Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058-India

(Autonomous College Affiliated to University of Mumbai)

End Semester Examination

July 2019

Max. Marks: 60

Class: S.E.

Course Code: IT41 / CE41

Name of the Course: Design And Analysis of Algorithm - Synoptic

Duration: 3 Hrs

Semester: IV

Branch: IT/COMP

Instructions:

(4) **All Questions are Compulsory.**

(5) **Draw neat diagrams.**

(6) **Assume suitable data if necessary.**

Question No.	Question	Max. Marks	CO
Q. 1 a)	i. For each function f(n) along the left side of the table, and for each function g(n) across the top, write O, Ω, or Θ in the appropriate space, depending on whether f(n) = O(g(n)), f(n) = Ω(g(n)), or f(n) = Θ(g(n)). If more than one such relation holds between f(n) and g(n), write only the strongest one. The first row is a demo solution for $f(n) = n^2$.	03	CO1

		g(n)		
		n	n log n	n^2
f(n)	n^2	Ω	Ω	θ
	n^4	Ω	Ω	Ω
	log n	O	O	O

For each correct answer 0.5 Marks * 6 = 3 Marks

Question No.	Question	Max. Marks	CO
	ii. Strategy in which problem can be solved by combining solutions of non-overlapping sub problems, strategy is called **Divide & Conquer**.	01	CO1
Q.1.b)	i. Use the using substitution method to solve the given recurrence equation: $T(n)= 2T(n/2) + n$	02	CO1

For guessing correct solution: 1 Mark , for correct answer:1 Mark

We are trying to prove that $T(n) \le cn\lg(n)$. So we start by assuming that the bound holds for $\frac{n}{2}$:

$$\begin{aligned} T(n) &\le c\left(\frac{n}{2}\right)\lg\left(\frac{n}{2}\right) \\ &\le cn\lg\left(\frac{n}{2}\right) + n \\ &= cn\lg(n) - cn\lg(2) + n \\ &\le cn\lg(n). \end{aligned}$$

Since the last step holds for any $c > 1$, we are done. □

	ii. Solve the given recurrences using master theorem method $T(n) = 4T(n/2) + n^3$ **For finding case 3 applicable: 1 mark , For regularity condition: 1 Mark** $T(n) = 4T(n/2) + n^3$ Reading from the equation, $a = 4$, $b = 2$, and $f(n) = n^3$. Is $n^3 = \Omega(n^{\log_2 4+\epsilon}) = \Omega(n^{2+\epsilon})$? Yes, for $0 < \epsilon 1$, so case 3 *might* apply. Is $4(n/2)^3 \leq c \cdot n^3$? Yes, for $c \geq 1/2$, so there exists a $c < 1$ to satisfy the regularity condition, so case 3 applies and $T(n) = \Theta(n^3)$.	02	

Q. 1 c)	Write an algorithm to perform binary search using divide and conquer strategy. Apply it to search a given number say 63 on following array. Also derive its time complexity.
	5, 13, 27, 30, 50, 57, 63, 76
	Algorithm: 2 Marks , Searching number 63: 2 Marks , Coplexity: 2 Marks
	
	`Algorithm: Binary-Search(numbers[], x, l, r)`
	`if l = r then`
	`   return l`
	`else`
	`   m := ⌊(l + r) / 2⌋`
	`   if x ≤ numbers[m]  then`
	`      return Binary-Search(numbers[], x, l, m)`
	`   else`
	`      return Binary-Search(numbers[], x, m+1, r)`
	
	$T(n) = \begin{cases} 0 & if\, n = 1 \\ T(\frac{n}{2}) + 1 & otherwise \end{cases}$
	
	Time complexity O (log n)
		06	CO2

First m is determined and the element at index m is compared to x.

5	13	27	30	50	57	63	76
l=0			m=3				r=7

As x > numbers[3], the element may reside in numbers[4..7]. Hence, the first half is discarded and the values of l, m and r are updated as shown below.

5	13	27	30	50	57	63	76
				L=4	m=5		r = 7

Now the element x needs to be searched in numbers[4..7]. As x > numbers[5], new values of l, m and r are updated in a similar way.

5	13	27	30	50	57	63	76
						l=m=6	r = 7

Now, comparing x with numbers[6], we get the match. Hence, the position of x = 63 have been determined.

