Numerical: Refrigerator Insulation Thickness

Problem: Design a small refrigerator for a dorm room. Determine the minimum thickness of insulation (k= 0.03 W/(m*K)) required to maintain a desired temperature difference (ΔT) of 15°C between the inside (5°C) and outside (20°C) of the refrigerator. The total surface area of the refrigerator (excluding the door) is 1.5 m^2. The rate of heat transfer (Q) through the insulation needs to be limited to 50 W.

Solution:

1. **Heat Transfer Equation:** We can use the Fourier's Law of heat conduction to find the heat transfer rate:

$Q = k * A * (\Delta T) / L$

where: - Q is the heat transfer rate (W)

- k is the thermal conductivity of the insulation (W/(m*K))
- A is the surface area of the refrigerator (m^2)
- ΔT is the temperature difference (°C)
- L is the thickness of the insulation (m)

2. **Rearrange the equation for L:**
   $L = k * A * (\Delta T) / Q$
3. **Plug in the values:**
   L = 0.03 W/(m*K) * 1.5 m^2 * (15°C) / 50 W
4. **Solve for L:**
   $L \approx 0.0675$ m $\approx$ 6.75 cm

Therefore, the minimum thickness of insulation required is approximately 6.75 cm.

Additional Considerations:

- This is a simplified model and doesn't account for factors like door insulation, heat generated by the compressor, or leakage around the door seal.
- In real-world applications, additional safety factors are added to account for these uncertainties.
- Students can explore how the thickness changes with different materials (varying thermal conductivity) or desired temperature difference.

Refrigerator Design Numerical

Problem: Designing a new refrigerator for a dorm room. The desired volume for food storage is 5 cubic feet (ft^3). The refrigerator will have walls with a thickness of 4 inches (in) all around. To minimize energy consumption, you want to use the most efficient insulation material available, which has a thermal conductivity (k) of 0.03 BTU/(hr ft °F). The desired temperature difference between the inside (T_i) and outside (T_o) of the refrigerator is 30°F (T_i = 40°F, T_o = 70°F).

Objective:

1. Determine the total interior volume of the refrigerator after accounting for the wall thickness.
2. Calculate the surface area of the refrigerator needed for heat transfer.
3. Estimate the rate of heat transfer (Q) through the refrigerator walls.

Solution:

1. **Interior Volume:**

- First, convert the wall thickness from inches to feet: 4 in / (12 in/ft) = 1/3 ft
- Subtract the wall thickness from each dimension of the refrigerator's desired interior space to find the actual usable space. Since it's a cubic volume, we can subtract from a single side length:
  - Usable side length = 5 ft - (2 * 1/3 ft) = 4 1/3 ft
- Calculate the final usable volume:
  - Usable Volume = (4 1/3 ft)^3 ≈ 72.9 ft^3

2. **Surface Area for Heat Transfer:**

- To calculate the surface area, we need to consider the six sides of the refrigerator. However, since the walls have a constant thickness, the top and bottom areas will be the same as the sides, and the front and back will also be the same size (assuming a rectangular design).
- We can represent the sides, top/bottom, and front/back with separate variables for simplicity:
  - Side area (each): L x W (where L and W are the length and width of the usable space)
  - Top/Bottom area (each): L x W
  - Front/Back area (each): H x W (where H is the height of the usable space)
- Total surface area:
  - Surface Area = 2 (L x W) + 2 (L x W) + 2 (H x W)
  - Simplifying: Surface Area = 6 (L x W + H x W)

3. **Rate of Heat Transfer:**

- We can use the formula for heat conduction through a flat wall:
  - Q = k * A * (T_i - T_o) / L
  - Where:
    - Q = Rate of heat transfer (in BTU/hr)
    - k = Thermal conductivity of the insulation (0.03 BTU/(hr ft °F))
    - A = Surface area of the refrigerator walls (calculated in step 2)

- T_i = Interior temperature (40°F)
- T_o = Exterior temperature (70°F)
- L = Wall thickness (converted to feet in step 1)

Note: This is a simplified model and doesn't account for factors like door seals, air circulation inside the refrigerator, or the efficiency of the cooling system.

Solution :

Surface Area: Calculate the surface area based on your estimated dimensions (L, W, and H) of the usable refrigerator space.

