0/1 Knapsack problem

Profit	10	10	12	18
weight	2	4	6	9

m=15 / n=4

Given n items of known weights w_i & values v_i, $i = 1, 2, \ldots n$ & knapsack of capacity W, find the most valuable subset of items that ~~fino~~ fit in the knapsack

→ It is convenient to order the items of a given instance in descending order of their value to weight ratios

→ First item gives the max. profit & the last one gives the least profit. The items are arranged as shown below.

$$v1/w1 \geq v2/w2 \geq \ldots \geq v_n/w_n$$

State space tree:

→ Each node on the ith level of this tree, $0 \leq i \leq n$, represents all the subsets of n consisting of the items selected.

→ A selection is determined by the path from the root to the node where a branch going to the left indicates the inclusion of the next item, & a branch going to the right indicates exclusion.

→ For a particular selection, record the total weight & the total profit value v in the node along with some upper bound ub.

~~ub~~

ub Upper bound calculation:

~~ub = v + (W-w)((v_i + 1)/(w_i + 1))~~

$$\boxed{ub = v + (W-w)(v_{(i+1)} / w_{(i+1)})}$$

where:

ub = upper bound

(W-w) = remaining capacity of knapsack

v = total profit or value of all objects placed into knapsack

W = maximum capacity of the knapsack

Capacity of knapsack : $\boxed{W: 10}$

Item	Weight	Value	Value/weight
1	4	40	10
2	7	42	6
3	5	25	5
4	3	12	4

i = 0
w = 0
v = 0

$ub = v + (W-w)*(v1/w1)$
