Design and Analysis of Algorithms

Lecture # 9

STRING MATCHING

String Matching

- **Introduction**
- **The Naive String Matching Algorithm**
- **The Rabin-Karp Algorithm**
- **String Matching with Finite Automata**
- **The Knuth-Morris-Pratt Algorithm**

Introduction

- Text-editing programs frequently need to find all occurrences of a pattern in the text.
- Efficient algorithms for this problem is called String-Matching Algorithms.
- Among its many applications, “String-Matching” is highly used in Searching for patterns in DNA and Internet search engines.
- Assume that the text is represented in the form of an array $\boldsymbol{T[1 \ldots n]}$ and the pattern is an array $\boldsymbol{P[1 \ldots m]}$.

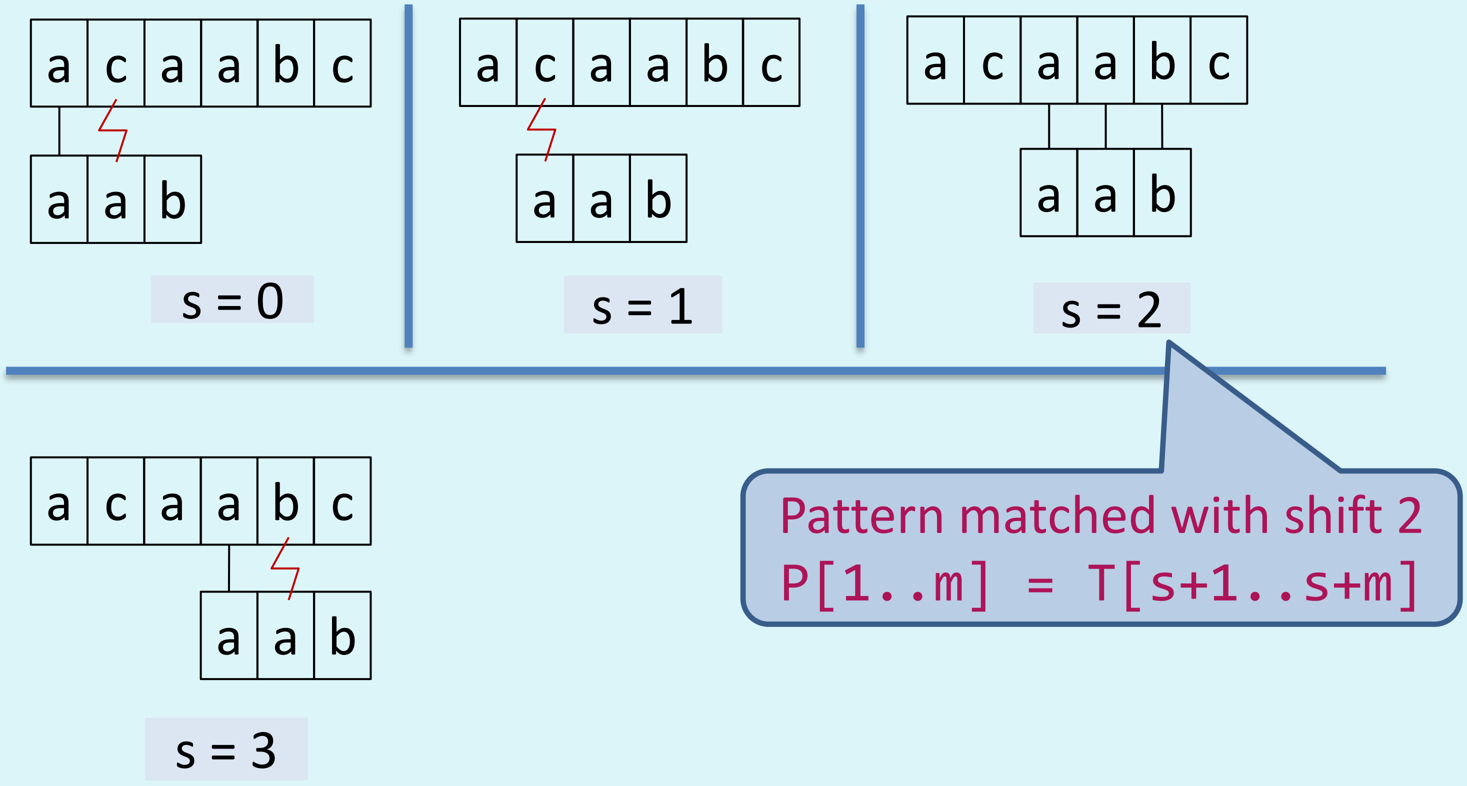
Text T[1..13]	a	b	c	a	b	a	a	b	c	a	b	a	c

Pattern P[1..4]	a	b	a	a

NAIVE STRING MATCHING ALGORITHM

Naive String Matching - Example

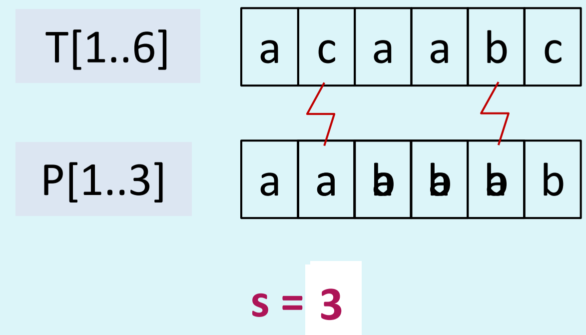
- The naive algorithm finds all valid shifts using a loop that checks the condition **P[1..m] = T[s+1..s+m]**



Naive String Matching - Algorithm

```
NAIVE-STRING MATCHER (T,P)
n = T.length
m = P.length
for s = 0 to n-m
      if   p[1..m] == T[s+1..s+m]
                print “Pattern occurs with
                      shift” s
