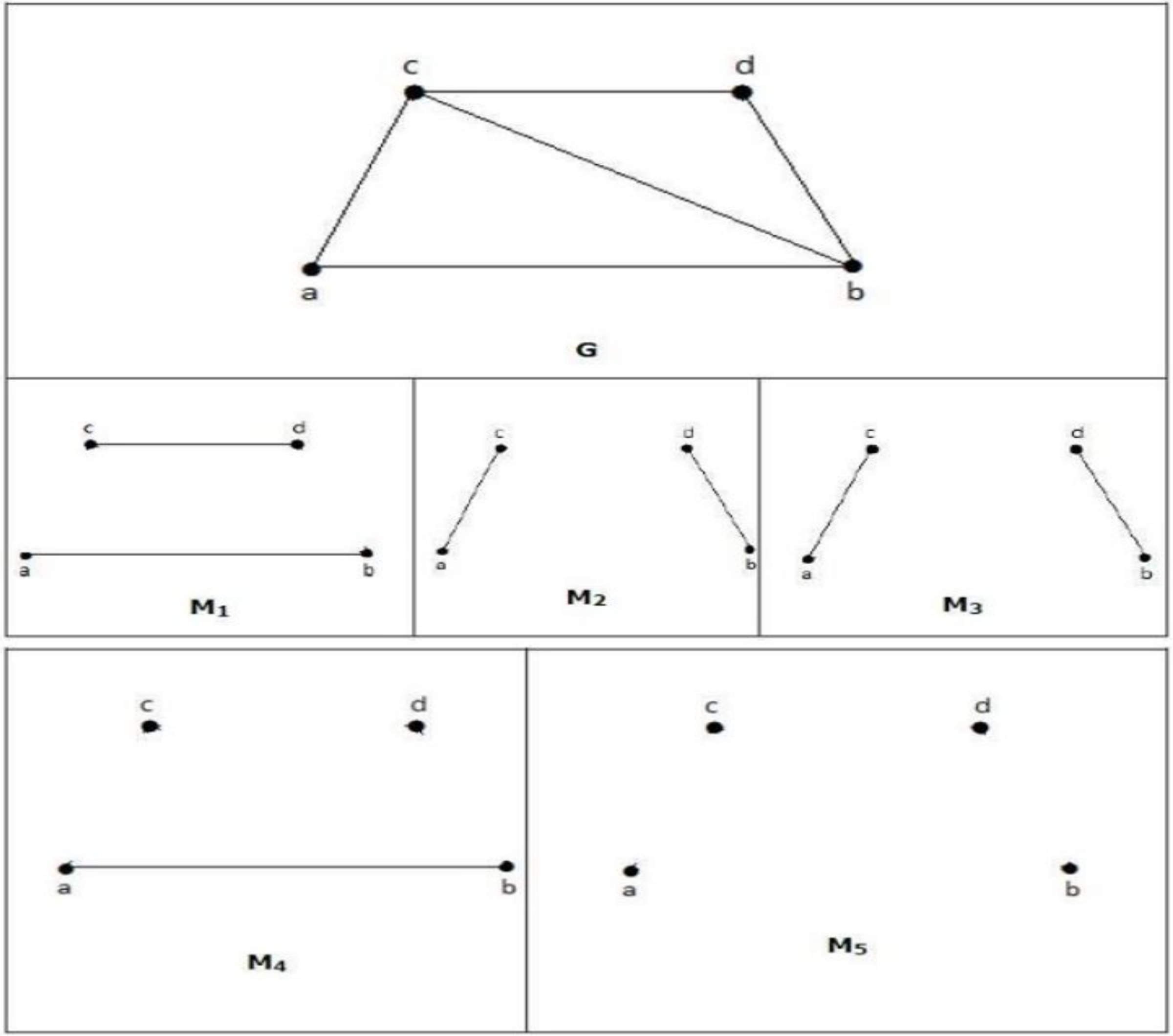
Graph connectivity

Matching Problems

Matching in a graph



Given an **undirected graph**, a **matching** is a set of edges, such that no two edges share the same vertex.

In other words, matching of a graph is a subgraph where each node of the subgraph has either **zero or one edge** incident to it.

