

D.C. MACHINES

$$1. E_b = \frac{\phi Z N P}{60 A}$$

E_b : Back EMF

ϕ : Flux per pole

Z : Total no. of armature conductor

N : Speed of armature in RPM.

P : No. of poles

A : No. of parallel paths

2. Eq. ckt of motor:

Motor \rightarrow electrical energy into mechanical.

Principle : When a current carrying conductor is placed in a magnetic field, it experiences a mech force whose direction is given by Fleming's left-hand Rule whose magnitude is given by $F = B I L N$

There's constructionally no diff between D.C. Generator & D.C. Motor. Hence, the same D.C. Machine can be used interchangeably as a generator or as a motor.

When operating as a generator, it is driven by a mech machine & it develops voltage which in turn produces a current flow in an electric ckt. When operating as a motor, it is supplied by electric current & it develops a torque which in turn develops mech rotation.

When its field magnets are excited & its armature conductor are supplied with current from the supply mains, experience a force tending to rotate the armature. Armature conductors under N-pole are assumed to carry current downward & those under S-poles to carry current up.

By applying Fleming's Left Hand Rule, the direction of force on each conductor can be found. It is shown seen that each conductor experiences a force F which tends to rotate the armature in anticlock direction. These forces collectively produce a driving torque which sets armature rotating.

→ BACK EMF:

- When motor armature rotates, the conductor also rotates & hence cut the flux.
- In accordance with the laws of EMI, emf is induced in them whose direction, as found by Fleming's Right Hand Rule is in opposition to applied voltage.
- Because of its opposing direction, it is referred as counter emf or back emf E_b .

- The rotating armature generating back emf E_b is like battery. The power reqd to overcome opposition of E_b is $E_b I_a$.
- $E_b = \frac{(\phi Z N P)}{A} v$. N is in rps.
- Back emf depends on armature speed. Speed \uparrow , $E_b \uparrow$ $\Rightarrow I_a$ small.
- Speed is less, E_b is less, more current flows which develops motor torque.
- E_b makes a motor self-regulating so that it draws as much current as it is necessary.

\rightarrow VOLTAGE EQUATION OF MOTOR:

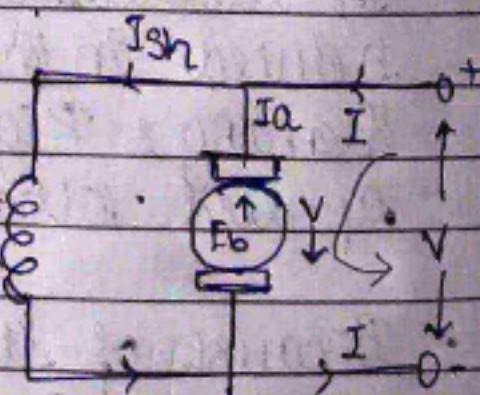
1. Voltage applied across the motor armature has to

- overcome back emf E_b &
- Supply the armature ohmic drop $I_a R_a$.

2. $V = E_b + I_a R_a$

3. $V I_a = E_b I_a + I_a^2 R_a$

Electrical input to armature $\xrightarrow{\text{equivalent of mech power}}$ Cu loss in armature



4. Motor efficiency is given by the ratio of power developed by the armature to its input i.e. $\eta = \frac{E_b I_a}{V I_a}$

5. Condition for max power: $E_b = \frac{V}{2}$

Principle: Electromechanical energy conversion takes place whenever a change of flux linkages is associated with mechanical motion.

Speed voltage is generated when there is relative motion between coil & field.

Cylindrical structure.

Field windings

- i) Primary source of flux
- ii) Arranged to produce cyclic north-south distri

Armature windings

- i) Series/parallel or star/delta 3-phase connection.
- ii) Gives desired voltage to supply rated current

Interaction

- Produce EM torque
- which aligns the 2 fields

current passes the coils.

