2

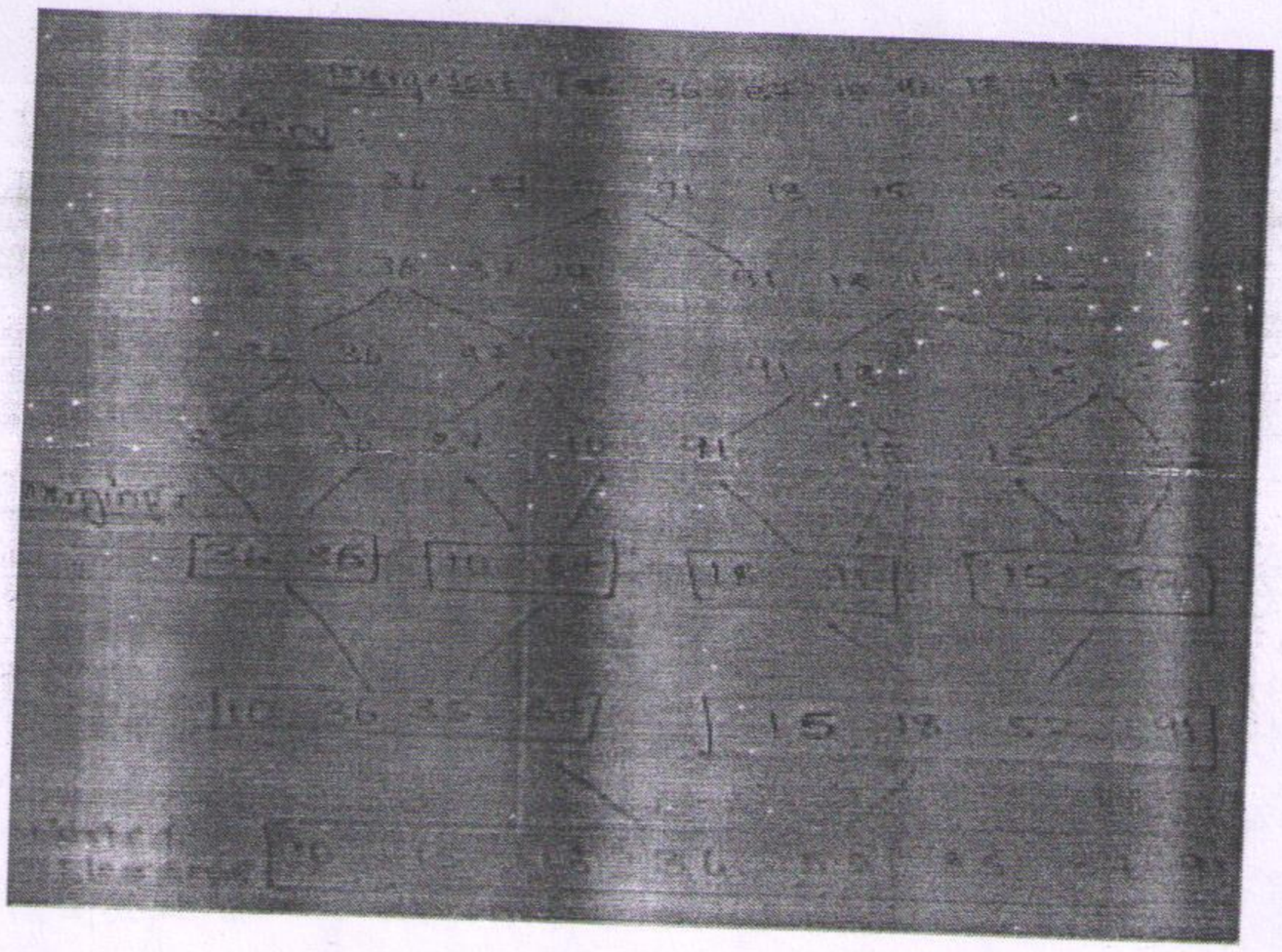
Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058-India
(Autonomous College Affiliated to University of Mumbai)

