = \frac{Cov(A, B)}{\sigma_A \sigma_B}$

where n is the number of tuples, \bar{A} and \bar{B} are the respective mean or **expected values** of A and B, σ_A and σ_B are the respective standard deviation of A and B.

- **Positive covariance:** If $Cov_{A,B} > 0$, then A and B both tend to be larger than their expected values.
- **Negative covariance:** If $Cov_{A,B} < 0$ then if A is larger than its expected value, B is likely to be smaller than its expected value.
- **Independence:** $Cov_{A,B} = 0$ but the converse is not true:
 - Some pairs of random variables may have a covariance of 0 but are not independent. Only under some additional assumptions (e.g., the data follow multivariate normal distributions) does a covariance of 0 imply independence

Co-Variance: An Example

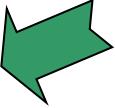
$$Cov(A, B) = E((A - \bar{A})(B - \bar{B})) = \frac{\sum_{i=1}^n (a_i - \bar{A})(b_i - \bar{B})}{n}$$

- It can be simplified in computation as

$$Cov(A, B) = E(A \cdot B) - \bar{A}\bar{B}$$

- Suppose two stocks A and B have the following values in one week: (2, 5), (3, 8), (5, 10), (4, 11), (6, 14).
- Question: If the stocks are affected by the same industry trends, will their prices rise or fall together?
 - $E(A) = (2 + 3 + 5 + 4 + 6)/ 5 = 20/5 = 4$
 - $E(B) = (5 + 8 + 10 + 11 + 14) /5 = 48/5 = 9.6$
 - $Cov(A,B) = (2 \times 5 + 3 \times 8 + 5 \times 10 + 4 \times 11 + 6 \times 14) /5 - 4 \times 9.6 = 4$
- Thus, A and B rise together since $Cov(A, B) > 0$.

Chapter 3: Data Preprocessing

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Data Reduction Strategies

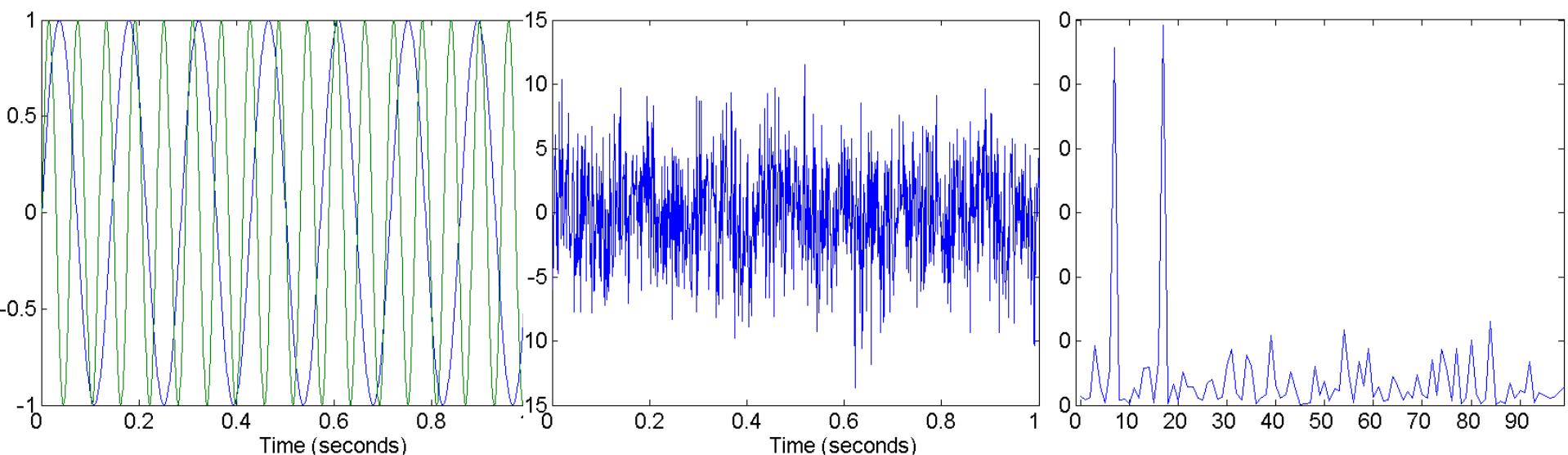
- **Data reduction:** Obtain a reduced representation of the data set that is much smaller in volume but yet produces the same (or almost the same) analytical results
- Why data reduction? — A database/data warehouse may store terabytes of data. Complex data analysis may take a very long time to run on the complete data set.
- Data reduction strategies
 - Dimensionality reduction, e.g., remove unimportant attributes
 - Wavelet transforms
 - Principal Components Analysis (PCA)
 - Feature subset selection, feature creation
 - Numerosity reduction (some simply call it: Data Reduction)
 - Regression and Log-Linear Models
 - Histograms, clustering, sampling
 - Data cube aggregation
 - Data compression

Data Reduction 1: Dimensionality Reduction

- **Curse of dimensionality**
 - When dimensionality increases, data becomes increasingly sparse
 - Density and distance between points, which is critical to clustering, outlier analysis, becomes less meaningful
 - The possible combinations of subspaces will grow exponentially
- **Dimensionality reduction**
 - Avoid the curse of dimensionality
 - Help eliminate irrelevant features and reduce noise
 - Reduce time and space required in data mining
 - Allow easier visualization
- **Dimensionality reduction techniques**
 - Wavelet transforms
 - Principal Component Analysis
 - Supervised and nonlinear techniques (e.g., feature selection)

