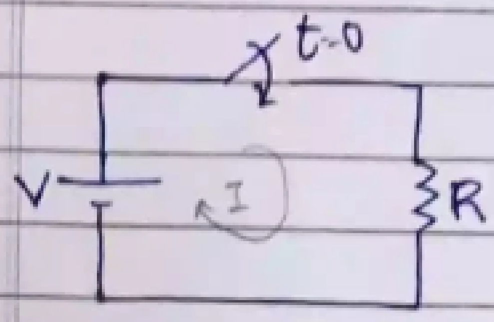
Chp: TRANSIENT ANALYSIS

1. BEHAVIOR OF RESISTOR:



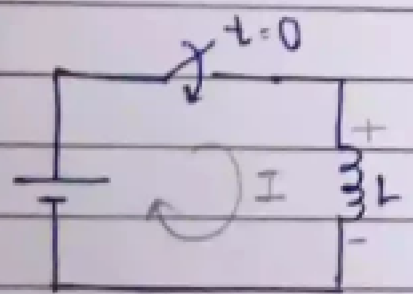
$t=0^-$ $R \rightarrow I=0$

$I=\frac{V}{R}$

$t=0^+$ $I_R=\frac{V}{R}$

2. BEHAVIOUR OF INDUCTOR:

a)



$t=0$

$V_L=L\cdot\frac{di}{dt}$

$dt=0$ $\frac{di}{dt}\rightarrow\infty$

$V_L\rightarrow\infty$ practically not possible

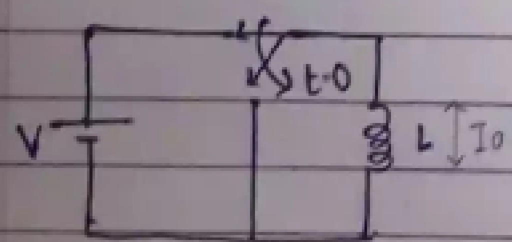
Inductor opposes the instantaneous change of current

$t=0^- \Rightarrow t=0^+$

$i_L(0^-)=0 \Rightarrow i_L(0^+)=0$

At $t=0^+ \Rightarrow$ Open Ckt

b) If a voltage is applied across the inductor continuously.



At $t=0$ voltage source is removed

$t=0^- \rightarrow I_0$

$t=0^+ \rightarrow I_0$

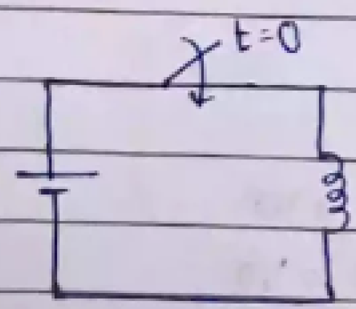
During transition it will act as a current source with I_0 current

$I_L(0^+) = I_L(0^-) = I_0$

$V_L = L \cdot \frac{di}{dt}$

$t = \infty$. no change in current

$\therefore$ $V_L = 0 \Rightarrow$ Sht. Ckt

c) 

$t = \infty$

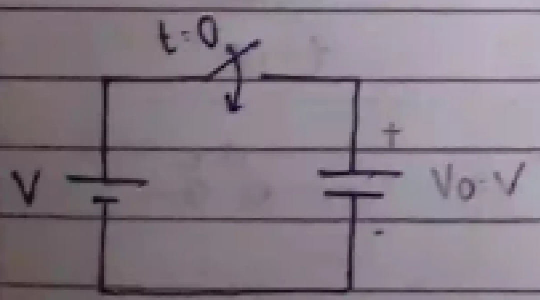
$V_L = L \cdot \frac{di}{dt} = 0$

$\frac{di}{dt} \to 0$

$t = \infty \Rightarrow$ O.C.

