

# INDUCTION MOTOR

→ Classification of AC Motor:

- 1. Principle of operation ↗ Synchronous  
Asynchronous
- 2. Type of current ↗ Single-phase  
3-phase
- 3. Speed ↗ Constant speed  
Variable speed

4. Structural features.

Induction motor is a type of asynchronous motor

→ WORKING PRINCIPLE

- 1. As a general rule, conversion of electric power into mechanical power takes place in the rotating part of an electric motor.
- 2. In D.C. Motor, the electric power is converted conducted directly to armature (i.e. rotating part) through brushes & commutator. Hence, in this sense, a d.c motor can be called as conduction motor.
- 3. However, in a.c. motors, the rotor does not receive any electric power by conduction, but by induction in exactly the same way as the secondary of a - winding transformer receives its power from the primary. Hence they are called induction motors.

4. An induction motor can be treated as a rotating transformer i.e. one in which primary winding is stationary but secondary is free to rotate.

→ CONSTRUCTION:

1. Stator & rotor.

2. ↗

1. STATOR:

Stator is made up of no. of windings stampings, which are slotted to receive the windings.

Stator carries a 3-phase-winding & is fed from a 3-phase supply. It is wound for a definite no. of poles.

Greater the speed, lesser the no. of poles.

$$P = 2n, n = \text{no. of stator slots/pole}$$

2. ROTOR:

- a) Squirrel-cage
- b) Phase-wound.

a) Squirrel Cage Rotor:

Simplest structure & is almost indestructible.

Rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors. The rotor conductors bars are permanently short-circuited on themselves. Hence it is not possible to add any ext resistance in series with the rotor circuit.

The rotor slots are slightly skewed. This is useful as:

- 1) It helps the motor run without magnetic hum.
- 2) It helps in reducing locking tendency of the motor.

→ PRODUCING THE ROTATING FIELD.

1. When three phase current flows through the stator windings, a magnetic flux is produced that rotates at synchronous speed  $N_s = \frac{120f}{P}$
2. The rotor is an iron laminated cylinder with large embedded conductors in the form of copper or aluminium bars in semi closed slots.
3. The slots are usually not made parallel to the axis but are given a slight twist hence known as skewed rotor.
4. The bars are shorted at each end by a conducting ring or plate. The conducting ring or plate ends of the shorting rings look like a squirrel cage.
5. The air gap between the rotor & the stator is uniform & is made as small as possible mechanically.
6. In a 3-phase supply, the magnitude of resultant flux is constant =  $\phi_m$ .

→ SLIP

1. The rotor never really catches up with the stator field. When an emf is generated between the rotating flux of stationary conductors, rotor current tries to oppose this emf, according to Lenz's Law.

2. The relative velocity between the rotating plane of the stator & the rotor conductors. Hence, to reduce the relative speed, the rotor starts running in the same direction as that of the flux to catch up with stator field.
3. But, if the rotor does really catch up with stator field then there would be no rotor emf & so no torque.  
 $\therefore \text{rotor speed} < \text{stator speed}$ .
4. The diff between the synchronous speed  $N_s$  & the actual speed  $N$  of rotor is known as slip speed  $\Delta N = N_s - N$ .
5. Generally slip is expressed as per unit or fraction of the synchronous speed. Hence  $S = \Delta N / N_s = [N_s - N]$ .  
 For given slips, rotor speed,  $N = N_s [1 - S] \quad N_s$
6.  $\therefore \text{Slip (S)} = \frac{N_s - N}{N_s} \times 100$
7. When motor is at standstill, rotor  $N$  is zero, hence  $S = 1$ .
8. Generally, slip is from 1% to 3% from no load to full load.
9. Negative value of slip indicates generating action.

→ FREQUENCY OF ROTOR CURRENTS

1. When induction motor is at standstill, the frequency of the currents induced in the rotor winding is same as the supply frequency.
2. When motor runs, the frequency of rotor currents depends upon the relative speed or slip speed.