$= 0 + (10)(40/4)$
$= 100$

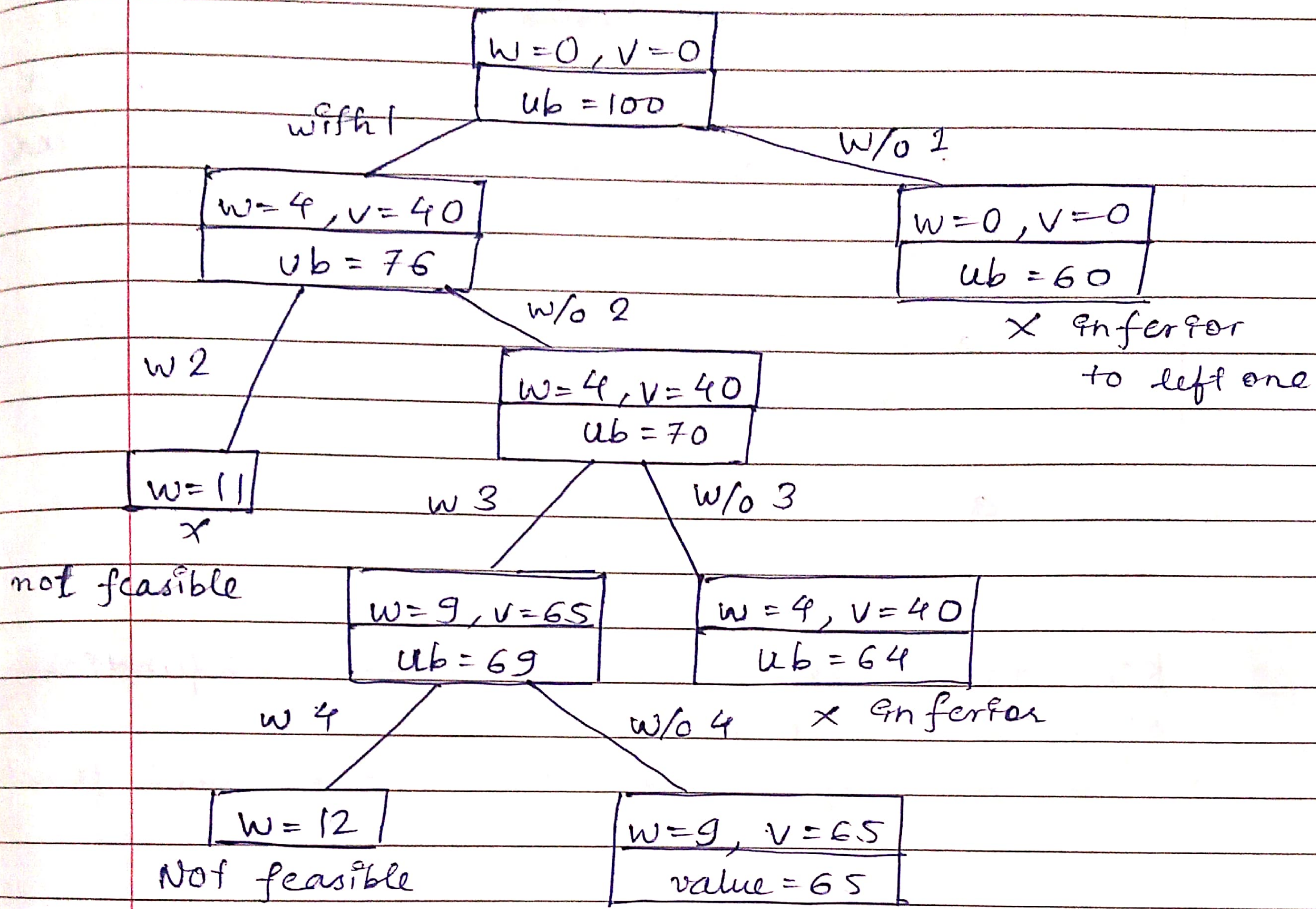
left i = 1

$ub = v + (W-w)(v2/w2)$
$= 40 + (10-4)(6)$
$= 76$

right i = 1

$ub = v + (W-w)(v2/w2)$
$= 0 + (10-0)(6)$
$= 60$

In this algo., a partial soln. is eliminated if its upper bound is less than or equal to current best soln.



So final soln. {1, 3}

T.C. = O(n)

~~Traa~~ Travelling Salesman problem

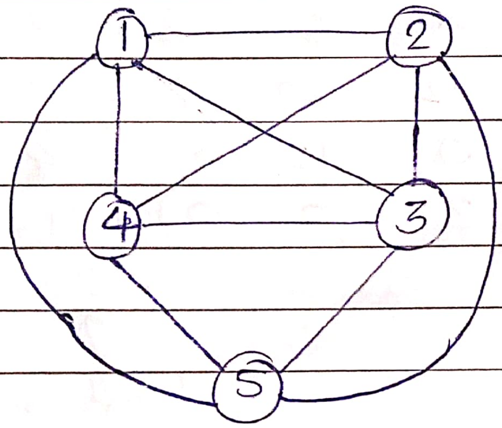
Given a list of cities & distance b/w each pair of cities, find the shortest possible route that visits each city exactly once & returns to the origin city.

Hamiltonian ckt. + length constraint

For each city i, $1 \le i \le n$, find the sum s_i of the distances from city i to the 2 nearest cities; compute the sum s of these n numbers, divide the result by 2, & if all the distances are integers, round up the result to the nearest integer.

$$lb = \lceil s/2 \rceil$$

E.g. Graph



	1	2	3	4	5	Min. from each row
1	∞	20	30	10	11	10
2	15	∞	16	4	2	2
3	3	5	∞	2	4	2
4	19	6	18	∞	3	3
5	16	4	7	16	∞	4

~~Sett Sub I Si~~ Subtract the min. value from each row

	1	2	3	4	5	
1	∞	10	20	0	1	10
2	13	∞	14	2	0	2
3	1	3	∞	0	2	2
4	16	3	15	∞	0	3
5	12	0	3	12	∞	4
Min. of each column	1	0	3	0	0	21

Total cost of reduction of rows

Subtract the min. value from each column

$$
\begin{array}{c|ccccc}
1 & \infty & 10 & 20 & 0 & 1 \\
2 & 12 & \infty & 14 & 2 & 0 \\
3 & 0 & 3 & \infty & & \\
4 & & & & & \\
5 & & & & &
\end{array}
$$

