

# Basic Electrical Engineering: AC Circuits

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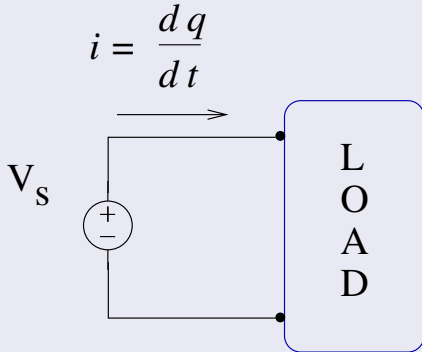
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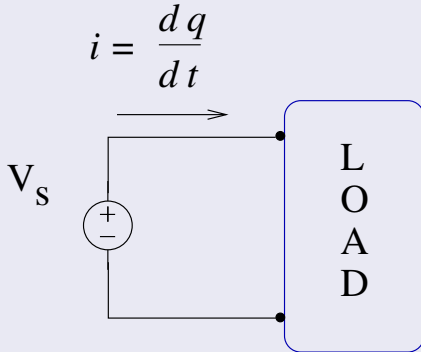
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# Introduction



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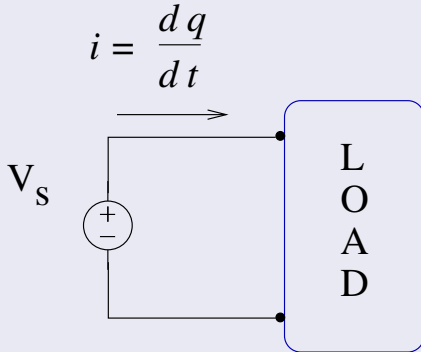
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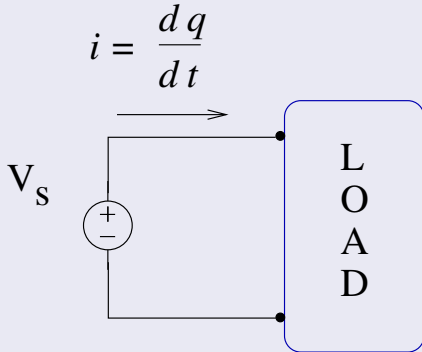


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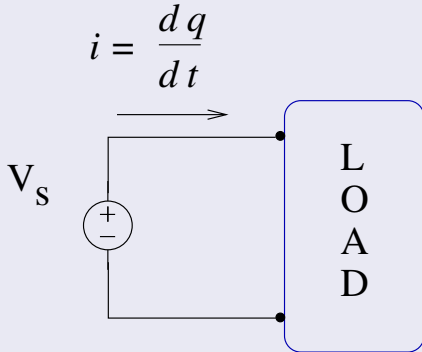
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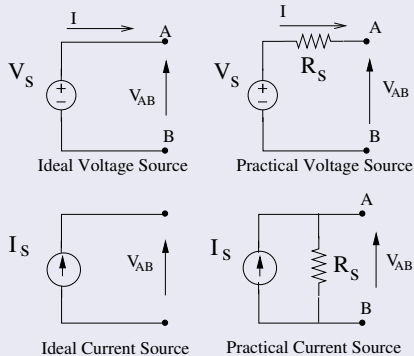


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# Voltage and Current Sources

## Independent Sources

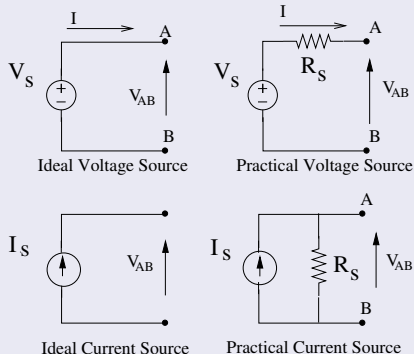


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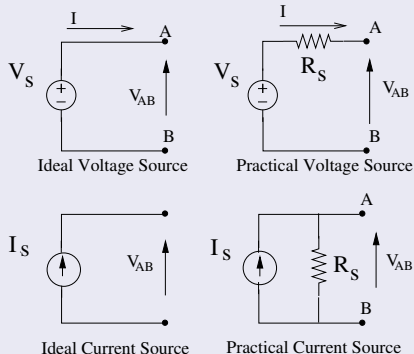


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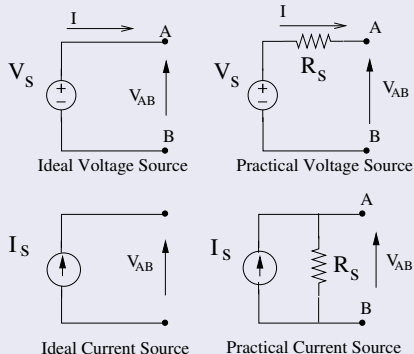


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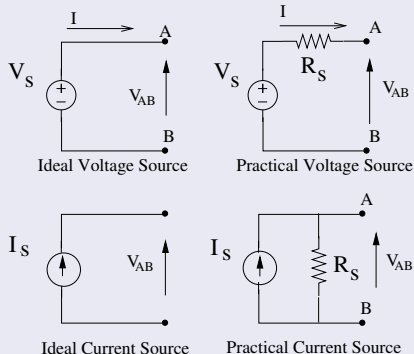


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# Electrical Quantities and their dimesions

**Current** is denoted by symbol  $I$  is the amount of charge per unit time,  $i(t) = \frac{dq(t)}{dt}$  OR  $I = \frac{Q}{t}$ , the unit is  $\frac{\text{Coulomb}}{\text{sec}}$  OR *Ampere*



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**Electric Power** (Unit: Watt):  $P = \text{Voltage} \cdot \text{Current}$   
 $V = \frac{P}{I}$        $V = \frac{\text{Work}}{\text{time}} \cdot \frac{\text{time}}{Q}$ ,  $V = \frac{W}{Q}$

**Electro Motive Force** (Unit Volt) does not have unit of force, but is a driving force to produce electric current when connected to electrical load. Typically, it is the terminal voltage of an open circuited Battery difference in Potentials between the two points. Its unit is *Volt*. The potential sources of e.m.f. are:

- ① The electrodes of dissimilar materials immersed in an electrolyte, as in primary and secondary cells, i.e. batteries.
- ② The relative movement of a conductor and a magnetic flux, as in electric generators; this source can, alternatively, be expressed as the variation of magnetic flux linked with a coil.
- ③ The difference of temperature between junctions of dissimilar metals, as in thermo-junctions.

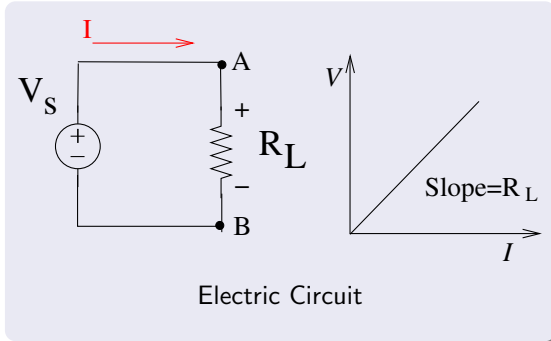
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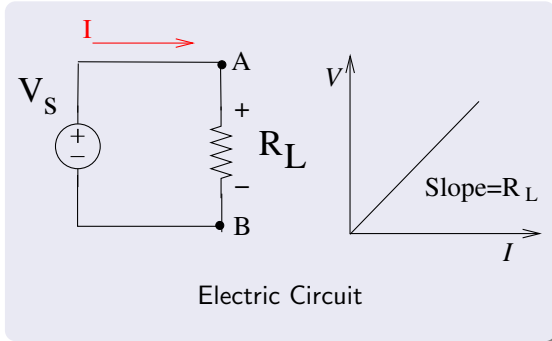
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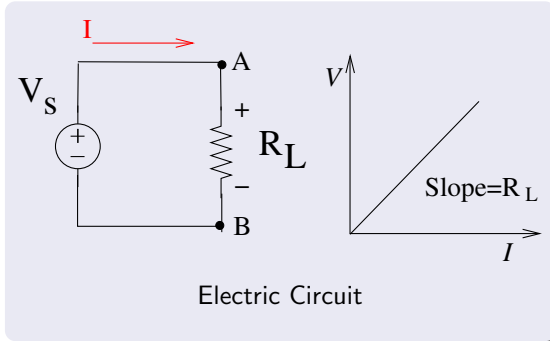
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- The resistance of a wire is expressed as  $R = \frac{\rho \cdot \text{length}}{\text{Area}}$ , where  $\rho$  is a constant for given conductor material (metal) and is called **Resistivity**, ( $\Omega \cdot \text{meters}$ ), **temperature dependent**



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**Q:** Why equivalent resistance needs to be known to analyse the network?

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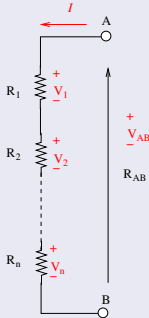
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Let  $I$  the net current flowing through the circuit. Therefore

$$R_{AB} = \frac{V_{AB}}{I} = \frac{V_1 + V_2 + \dots + V_n}{I}$$

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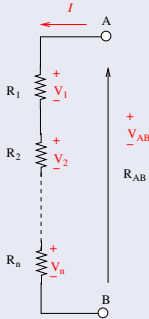


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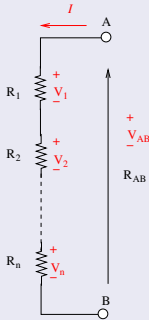
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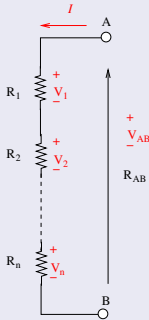
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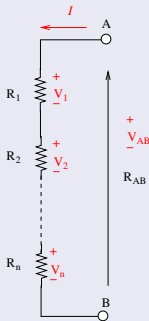


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$$V_1 = I \cdot R_1 = \frac{R_1}{R_{AB}} \cdot V_{AB}$$

$$V_2 = \frac{R_2}{R_{AB}} \cdot V_{AB} \dots V_n = \frac{R_n}{R_{AB}} \cdot V_{AB}$$

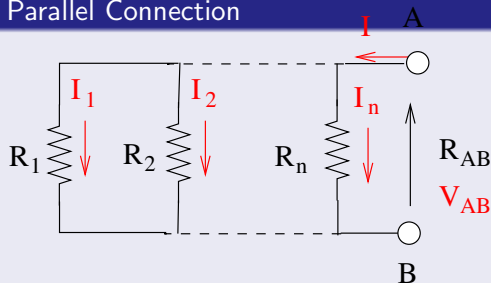
## Voltage Sharing Formula

# To find Equivalent Resistance

In Parallel connection, the Voltage across each resistor is same but current would be different. Let  $I$  the net current flowing through the circuit. Therefore, let equivalent resistance of the network is:  $R_{AB}$

$$R_{AB} = \frac{V_{AB}}{I} = \frac{V_{AB}}{I_1 + I_2 + \dots + I_n}$$

## Parallel Connection



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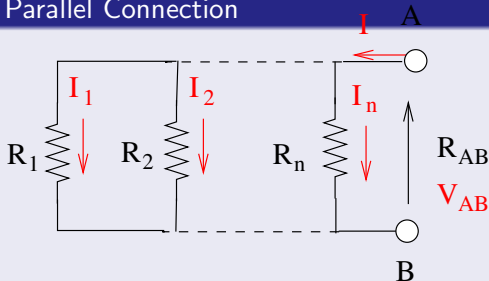
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$$R_{AB} = \frac{1}{\frac{I_1}{V_{AB}} + \frac{I_2}{V_{AB}} + \dots + \frac{I_n}{V_{AB}}}$$

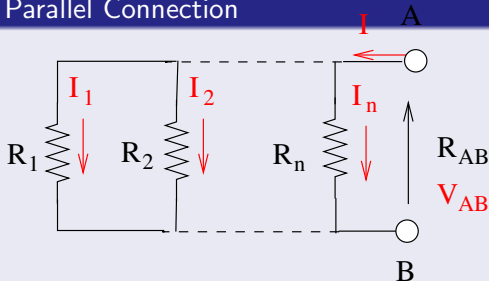
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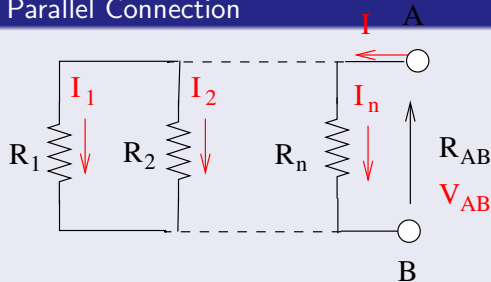
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$$R_{AB} = \frac{1}{\frac{I_1}{V_{AB}} + \frac{I_2}{V_{AB}} + \dots + \frac{I_n}{V_{AB}}}$$

$$R_{AB} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

$$I_1 = I \cdot \frac{R_{AB}}{R_1}; I_2 = I \cdot \frac{R_{AB}}{R_2}; \dots I_n = I \cdot \frac{R_{AB}}{R_n}$$

## Current Sharing Rule

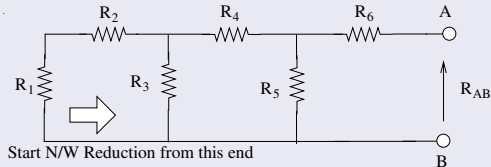
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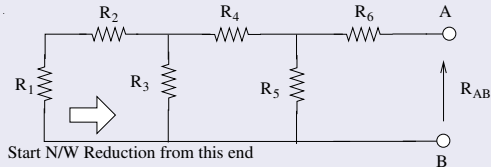


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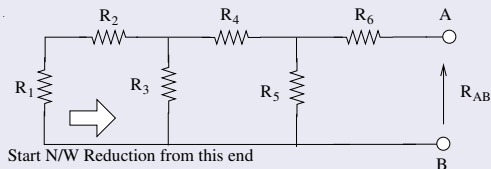




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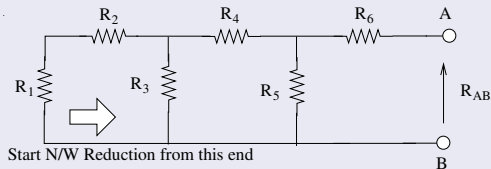


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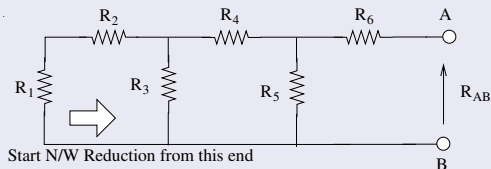


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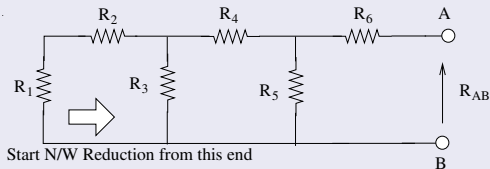


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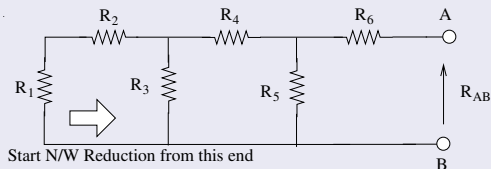


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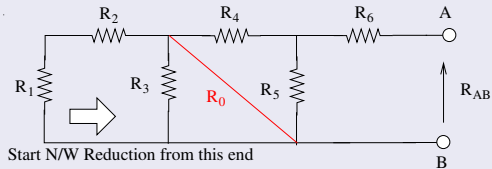
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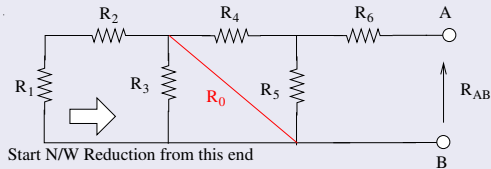


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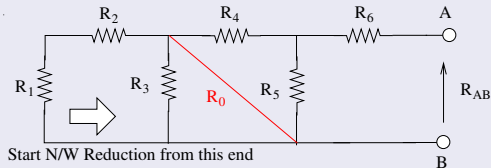




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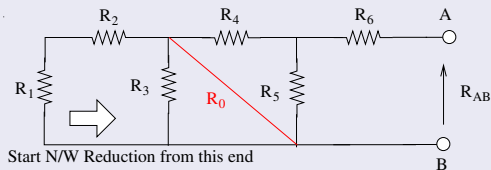


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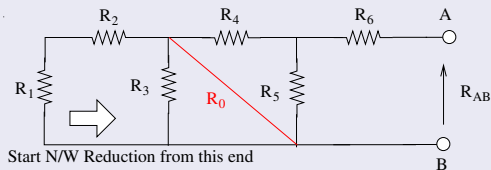


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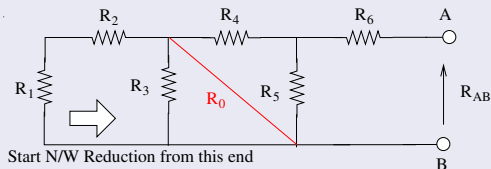


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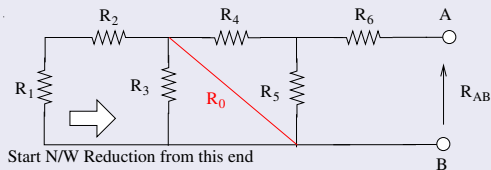


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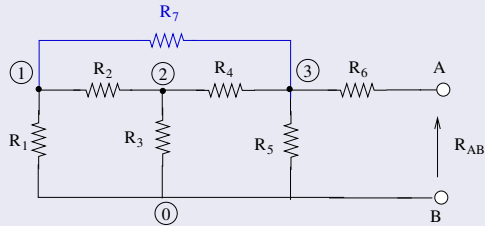
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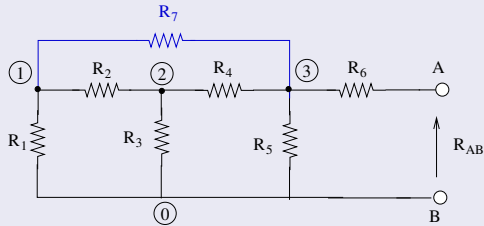