Mapping Data to a New Space

- Fourier transform
- Wavelet transform



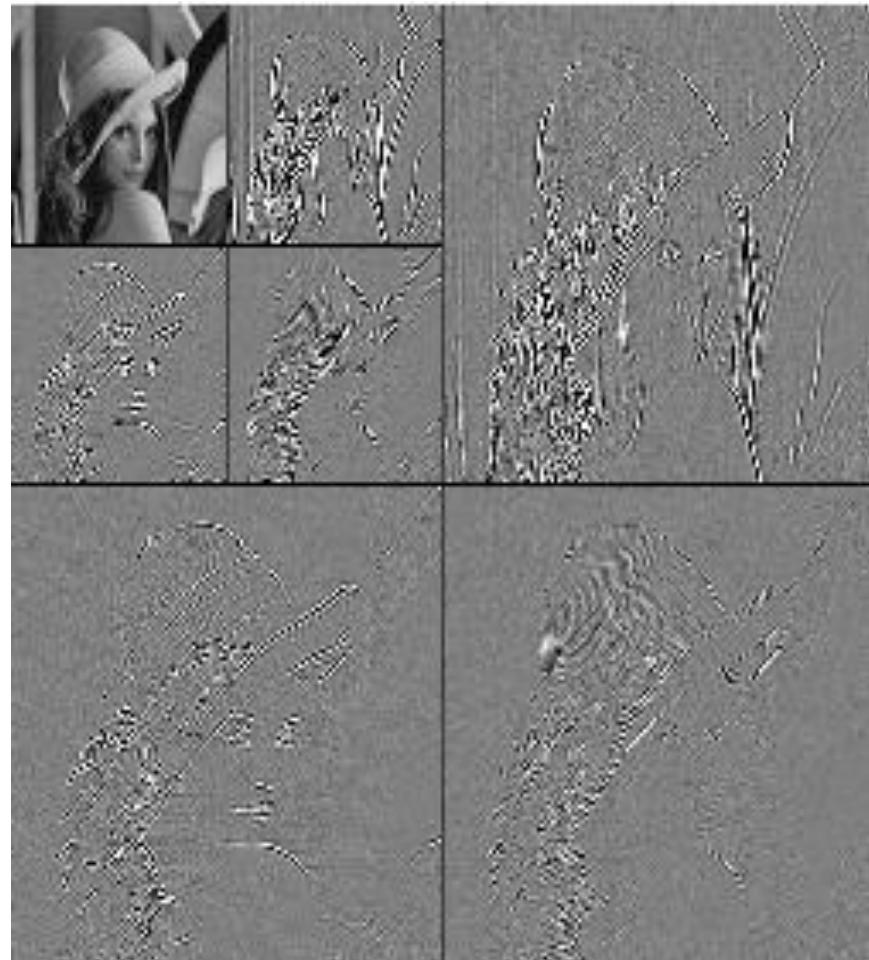
Two Sine Waves

Two Sine Waves + Noise

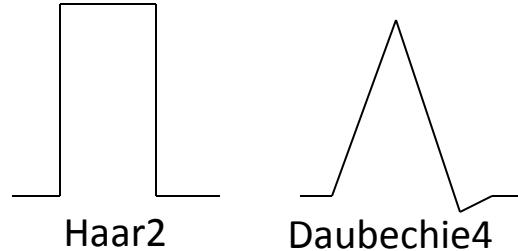
Frequency

What Is Wavelet Transform?

- Decomposes a signal into different frequency subbands
 - Applicable to n-dimensional signals
- Data are transformed to preserve relative distance between objects at different levels of resolution
- Allow natural clusters to become more distinguishable
- Used for image compression



Wavelet Transformation



- Discrete wavelet transform (DWT) for linear signal processing, multi-resolution analysis
- Compressed approximation: store only a small fraction of the strongest of the wavelet coefficients
- Similar to discrete Fourier transform (DFT), but better lossy compression, localized in space
- Method:
 - Length, L , must be an integer power of 2 (padding with 0's, when necessary)
 - Each transform has 2 functions: smoothing, difference
 - Applies to pairs of data, resulting in two set of data of length $L/2$
 - Applies two functions recursively, until reaches the desired length

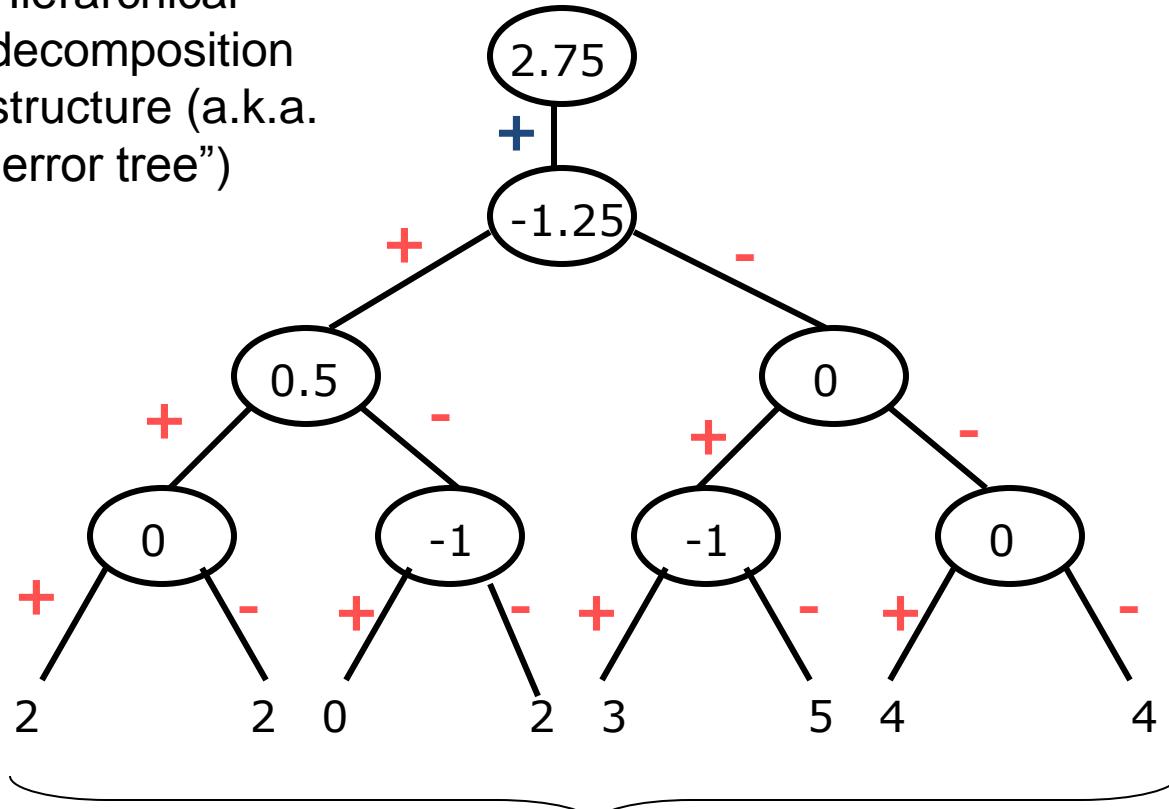
Wavelet Decomposition

- Wavelets: A math tool for space-efficient hierarchical decomposition of functions
- $S = [2, 2, 0, 2, 3, 5, 4, 4]$ can be transformed to $S_\wedge = [2^3/4, -1^1/4, 1/2, 0, 0, -1, -1, 0]$
- Compression: many small detail coefficients can be replaced by 0's, and only the significant coefficients are retained