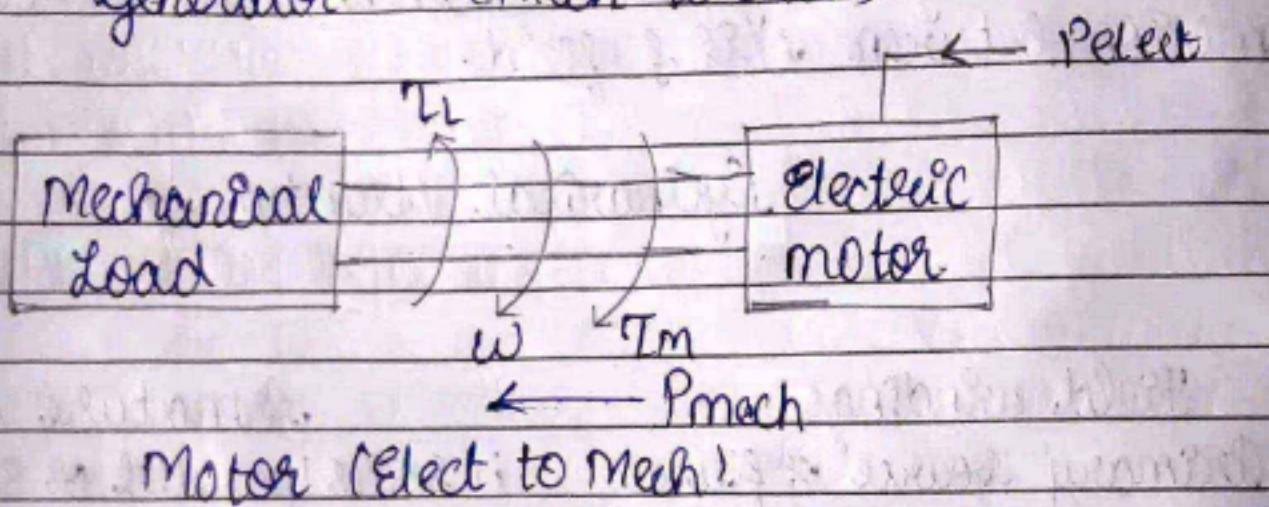
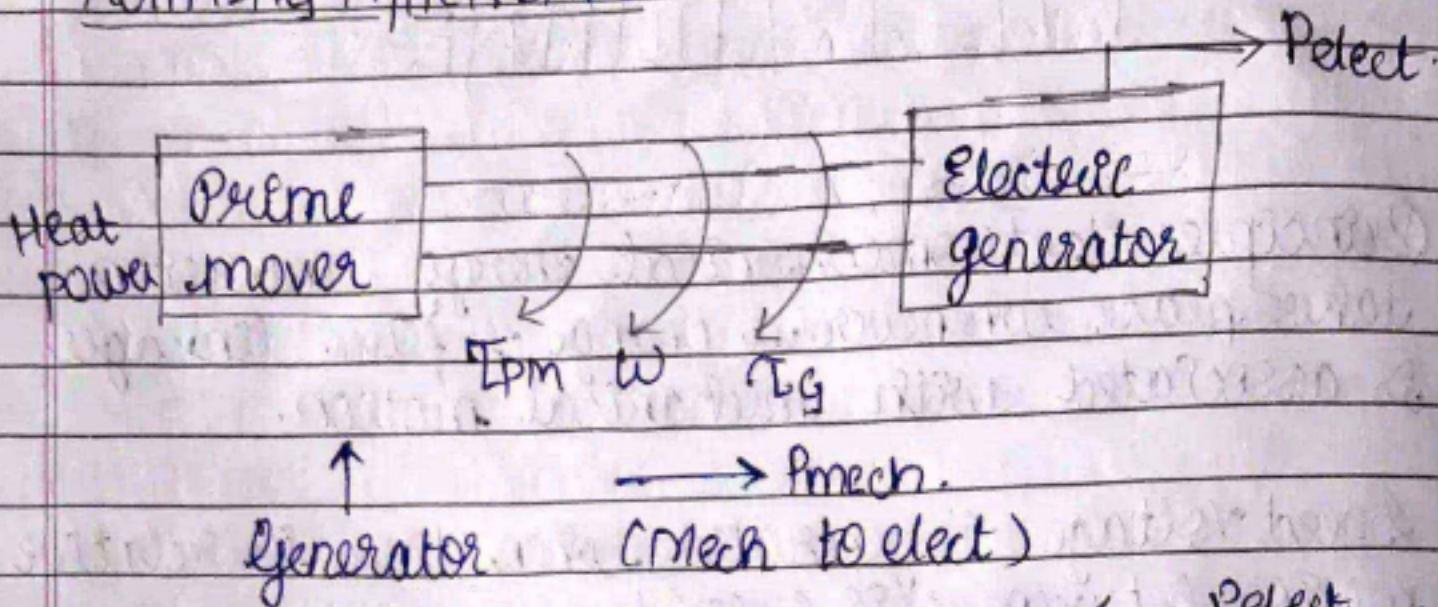
Stator (stationary)

- i) Annular portion of cylinder in which cylindrical rotor rotates

Rotor (rotating)

- i) Housed with an axle on two sides of stator
- ii) High permeability of magnetic material

→ ROTATING MACHINES:



where T_{pm} = Prime move torque & $T_{pm} = T_g$

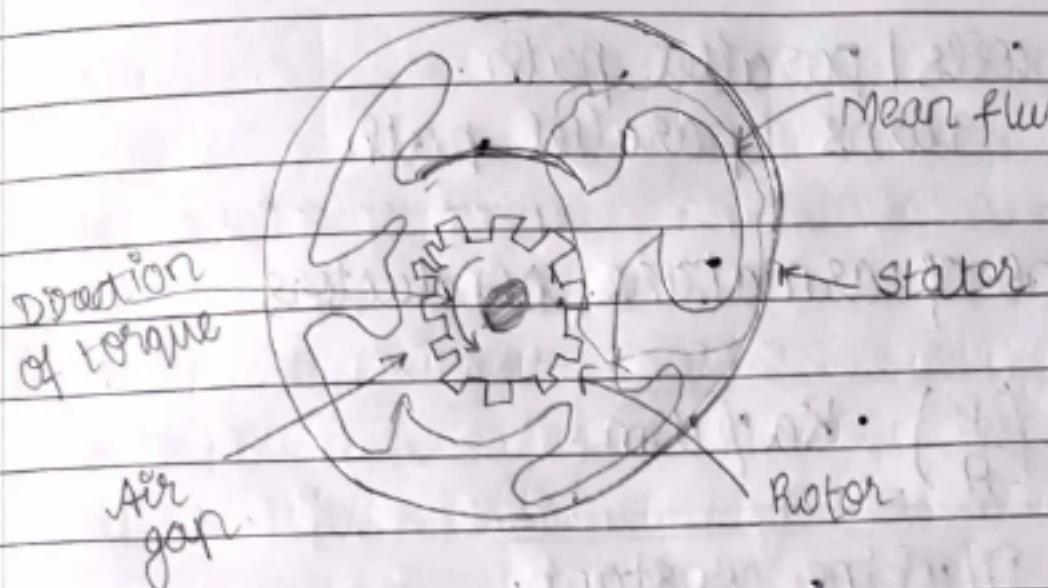
T_g = generator torque

T_L = load torque & $T_L = T_m$

T_m = Motor torque

→ CONSTRUCTION & WORKING FEATURES:

- i) Stator & rotor separated by air gap.
- ii) Field windings are primary source of flux.
- iii) Rotor is housed in slots uniformly cut around the periphery & are connected to armature winding.
- iv) Relative motion of field winding & armature winding induces emf which exchanges power.
- v) Interaction produces torque of EM origin resulting in conversion.



→ ARMATURE WINDINGS & COMMUTATOR:

1. Accommodated in slots of two layers.
2. One side of the coil is in top in one slot & its 2nd side is in bottom of a slot ($\pi/2$ rad) away.
3. There are as many commutator segments as the no. of armature coils.

→ EMF AND TORQUE:

$$\phi = \text{flux/pole}$$

$$\lambda_1 = Nc\phi, \lambda_2 = -Nc\phi \text{ (after reversing one pole switch)}$$

$$\Delta\lambda = -2Nc\phi$$

Time of travel the one pole switch.

$$\Delta t = \frac{2\pi}{\omega_m} \cdot s$$

$$E_c = \frac{-\Delta\lambda}{\Delta t} = \phi \omega_m N c P$$

$$\text{Armature EMF: } E_a = \frac{\phi \omega_m (C_p N c) P}{\pi}$$

where E_p = coils / parallel path.

$$C_p N_c = \frac{Z}{2A} \text{ turns / parallel path}$$

Z = Total no. of armature conductors

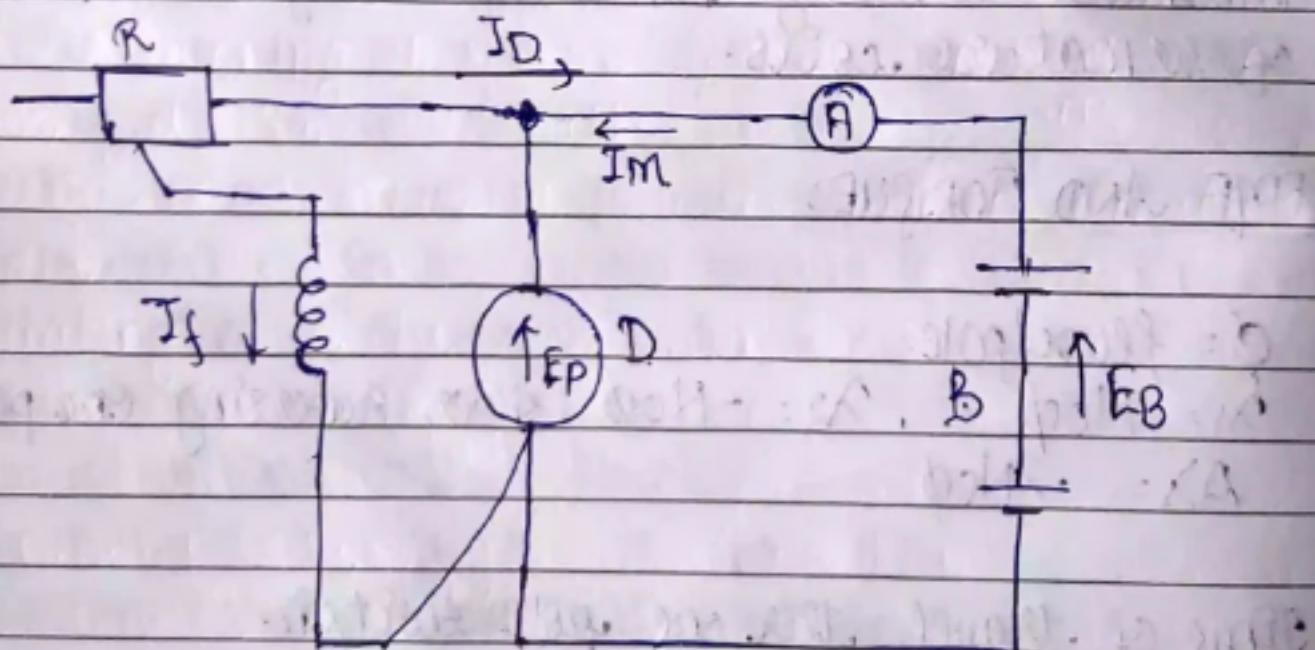
$$E_a = \frac{\phi \omega m Z}{2\pi} \left(\frac{P}{A} \right) = K_a \phi \omega m$$

$$K_a = \frac{Z P}{2\pi A} = \text{Machine constant.}$$

$$E = \frac{\phi Z N P}{60 A}$$

→ TYPES OF DC MACHINES

1.



Shunt-Wound Machine.

Works as a generator when: current I_D is in the same direction as emf E_B .