Q. 2 a)

i. Write a program in C for finding k^{th} smallest element from an array using divide and conquer approach. Show the output of your program for the given set of inputs to find 5^{th} smallest element : — 03 — CO2

22 13 -5 -8 15 60 17 31 47

For Correct program: 3 Marks

Showing All passes correctly: **1 Mark** — 01

Finding 5th element : 22 **1 Mark**

ii. Analyze it's time complexity by stating its recurrence relation in worst case. — 02

Time complexity O(n^2) 1 Mark

Q2. b)

i. State the steps to be followed to develop a dynamic programming solution for 0/1 knapsack problem — 03 — CO3

For each correct step : 0.5 Mark * 6 = 3 Marks

ii. Apply the Dynamic Programming approach to find Optimal solution for following 0/1 knapsack problem. Capacity of knapsack is **8 .** — 03

Item i	Value v_i	Weight w_i
1	15	1
2	10	5
3	9	3
4	5	4

For each correct row 0.5 Marks * 4 = 2 Marks, Constructing the

solution 1 Mark

		Capacity remaining g=0	g=1	g=2	g=3	g=4	g=5	g=6	g=7	g=8
k=0	f(0,g) =	0	0	0	0	0	0	0	0	0
k=1	f(1,g) =	0	15	15	15	15	15	15	15	15
k=2	f(2,g) =	0	15	15	15	15	15	25	25	25
k=3	f(3,g) =	0	15	15	15	24	24	25	25	25
k=4	f(4, g) =	0	15	15	15	24	24	25	25	<u>29</u>

OR

i. State and apply the steps to be followed to develop a dynamic programming solution to the Longest common Subsequence — *03*

For each step 0.5 * 6 = 3 Marks. — *03*

ii. Apply the Dynamic Programming approach to find LCS for following two strings.

X = ABCAB and Y = AABACA.

0	ϕ	A	B	C	A	B
ϕ	0	0	0	0	0	0
A	0	1	1	1	1	1
A	0	1	1	1	2	2
B	0	1	2	2	2	3
A	0	1	2	2	3	3
C	0	1	2	3	3	3
A	0	1	2	3	4	4

LCS = "ABCA"

For table – 3 Marks , For LCS construction – 3 Marks

Q .3a)	Explain branch and bound strategy in general and how it can be used to find solution of TSP along with example **For strategy = 03 Marks , TSP problem : 3 Marks**	*06*	CO4