3. The rotor speed  $N$  of the synchronous speed  $N_s$ , then the frequency  $f_r$  of the rotor currents is given by

$$\frac{N_s - N}{P} = 120 f_r$$

$$4. \therefore \frac{N_s - N}{N_s} = \frac{f_r}{f} \text{ ie. } S = \frac{f_r}{f} \text{ or } f_r = Sf$$

$$5. \frac{120 f_r}{P} = \frac{120 f}{P} = S N_s. \text{ In India, } f = 50 \text{ Hz and } P_{\min} = 2. \therefore N_s \text{ max } 3000 \text{ rpm}$$

#### → SPEED OF ROTATION OF ROTOR FIELD

1. The frequency  $f_r$  of the rotor current is  $Sf$ , the speed of rotating field is  $S N_s$  wrt to rotor winding. But rotor is running at a speed  $N$  wrt stator.

2. Speed of rotor field in space = speed of rotor field relative to the rotor + speed of rotor relative to stator

$$\therefore S N_s - N = S N_s + N_s(1-S)$$

$$S N_s - N = S N_s + N_s - S N_s$$

$$N = N_s(S+1) \rightarrow N = N_s(S-1)$$

3. Even though the rotor is not rotating at synchronous speed but rotor field rotates at the synchronous speed i.e. rotor field remains locked with the stator field irrespective of the rotor speed

#### → TORQUE

1. In case of a d.c. motor, the torque  $T_d \propto \phi I_a$ .

2. Induction motor:- Torque is proportional to the product of flux per stator pole or the rotor current & power factor of motor.

$$T \propto \phi I_2^{\cos\phi_2} \text{ or } T = R_1 \phi I_2 \cos\phi_2$$

$I_2$  = rotor current at standstill.

$\phi_2$  = angle between rotor emf & rotor current.

$R_1$  = a constant.

3. Denoting rotor emf at standstill by  $E_2$ , we have  $E_2 \propto \phi$

$$T \propto E_2 I_2 \cos\phi_2 \quad | \quad R_1 = \text{another constant}$$

$$T = R_1 E_2 I_2 \cos\phi_2$$

### → STARTING TORQUE:

Torque developed by the motor at an instant of starting is called starting torque.

Let  $E_2$  = rotor emf per phase at standstill.

$R_2$  = rotor resistance / phase.

$X_2$  = rotor reactance / phase at standstill.

$Z_2 = \sqrt{R_2^2 + X_2^2}$  : rotor impedance / phase at standstill.

$$\text{Then } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}, \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

Standstill or starting torque.  $T_{st} = R_1 E_2 I_2 \cos\phi_2$ .

$$\text{or } T_{st} = R_1 E_2 \cdot \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{R_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

If supply voltage  $V$  is constant, then the flux  $\phi$  & hence  $E_2$  both are constant.

$$T_{st} = \frac{R_1 R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{R_1 R_2}{X_2^2} \text{ where } R_2 \text{ is some other constant.}$$

$$\text{Now, } R_1 = \frac{3}{2\pi N_b} \quad \therefore T_{st} = \frac{3}{2\pi N_b} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

$N_s \rightarrow$  synchronous speed in rps

→ CONDITION FOR MAX STARTING TORQUE

$$\text{From, } T_{st} = \frac{R_2 R_2}{R_2^2 + X_2^2} \quad \therefore \frac{dT_{st}}{dR_2} = \frac{R_2}{R_2^2 + X_2^2} \left[ \frac{1}{R_2} - \frac{R_2(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

$$\therefore R_2^2 + X_2^2 = 2R_2^2$$

$$\therefore X_2 = R_2$$

→ EFFECT OF CHANGE OF SUPPLY VOLTAGE

$$T_{st} = \frac{R_1 E_2^2 R_2}{R_2^2 + X_2^2}, \text{ Now } E_2 = \text{Supply voltage } V$$

$$\underline{T_{st}} = \frac{R_2 V^2 R_2}{R_2^2 + X_2^2} = \frac{R_2 V^2 R_2}{R_2^2 + \underline{R_2^2}} = \underline{\underline{R_2^2}} \quad \text{where } R_3 = \text{another constant}$$

$$T_{st} = V^2$$

Torque is very sensitive to changes in supply voltage.