```

Naive String Matcher takes time O((n-m+1)m)



Pattern occurs with shift 2

RABIN-KARP ALGORITHM

Introduction

Text T

3	1	4	1	5	9	2	6	5	3	5

Pattern P

2	6

Choose a random prime number q = 11

Let, p = P mod q
= 26 mod 11 = 4

Let t_s denotes modulo q for text of length m

3	1	4	1	5	9	2	6	5	3	5

Rabin-Karp Algorithm

Pattern P

2	**6**

p = P mod q = 26 mod 11 = 4

Text T

3	**1**	**4**	**1**	**5**	**9**	**2**	**6**	**5**	**3**	**5**

t_s

9	3	8	4	4	4	4	10	9	2

Spurious Hit

Valid match

if t_s == p
    if P[1..m] == T[s+1..s+m]
        print "pattern occurs with shift" s

Rabin-Karp Algorithm

- We can compute t_s using following formula

$$t_{s+1} = 10(t_s - 10^{m-1}T[s+1]) + T[s + m + 1]$$

3	1	4	1	5	9	2	6	5	3	5

For m=2 and s=0 $t_s = 31$

We wish to remove higher order digit T[s+1]=3 and bring the new lower order digit T[s+m+1]=4

$$\begin{aligned} t_{s+1} &= 10(31-10\cdot 3) + 4 \\ &= 10(1) + 4 = \mathbf{14} \end{aligned}$$

$$\begin{aligned} t_{s+2} &= 10(14-10\cdot 1) + 1 \\ &= 10(4) + 1 = \mathbf{41} \end{aligned}$$

Rabin-Karp-Matcher

RABIN-KARP-MATCHER(T, P, d, q)

$n \leftarrow$ length[T];

$m \leftarrow$ length[P];

$h \leftarrow d^{m-1} \bmod q$;

$p \leftarrow 0$;

$t_0 \leftarrow 0$;