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## Need of Delta-Star Transformation



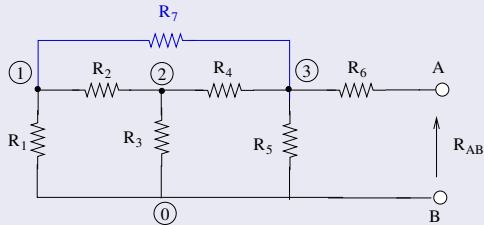


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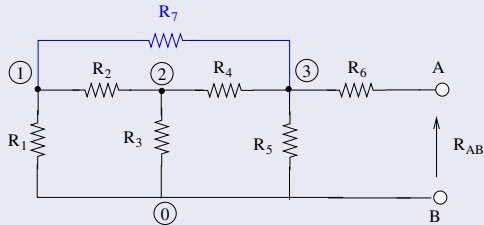
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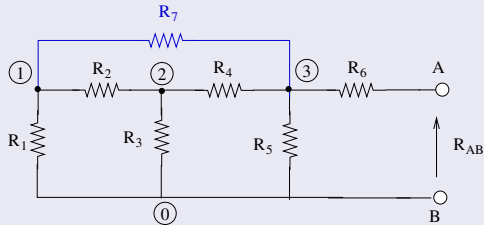


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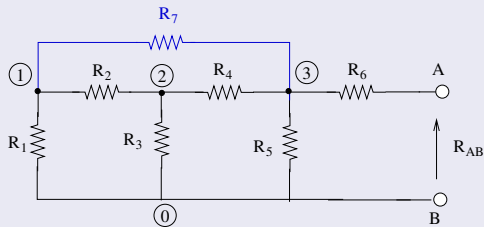


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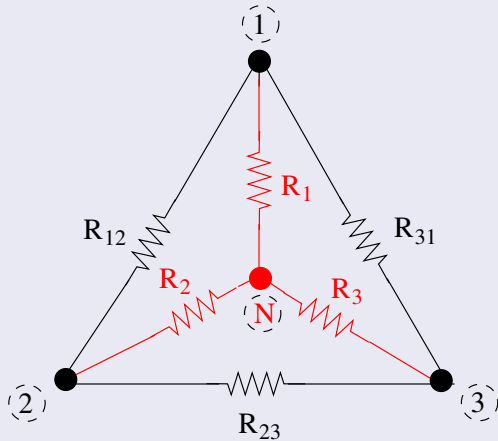
## Need of Delta-Star Transformation



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- 2 The resistors  $R_7$  can not be seen as parallel to resistor  $R_2$  OR resistor  $R_4$
- 3 Therefore, the series parallel technique of network reduction fails.
- 4 We can use **Delta-Star Transformation** or **Star-Delta Transformation**
- 5 Here, resistor  $R_7$  forms Delta (nodes 1,2 and 3) with resistor  $R_2$  and  $R_4$ . Then, converting this into equivalent Delta can solve the problem. OR Resistor  $R_2, R_3$  and  $R_4$  forms a Star (nodes 1,3 and 0), transform Star-Delta

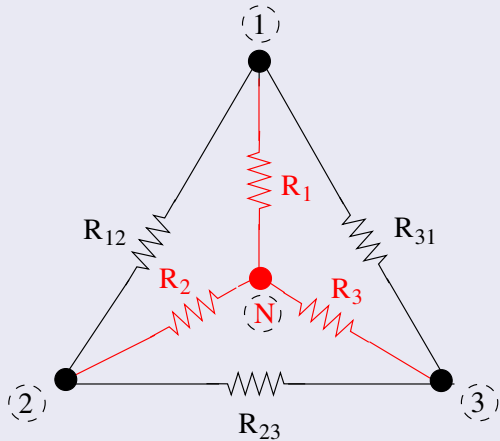
# To find Equivalent Resistance

## Delta-Star Transformation



# To find Equivalent Resistance

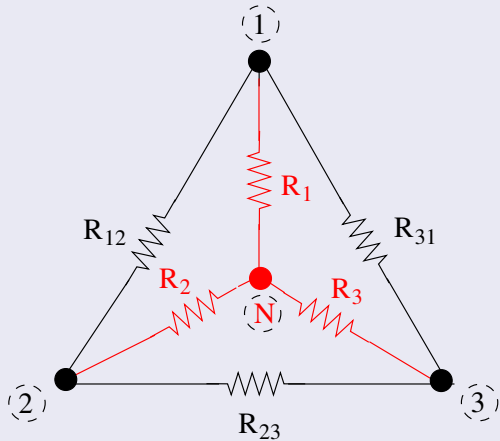
## Delta-Star Transformation



Looking at nodes 1 and 2, we write and equate equivalent resistor from both Delta and Star Network as:

# To find Equivalent Resistance

## Delta-Star Transformation

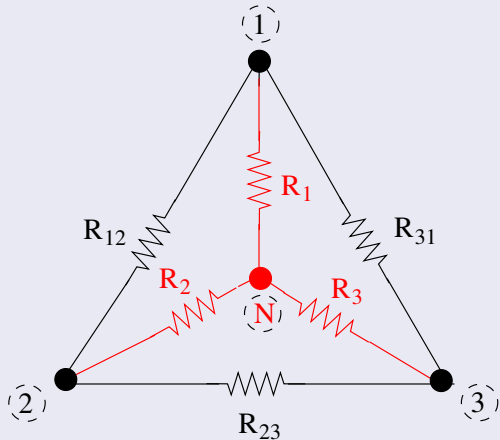


Looking at nodes 1 and 2, we write and equate equivalent resistor from both Delta and Star Network as:

$$R_1 + R_2 = \frac{R_{12} \cdot (R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})} \quad (1)$$

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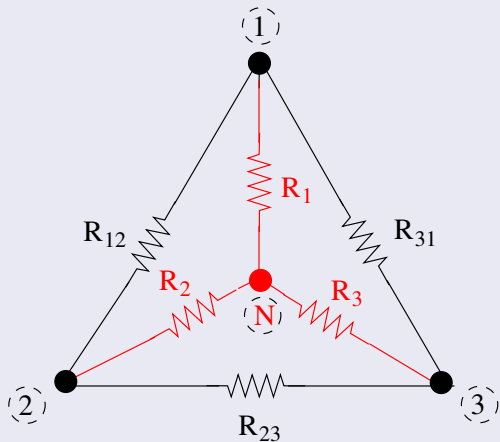
Similarly, for node pairs 2-3 and 3-1, we write:

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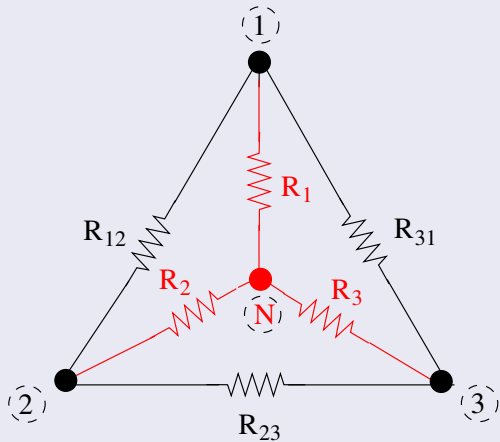
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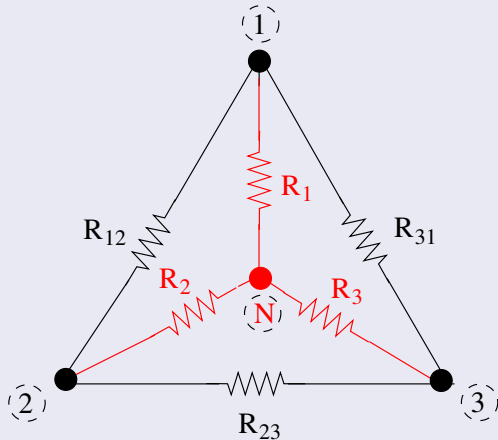
## Delta-Star Transformation



# To find Equivalent Resistance

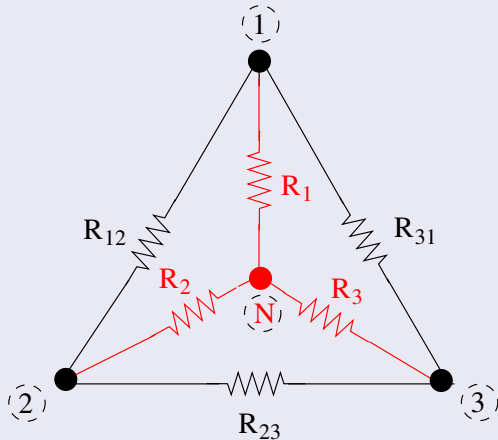
Then Perform the operation on equations:  
(1)-(2)+(3)

## Delta-Star Transformation



# To find Equivalent Resistance

## Delta-Star Transformation



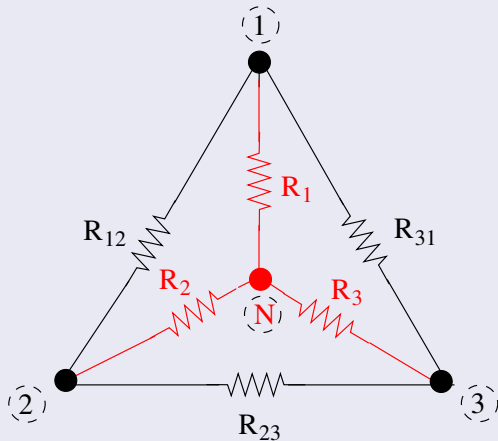
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## Delta-Star Transformation

$$R_1 = \frac{R_{12} \cdot R_{31}}{(R_{12} + R_{23} + R_{31})}$$

# To find Equivalent Resistance

## Delta-Star Transformation



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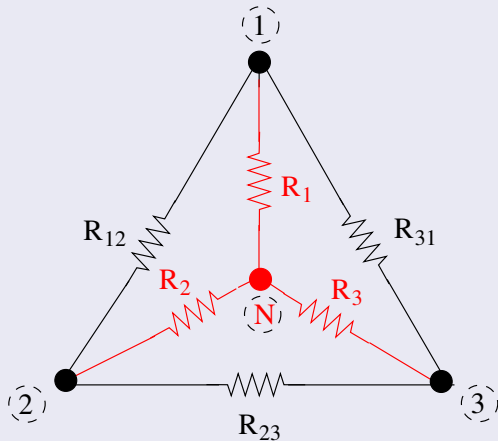
## Delta-Star Transformation

$$R_1 = \frac{R_{12} \cdot R_{31}}{(R_{12} + R_{23} + R_{31})} \quad (4)$$

Similarly, we can perform (2)-(3)+(1) and (3)-(1)+(2), we write:

# To find Equivalent Resistance

## Delta-Star Transformation



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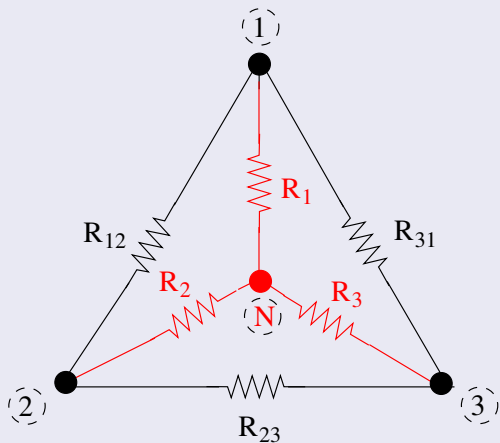
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## Delta-Star Transformation



Then Perform the operation on equations:  
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## Delta-Star Transformation

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$$R_2 = \frac{R_{12} \cdot R_{23}}{(R_{12} + R_{23} + R_{31})} \quad (5)$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{(R_{12} + R_{23} + R_{31})} \quad (6)$$

# To find Equivalent Resistance

Following operations are performed  
on equations: (4), (5) and (6)



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$$\begin{aligned} R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1 &= \\ &= \frac{(R_{12} \cdot R_{23} \cdot R_{31})}{(R_{12} + R_{23} + R_{31})} \end{aligned}$$

# To find Equivalent Resistance

Following operations are performed on equations: (4), (5) and (6)  
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Therefore substituting for  $R_3$  as the term shown with red color, we can write:

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$$R_{12} = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_3}$$

# To find Equivalent Resistance

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## Star-Delta Transformation

$$R_{12} = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_3} \quad (7)$$

Similarly,

$$R_{23} = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_1}$$

# To find Equivalent Resistance

Following operations are performed on equations: (4), (5) and (6)  
 $(4) \cdot (5) + (5) \cdot (6) + (6) \cdot (4)$

$$\begin{aligned} R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1 &= \\ &= \frac{(R_{12} \cdot R_{23} \cdot R_{31})}{(R_{12} + R_{23} + R_{31})} \end{aligned}$$

Therefore substituting for  $R_3$  as the term shown with red color, we can write:

## Star-Delta Transformation

$$R_{12} = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_3} \quad (7)$$

Similarly,

$$R_{23} = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_1} \quad (8)$$

$$R_{31} = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_2}$$

# To find Equivalent Resistance

Following operations are performed on equations: (4), (5) and (6)  
 $(4) \cdot (5) + (5) \cdot (6) + (6) \cdot (4)$

$$\begin{aligned} R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1 &= \\ &= \frac{(R_{12} \cdot R_{23} \cdot R_{31})}{(R_{12} + R_{23} + R_{31})} \end{aligned}$$

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# To find Equivalent Resistance: Example-1

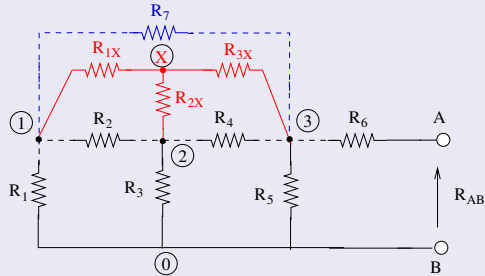
Now we shall solve this problem by existing Delta: ( $R_2, R_4$  &  $R_7$ ) connections to Star: ( $R_{1X}, R_{2X}$  &  $R_{3X}$ ) Network by Delta-Star Transformation.



# To find Equivalent Resistance: Example-1

Now we shall solve this problem by existing Delta: ( $R_2, R_4$  &  $R_7$ ) connections to Star: ( $R_{1X}, R_{2X}$  &  $R_{3X}$ ) Network by Delta-Star Transformation.

## Delta-Star Transformation

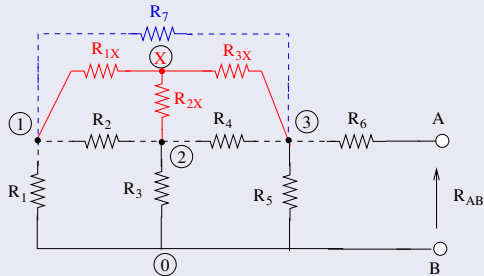


# To find Equivalent Resistance: Example-1

Now we shall solve this problem by existing Delta: ( $R_2, R_4$  &  $R_7$ ) connections to Star: ( $R_{1X}, R_{2X}$  &  $R_{3X}$ ) Network by Delta-Star Transformation.

By using equations: (4), (5) and (6)

## Delta-Star Transformation

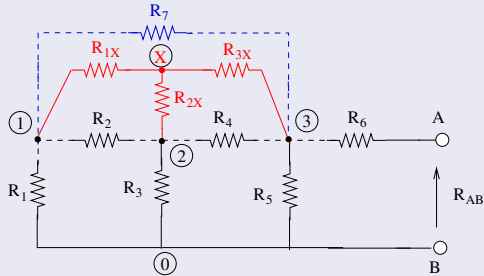


$$R_{1X} = \frac{R_2 \cdot R_7}{R_2 + R_4 + R_7}$$

# To find Equivalent Resistance: Example-1

Now we shall solve this problem by existing Delta: ( $R_2, R_4$  &  $R_7$ ) connections to Star: ( $R_{1X}, R_{2X}$  &  $R_{3X}$ ) Network by Delta-Star Transformation.

## Delta-Star Transformation



By using equations: (4), (5) and (6)

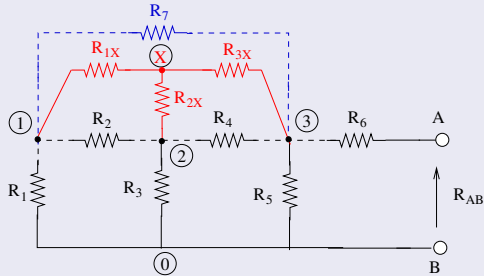
$$R_{1X} = \frac{R_2 \cdot R_7}{R_2 + R_4 + R_7}$$

$$R_{2X} = \frac{R_2 \cdot R_4}{R_2 + R_4 + R_7}$$

# To find Equivalent Resistance: Example-1

Now we shall solve this problem by existing Delta: ( $R_2, R_4$  &  $R_7$ ) connections to Star: ( $R_{1X}, R_{2X}$  &  $R_{3X}$ ) Network by Delta-Star Transformation.

## Delta-Star Transformation



By using equations: (4), (5) and (6)

$$R_{1X} = \frac{R_2 \cdot R_7}{R_2 + R_4 + R_7}$$

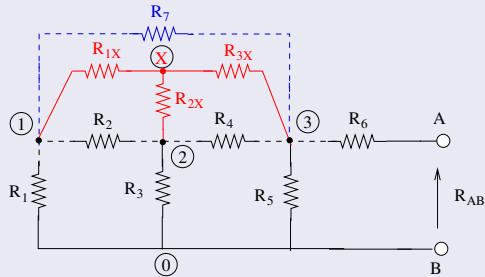
$$R_{2X} = \frac{R_2 \cdot R_4}{R_2 + R_4 + R_7}$$

$$R_{3X} = \frac{R_4 \cdot R_7}{R_2 + R_4 + R_7}$$

# To find Equivalent Resistance: Example-1

Now we shall solve this problem by existing Delta: ( $R_2, R_4$  &  $R_7$ ) connections to Star: ( $R_{1X}, R_{2X}$  &  $R_{3X}$ ) Network by Delta-Star Transformation.

