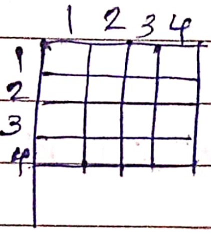
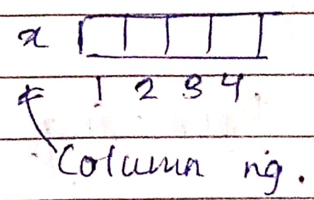
N - Queen's Problem

Place N queens in a $n \times n$ chessboard such that no two of them can "attack", i.e. no two of them are in the same row, column or diagonal.

E.g.
4 queens

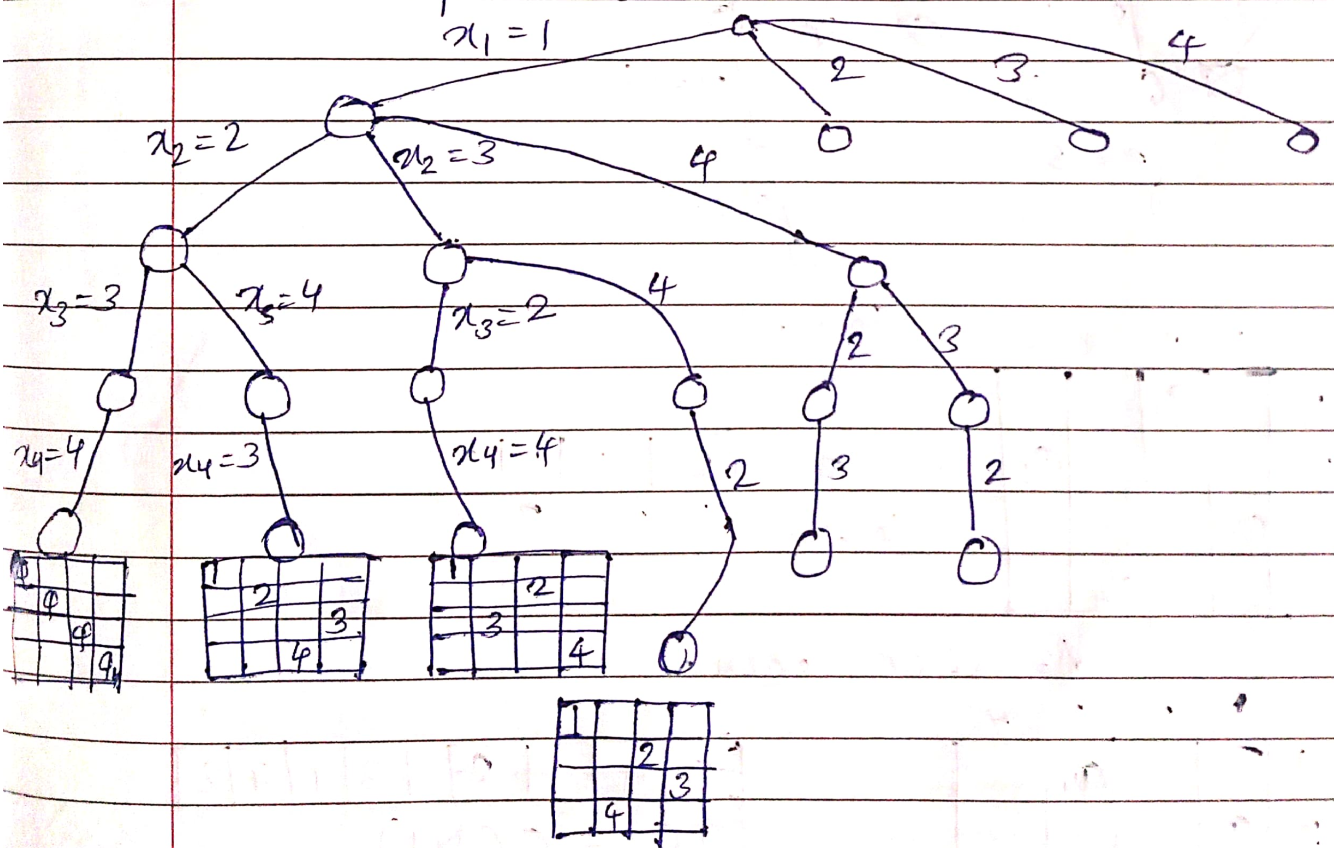




q_1, q_2, q_3, q_4

4 queens can be placed on a 4×4 chessboard by ${}^{16}C_4$ ways

State space tree



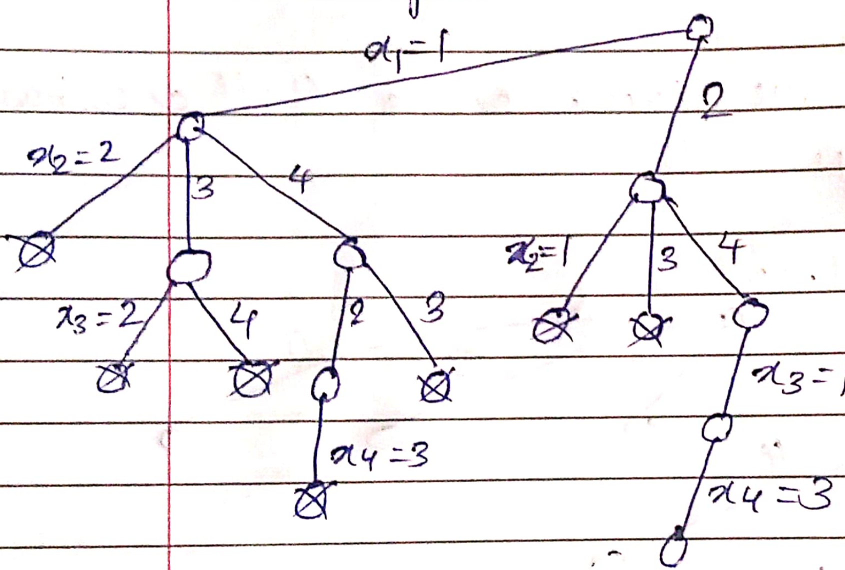
Total no. of nodes $= 1+4+4\times3+4\times3\times2+4\times3\times2\times1$

$$= 1+\sum_{i=0}^{3}\left[\prod_{j=0}^{i}(4-j)\right]$$

$$= \boxed{1+\sum_{i=0}^{N-1}\left[\prod_{j=0}^{i}(N-j)\right]}$$

Bounding funcⁿ : Not in same row/col/dig.

State space tree



Soln.

	1	2	3	4
1		Q_1		
2				Q_2
3	Q_3			
4			Q_4	

x	2	4	1	3
Col. no.	1	2	3	4

Another soln.

	1	2	3	4
1			Q_1	
2	Q_2			
3				Q_3
4		Q_4		

~~[3 | 4 | 1 | 2]~~ [3 | 1 | 4 | 2]

$$\boxed{\begin{array}{l} T.C. = O(N!) \\ S.C. = O(N^2) \end{array}}$$

Time complexity = $O(2^n)$

Sum of Subsets:

→ We are given n distinct +ve no.s (usually called weights)

→ And we are desired to find all combination of these no.s whose sums are m.

$$w[1:6] = \{\overset{1}{5}, \overset{2}{10}, \overset{3}{12}, \overset{4}{13}, \overset{5}{15}, \overset{6}{18}\}$$

