F - Test

The F-test is named after R.A. Fischer.

Obj. of F-Test

To find out whether the 2 indep. est. of population variance differ significantly.

OR

To find out whether the 2 samples may be regarded as drawn from the ~~or~~ normal pop. having same variance

→ So to carry of test of significance, we have to cal. ratio of F

$$\boxed{f = \frac{\sigma_1^2}{\sigma_2^2}} \quad \text{or} \quad \boxed{f = \frac{S_1^2}{S_2^2}}$$

$$\sigma_1^2 > \sigma_2^2 \qquad S_1^2 > S_2^2$$

Var. is square of S.D.

$$\boxed{\sigma^2 = \frac{\Sigma(x-\mu)^2}{n}}, \quad \boxed{S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}}$$

In short,

$$F = \frac{\text{Larger estimate of variance}}{\text{Smaller estimate of variance}}$$

D.O.F (v)

v_1 (numerator) $= n_1 - 1$ (larger var.)
v_2 (denominator) $= n_2 - 1$ (Smaller var.)

Now, cal. F-value is comp. wit table. F value for v_1 & v_2 at 1% or 5% ~~L.O.S~~ level of sig.

① $F_{cal.} < F_{Table.}$

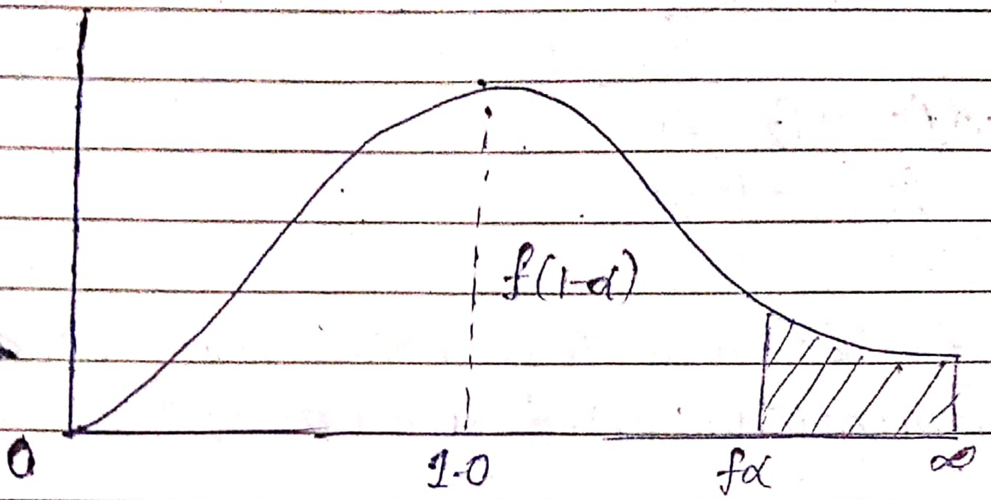
Ho acc. & No sig. diff. betn 2 var. ~~var~~ (S^2)

② $F_{cal.} > F_{Table.}$

Ho ~~acc~~ rejected & sig. diff b/w 2 var. (S^2

∴ F-test is based on ratio of var., its called "Var. Ratio Test".

Prop. of F- distr.



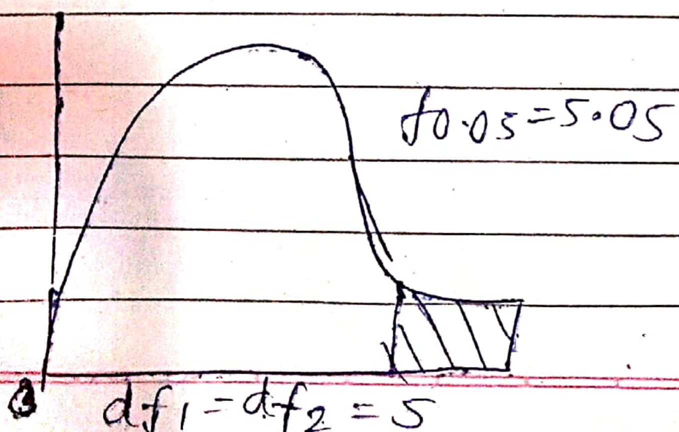
F- Distr. curve for n. d.o.f

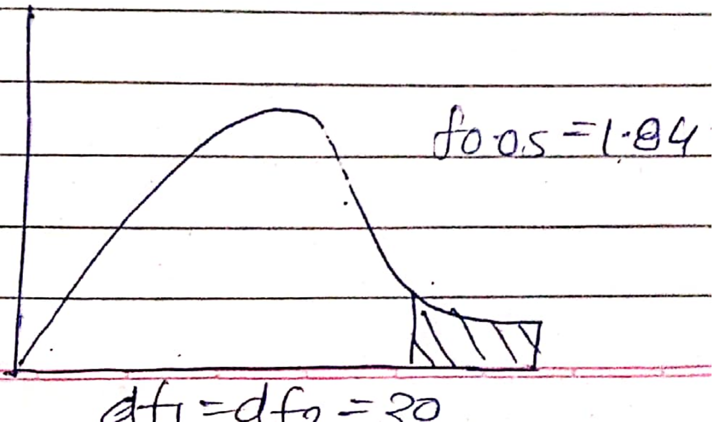
→ F-Distr. curve is skewed towards right with range 0 to ∞ & having roughly median value 1.