Resolution	Averages	Detail Coefficients
8	$[2, 2, 0, 2, 3, 5, 4, 4]$	
4	$[2, 1, 4, 4]$	$[0, -1, -1, 0]$
2	$[1\frac{1}{2}, 4]$	$[\frac{1}{2}, 0]$
1	$[2\frac{3}{4}]$	$[-1\frac{1}{4}]$

Haar Wavelet Coefficients

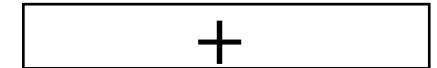
Hierarchical decomposition structure (a.k.a. “error tree”)



Original frequency distribution

Coefficient “Supports”

2.75



-1.25



0.5



0



0



-1



-1



0

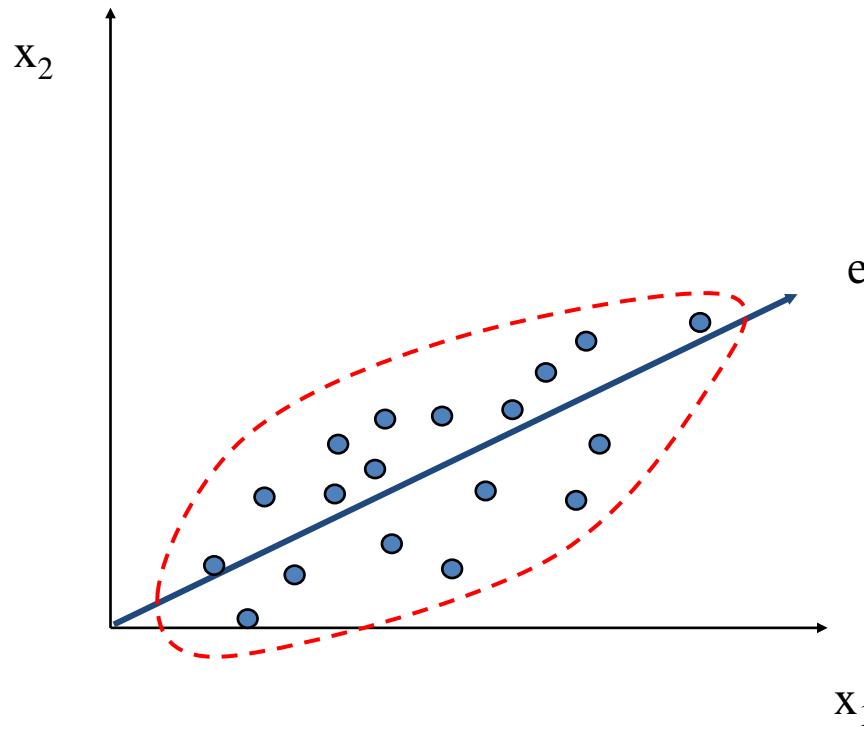


Why Wavelet Transform?

- Use hat-shape filters
 - Emphasize region where points cluster
 - Suppress weaker information in their boundaries
- Effective removal of outliers
 - Insensitive to noise, insensitive to input order
- Multi-resolution
 - Detect arbitrary shaped clusters at different scales
- Efficient
 - Complexity $O(N)$
- Only applicable to low dimensional data

Principal Component Analysis (PCA)

- Find a projection that captures the largest amount of variation in data
- The original data are projected onto a much smaller space, resulting in dimensionality reduction. We find the eigenvectors of the covariance matrix, and these eigenvectors define the new space



Principal Component Analysis (Steps)

- Given N data vectors from n -dimensions, find $k \leq n$ orthogonal vectors (*principal components*) that can be best used to represent data
 - Normalize input data: Each attribute falls within the same range
 - Compute k orthonormal (unit) vectors, i.e., *principal components*
 - Each input data (vector) is a linear combination of the k principal component vectors
 - The principal components are sorted in order of decreasing “significance” or strength
 - Since the components are sorted, the size of the data can be reduced by eliminating the *weak components*, i.e., those with low variance (i.e., using the strongest principal components, it is possible to reconstruct a good approximation of the original data)
- Works for numeric data only

Attribute Subset Selection

- Another way to reduce dimensionality of data
- Redundant attributes
 - Duplicate much or all of the information contained in one or more other attributes
 - E.g., purchase price of a product and the amount of sales tax paid
- Irrelevant attributes
 - Contain no information that is useful for the data mining task at hand
 - E.g., students' ID is often irrelevant to the task of predicting students' GPA

Heuristic Search in Attribute Selection

- There are 2^d possible attribute combinations of d attributes
- Typical heuristic attribute selection methods:
 - Best single attribute under the attribute independence assumption: choose by significance tests
 - Best step-wise feature selection:
 - The best single-attribute is picked first
 - Then next best attribute condition to the first, ...
 - Step-wise attribute elimination:
 - Repeatedly eliminate the worst attribute
 - Best combined attribute selection and elimination
 - Optimal branch and bound:
 - Use attribute elimination and backtracking

Attribute Creation (Feature Generation)

- Create new attributes (features) that can capture the important information in a data set more effectively than the original ones
- Three general methodologies
 - Attribute extraction
 - Domain-specific
 - Mapping data to new space (see: data reduction)
 - E.g., Fourier transformation, wavelet transformation, manifold approaches (not covered)
 - Attribute construction
 - Combining features (see: discriminative frequent patterns in Chapter 7)
 - Data discretization

Data Reduction 2: Numerosity Reduction

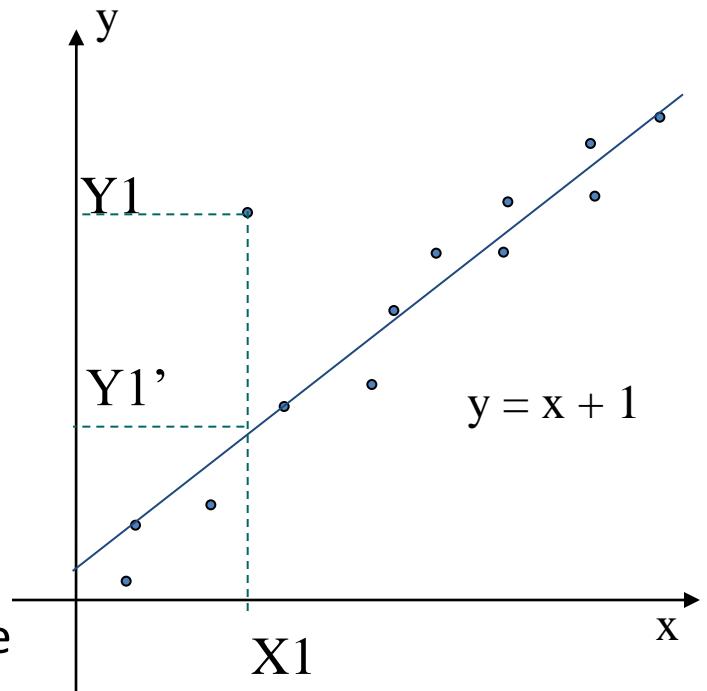
- Reduce data volume by choosing alternative, *smaller forms* of data representation
- **Parametric methods** (e.g., regression)
 - Assume the data fits some model, estimate model parameters, store only the parameters, and discard the data (except possible outliers)
 - Ex.: Log-linear models—obtain value at a point in m -D space as the product on appropriate marginal subspaces
- **Non-parametric** methods
 - Do not assume models
 - Major families: histograms, clustering, sampling, ...