RE-Examination Synoptic
July 2019

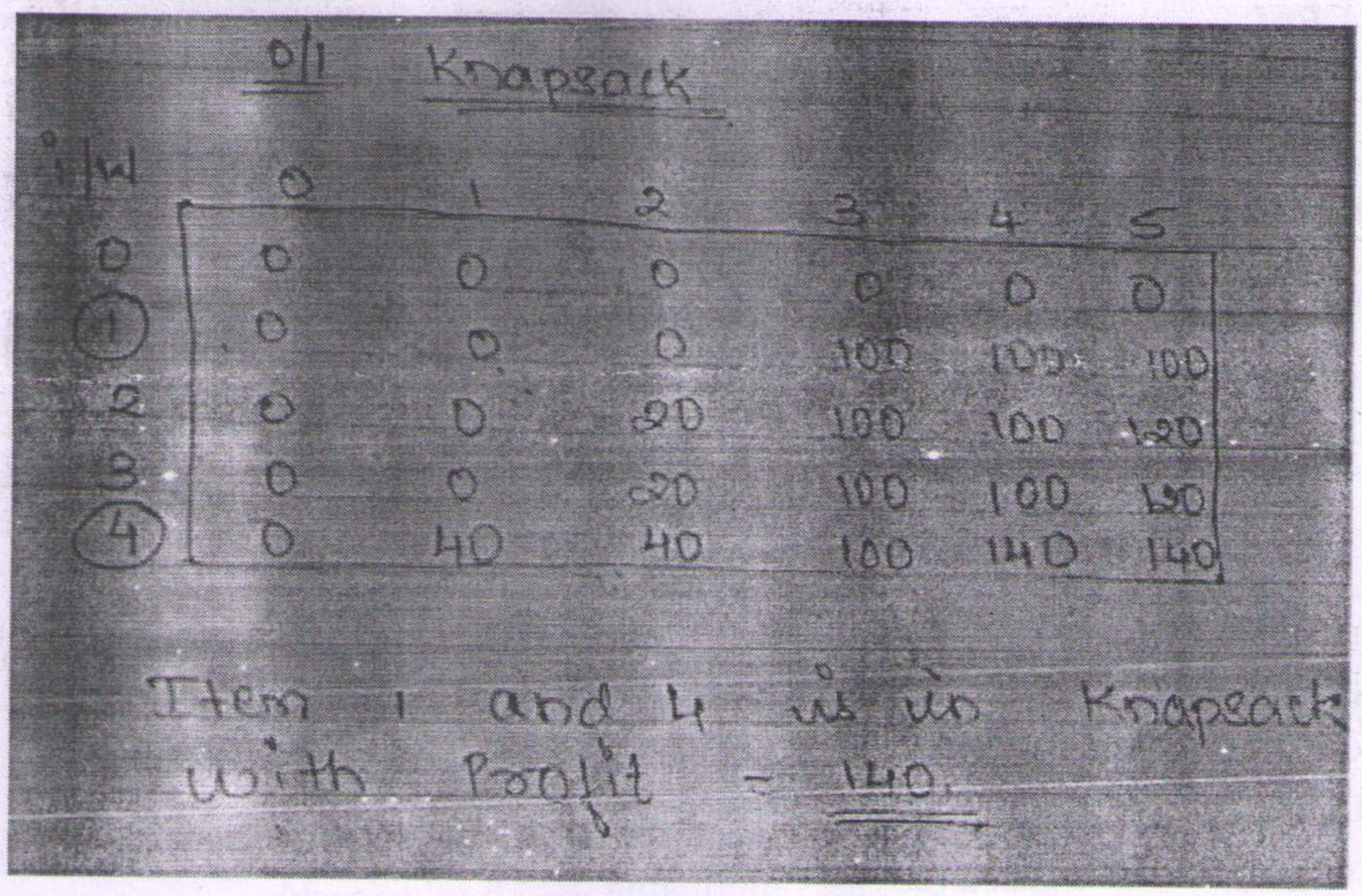
Max. Marks: 100 | **Duration: 3 Hrs**
Class: S.E. | **Semester: IV**
Course Code: CE41 | **Branch: COMP**
Name of the Course: Design And Analysis of Algorithm

Instructions:
(1) All Questions are Compulsory
(2) Draw neat diagrams
(3) Assume suitable data if necessary

Q. No.	Question	Max. Mks
Q1 a)	i) $T(n) = 3T(n/2) + n^2 \Rightarrow T(n) = \Theta(n^2)$ (Case 3) ii) $T(n) = T(n/2) + 2^n \Rightarrow \Theta(2^n)$ (Case 3) **Marks Distribution:** Solved correctly and also stated the cases applicable ---- 2.5 mks (each)	*05*
Q1 c)	 • Solved correctly with dividing and merging steps shown-------- 07mks • Derived the time complexity correctly------ 03mks	*10*
Q2 a)	**Marks Distribution:** • Solved correctly with the calculations steps shown and with correct matrix	

value------10mks

- Solved correctly with only the correct matrix value------06mks



10

OR

$g(2,\emptyset) = c_{21} = 1$
$g(3,\emptyset) = c_{31} = 15$
$g(4,\emptyset) = c_{41} = 6$

k = 1, consider sets of 1 element:

