Student's t-test

Theoretical work on t- distr. was done by W.S. Gosset, he has published his findings under the pen name "Student". That's why the name Students t-test.

→ Used when sample size is 30 or less than 30 & pop. S.D. is unknown.

→ t-distr. has been derived mathematically under the ass. of a normally distributed population.

~~f(t~~

$$\boxed{f(t) = C\left(1 + \frac{t^2}{v}\right)^{\left(-\frac{v+1}{2}\right)}}$$

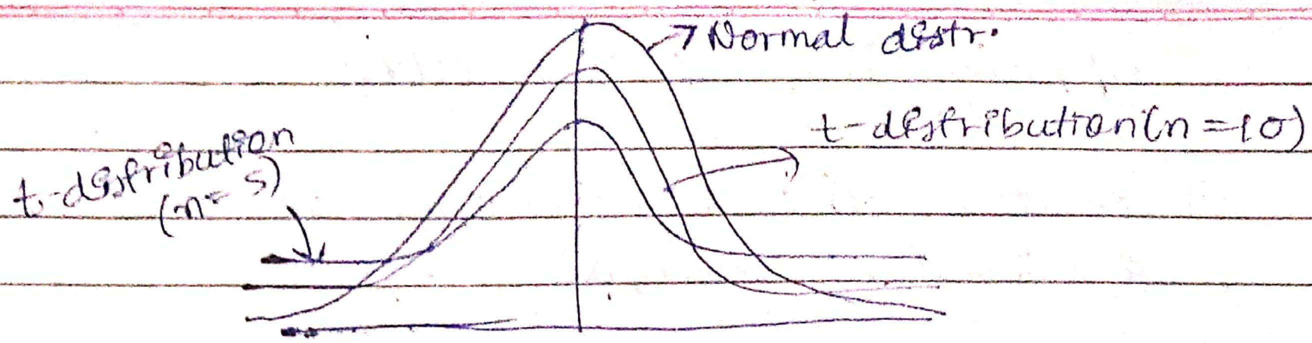
C = const. to make the area under the curve to unity

v = Degree of freedom

Prop. of t- distr.

1) The variable t-distribution ranges ~~fro~~ from $-\infty$ to $+\infty$ $(-\infty < t < +\infty)$

2) t-distr. would be symm. like normal distr., if power of t is even in pdf



(3) For large values of v (i.e. inc. sample size n); the t-distribution ~~tends~~ tends to a standard normal distribution. This implies that for diff v values and the shape of t-distribution differs:

Sample size (n)	d.o.f (v)	t-value
5	4	2.776
10	9	2.262
30	29	2.045
∞	-	1.96

(4) The t-distr. is less peaked than normal distribution at the center & higher peaked in the tails

(5) The value of y (peak height) attains highest at $t=0$.

t-distribution table

Gives t-value for diff. level of significance & diff d.o.f.

Applications & Formulas:

① To test the significance of the mean of a random sample

$$\boxed{t = \frac{(\bar{x}-\mu)\cdot\sqrt{n}}{S}} \qquad \boxed{S = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}}$$

$n-1 \rightarrow \nu$

$\bar{x}$ = mean of sample.
μ = mean of population
n = sample size
S = Std. deviation of sample.

σ: Std. dev. of population

$$\boxed{\sigma = \sqrt{\frac{\sum(x-\mu)^2}{n}}} \qquad \boxed{\nu = n-1}$$

Confidence interval estimate (for α level of significance)

One tailed test: $\boxed{\bar{x} \pm t_{\alpha}\frac{S}{\sqrt{n}}}$

Two tailed test: $\boxed{\bar{x} \pm t_{\alpha/2}\frac{S}{\sqrt{n}}}$

② To test the diff. betn means of the two samples (Independent Samples)

$$\boxed{t = \frac{(\bar{x}_1 - \bar{x}_2)}{S} \times \sqrt{\frac{n_1 \times n_2}{n_1 + n_2}}}$$

$$S.E. = S \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\boxed{S = \sqrt{\frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{\underbrace{n_1 + n_2 - 2}_{v}}}}$$

$\bar{x}_1$ = Mean of the sample 1

$\boxed{v = n_1 + n_2 - 2}$

$\bar{x}_2$ = " 2

n_1 = Sample size of sample (1)

n_2 = " " (2)

S = Standard deviation (combined)