Parametric Data Reduction: Regression and Log-Linear Models

- **Linear regression**
 - Data modeled to fit a straight line
 - Often uses the least-square method to fit the line
- **Multiple regression**
 - Allows a response variable Y to be modeled as a linear function of multidimensional feature vector
- **Log-linear model**
 - Approximates discrete multidimensional probability distributions

Regression Analysis

- Regression analysis: A collective name for techniques for the modeling and analysis of numerical data consisting of values of a **dependent variable** (also called **response variable** or **measurement**) and of one or more **independent variables** (aka. **explanatory variables** or **predictors**)
- The parameters are estimated so as to give a "best fit" of the data
- Most commonly the best fit is evaluated by using the **least squares method**, but other criteria have also been used



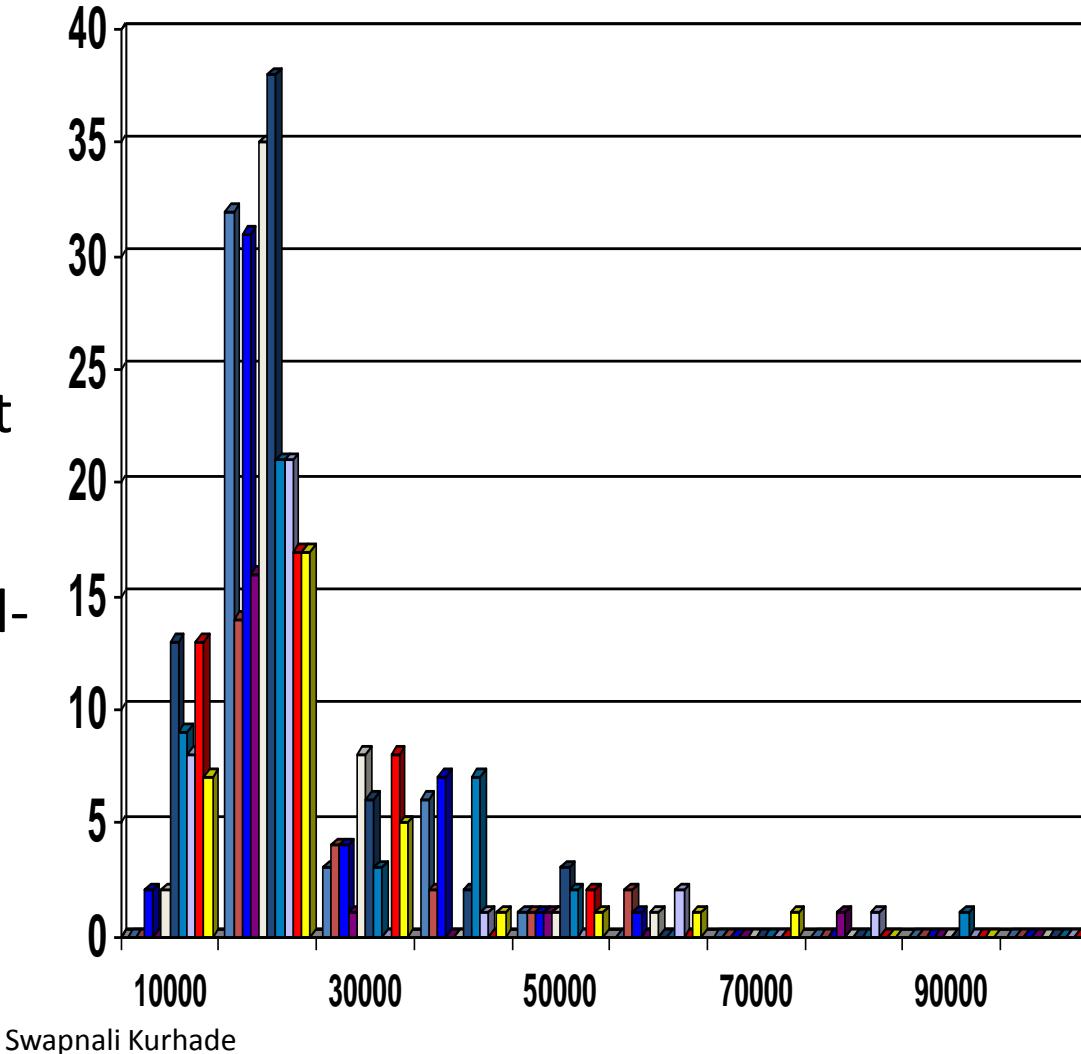
- Used for prediction (including forecasting of time-series data), inference, hypothesis testing, and modeling of causal relationships

Regress Analysis and Log-Linear Models

- Linear regression: $Y = w X + b$
 - Two regression coefficients, w and b , specify the line and are to be estimated by using the data at hand
 - Using the least squares criterion to the known values of $Y_1, Y_2, \dots, X_1, X_2, \dots$
- Multiple regression: $Y = b_0 + b_1 X_1 + b_2 X_2$
 - Many nonlinear functions can be transformed into the above
- Log-linear models:
 - Approximate discrete multidimensional probability distributions
 - Estimate the probability of each point (tuple) in a multi-dimensional space for a set of discretized attributes, based on a smaller subset of dimensional combinations
 - Useful for dimensionality reduction and data smoothing

Histogram Analysis

- Divide data into buckets and store average (sum) for each bucket
- Partitioning rules:
 - Equal-width: equal bucket range
 - Equal-frequency (or equal-depth)



Clustering

- Partition data set into clusters based on similarity, and store cluster representation (e.g., centroid and diameter) only
- Can be very effective if data is clustered but not if data is “smeared”
- Can have hierarchical clustering and be stored in multi-dimensional index tree structures
- There are many choices of clustering definitions and clustering algorithms
- Cluster analysis will be studied in depth in Chapter 10