Let ‘G’ = (V, E) be a graph. A subgraph is called a matching M(G), if each vertex of G is incident with at most one edge in M, i.e.,

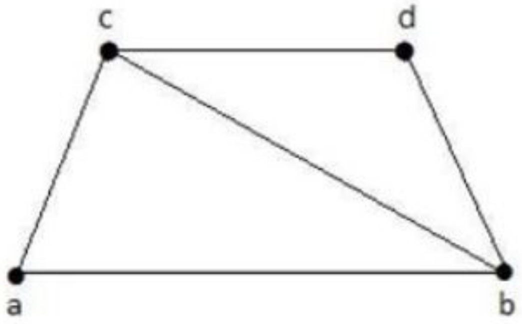
$\deg(V) \leq 1 \ \forall \ V \in G$

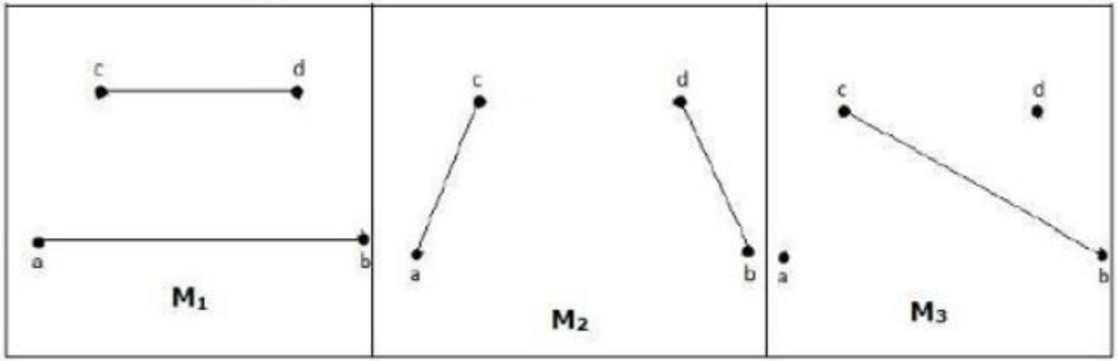
which means in the matching graph M(G), the vertices should have a degree of 1 or 0, where the edges should be incident from the graph G.

Maximal Matching

A matching M of graph ‘G’ is said to maximal **if no other edges of ‘G’ can be added to M**.

Example

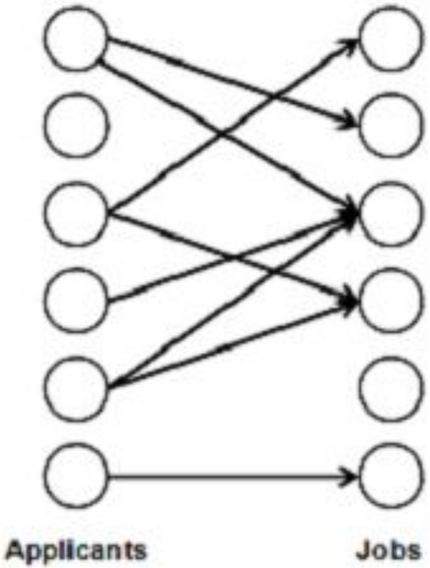


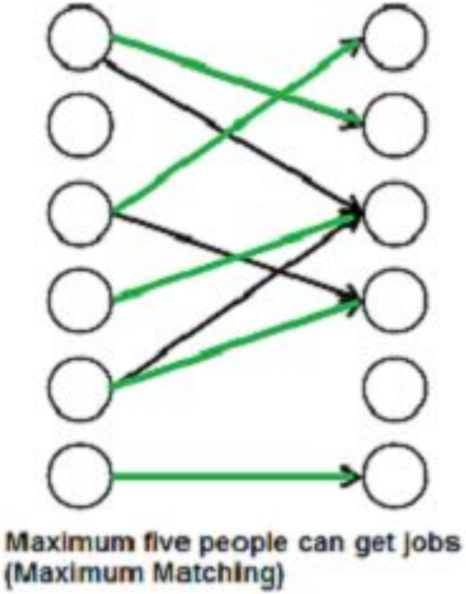


M1, M2, M3 from the above graph are the maximal matching of G.

Maximal matching in a bipartite graph

There are M job applicants and N jobs. Each applicant has a subset of jobs that he/she is interested in. Each job opening can only accept one applicant and a job applicant can be appointed for only one job. Find an assignment of jobs to applicants in such that as many applicants as possible get jobs.

