Function

Composition of Functions

$f(x) = 3x + 4$

$g(x) = 2x - 5$

$(f \circ g)(3) = 7$

$f(g(3)) = 7$

$f(1) = 7$

Composition of Functions

$$(g \circ f)(3) = 21$$

$$g(f(3)) = 21$$

$$g(13) = 21$$

Composition of Functions

Find:

1. $(f \circ g)(x)$

Given:

$f(x) = 2x^2$

$g(x) = 4x$

$= f[g(x)]$

$= f[\,4x\,]$

$= 2(4x)^2$

$= 2(16x^2)$

$= 32x^2$

Composition of Functions

Find:

2. $(g \circ f)(x)$

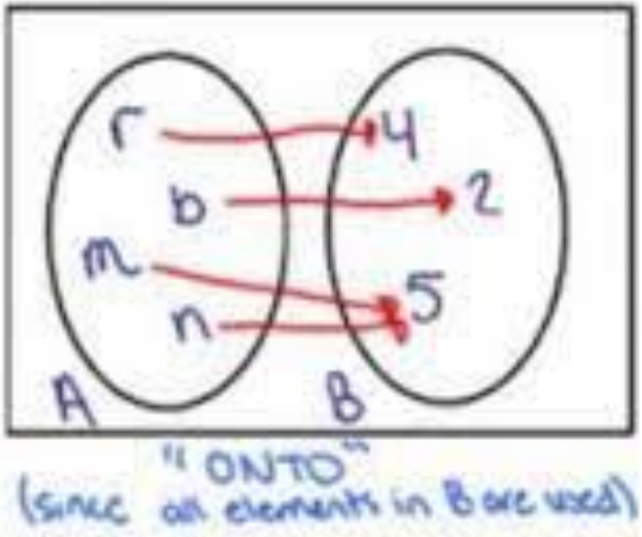
$$= g[f(x)]$$

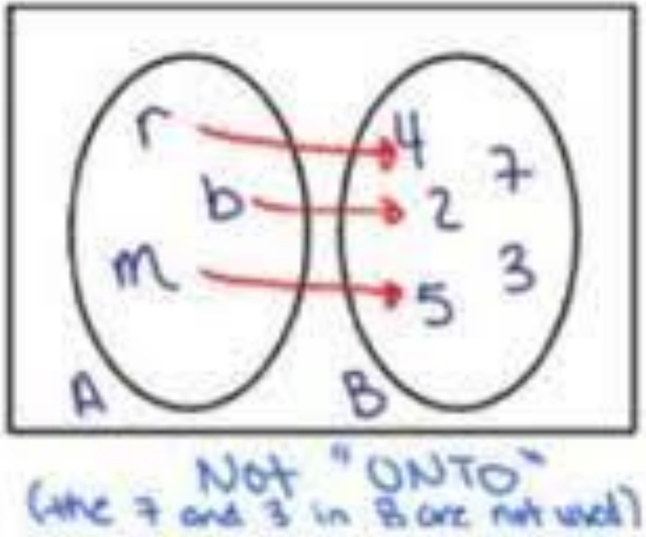
$$= g[2x^2]$$

$$= 4(2x^2)$$

$$= 8x^2$$

Invertible function





Invertible Function

$f : X \longrightarrow Y$ Invertible

$g : Y \longrightarrow X$

$g \circ f = I_X \qquad f \circ g = I_Y$

g : inverse of f

$g = f^{-1}$

$\underset{\text{Invertible}}{f} \Rightarrow f \text{ is } \begin{Bmatrix} \text{one} - \text{one} \\ \text{onto} \end{Bmatrix}$

$f \text{ is } \begin{Bmatrix} \text{one} - \text{one} \\ \text{onto} \end{Bmatrix} \Rightarrow f \text{ Invertible}$

Consider f: **R** → **R** given by f(x) = 4x+ 3. Show that f is invertible. Find the inverse of f.

f is invertible if f is **one-one** and **onto**

Checking one-one

$f(x_1) = 4x_1 + 3$

$f(x_2) = 4x_2 + 3$

Putting $f(x_1) = f(x_2)$

$4x_1 + 3 = 4x_2 + 3$

$4x_1 = 4x_2$

$x_1 = x_2$

If $f(x_1) = f(x_2)$, then $x_1 = x_2$

∴ f is one-one

Rough

One-one Steps:

1. *Calculate $f(x_1)$*
2. *Calculate $f(x_2)$*
3. *Putting $f(x_1) = f(x_2)$*
   we have to prove $x_1 = x_2$

Checking onto

$f(x) = 4x + 3$

Let f(x) = y, where y ∈ Y

$y = 4x + 3$

$y - 3 = 4x$

$4x = y - 3$

$x = \frac{y-3}{4}$

Here, y is a real number

So, $\frac{y-3}{4}$ is also a real number

So, x is a real number

Thus, f is onto

Since f is one-one and onto

f is invertible

Consider f: **R** → **R** given by f(x) = 4x+ 3. Show that f is invertible.
Find the inverse of f.