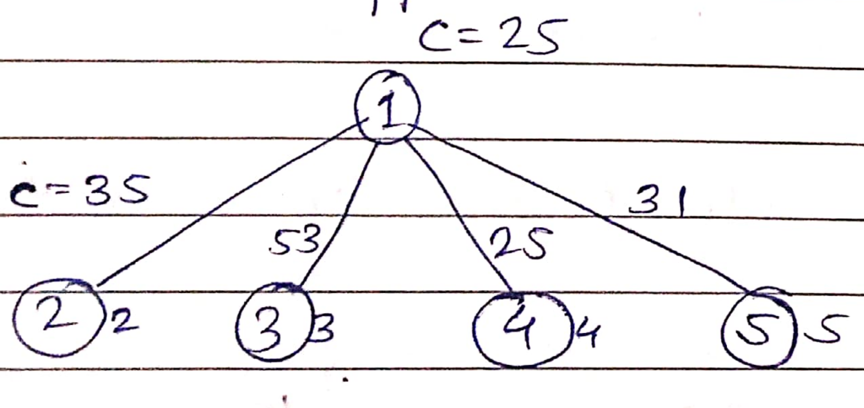
$$
\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & 10 & 17 & 0 & 1 & 10 \\
2 & 12 & \infty & 11 & 2 & 0 & 2 \\
3 & 0 & 3 & \infty & 0 & 2 & 2 \\
4 & 15 & 3 & 12 & \infty & 0 & 3 \\
5 & 11 & 0 & 0 & 12 & \infty & 4 \\
\hline
 & (1 & 0 & 3 & 0 & 0) & +21
\end{array}
$$

$= 25$

Total cost of reduction

∴ Reduced cost = 25, ∴ The cost of tour would be atleast 25

Consider upper to be ∞ initially



Consider $1 \rightarrow 2$

As we are going from 1^{st} row to 2^{nd} column. make 1^{st} row & 2^{nd} column ∞, as we won't return from $2 \rightarrow 1$ make $(2,1)$ as ∞

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \end{array}$$

As each row has 0 & each col. has 0, the matrix is reduced

reduced cost

cost of extra reduction

Cost now $= C(1,2) + r + \hat{r}$

$= 10 + 25 + 0$

$= 35$

PAGE NO.:

Consider 1→3

So make 1^{st} row & 3^{rd} col. as ∞

$$\begin{array}{c|ccccc|c} & 1 & 2 & 3 & 4 & 5 & \text{Min. of each row} \\ \hline 1 & \infty & \infty & \infty & \infty & \infty & \\ 2 & 12 & \infty & \infty & 2 & 0 & 0 \\ 3 & \infty & 3 & \infty & 0 & 2 & 0 \\ 4 & 15 & 3 & \infty & \infty & 0 & 0 \\ 5 & 11 & 0 & \infty & 12 & 0 & 0 \\ \hline \text{Min. of each col.} & 11 & 0 & 0 & 0 & 0 & \end{array}$$

∴ As we can see min. value of col. 1 is not 0, ∴ reduce column 1 (i.e. ~~sett~~ subtract 11 from col. 1)

$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & 1 & \infty & \infty & 2 & 0 \\ 3 & \infty & 3 & \infty & 0 & 2 \\ 4 & 4 & 3 & \infty & \infty & 0 \\ 5 & 0 & 0 & \infty & 12 & 0 \end{array}$$

∴ Cost $= C(1,3) + r + \hat{r}$ → We reduced 1^{st} col. with a cost of 11

$= 17 + 25 + 11$

$\boxed{\text{Cost} = 53}$

Consider 1→4

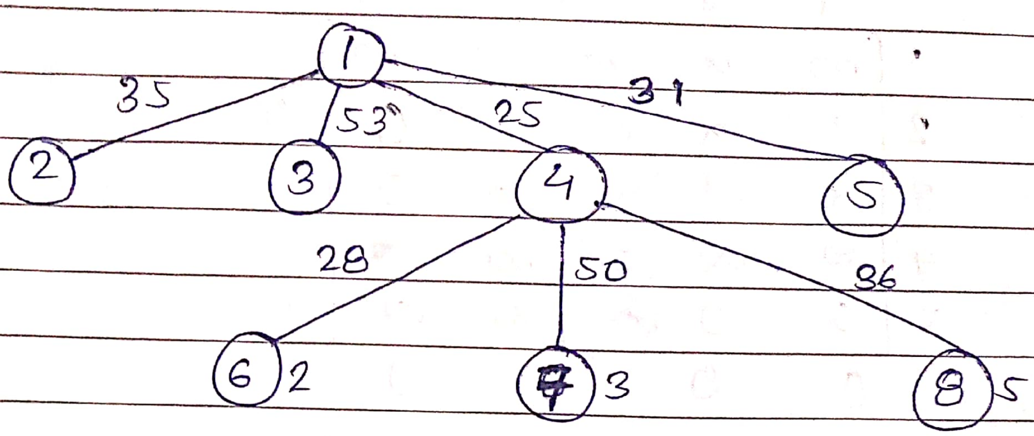
$$\begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & 12 & \infty & 11 & \infty & 0 \\ 3 & 0 & 3 & \infty & \infty & 2 \\ 4 & \infty & 3 & 12 & \infty & 0 \\ 5 & 11 & 0 & 0 & \infty & \infty \end{array}$$