→ ROTOR EMF UNDER RUNNING CONDITION.

1. Let  $E_2$  = standstill rotor induced emf / phase,  $X_2$  = standstill rotor reactance / phase,  $f_2$  = rotor current frequency at standstill.
2. When rotor is stationary i.e.  $s=1$ , the frequency of rotor emf is the same as that of the stator supply frequency.
3. The value of emf induced in rotor at standstill is maximum because of the relative speed between the rotor & the revolving stator flux is max.
4. When rotor starting running, the relative speed between it & the rotating stator flux is decreased. Hence, the rotor induced emf, which is directly proportional to this speed, is also decreased.
5. Hence, for a slip  $s$ , the rotor induced emf will be  $s$  times the induced emf at standstill.

Under running conditions  $E_R = SE_2$ . The frequency of the induced e.m.f will likewise become  $f_R = Sf_2$ . Due to decrease in frequency of rotor emf, the rotor reactance will also decrease.  $\therefore X_R = S X_2$

$E_R$  = rotor emf under ~~no~~ running,  $X_R$  = Reactance under running.

### TORQUE UNDER RUNNING CONDITION

$$T \propto E_R I_R \cos \phi_2 \text{ or } T \propto \phi I_R \cos \phi_2$$

where  $E_R$  = rotor emf / phase under running conditions

$I_R$  = rotor current / phase under running conditions.

Now.  $E_R = SE_2$

$$I_R = \frac{E_R}{Z_R} = \frac{SE_2}{\sqrt{R_2^2 + (S X_2)^2}}, \cos \phi_2 = \frac{R_2}{\sqrt{R_2^2 + (S X_2)^2}}$$

$$T \propto \frac{S \phi E_2 R_2}{R_2^2 + (S X_2)^2} = \frac{R \phi \cdot S \cdot E_2 R_2}{R_2^2 + (S X_2)^2}$$

$$\text{Also } T = \frac{R_1 S E_2^2 R_2}{R_2^2 + (S X_2)^2} \quad \because E_2 \propto \phi$$

$$T = \frac{3}{2\pi N_s} \cdot \frac{S E_2^2 R_2}{R_2^2 + (S X_2)^2}$$

$$T = \frac{3}{2\pi N_s} \cdot \frac{S E_2^2 R_2}{X_R^2}$$

### CONDITION FOR MAX TORQUE UNDER RUNNING CONDITION

$$\underline{R_2 = S X_2} \quad T_{max} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2 X_2} \text{ N-m}$$

## → INFERENCES

From the above, it is found:

1. The max torque is independent of rotor resistance as such
2. The speed or slip at which max torque occurs is determined by rotor resistance.
3. The max torque varies inversely as standstill reactance. Hence, it should be kept as small as possible.
4. Max torque varies directly as the square of app voltage
5. For obtaining max torque at starting ( $s=1$ ),  $r_{rotor} \approx$  resistance must be rotor reactance.

## → ROTOR TORQUE & BREAKDOWN TORQUE

The motor torque at any slip can be expressed in terms of maximum (or breakdown) torque  $T_S$ :-

$$T = T_S \left[ \frac{2}{\left( \frac{s}{s_b} \right) + \left( \frac{s_b}{s} \right)} \right]$$

where  $s_b$  is breakdown or pull-out slip.

- Q. Calc the torque exerted by an 8-pole, 50Hz, 3-φ induction motor operating with a 4% slip which develops a  $T_{max}$  = 150 Nm of a speed of 660 rpm. The resistance / phase is 0.5Ω

\* Method 1:

$$N_S = \frac{120 \times 50}{8} = 750 \text{ rpm.}$$

Speed at  $T_{max}$  = 660 rpm.

$$\text{Core slip } s_b = \frac{750 - 660}{750} = 0.12.$$

$\frac{750}{660}$

For  $T_{max} = R_2 = S_b X_2$ .