$t = 0$	$t = 0^+$	$t = \infty$
L	O.C	S.C
I_0	I_0	I_0

3. BEHAVIOUR OF CAPACITOR.



$Q = CV$

$\frac{dQ}{dt} = C \cdot \frac{dV}{dt}$

$i_c = C \cdot \frac{dV}{dt}$

$dt \to 0$, $\frac{dV}{dt} \to \infty$

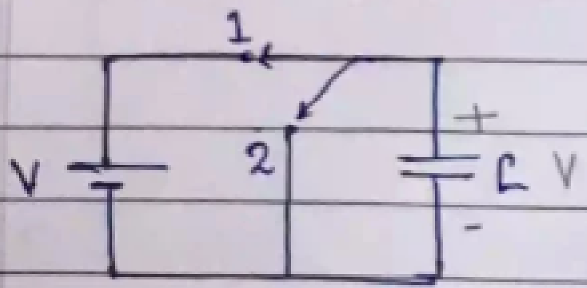
$i_c \to \infty$

Capacitor opposes inst change of voltage.

$t=0^- \Rightarrow t=0^+$

$\downarrow 0 \qquad V_C(0^+) = 0$

b) Voltage is applied continuously.



$t=0^+$

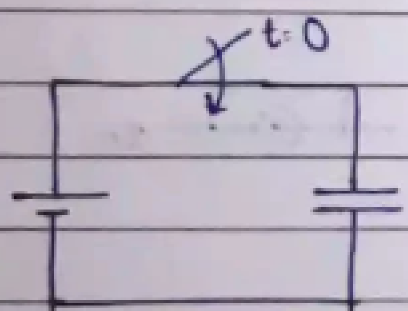
$t=0^- \rightarrow V_C(0^-) = V_0$

$t=0^+ \rightarrow V_C(0^+) = V_0$

$i = C \cdot \frac{dV}{dt} \rightarrow 0 \qquad i_C = 0$

$t=\infty \quad i_C = 0 \quad$ O.C.

c)



$t=\infty$

$i = C \cdot \frac{dV}{dt} \; 0$

$i_C = 0 \Rightarrow$ O.C.

	Initial	Final
$t=0$	$t=0^+$	$t=\infty$
C	S.C	O.C
$+ \; V_0 \; -$	V_0	O.C.

LINEAR DIFFERENTIAL EQ^n.

$\frac{di}{dt} + Pi = Q$

P → const

Q → Forced excitation

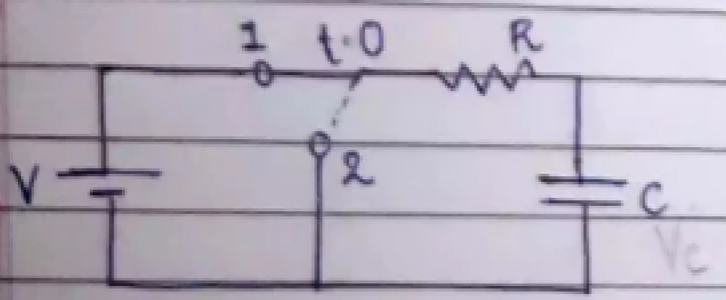
- → AC
- → DC
- → Any other

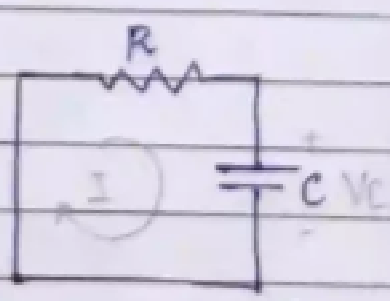
SOL^n: PI + CF

PI → Forced excitation → Steady State

CF → Source free response → Transient

→ SOURCE FREE RESPONSE OF RC CIRCUIT.





at $t=0^+$

$t=0^-$, $V_c = V$

$t=0^+$, $V_c = V$

$-IR - V_c = 0$

$IR + V_c = 0$

$R\left[C \cdot \frac{dV}{dt}\right] + V_c = 0$

$\frac{dV_c}{dt} + \frac{1}{RC} V_c = 0 \quad \left(\because \frac{di}{dt} + Pi = Q\right)$