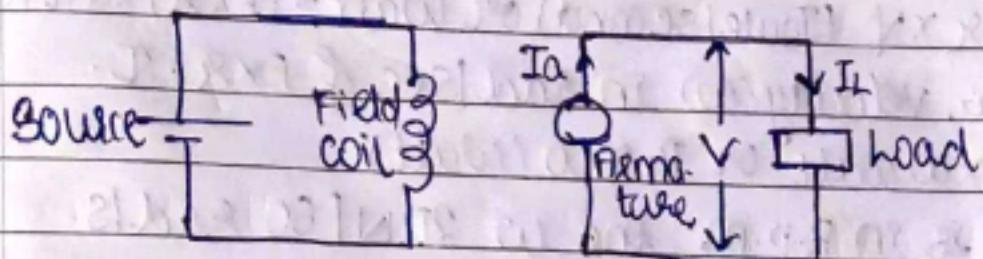
As generator: I_D is in the same direction as E_B and $V < E$

$$\therefore V = E - I_a R_a$$

Works as a motor when $\therefore I_m$ is in reverse direction
as emf E_d .

$$\text{As motor: } V > E \quad \therefore N = E + I_a R_a.$$

2. Separately excited generators:



$$I_a - I_L = I.$$

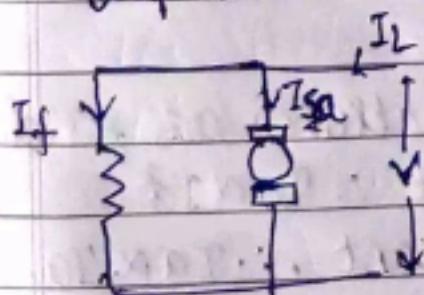
$$V = E - I R_a.$$

$$\text{Power developed} = EI.$$

$$\text{Power delivered} = VI.$$

→ SHUNT EXCITED GENERATORS

→ Q. A 4-pole 220-V shunt motor has 540 lap-wound conductor ft takes 32A from supply & develops output power of ~~5.595~~ 5.595 kW. The field winding takes 1A. The armature resistance is 0.09Ω & flux per pole is 30mWb. Calculate i) speed ii) torque (Nm)
lap wound \rightarrow 11 parallel = no. poles



$$I_a = I_L + I_f$$

$$= 32 - 8$$

$$= 24 A$$

$$E_b = V - I_a R_a$$

$$= 220 - 24 \times 0.09$$

$$E_b = 217.21 V$$

$$E_b = \Phi Z N P$$

$$60 A$$

$$T_{sh} = \frac{q \cdot S \cdot X}{N}$$

$$217.21 = \frac{30 \times 10^{-3} \times 540 \times N}{60}$$

$$= 0.95 \times 5.595 \times 10^3$$

$$N = \frac{804.48 \text{ rpm}}{804.48}$$

$$T_{sh} = 6642 \text{ N-m.}$$

→ TORQUE

1. Turning or twisting moment of a force abt an axis
2. & Torque (T) : $T = F \times r$ Newton-metre (N-m)
3. Work done in 1 revolution = $F \times \theta \times r$ (Joule)
 Power : $F \times \theta \times r \times N$ (Joule/second) or Watt = $(F \times r) \times \omega \times N$ (Watt)
 ωN = Angular velocity (ω) in rad/sec & $F \times r = T$.
 ∴ Power :- $T \times \omega$ watt or $P = T \omega$ (Watt)
3. Moreover, if N is in R.p.m. then $\omega = 2\pi N / 60$ rad/s.
 $\therefore P = \frac{\omega T N}{60}$ or $P = \frac{2\pi}{60} N T = \frac{NT}{9.55}$

→ ARMATURE TORQUE OF A MOTOR

1. Let T_a be the torque developed by the armature of a motor running at N r.p.s. If T_a is in $\frac{Nm}{A}$, then power developed = $T_a \times \omega N$ watt.
2. Electric power converted to mech. power in armature :- $E_b I_a$ Watt.
3. Equating both $T_a \times \omega N = E_b I_a$.
 Subs $E_b = \phi Z N \times \left(\frac{P}{A}\right) V$,

$$T_a = 0.159 \cdot \phi Z I_a \times \left(\frac{P}{A}\right) \quad \therefore T_a \propto \phi I_a$$

4. For series motor, ϕ is $\propto I_a$ (by saturation), as field windings carry full armature current $\therefore T_a \propto I_a^2$
 For shunt motor, ϕ is practically constant, $\therefore T_a \propto I_a$.

→ SHAFT TORQUE OF A MOTOR

1. The torque which is available for doing useful work is

known as shaft torque $\cdot T_{sh}$.

2. Output : $T_{sh} \times 2\pi N$ Watt provided T_{sh} is in N-m & N- rps
3. $\therefore T_{sh}$ = Output in watts / $2\pi N$ N-m, N- rps .
 - = Output in watts / $2\pi N$ / 60 N-m, N- rpm .
 - = $\frac{\text{Output}}{N}$ = $\frac{9.55 \text{ Output}}{N}$ N-m.

4. The difference ($T_a - T_{sh}$) is known as lost torque & is due to iron & friction losses in motor.

\rightarrow SPEED OF DC MOTOR

$$E_b = V - I_a R_a \text{ or. } \frac{\phi Z \cdot N}{60 A} = V - I_a R_a$$

$$N = \frac{\gamma - I_a R_a \times (60A)}{\phi Z P} \text{ r.p.m.}$$

$$\gamma - I_a R_a = E_b \therefore N = \frac{E_b \times (60A)}{\phi Z P} \text{ r.p.m.} \text{ or } N = \frac{\gamma \cdot E_b}{\phi}$$

\rightarrow Series motor:

Let N_1 = speed in 1st case : I_{a1} = armature I in 1st case
 ϕ_1 = no flux per pole in the first case.