for $i \leftarrow 1$ to m do

  $p \leftarrow (d_p + P[i]) \bmod q$

  $t_0 \leftarrow (dt_0 + T[i]) \bmod q$

for $s \leftarrow 0$ to $n - m$ do

  if $p == t_s$ then

   if $P[1..m] == T[s+1..s+m]$ then

     print "pattern occurs with shift" s

  if $s < n-m$ then

   $t_{s+1} \leftarrow (d(t_s - T[s+1]h) + T[s+m+1]) \bmod q$

T	3	1	4	1	5	9	2	6	5	3	5

P	2 6	d	10	q	11
n	11	m	2	h	10
p	0 / 4	t_0	0 / 9		

STRING MATCHING WITH FINITE AUTOMATA

Introduction to Finite Automata

- Finite automaton (FA) is a simple machine, used to recognize patterns. It has a set of states and rules for moving from one state to another.
- It takes the string of symbol as input and changes its state accordingly. When the desired symbol is found, then the transition occurs.
- At the time of transition, the automata can either move to the next state or stay in the same state.
- When the input string is processed successfully, and the automata reached its final state, then it will accept the input string.
- The string-matching automaton is very efficient: it examines each character in the text exactly once and reports all the valid shifts.

Introduction to Finite Automata

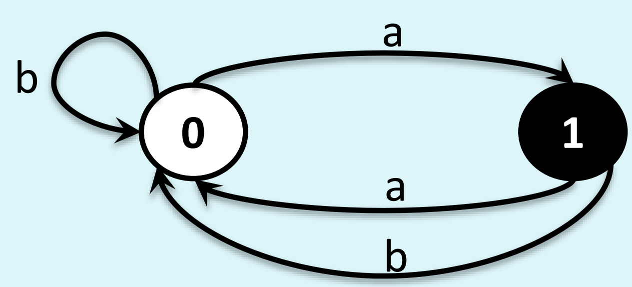
- A finite automaton M is a 5-tuple, which consists of,

$$(Q, q_0, A, \Sigma, \delta)$$

- Q is a finite set of **states**,
- $q_0 \epsilon\ Q$ is a **start state**,
- $A \subseteq Q$ set of **accepting states**,
- Σ is a finite **input alphabet**,
- δ is a **transition function** of M.

	Input	
State	a	b
0	1	0
1	0	0

Transition Table



Finite Automaton

Suffix of String

- Suffix of a string is any number of trailing symbols of that string. If a string $\boldsymbol{\omega}$ is a suffix of a string $\boldsymbol{x}$ then it is denoted by $\boldsymbol{\omega} \sqsupset \boldsymbol{x}$.

$P = ababa$	

Compute Transition Function

```
COMPUTE-TRANSITION-FUNCTION(P, Σ )
m ← length[P]
for q ← 0 to m do
   for each character α Є Σ do
          k ← min(m + 1, q + 2)
          repeat k ← k – 1 until P_k ⊐ P_qα
          δ(q, α) ← k
return δ
```

	1	2	3	4	5	6	7
Pattern	a	b	a	b	a	c	a
	1	2	3	4	5	6	7

$\Sigma = \{a, b, c\}$

$m = 7$

for q ← 0 to m **do**
  for each character ω ∈ Σ **do**
    k ← min(m + 1, q + 2)
    repeat k ← k – 1 **until** $P_k \sqsupset P_q\,\omega$
    δ(q, ω) ← k
return δ

input

State	a	b	c
0			
1			
2			
3			
4			
5			
6			
7			

q=0	ω=a	k=2	$P_2 \sqsupset P_0\omega$	ab⊐ϵa
		$\boxed{k=1}$	$P_1 \sqsupset P_0\omega$	a⊐ϵa
	ω=b	k=2	$P_2 \sqsupset P_0\omega$	ab⊐ϵb
		k=1	$P_1 \sqsupset P_0\omega$	a⊐ϵb
		$\boxed{k=0}$	$P_0 \sqsupset P_0\omega$	ϵ⊐ϵb
	ω=c	k=2	$P_2 \sqsupset P_0\omega$	ab⊐ϵc
		k=1	$P_1 \sqsupset P_0\omega$	a⊐ϵc
		$\boxed{k=0}$	$P_0 \sqsupset P_0\omega$	ϵ⊐ϵc