Page No.
Date

No force eqⁿ: → $Q = 0$

C.F only PI = 0

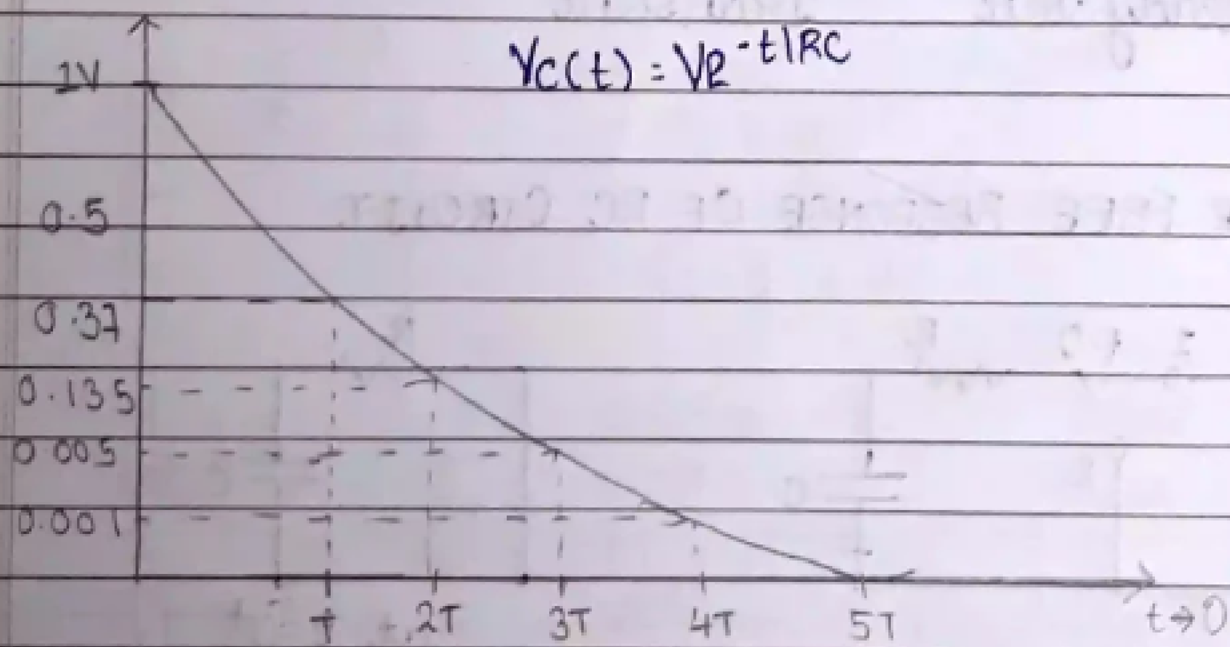
$$\frac{dV_c}{dt} + \frac{1}{RC} V_c = 0 \qquad \frac{di}{dt} + Pi = 0$$

$$C.F \to Ae^{-Pt}$$

$$V_c(t) = Ae^{-t/RC}$$
$$V_c(0^+) = Ae^{-0}$$
$$V_c(0^+) = A$$
$$V = A$$

$$V_c(t) = Ve^{-\frac{t}{RC}}$$

RC → Time constant
RC → T or τ



1T – 0.366V
2T – 0.135V
3T – 0.005V
4T – 0.001V
5T – 0.0007V

Scanned By ScanIt

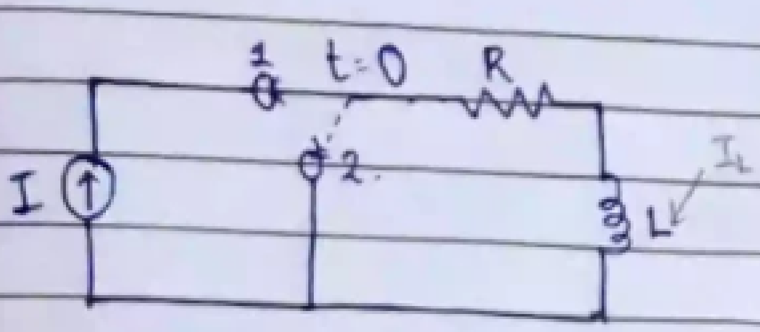
For eg if we want cap to get discharged in
$5\mu s \rightarrow 5RC$