N_2, I_{a2}, ϕ_2 = flux, speed, armature current in 2nd case.

$$N_1 = E_{b1} \text{ where } E_{b1} = \gamma - I_{a1} R_a$$

ϕ_1

$$N_2 = E_{b2} \text{ where } E_{b2} = \gamma - I_{a2} R_a$$

ϕ_2

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2} \times \phi_1}{E_{b1} \times \phi_2} \cdot \text{ Prior to magnetic poles saturation:}$$

$\phi = I_a$

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2} \times I_{a1}}{E_{b1} \times I_{a2}}$$

→ For shunt motor

In this case same eqn:-

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \quad \text{if } \phi_1 : \phi_2 = N_2 : E_{b2}$$

→ TORQUE & SPEED DC MOTOR

$$N = \frac{K(V - I_a R_a)}{\phi} = \frac{KE_b}{\phi}$$

$$T_a \propto \phi I_a \quad T_a \propto \phi I_a$$

1. Increase in flux would decrease the speed but increase the armature torque. It cannot be so as torque always tends to produce rotation.
2. If the flux of a motor is decreased by decreasing field current
 - a) Back emf $(E_b) = (N\phi)K$ drops instantly (speed remains const because of inertia of heavy armature)
 - b) Due to decrease in E_b , I_a is increased as $I_a = (V - E_b) / R_a$. Moreover, a small reduction in flux produces a proportionally large (\uparrow) in armature current.
 - c) ∵ The eqn $T_a \propto \phi I_a$, a small decrease in ϕ is more than counter balanced by a large increase in I_a with the result that there is a net increase in T_a .
3. It is ~~then~~ possible to (\uparrow) flux & at the same time (\uparrow) speed provided I_a is held const.

→ MOTOR CHARACTERISTICS

1. Shows relationship betw:

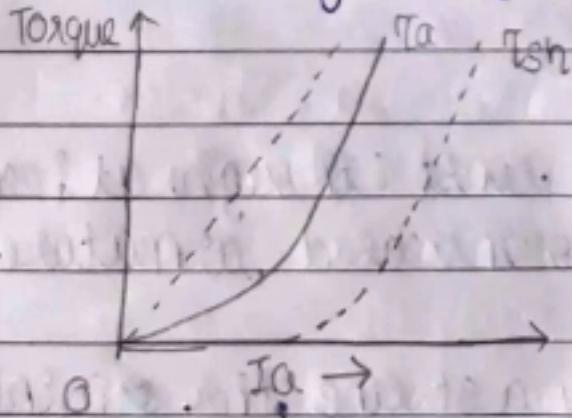
- a) Torque & armature current i.e. T_a/I_a characteristic: electrical characteristic.

- b) Speed & armature current ; e. N/I_a charac.
- c) Speed & torque i.e. N/I_a charac : Mech charac.

$$T_a = \frac{\phi I_a}{N} \propto \frac{E_b}{\phi}$$

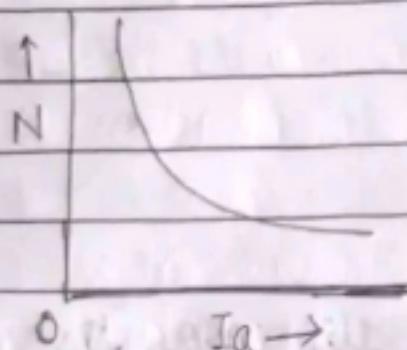
→ SERIES MOTOR:

- i) 1. T_a/I_a character.
2. $T_a \propto \phi I_a$.
3. In this case, as field winding also carry the armature current, ϕ rises upto pt of magnetic saturation. Hence, by saturation, $T_a \propto \phi I_a$ & $T_a \propto I_a^2$.
4. At light loads, I_a & hence ϕ is small. But as $I_a (\uparrow)$, $T_a (\uparrow)$ as $\propto I^2$. Hence T_a/I_a is parabola.
5. After saturation, ϕ is almost independent of I_a & hence T_a only. ∴ St line.
6. Eg: used in case where huge starting torque is required for accelerating heavy masses quickly in hoists & electric train

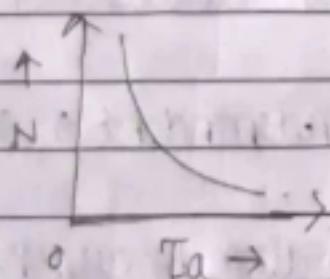


- ii) i. N/I_a charact.
2. ~~$E_b \propto N \alpha E_b / \phi$~~
3. When load is heavy, I_a is large. ∴ Speed low (this decrease E_b & allows more armature to flow). But when load current falls & hence I_a falls to a small value, speed becomes dangerously high.

4. Hence, a series motor should never be started without some mech (not belt-driven) load on it, else it may develop excessive speed & get damaged due to heavy centrifugal forces so produced.
5. It is variable speed motor.