$$X_2 = \frac{R_2}{S_b} = \frac{0.5}{0.12} = 4.17 \Omega$$

$$\frac{T_{max} \cdot R\phi E_2 S_b}{2R_2} = \frac{R\phi E_2 \times 0.12}{2 \times 0.5} = 0.12 R\phi E_2$$

$$T = \frac{R\phi E_2 \cdot S R_2}{R_2^2 + (S X_2)^2} \Rightarrow R\phi E_2 \left( \frac{0.04 \times 0.5}{0.5^2 + (0.04 \times 4.17)^2} \right)$$

$$T = \frac{0.02}{0.277} R\phi E_2$$

$$\therefore \frac{T}{T_{max}} = \frac{T}{150} = \frac{0.02 \times R\phi E_2 \times 1}{0.277 \times 0.12 \times R\phi E_2}$$

$$\therefore \frac{T}{T_{max}} = 0.602 \Rightarrow T = 0.602 \times 150 \text{ Rgm}$$

$$\boxed{T = 90.25 \text{ Rgm}}$$

\* Method 2: Use relation between  $T$  &  $T_b$ .

$$T_b = 150 \text{ Rgm}, S_b = 0.12, S = 4.1 \approx 0.04$$

$\rightarrow$  from step 1 of method 1

$$T = T_b \left[ \frac{2}{\left( \frac{S_b}{S} \right) + \left( \frac{S}{S_b} \right)} \right] = 150 \times \left[ \frac{2}{\left( \frac{0.12}{0.04} \right) + \left( \frac{0.04}{0.12} \right)} \right]$$

$$\boxed{T = 90.25 \text{ Rgm}}$$

$\rightarrow$  TORQUE-SLIP CURVE

When  $slip = 0$ , motor is running at synchronous speed or rated speed. When  $slip = 1$ , standstill. Slip ( $\uparrow$ ) Speed ( $\downarrow$ )

$$T = \frac{R\phi SE_2 R_2}{R_2^2 + (SX_2)^2}$$

at  $S=0$ ,  $T=0$

At normal speed  $S_1 \rightarrow SX_2$ , will be smaller as compared to  $R_2$   
 $\therefore$  We neglect  $SX_2$