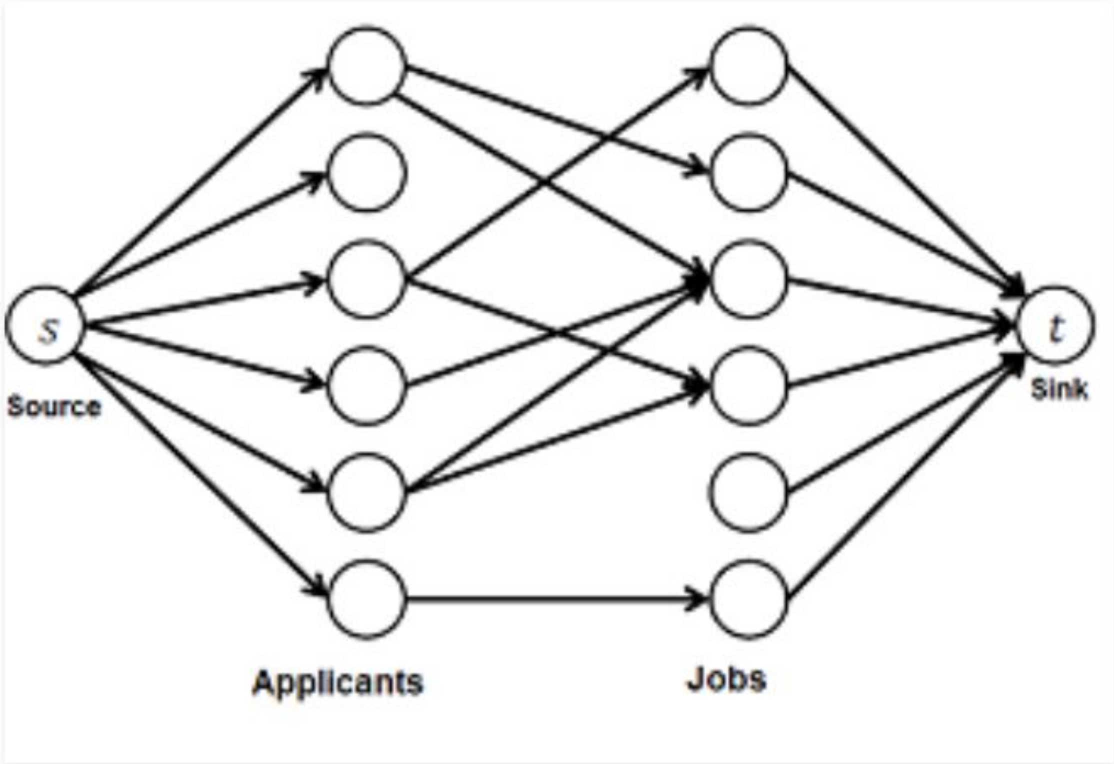




Maximum Bipartite Matching and Max Flow Problem

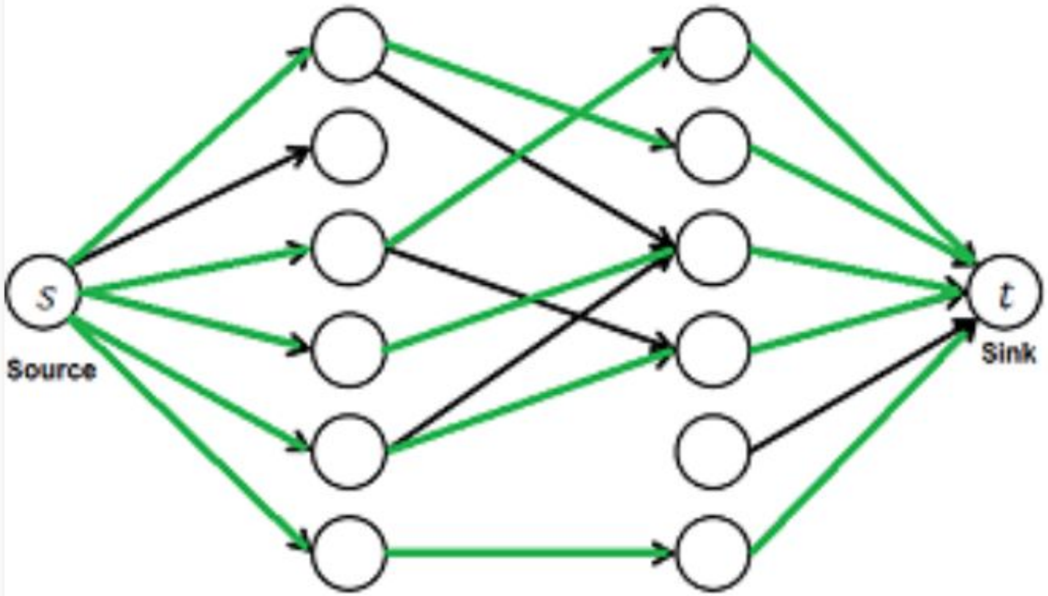
Maximum **B**ipartite **M**atching (**MBP**) problem can be solved by converting it into a flow network

Following are the steps.



1) Build a Flow Network

There must be a source and sink in a flow network. So we add a source and add edges from source to all applicants. Similarly, add edges from all jobs to sink. The capacity of every edge is marked as 1 unit.



The maximum flow from source to sink is five units. Therefore, maximum five people can get jobs.

