MOMENTS & M.G.F.

① **Raw moments or moments about origin**

$$\mu_r' = E(x^r)$$

$$\mu_1' = E(x) = \Sigma x P(X=x).$$

$$\mu_4' = E(x^4) = \Sigma x^4 P(X=x)$$

$$\mu_r' = E(x^r) = \Sigma x^r P(X=x).$$

② **Central moments or moments about mean**

$$\mu_r = E[(x-\bar{x})^r]$$

$\mu_0' = 1$, $\mu_0 = 1$, $\cancel{\mu_1} = \not{p}$

$\mu_1 = 0$, $\mu_2 = \sigma^2$, $\bar{x} = A + \mu_1'$

$$\mu_r = \mu_r' - {}^rC_1 \mu_{r-1}' \mu_1' + {}^rC_2 \mu_{r-2}' \mu_1'^2 - {}^rC_3 \mu_{r-3}' \mu_1'^3 + \dots + (-1)^r \mu_1'$$

$$\mu_r' = \mu_r + {}^rC_1 \mu_{r-1} \mu_1' + {}^rC_2 \mu_{r-2} \mu_1'^2 + {}^rC_3 \mu_{r-3} \mu_1'^3 + \dots + \mu_1'^r$$

(3) Moment about any arbitrary point a

$$\mu_{ar} = E[(x-a)^r]$$

(4) **M.G.F.**

M.G.F. about a r.v. X (about origin) having prob. funcⁿ f(x) is given by

$$\boxed{M_x(t) = E(e^{tx})}$$

r.v.

$$M_x(t) = \begin{cases} \sum e^{tx} f(x) & \rightarrow \text{d.r.v} \\ \int e^{tx} f(x)\,dx & \rightarrow \text{c.r.v.} \end{cases}$$

$$M_x(t) = E\left[1 + tx + \frac{t^2x^2}{2!} + \cdots + \frac{t^r x^r}{r!} + \cdots\right]$$

$$M_x(t) = 1 + tE(X) + \frac{t^2}{2!}E(X^2) + \cdots \frac{t^r}{r!}E(X^r) \cdots$$

$$M_x(t) = 1 + t\mu_1' + \frac{t^2}{2!}\mu_2' + \cdots + \frac{t^r}{r!}\mu_r'$$

where,

$$\mu_r' = E(x^r) = \Sigma x^r P(x)$$

$$\mu_r' = E(x^r) = \int x^r f(x)\, dx$$

Coeff. of $\frac{t^r}{r!}$ is μ_r'

$\mu_r' \rightarrow r^{th}$ moment of x about origin

M.G.F. about various points

① $M_x(t) = E(e^{tx})$ → about $x = 0$

② $M_{x=a}(t) = E(e^{t(x-a)})$ about $x = a$

$$= E\left[1 + t(x-a) + \frac{t^2}{2!}(x-a)^2 + \frac{t^3}{3!}(x-a)^3 + \cdots \cdots + \frac{t^r}{r!}(x-a)^r + \cdots\right]$$

$$= 1 + tE(x-a) + \frac{t^2}{2!}E(x-a)^2 + \cdots + \frac{t^r}{r!}E(x-a)^r$$

$$= 1 + t\mu_1'(a) + \frac{t^2}{2!}\mu_2'(a) + \frac{t^3}{3!}\mu_3'(a) + \cdots + \frac{t^r}{r!}\mu_r'(a)$$

where, ~~$\mu_r' t$~~

$$\boxed{\mu_r'(a) = E(x-a)^r}$$

$$M_{x=\bar{x}}(t) = E(e^{t(x-\bar{x})})$$

$$M_{x=\bar{x}}(t) = 1 + tE(x-\bar{x}) + \frac{t^2}{2!}E(x-\bar{x})^2 + \cdots$$
$$\cdots + \frac{t^r}{r!}E(x-\bar{x})^r + \cdots$$

$$= 1 + t\mu_1 + \frac{t^2}{2!}\mu_2 + \cdots + \frac{t^r}{r!}\mu_r + \cdots$$

μ_r = r^{th} mom. ~~of~~ about mean

$$\mu_r = E(x-\bar{x})^r$$

$r=1$

$$\mu_1 = E(X-\bar{X}) = 0$$

The dev. of a series about its mean is zero

$r=2$

$$\mu_2 = E(X-\bar{X})^2 = \sigma^2 = \text{Var.}$$

$$\mu_r' = \left[\frac{d^r}{dt^r} M_x(t)\right]_{t=0}$$

Prop.

① $M_{cx}(t) = M_x(ct)$

constant   r.v.

② <u>Change of origin</u>

$x \to$ r.v.