Checking inverse

Step 1

$f(x) = 4x + 3$

Let $f(x) = y$

$y = 4x + 3$

$y - 3 = 4x$

$4x = y - 3$

$x = \frac{y-3}{4}$

Rough

Checking inverse of $f: X \to Y$

***Step 1**: Calculate $g: Y \to X$*

***Step 2**: Prove $gof = I_X$*

***Step 3**: Prove $fog = I_Y$*

g is the inverse of f

Let g(y) = $\frac{y-3}{4}$

where g: **R** → **R**

Step 2:

gof = g(f(x))

= g(4x + 3)

$= \frac{(4x+3)-3}{4}$

$= \frac{4x+3-3}{4}$

$= \frac{4x}{4}$

= x

= I_R

Rough

Checking inverse of f:X → Y

Step 1*: Calculate g: Y → X*

Step 2*: Prove gof = I_X*

Step 3*: Prove fog = I_Y*

g is the inverse of f

Step 3:

fog = f(g(y))

$= f\left(\frac{y-3}{4}\right)$

$= 4\left(\frac{y-3}{4}\right) + 3$

$= y - 3 + 3$

$= y + 0$

$\mathbf{= y}$

$\mathbf{= I_R}$

Since *gof* $= I_R$ and *fog* $= I_R$,

f is invertible

& Inverse of f = g(y) = $\mathbf{\frac{y-3}{4}}$

Rough

Checking inverse of f:X → Y

Step 1*: Calculate g: Y → X*

Step 2*: Prove gof =* I_X

Step 3*: Prove fog =* I_Y

g is the inverse of f

Pigeonhole Principle Definition

In Discrete Mathematics, the pigeonhole principle states that if we must put N + 1 or more pigeons into N Pigeon Holes, then some pigeon holes must contain two or more pigeons.

Example:

If Kn+ 1 (where k is a positive integer) pigeons are distributed among n holes than some hole contains at least k + 1 pigeons.

The **Extended Pigeonhole Principle** states that if n objects (pigeons) are placed into m boxes (pigeonholes) then there are at least one box containing at least $\left\lceil \frac{n}{m} \right\rceil$ objects.

Pigeon Hole Principle

A bag contains beads of two colours: black and white. What is the smallest number of beads which must be drawn from the bag, without looking so that among these beads, two are of the same colour?

Find the minimum number of students in a class such that three of them are born in the same month?

Solution: Number of month $n = 12$

According to the given condition,

$K+1 = 3$

$K = 2$

$M = kn + 1 = 2*12 + 1 = 25.$

Suppose 5 pairs of socks are in a drawer. Picking minimum __ socks guarantees that at least one pair is chosen.

Show that in a group of 20 people and friendship is mutual, show that there exist two people who have the same number of friends?

Solution: Each person can have 0 to 19 friends. But if someone has 0 friends, then no one can have 19 friends and similarly you cannot have 19 friends and no friends. So, there are only 19 options for the number of friends and 20 people, so we can use pigeonhole.

suppose there are 35 different time periods during which classes at the local college can be scheduled. If there are 679 different classes, what is the minimum number of rooms that will be needed?

Let $n = 679$ (pigeons) and $m = 35$ (pigeonholes)

Then $\left\lceil \dfrac{679}{35} \right\rceil = \lceil 19.4 \rceil = 20$

suppose we want to know the minimum number of students required in a math class so that at least six students will receive the same letter grade (A, B, C, D, or F).

n = ?, m = 5, and 6 are the number of students who receive the same letter grade then $\left\lceil \frac{n}{5} \right\rceil = 6$

Which means the number of students in the class must equal 26, because $\left\lceil \frac{26}{5} \right\rceil = \lceil 5.2 \rceil = 6$

Show that if there are 30 students in a class, then there are at least two that have last names that begin with the same letter.

A bowl contains 10 red cubes and 10 blue cubes. A woman selects cubes at random without looking at them.

a. How many cubes must she select to be sure of having at least three cubes of the same color?

b. How many cubes must she select to be sure of having at least three blue cubes?

A bowl contains 10 red cubes and 10 blue cubes. A woman selects cubes at random without looking at them.

a. How many cubes must she select to be sure of having at least three cubes of the same color?

r b r b r b . . . 5th choice

⇒ 5 cubes

b. How many cubes must she select to be sure of having at least three blue cubes?

r r r r r r r r r r b b b

⇒ 13 cubes.

How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers adds up to 16?

$\{1, 15\}, \{3, 13\}, \{5, 11\}, \{7, 9\}$

15 13 11 9 ◯ $\Rightarrow$ 5

or

1 3 5 7 ◯

Suppose there are nine students in a discrete math class at a small college. Show that the class must have at least five male students or at least five female students.

Suppose to the contrary four or fewer males and four or fewer females. But then 4 males and 4 females would mean there are 8 people in the class. (There were 9 students.)