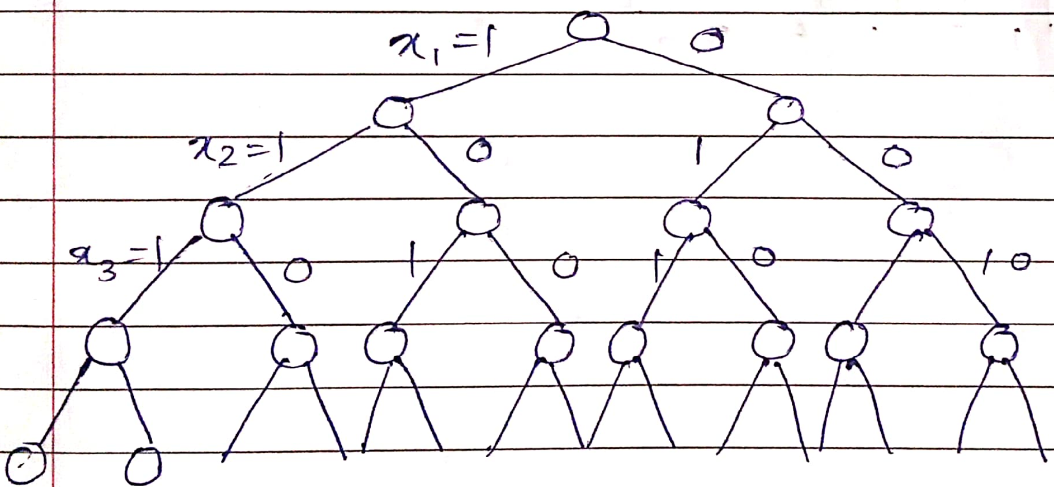
$n = 6$ $\quad m = 30$

$$x \; \begin{array}{|c|c|c|c|c|c|} \hline \; & \; & \; & \; & \; & \; \\ \hline \end{array}$$
$\quad\;\;$ 1 2 3 4 5 6

$x_i = 0/1$

↑ whether included or not

State Space Tree (Brute force)



Total paths = 2^n

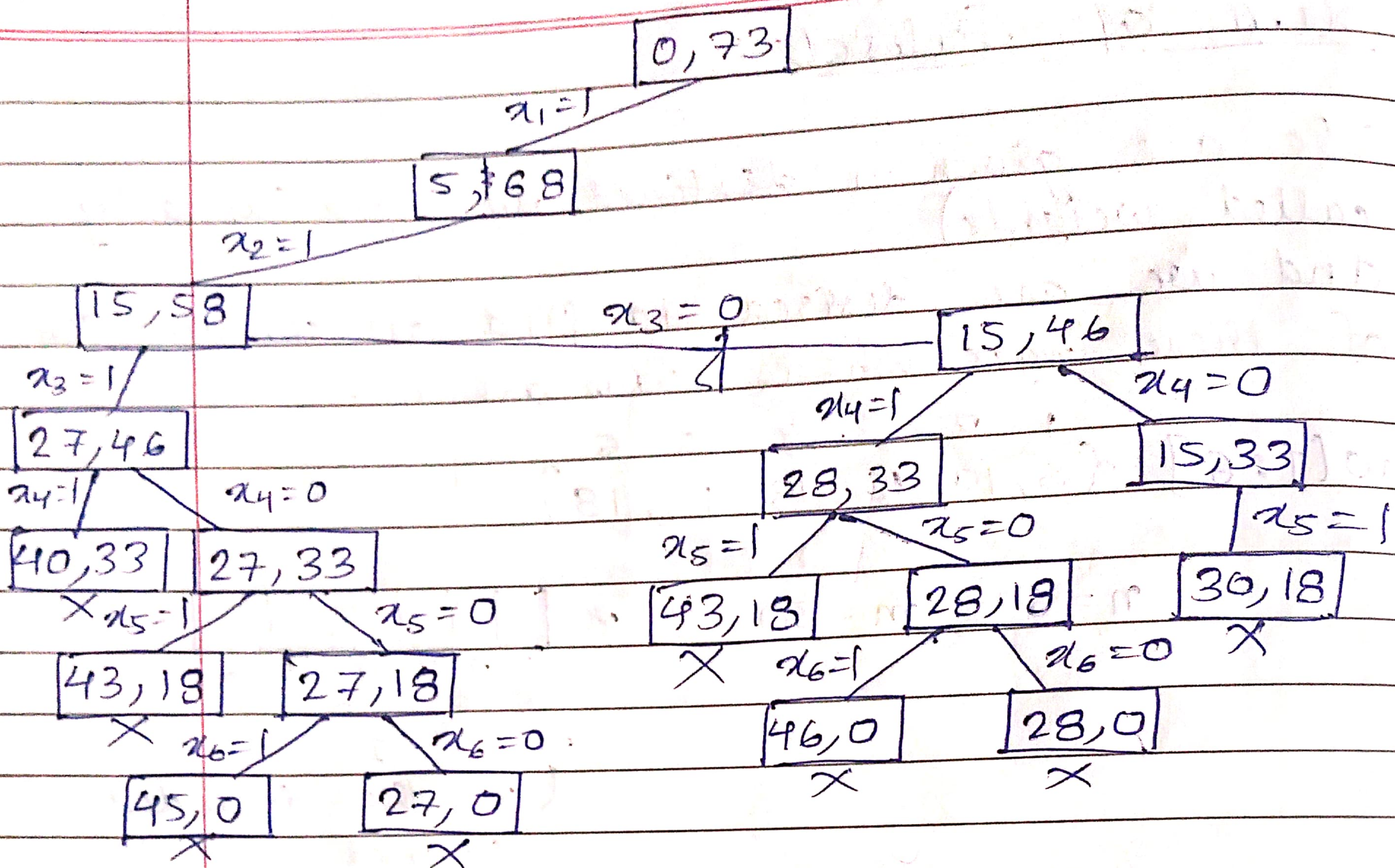
Bounding func^n = $\boxed{\sum_{i=1}^{k} w_i x_i + w_{k+1} \le m}$