## Delta-Star Transformation



By using equations: (4), (5) and (6)

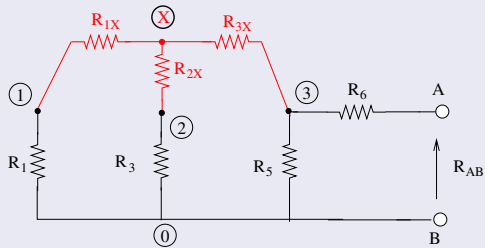
$$R_{1X} = \frac{R_2 \cdot R_7}{R_2 + R_4 + R_7}$$

$$R_{2X} = \frac{R_2 \cdot R_4}{R_2 + R_4 + R_7}$$

$$R_{3X} = \frac{R_4 \cdot R_7}{R_2 + R_4 + R_7}$$

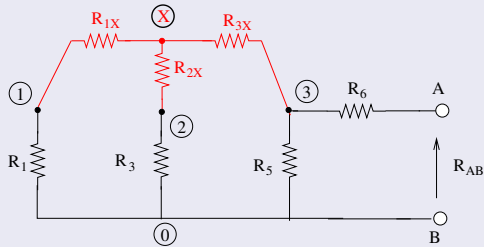
# To find Equivalent Resistance: Example-1

## Delta-Star Transformation



# To find Equivalent Resistance: Example-1

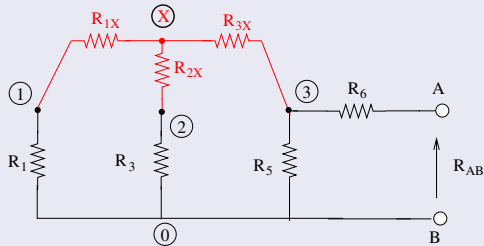
## Delta-Star Transformation



Now, we have to analyse new network and neglect the resistors ( $R_2, R_4$  &  $R_7$ ). A new node  $X$  is created.

# To find Equivalent Resistance: Example-1

## Delta-Star Transformation



Now, we have to analyse new network and neglect the resistors ( $R_2, R_4$  &  $R_7$ ). A new node  $X$  is created.

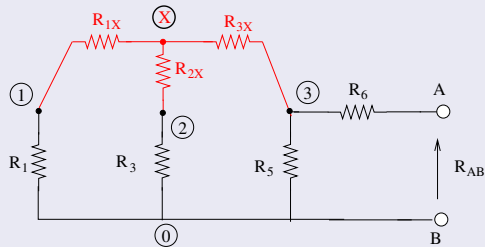
We can now use series-parallel technique

- $R_{11X} = R_1 + R_{1X}$ ,  $R_{23X} = R_3 + R_{2X}$



# To find Equivalent Resistance: Example-1

## Delta-Star Transformation



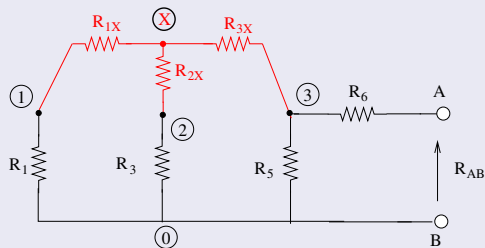
Now, we have to analyse new network and neglect the resistors ( $R_2, R_4$  &  $R_7$ ). A new node  $X$  is created.

We can now use series-parallel technique

- $R_{11X} = R_1 + R_{1X}$ ,  $R_{23X} = R_3 + R_{2X}$
- $R_{1123X} = \frac{R_{11X} \cdot R_{23X}}{R_{11X} + R_{23X}}$

# To find Equivalent Resistance: Example-1

## Delta-Star Transformation



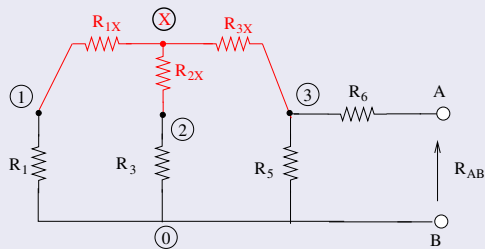
Now, we have to analyse new network and neglect the resistors ( $R_2, R_4$  &  $R_7$ ). A new node  $X$  is created.

We can now use series-parallel technique

- $R_{11X} = R_1 + R_{1X}$ ,  $R_{23X} = R_3 + R_{2X}$
- $R_{1123X} = \frac{R_{11X} \cdot R_{23X}}{R_{11X} + R_{23X}}$
- $R_{11233X} = R_{1123X} + R_{3X}$

# To find Equivalent Resistance: Example-1

## Delta-Star Transformation



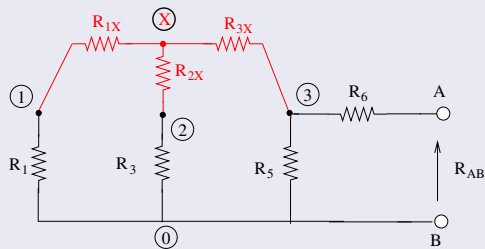
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We can now use series-parallel technique

- $R_{11X} = R_1 + R_{1X}$ ,  $R_{23X} = R_3 + R_{2X}$
- $R_{1123X} = \frac{R_{11X} \cdot R_{23X}}{R_{11X} + R_{23X}}$
- $R_{11233X} = R_{1123X} + R_{3X}$
- $R_{112335X} = \frac{R_{11233X} \cdot R_5}{R_{11233X} + R_5}$

# To find Equivalent Resistance: Example-1

## Delta-Star Transformation



Now, we have to analyse new network and neglect the resistors ( $R_2, R_4$  &  $R_7$ ). A new node  $X$  is created.

We can now use series-parallel technique

- $R_{11X} = R_1 + R_{1X}$ ,  $R_{23X} = R_3 + R_{2X}$
- $R_{1123X} = \frac{R_{11X} \cdot R_{23X}}{R_{11X} + R_{23X}}$
- $R_{11233X} = R_{1123X} + R_{3X}$
- $R_{112335X} = \frac{R_{11233X} \cdot R_5}{R_{11233X} + R_5}$
- $R_{AB} = R_{112335X} + R_6$

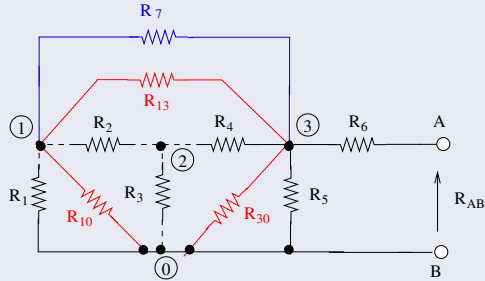
## To find Equivalent Resistance: Example-2

Now we shall solve this problem by existing Star: ( $R_2, R_3$  &  $R_4$ ) connections to new Delta: ( $R_{10}, R_{30}$  &  $R_{13}$ ) Network by Star-Delta Transformation.

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## Delta-Star Transformation

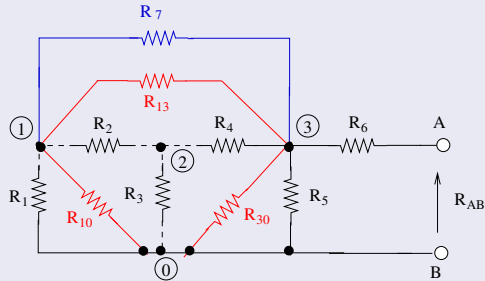


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By using equations: (7), (8) and (9)

## Delta-Star Transformation

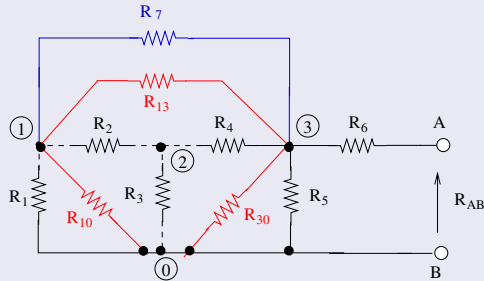


# To find Equivalent Resistance: Example-2

Now we shall solve this problem by existing Star: ( $R_2, R_3$  &  $R_4$ ) connections to new Delta: ( $R_{10}, R_{30}$  &  $R_{13}$ ) Network by Star-Delta Transformation.

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## Delta-Star Transformation



## Star-Delta Transformation

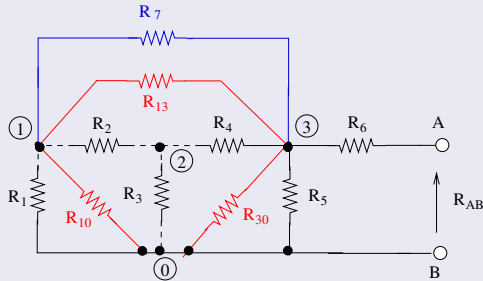
$$R_{10} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_4}$$



# To find Equivalent Resistance: Example-2

Now we shall solve this problem by existing Star: ( $R_2, R_3$  &  $R_4$ ) connections to new Delta: ( $R_{10}, R_{30}$  &  $R_{13}$ ) Network by Star-Delta Transformation.

## Delta-Star Transformation



By using equations: (7), (8) and (9)

## Star-Delta Transformation

$$R_{10} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_4} \quad (10)$$

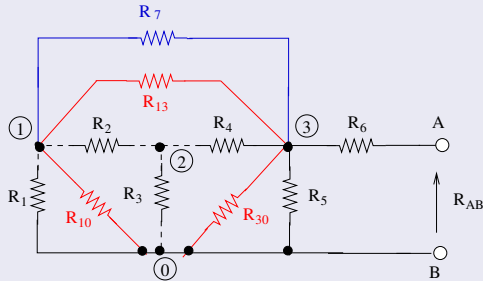
Similarly,

$$R_{30} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_2}$$

# To find Equivalent Resistance: Example-2

Now we shall solve this problem by existing Star: ( $R_2, R_3$  &  $R_4$ ) connections to new Delta: ( $R_{10}, R_{30}$  &  $R_{13}$ ) Network by Star-Delta Transformation.

## Delta-Star Transformation



By using equations: (7), (8) and (9)

## Star-Delta Transformation

$$R_{10} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_4} \quad (10)$$

Similarly,

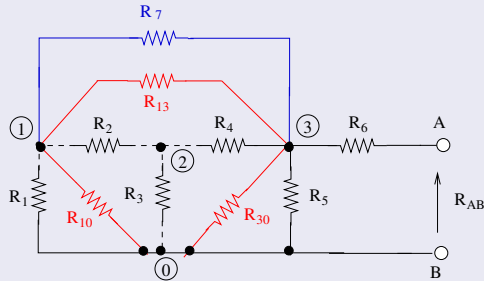
$$R_{30} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_2} \quad (11)$$

$$R_{13} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_3}$$

# To find Equivalent Resistance: Example-2

Now we shall solve this problem by existing Star: ( $R_2, R_3$  &  $R_4$ ) connections to new Delta: ( $R_{10}, R_{30}$  &  $R_{13}$ ) Network by Star-Delta Transformation.

## Delta-Star Transformation



By using equations: (7), (8) and (9)

## Star-Delta Transformation

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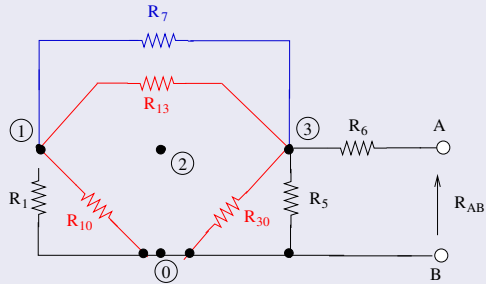
Similarly,

$$R_{30} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_2} \quad (11)$$

$$R_{13} = \frac{R_2 \cdot R_3 + R_3 \cdot R_4 + R_2 \cdot R_4}{R_3} \quad (12)$$

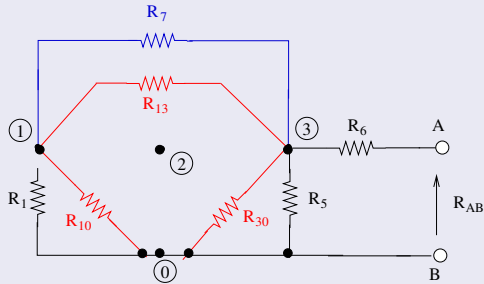
# To find Equivalent Resistance: Example-2

## Star-Delta Transformation



# To find Equivalent Resistance: Example-2

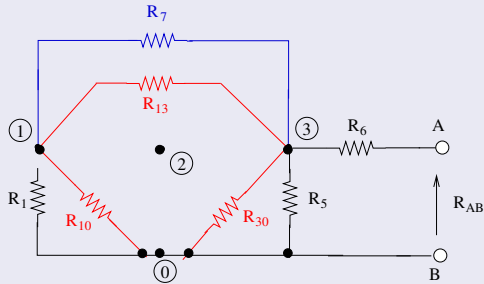
## Star-Delta Transformation



Now, we have to analyse new network and neglect the resistors ( $R_2, R_3$  &  $R_4$ ). The node (2) will vanish now in the new network.

# To find Equivalent Resistance: Example-2

## Star-Delta Transformation



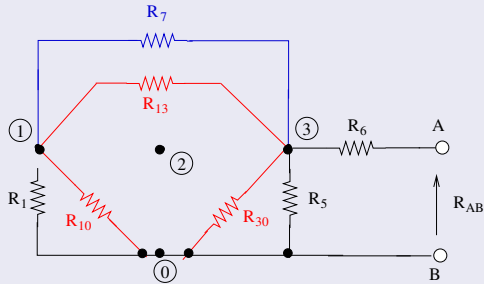
Now, we have to analyse new network and neglect the resistors ( $R_2, R_3$  &  $R_4$ ). The node (2) will vanish now in the new network.

We can now use series-parallel technique

- $R_{137} = \frac{R_{13} \cdot R_7}{R_{13} + R_7}$

# To find Equivalent Resistance: Example-2

## Star-Delta Transformation



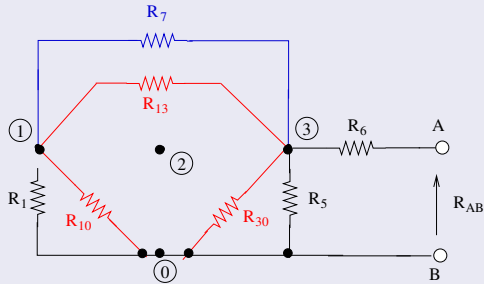
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- $R_{137} = \frac{R_{13} \cdot R_7}{R_{13} + R_7}$
- $R_{101} = \frac{R_{10} \cdot R_1}{R_{10} + R_1}$

# To find Equivalent Resistance: Example-2

## Star-Delta Transformation



Now, we have to analyse new network and neglect the resistors ( $R_2, R_3$  &  $R_4$ ). The node (2) will vanish now in the new network.

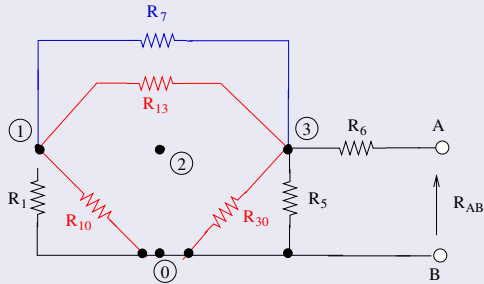
We can now use series-parallel technique

- $R_{137} = \frac{R_{13} \cdot R_7}{R_{13} + R_7}$
- $R_{101} = \frac{R_{10} \cdot R_1}{R_{10} + R_1}$
- $R_{305} = \frac{R_{30} \cdot R_5}{R_{30} + R_5}$



# To find Equivalent Resistance: Example-2

## Star-Delta Transformation



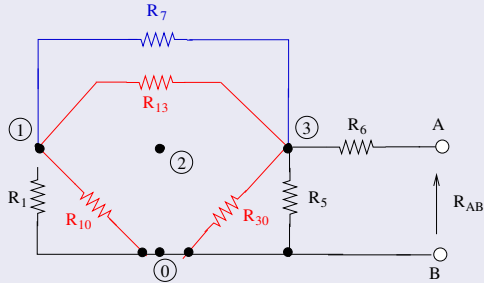
Now, we have to analyse new network and neglect the resistors ( $R_2, R_3$  &  $R_4$ ). The node (2) will vanish now in the new network.

We can now use series-parallel technique

- $R_{137} = \frac{R_{13} \cdot R_7}{R_{13} + R_7}$
- $R_{101} = \frac{R_{10} \cdot R_1}{R_{10} + R_1}$
- $R_{305} = \frac{R_{30} \cdot R_5}{R_{30} + R_5}$
- $R_{137101} = R_{137} + R_{101}$

# To find Equivalent Resistance: Example-2

## Star-Delta Transformation



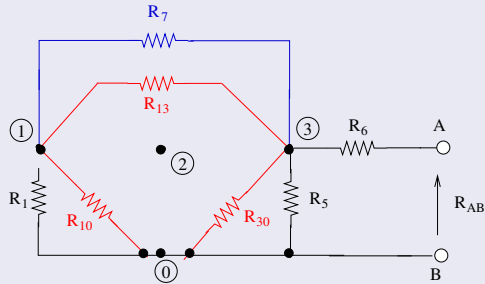
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- $R_{137} = \frac{R_{13} \cdot R_7}{R_{13} + R_7}$
- $R_{101} = \frac{R_{10} \cdot R_1}{R_{10} + R_1}$
- $R_{305} = \frac{R_{30} \cdot R_5}{R_{30} + R_5}$
- $R_{137101} = R_{137} + R_{101}$
- $R_{137101305} = \frac{R_{137101} \cdot R_{305}}{R_{137101} + R_{305}}$

# To find Equivalent Resistance: Example-2

## Star-Delta Transformation



Now, we have to analyse new network and neglect the resistors ( $R_2, R_3$  &  $R_4$ ). The node (2) will vanish now in the new network.