Set {2}: $g(3,\{2\}) = c_{32} + g(2,\emptyset) = c_{32} + c_{21} = 7 + 1 = 8$ $p(3,\{2\}) = 2$
$g(4,\{2\}) = c_{42} + g(2,\emptyset) = c_{42} + c_{21} = 3 + 1 = 4$ $p(4,\{2\}) = 2$

Set {3}: $g(2,\{3\}) = c_{23} + g(3,\emptyset) = c_{23} + c_{31} = 6 + 15 = 21$ $p(2,\{3\}) = 3$
$g(4,\{3\}) = c_{43} + g(3,\emptyset) = c_{43} + c_{31} = 12 + 15 = 27$ $p(4,\{3\}) = 3$

Set {4}: $g(2,\{4\}) = c_{24} + g(4,\emptyset) = c_{24} + c_{41} = 4 + 6 = 10$ $p(2,\{4\}) = 4$
$g(3,\{4\}) = c_{34} + g(4,\emptyset) = c_{34} + c_{41} = 8 + 6 = 14$ $p(3,\{4\}) = 4$

k = 2, consider sets of 2 elements:

Set {2,3}: $g(4,\{2,3\}) = \min\{c_{42} + g(2,\{3\}), c_{43} + g(3,\{2\})\} = \min\{3+21, 12+8\} = \min\{24, 20\} = 20$
$p(4,\{2,3\}) = 3$

Set {2,4}: $g(3,\{2,4\}) = \min\{c_{32} + g(2,\{4\}), c_{34} + g(4,\{2\})\} = \min\{7+10, 8+4\} = \min\{17, 12\} = 12$
$p(3,\{2,4\}) = 4$

Set {3,4}: $g(2,\{3,4\}) = \min\{c_{23} + g(3,\{4\}), c_{24} + g(4,\{3\})\} = \min\{6+14, 4+27\} = \min\{20, 31\} = 20$
$p(2,\{3,4\}) = 3$

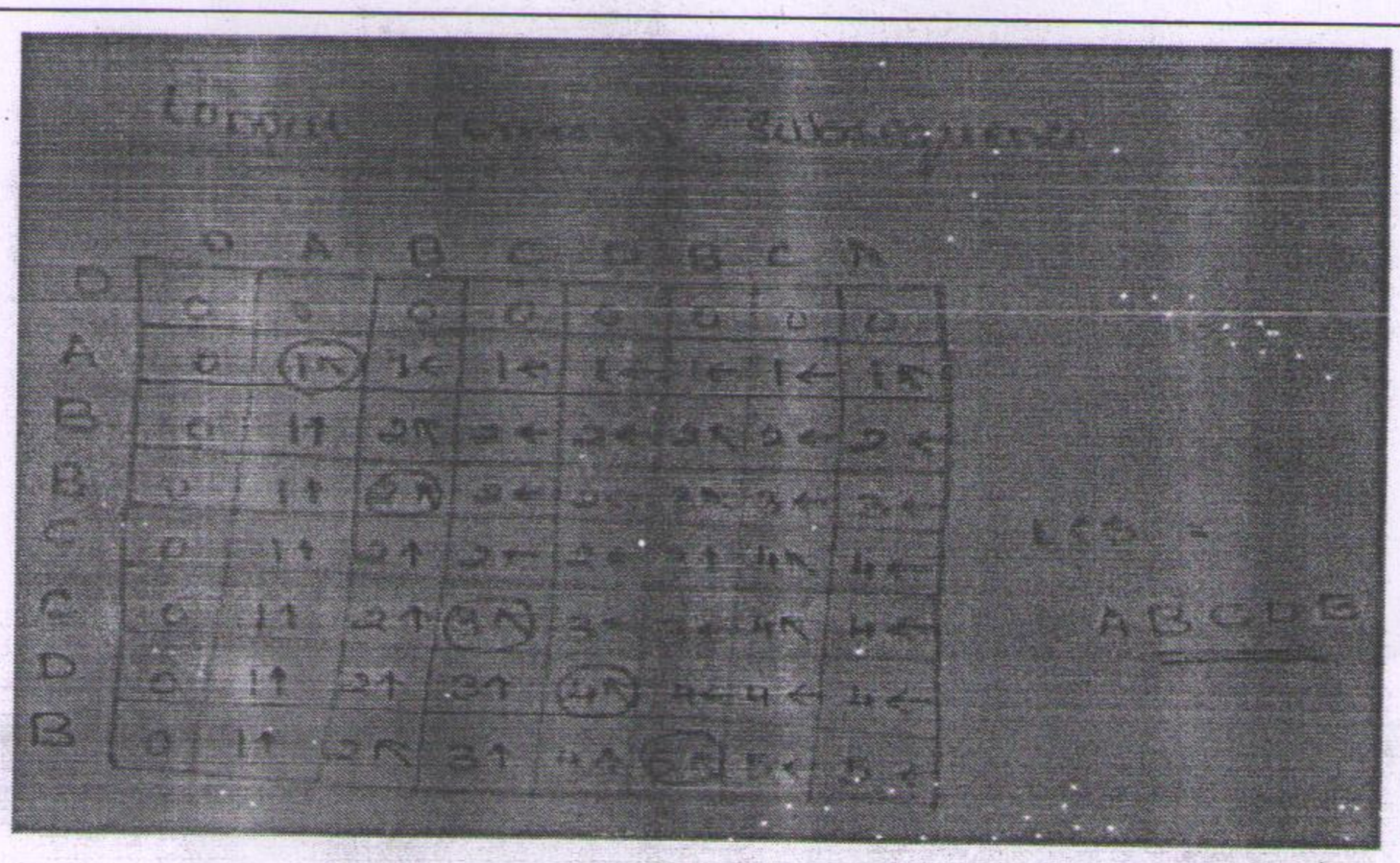
Length of an optimal tour:

$f = g(1,\{2,3,4\}) = \min \{ c_{12} + g(2,\{3,4\}), c_{13} + g(3,\{2,4\}), c_{14} + g(4,\{2,3\}) \}$
$= \min \{2 + 20, 9 + 12, 10 + 20\} = \min \{22, 21, 30\} = 21$

Successor of node 1: $p(1,\{2,3,4\}) = 3$
Successor of node 3: $p(3, \{2,4\}) = 4$
Successor of node 4: $p(4, \{2\}) = 2$

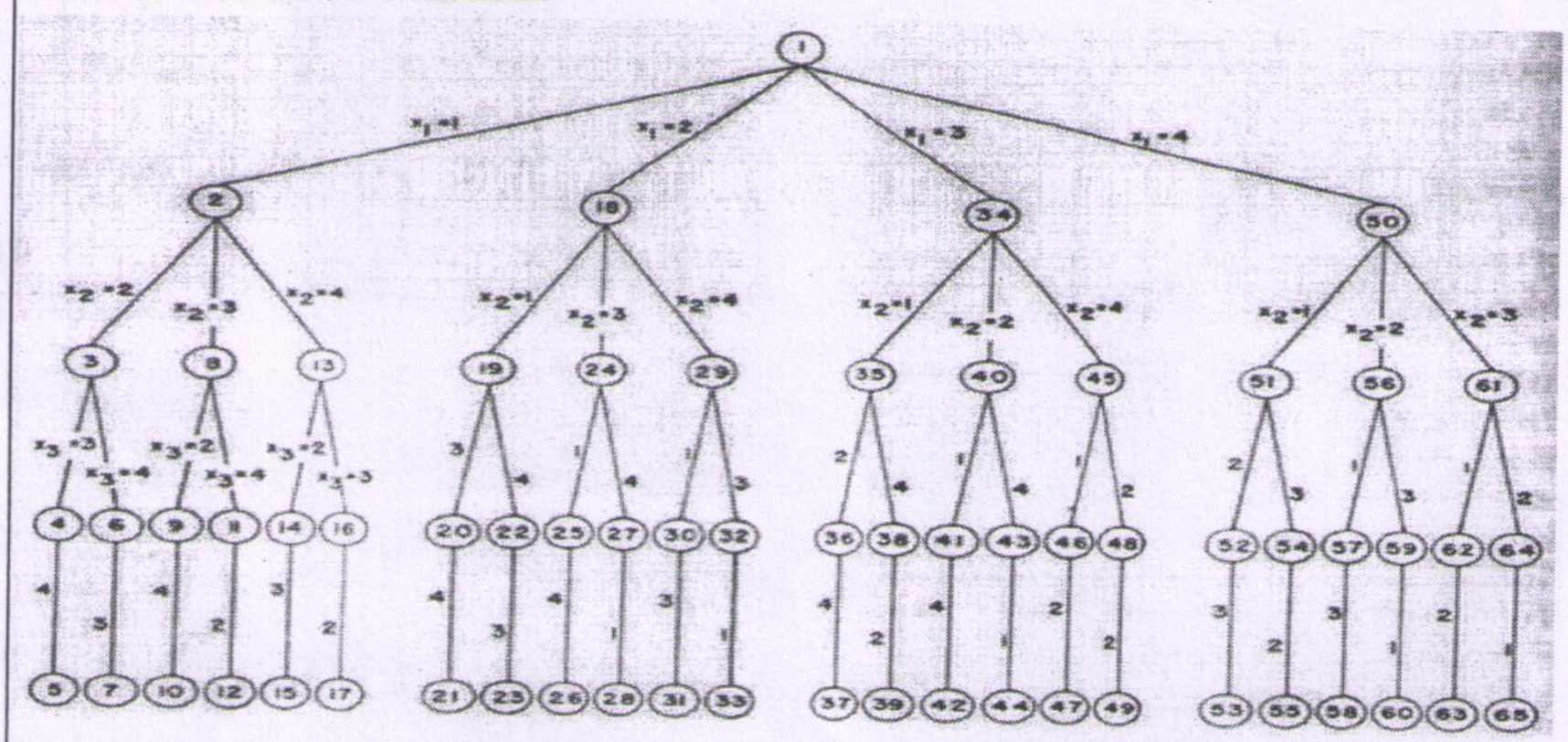
Optimal TSP tour: $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$