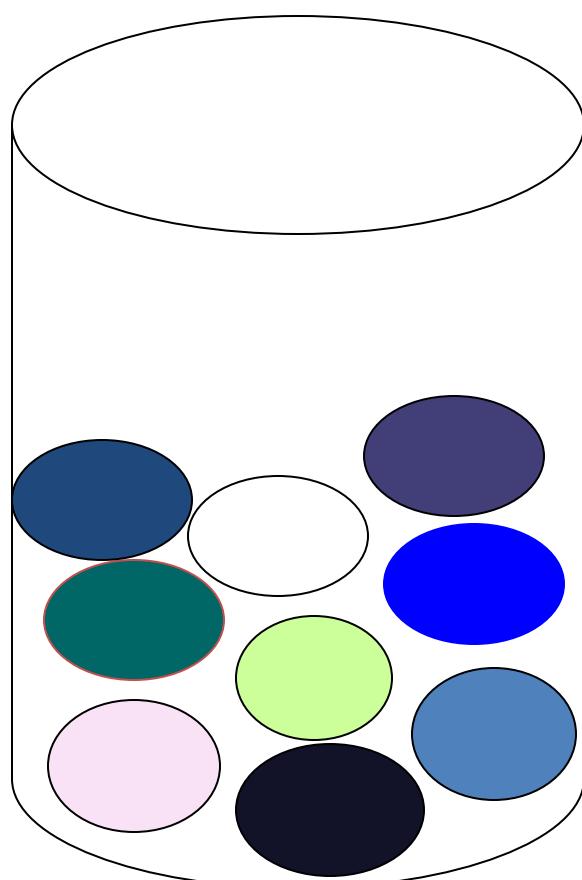
Sampling

- Sampling: obtaining a small sample s to represent the whole data set N
- Allow a mining algorithm to run in complexity that is potentially sub-linear to the size of the data
- Key principle: Choose a **representative** subset of the data
 - Simple random sampling may have very poor performance in the presence of skew
 - Develop adaptive sampling methods, e.g., stratified sampling:
- Note: Sampling may not reduce database I/Os (page at a time)

Types of Sampling

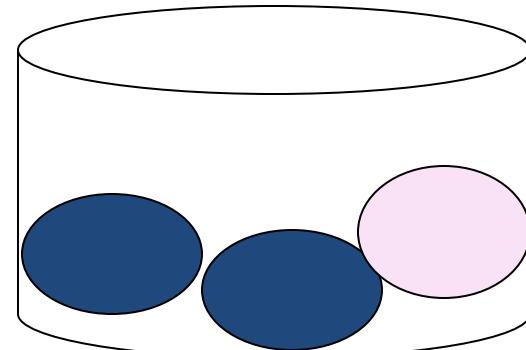
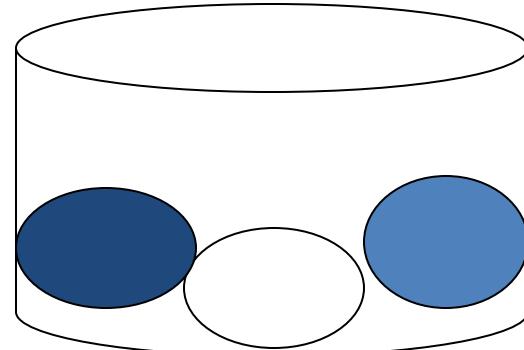
- **Simple random sampling**
 - There is an equal probability of selecting any particular item
- **Sampling without replacement**
 - Once an object is selected, it is removed from the population
- **Sampling with replacement**
 - A selected object is not removed from the population
- **Stratified sampling:**
 - Partition the data set, and draw samples from each partition (proportionally, i.e., approximately the same percentage of the data)
 - Used in conjunction with skewed data

Sampling: With or without Replacement



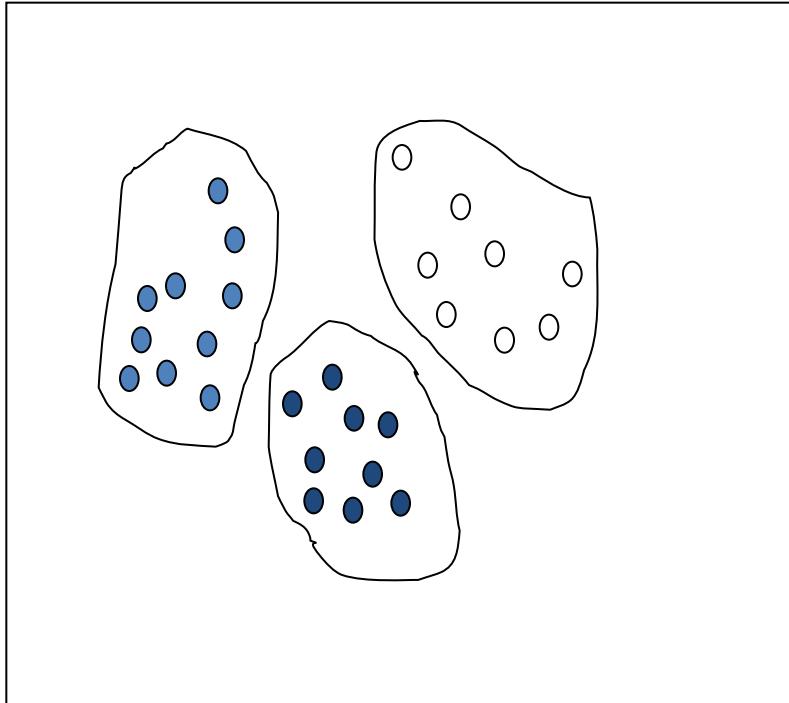
SRSWOR
(simple random sample without replacement)