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- $R_{137} = \frac{R_{13} \cdot R_7}{R_{13} + R_7}$
- $R_{101} = \frac{R_{10} \cdot R_1}{R_{10} + R_1}$
- $R_{305} = \frac{R_{30} \cdot R_5}{R_{30} + R_5}$
- $R_{137101} = R_{137} + R_{101}$
- $R_{137101305} = \frac{R_{137101} \cdot R_{305}}{R_{137101} + R_{305}}$
- $R_{AB} = R_{137101305} + R_6$

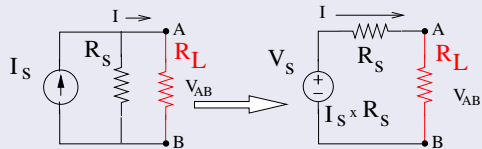
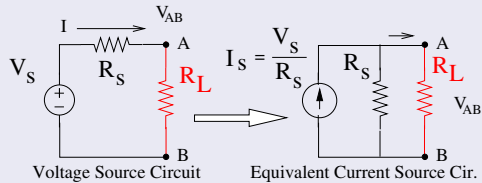
# Circuit Analysis by Source Transformation

**Voltage to Current Source Transformation:** Let us consider a primary circuit as Voltage source with voltage  $V_s$  and having a series resistor having value  $R_s$ .

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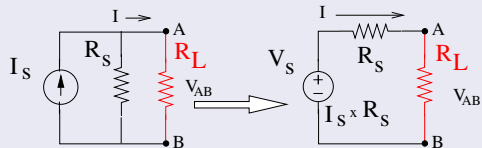
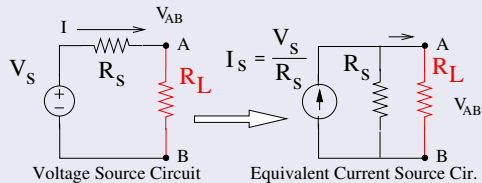
## Source Transformation



# Circuit Analysis by Source Transformation

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## Source Transformation

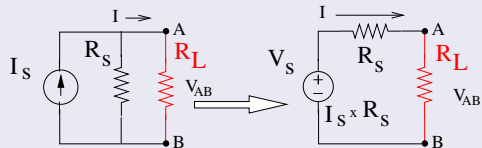
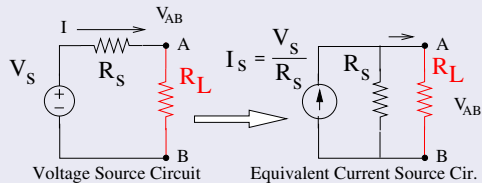


Let us assume that this voltage source circuit can be transformed to an equivalent current source circuit with a parallel resistor of value  $R_S$ . Due to equivalence, the terminal current in both the circuit should be equal.

# Circuit Analysis by Source Transformation

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## Source Transformation



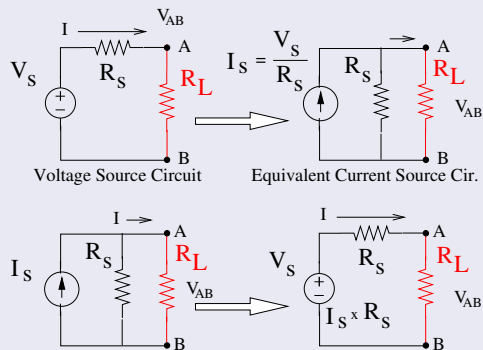
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Let an external load resistor  $R_L$  be connected at the terminal A-B of both the circuits.

# Circuit Analysis by Source Transformation

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## Source Transformation



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Let an external load resistor  $R_L$  be connected at the terminal A-B of both the circuits. If we equate the current produced by both the circuits in the load resistor, we get:

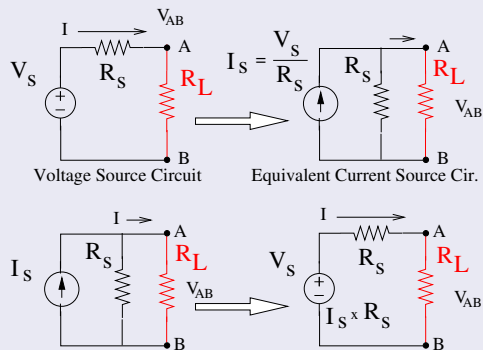
$$\frac{V_s}{R_s + R_L} = I_s \cdot \frac{R_s}{R_s + R_L}$$



# Circuit Analysis by Source Transformation

**Voltage to Current Source Transformation:** Let us consider a primary circuit as Voltage source with voltage  $V_s$  and having a series resistor having value  $R_s$ .

## Source Transformation



Let us assume that this voltage source circuit can be transformed to an equivalent current source circuit with a parallel resistor of value  $R_s$ . Due to equivalence, the terminal current in both the circuit should be equal.

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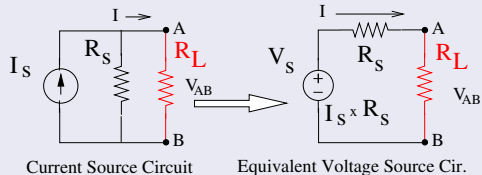
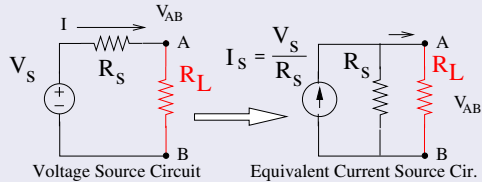
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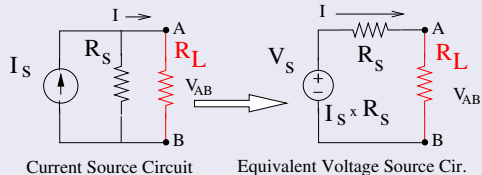
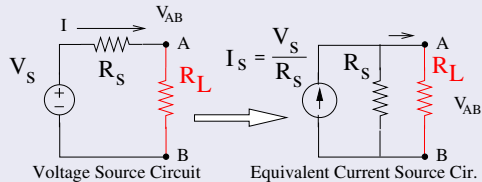
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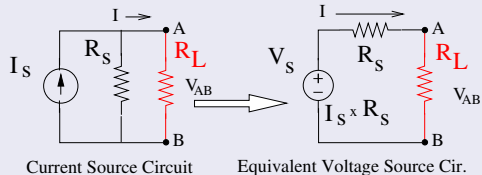
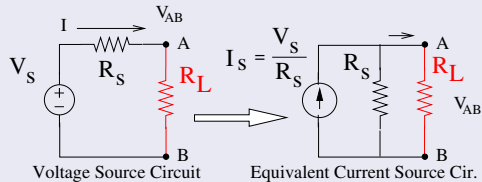


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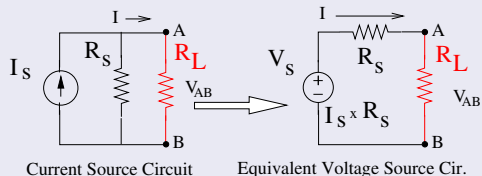
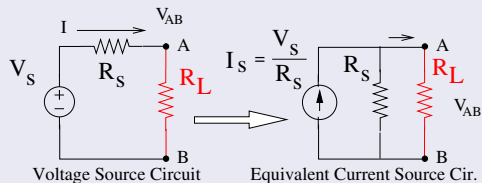
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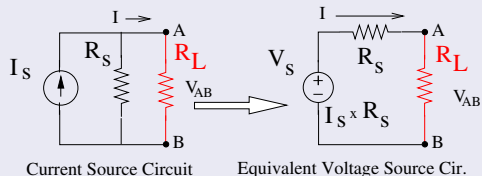
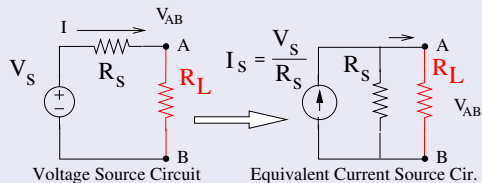
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# Circuit Analysis by Source Transformation: Example

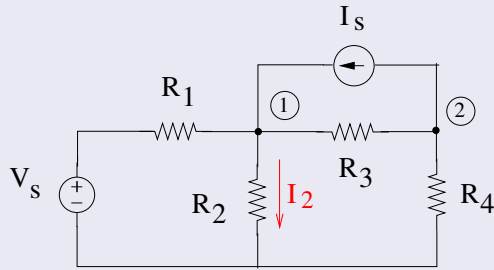
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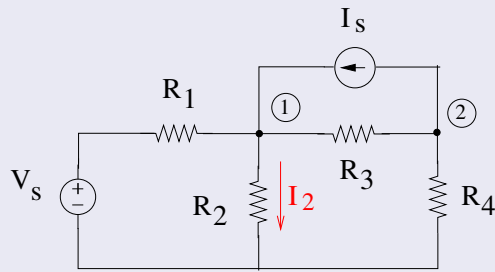
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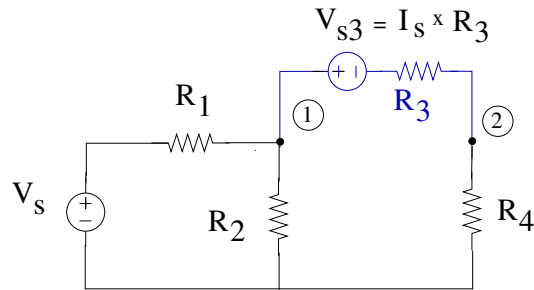
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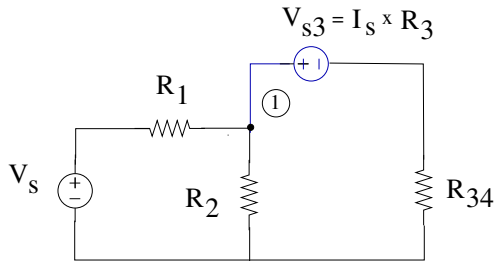


Now let us transform the current source  $I_s$  along with parallel resistor  $R_3$  to an equivalent voltage source  $V_{s3}$  and a series resistor  $R_3$ . We can write  $V_{s3} = I_s \cdot R_3$ .



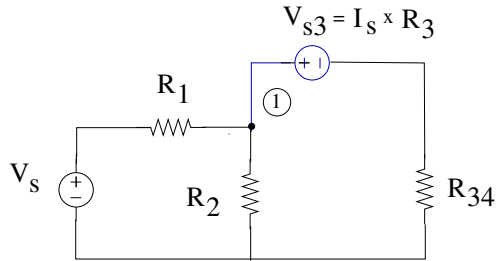
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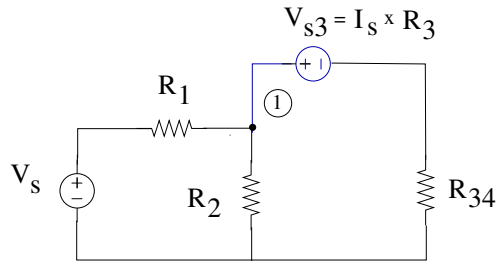
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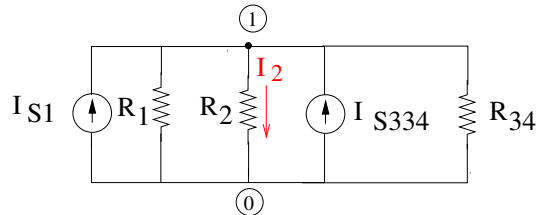
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Now transform the voltage source  $V_{s3}$  alongwith series resistor  $R_{34}$ , to a current source  $I_{s334} = \frac{V_{s3}}{R_{34}}$  with a parallel resistor  $R_{34}$ .

Now, transform voltage source  $V_s$  into an equivalent current source  $I_{s1}$  with a parallel resistor  $R_1$ .

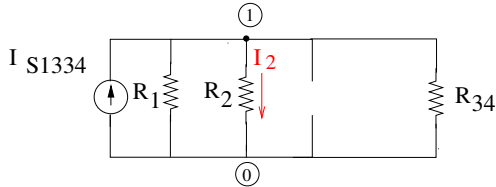


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Since, both current sources are connected between same pair of nodes and they are in the same direction, we can get resultant current as  $I_{S1334} = I_{s1} + I_{s334}$ .

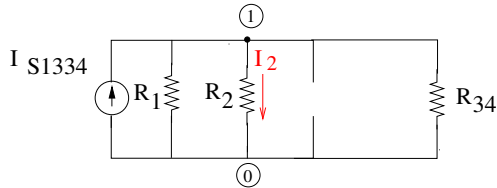
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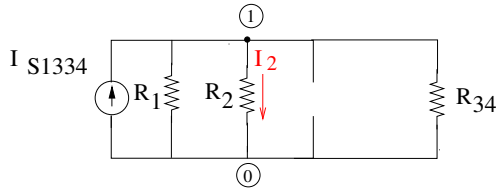


Calculating total equivalent parallel resistor across  $R_2$  as:  $R_{134} = \frac{R_1 \cdot R_{34}}{R_1 + R_{34}}$



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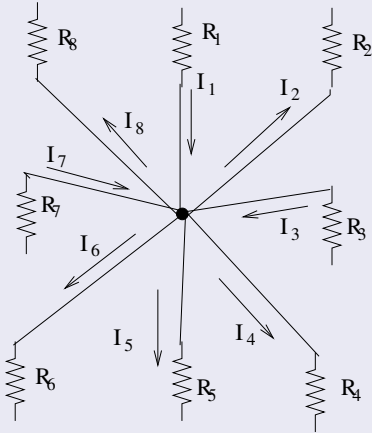
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Finally, the current  $I_2$  can be computed from following equation:

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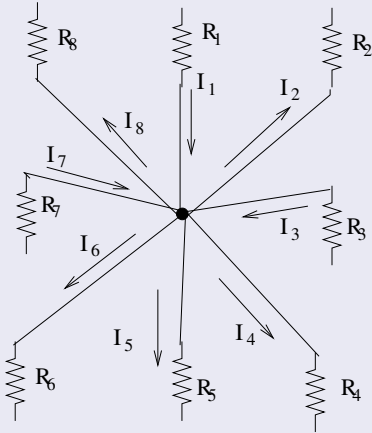
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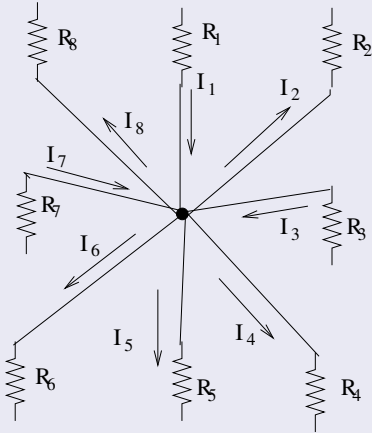
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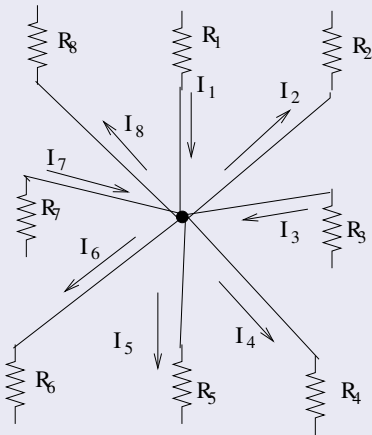


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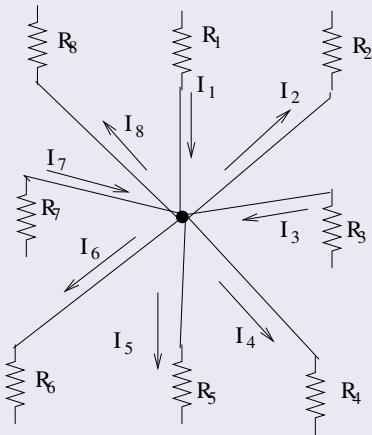
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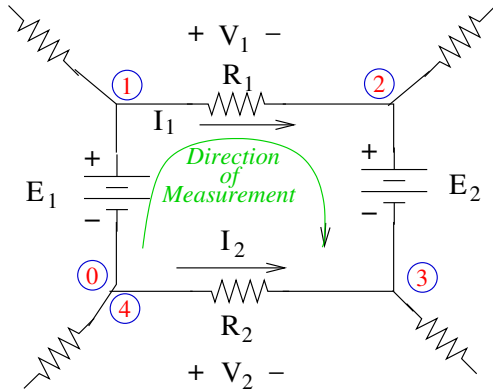
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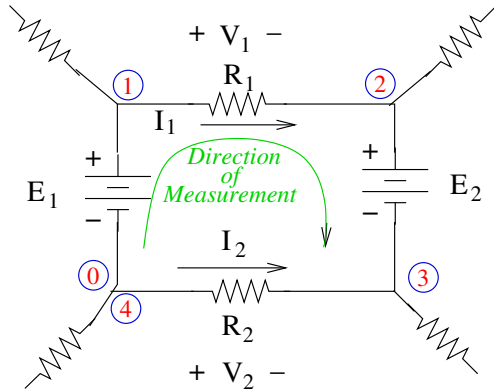
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**Sign-Convention** Entering current is positive (+ve) and leaving current is negative (-ve).

