Chi Square (χ^2) Test:

→ Various significance tests like Z-test, t-test or f-test were based on ass. that the samples were drawn from the normally distributed populations.

Parametric tests: Testing Procedure req. the ass. about type or parameters of pop. E.g. Z-test, f-test

Non-Parametric tests: → It is applied when no exact info. is avail. about the pop. distr., whether it's binomial, poisson or normal

→ These are distn free testing methods
→ χ^2 test is commonly used non-param. test

χ^2 test:

→ First used by Karl Pearson in 1900.
→ Describes mag. of discrepancy b/w obs. & theory

Formula: ~~χ^2~~

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

O = Observed freq., E = Expected freq.

$$E = \frac{RT \times CT}{N}$$

RT = Row total for row cont. cells

CT: Col. total for col. cont. cells

$v = (r-1)(c-1)$

v = d.o.f
r = no. of rows
c = no. of col.

Conditions:

→ Each cell should contain atleast 5 obs. (Generally pref. 10 obs.),
∵ if $E_{ij} < 5$ then χ^2 will be overestimated which leads to rejection of null hyp.

→ All individual obs. should be indep. & completely random

→ The total sample size must be atleast 50 obs.

→ The data should be expressed in orig. units. Units should not be expressed in % or ratio

Applications

(1) → Test for Independence of Attr.

→ We can find out whether 2 or more attr. are associated or not

(2) As Goodness of fit

→ On several occassions the decision maker needs to understand whether an actual sample distribution matches or coincides with a known prob. distribution such as poisson, binomial or normal

→ χ^2-test for goodness of fit enables us to determine the extent to which the theoretical prob. distribution coincides with empirical sample distribution.

(3) For Yate's corr. of continuity :

→ The distribution of χ^2 test stat. is continuous but data under ~~test~~ test is categorical which is discrete.

→ It causes errors due to discrete data & if it is a 2X2 contingency table then we can apply Yate's corr. for continuity

(4) For Pop. variance

→ Considered as parametric test

→ Ass. underlying ~~pop.~~ χ^2 test is that the pop. from which the samples are drawn is normally distributed.

$$\chi^2 = \frac{\sigma_s^2}{\sigma_p^2} \times (n-1) \qquad v = n-1$$

σ_s = variance of sample n = sample size

σ_p = variance of pop.

(5) Test for Homogeneity

→ ~~Use~~ Useful in a case when we intend to verify whether several pop. are homogenous w.r.t. some char. of interest

→ E.g. The milk supplied by various suppliers has a particular ingredient in common (e.g. lactose) or not.

Q.1 In an anti-malarial campaign, Quinine was adm. to 500 persons out of tot. pop. of 2000. The no. of fever cases are shown below.

Treatment	Fever	No Fever	Total
Quinine	20	480	500
No Quinine	100	1400	1500
Total	120	1880	2000

Discuss the usefulness of quinine in checking malaria.

Soln: Ho : Quinine Not eff. in check malaria

Ha : " " ≠ " "

Treatment	Fever	(1) Exp. val.	No fever	(2) Exp. val.	Total
1) Quinine	20	30	480	470	500
2) No Quinine	100	90	1400	1410	1500
3) Total	120		1880		2000

$$E_{11} = \frac{RT \times CT}{N} = \frac{500 \times 120}{2000} = 30$$

$$E_{12} = \frac{RT \times CT}{N} = \frac{1500 \times 120}{2000} = 90$$

Col × Row

$$E_{21} = \frac{RT \times CT}{N} = \frac{500 \times 1800}{2000} = 470$$

$$E_{22} = \frac{1500 \times 1880}{2000} = 1410$$

Calc. of χ^2:

O	E	(O-E)	$(O-E)^2$	$(O-E)^2/E$
20	30	-10	100	3.33
100	90	+10	100	1.11
480	470	+10	100	0.21
1000				
1400	1410	-10	100	0.07
			$\Sigma \frac{(O-E)^2}{E}$	= 4.72

$$\chi^2_{cal.} = \Sigma \frac{(O-E)^2}{E} = 4.72$$

$$v = (c-1)(r-1) = 1 \times 1 = 1$$

Level of sig. = 5%.

$$\chi^2_{0.05} = 3.84$$

∴ H_0 is failed & rejected

∴ Quinine is useful in checking malaria

Q.2 A drug X claimed to be eff. in curing colds. In an exp. of 500 persons with cold, half of them were given the drug X & half of them were given placebo (sugar pills).

Table of patients reactions:

Treat.	Helped	Rxn	No eff.	Total
Drug	150	30	70	250
Placebo	130	40	80	250
Total	280	70	150	500

On the basis of data, can it be concluded that there is a sig. difference in effect of drug X & placebo.

Soln:

Ho : Drug = Placebo

Ha : Drug ≠ Placebo

Given:

Treat.	Help.	E	Rxn	E	No eff.	E	Total
Drug	150	140	30	35	70	75	250
Placebo	130	140	40	35	80	75	250
	280		70		150		500

$E_{11} = 140$ $E_{21} = 35$ $E_{31} = 75$

O	E	(O-E)	$(O-E)^2$	$(O-E)^2/E$
150	140	+10	100	0.714
130	140	-10	100	— 11 —
30	35	-5	25	— 11 —
40	35	+5	25	— 11 —
70	75	-5	25	0.333
80	75	+5	25	— 11 —

$$\sum \frac{(O-E)^2}{E} = 3.522$$

$\chi^2 = 3.522$ $\nu = (3-1)(2-1) = 2$

$\chi^2_{0.05} = 5.99$

$\therefore$ H_0 is accepted