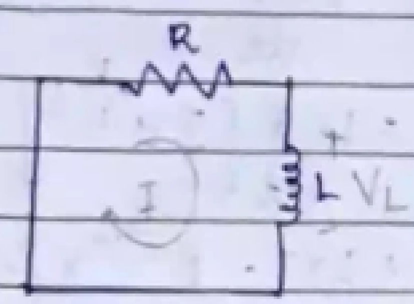
$RC \rightarrow 1\mu s$

$R = 1k\Omega$

$C = \frac{1\mu s}{1k\Omega} = 10^{-9}F$

→ SOURCE FREE RESPONSE OF RL CIRCUIT





at $t = 0^+$

at: $t = 0^-$, $I_L = I$

$t = 0^+$, $I_L = I$

Applying KVL,

$-iR - v_L = 0$

$iR + L\frac{di}{dt} = 0$

$\frac{di}{dt} + \frac{R}{L} i = 0$ $\left(\because \frac{di}{dt} + Pi = Q\right)$

$i(t) = Ae^{-\frac{Rt}{L}}$

$I_L = I$ at $t = 0^+$

$i(0^+) = Ae^{-0}$

$i(0^+) = A$

$i(t) = Ie^{\cancel{-R/t}\,-\frac{R}{L}t}$

$\tau = RC.$

$\frac{L}{R}$ = Time constant = τ

$$i(t) = I e^{-\frac{R}{L}t} = I e^{-t/\tau}$$

τ or $T = \frac{L}{R}$

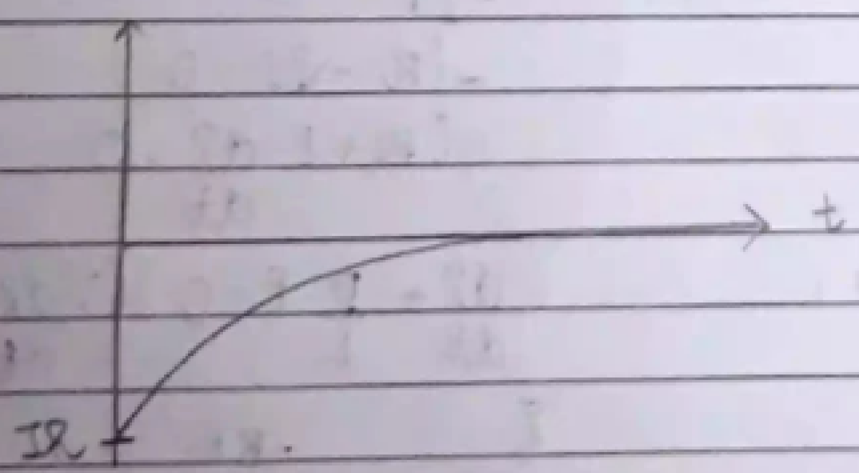
→ Inductor voltage

$$V_L = L \cdot \frac{di}{dt}$$

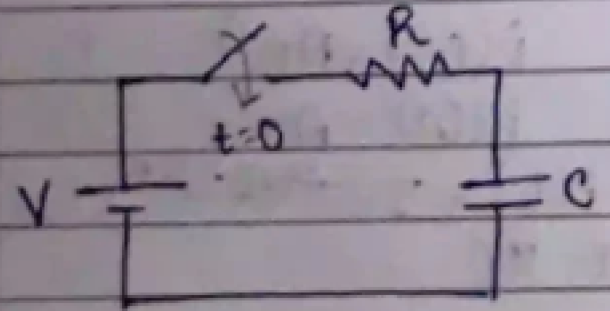
$$= L \cdot \frac{d}{dt}\left[I e^{-\frac{R}{L}t}\right]$$