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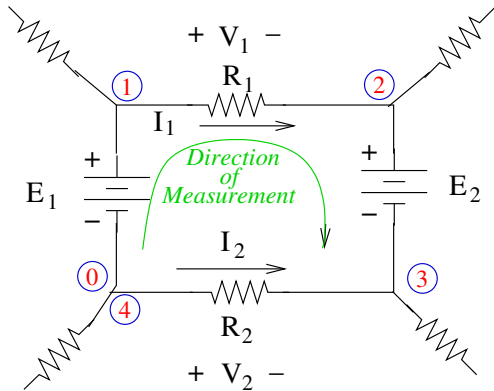
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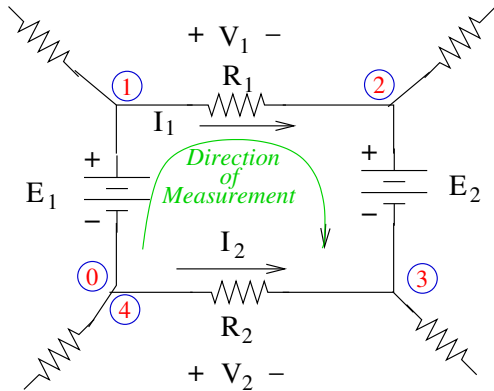
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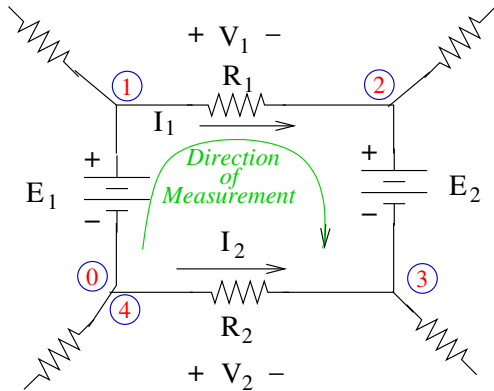
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**Sign-Convention** In the direction of measurement, if a voltage is meared from -ve polarity to +ve polarity, considered as (+ve) and vice-versa (as negative (-ve)).

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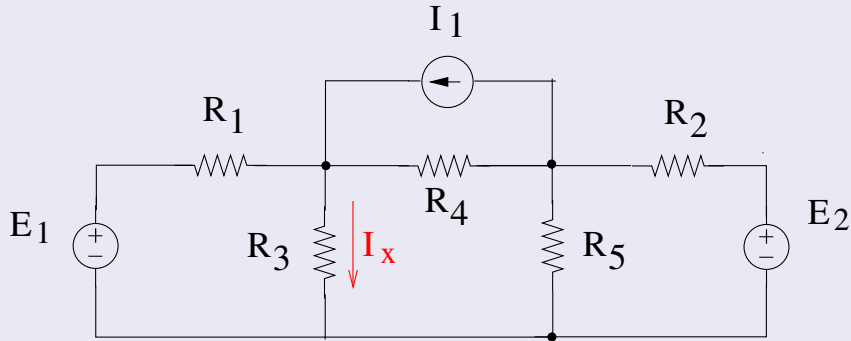
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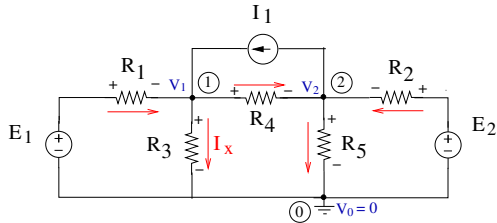
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**Example:** To find the current  $I_x$  as shown in the figure below by Nodal Analysis.



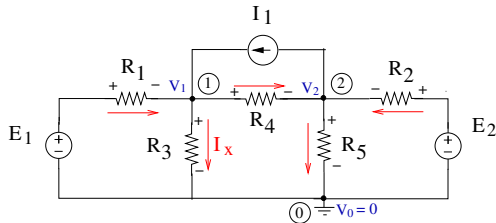
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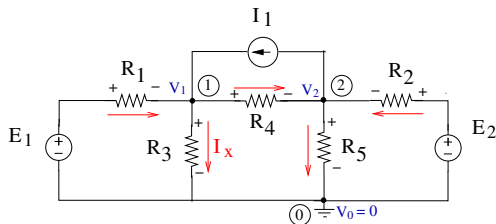
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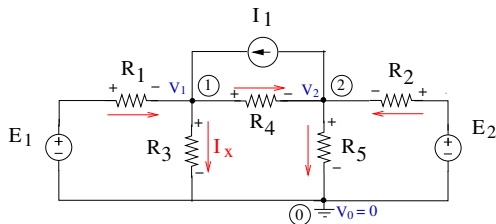
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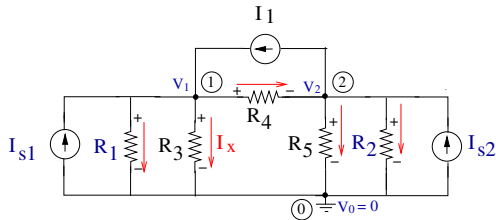


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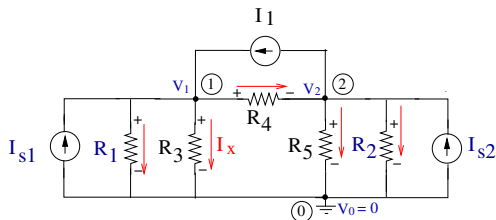
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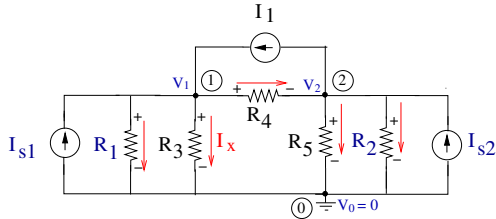
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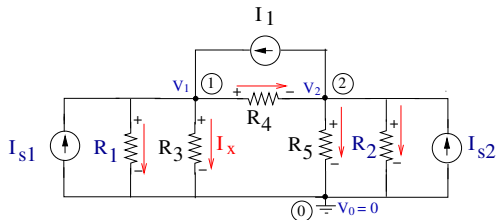
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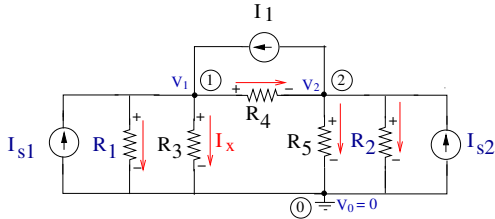
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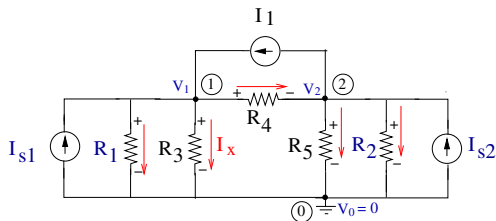
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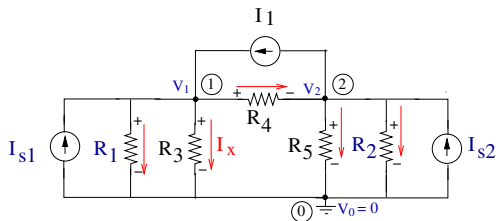
Applying KCL at Node-1 will results in equation (13) and KCL at Node-2 will results in equation (14).

$$I_{s1} - \frac{V_1}{R_1} - \frac{V_1}{R_3} - \frac{V_1 - V_2}{R_4} + I_1 = 0 \quad (13)$$

$$I_{s2} - \frac{V_2}{R_2} - \frac{V_2}{R_5} + \frac{V_1 - V_2}{R_4} - I_1 = 0 \quad (14)$$

# Example: Nodal Analysis

## Example



- 1 Write KCL equations for each independent node voltages. Form simultaneous linear equations in terms of node voltages as unknown quantities.

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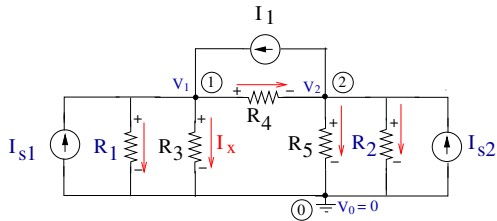
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Please note that here, only unknown quantities are node voltages  $V_1$  and  $V_2$ .

# Example: Nodal Analysis

## Example

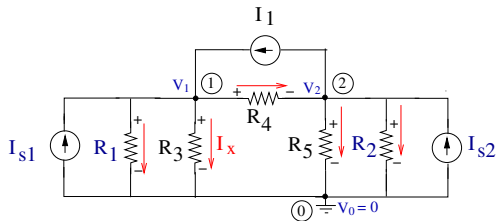


- 1 Solving equations (13) and (14), simultaneously, we can find the unknown node voltages  $V_1$  and  $V_2$ .



# Example: Nodal Analysis

## Example



- 1 Solving equations (13) and (14), simultaneously, we can find the unknown node voltages  $V_1$  and  $V_2$ .
- 2 Finally we can find current through any branch of the network. However, we can find current  $I_x = \frac{V_1}{R_3}$  since it is specifically asked in the exercise.

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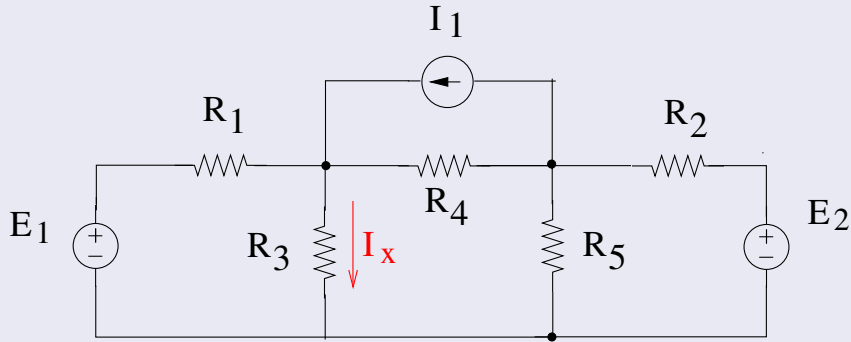
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# Example: Mesh Analysis

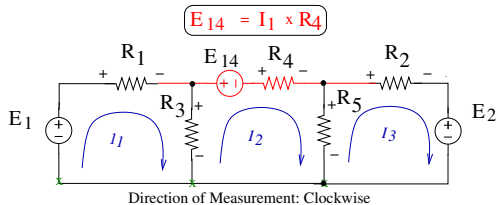
## Example

**Example:** To find the current  $I_x$  as shown in the figure below by Mesh Analysis.



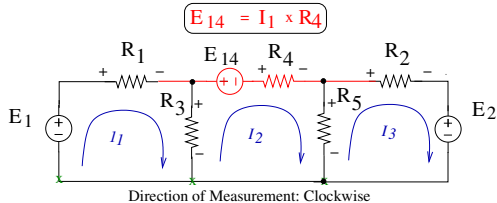
# Example: Mesh Analysis

## Example



# Example: Mesh Analysis

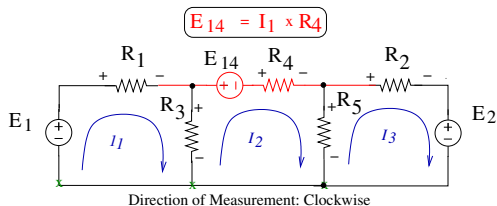
## Example



- 1 The circuit has three independent meshes/windows (1-3).

# Example: Mesh Analysis

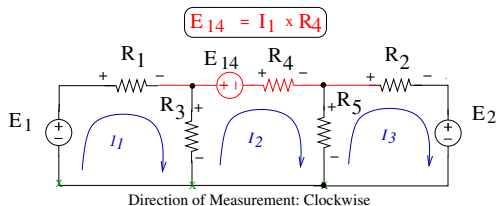
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- 1 The circuit has three independent meshes/windows (1-3).
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# Example: Mesh Analysis

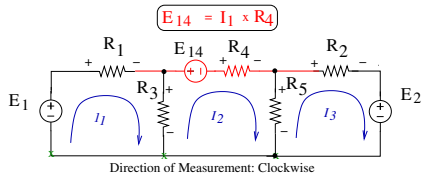
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- 2 We assign  $I_1, I_2$  and  $I_3$  as mesh currents corresponding to these meshes and assume that  $I_1 > I_2 > I_3$  as inequalities between them.
- 3 We then transform the current source  $I_1$  to the equivalent voltage source  $E_{14} = I_1 \cdot R_4$
- 4 Now mark the polarity voltage drop across each resistor depending on the direction of the mesh current

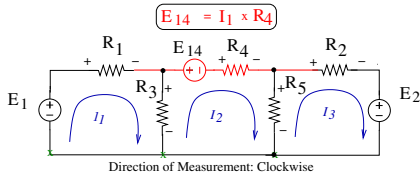
# Example: Mesh Analysis

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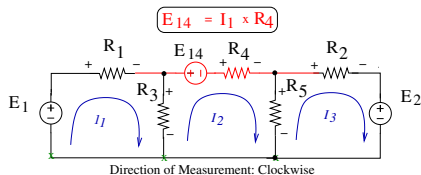


- Now, start writing KVL equation for each mesh by assuming direction of, the resulting equations for Mesh-1 to Mesh-3 are given below, sequentially.



# Example: Mesh Analysis

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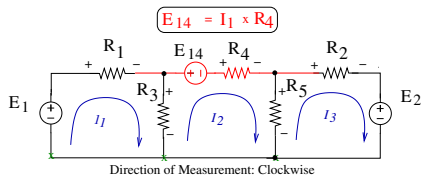


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$$\begin{aligned} E_1 - I_1 \cdot R_1 - (I_1 - I_2) \cdot R_3 &= 0 \\ (I_1 - I_2) \cdot R_3 - E_{14} - I_2 \cdot R_4 - (I_2 - I_3) \cdot R_5 &= 0 \\ (I_2 - I_3) \cdot R_5 - I_3 \cdot R_2 - E_2 &= 0 \end{aligned}$$

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Now, by solving above three equations, simultaneously, the three unknown mesh currents can be computed. The current  $I_x$  can be computed by solving  $(I_1 - I_2)$ .

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- ④ **Deactivation of Voltage Source:** For Voltage source, replace it by short wire with zero resistance

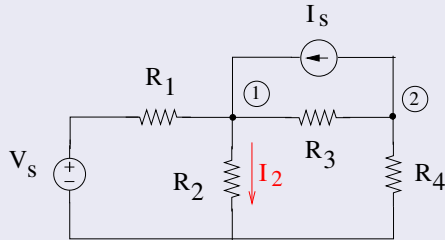
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- 4 **Deactivation of Voltage Source:** For Voltage source, replace it by short wire with zero resistance
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# Superposition Theorem

**Example-1:** To find current  $I_2$  in the circuit below

## Superposition Theorem: Example



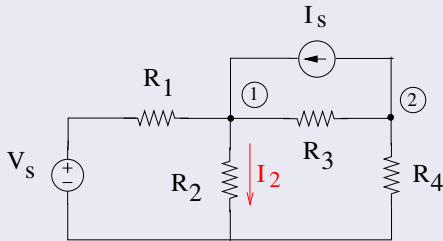
$$I_2 = I_{2X} + I_{2Y}$$



# Superposition Theorem

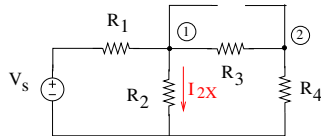
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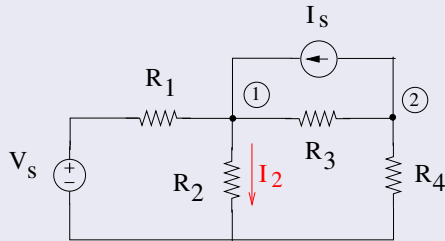
**Considering Voltage source  $V_s$  only**



# Superposition Theorem

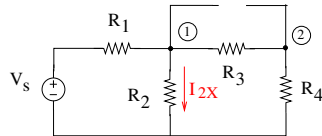
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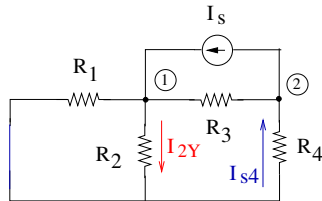


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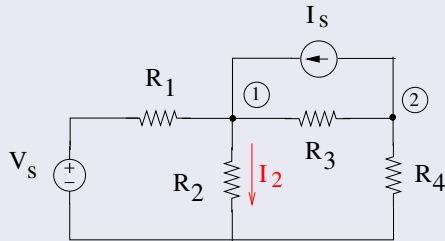
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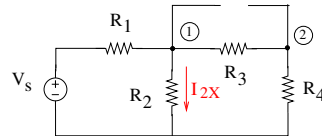
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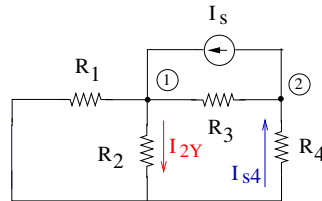


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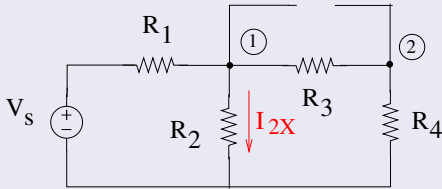
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# Superposition Theorem

**Circuit-1:** To compute current  $I_{2X}$  in the circuit below

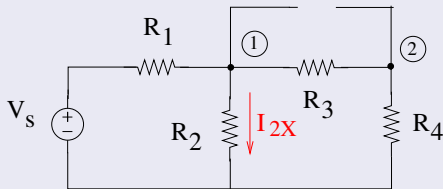
## Superposition Theorem: Example



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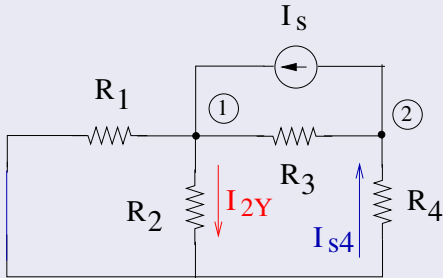
**Considering Current  $V_s$  only**  
**Compute  $I_{2X}$ :**

$$R_{234} = \frac{R_2 \cdot (R_3 + R_4)}{R_2 + R_3 + R_4}$$
$$I_{2X} = \frac{1}{R_2} \cdot V_s \cdot \frac{R_{234}}{R_1 + R_{234}}$$