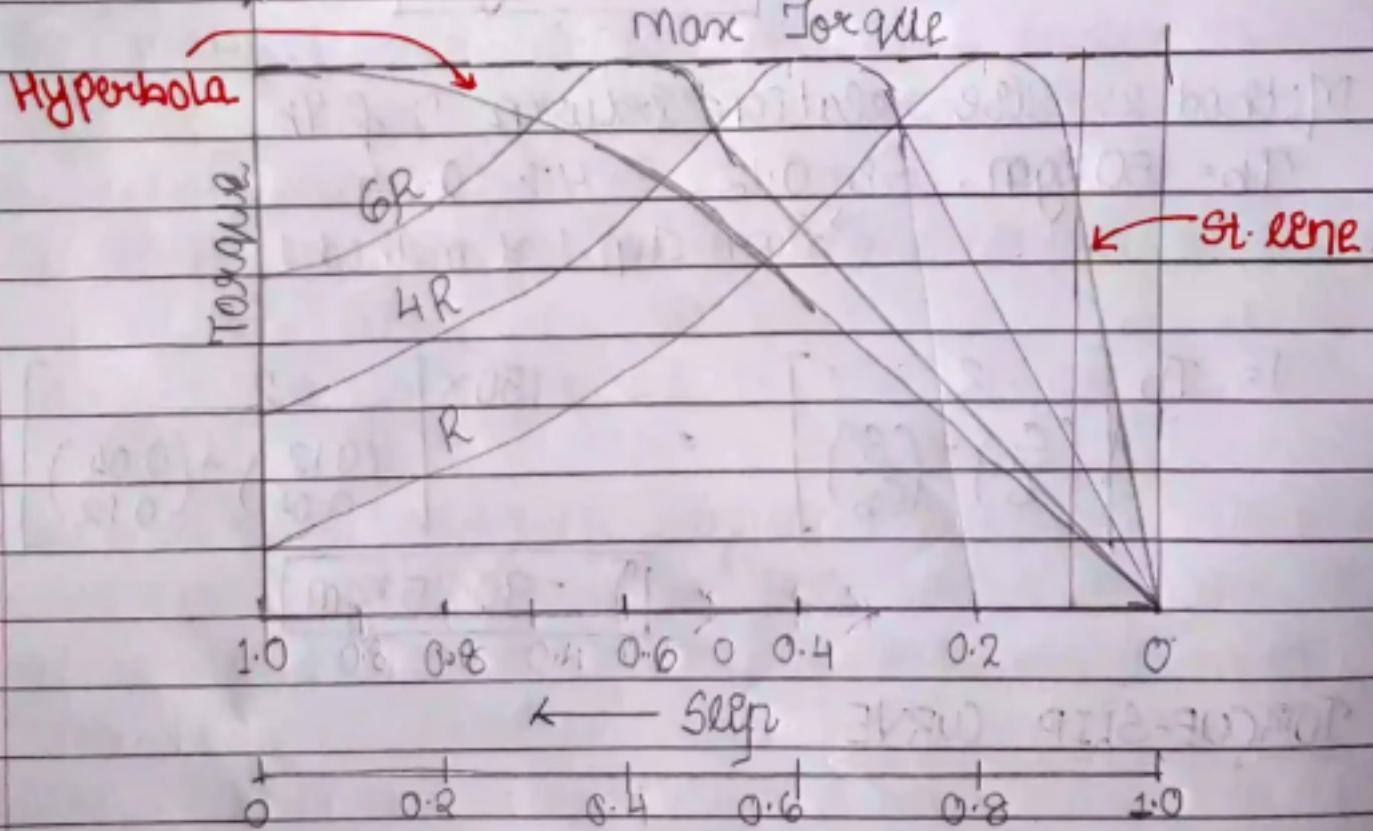
$$T = \frac{R\phi SE_2 R_2}{R_2^2} = \frac{R\phi SE_2}{R_2}; T \propto S; T \propto S \rightarrow St. line$$

