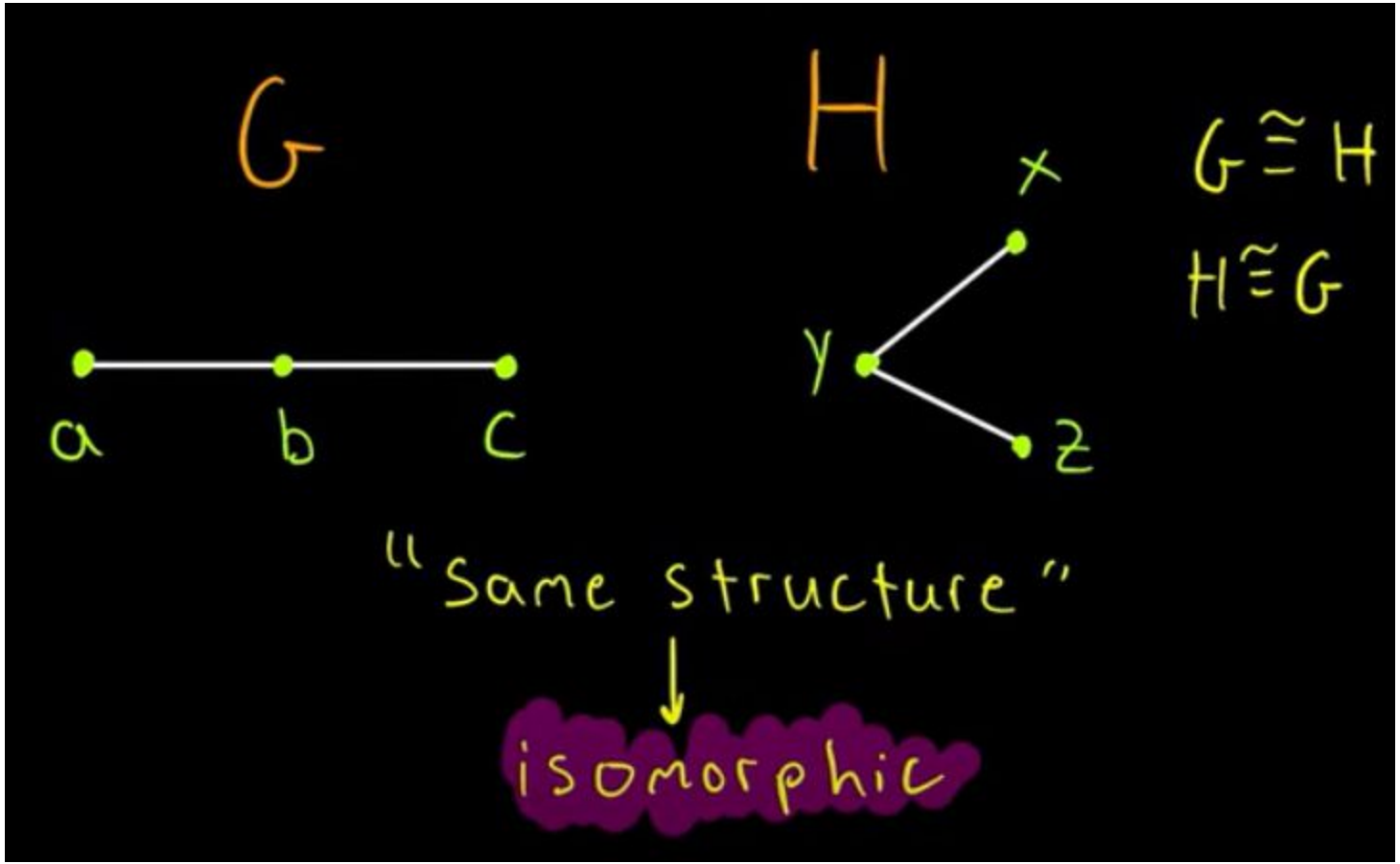
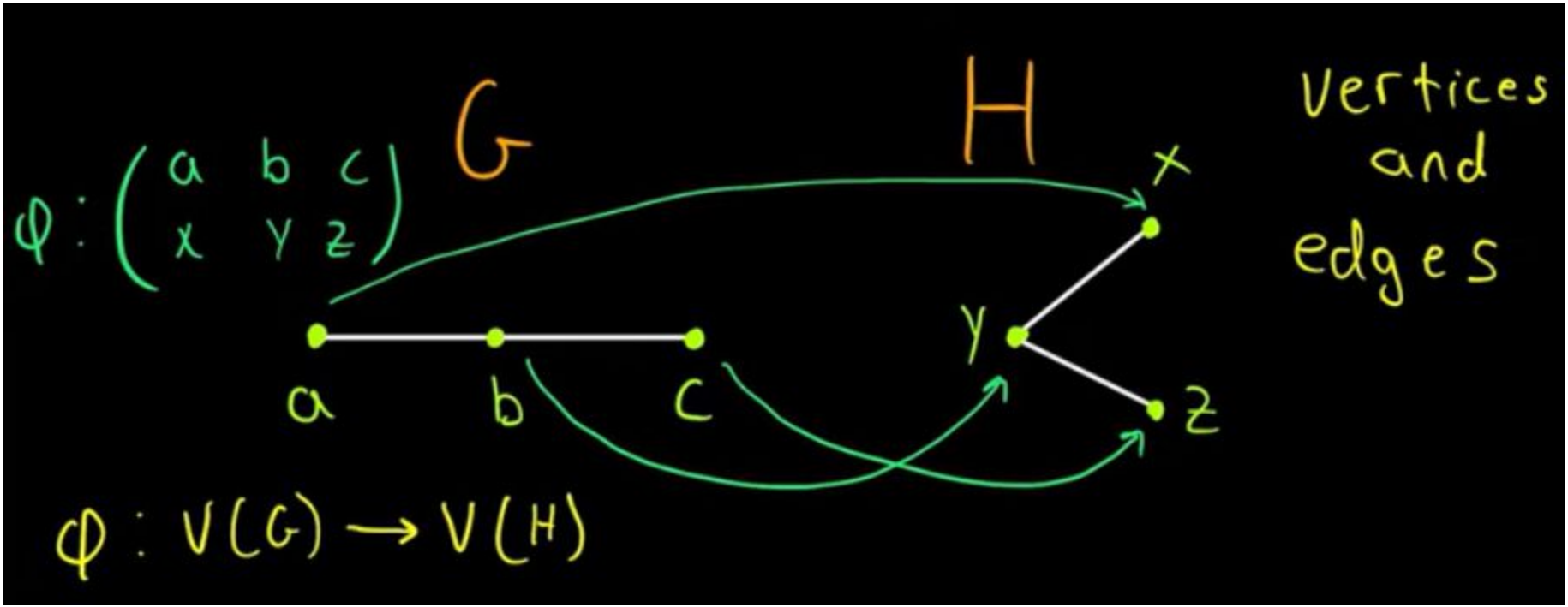