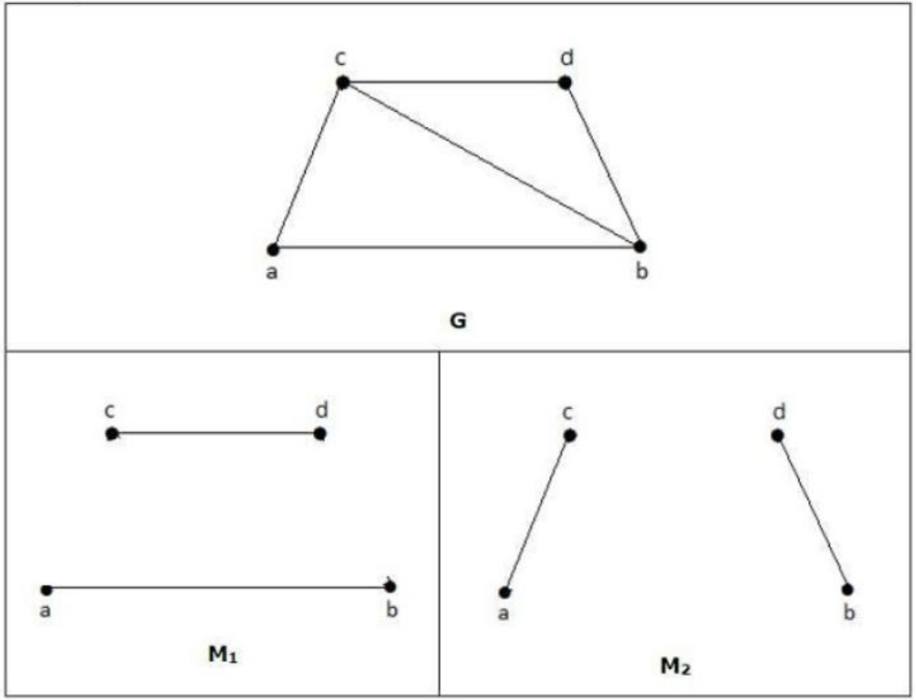
We use Ford-Fulkerson algorithm to find the maximum flow in the flow network

Maximum Matching

It is also known as largest maximal matching. Maximum matching is defined as the maximal matching with maximum number of edges.

The number of edges in the maximum matching of 'G' is called its **matching number**.

Example



For a graph given in the above example, M1 and M2 are the maximum matching of 'G' and its matching number is 2. Hence by using the graph G, we can form only the subgraphs with only 2 edges maximum. Hence we have the matching number as two.

Perfect Matching

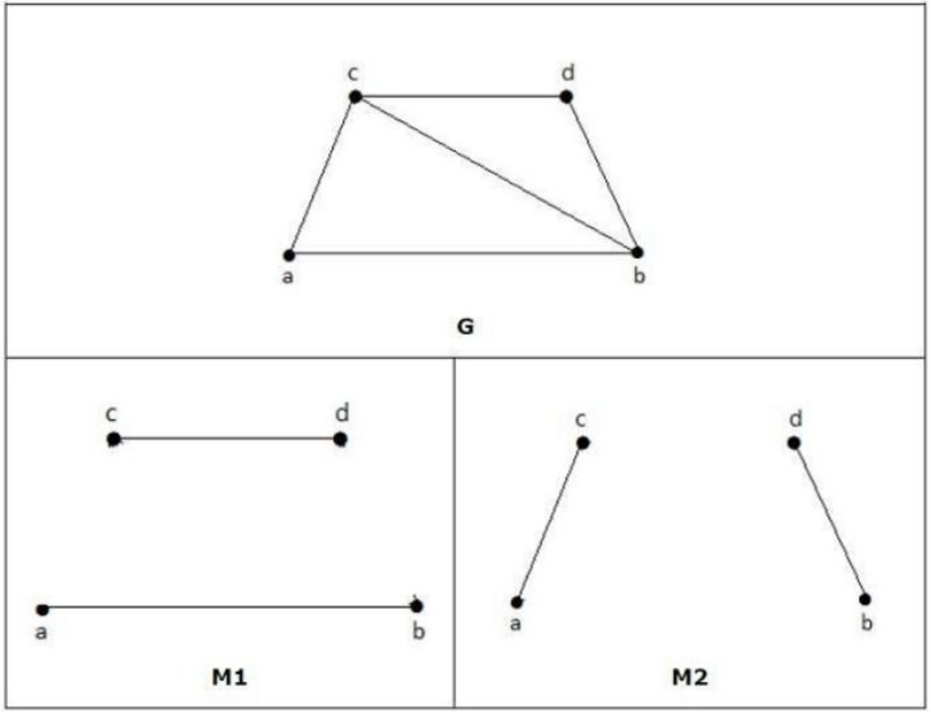
A matching (M) of graph (G) is said to be a perfect match, if every vertex of graph g (G) is incident to exactly one edge of the matching (M), i.e.,

$\deg(V) = 1 \; \forall \; V$

The degree of each and every vertex in the subgraph should have a degree of 1.