Rate of Heat Transfer: Plug the values for k, A (calculated surface area), T_i, T_o, and L into the formula and solve for Q. The result will be the estimated rate of heat transfer through the refrigerator walls in BTU/hr.

This numerical exercise helps you understand the relationship between design choices (wall thickness, insulation), refrigerator size, and energy consumption.

Refrigerator Design Numerical

Problem: A student is designing a dorm-sized refrigerator. The desired internal volume of the refrigerator is 3.5 cubic feet (ft^3). The walls, including the door, will be constructed with 2 inches of rigid foam insulation (k = 0.025 BTU/(hr*ft*°F)) sandwiched between two thin metal sheets (assumed negligible thermal resistance). The target temperature inside the refrigerator is 40°F, while the room temperature is expected to be 70°F.

Task:

1. Determine the minimum total surface area of the refrigerator needed to maintain the desired temperature difference.
2. Considering a worst-case scenario where the refrigerator door is opened 10 times a day for 1 minute each time, calculate the additional heat gain due to door openings.
3. Suggest an appropriate compressor size based on the total heat gain.

Solution:

1. **Surface Area:**

- We need to find the heat transfer through the walls to maintain the temperature difference. We can use the formula for conduction heat transfer:

$$Q = k * A * (T_h - T_c) / L$$

Where:

- Q = heat transfer rate (BTU/hr)
- k = thermal conductivity of insulation (0.025 BTU/(hr*ft*°F))
- A = surface area of the walls (ft^2)
- T_h = hot side temperature (room temperature, 70°F)
- T_c = cold side temperature (refrigerator temperature, 40°F)
- L = thickness of insulation (2 inches, convert to ft: 2 in / 12 in/ft = 0.167 ft)

We know the desired internal volume ($3.5\ ft^3$) but not the surface area (A). However, we can manipulate the formula to solve for A:

$A = Q * L / (k * (T_h - T_c))$

Heat Transfer Rate (Q):

- Refrigerators typically aim to remove 100 BTU per cubic foot of volume per day to maintain proper cooling. So, for a $3.5\ ft^3$ refrigerator, the daily heat removal is:

Daily heat removal = $100\ BTU/ft^3 * 3.5\ ft^3 = 350$ BTU

We need to convert this to heat transfer rate per hour:

Q = Daily heat removal / 24 hours = 350 BTU / 24 hr = 14.58 BTU/hr

Surface Area Calculation:

$A = 14.58\ BTU/hr * 0.167\ ft / (0.025\ BTU/(hr*ft*°F)) * (70°F - 40°F)$

$A \approx 38.2\ ft^2$

Therefore, the minimum total surface area of the refrigerator needed is approximately $38.2\ ft^2$.

2. **Heat Gain from Door Openings:**

- Heat gain per door opening can be estimated using the formula:

$Q_{door} = V_{door} * Cp * (T_h - T_c)$

Where:

- Q_{door} = heat gain per door opening (BTU)
- V_{door} = volume of air entering the refrigerator with each opening (estimated, assume $0.1\ ft^3$)
- Cp = specific heat capacity of air (around 0.024 BTU/(lb*°F))
- T_h = hot side temperature (room temperature, 70°F)
- T_c = cold side temperature (refrigerator temperature, 40°F)

$Q_{door} = 0.1\ ft^3 * 0.024\ BTU/(lb*°F) * (70°F - 40°F) \approx 0.72$ BTU

- Total heat gain due to door openings per day:

Total heat gain_door = Q_door * number of openings = 0.72 BTU/opening * 10 openings/day = 7.2 BTU/day

3. **Compressor Size:**

- We need to account for both the continuous heat leak through the walls and the additional heat gain from door openings.
- Total daily heat gain:

Total heat gain_day = Daily heat removal + Total heat gain_door = 350 BTU + 7.2 BTU = 357.2 BTU

- Safety factor: It's wise to consider a safety

Refrigerator Design Numerical

Problem: You are designing a new refrigerator with a single compartment. The desired internal volume of the refrigerator is 200 liters (L). The insulation material you plan to use has a thermal conductivity (k) of 0.03 W/(m·K). The desired temperature difference between the inside (T_i) and outside (T_o) of the refrigerator is 15°C (T_i = 5°C, T_o = 20°C). The average surface area (A) of the refrigerator exposed to the surrounding air is estimated to be 3 m^2.

Objective:

1. Determine the minimum thickness (x) of the insulation required to maintain the desired temperature difference.
2. Calculate the heat leak (Q) into the refrigerator per hour.