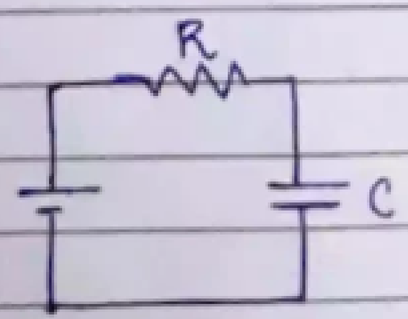
$$= L \times I \cdot e^{-\frac{R}{L} \cdot t} \times \left(-\frac{R}{L}\right)$$

$$V_L = -IR e^{-\frac{R}{L}t}$$



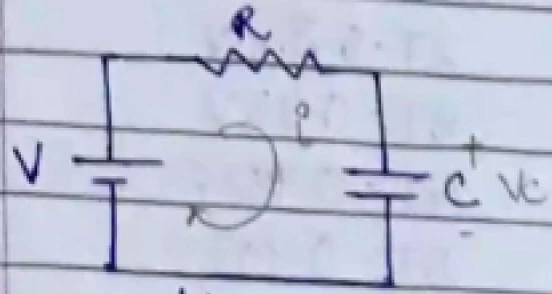
→ FORCED RESPONSE OF RC CIRCUIT





At $t = 0^+$

At $t = 0^-$; $V_C = 0$
$t = 0^+$; $V_C = 0$



At $t = 0^+$

Applying KVL

$V - iR - V_C = 0$

$V = iR + V_C$

$V = R\left[C \cdot \frac{dV_C}{dt}\right] + V_C$

$\frac{dV_C}{dt} + \frac{1}{RC} V_C = \frac{V}{RC}$

$\frac{di}{dt} + Pi = Q$

$P = \frac{1}{RC}$, $Q = \frac{V}{RC}$

Soln: $P.I + C.F$
$t = \infty$; Source free

$P.I \rightarrow V_C (t \rightarrow \infty)$, $V_C = V$

$P.I = V$

$C.F \rightarrow Ae^{-Pt}$, $P = \frac{1}{RC}$

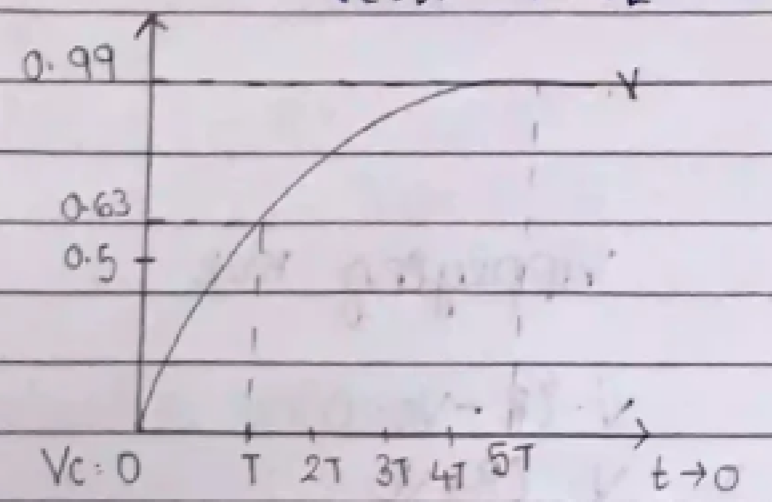
$= Ae^{-\frac{1}{RC}t}$

$V_C(t) = V + Ae^{-\frac{t}{RC}}$

$V_C(0^+) = V + Ae^{-0}$

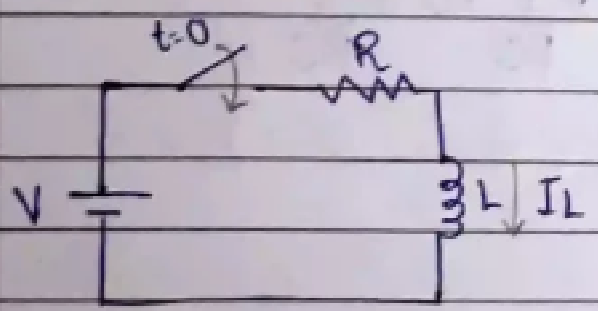
$V_C(0^+) = V + A$ $\Rightarrow$ $0 = V + A$ $\therefore$ $\boxed{A = -V}$