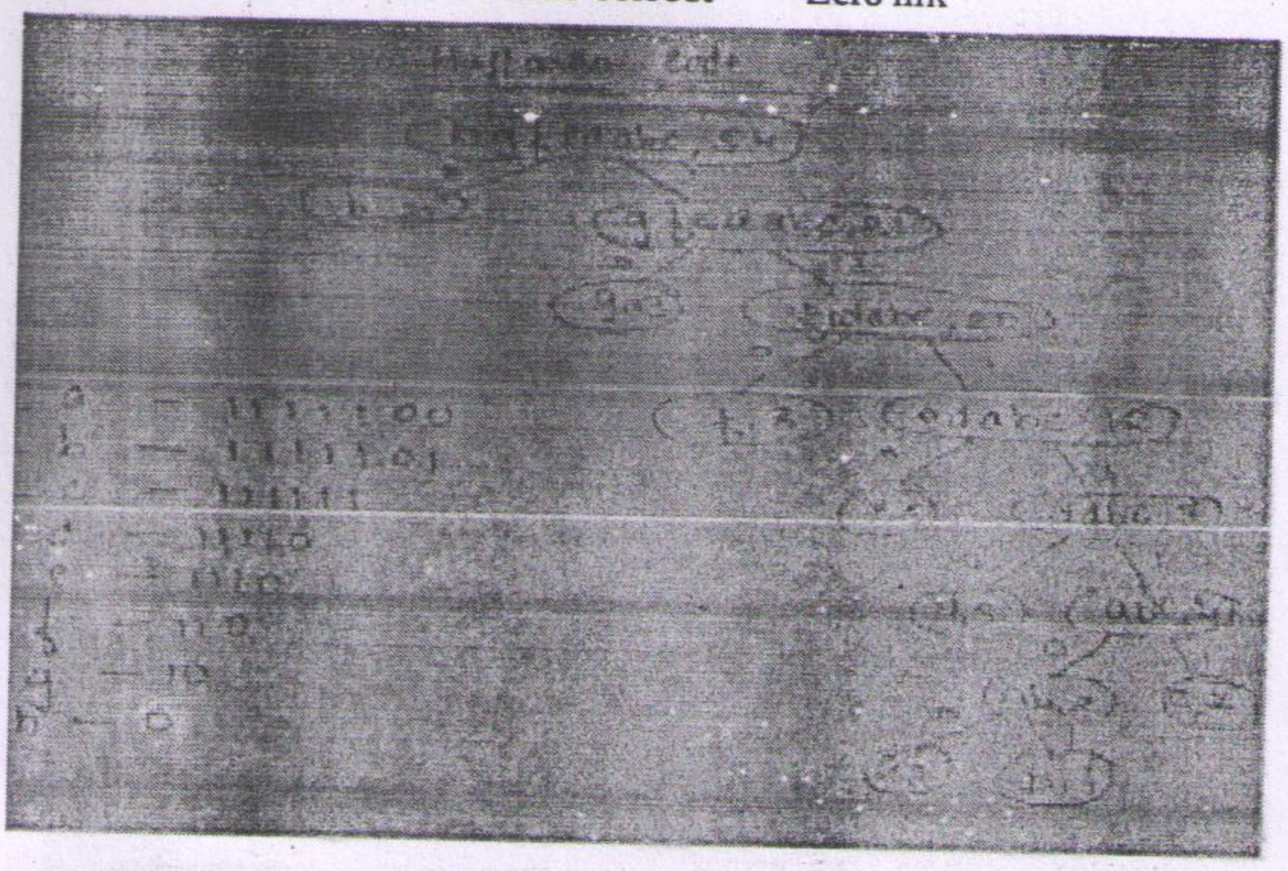