Solution:

1. **Minimum Insulation Thickness:** We can use the formula for heat conduction through a flat wall to determine the minimum insulation thickness:

$$Q = k * A * (T_o - T_i) / x$$

where:

- Q is the heat transfer rate (W)
- k is the thermal conductivity (W/(m·K))
- A is the surface area (m^2)
- T_o is the outside temperature (K)
- T_i is the inside temperature (K)
- x is the thickness of the insulation (m)

We want to find the minimum thickness (x) that will achieve the desired heat transfer rate (Q). In a refrigerator, we want to minimize heat entering the cold compartment (Q should be negative). Let's assume a heat leak of -10 W (negative to indicate heat entering the refrigerator).

Converting temperatures to Kelvin:

- T_o = 20°C + 273.15 = 293.15 K
- T_i = 5°C + 273.15 = 278.15 K

Plugging in the values:

$$-10\ W = 0.03\ W/(m\cdot K) * 3\ m^2 * (293.15\ K - 278.15\ K) / x$$

Solving for x:

$$x = (0.03\ W/(m\cdot K)) * 3\ m^2 * (15\ K) / (-10\ W)$$

$$x \approx 0.135\ m \text{ (or 13.5 cm)}$$

Therefore, the minimum thickness of the insulation required is approximately 13.5 cm.

2. **Heat Leak Calculation:**

Now that we have the minimum thickness, we can calculate the actual heat leak into the refrigerator per hour.

Using the same formula with the calculated thickness ($x = 0.135$ m) and assuming the desired temperature difference:
$Q = 0.03 \text{ W/(m·K)} * 3 \text{ m}^2 * (293.15 \text{ K} - 278.15 \text{ K}) / 0.135 \text{ m}$
$Q \approx -7.4 \text{ W}$

The negative value indicates heat entering the refrigerator. The actual heat leak into the refrigerator is approximately 7.4 W.
Note: This is a simplified example and doesn't account for factors like door opening, efficiency of the cooling system, or variations in ambient temperature.

Refrigerator Design Numerical
Problem: You are designing a new refrigerator for a dorm room. The desired internal volume of the refrigerator is 3.5 cubic feet (ft^3). The walls, including the door, will be constructed with 2 inches of rigid foam insulation (k = 0.025 Btu/hr*ft*°F). The internal temperature (T_{in}) needs to be maintained at 40°F, while the room temperature (T_{out}) is expected to be 70°F.
Tasks:

1. Calculate the minimum surface area of the refrigerator needed for efficient cooling.
2. Determine the theoretical heat leak rate into the refrigerator.

Solution:

1. **Surface Area Calculation:**
- We can simplify the shape of the refrigerator to a rectangular box. Let L, W, and H be the length, width, and height of the refrigerator (all in feet).
- The total surface area (A) of the box is:
  - $A = 2(LW + LH + WH)$
- We know the volume (V) of the refrigerator is 3.5 ft^3:
  - $V = LWH = 3.5 \text{ ft}^3$
- We can solve for one dimension (let's say L) in terms of the other two:
  - $L = V / (WH)$
- Since the walls are insulated, we need to consider the total thickness of the insulation (4 inches = 1/3 ft) when calculating the outer dimensions.

2. **Heat Leak Rate Calculation:**
- Once we have the surface area (A) from step 1, we can use the following formula to find the heat leak rate (Q) into the refrigerator:
  - $Q = U * A * (T_{out} - T_{in})$

- U is the overall heat transfer coefficient, which depends on the insulation material and thickness. We can assume steady-state conduction through the walls and neglect convection for simplicity.
    - $U = 1 / (thickness / k\_insulation) = 1 / (1/3\ ft / 0.025\ Btu/hrft°F) = 0.075\ Btu/hrft^2°F$

Solving for the unknowns:

1. We can't solve for all three dimensions (L, W, H) uniquely with the given information. However, we can find the relationship between them and the total surface area.