Cost $= C(1,4) + r + \hat{r}$

$= 0 + 25 + 0$

$= 25$

As vertex 4 has the least cost explore vertex 4



⑥ Consider $4 \rightarrow 2$ (Continue from the matrix obtained in $1 \rightarrow 4$)

Here we should not go back from $2 \rightarrow 4$ & $4 \rightarrow 1$, so make both of them as ∞

Also make 4^{th} row & 2^{nd} col. as ∞

We should also not go back from $2 \rightarrow 1$

$\therefore$ make it as ∞

$$
\begin{array}{c|c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} & \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} & \begin{matrix} \\ 0 \\ 0 \\ \\ 0 \end{matrix} \\
 & \begin{matrix} 0 & & 0 & & 0 \end{matrix} &
\end{array}
$$

Matrix is already reduced $\therefore \hat{r} = 0$

$\therefore$ Cost $= c(4,2) + c(4) + \hat{r}$

$= 3 + 25 + 0 = 28$

$\quad \quad \downarrow$ cost of $1 \rightarrow 4$

⑦ Consider $4 \to 3$

$$
\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & \infty & \infty & \infty & \infty & \\
2 & 1 & \infty & \infty & \infty & 0 & 0 \\
3 & \infty & 1 & \infty & \infty & 0 & 0 \\
4 & \infty & \infty & \infty & \infty & \infty & \\
5 & 0 & 0 & \infty & \infty & \infty & 0 \\
\hline
 & 0 & 0 & & & 0 &
\end{array}
$$

$\therefore$ Already reduced, $\therefore \hat{r} = 0$

$\therefore$ cost $= c(4) + c(4,3) + \hat{r}$

$= 25 + 12$

$$
\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & \infty & \infty & \infty & \infty & \\
2 & 12 & \infty & \infty & \infty & 0 & 0 \\
3 & \infty & 3 & \infty & \infty & 2 & 2 \\
4 & \infty & \infty & \infty & \infty & \infty & \\
5 & 11 & 0 & \infty & \infty & \infty & \\
\hline
 & 11 & 0 & & & 0 &
\end{array}
$$

After reducing 3rd row & 1st col.

$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \infty & \infty & \infty & \infty & \infty \\
2 & 1 & \infty & \infty & \infty & 0 \\
3 & \infty & 1 & \infty & \infty & 0 \\
4 & \infty & \infty & \infty & \infty & \infty \\
5 & 0 & 0 & \infty & \infty & \infty
\end{array}
$$

$\therefore \hat{r} = 11 + 2 = 13$

Cost $= \text{cost}(4) + \text{cost}(4,3) + \hat{r}$

$= 25 + 12 + 13 = 50$

Consider $4 \to 5$ 8

$$
\begin{array}{c}
\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & \infty & \infty & \infty & \infty & \\
2 & 12 & \infty & 11 & \infty & \infty & 11 \\
3 & 0 & 3 & \infty & \infty & \infty & 0 \\
4 & \infty & \infty & \infty & \infty & \infty & \\
5 & \infty & 0 & 0 & \infty & \infty & 0 \\
\hline
 & 0 & 0 & 0 & & &
\end{array}
\end{array}
\qquad \hat{r} = 11
$$