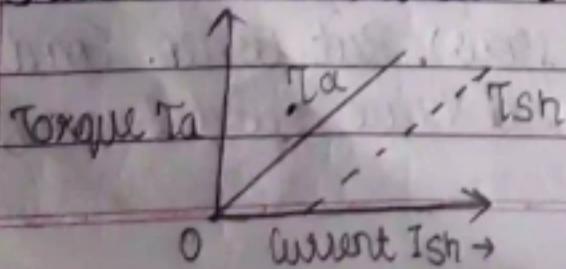


iii) $N \propto T_a$ or ~~\propto~~ mech
When speed is high, torque is low & vice-versa.

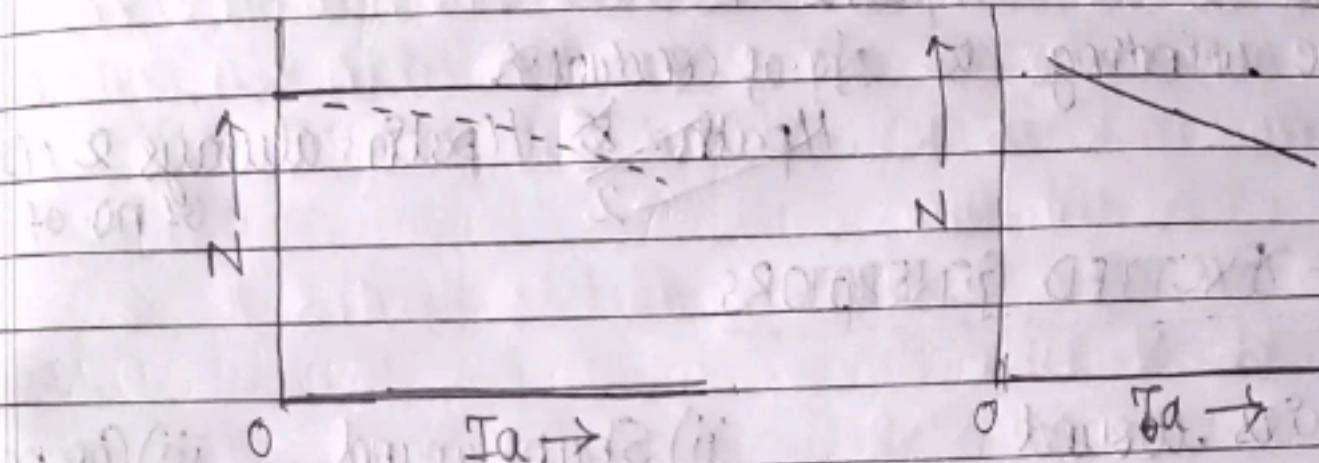


→ SHUNT MOTOR:

- i) 1. $T_a \propto I_a$ charac.
2. Assuming ϕ to be practically const (though at heavy loads, ϕ decreases somewhat due to increased armature reactn), we find that $T_a \propto I_a$.
3. ∵ Electrical charact. is practically a st line thru origin. Shaft torque is shown ~~is~~ dotted.
4. Since heavy starting will need heavy starting current, shunt motor should never be started on (heavy load).



ii) $N|I_a$ charact.



iii) $N|T_a$

→ Constant Speed motor.

→ COMPOUND MOTOR:

i) Have both series & shunt windings.

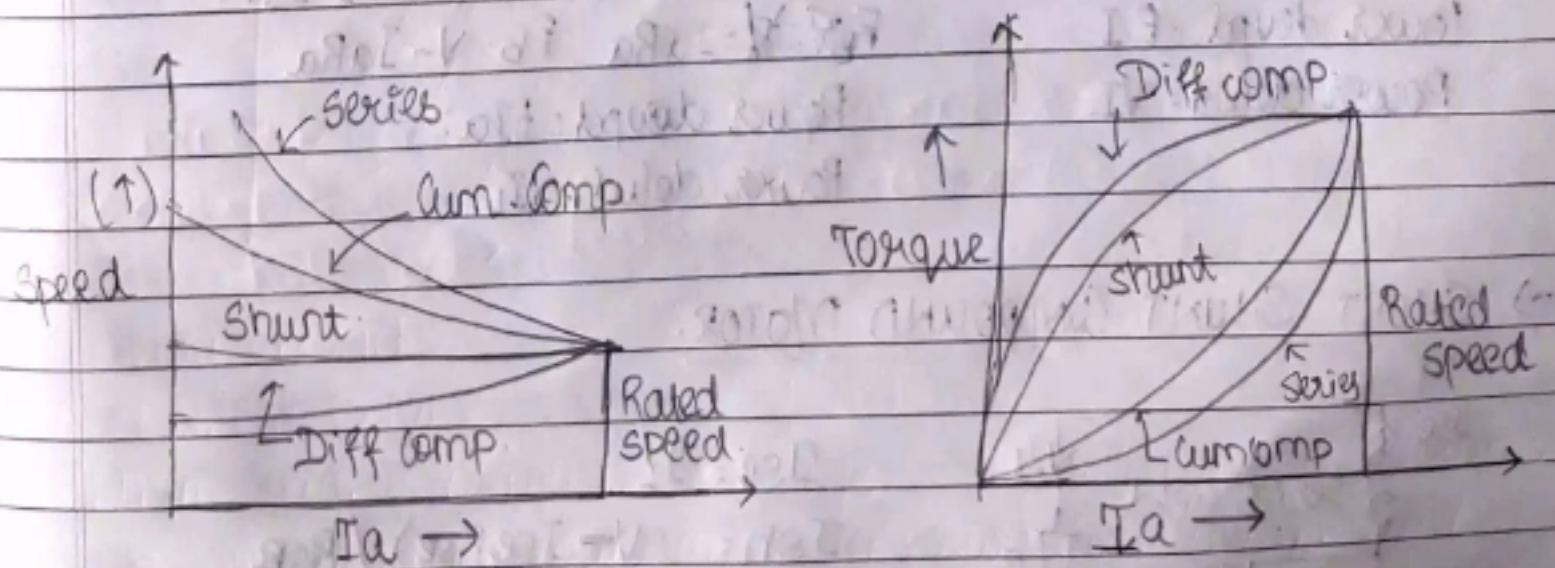
Compound.