	1	2	3	4	5	6	7
Pattern	a	b	a	b	a	c	a
	1	2	3	4	5	6	7

$\Sigma = \{a, b, c\}$

$m = 7$

for $q \leftarrow 0$ to m **do**

for each character $\omega \in \Sigma$ **do**

$k \leftarrow \min(m + 1, q + 2)$

repeat $k \leftarrow k - 1$ **until** $P_k \sqsupset P_q\omega$

$\delta(q, \omega) \leftarrow k$

return δ

input

State	a	b	c
0	1	0	0
1			
2			
3			
4			
5			
6			
7			

q=1	ω=a	k=3	$P_3 \sqsupset P_1\omega$	aba⊐aa
		k=2	$P_2 \sqsupset P_1\omega$	ab⊐aa
		$\boxed{k=1}$	$P_1 \sqsupset P_1\omega$	a⊐aa
	ω=b	k=3	$P_3 \sqsupset P_1\omega$	aba⊐ab
		$\boxed{k=2}$	$P_2 \sqsupset P_1\omega$	ab⊐ab
	ω=c	k=3	$P_3 \sqsupset P_1\omega$	aba⊐ac
		k=2	$P_2 \sqsupset P_1\omega$	ab⊐ac
		k=1	$P_1 \sqsupset P_1\omega$	a⊐ac
		$\boxed{k=0}$	$P_0 \sqsupset P_1\omega$	ϵ⊐ac

	1	2	3	4	5	6	7
Pattern	a	b	a	b	a	c	a
	1	2	3	4	5	6	7

$\Sigma = \{a, b, c\}$

$m = 7$

for q ← 0 to m **do**

for each character ω Є Σ **do**

k ← min(m + 1, q + 2)

repeat k ← k – 1 **until** $P_k \sqsupset P_q\omega$

δ(q, ω) ← k

return δ

input

State	a	b	c
0	1	0	0
1	1	2	0
2			
3			
4			
5			
6			
7			

q=2	ω=a	k=4	$P_4 \sqsupset P_2\omega$	abab⊐aba
		k=3	$P_3 \sqsupset P_2\omega$	aba⊐aba
	ω=b	k=0	$P_0 \sqsupset P_2\omega$	ϵ⊐abb
	ω=c	k=0	$P_0 \sqsupset P_2\omega$	ϵ⊐abc

q=3	ω=a	k=1	$P_1 \sqsupset P_3\omega$	a⊐abaa
	ω=b	k=4	$P_4 \sqsupset P_3\omega$	abab⊐abab
	ω=c	k=0	$P_0 \sqsupset P_3\omega$	ϵ⊐abac

Finite Automata Matcher

```
FINITE-AUTOMATON MATCHER(T, δ, m)
n ← length[T]
q ← 0
for i ← 1 to n do
    q ← δ(q, T[i])
    if q == m then
    print "Pattern occurs with shift" i – m
```

i = 7

q = 5

$q = \delta(0, a) = 1$

$q = \delta(1, b) = 2$

$q = \delta(2, a) = 3$

$q = \delta(3, b) = 4$

$q = \delta(4, a) = 5$

$q = \delta(5, b) = 4$

$q = \delta(4, a) = 5$

State	a	b	c
0	1	0	0
1	1	2	0
2	3	0	0
3	1	4	0
4	5	0	0
5	1	4	6
6	7	0	0
7	1	2	0

(input)

	1	2	3	4	5	6	7
Pattern	a	b	a	b	a	c	a

	1	2	3	4	5	6	7	8	9	10	11
Text	a	b	a	b	a	b	a	c	a	b	a

Finite Automata Matcher

```
FINITE-AUTOMATON MATCHER(T, δ, m)
n ← length[T]
q ← 0
for i ← 1 to n do
    q ← δ(q, T[i])
    if q == m then
    print "Pattern occurs with shift" i – m
```

i = 9

q = 7

$q = \delta(0, a) = 1$
$q = \delta(1, b) = 2$
$q = \delta(2, a) = 3$
$q = \delta(3, b) = 4$
$q = \delta(4, a) = 5$
$q = \delta(5, b) = 4$
$q = \delta(4, a) = 5$
$q = \delta(5, c) = 6$
$q = \delta(6, a) = 7$

State	input a	input b	input c
0	1	0	0
1	1	2	0
2	3	0	0
3	1	4	0
4	5	0	0
5	1	4	6
6	7	0	0
7	1	2	0