After reducing

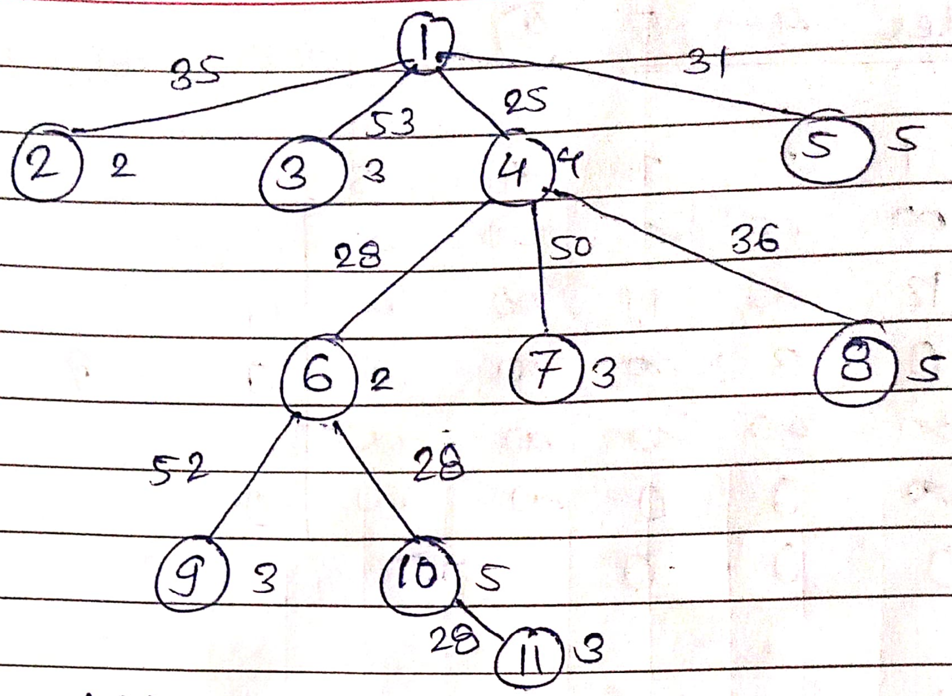
$$
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
1 & \infty & 0 & \infty & \infty \\
0 & 3 & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & 0 & 0 & \infty & \infty
\end{bmatrix}
\end{array}
$$

$$
\begin{aligned}
\text{Cost} &= C(4) + C(4,5) + \hat{r} \\
&= 25 + 0 + 11 \\
&= 36
\end{aligned}
$$

Now Node 6 is the min., so go to 6

9 Consider $2 \to 3$

$$
\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \\
\hline
1 & \infty & \infty & \infty & \infty & \infty & \\
2 & \infty & \infty & \infty & \infty & \infty & \\
3 & \infty & \infty & \infty & \infty & 2 & 2 \\
4 & \infty & \infty & \infty & \infty & \infty & \\
5 & 11 & \infty & \infty & \infty & \infty & 11
\end{array}
\qquad
\begin{aligned}
\hat{r} &= 11 + 2 \\
&= 13
\end{aligned}
$$



After reducing

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & \infty \\ 3 & \infty & \infty & \infty & \infty & 0 \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & 0 & \infty & \infty & \infty & \infty \end{array}$$

$$\text{Cost} = c(2,3) + c(6) + 13$$
$$= 11 + 28 + 13$$
$$= 52$$

Consider $2 \to 5$ (10)

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & \infty \\ 3 & 0 & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & \infty & \infty & 0 & \infty & \infty \end{array}$$