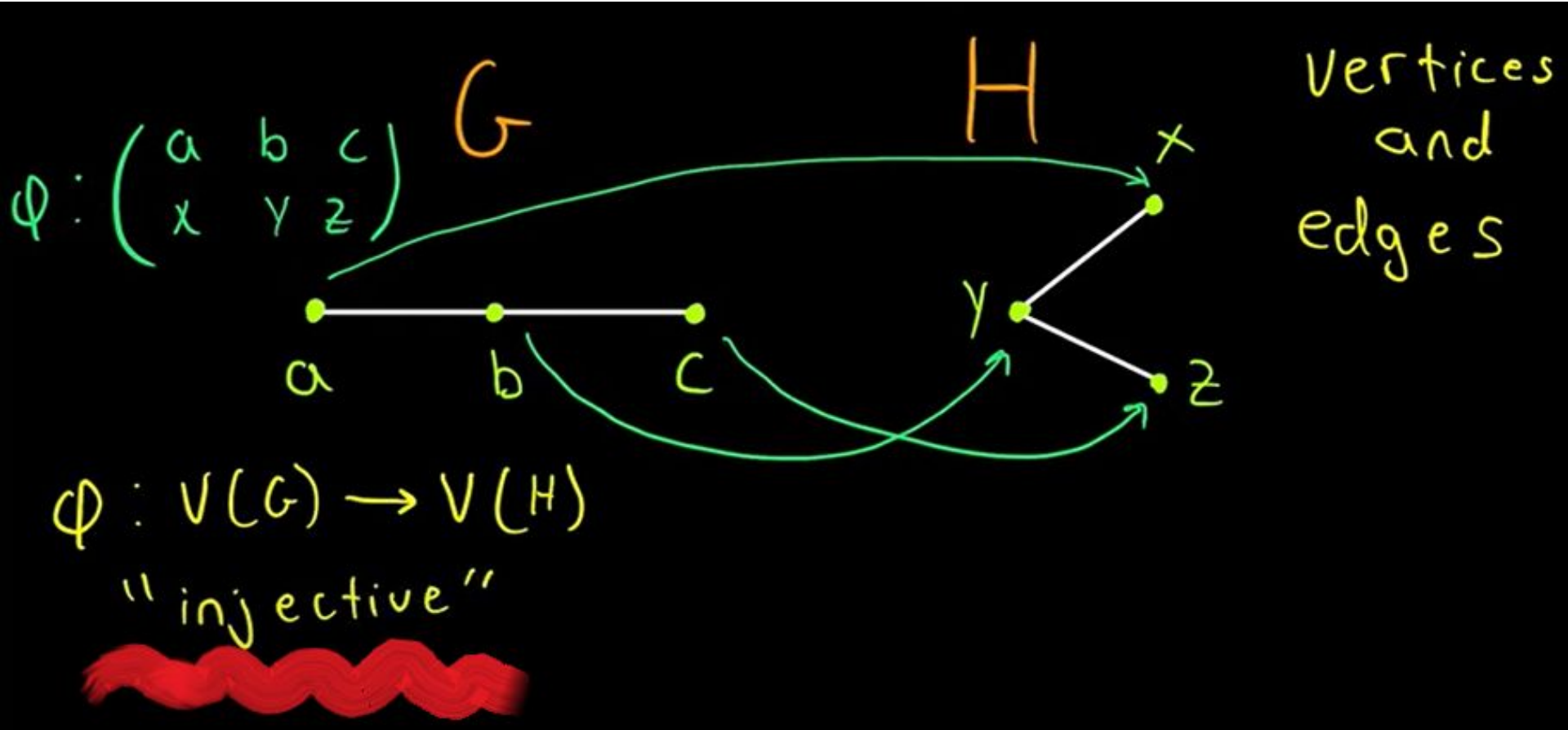
Isomorphic Graphs