	1	2	3	4	5	6	7
Pattern	a	b	a	b	a	c	a

	1	2	3	4	5	6	7	8	9	10	11
Text	a	b	a	b	a	b	a	c	a	b	a

Suffix & Prefix of a String

Suffix of a string	
$P \ = string$	
$P_1 = g$	$\boldsymbol{P_1 \sqsupset P}$
$P_2 = ng$	$\boldsymbol{P_2 \sqsupset P}$
$P_3 = ing$	$\boldsymbol{P_3 \sqsupset P}$
$P_4 = ring$	$\boldsymbol{P_4 \sqsupset P}$
$P_5 = tring$	$\boldsymbol{P_5 \sqsupset P}$

Prefix of a string	
$P \ = string$	
$P_1 = s$	$\boldsymbol{P_1 \sqsubset P}$
$P_2 = st$	$\boldsymbol{P_2 \sqsubset P}$
$P_3 = str$	$\boldsymbol{P_3 \sqsubset P}$
$P_4 = stri$	$\boldsymbol{P_4 \sqsubset P}$
$P_5 = strin$	$\boldsymbol{P_5 \sqsubset P}$

STRING MATCHING WITH KNUTH-MORRIS-PRATT ALGORITHM

Introduction

- The KMP algorithm relies on prefix function (π).
- Proper prefix: All the characters in a string, with one or more cut off the end. “S”, “Sn”, “Sna”, and “Snap” are all the proper prefixes of “Snape”.
- Proper suffix: All the characters in a string, with one or more cut off the beginning. “agrid”, “grid”, “rid”, “id”, and “d” are all proper suffixes of “Hagrid”.
- KMP algorithm works as follows:
  - Step-1: Calculate Prefix Function
  - Step-2: Match Pattern with Text

Longest Common Prefix and Suffix

	1	2	3	4	5	6	7
Pattern	a	b	a	b	a	c	a
Prefix(π)							

ababa

Possible prefix = a, ab, aba, abab

Possible suffix = a, ba, aba, baba

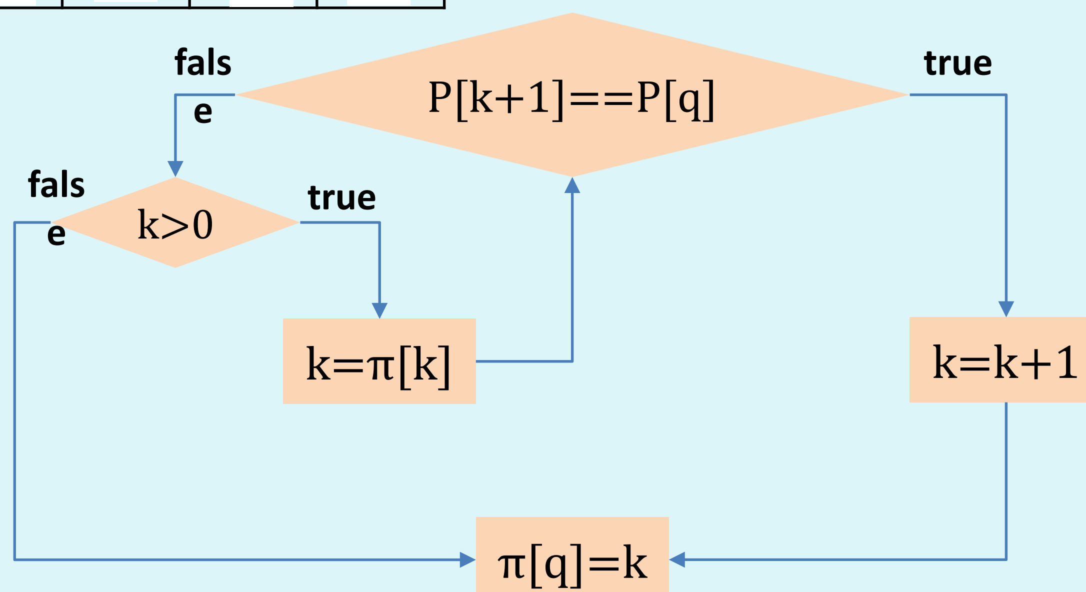
Calculate Prefix Function - Example

	k+1	q					q
	1	2	3	4	5	6	7
P	a	c	a	c	a	g	t
π							

$k = 0$

$q = 7$

Initially set **π[1] = 0**
k is the longest prefix found
q is the current index of pattern