$x-a$ = change origin

$$M_x(t) = E(e^{tx})$$

~~we~~ if we change x to $x-a$

$$M_{x-a}(t) = E(e^{t(x-a)})$$

$$= \cancel{E(e^{tx} - e^{at})}$$

$$= \cancel{E(e^{tx}, e^{-})}$$

$$= E(e^{tx-ta})$$

$$= E(e^{tx} \cdot e^{-ta})$$

$$= e^{-at} \cdot E(e^{tx})$$

$$\boxed{M_{x-a}(t) = e^{-at} M_x(t)}$$

③ Change of ~~sate~~ scale

$x \to r.v$

$ax \to$ change scale

$$M_x(t) = E(e^{tx}) \quad -①$$

we change x to ax

$$M_{ax}(t) = E(e^{tax})$$

$$\boxed{M_{ax}(t) = M_x(ta)}$$

④ Effect of change of origin & scale both

$X \to r.v.$

$\frac{x-a}{b} \to$ change of origin & scale

$$M_{\frac{x-a}{b}}(t) = e^{-at/b} M_x\left(\frac{t}{b}\right)$$

⑤ $M_{x_1+x_2+x_3+\cdots x_n}(t) = M_{x_1}(t) \cdot M_{x_2}(t) \cdots M_{x_n}(t)$

P.M.F.

X	x_1	x_2	x_3	x_4	----	x_k
$P(X)$	P_1	P_2	P_3	P_4		

Cumulative distribution funcn:

X	x_1	x_2	x_3	----	x_k
$F(x_i)$	$p(x_i)$	$\sum_1^2 p(x_i)$	$\sum_1^3 p(x_i)$	----	

Imp. prop. of distribution function:

1) $0 \leq F(x) \leq 1$.

2) $F(x) = 0$ for $x < a$ & $F(x) = 1$ for $x > b$

3) $F(x)$ is a step funcn

P.D.F.

Prop: $\rightarrow f(x)$ is integrable

$\rightarrow f(x) \geq 0$

$\rightarrow \int_a^b f(x)\,dx = 1$ X lies in $[a,b]$

$\rightarrow \int_\alpha^\beta f(x)\,dx = P(\alpha \leq X \leq \beta)$ where $a < \alpha < \beta < b$

$\rightarrow 0 \leq F(x) \leq 1$

Joint ~~p.d~~ ~~p.d.f~~ prob. distribution

$\boxed{p(x_i, y_j) \geq 0}$ & $\boxed{\sum_{j=1}^{\infty} \sum_{i=1}^{\infty} p(x_i, y_j) = 1}$

Marginal ~~p.d~~ prob. distribution

The prob. that X will take a particular value irrespective of the values of Y. vice ~~ver~~ is called marginal prob. distribution of X.

Y \ X	y_1	y_2	y_3	Total
x_1	P_{11}	P_{12}	P_{13}	P_1
x_2	P_{21}	P_{22}	P_{23}	P_2
x_3	P_{31}	P_{32}	P_{33}	P_3
Total	P_1'	P_2'	P_3'	1

Marginal prob. dist. of X:

X	x_1	x_2	x_3	...	x_k	Sum
P(X)	P_1	P_2	P_3		P_k	1

$\cancel{P(X_1 = x_1)}$ $P(X=x, Y=y) = \cancel{P(x,y)}\ P(x,y)$

$P(X=x) = \sum_{y=0}^{n} P(x,y)$ $\quad E(X+Y) = \sum\sum p_{ij}(x_i + y_j)$

$E(XY) = \sum\sum p_{ij} x_i y_j$

$\cancel{P(x=y)}$

$\cancel{P(x=}\ P(Y=y) = \sum_{x=0}^{n} P(x,y)$

Two dim. continuous prob. dist.

(i) $f_{XY}(x,y) \geq 0$ (ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1$

(iii) $\int_{a}^{b}\int_{c}^{d} f_{XY}(x,y)\,dx\,dy$

$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$ $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$

Expectation : $E(X)$

d.r.v. $\rightarrow$ $E(X) = \sum p_i x_i$

c.r.v. $\rightarrow$ $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

For any funcn $g(x)$ of X

$E[g(x)] = \sum p_i\, g(x_i)$ $\rightarrow$ d.r.v

$E[g(x)] = \int_{-\infty}^{\infty} g(x)\cdot f(x)\,dx$ $\rightarrow$ c.r.v.

$\rightarrow E(X) \geq 0$ $\longrightarrow E(aX+b) = aE(X)+b$

$\rightarrow E(X \pm Y) = E(X) \pm E(Y)$

$E(X_1 \pm X_2 \pm X_3 \pm \dots \pm X_n) = E(X_1) \pm E(X_2) \pm E(X_3) \pm \dots \pm E(X_n)$

$\rightarrow E(X, Y, Z, \dots, W) = E(X)\cdot E(Y)\cdot E(Z) \dots E(W)$

Median :

$$\int_a^M f(x)\,dx = \int_M^b f(x)\,dx = \frac{1}{2}$$

Mode

$\frac{dy}{dx} = 0$ i.e. $f'(x) = 0$

& $f''(x) < 0$

Prop. of Variance :

→ $V(ax+b) = a^2 V(x)$

→ $\cancel{V(a_1x_1 + a_2x_2) = a_1^2 V_1(x}$

→ $V(a_1x_1 + a_2x_2) = a_1^2 V(x_1) + a_2^2 V(x_2)$

→ $V(x_1 \pm x_2) = V(x_1) \pm V(x_2)$