(3) To test the difference betⁿ means of two samples (Dep. samples or matched pairs obs.)

$$\boxed{t = \frac{\bar{d}\sqrt{n}}{S}}$$

$$\boxed{S = \sqrt{\frac{\Sigma(d - \bar{d})^2}{\underbrace{(n-1)}_{d.o.f(v)}}}}$$

$\bar{d}$ = mean of difference

$\boxed{v = n - 1}$

S = Standard dev. of differences

n = size of the sample

(4) Testing the significance of an observed co-relation coefficient

$$\boxed{t = \frac{r}{\sqrt{1-r^2}} \times \sqrt{n-2}} \rightarrow \nu$$

r = co-relation coeff. n = size of sample

(1) A manufacturer of a certain LED bulb claims that his bulbs have a mean life of 20 months. A random sample of 7 such ~~bulb~~ bulbs gave foll. values:

Life of bulbs in months: 19, 21, 25, 16, 17, 14, 21

Can you regard the producer's claim to be valid at 5% level of significance

Given:
Pop. mean (μ) = 20 months

Life of bulbs (in months) = 19, 21, 25, 16, 17, 14, 21

Level of sig. = 5%

$H_0 = \mu = \bar{x}$

$H_a = \mu \neq \bar{x}$

$$t = \frac{|\bar{x} - \mu|}{S} \times \sqrt{n} \quad ; \quad S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

Calc. of $\bar{x}$ & S

$$\bar{x} = \frac{\Sigma x}{n} = \frac{133}{7} = 19$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
19	0	0
21	2	4
25	6	36
16	-3	9
17	-2	4
14	-5	25
21	2	4
133		82

$$S = \sqrt{\frac{82}{7-1}} = \sqrt{\frac{82}{6}}$$

$$S = \sqrt{13.67} = 3.7$$

$$t = \frac{|19-20|}{3.7} \times \sqrt{7}$$

$$\boxed{t = 0.716 \text{ (cal. value)}}$$

$$v = n-1 = 6$$

$t_{0.001}$ (tabulated value)

$$\boxed{t_{0.005} = 3.707}$$

Ho is passed & ~~accepted~~ accp. There is no diff. betⁿ the sample mean & pop. mean life of bulbs. Claim of producer is correct.

2) A random sample of size 15 has 50 as mean, the sum of the squares of dev. taken from the mean is 130. Can this sample be regarded as taken from the pop. having 53 as mean?

Obtain 95% & 99% conf limits of the mean for the pop.

Given:

$\bar{x} = 50, \mu = 53, n = 15, \Sigma(x-\bar{x})^2 = 130$

$H_0: \mu = \bar{x}$

$H_a: \mu \neq \bar{x}$

$t = \frac{|\bar{x} - \mu|}{s} \times \sqrt{n}$ $\quad s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$

$s = \sqrt{\frac{130}{15-1}} = \sqrt{\frac{130}{14}} = 3.05$

$\boxed{s = 3.05}$

$t = \frac{|50-53|}{3.05} \times \sqrt{15} = \frac{3 \times 3.87}{3.05}$

$= \frac{11.61}{3.05} = 3.31$

$\boxed{t = 3.31}$ (Cal. t value)

$\nu = n-1 = 14$

$v = n-1 = 15-1 = 14$ $\qquad t_{0.05} = 2.145$

$\qquad\qquad\qquad\qquad\qquad t_{0.01} = 2.977$

Sample mean for 95% confidence limit

$$= \bar{x} \pm \frac{S}{\sqrt{n}} \times t_{0.05}$$

$$= 50 \pm \frac{3.05}{\sqrt{15}} \times 2.145 = 50 \pm \frac{3.05}{3.87} \times 2.145$$

$$= 50 \pm 1.69$$

$$\boxed{\text{limit} = 48.31 \text{ to } 51.69}$$

Sample mean for 99% conf. limit

$$\bar{x} \pm \frac{S}{\sqrt{n}} \times t_{0.01}$$

$$= 50 \pm \frac{2.05}{\sqrt{15}} \times 2.977 = 50 \pm \frac{3.05}{3.87} \times 2.977$$

$$= 50 \pm 2.35$$

$$\boxed{\text{limit} = 47.65 \text{ to } 52.35}$$

$\because$ t cal is higher than table value of t we reject null hyp. in both ~~case~~ cases

3) 2 types of drugs were used on 6 & 5 patients for reducing their weight. Drug A was imported & drug B was indigenous. The inc. in wt. after using drugs for 90 days was given below

Drug A: 8, 10, 12, 9, 14, 13
—"— B: 7, 9, 14, 12, 8

Is there a sig. diff. in efficacy of drugs?