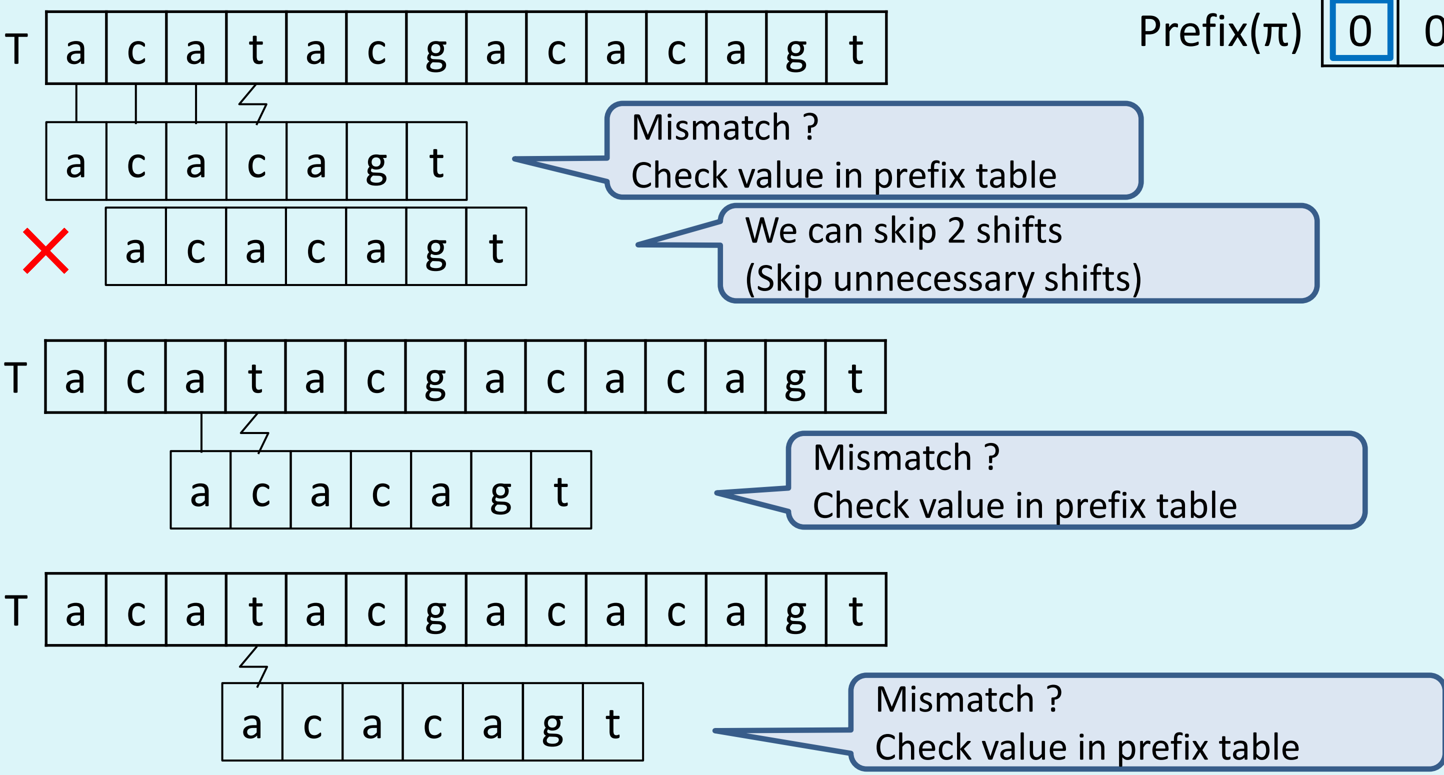


KMP- Compute Prefix Function

```
COMPUTE-PREFIX-FUNCTION(P)
    m ← length[P]
    π[1] ← 0
    k ← 0
    for q ← 2 to m
        while k > 0 and P[k + 1] ≠ P[q]
                k ← π[k]
        end while
        if P[k + 1] == P[q] then
                k ← k + 1
        end if
        π[q] ← k
    return π
```