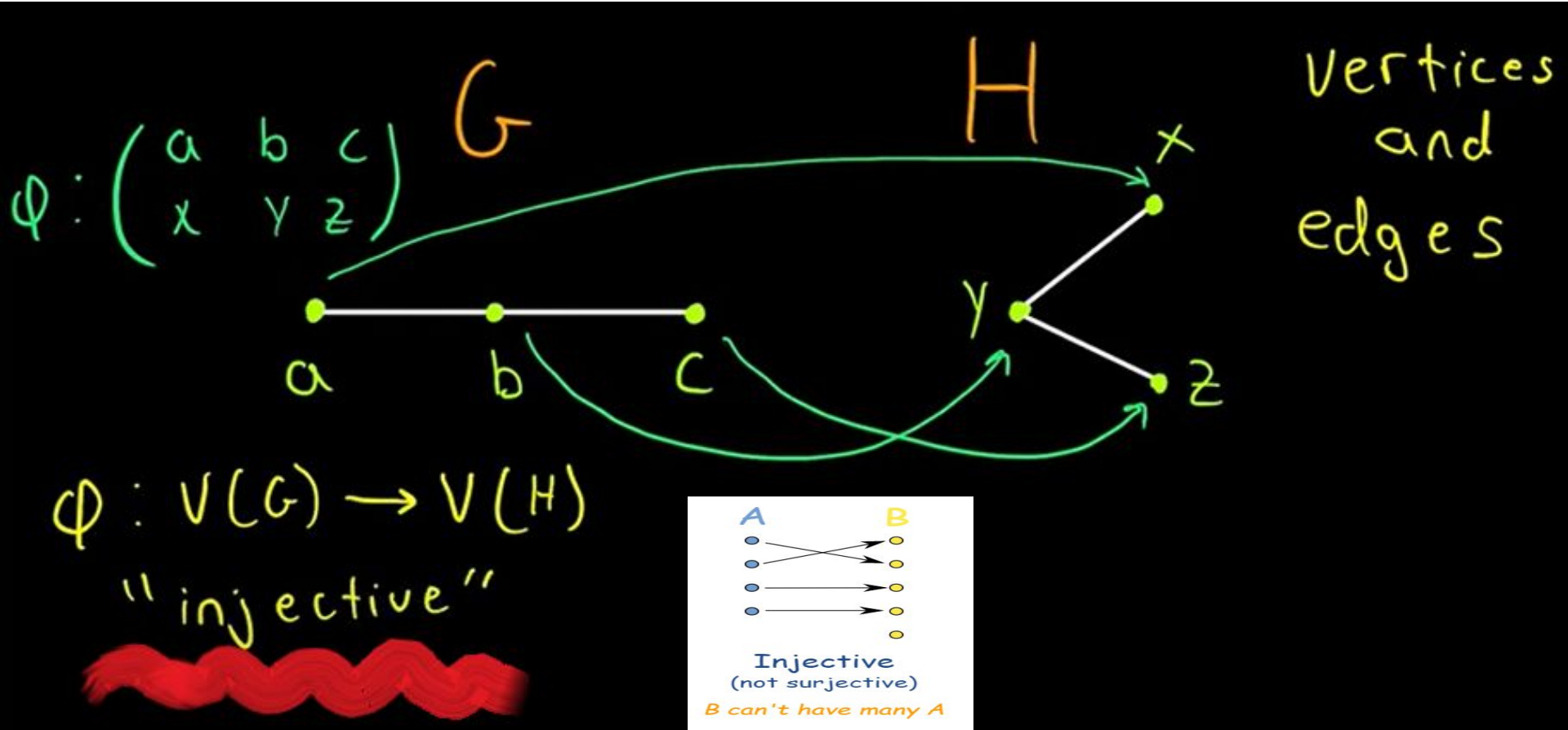

G
H
x
y
z
a
b
c
G ≅ H
H ≅ G
"same structure"
isomorphic


G
H
Vertices and edges
φ : (a b c / x y z)
x
y
z
a
b
c
φ : V(G) → V(H)

Each distinct vertex of graph G should be mapped to each distinct vertex of Graph H.