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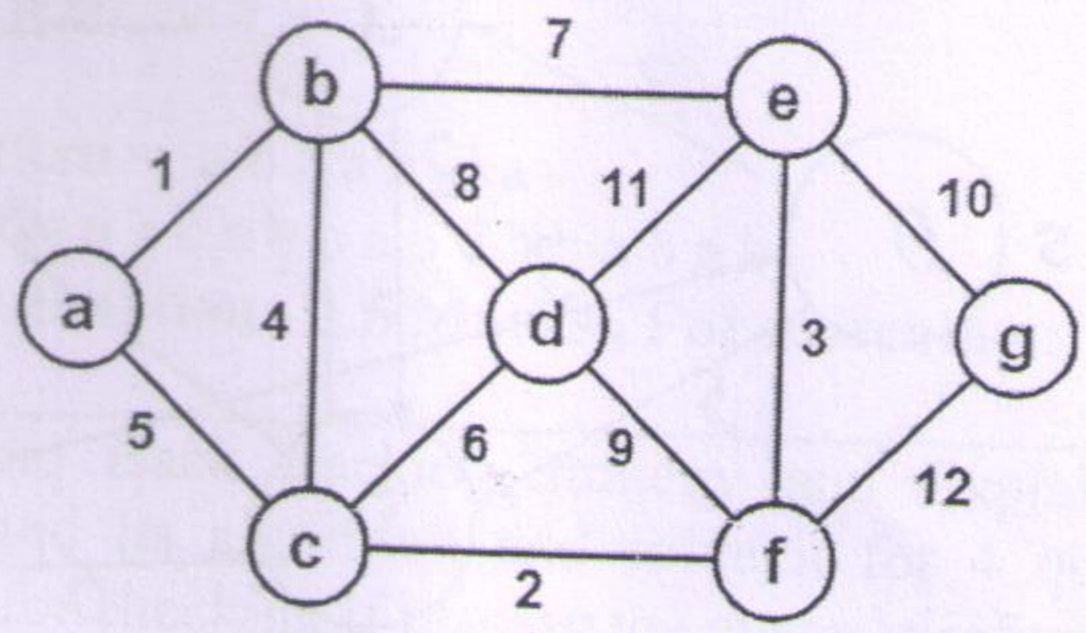
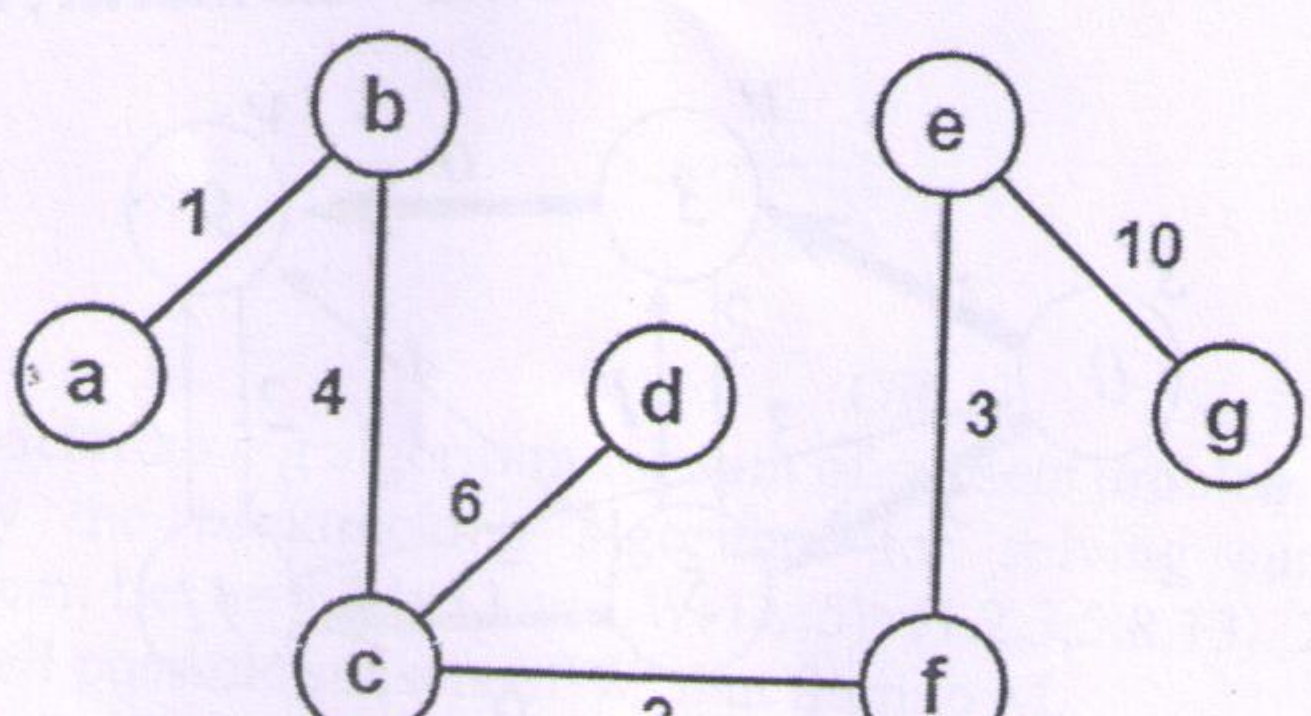
Q.3 b)	Construct the minimum spanning tree (MST) for the given graph using Prim's Algorithm. starting node is 1	*6*	CO3