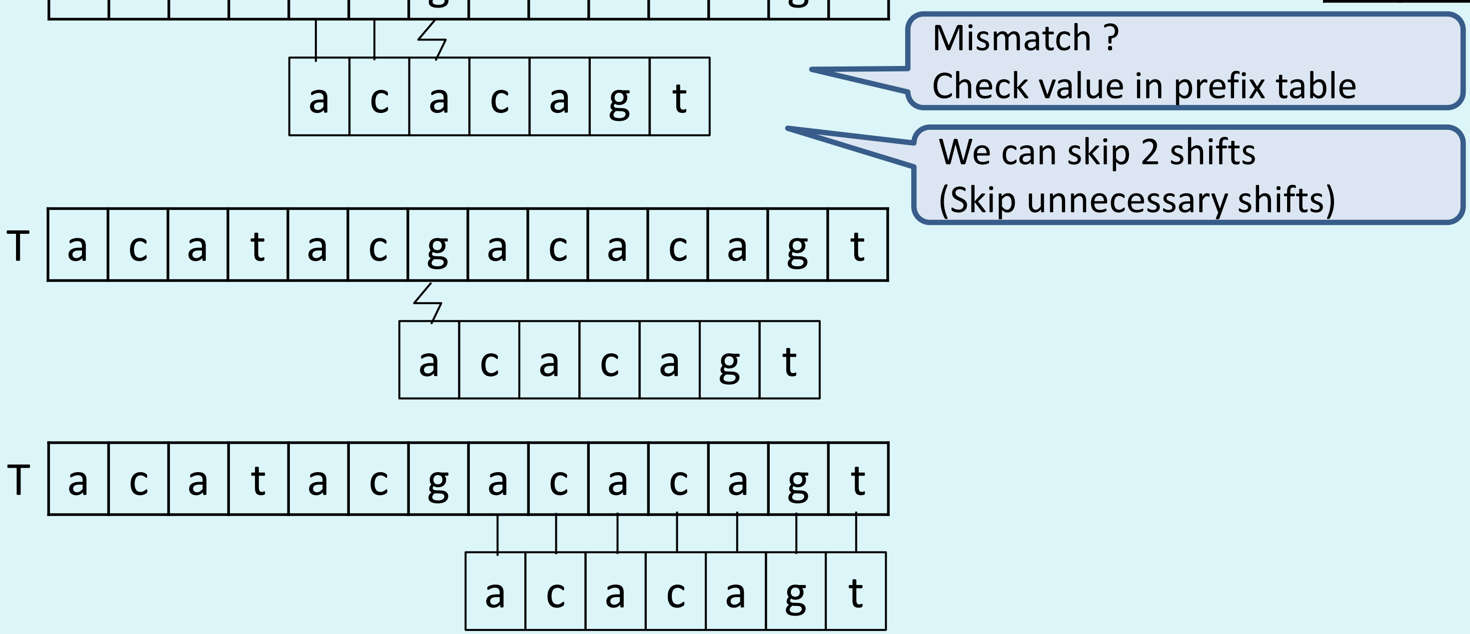
KMP String Matching

	1	2	3	4	5	6	7
Pattern	a	c	a	c	a	g	t
Prefix(π)	$\boxed{0}$	0	$\boxed{1}$	2	3	0	0



KMP String Matching

	1	2	3	4	5	6	7
Pattern	a	c	a	c	a	g	t
Prefix(π)	0	$\boxed{0}$	1	2	3	0	0



Pattern matches with shift $i - m$

KMP-MATCHER

```
KMP-MATCHER(T, P)
n ← length[T]
m ← length[P]
π ← COMPUTE-PREFIX-FUNCTION(P)
q ← 0                        //Number of characters matched.
for i ← 1 to n               //Scan the text from left to right.
    while q > 0 and P[q + 1] ≠ T[i]
        q ← π[q]    //Next character does not match.
    if P[q + 1] == T[i] then
        then q ← q + 1      //Next character matches.
    if q == m then          //Is all of P matched?
        print "Pattern occurs with shift" i - m
        q ← π[q]    //Look for the next match.
```