SRSWR

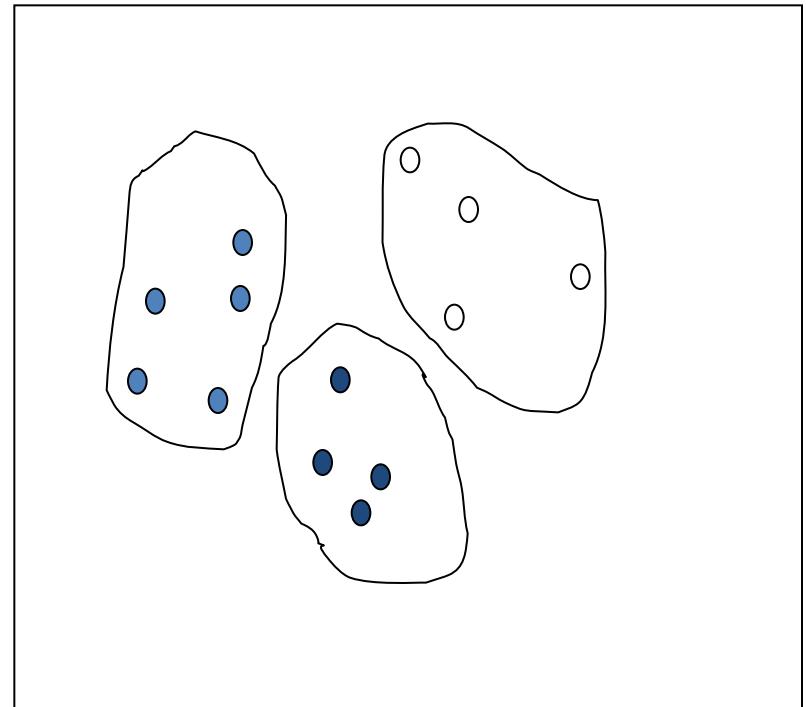


Sampling: Cluster or Stratified Sampling

Raw Data



Cluster/Stratified Sample



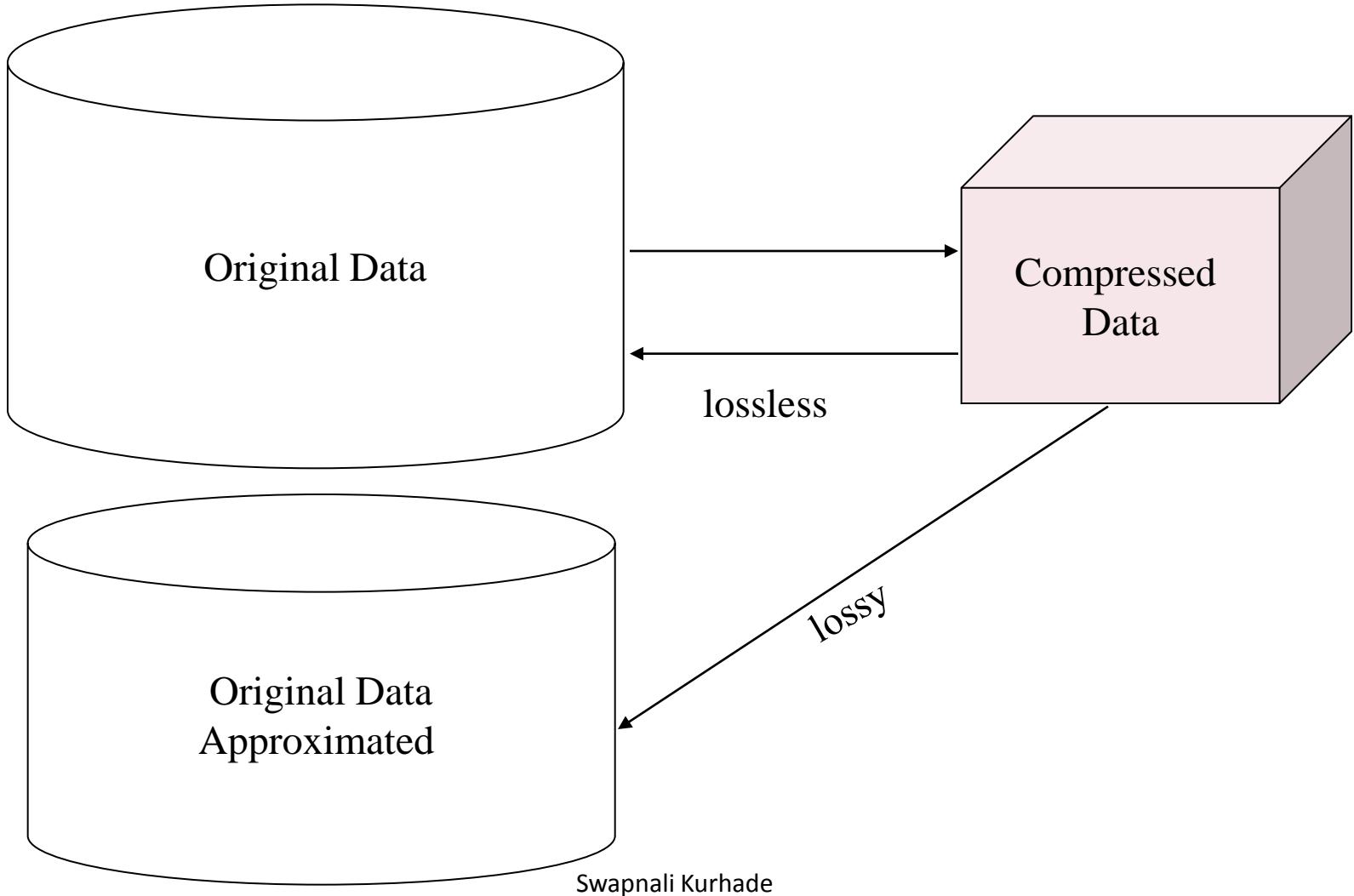
Data Cube Aggregation

- The lowest level of a data cube (base cuboid)
 - The aggregated data for an **individual entity of interest**
 - E.g., a customer in a phone calling data warehouse
- Multiple levels of aggregation in data cubes
 - Further reduce the size of data to deal with
- Reference appropriate levels
 - Use the smallest representation which is enough to solve the task
- Queries regarding aggregated information should be answered using data cube, when possible

Data Reduction 3: Data Compression

- String compression
 - There are extensive theories and well-tuned algorithms
 - Typically lossless, but only limited manipulation is possible without expansion
- Audio/video compression
 - Typically lossy compression, with progressive refinement
 - Sometimes small fragments of signal can be reconstructed without reconstructing the whole
- Time sequence is not audio
 - Typically short and vary slowly with time
- Dimensionality and numerosity reduction may also be considered as forms of data compression

Data Compression

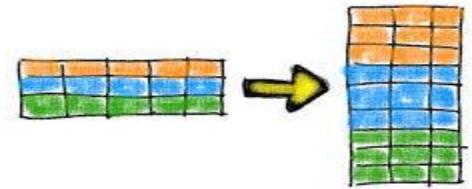


Chapter 3: Data Preprocessing

- Data Preprocessing: An Overview
 - Data Quality
 - Major Tasks in Data Preprocessing
- Data Cleaning
- Data Integration
- Data Reduction
- Data Transformation and Data Discretization
- Summary



Data Transformation



A function that maps the entire set of values of a given attribute to a new set of replacement values i.e. each old value can be identified with one of the new values

- Methods
 1. Smoothing: Remove noise from data
 2. Attribute/feature construction
 - New attributes constructed from the given ones
 3. Aggregation: Summarization, data cube construction
 4. Normalization: Scaled to fall within a smaller, specified range
 - min-max normalization
 - z-score normalization
 - normalization by decimal scaling
 5. Discretization: Concept hierarchy climbing

Normalization

- **Min-max normalization:** to $[new_min_A, new_max_A]$

$$v' = \frac{v - min_A}{max_A - min_A} (new_max_A - new_min_A) + new_min_A$$

- Ex. Let income range \$12,000 to \$98,000 normalized to [0.0, 1.0].