Example

In the following graphs, M1 and M2 are examples of perfect matching of G.



1. Every perfect matching of graph is also a maximum matching of graph

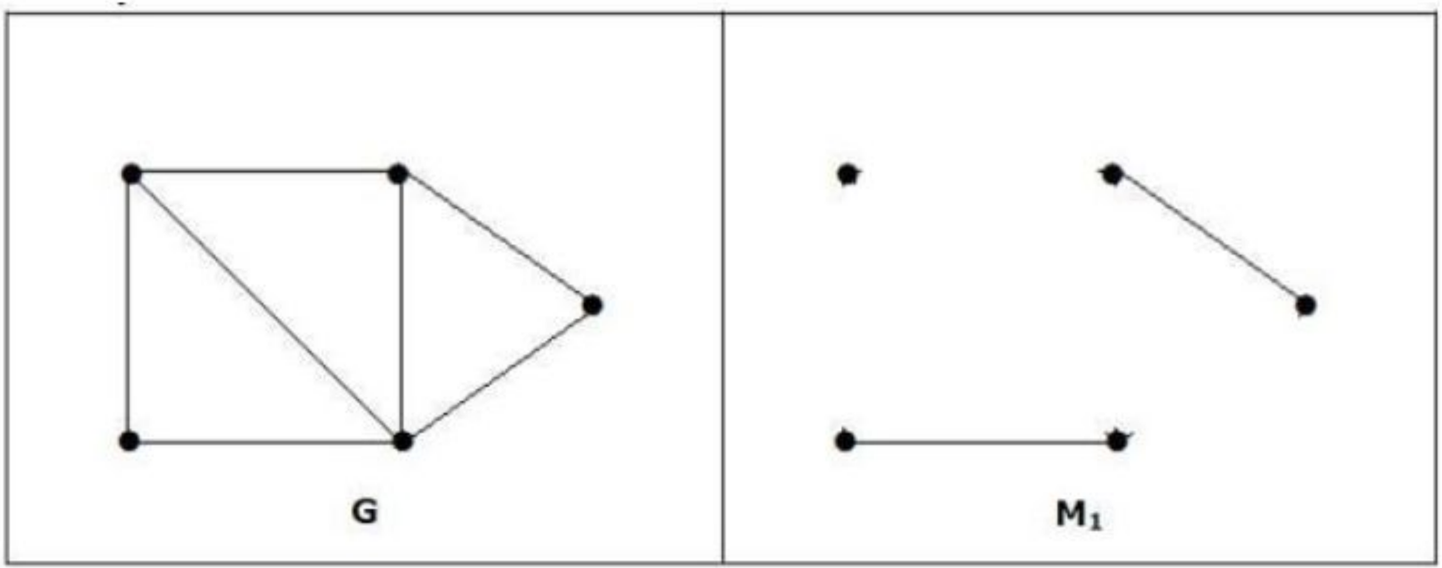
2. A maximum matching of graph need not be perfect.