Cumulatively

Differentially

Series flux is in same direction as shunt field flux

Series flux opposes shunt flux



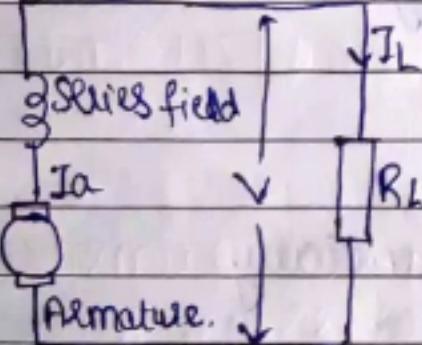
→ Lap winding: $\alpha = \text{No. of conductors}$, $P = \text{poles}$
 $\text{No. paths} = P$

→ Wave winding: $\alpha = \text{No. of conductors}$
 $\text{No. paths} = \frac{\alpha}{2}$ // path: always 2 (irrespective of no. of poles).

→ SELF-EXCITED GENERATORS

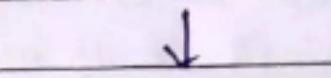
i) Series wound

field coils in series
with armature ckt



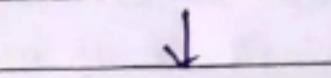
ii) Shunt wound

field coils connected
across armature ckt



iii) Compound wound

one in parallel
other in series



$$I_a = I_{se} = I_L = I$$

$$E_b - V = I_a(R_a + R_s)$$

Power delvd - $E I$

Power delvd = $V I$

$$I_{sh} = V / R_{sh}$$

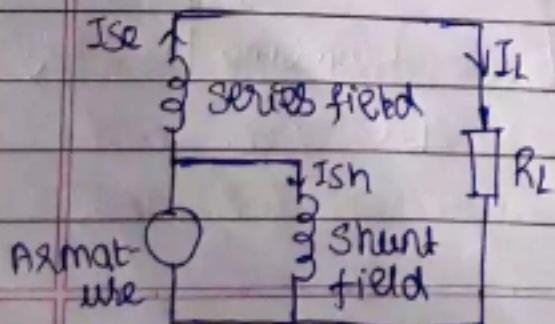
$$I_a = I_{sh} + I_L \quad I_a = I_L - I_{sh}$$

$$E_b - V = I_a R_a \quad E_b = V - I_a R_a$$

Power delvd: $E I_a$

Power delvd: $V I_L$

→ SHORT-SHUNT COMPOUND MOTOR.