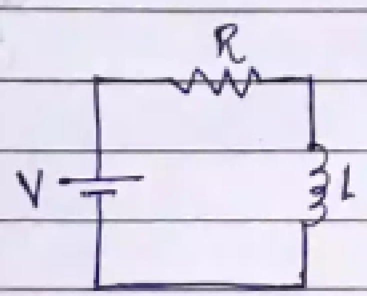
$$V_c(t) = V - Ve^{-t/RC}$$



1T - 0.63V
2T - 0.865V
3T - 0.95V
4T - 0.982V
5T - 0.993V

→ FORCED RESPONSE FOR RL CIRCUIT

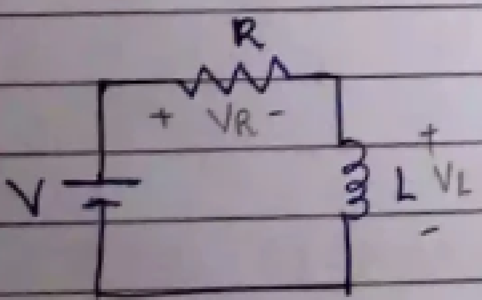




At $t = 0^+$

At $t = 0^-$; $I_L = 0$

At $t = 0^+$; $I_L = 0$



At $t = 0^+$

Applying KVL

$V = V_R + V_L$

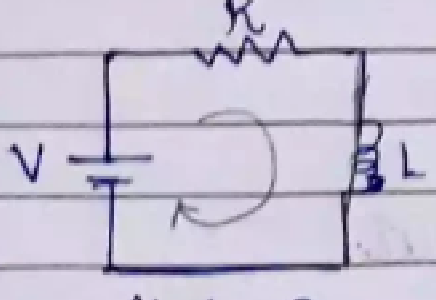
$= iR + L\frac{di}{dt}$

$$\frac{di}{dt} + \frac{Ri}{L} = \frac{V}{L}$$

$$P = \frac{R}{L}, \quad Q = \frac{V}{L}.$$

Sol^n: P.I + C.F

$t = \infty$ $\quad$ $Q = 0$



At $t = \infty$

At steady state → Sht. ckt

$P.I : I_L(t=\infty)$

$I_L = \frac{V}{R}$

$P.I = \frac{V}{R}$

$C.F : \quad Ae^{-Pt}$

$\quad\quad Ae^{-\frac{R}{L}t}$

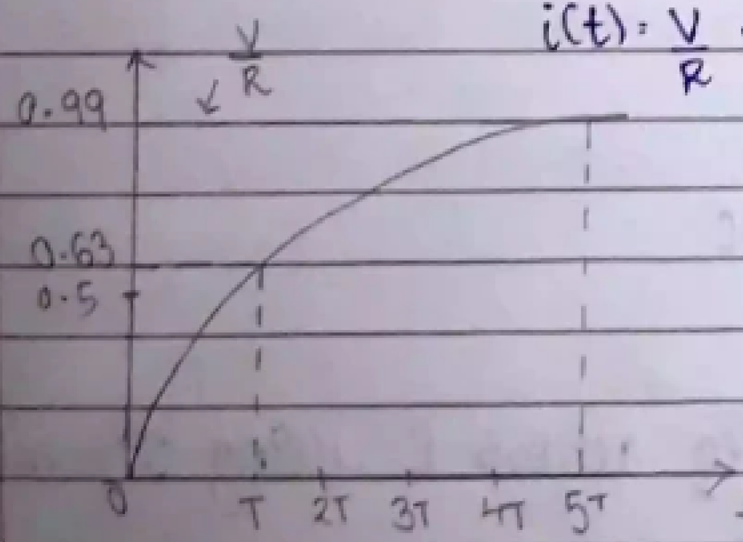
$i(t) = \frac{V}{R} + Ae^{-\frac{R}{L}t}$

$i(0^+) = \frac{V}{R} + A \Rightarrow A = -\frac{V}{R}$

$\downarrow$

0

$i(t) = \frac{V}{R} - \frac{V}{R} e^{-R/L\, t}$



Page No.
Date

1. $V_C(t) = V - Ve^{-t/RC}$: Forced excitation.