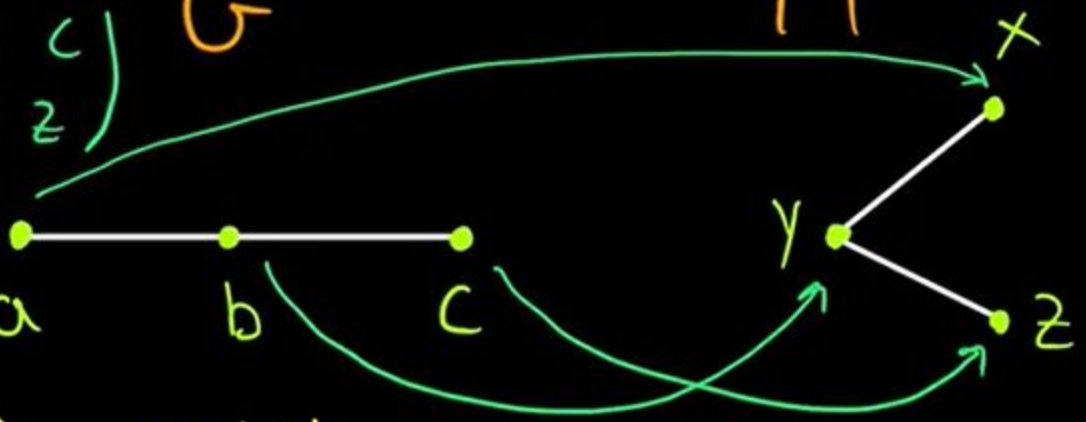
Every vertex in graph H, should get mapped to some vertex in graph G.