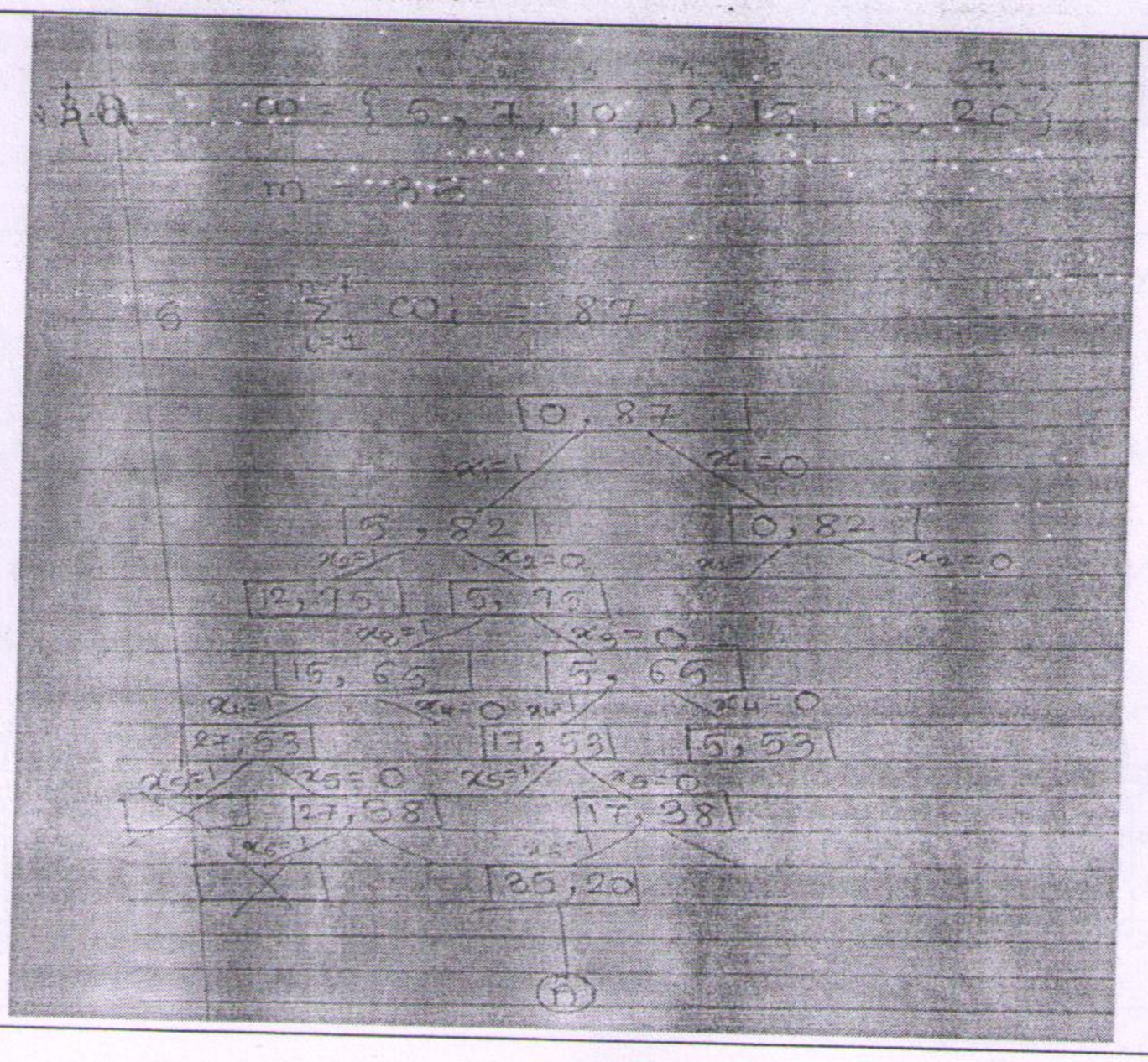
Q2 b) — 10