2. $V_C(t) = Ve^{-t/RC}$: Source free response.

3. $i_L(t) = Ie^{-\frac{R}{L}t}$: Source free

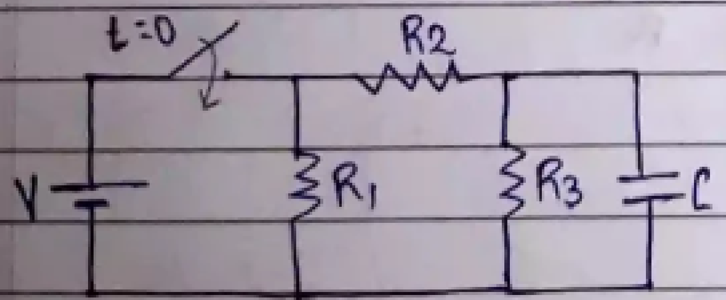
4. $i_L(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{Rt}{L}}$: Forced input.

1. $V_C(t)$ or $i_L(t)$ $= [\text{Final value}]_{t \to \infty} + [\text{Initial} - \text{final}]e^{-\frac{t}{T}}$

$$= V + [0 - V]e^{-t/T} \quad , \; T = RC$$
$$= V - Ve^{-t/RC}$$

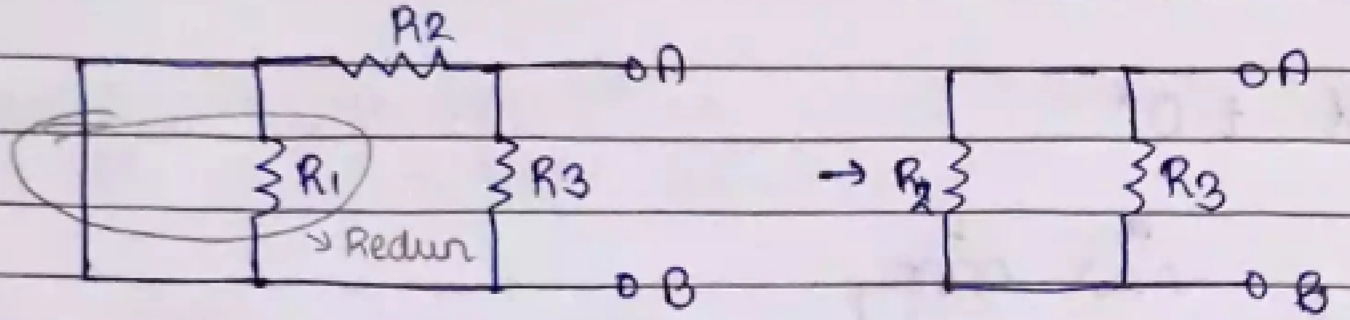
Source Free : $[0] + [V - 0]e^{-t/T}$
$$= Ve^{-t/RC}$$

If there are more than 1 elements.



In such case, find Req across C using Thevenin's eq ckt at ~~t~~ $t = 0^+$

Scanned By ScanIt



∴ $R_{eq} = R_2 \| R_3$

∴ τ or $\tau = RC$

$= R_{eq} \times C$

$= \left(\dfrac{R_2 \cdot R_3}{R_2 + R_3}\right) \times C$.

★ IN SHORT (STEPS)

1. Draw the circuit at $t = 0^+$
2. Find initial conditions: $V_C(0^+)$ or $I_L(0^+)$
3. Find R_{eq} & (C_{eq} or L_{eq}) of ckt.
   Time constant $(\tau) = R_{eq} \times C_{eq}$
   $(\tau) = \dfrac{L_{eq}}{R_{eq}}$
4. Find final values of V_C or I_L (at time $\to \infty$)
5. Find solution = P.I + C.F
6. Total solution = Final value + (Initial − Final) $e^{-t/\tau}$

Scanned By ScanIt