3. If a graph ‘G’ has a perfect match, then the number of vertices $|V(G)|$ is even.

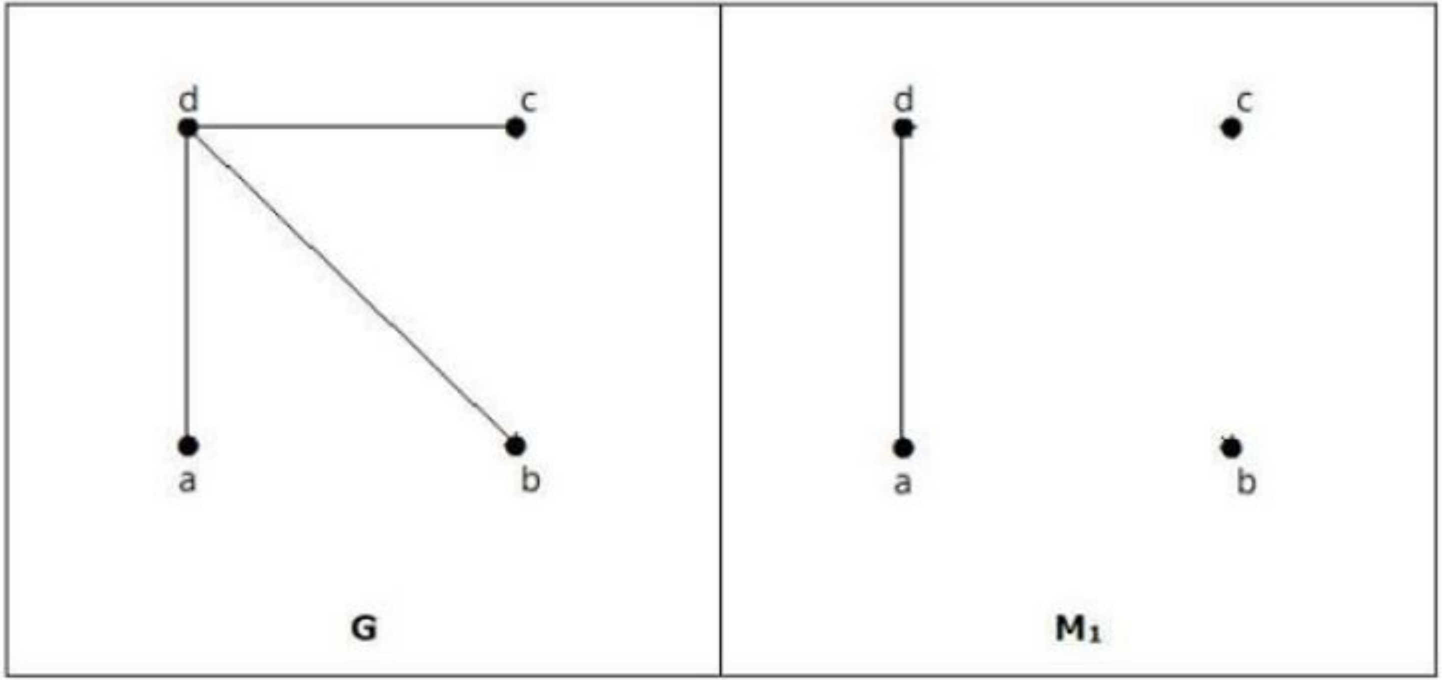
Vice versa not true

4. If G has even number of vertices, then Matching need not be perfect.

A maximum matching of graph need not be perfect.



Not a perfect match, even though it has even number of vertices.