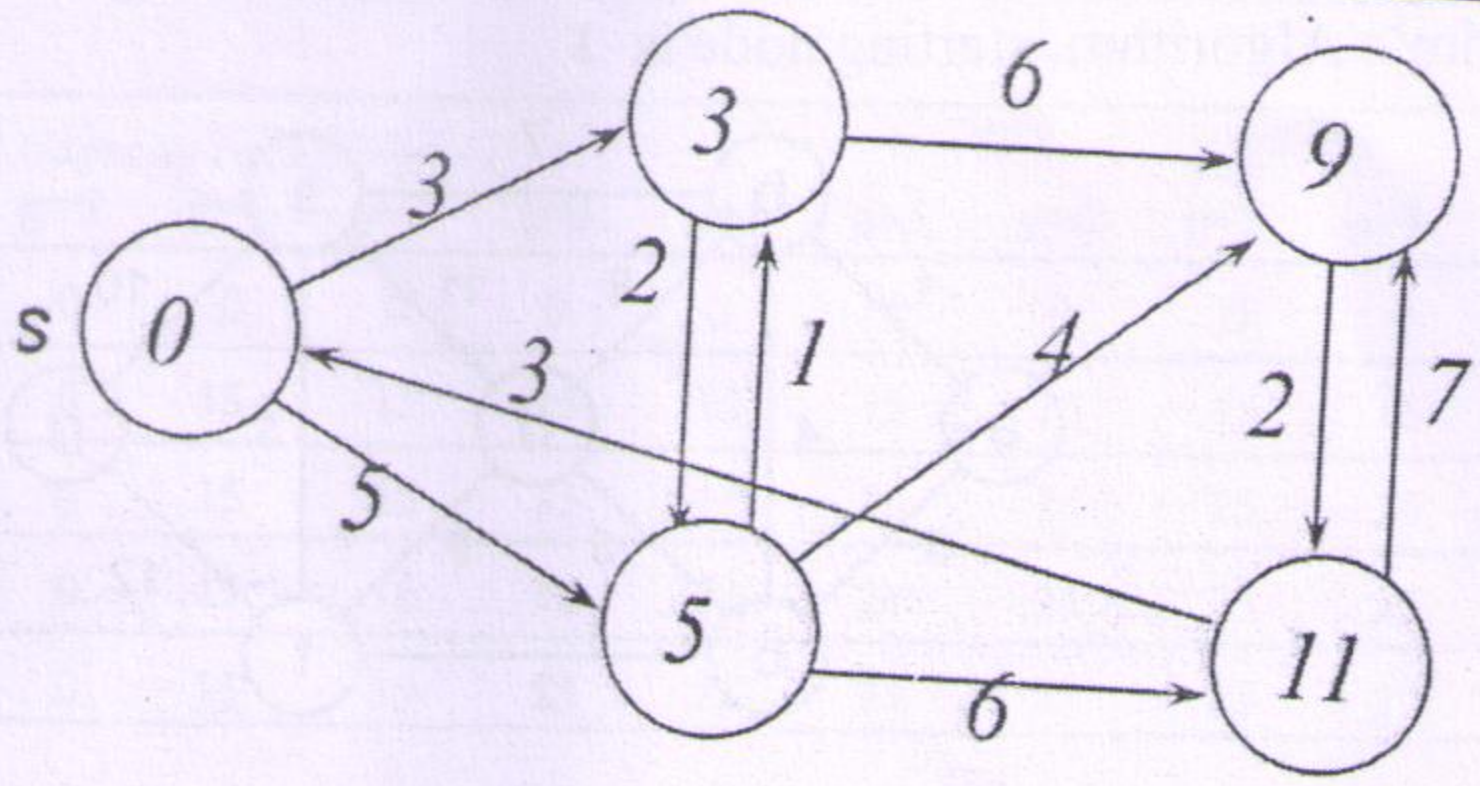


For each correct edge selection and explanation 1 Marks

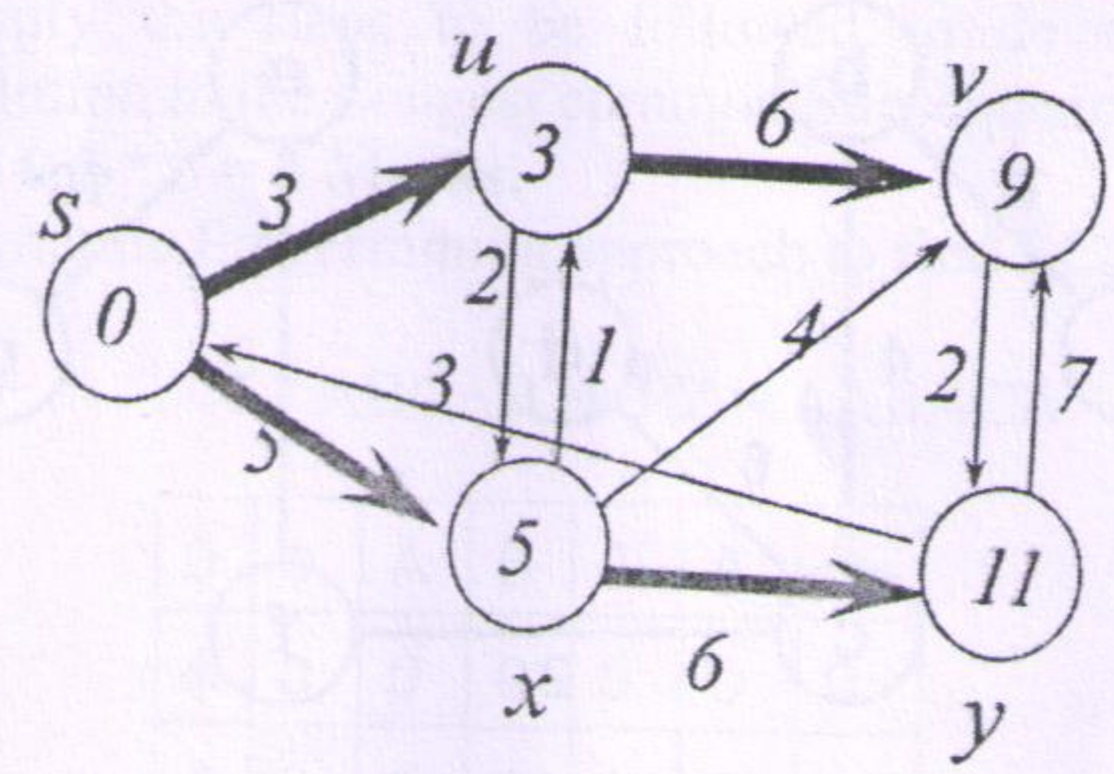


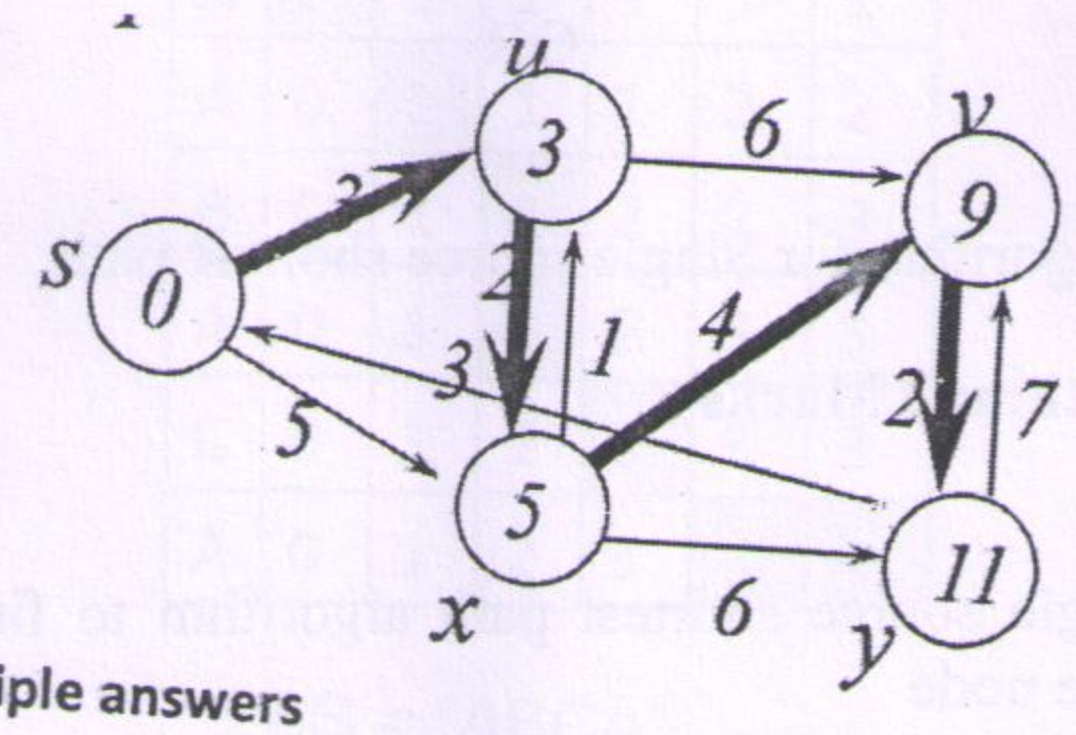
OR

iii.	Write an algorithm for Single source shortest path. **For algorithm: 3 Marks**	*03*	
iv.	Apply Single source shortest path algorithm to find shortest path from source node	*03*	



