Sardar Patel Institute of Technology
Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India
(Autonomous College Affiliated to University of Mumbai)

Mid Semester Examination
March 2019

Course Code:CE41/IT41

Branch:Computer/IT

Name of the Course:**Design and Analysis of algorithm Synoptic**

Q.1

1. Define Asymptotic notation Ω notation with suitable diagram — 1 — CO1

Solution: For a given function g(n), we denote by Ω(g(n)) the set of functions.

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Ω(g(n)) = {f(n): there exist positive constants c and
              n0 such that 0 <= c*g(n) <= f(n) for
              all n >= n0}
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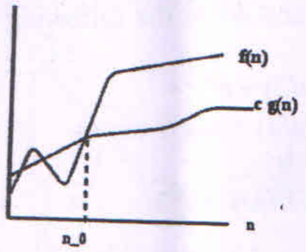


f(n) = Omega(g(n))

For definition 0.5 Marks , Diagram : 0.5 Marks

2. If all the elements in an input array is equal for example {5,5,5,5,5,5}, What would be the running time of the Insertion sort algorithm? — 1 — CO1

Solution: c) O(n) 1 Mark

3. State True or False and justify. — 1 — CO4

Dijkstra's algorithm may not terminate if the graph contains negative-weight edges.

Solution: False.

Justification: It always terminates after |E| relaxations and |V |+|E| priority queue operations, but may produce incorrect results **0.5 Mark – False , justification – 0.5 Marks**

4.What is the time complexity of Huffman Coding? — 1 — CO4

a O(n) **b. O(n log n)** c.O(n (log n)^2 d.O(n^2)

~~0.5~~ Mark – O(n log n) ~~0.5 for explanation~~.

5.Strassen's algorithm needs_____many multiplications to multiply two 2×2 matrices. — 1 — CO2

a. 8 b.9 c.7 d.3 Ans. 7. 1 mark.

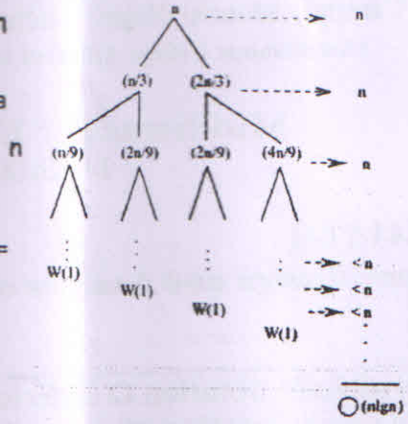
~~**0.5 Marks for 7 and 0.5 for justification**~~

Q.2 Solve the recurrence equation using recursive tree method

$$T(n) = T(n/3) + T(2n/3) + O(n).$$

For recurrence tree – 1Marks, Each level cost – 1 Mark , Total cost of all levels – 1 M , correct time complexity with justification– 2 Marks

$W(n) = W(n/3) + W(2n/3) + n$



- The longest path from the root to a leaf is:
  $n \to (2/3)n \to (2/3)^2 n \to \ldots \to 1$
- Subproblem size hits 1 when $1 = (2/3)^i n \Leftrightarrow i = \log_{3/2} n$
- Cost of the problem at level i = n
- Total cost:

$$W(n) < n + n + \ldots = n(\log_{3/2} n) = n\frac{\lg n}{\lg\frac{3}{2}} = O(n\lg n)$$

$$\Rightarrow W(n) = O(n\lg n)$$

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OR

State master theorem all cases. and solve the following recurrence using master method

i $T(n) = 2T(n/2) + n^2$

ii. $T(n) = T(n/2) + n(2 - \cos n)$

5 CO1

master theorem for each case 1 Mark (1*3)

Master Theorem

The Master Theorem applies to recurrences of the following form:

$$T(n) = aT(n/b) + f(n)$$

where $a \geq 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive function.

There are 3 cases:

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a} \log^k n)$ with[1] $k \geq 0$, then $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ with $\epsilon > 0$, and $f(n)$ satisfies the regularity condition, then $T(n) = \Theta(f(n))$.
   Regularity condition: $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n.

i. For solving each recurrence 1 Marks

Solution : $a = 2$, $b = 2$, $f(n) = n^2$

$n^{\log_b a} = n^1$

$f(n) > n^{\log_b a}$

So case 3 applies answer =

$T(n) = \text{theta}(n^2)$

Only answer written: 0.5 marks

Case specified with justification:0.5 marks

ii) $T(n) = T(n/2) + n(2 - \cos n)$

$\Longrightarrow$ Does not apply. We are in Case 3, but the regularity condition is violated. (Consider $n = 2\pi k$, where k is odd and arbitrarily large. For any such choice of n, you can show that $c \geq 3/2$, thereby violating the regularity condition.)

Only answer written: 0.5 marks

Case specified with justification:0.5 marks

Q.3 Find the optimal solution for the fractional knapsack problem making use of greedy approach. **5 CO4**

Consider- n = 5 , w = 60 kg ,
(w1, w2, w3, w4, w5) = (5, 10, 15, 22, 25)
(b1, b2, b3, b4, b5) = (30, 40, 45, 77, 90)
A thief enters a house for robbing it. He can carry a maximal weight of 60 kg into his bag. There are 5 items in the house with the following weights and values. What items should thief take if he can even take the fraction of any item with him?

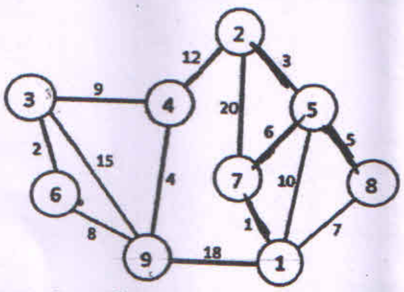
1 Mark - Compute the value / weight ratio for each item
1Mark - Sort all the items in the decreasing order of their value / weight ratios.
1 Mark –intermediate steps calculation

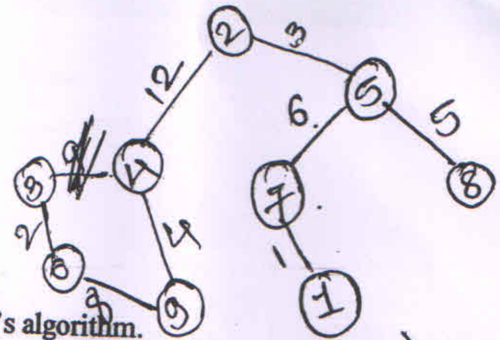
our knapsack finally will contain the items- 0.5 marks
< I1 , I2 , I5 , (20/22) of I4 >

1.5 **Marks for profit calculations** Total cost of the knapsack
= 160 + (20/22) x 77 = 160 + 70 = 230 units.

OR

Solve the following problem to obtain minimum spanning tree using Prim's algorithm **5 CO4**





Also comment on time complexity of Prim's algorithm.
For each correct step – **0.5 Marks (upto 4 Marks)**
For time complexity – **1 Marks .**

=4

Q.4 Analyze the time complexity of Quicksort for all cases by specifying recurrence equations. and justify it. **5 CO2**

Solution:

Best case time complexity – 2Marks T(n) = 2T(n/2) + n - Theta(n log n),
Worst Case time complexity 2Marks T(n) = T(n-1) + n - O(n^2) ,
Average case time complexity – 1 Marks T(n) = T(n/10) + T(9n/10) + n = Theta(n^2)

* only stated Time complexities all 3 then 0.5 marks for each correct statement