~~So~~ Soln:

Ho: Drug A = Drug B

Ha: Drug A ≠ Drug B

in kg A: 8, 10, 12, 9, 14, 13 B: 7, 9, 14, 12, 8

$n_1 = 6$ $n_2 = 5$

x_1	$(x_1-\bar{x}_1)$	$(x_1-\bar{x}_1)^2$	x_2	$(x_2-\bar{x}_2)$	$(x_2-\bar{x}_2)^2$
8	-3	9	7	-3	9
10	-1	1	9	-1	1
12	+1	1	14	+4	16
9	-2	4	12	+2	4
14	+3	9	8	-2	4
13	2	4			
66	$\Sigma(x_1-\bar{x}_1)^2 = 28$		50	$\Sigma(x_2-\bar{x}_2)^2 = 34$	

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = 11$ $\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{50}{5} = 10$

$t = \frac{|\bar{x}_1 - \bar{x}_2|}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$, $S = \sqrt{\frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}$

$t = \frac{|11 - 10|}{2.62} \times \sqrt{\frac{6 \times 5}{6+5}}$ $S = \sqrt{\frac{28 + 34}{6+5-2}}$

$t = \frac{1.65}{2.62} = 0.63$ $S = \sqrt{66/9} = 2.62$

$\boxed{t = 0.63}$ (cal. t value)

$v = n_1 + n_2 - 2 = 6 + 5 - 2 = 9$

$t_{0.05} = 2.262$
(tab.)

As t_{cal} is less than t_{tab}.
Ho is accp. i.e there is no significant diff. betn efficacy of A & B.

Date.

4) A drug is given to 8 patients & diff. in their blood pressures was recorded to be:

~~Bef~~ Before drug A: 112, 113, 118, 120, 119, 113, 110, 122

After drug A: 116, 120, 117, 125, 126, 111, 111, 117

Is it reasonable to believe that the drug has no effect on change of blood pressure

Soln: cal. of $\bar{d}$ & S

Before A	After A	d	$(d-\bar{d})$	$(d-\bar{d})^2$
112	116	4	2	4
113	120	7	5	25
118	117	-1	-3	9
120	125	5	3	9
119	126	7	5	25
113	111	-2	-4	16
110	111	1	-1	1
122	117	-5	-7	49
	$\Sigma d = 16$		$\Sigma(d-\bar{d})^2 = 138$	

$$\bar{d} = \frac{\Sigma d}{n} = \frac{16}{8} = 2$$

$$\boxed{\bar{d} = 2}$$

$$t = \frac{\bar{d} \cdot \sqrt{n}}{S}, \quad S = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}}$$

$$S = \sqrt{\frac{138}{8-1}} = \sqrt{\frac{138}{7}} = 4.44$$

$$t = \frac{2\sqrt{8}}{4.4} = 1.912$$

$$\boxed{t_{cal} = 1.912}$$

$$\nu = n-1 = 7$$

$t_{0.05} = 2.365$
(table)

Ho is correct & accepted. So drug has no significant role in change of BP.

Ho : BP before A = BP After A

Ha : — " ———— ≠ — " ————

(5) A random sample of 27 pairs of obs. from a normal pop. gives a correlation coeff. of 0.55. Is it likely that the variables in the pop. are uncorrelated

~~n = 27~~ Ho = Correlation = Not significant

Ha : — " ———— ≠ — " ———— " —

Given: $n = 27$ $r = 0.55$

$$t = \frac{r}{\sqrt{1-r^2}} \times \sqrt{n-2}$$

$$t = \frac{0.55}{\sqrt{1-0.55^2}} \times \sqrt{27-2} = \frac{0.55 \times 5}{\sqrt{0.6975}}$$

$$\boxed{t = 3.293} \text{ (Cal.)}$$

$$v = n-2 = 27-2 = 25$$

~~to~~ $\boxed{t_{tab.} = 2.060}$

∴ Ho is failed & rejected so variable in pop. is unco-related.