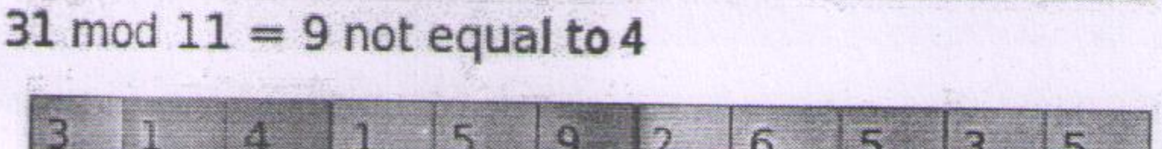
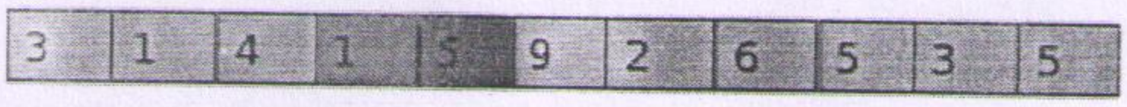
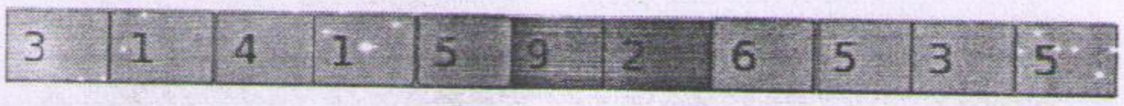
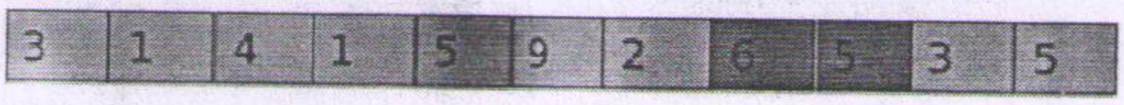
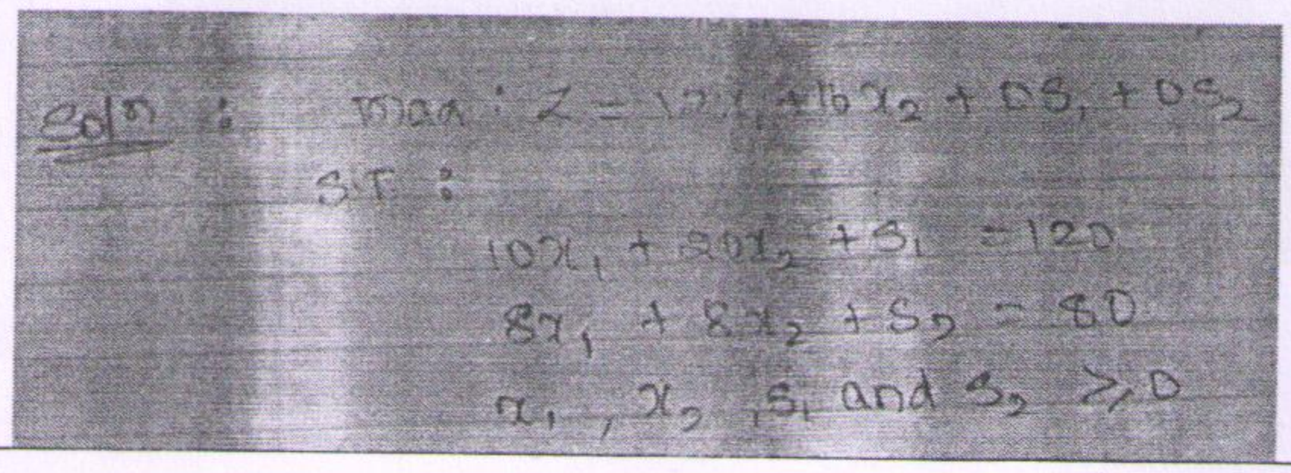