# Superposition Theorem

**Circuit-2:** To compute current  $I_{2Y}$  in the circuit below

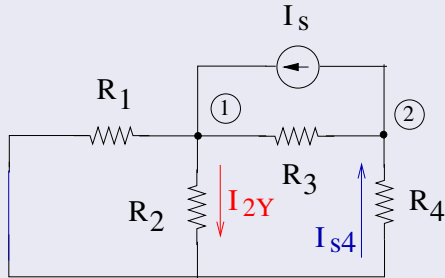
## Superposition Theorem: Example



# Superposition Theorem

**Circuit-2:** To compute current  $I_{2Y}$  in the circuit below

## Superposition Theorem: Example



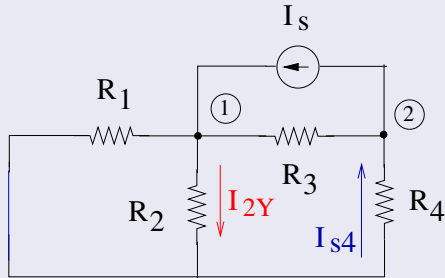
**Considering Current  $I_s$  only**  
**Compute  $I_{2X}$ :**

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

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**Circuit-2:** To compute current  $I_{2Y}$  in the circuit below

## Superposition Theorem: Example



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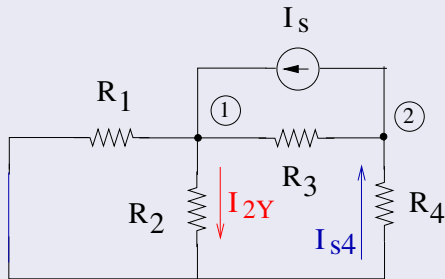
$$R_{124} = R_{12} + R_4$$



# Superposition Theorem

**Circuit-2:** To compute current  $I_{2Y}$  in the circuit below

## Superposition Theorem: Example



**Considering Current  $I_s$  only**  
**Compute  $I_{s4}$ :**

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

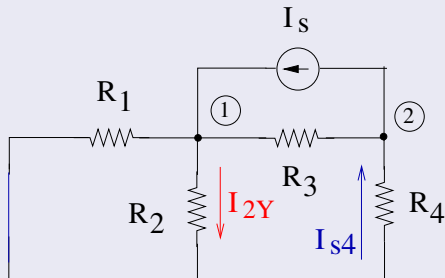
$$R_{124} = R_{12} + R_4$$

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**Circuit-2:** To compute current  $I_{2Y}$  in the circuit below

## Superposition Theorem: Example



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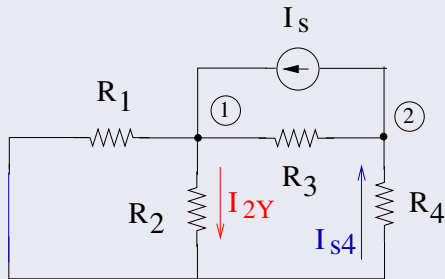
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Finally, By superposition theorem,

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**Circuit-2:** To compute current  $I_{2Y}$  in the circuit below

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**Considering Current  $I_s$  only**  
**Compute  $I_{2X}$ :**

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$$R_{124} = R_{12} + R_4$$

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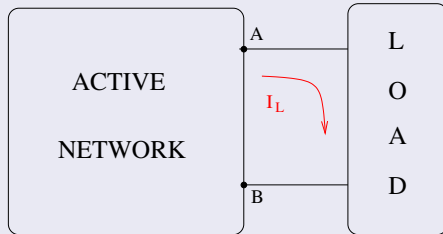
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# Thevenin's and Norton's Theorem

## Generalised Active Electrical Network

An active network consists of one or more energy sources. A load branch may be passive circuit branch or active branch. Both Thevenin and Norton Theorem are applied to simplify and evaluate the given network.

### Generalised Network

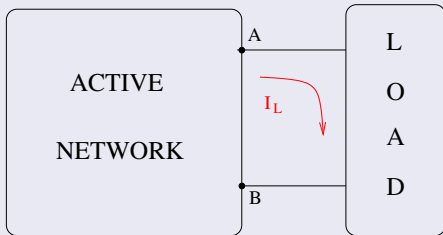


# Thevenin's and Norton's Theorem

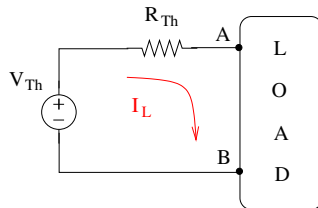
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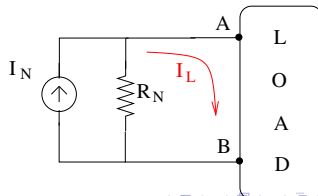
## Generalised Network



## Thevenin's Equivalent Circuit



## Norton's Equivalent Circuit



# Thevenin's and Norton's Theorem

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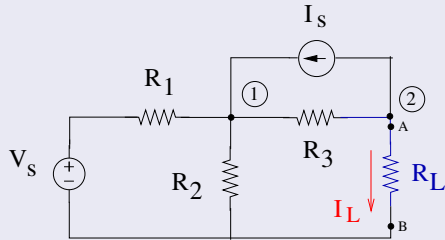
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- ③ The Norton's equivalent resistance  $R_N$  has same meaning and value as that of  $R_{Th}$ .
- ④ The Norton's equivalent Current source  $I_N$  means the short-circuit current appearing at A and B terminals of the active network (AN), when load branch is removed from the original AN.

# Thevenin's and Norton's Theorem

**Circuit-1:** To compute current  $I_L$  in the circuit below

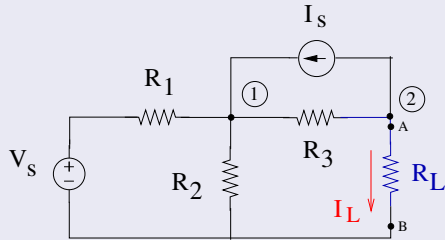
## Sample Example



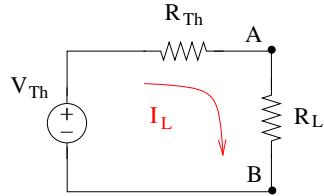
# Thevenin's and Norton's Theorem

**Circuit-1:** To compute current  $I_L$  in the circuit below

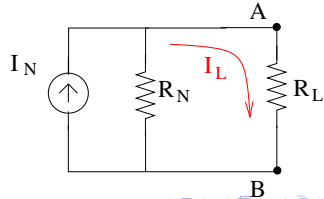
## Sample Example



## Thevenin's Equivalent Circuit



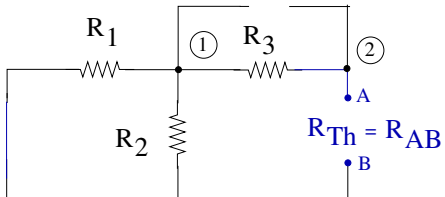
## Norton's Equivalent Circuit



# Thevenin's Theorem

## Sample Example:1

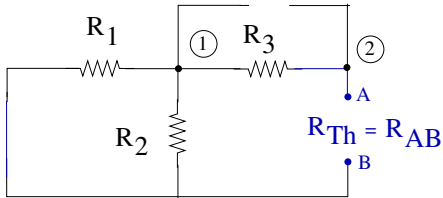
To find  $R_{Th}$ , Thevenin's Resistance



# Thevenin's Theorem

## Sample Example:1

To find  $R_{Th}$ , Thevenin's Resistance

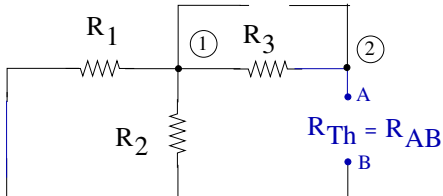


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# Thevenin's Theorem

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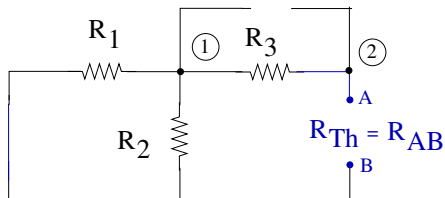
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$$R_{Th} = R_{AB} = R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2}$$

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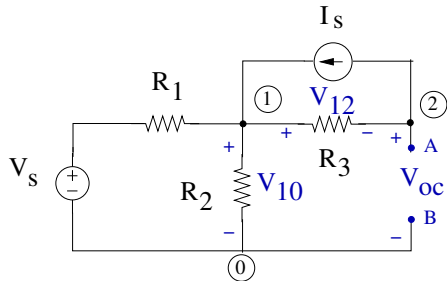
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# Thevenin's Theorem

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To find  $V_{Th}$ , Thevenin's Voltage Source

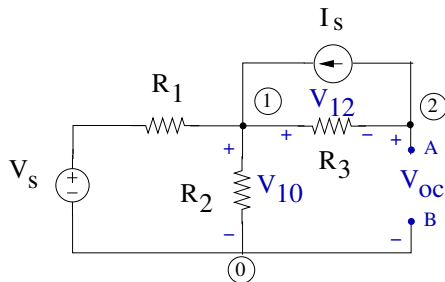




# Thevenin's Theorem

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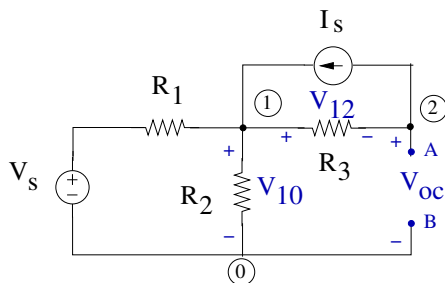
By Kirchoff's Voltage Law

$$\begin{aligned} V_{Th} = V_{oc} &= V_{10} - V_{12} \\ &= V_s \cdot \frac{R_2}{R_1 + R_2} - I_s \cdot R_3 \end{aligned}$$

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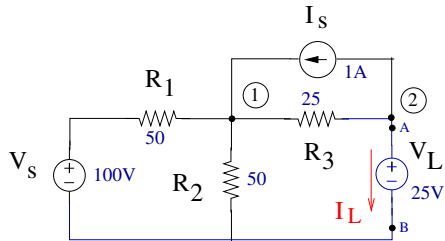
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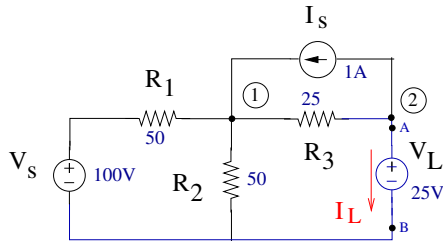
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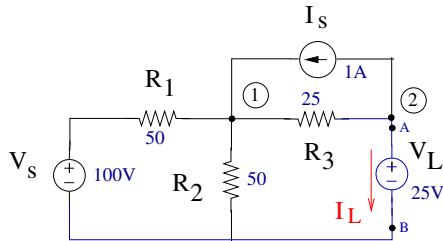


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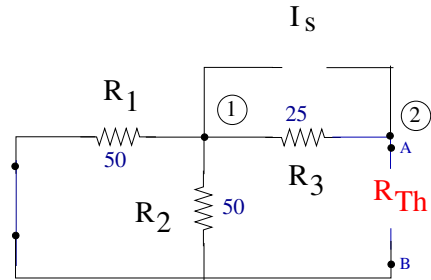
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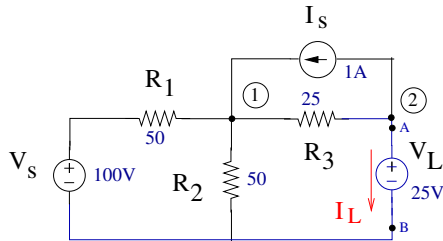


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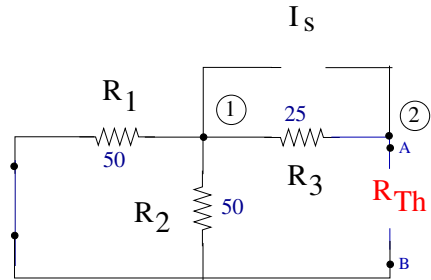
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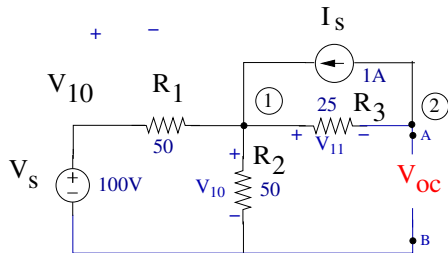
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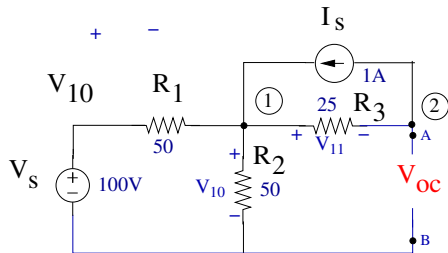
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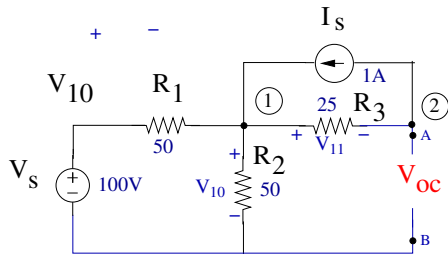
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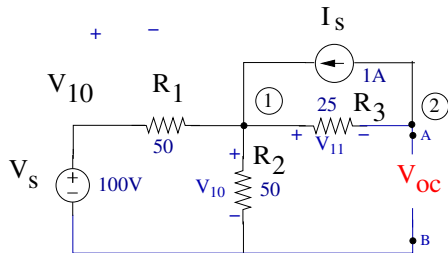


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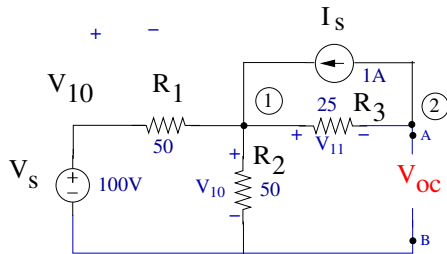
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Solving Thevenin's Equivalent Circuit

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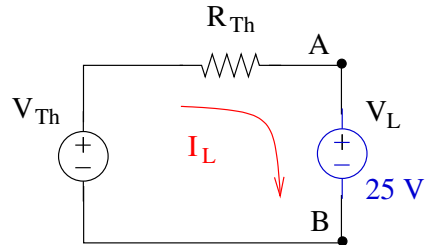
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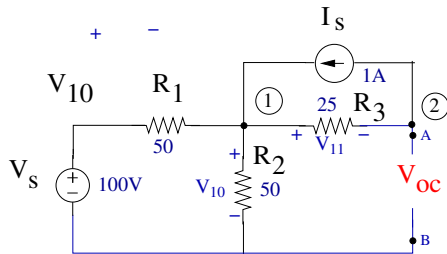
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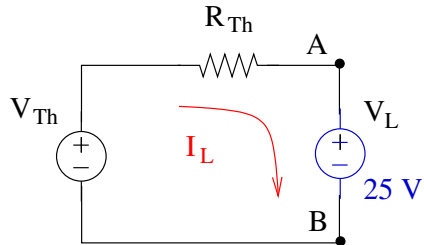
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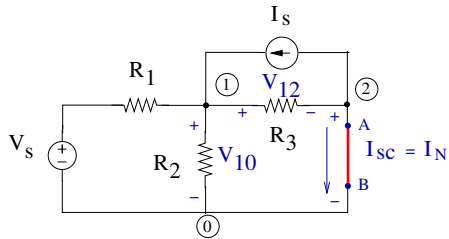
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$$\begin{aligned} I_L &= \frac{V_{Th} - V_L}{R_{Th}} \\ &= \frac{75}{50} = 1.5A \end{aligned}$$

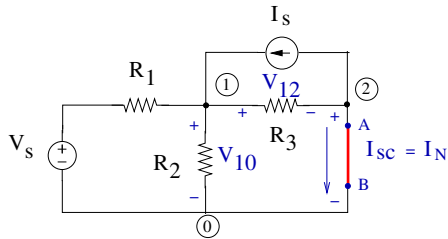
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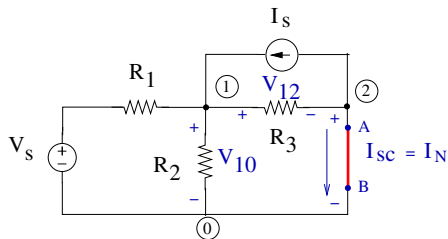


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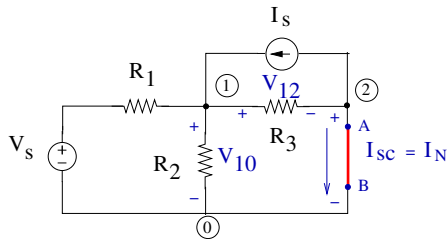


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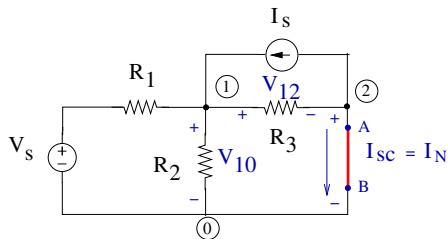
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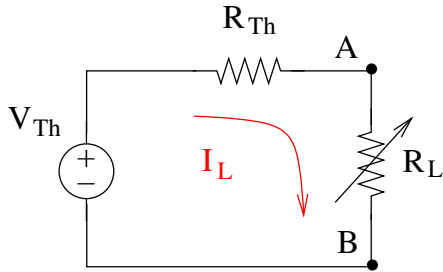
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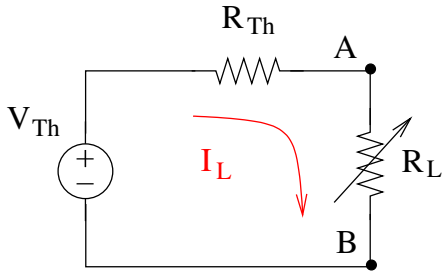
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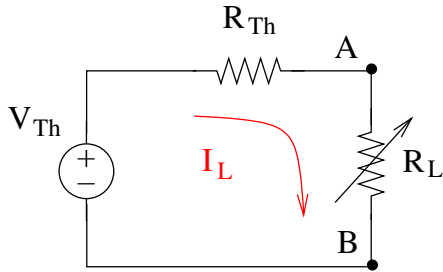


If any active network is replaced by Thevenin's equivalent circuit, then there should be a unique value of  $R_L$  for which maximum power transfer would take place from the active network to the load resistor. Therefore, the power transferred to the load resistor is given as  $I_L^2 \cdot R_L$ .