Then \$73,000 is mapped to

$$\frac{73,600 - 12,000}{98,000 - 12,000} (1.0 - 0) + 0 = 0.716$$

- **Z-score normalization** (μ : mean, σ : standard deviation):

$$v' = \frac{v - \mu_A}{\sigma_A}$$

- Ex. Let $\mu = 54,000$, $\sigma = 16,000$. Then

$$\frac{73,600 - 54,000}{16,000} = 1.225$$

- **Normalization by decimal scaling**

$$v' = \frac{v}{10^j} \quad \text{Where } j \text{ is the smallest integer such that } \text{Max}(|v'|) < 1$$

Discretization

- Three types of attributes
 - Nominal—values from an unordered set, e.g., color, profession
 - Ordinal—values from an ordered set, e.g., military or academic rank
 - Numeric—real numbers, e.g., integer or real numbers
- Discretization: Divide the range of a continuous attribute into intervals
 - Interval labels can then be used to replace actual data values
 - Reduce data size by discretization
 - Supervised vs. unsupervised
 - Split (top-down) vs. merge (bottom-up)
 - Discretization can be performed recursively on an attribute
 - Prepare for further analysis, e.g., classification

Data Discretization Methods

- Typical methods: All the methods can be applied recursively
 - Binning
 - Top-down split, unsupervised
 - Histogram analysis
 - Top-down split, unsupervised
 - Clustering analysis (unsupervised, top-down split or bottom-up merge)
 - Decision-tree analysis (supervised, top-down split)
 - Correlation (e.g., χ^2) analysis (unsupervised, bottom-up merge)

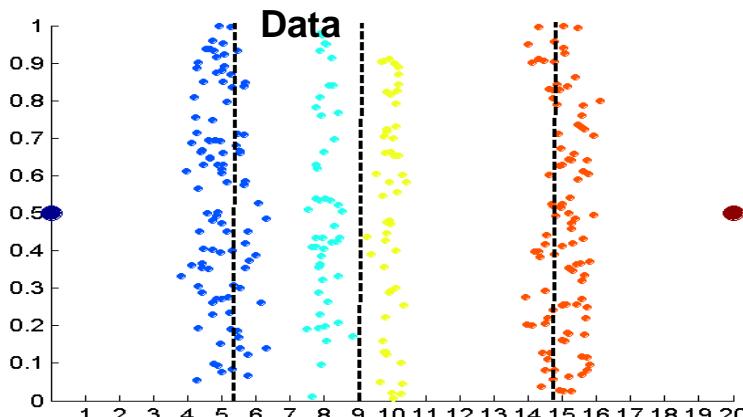
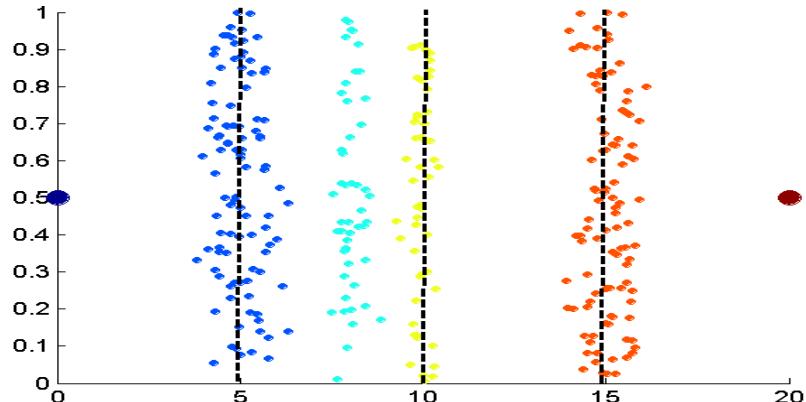
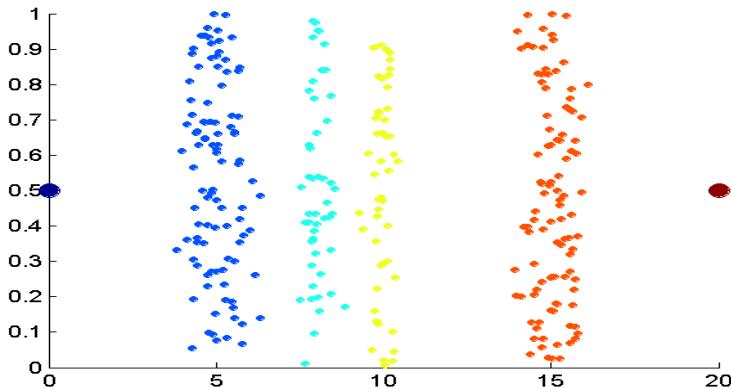
Simple Discretization: Binning

- Equal-width (distance) partitioning
 - Divides the range into N intervals of equal size: uniform grid
 - if A and B are the lowest and highest values of the attribute, the width of intervals will be: $W = (B - A)/N$.
 - The most straightforward, but outliers may dominate presentation
 - Skewed data is not handled well
- Equal-depth (frequency) partitioning
 - Divides the range into N intervals, each containing approximately same number of samples
 - Good data scaling
 - Managing categorical attributes can be tricky