wt. included till now

next wt.

$$\boxed{\sum_{i=1}^{k} w_i x_i + \sum_{i=k+1}^{n} w_i > m}$$

sum of remaining wt.

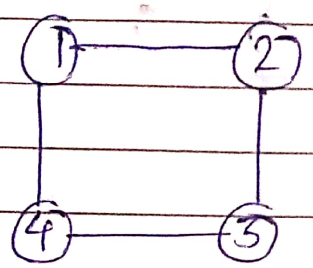


x	1	1	0	0	1	0
	1	2	3	4	5	6

Graph Coloring

Let G be a graph & m be the no. of colors, we need to find out whether the nodes of G can be colored in such a way that no two adjacent nodes have same color.

E.g.

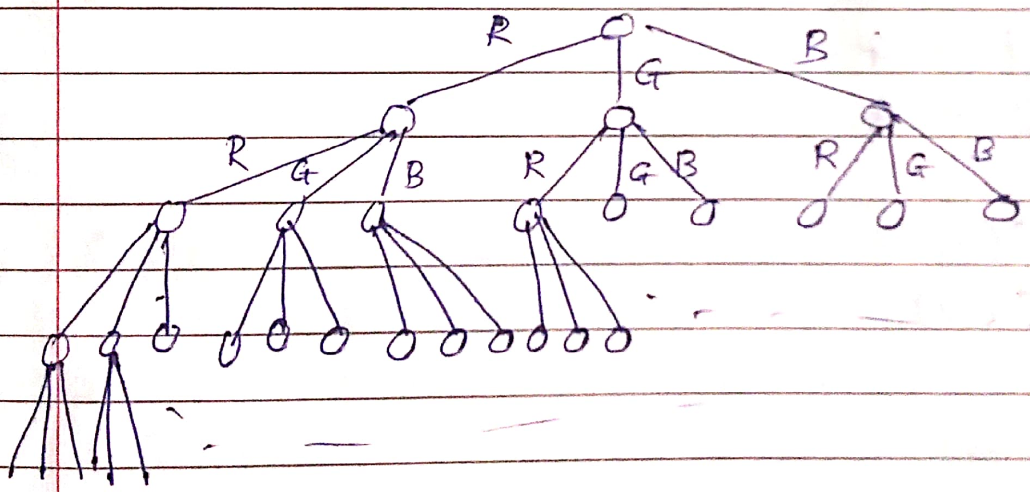


$m = 3$

$\{R, G, B\}$

Without cond.