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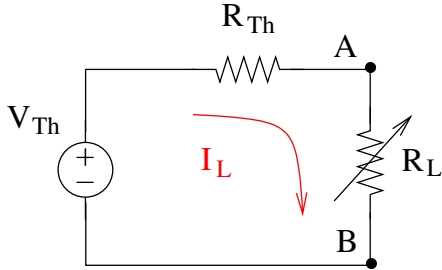


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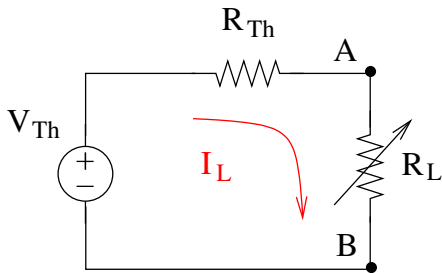
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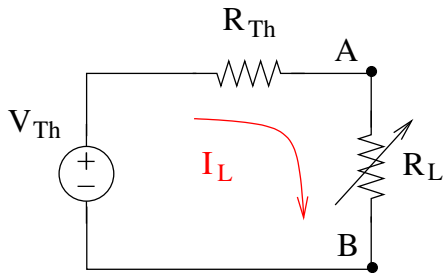
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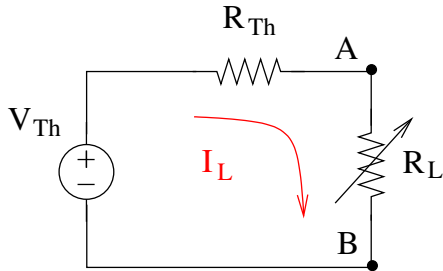
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The leads to the Condition for Maximum Power Transfer

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# Maximum Power Transfer Theorem: Example

## Maximum Power in a load resistor by Theven's Theorem

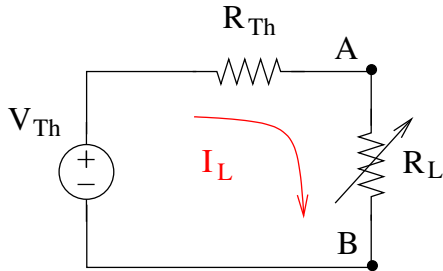


Let  $R_{Th} = 100$ ,  $V_{Th} = 12V$ ,  $R_L$  is varying from 0 to 300. The power consumed in  $R_L$  is plotted against the load Resistor.

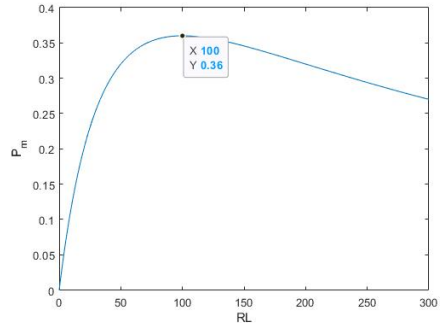


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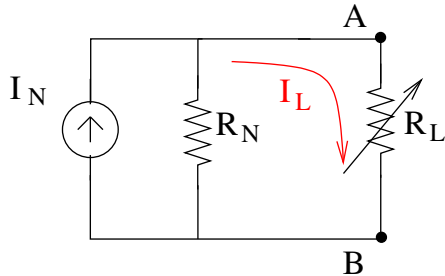
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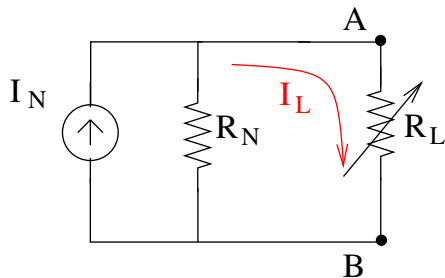
## Maximum Power in a load resistor by Norton's Theorem



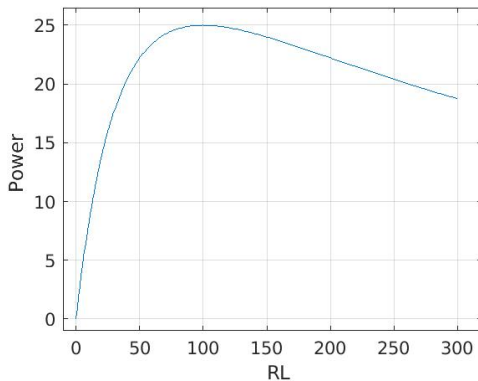
Let  $R_{100} = 100$ ,  $I_N = 1$  A,  $R_L$  is varying from 0 to 300. The power consumed in  $R_L$  is plotted against the load Resistor.

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# DC Transients in R-L and R-C Circuit

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- ⑥ Sometimes we may use conditions at  $t = \infty$ ; these are called final conditions. Final conditions are normally described as the values of the state variable after long time from the instance of switching and therefore describes the new network equilibrium condition.

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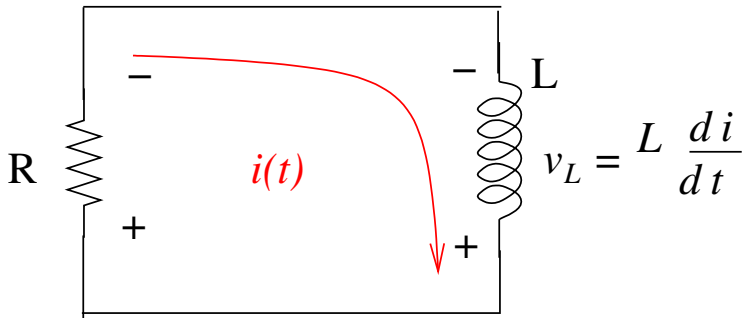
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- ④ Initial conditions in an electric circuit depend on the state of the network prior to  $t = 0^+$ , and the network structure at  $t = 0^+$ , after switching.

# Natural Response in R-L Circuit

The **source-free** response is called as natural response.

Consider a simple R-L circuit as shown below. The network has no excitation, but assumed that there is some initial current  $I_0$  present in the network prior to  $t=0$  where the analysis starts, as inductor is an energy storing device,



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This equation is in the form of homogeneous linear differential equation with constant co-efficients. Let us assume that the solution is obtained in general form:

$$i(t) = A \cdot e^{\alpha t} \quad (16)$$

At  $t = 0, i(t) = I_0$ , substituting in eqn:(16)

$$i(0) = I_0 = A$$

Which will make:

$$i(t) = I_0 \cdot e^{\alpha t} \quad (17)$$

# Natural Response in R-L Circuit

By applying Kirchhoff's Voltage Law (KVL),

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Let  $I_0 = 1 \text{ A}$ ,  $R = 10 \text{ } \Omega$ ,  $L = 79.57 \text{ } \mu$

Then let us define  $\tau = \frac{L}{R}$ , Therefore, here,  $\tau = 7.957 \text{ } \mu \text{ sec}$ . The symbol  $\tau$  is called as **Time-Constant** of R-L Circuit. Following is the plot for  $i(t)$  at above parameters.

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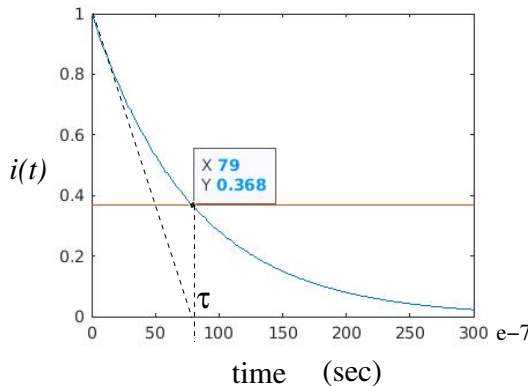
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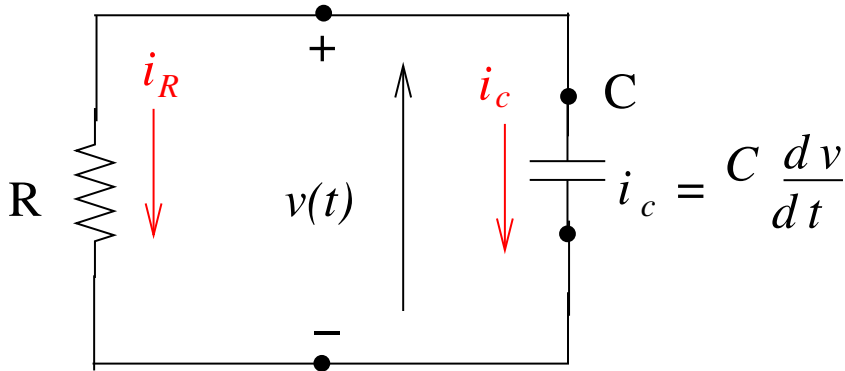
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Draw a tangent to  $i(t)$  curve at  $t=0$ , the line cuts the time-axis at  $t = \tau$ .



# Natural Response in R-C Circuit

Consider a simple R-C circuit as shown below. The network has no excitation, but assume that there is some initial voltage  $V_0$  present across the capacitor prior to  $t=0$ , where the analysis starts, as the capacitor is an energy storing device.



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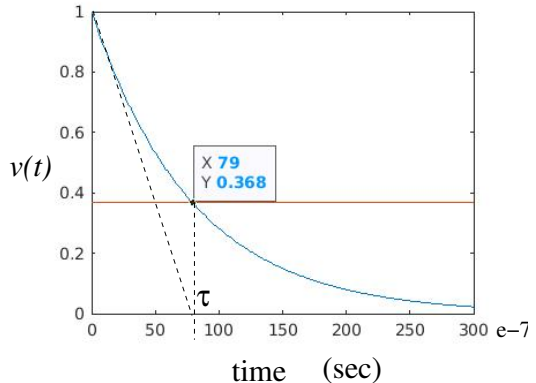
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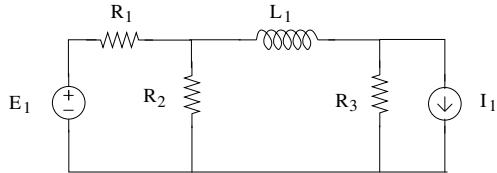
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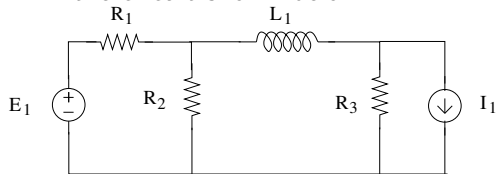
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**Example-1** Find the time constant of the circuit shown below.



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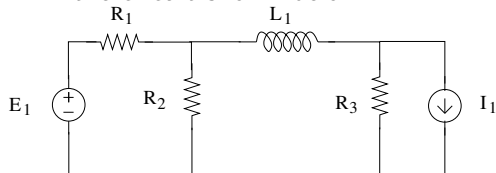
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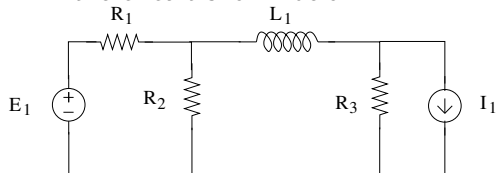


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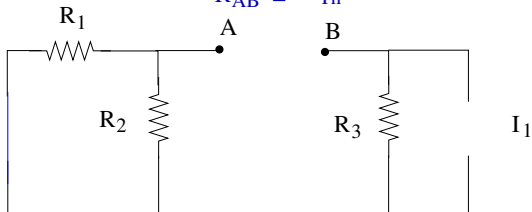
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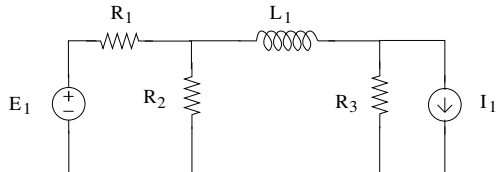
Passive equivalent circuit to find Thevenin's resistance

$$R_{AB} = R_{Th}$$



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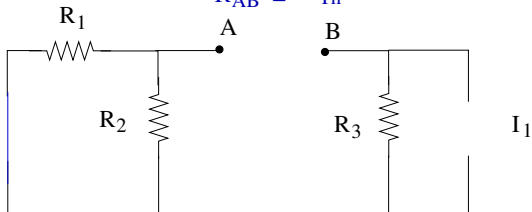
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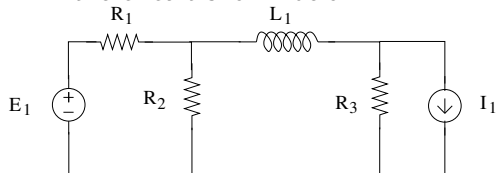
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# Examples on Time Constant

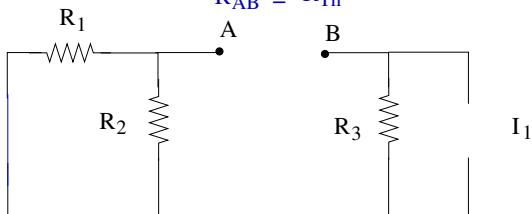
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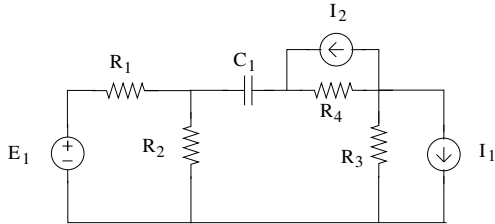


$$R_{Th} = R_{AB} = R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Therefore time-constant of the given circuit is given as:  $\tau = \frac{L_1}{R_{Th}}$ .

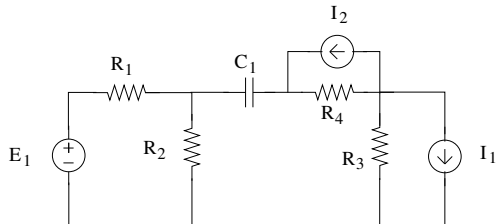
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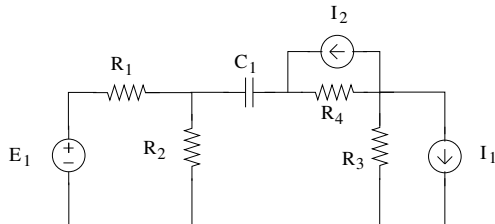
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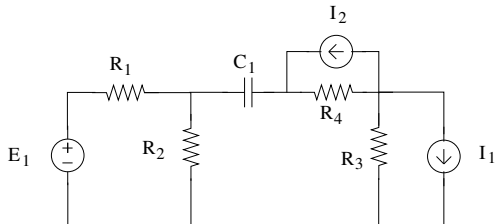
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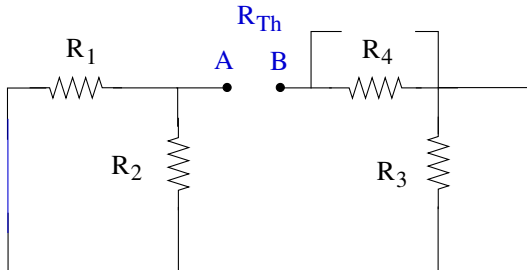
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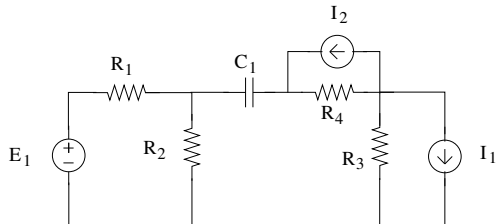
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Passive equivalent circuit to find Thevenin's resistance



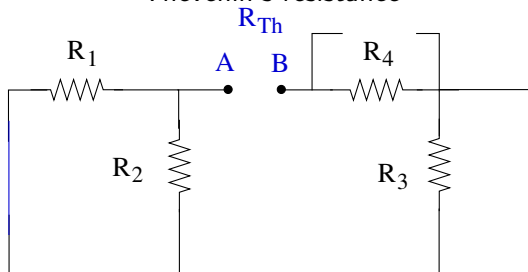
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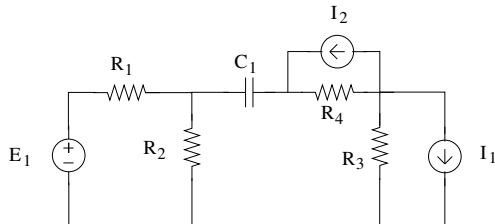


$$R_{Th} = R_{AB} = R_4 + R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2}$$