$\varphi : \begin{pmatrix} a & b & c \\ x & y & z \end{pmatrix}$

G

H

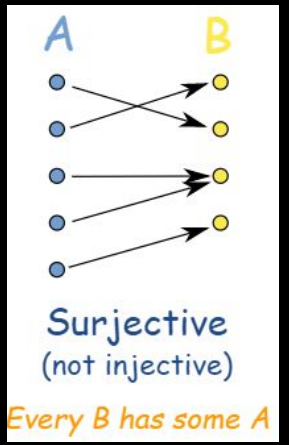


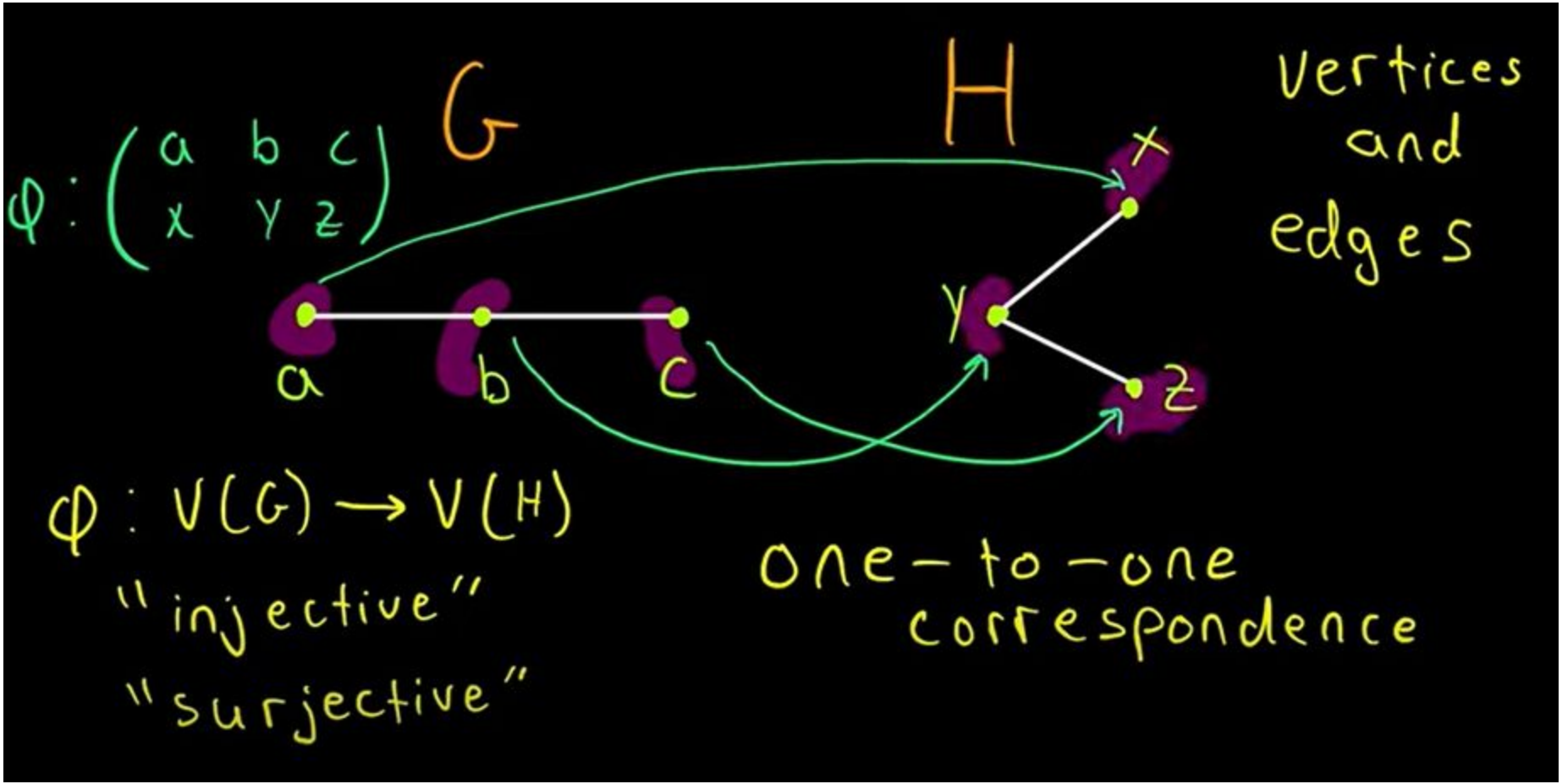
Vertices and edges

$\varphi : V(G) \rightarrow V(H)$

"injective"