When slip ( $\uparrow$ ),  $T(\uparrow)$ ,  $T_{max}$  at  $R_2 = SX_2$

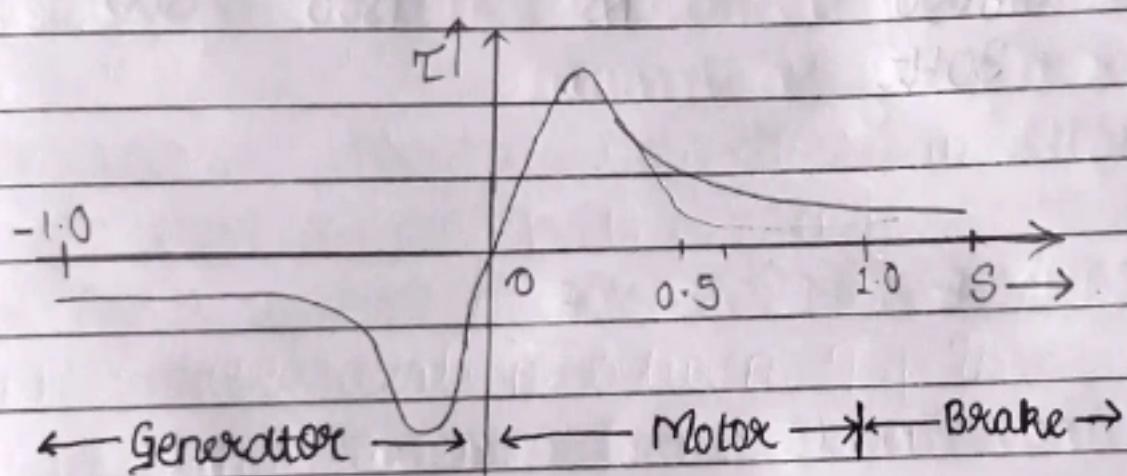
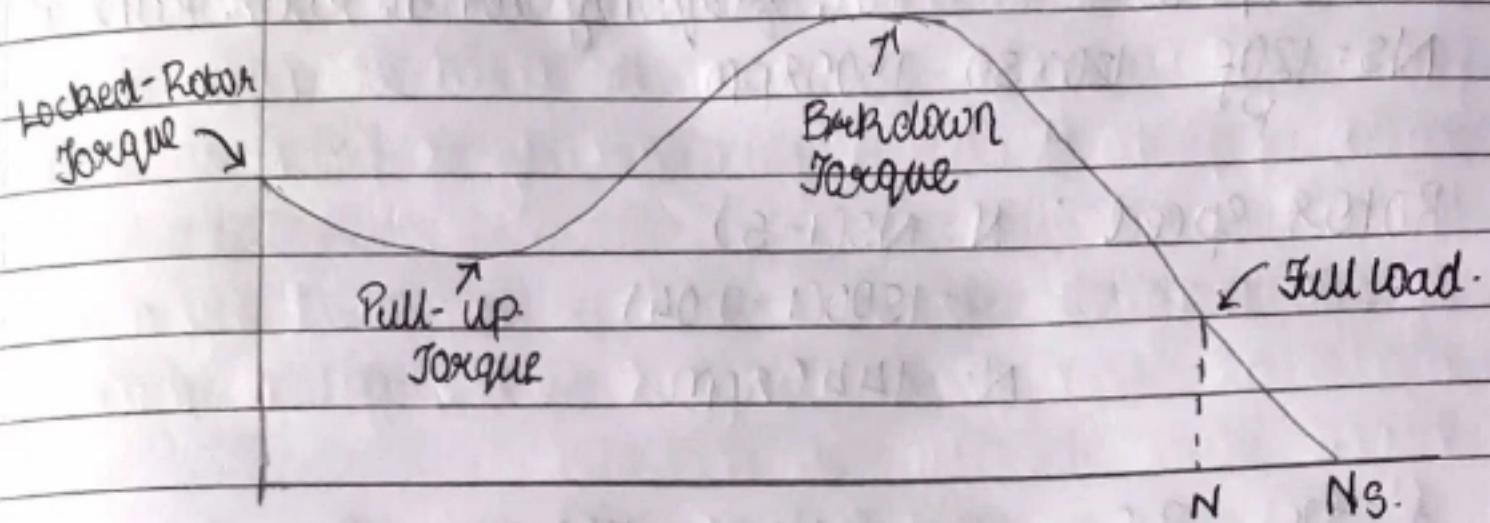
$$S = \frac{R_2}{X_2}$$

When slip ( $\uparrow$ ) more, we can now neglect  $R_2$ .  $R_2 \ll SX_2$

$$T \propto S \Rightarrow T \propto \frac{1}{S} \rightarrow \text{Rectangular hyperbola}$$



## → TORQUE SPEED CURVE



At full load, the motor runs at a speed  $N_f$ . When load ( $T$ ), motor speed ( $\downarrow$ ), till the motor torque again becomes equal to the load torque

- Q. A stator of a 3-φ induction motor has 3-slots per pole per phase. If supply frequency is 50 Hz, calculate i) no. of stator poles produced & total no. of slots on the stator ii) Speed of the rotating stator flux (or magnetic field).

$$i) P = 2 \cdot n = 2 \times 3 = 6$$

Total no slot = 3 slots per 1 pole / phase

$$= 3 \times 6 \times 3 = 54$$

$$ii) N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

→ Q. A 3-φ induction motor is wound 4 poles & is supplied from 50 Hz system. Calculate i) the synchronous speed ii) Rotor speed when slip is 4% iii) Rotor frequency when motor runs at 600 rpm

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{ROTOR speed : } N = N_s(1-s) \\ = 1500(1 - 0.04)$$

$$N = 1440 \text{ rpm}$$

$$f' = f_r = S_f \quad ; - S = \frac{N_s - N}{N_s} = \frac{1500 - 600}{1500} = \frac{900}{1500} = 0.6 \\ = 0.6 \times 50 \\ = 30 \text{ Hz}$$