Q3 a) — 10

State Space tree for 4 Queen



	Marks Distribution: Backtracking algorithm for N queen Problem ---— **05mks** State Space Tree —--- **05mks**	
Q3 b)	**Marks Distribution:** • Explain Huffman code algorithm---------- 05mks • Tree drawn correct and Huffman Code written ----- 05mks • Tree drawn correct and NO Huffman Code written----- 03mks • Incorrect Tree and Huffman code correct------Zero mk 	*10*
Q4 a)		*10*

	Marks Distribution: Backtracking algorithm for sum of subset problem----------05 marks Correctly drawn state space tree----------05 marks	
Q4 b)	• Given T = 31415926535 and P = 26 • We choose q = 11 • P mod q = 26 mod 11 = 4  **31** mod 11 = 9 not equal **to 4** **14** mod 11 = 3 not equal **to 4**  **41** mod 11 = 8 not equal **to 4**  15 mod **11** = 4 equal **to** 4 -> spurious hit  59 mod **11** = 4 equal **to** 4 -> spurious hit  92 mod **11** = 4 equal **to** 4 -> spurious hit  26 mod **11** = 4 equal **to** 4 -> an exact match!!  65 mod **11** = 10 not equal to 4	*10*
Q5 a)	 Soln : Max : $Z = 120x_1 + 80x_2 + 0S_1 + 0S_2$ S.T : $10x_1 + 8x_2 + S_1 = 120$ $8x_1 + 8x_2 + S_2 = 80$ x_1, x_2, S_1 and $S_2 \geq 0$	*10*

Initial Simplex Table

	CBi	Cj	12	16	0	0	Sol^n	Ratio
		Bas Vec	x_1	x_2	S_1	S_2		
	0	S_1	10	20	1	0	120	120/20 = 6
	0	S_2	8	8	0	1	80	80/8 = 10
		Z_j	0	0	0	0	0	
		$C_j - Z_j$	12	16	0	0		

$Z_j = \sum_{i=1} (CB_i)(a_{ij})$ If $C_j - Z_j \leq 0$ Then Max

If $C_j - Z_j \geq 0$ Then Min

Iteration 1

	CBi	Cj	12	16	0	0	Sol^n	Ratio
		Bas Vec	x_1	x_2	S_1	S_2		
	16	x_2	1/2	1	1/20	0	6	6/1/2 = 12
	0	S_2	4	0	-2/5	1	32	32/4 = 8
		Z_j	8	16	4/5	0	96	
		$C_j - Z_j$	4	0	-4/5	0		

$\Rightarrow$ New value = Old value − $\frac{\text{Corr. Key Col. Val} \times \text{Corr. Key Row Value}}{\text{Key Element}}$

$8 - \frac{8 \times 10}{20} = 8 - \frac{80}{20} = 8 - 4 = 4$

$8 - \frac{8 \times 20}{20} = 8 - \frac{160}{20} = 8 - 8 = 0$

$1 - \frac{8 \times 0}{20} = 1 - 0 = 1$

$80 - \frac{8 \times 120}{20} = 80 - 960/20 = 80 - 48 = \underline{32}$

Iteration II

CBi	Cj	12	16	0	0	Soln	Ratio
	B.V.	x_1	x_2	S_1	S_2		
16	x_2	0	1	1/10	-1/8	2	
12	x_1	1	0	-1/10	1/4	8	
	Zj	12	16	2/5	1	128	
	Cj-Zj	0	0	-2/5	-1		

$1/2 - \left(\frac{1/2 \times 4}{4}\right) = 0$ | $0 - \left(\frac{1/2 \times 1}{4}\right) = -1/8$

$1 - \left(\frac{1/2 \times 0}{4}\right) = 1$ | $6 - \left(\frac{1/2 \times 32}{4}\right) = 2$

$1/20 - \left(\frac{1/2 \times -2/5}{4}\right) = -1/10$ |

$\therefore \quad x_1 = 12 \qquad x_2 = 16$

$Z(\text{optimum}) = 128$

Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058-India

(Autonomous College Affiliated to University of Mumbai)

Q5 b)

(i)

Defining the Decision Variables:

Let Z = Profit

Let X_1= number of deluxe jackets to produce

Let X_2= number of regular jackets to produce

Complete LP Model would be written as:

Max Z = $\$18X_1 + \$12\ X_2$ ---------- Equation 1

Subject To:

$0.16X_1 + 0.15X_2 \leq 40$ hrs ---------- Equation 2

$0.47X_1 + 0.28X_2 \leq 80$ hrs---------- Equation 3

$0.40X_1 + 0.14X_2 \leq 55$ hrs---------- Equation 4

$X_1, X_2 \geq 0$

(ii) **<u>Slack Form</u>**

Max $Z = 18X_1 + 12\ X_2 + 0S_1 + 0S_2 + 0S_3$

Subject To:

$0.16X_1 + 0.15X_2 + S_1 = 40$ hrs

$0.47X_1 + 0.28X_2 + S_2 = 80$ hrs

$0.40X_1 + 0.14X_2 + S_3 = 55$ hrs

$X_1, X_2, S_1, S_2, S_3 \geq 0$

iii) **Dual Problem**

<u>Marks distribution:</u>

i) **01mk** ------Decision Variable Defined

04mks ------ All four equation written correctly

ii) **02mks** ------ For correct slack form.

iii) **03mks**------- For correct Dual Form

10

Q5 b)