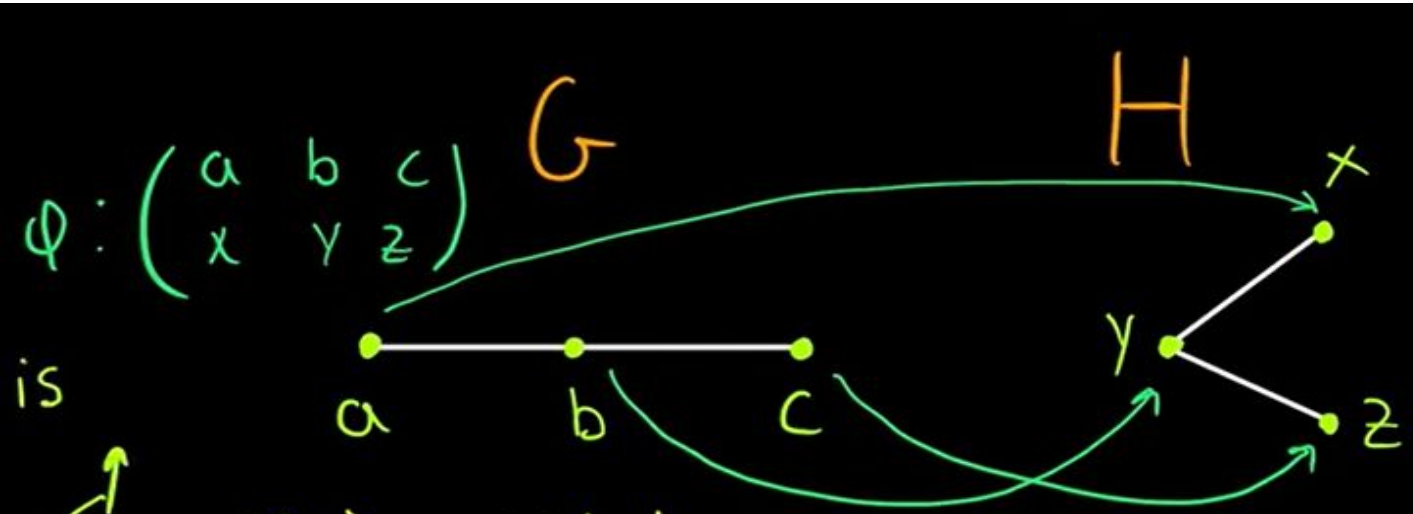
"surjective"




φ : (a b c / x y z)
G
H
Vertices and edges
a
b
c
x
y
z
φ : V(G) → V(H)
"injective"
"surjective"
one-to-one correspondence

Function □ is called isomorphism.