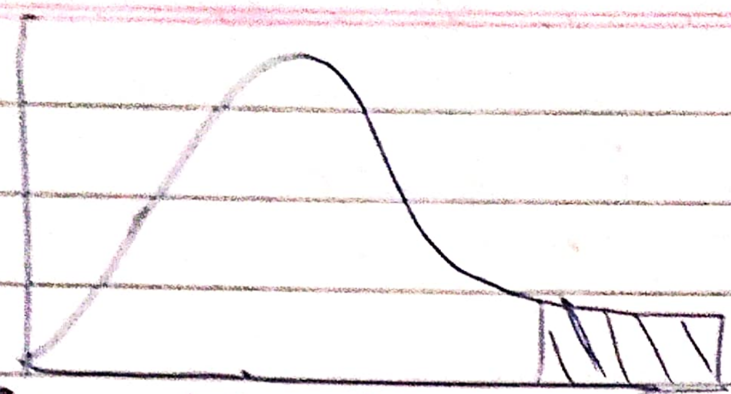
→ Value of F will always be more than 0

→ Shape of F-distr. curve is dep. on
  → d.o.f of num.
  → d.o.f of den.

→ F-distr. curve is never ~~symm.~~ symm., but if d.o.f will be inc. then it will be more similar to the symm. shape







0 $df_1 = df_2 = 60$

$f_{0.05} = 1.53$

→ Deg. of skewness inc. with inc. in d.o.f. (ν) for numerator (ν_1) & den. (ν_2)

→ Shape of curve will be more symm. by inc. in d.o.f

→ In F-test ~~=~~ var. would be compared from randomly drawn samples & the obs. are independent

Q.1 2 random samples were drawn from 2 normal pop. & their values are:

A : 16, 17, 25, 26, 32, 34, 38, 40, 42

B : 14, 16, 24, 28, 32, 35, 37, 42, 43, 45, 47

Test whether 2 pop. have same var. at 5% level of significance

A : $n = 9$ B : $n = 11$

$\cancel{A}$:

$H_0 = \sigma_1^2 = \sigma_2^2$ $H_a = \sigma_1^2 > \sigma_2^2$

Calculation Table:

A	$(x - \bar{x}_A)$	$(x - \bar{x}_A)^2$	B	$(x - \bar{x}_B)$	$(x - \bar{x}_B)^2$
16	-14	196	14	-19	361
17	-13	169	16	-17	289
25	-5	25	24	-9	81
26	-4	16	28	-5	25
32	+2	04	32	-1	1
34	+4	16	35	+2	4
38	+8	64	37	+4	16
40	+10	100	42	+9	81
42	+12	144	43	+10	100
270	$\Sigma(x - \bar{x}_A)^2 = 734$		45	+12	144
			47	+14	196
			363	$\Sigma(x - \bar{x}_B)^2 = 1298$	

$\bar{x}_A = \frac{\Sigma x_A}{n_A} = \frac{270}{9} = 30$

$x_B = \frac{363}{11} = 33$

$F = \frac{S_1^2}{S_2^2}$, $S_1^2 > S_2^2$ $S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

$S_2^2 = \frac{734}{9-1} = \frac{734}{8} = 91.75$

$S_1^2 = \frac{1298}{11-1} = \frac{1298}{10} = \cancel{19}\,129.8$

$F = \frac{129.8}{91.75} = 1.4147$ (Cal. F value)

$v_1 = 11-1 = 10$ $v_2 = 9-1 = 8$

$F_{0.05} = 3.35$ (Tab. value)

∵ Cal. F val. is smaller than Tab. F val., ∴ H_0 is acc.

2) In a sample of 9 obs., the sum of sq. dev. of items from the mean was 64.
In another sample of 11 obs., the value was found to be 88.
$l.o.s = 5\%$

Given: $n_A = 9$, $\Sigma(x-\bar{x}_A)^2 = 64$

$n_B = 11$, $\Sigma(x-\bar{x}_B)^2 = 88$

Let's take the hyp. ~~as~~ that variance of the two samples are not significant.

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_a : \sigma_1^2 > \sigma_2^2$

$F = \frac{S_1^2}{S_2^2}$, $S_1^2 > S_2^2$ $S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

$S_2^2 = \frac{64}{9-1} = 8$ ~~S_2~~ $S_1^2 = \frac{88}{11-1} = 8.8$

$F = \frac{8.8}{8} = 1.1$ (cal. F value)

$v_1 = n_1 - 1 = 11 - 1 = 10$
$v_2 = n_2 - 1 = 9 - 1 = 8$

$F_{0.05} = 3.35$
(Tab. F value)

Cal. F value is smaller than tab. F val., so H_0 is accepted. Hence, the diff. in variance of 2 samples is not significant at 5% l.o.s