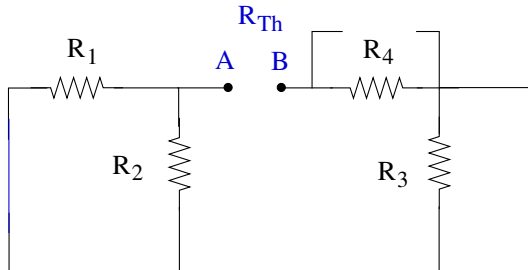
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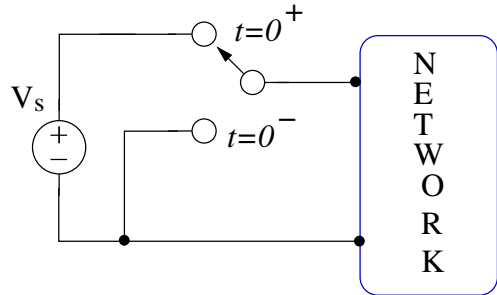
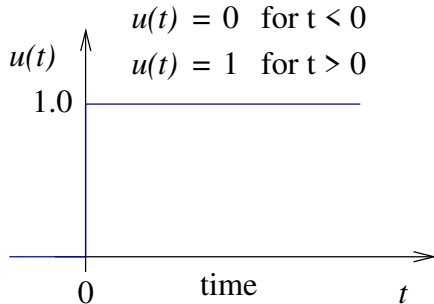
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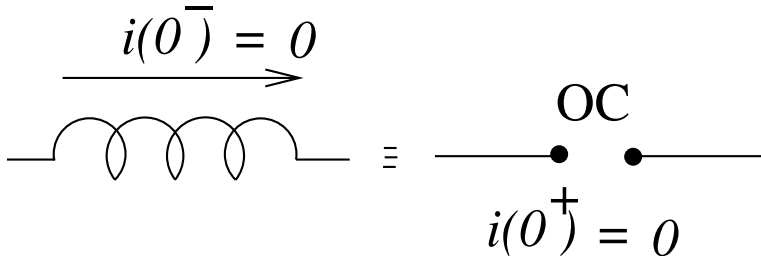
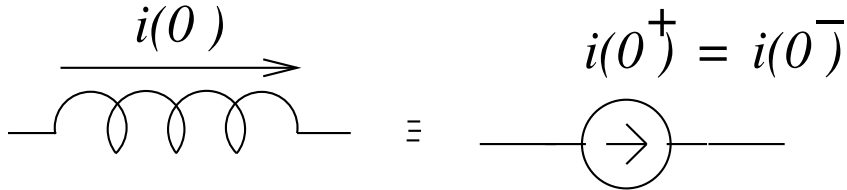


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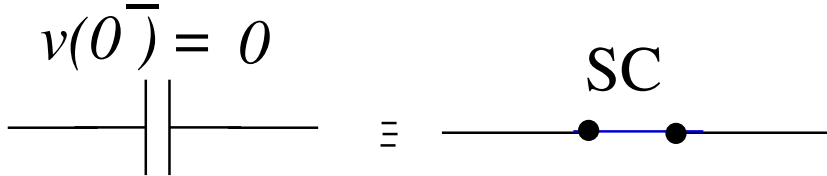
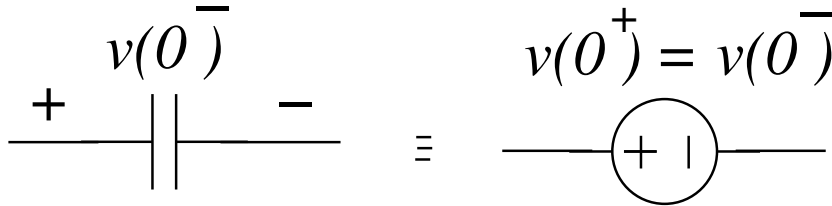


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During steady state (SS), a Capacitor follows the following equation:

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# Steady state Behaviour of an Inductor and a Capacitor

During steady state (SS), an inductor follows the following equation:

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During steady state (SS), a Capacitor follows the following equation:

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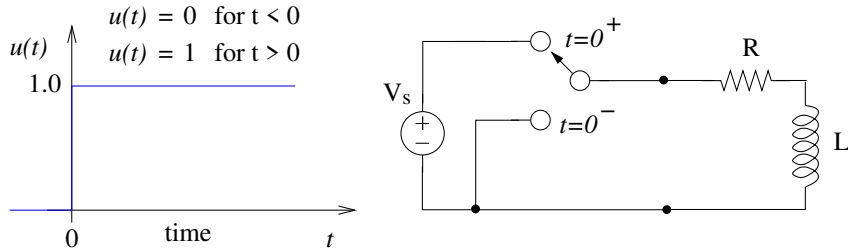
During SS, under DC excitation, the change in capacitor voltage with respect to time is zero. Therefore, during SS it acts as an open circuit.

# Analysis of Step Response in R-L Circuit

Let us analyse the time domain response of R-L Circuit described in following figure:

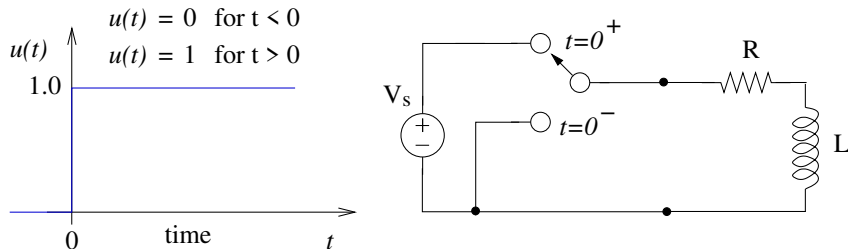
# Analysis of Step Response in R-L Circuit

Let us analyse the time domain response of R-L Circuit described in following figure:



# Analysis of Step Response in R-L Circuit

Let us analyse the time domain response of R-L Circuit described in following figure:



The assumption is the initial inductor current prior to  $t=0$  is zero. Therefore.

$$i(0)^- = i(0)^+ = 0.$$



# Analysis of Step Response in R-L Circuit

At  $t=0$  and onwards, the current  $i(t)$  can be solved by following differential equation obtained by applying KVL:

# Analysis of Step Response in R-L Circuit

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- 1 First obtain the solution of Homogeneous part of the equation: (26)

# Analysis of Step Response in R-L Circuit

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- 2 Apply the initial and steady state condition to get the particular solution.

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# Analysis of Step Response in R-L Circuit

At  $t=0$  and onwards, the current  $i(t)$  can be solved by following differential equation obtained by applying KVL:

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## The Natural Response

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$$L \cdot \frac{di(t)}{dt} + R \cdot i(t) = 0 \quad (22)$$

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$$L \cdot \frac{di(t)}{dt} + R \cdot i(t) = 0 \quad (22)$$

The solution to homogeneous differential equation with constant coefficient is given as:

$$i_n = A \cdot e^{\frac{-R}{L} \cdot t} = A \cdot e^{\frac{-t}{\tau}}$$

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## The Forced Response

# Analysis of Step Response in R-L Circuit

## The Natural Response

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The solution to homogeneous differential equation with constant coefficient is given as:

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## The Forced Response

Since the forcing function is constant for  $t > 0$ , the particular solution is assumed be constant as  $i_t = I$ . Therefore, substituting this in eq:(26),

$$L \cdot \frac{dI}{dt} + R \cdot I = V_s$$

# Analysis of Step Response in R-L Circuit

## The Natural Response

$$L \cdot \frac{di(t)}{dt} + R \cdot i(t) = 0 \quad (22)$$

The solution to homogeneous differential equation with constant coefficient is given as:

$$i_n = A \cdot e^{\frac{-R}{L} \cdot t} = A \cdot e^{\frac{-t}{\tau}}$$

## The Forced Response

Since the forcing function is constant for  $t > 0$ , the particular solution is assumed be constant as  $i_t = I$ . Therefore, substituting this in eq:(26),

$$L \cdot \frac{dI}{dt} + R \cdot I = V_s$$

Since  $I$  is constant,  $\frac{dI}{dt} = 0$ ,  $I = \frac{V_s}{R} = I_f$  is called as Particular Solution.

## The Complete Response

$$i(t) = i_n + i_f = A \cdot e^{\frac{-t}{\tau}} + \frac{V_s}{R} \quad (23)$$

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$$i(t) = i_n + i_f = A \cdot e^{\frac{-t}{\tau}} + \frac{V_s}{R} \quad (23)$$

Now, substituting initial condition as  $i0^- = i0^+ = 0$

$$i(0) = A + \frac{V_s}{R} = 0$$



# Analysis of Step Response in R-L Circuit

## The Complete Response

$$i(t) = i_n + i_f = A \cdot e^{\frac{-t}{\tau}} + \frac{V_s}{R} \quad (23)$$

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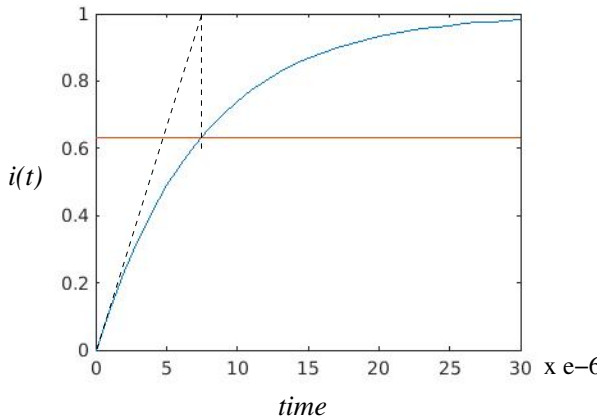
$$i(0) = A + \frac{V_s}{R} = 0$$

Therefore,  $A = -\frac{V_s}{R}$ , hence, substituting this in eq:(24), we get final solution as:

$$i(t) = \frac{V_s}{R} \cdot \left(1 - e^{\frac{-R}{L} \cdot t}\right)$$

Let  $V_s = 1 \text{ V}$ ,  $R=1 \Omega$ ,  $L=79.57 \mu\text{H}$

$$i(t) = \left(1 - e^{\frac{-1}{79.57 \mu}}\right)$$



# Analysis of Step Response in R-L Circuit

## The Complete Response with non-zero initial condition

$$i(t) = i_n + i_f = A \cdot e^{\frac{-t}{\tau}} + \frac{V_s}{R} \quad (24)$$

# Analysis of Step Response in R-L Circuit

## The Complete Response with non-zero initial condition

$$i(t) = i_n + i_f = A \cdot e^{\frac{-t}{\tau}} + \frac{V_s}{R} \quad (24)$$

Now, substituting initial condition as  $i0^- = i0^+ = I_0$

$$i(0) = A + \frac{V_s}{R} = I_0$$

# Analysis of Step Response in R-L Circuit

**The Complete Response with non-zero initial condition**

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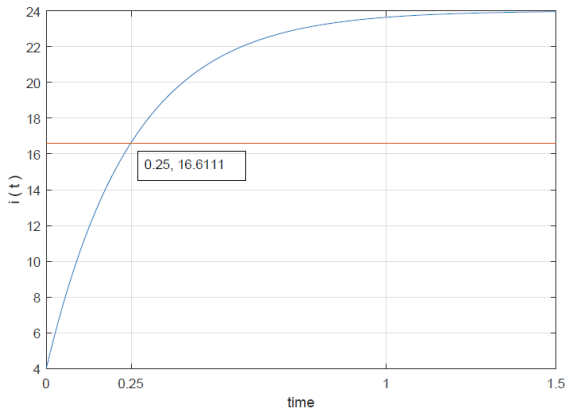
$$i(0) = A + \frac{V_s}{R} = I_0$$

Therefore,  $A = -\frac{V_s}{R} + I_0$ , hence, substituting this in eq:(24), we get final solution as:

$$i(t) = \frac{V_s}{R} \cdot \left(1 - e^{\frac{-R}{L} \cdot t}\right) + I_0 \cdot e^{\frac{-R}{L} \cdot t}$$

Let  $V_s = 40\text{V}$ ,  $I_0 = 4\text{A}$ ,  $R=2\ \Omega$ ,  $L=0.5\text{H}$

$$i(t) = 4 e^{\frac{-t}{0.25}} + 20 \left(1 - e^{\frac{-t}{0.25}}\right) .$$



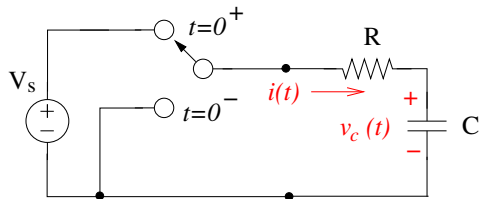
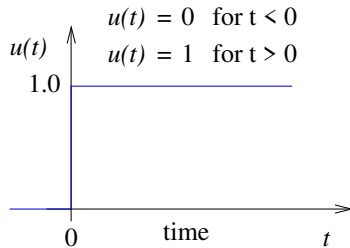
**Transient Response of R-L Circuit**

# Analysis of Step Response in R-C Circuit

Let us analyse the time domain response of R-C Circuit described in following figure:

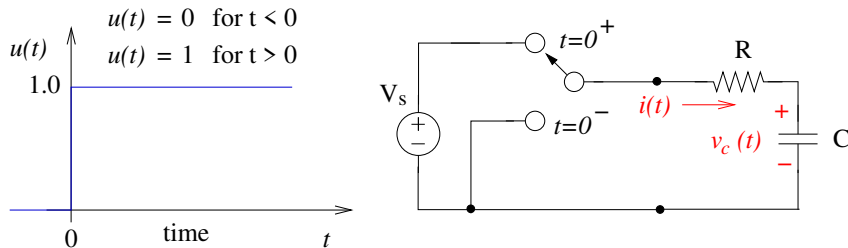
# Analysis of Step Response in R-C Circuit

Let us analyse the time domain response of R-C Circuit described in following figure:



# Analysis of Step Response in R-C Circuit

Let us analyse the time domain response of R-C Circuit described in following figure:



The assumption is the initial capacitor voltage prior to  $t=0$  is zero. Therefore.  
$$v_c(0)^- = v_c(0)^+ = 0.$$

# Analysis of Step Response in R-C Circuit

At  $t=0$  and onwards,  $i = C \cdot \frac{dv_c}{dt}$ . The voltage  $v_c(t)$  can be solved by following differential equation obtained by applying KVL:



# Analysis of Step Response in R-C Circuit

At  $t=0$  and onwards,  $i = C \cdot \frac{dv_c}{dt}$ . The voltage  $v_c(t)$  can be solved by following differential equation obtained by applying KVL:

$$R \cdot i(t) + v_c(t) = V_s \quad (25)$$

$$R \cdot C \cdot \frac{dv_c}{dt} + v_c = V_s \quad (26)$$

# Analysis of Step Response in R-C Circuit

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This is a non-homogeneous differential equation with constant coefficients.

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## The Natural Response



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The solution to homogeneous differential equation with constant coefficient is given as:

$$v_n = A \cdot e^{\frac{-t}{R \cdot C}} = A \cdot e^{\frac{-t}{\tau}}$$

# Analysis of Step Response in R-C Circuit

## The Natural Response

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The solution to homogeneous differential equation with constant coefficient is given as:

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Since the forcing function is constant for  $t > 0$ , the particular solution is assumed be constant as  $v_c t = V_c$ . Therefore, substituting this in eq:(26),

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# Analysis of Step Response in R-C Circuit

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$$R \cdot C \cdot \frac{dV_c}{dt} + R \cdot V_c = V_s$$

Since  $V_c$  is constant,  $\frac{dV_c}{dt} = 0$ ,  $V_c = V_s = v_f$  is called as Particular Solution.

# Analysis of Step Response in R-C Circuit

## The Complete Response with zero Initial Condition

$$v_c(t) = v_n + v_f = A \cdot e^{\frac{-t}{\tau}} + V_s \quad (28)$$

## The Complete Response with zero Initial Condition

$$v_c(t) = v_n + v_f = A \cdot e^{\frac{-t}{\tau}} + V_s \quad (28)$$

Now, substituting initial condition as  $v_c 0^- = v_c 0^+ = 0$

$$v_c(0) = A + V_s = 0$$

# Analysis of Step Response in R-C Circuit

## The Complete Response with zero Initial Condition

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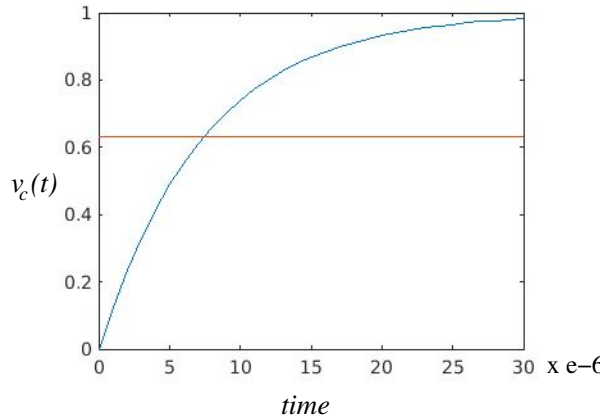
$$v_c(0) = A + V_s = 0$$

Therefore,  $A = -V_s$ , hence, substituting this in eq:(29), we get final solution as:

$$v_c(t) = V_s \cdot \left(1 - e^{\frac{-t}{R \cdot C}}\right)$$

Let  $V_s = 1 \text{ V}$ ,  $R=1 \text{ } \Omega$ ,  $C=79.57 \mu\text{H}$

$$v_c(t) = \left(1 - e^{\frac{-1}{79.57 \mu}}\right)$$





# Analysis of Step Response in R-C Circuit

## The Complete Response with Non-zero Initial Condition

$$v_c(t) = v_n + v_f = A \cdot e^{\frac{-t}{\tau}} + V_s \quad (29)$$

# Analysis of Step Response in R-C Circuit

## The Complete Response with Non-zero Initial Condition

$$v_c(t) = v_n + v_f = A \cdot e^{\frac{-t}{\tau}} + V_s \quad (29)$$

Now, substituting initial condition as  $v_c 0^- = v_c 0^+ = V_0$

$$v_c(0) = A + V_s = V_0$$

# Analysis of Step Response in R-C Circuit

## The Complete Response with Non-zero Initial Condition

$$v_c(t) = v_n + v_f = A \cdot e^{\frac{-t}{\tau}} + V_s \quad (29)$$

Now, substituting initial condition as  $v_c 0^- = v_c 0^+ = V_0$

$$v_c(0) = A + V_s = V_0$$

Therefore,  $A = V_0 - V_s$ , hence, substituting this in eq:(29), we get final solution as:

$$v_c(t) = V_s \cdot \left(1 - e^{\frac{-t}{R \cdot C}}\right) + V_0 \cdot e^{\frac{-t}{R \cdot C}}$$

Let  $V_s = 100\text{V}$ ,  $V_0 = 12\text{V}$   $R=10\ \Omega$ ,  
 $C=100\mu\text{H}$

$$v_c(t) = 100 \left(1 - e^{\frac{-1}{1e-3}}\right) + 12 e^{\frac{-1}{1e-3}}$$