- Substitute L = V / (WH) into the formula for surface area:
    - $A = 2(V/(WH) * W + V/(WH) * H + WH)$
    - This simplifies to: $A = 10 * (V^{(2/3)})$

2. Now we can find the minimum surface area for a given volume ($3.5\ ft^3$):
    - $A\_min = 10 * (3.5^{(2/3)}) \approx 21.5\ ft^2$

Theoretical Heat Leak Rate:

- Using the calculated minimum surface area and the given parameters:
    - $Q = 0.75\ Btu/hrft^2°F * 21.5\ ft^2 * (70°F - 40°F) \approx 11\ Btu/hr$

Note:

- This is a simplified calculation and doesn't account for factors like door seals, internal components (shelves, lights), or the efficiency of the cooling system itself.
- The actual heat leak rate and surface area requirements might be slightly higher in a real refrigerator.

This numerical exercise helps undergraduate students understand the relationship between refrigerator size, insulation properties, and heat transfer. It also introduces them to basic heat transfer calculations and their application in appliance design.

Refrigerator Design Numerical: Optimizing Insulation Thickness

Problem: You are designing a new refrigerator for a dorm room. To minimize energy consumption, you want to optimize the thickness of the insulation layer in the refrigerator walls.

Given:

- Desired internal temperature (T_i): 5°C
- Ambient room temperature (T_a): 25°C
- Thermal conductivity of insulation (k): 0.03 W/(m*K)
- Convection heat transfer coefficient at inner and outer surfaces (h_i & h_o): 10 W/(m^2*K) (assumed equal for simplicity)
- Refrigerator wall area (A_{wall}): 2 m^2

Objective:

1. Determine the heat transfer rate (Q) through the refrigerator walls.
2. Find the optimal thickness (L) of the insulation layer that minimizes energy consumption.

Solution:

1. **Heat Transfer Rate:**
- We can use the formula for conduction heat transfer through a single layer:

  Q = (k * A_wall * (T_a - T_i)) / (L)
- Substitute the given values:

  Q = (0.03 W/(m*K) * 2 m^2 * (25°C - 5°C)) / (L)
- To solve for Q, we need the insulation thickness (L). We will calculate it for different thicknesses later.

2. **Optimizing Insulation Thickness:**
- Thicker insulation reduces heat transfer but also increases material cost and wall thickness. We can find the optimal thickness by considering the total energy consumption over a period.
- Assume the refrigerator operates continuously and the cost of electricity is 0.10perkWh.−TotalEnergyConsumption(E)foraperiod(t):E=Q∗t/(3600s/hr)∗(1kWh/1000 Wh)∗(/kWh)
- We want to minimize E with respect to L. However, E depends on Q, which depends on L. This is where it gets interesting!

Solution Approach:

- We can't solve for a single optimal thickness analytically. Here's a step-by-step approach for undergraduates:
  1. Choose multiple values for insulation thickness (L) - e.g., 2 cm, 5 cm, 10 cm.
  2. For each chosen L, calculate the heat transfer rate (Q) using the formula from step 1.
  3. Choose a specific operation time (t) - e.g., 24 hours (86400 seconds).
  4. Calculate the total energy consumption (E) for each L using the formula from step 2.
  5. Analyze the results. The thickness with the lowest E is the optimal choice for this scenario.

Insulation Thickness (L)	**Heat Transfer Rate (Q)**	**Energy Consumption (E)**	**Cost**
2 cm	Q = (0.03 * 2 * 20) / 0.02 = 60 W	E = 60 W * 24h = 1.44 kWh	$0.144
5 cm	Q = (0.03 * 2 * 20) / 0.05 = 24 W	E = 24 W * 24h = 0.576 kWh	$0.0576
10 cm	Q = (0.03 * 2 * 20) / 0.10 = 12 W	E = 12 W * 24h = 0.288 kWh	$0.0288
15 cm	Q = (0.03 * 2 * 20) / 0.15 = 8 W	E = 8 W * 24h = 0.192 kWh	$0.0192

Optimal Thickness Determination

From our calculations, 15 cm provides:

- Lowest energy consumption: 0.192 kWh

- Lowest operational cost: $0.0192 per day

Benefits:

This approach allows students to:

- Apply heat transfer principles to a practical design problem.
- Understand the trade-off between insulation thickness and energy consumption.
- Practice with numerical calculations and explore solutions for different scenarios (e.g., varying electricity cost, operation time).

Additional Considerations:

- This is a simplified model. Real refrigerators have multiple layers of insulation and other factors affecting heat transfer.
- Students can explore more complex models with multiple layers or consider additional factors like door opening frequency.

This numerical provides a foundation for understanding how to optimize refrigerator design through calculations, making it a valuable exercise for undergraduate students.