$$\text{Cost} = c(2,5) + c(6) + 0$$
$$= 0 + 28 + 0 = 28$$

III^{ly} finding cost for node 11 it comes out to be 28

$$c(11) = 28$$

$$1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3$$

15 puzzle problem - Least Cost Branch & Bound

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

Initial

$\longrightarrow$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

~~final~~ Goal

$$\boxed{\hat{c}(x) = f(x) + \hat{g}(x)}$$

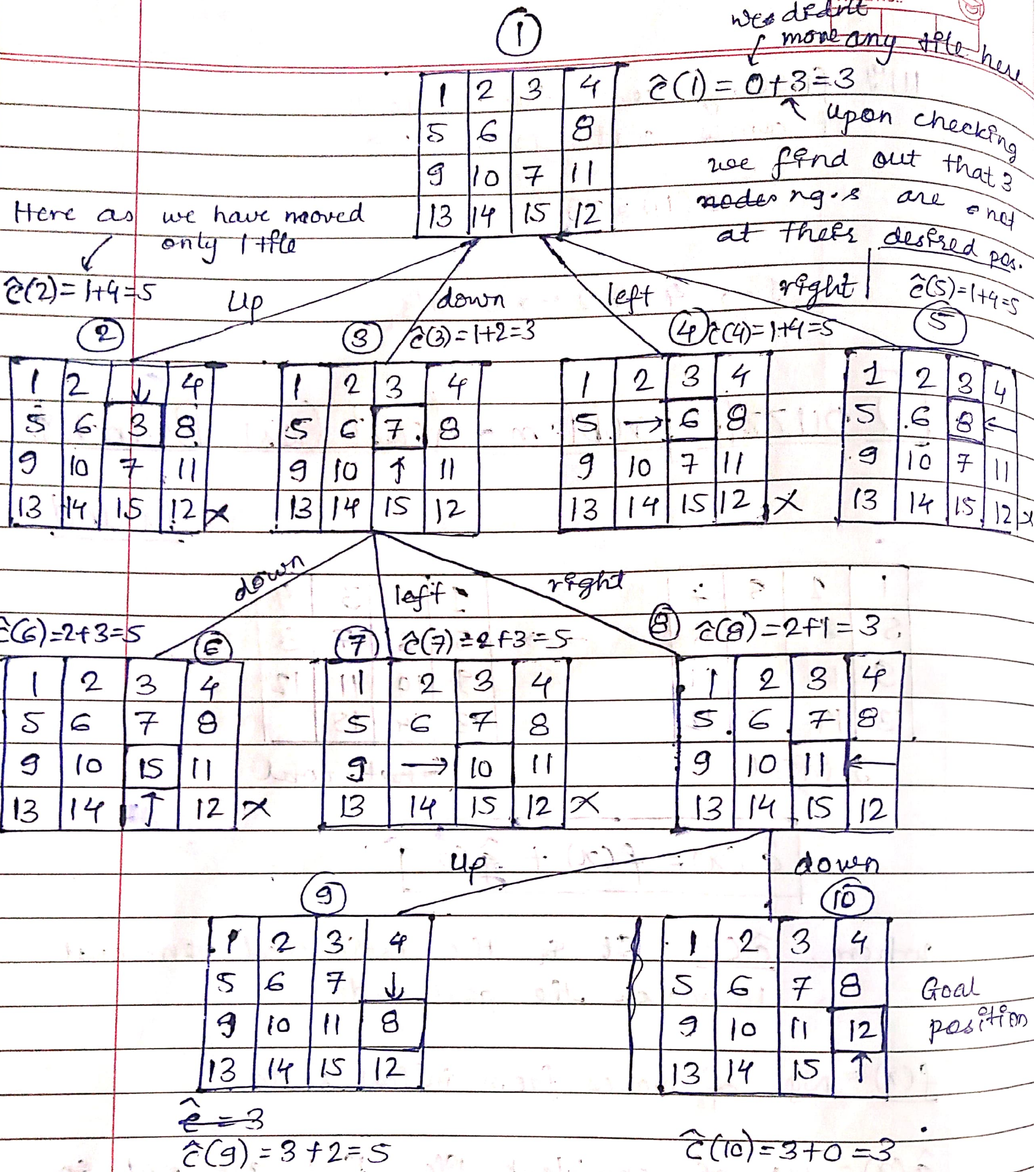
where, $\hat{c}(x)$ = It is the estimated min-cost to search the goal node

$f(x)$ = No. of moves from initial state

$\hat{g}(x)$ = No. of non-blank tiles that are not in their goal position

T.C = $O(n^2 \times n!)$ S.C. = $O(n^2)$

State Space Tree



Note: → We can't move tiles of the node in opp. direction of previous move else we reach the position.

→ When $\hat{g}(x) = 0$, stop, you have reached your goal

N*N-1 puzzle problem solvable or not

→ N is odd
→ Number of inversions even in input state

→ N is even
→ blank ~~of~~ on even row counting from bottom
& No. of inversions is odd

→ blank on odd row counting from bottom
& No. of inversions is even

Else puzzle not solvable