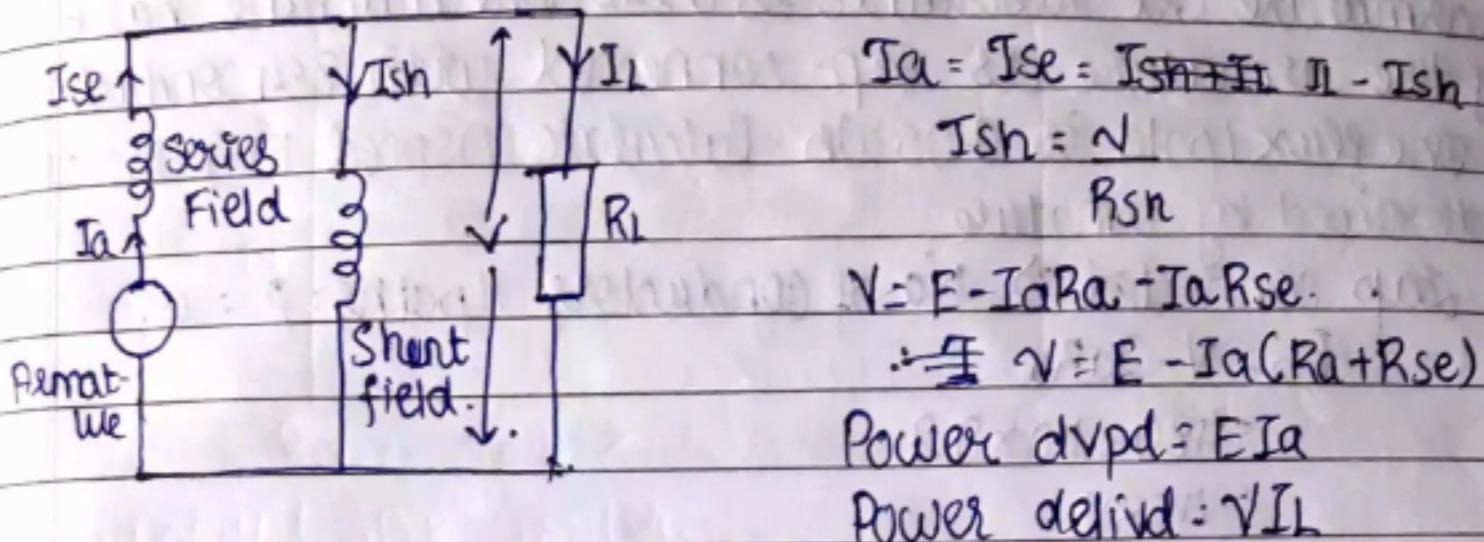
$$I_{se} = I_L$$

$$I_{sh} = (V + I_{se} R_{se}) / R_{sh}$$

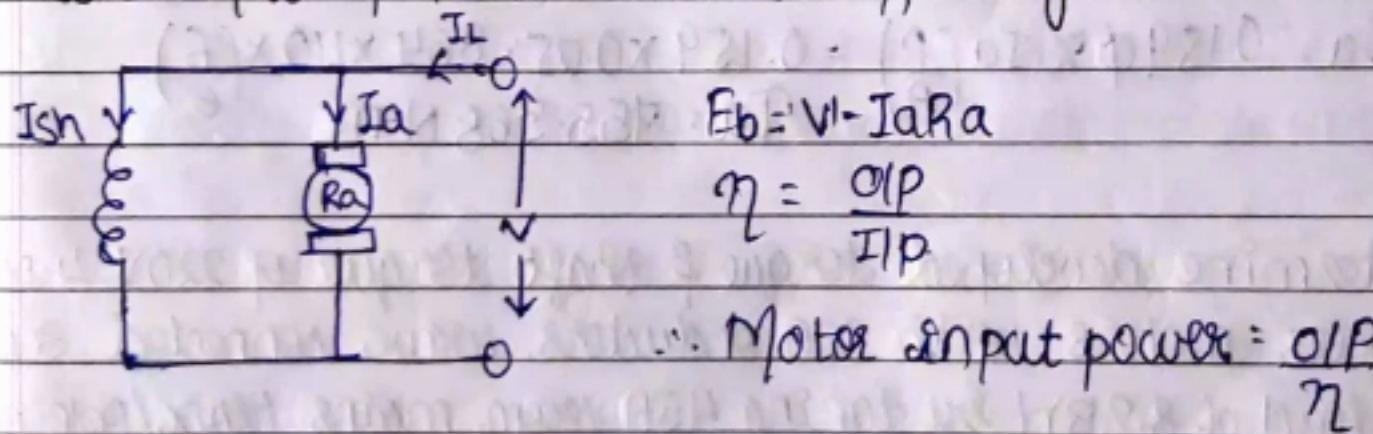
$$I_a = I_{sh} + I_L \quad I_a = I_L - I_{sh}$$

$$V = E - I_a R_a = I_{se} R_{se}$$

$$\text{Power delv} = E I_a, \quad \text{Power delv} = V I_L$$

LONG-SHUNT

→ Q.1 A 440V shunt motor has armature has resist of 0.8Ω & field resistance of 200Ω . Determine back emf when giving an output of 7.46 kW at 85% efficiency.



$$\text{Motor input current} = \frac{7.46 \times 10^3}{440} = 16.5 \text{ A}$$

$$I_L = 20 \text{ A}$$

$$\eta = 0.85$$

$$IIP = 8.776 \text{ kW}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{200} = 2.2 \text{ A}$$

$$I_a = I_L - I_{sh} = 20 - 2.2 = 17.8 \text{ A}$$

$$E_b = V - I_a R_a$$

$$= 440 - 17.8 \times 0.8$$

$$E_b = 425.76 \text{ V}$$

→ Q.2. A D.C. Motor takes an armature current at 110A at 480V. The armature ckt resistance is 0.2Ω. The machine has 6-poles & the armature is lap-connected with 864 conductors. The flux/pole is 0.05Wb. Calculate i) speed ii) Gauss torque developed by armature.

Lap connected \Rightarrow No. of conductors / paths = $P = 6$.

$$E_b = V - I_a R_a$$

$$= 480 - 110 \times 0.2$$

$$= 458V$$

$$E_b = \frac{\phi Z N P}{60 A} = \frac{0.05 \times 864 \times N \times \left(\frac{6}{6}\right)}{60} = 458$$

$$\therefore N = 636.11 \text{ rpm}$$

$$T_a = \frac{0.159 \phi Z I_a (P)}{A} = \frac{0.159 \times 0.05 \times 864 \times 110 \times \left(\frac{6}{6}\right)}{A} \\ T_a = 455.568 \text{ N-m}$$

→ Q.3 Determine developed torque & shaft torque of 220V, 4-pole series motors with 800 conductors, wave connected, supplying a load of 8.2kW by taking 45A from mains. Flux/pole is 25mWb & its armature ckt resistance is 0.6Ω.

Wave connected = 1/paths - $\frac{Z}{2} = 400$

$$T_a = 0.159 \times \phi \times Z \times I_a \times \left(\frac{P}{A}\right)$$

$$= 0.159 \times 25 \times 10^{-3} \times 800 \times 45 \times \left(\frac{4}{400}\right)$$

$$T_a = 1431.$$

$$E_b = V - I_a R_a$$

$$= 220 - 45 \times 0.6$$

$$E_b = 193V$$

$$T_{sh} =$$