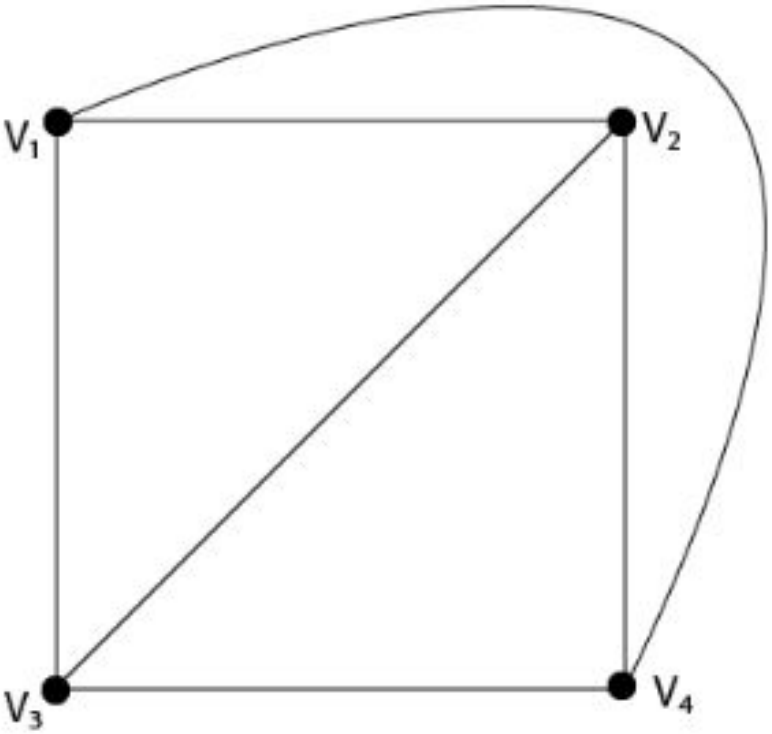
Isomorphic Graph

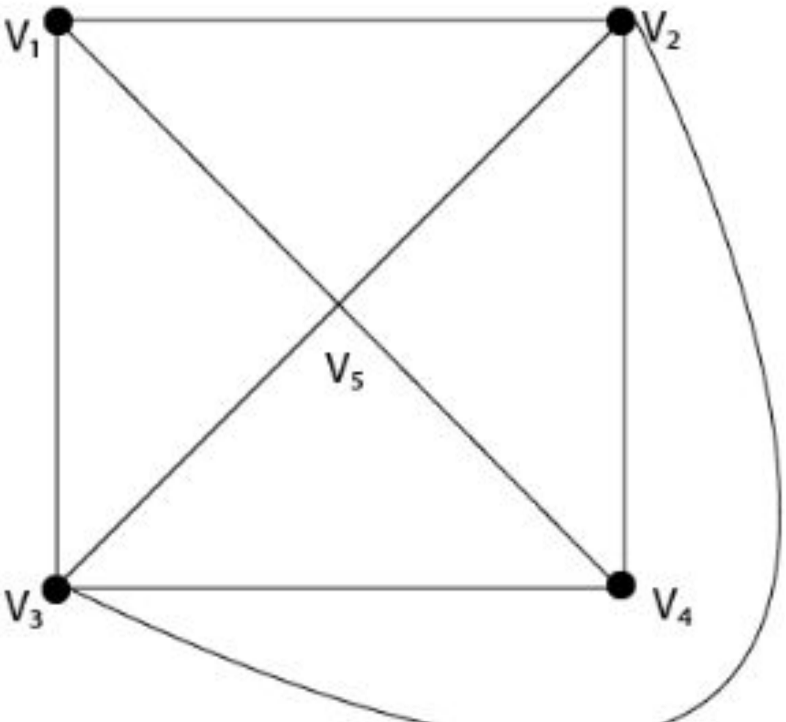
Planar Graph

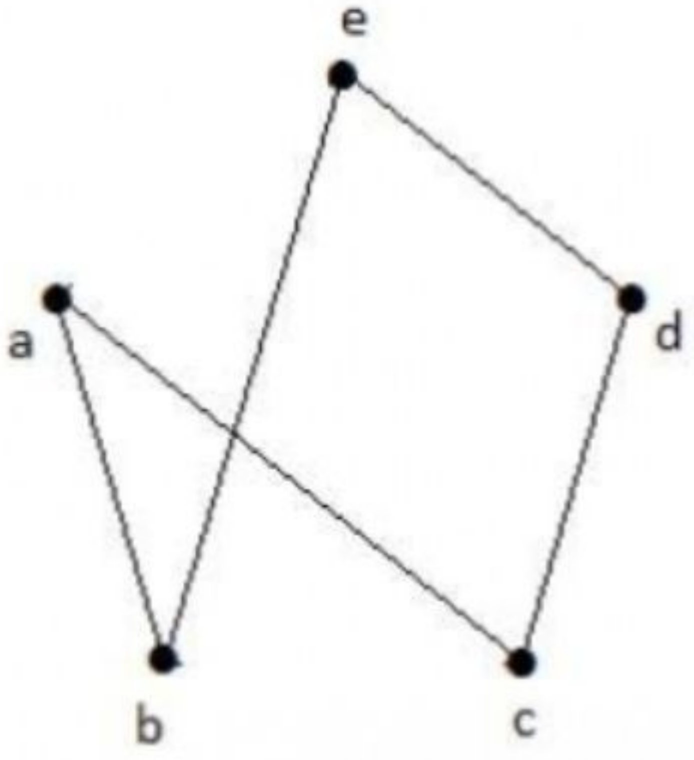
Planar Graph:

A graph is said to be planar if it can be drawn in a plane so that no edge cross.

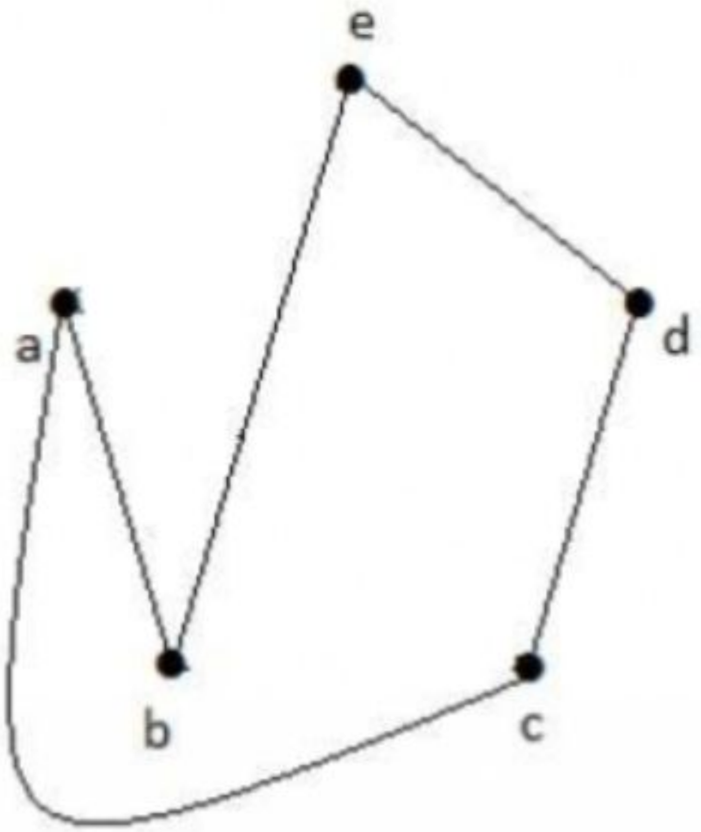
Example: The graph shown in fig is planar graph.