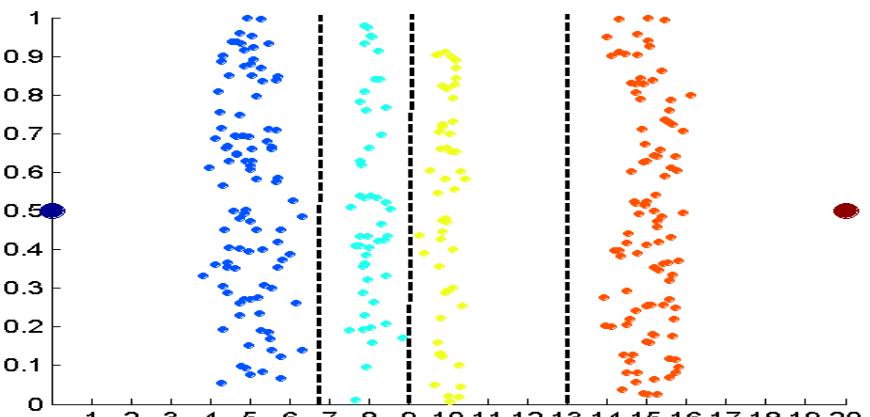
Binning Methods for Data Smoothing

- Sorted data for price (in dollars): 4, 8, 9, 15, 21, 21, 24, 25, 26, 28, 29, 34
 - * Partition into equal-frequency (**equi-depth**) bins:
 - Bin 1: 4, 8, 9, 15
 - Bin 2: 21, 21, 24, 25
 - Bin 3: 26, 28, 29, 34
 - * Smoothing by **bin means**:
 - Bin 1: 9, 9, 9, 9
 - Bin 2: 23, 23, 23, 23
 - Bin 3: 29, 29, 29, 29
 - * Smoothing by **bin boundaries**:
 - Bin 1: 4, 4, 4, 15
 - Bin 2: 21, 21, 25, 25
 - Bin 3: 26, 26, 26, 34

Discretization Without Using Class Labels (Binning vs. Clustering)



Equal frequency (binning)



K-means clustering leads to better results

Discretization by Classification & Correlation Analysis

- Classification (e.g., decision tree analysis)
 - Supervised: Given class labels, e.g., cancerous vs. benign
 - Using *entropy* to determine split point (discretization point)
 - Top-down, recursive split
 - Details to be covered in Chapter 7
- Correlation analysis (e.g., Chi-merge: χ^2 -based discretization)
 - Supervised: use class information
 - Bottom-up merge: find the best neighboring intervals (those having similar distributions of classes, i.e., low χ^2 values) to merge
 - Merge performed recursively, until a predefined stopping condition

Concept Hierarchy Generation

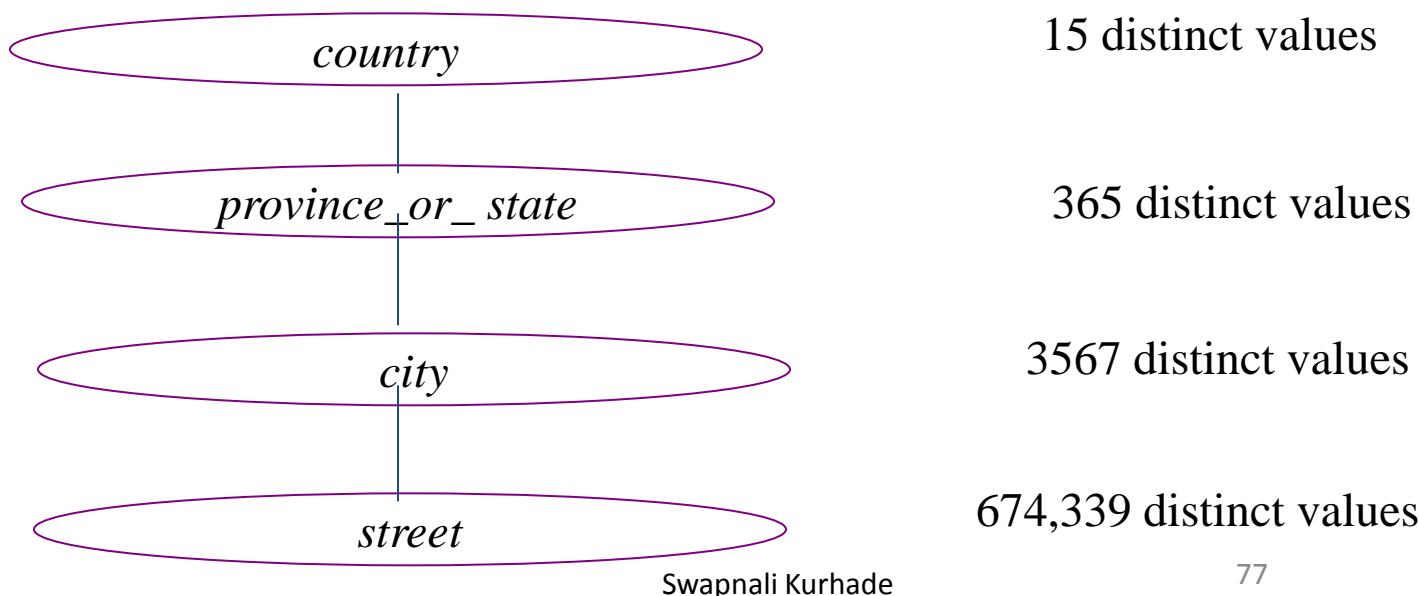
- **Concept hierarchy** organizes concepts (i.e., attribute values) hierarchically and is usually associated with each dimension in a data warehouse
- Concept hierarchies facilitate drilling and rolling in data warehouses to view data in multiple granularity
- Concept hierarchy formation: Recursively reduce the data by collecting and replacing low level concepts (such as numeric values for *age*) by higher level concepts (such as *youth*, *adult*, or *senior*)
- Concept hierarchies can be explicitly specified by domain experts and/or data warehouse designers
- Concept hierarchy can be automatically formed for both numeric and nominal data. For numeric data, use discretization methods shown.

Concept Hierarchy Generation for Nominal Data

- Specification of a partial/total ordering of attributes explicitly at the schema level by users or experts
 - $\text{street} < \text{city} < \text{state} < \text{country}$
- Specification of a hierarchy for a set of values by explicit data grouping
 - $\{\text{Urbana, Champaign, Chicago}\} < \text{Illinois}$
- Specification of only a partial set of attributes
 - E.g., only $\text{street} < \text{city}$, not others
- Automatic generation of hierarchies (or attribute levels) by the analysis of the number of distinct values
 - E.g., for a set of attributes: $\{\text{street}, \text{city}, \text{state}, \text{country}\}$

Automatic Concept Hierarchy Generation

- Some hierarchies can be automatically generated based on the analysis of the number of distinct values per attribute in the data set
 - The attribute with the most distinct values is placed at the lowest level of the hierarchy
 - Exceptions, e.g., weekday, month, quarter, year



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Summary

- **Data quality:** accuracy, completeness, consistency, timeliness, believability, interpretability
- **Data cleaning:** e.g. missing/noisy values, outliers
- **Data integration** from multiple sources:
 - Entity identification problem
 - Remove redundancies
 - Detect inconsistencies
- **Data reduction**
 - Dimensionality reduction
 - Numerosity reduction
 - Data compression
- **Data transformation and data discretization**
 - Normalization
 - Concept hierarchy generation