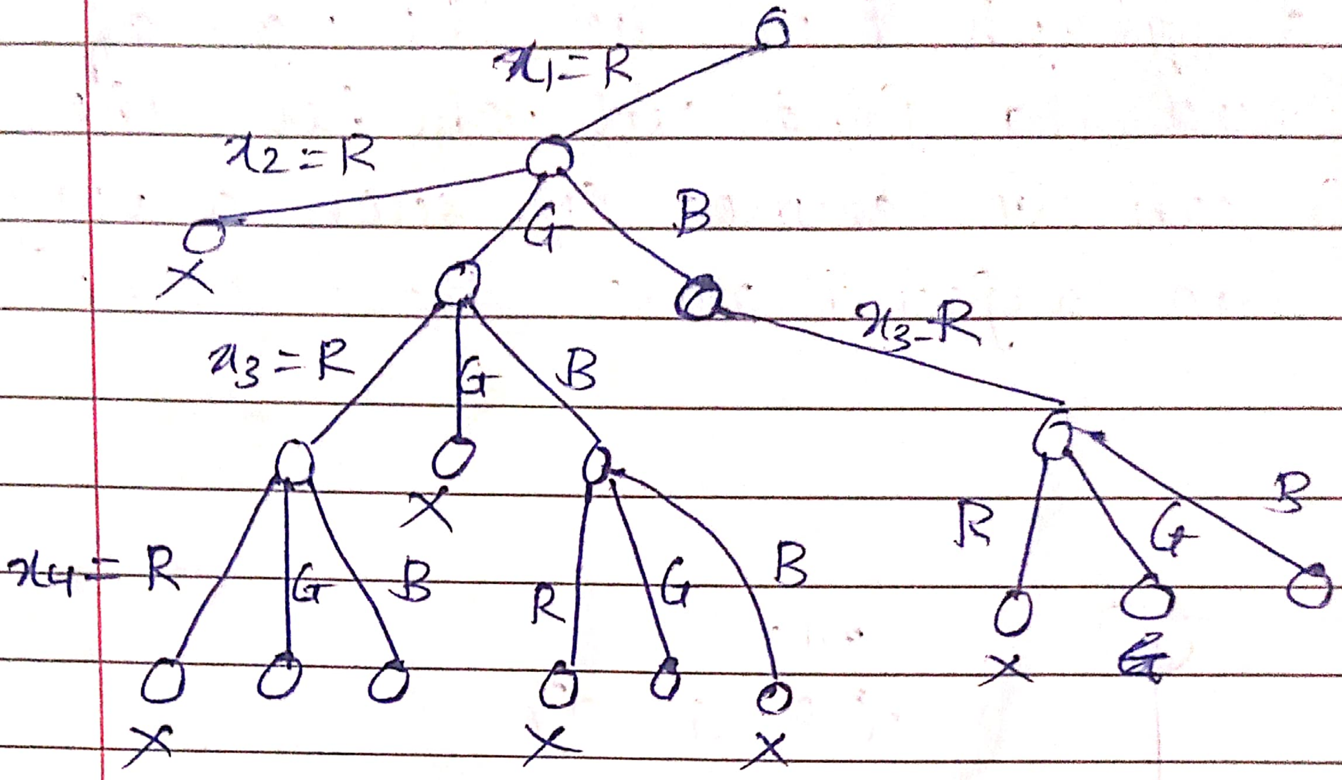
State space tree



$= 1 + 3 + 3 \times 3 + 3^3 + 3^4 + 3^5 \ldots$

$= \dfrac{3^{4+1} - 1}{2}$

Bounding funcⁿ:



Soln:

1) R, G, R, G
2) R, G, R, B
3) R, G, B, G
4) R, B, R, G
5) R, B, R, B

$T.C. = O(m^V)$

$S.C. = O(V)$