For each correct step 0.5 Marks * 5 Marks = 2.5 Marks , Final answer – 0.5Marks





There can be multiple answers

Q.4 a)	i. Construct the string matching automaton for the given pattern and show the sequences of states it enters in for the given text and also show occurrences of pattern in the text Pattern : a b c a b c a Text : a a a b a b c a b c a a b a a b **For table: 1.5 Marks , For matching : 1.5 marks**	03	CO5

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	ii. Compute KMP prefix function for the given pattern and check if it is present in given text. Pattern = a b c a b d a Text = a b c a b c a b d a b a b a b **For table: prefix function - 1.5 Marks , For matching : 1.5 marks**	*03*	
Q.4 b)	i. Explain the Back tracking strategy and Explain the N queen problem and its algorithm and solve it for 4 queen. Derive the condition for checking if placing the queen is safe or not ? **Strategy : 2 Marks , N queen algorithm and solution for 4 queen: 2 Marks , Condition checking 2 Marks** OR i. Write a backtracking algorithm for sum of subsets problem. 2M ii. Apply the backtracking algorithm for solving sum of subset problem. Let n=6, M=13 and W(1…5)=(1,2,3,5,8,13). 2M Find all possible subsets of W that Sum to M. iii. Draw the portion of state space tree for fixed tuple size solution. 2M	*06* *02* *02* *02*	CO4
Q.5a)	Make a linear programming model and identify the objective function and constraints and also formulate it into it's standard form: A woodworker builds and sells band-saw boxes. He manufactures two types of boxes using a combination of three types of wood, maple, walnut and cherry. To construct the Type I box, the carpenter requires 2 board foot (bf) (The board foot is a specialized unit of measure for the volume of lumber. It is the volume of a one-foot length of a board one foot wide and one inch thick) maple and 1 bf walnut. To construct the Type II box, he requires 3 bf of cherry and 1 bf of walnut. Given that he has 10 bf of maple, 5 bf of walnut and 11 bf of cherry and he can sell Type I of box for \$120 and Type II box for \$160, how many of each box type should he make to maximize his revenue? Assume that the woodworker can build the boxes in any size, therefore fractional solutions are acceptable **For each inequation 1 Mark * 4 = 4 Marks**	*04*	CO6

	Solution The decision variables in this problem are the number of Type I and II boxes to be built. They are denoted by x1 and x2 respectively. Since the goal is to maximize revenues and the revenues are a function of the number of boxes of each type sold, we can represent the objective function as max z =120x1 +160x2 One of the constraints in this problem is availability of different types of wood. Therefore, based on the number of boxes produced, the sum of the total wood requirement must be less than or equal to the available amount of wood for each type. We can represent this type of constraint with three inequalities referring to maple, cherry and walnut respectively as follows: $2x_1 \le 10$ $3x_2 \le 11$ $x_1 + x_2 \le 5$ In addition, there are the non-negativity constraints which ensure that our solution does not have negative number of boxes. These constraints are shown as x1, x2 ≥ 0 Standard Form: $\max z = 120x_1 + 160x_2$ $s.t.\quad 2x_1 + s_1 = 10$ $+3x_2 + s_2 = 11$ $x_1 + x_2 + s_3 = 5$ $x_1, x_2, s_1, s_2, s_3 \ge 0$		
Q. 5. b)	Solve the following problem using SIMPLEX maximize: P = 2x + 3y+ 4z subject to: 3x +2y + z ≤ 10 2x + 5y+ 3z ≤15 x, y ≥ 0 sol: P = 20 , x = 0, y = 0 and z = 5 **For each correct step 1 Marks and final solution 1 Mark**	*6*	**CO6**