NON - PLANAR GRAPH

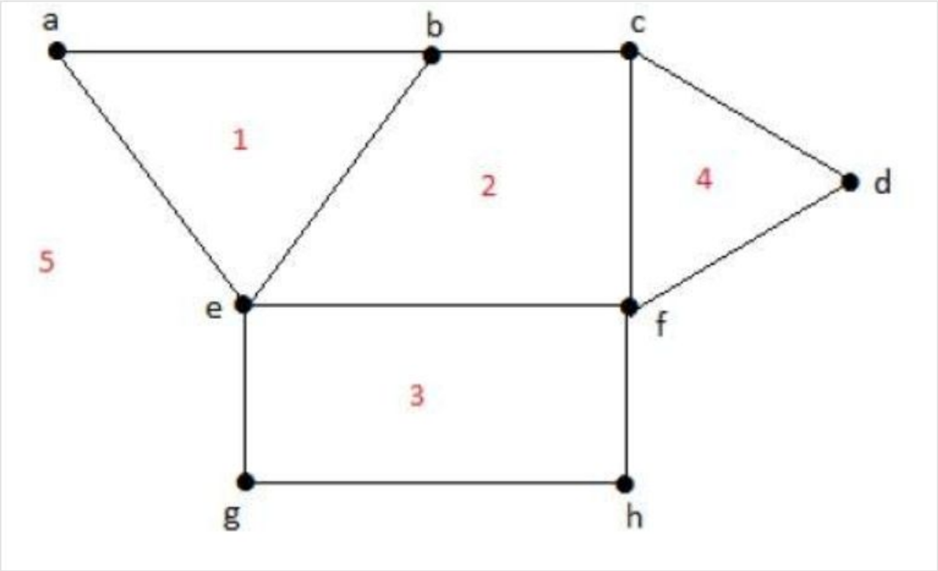


PLANAR GRAPH

Regions

Every planar graph divides the plane into connected areas called regions.

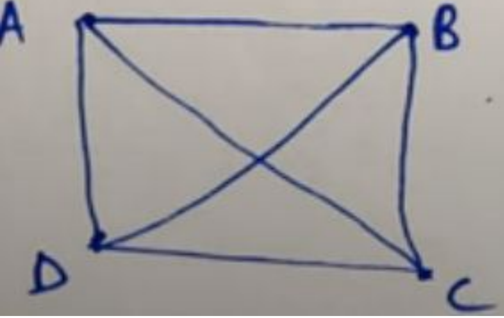
Example



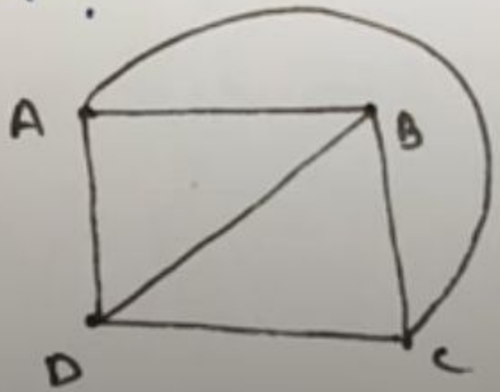
Degree of a bounded region **r = deg(r)** = Number of edges enclosing the regions **r**.

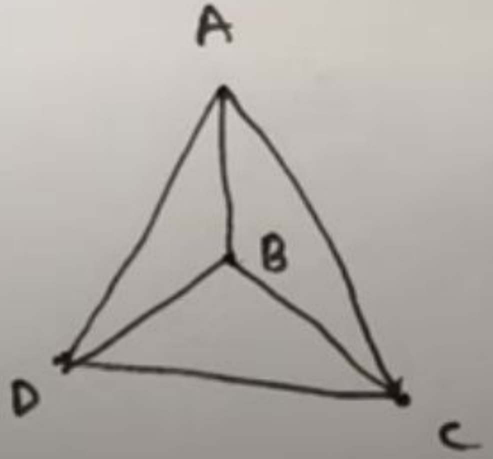
```
deg(1) = 3
deg(2) = 4
deg(3) = 4
deg(4) = 3
deg(5) = 8
```

the graph planar?



planar





In a planar graph,

- If degree of each region is K, then the sum of degrees of regions is

$$K|R| = 2|E|$$

- If the degree of each region is at least $K(\geq K)$, then

$$K|R| \leq 2|E|$$

- If the degree of each region is at most $K(\leq K)$, then

$$K|R| \geq 2|E|$$

Solved example in a notebook

Properties of Planar Graphs:

1. If a connected planar graph G has e edges and r regions, then $r \leq \frac{2}{3}$ e.
2. If a connected planar graph G has e edges, v vertices, and r regions, then $v-e+r=2$.
3. If a connected planar graph G has e edges and v vertices, then $3v-e\geq6$.
4. A complete graph K_n is a planar if and only if $n<5$.
5. A complete bipartite graph K_{mn} is planar if and only if $m<3$ or $n>3$.