$\varphi : \begin{pmatrix} a & b & c \\ x & y & z \end{pmatrix}$

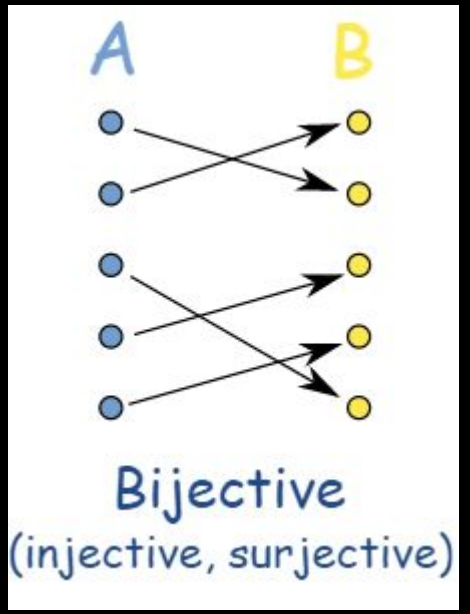


is

$\varphi : V(G) \to V(H)$

"injective" } → bijective
"surjective" }

$v_1v_2 \in E(G)$ if and only if $\varphi(v_1)\,\varphi(v_2) \in E(H)$



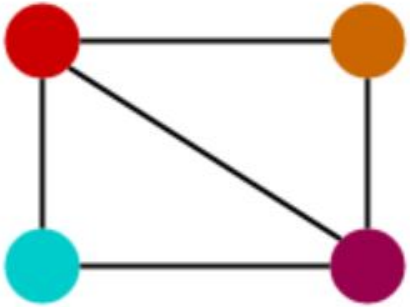
For any two graphs to be isomorphic, following 4 conditions must be satisfied-

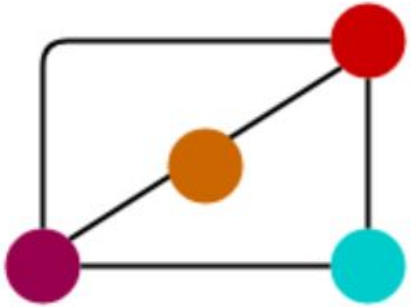
- Number of vertices in both the graphs must be same.
- Number of edges in both the graphs must be same.
- Degree sequence of both the graphs must be same.
- If a cycle of length k is formed by the vertices $\{ v_1 , v_2 , \ldots , v_k \}$ in one graph, then a cycle of same length k must be formed by the vertices $\{ f(v_1) , f(v_2) , \ldots , f(v_k) \}$ in the other graph as well.

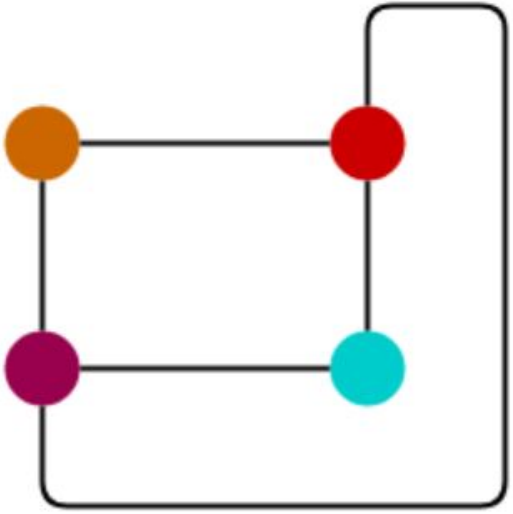
Some facts

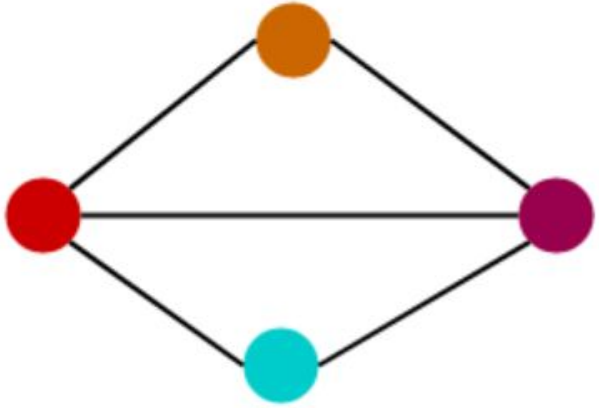
- Two graphs are isomorphic if and only if their complement graphs are isomorphic.
- Two graphs are isomorphic if their adjacency matrices are same.
- Two graphs are isomorphic if their corresponding sub-graphs obtained by deleting some vertices of one graph and their corresponding images in the other graph are isomorphic.

Graph Isomorphism Example-

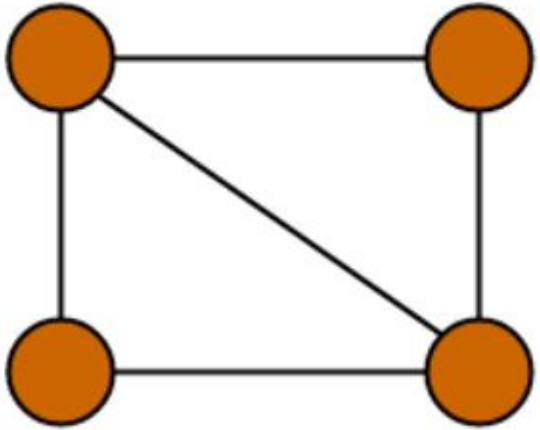




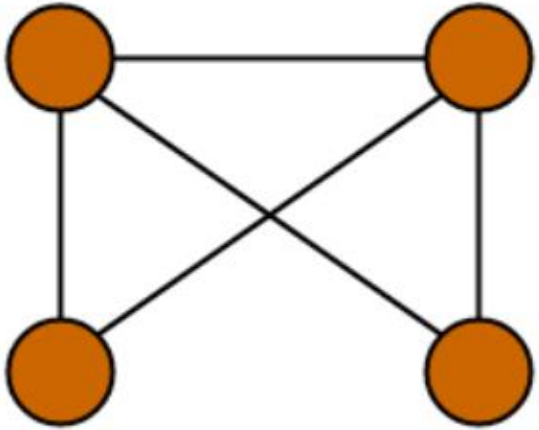




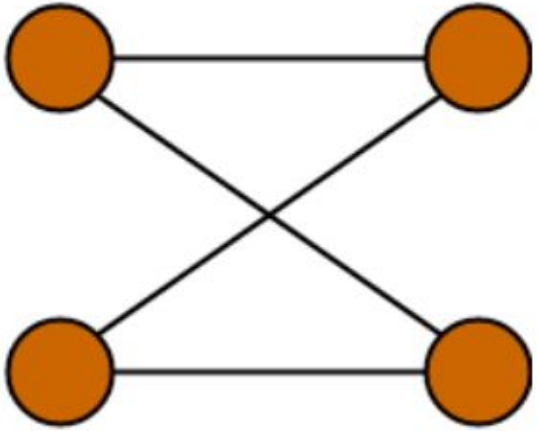
Which of the following graphs are isomorphic?



G1



G2



G3

G3 is neither isomorphic to G1 nor G2.

Graphs G1 and G2 are isomorphic graphs.

Which of the following graphs are isomorphic