Dsip imp sums imp notes

Computer Engineering (University of Mumbai)

DSP NOTES

PLITU

Important Formulaes

- Energy

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\boxed{\begin{array}{l} E \text{ is finite} \\ P \text{ is } 0 \end{array}}$$

- Power

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$\boxed{\begin{array}{l} P \text{ is finite} \\ E \text{ is } \infty \end{array}}$$

- $$\frac{\omega}{2\pi} = \frac{K}{N}$$

- Linear Convolution

$$y(n) = \sum_{m=-\infty}^{\infty} x(m) h(n-m)$$

$m_L = x_L$ $\quad$ $n_L = x_L + h_L$

$m_H = x_H$ $\quad$ $n_H = x_H + h_H$

- Correlation

$$r_{xy}(n) = \sum_{m=-\infty}^{\infty} x(m) h(m-n)$$

$n_L = x_L - (y_L + N_2 - 1)$ $\quad$ $m_L = x_L$

$n_H = n_L + (N_1 + N_2 - 2)$ $\quad$ $m_H = x_H$

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- DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi nk}{N}} \quad \text{where } 0 \le k \le N-1$$

- IDFT

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi nk}{N}}$$

- Karl Pearson's Coefficient

$$\boxed{r = \frac{\sum XY}{\sqrt{\sum X^2}\sqrt{\sum Y^2}}}$$

Values to Remember.

- $e^0 = 1$
- $e^{-\frac{j\pi n}{2}} = (-j)^n$ • $e^{j\pi/2 k} = (j)^k$
- $e^{-j\pi n} = (-1)^n$ • $e^{j\pi k} = (-1)^k$
- $e^{-j\frac{3\pi n}{2}} = (j)^n$ • $e^{j\frac{3\pi}{2}k} = (-j)^k$

$W_2^0 = 1$ $\quad W_8^0 = 1$ $\quad W_8^3 = -\frac{(1+j)}{\sqrt{2}}$

$W_4^0 = 1$ $\quad W_8^1 = \frac{1-j}{\sqrt{2}}$

$W_4^1 = -j$ $\quad W_8^2 = (-j)$

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44	27	28	29	30	31	·	·

3 Wednesday
SEPTEMBER
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08 BIBO Stability Condition

09
$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

10

2014 Energ & Power wala.

Q: $x(n) = \cos(3\pi n/4)$ State whether Energy or Power signal?

Step I → Energy(E) $= \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} \left|\cos\left(\frac{3\pi n}{4}\right)\right|^2$$

$$= \sum_{n=-\infty}^{\infty} \cos^2\left(\frac{3\pi n}{4}\right)$$

$\cos 2\theta = 2\cos^2\theta - 1$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2}\left(\cos 2\left(\frac{3\pi n}{4}\right) + 1\right)$$ using

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(1 + \cos\frac{3\pi n}{2}\right)$$

$$= \frac{1}{2} * \sum_{n=-\infty}^{\infty} 1 + \frac{1}{2} * \sum_{n=-\infty}^{\infty} \cos\frac{3\pi n}{2}$$

using $\sum_{-\infty}^{\infty} \cos\frac{3\pi n}{2} = 0$

$$= \infty + 0$$

$E = \infty$

Step II → Power(P) $= \cancel{\sum_{N=-\infty}^{\infty}} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\cos\left(\frac{3\pi n}{4}\right)\right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{3\pi n}{4}\right)$$

$\cos 2\theta = 2\cos^2\theta - 1$

using

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$$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1}{2}\left(1+\cos\frac{3\pi n}{2}\right)$$

$$\cancel{\lim_{N\to\infty}}\ \frac{1}{2}\cdot\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(1+\cos\frac{3\pi n}{2}\right)$$

$$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{N} 1 + \sum_{n=-N}^{N} \cos\frac{3\pi n}{2}\right)$$

$$= \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \left(2N+1 + \sum_{n=-N}^{N} \cos\frac{3\pi n}{2}\right)$$

$$= \frac{1}{2} \lim_{N\to\infty} \left(1 + \frac{1}{2N+1} \sum_{n=-N}^{N} \cos\frac{3\pi n}{2}\right)$$

Putting $N=\infty$.

$$\sum_{n=-\infty}^{\infty} \cos\frac{3\pi n}{2} = 0$$

$$\frac{1}{2}\cancel{\lim_{N\to\infty}}\left(1 + \frac{1}{2\ast\infty+1} \cdot \sum_{n=-\infty}^{\infty} \cos\frac{3\pi n}{2}\right)$$

$$\frac{1}{2}\cancel{\lim_{N\to\infty}}\left(1 + 0\cdot 0\right)$$

$$\boxed{P = 1/2}$$

$\therefore$ Energy is infinite & Power is finite

The above signal is a Power signal.

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Golf is an easy game...It's just hard to ...

CROSS Correlation

Q. $x(n) = \{2, 3, 1, 4\}$

$y(n) = 3\delta(n-3) - 2\delta(n) + \delta(n-1) + 4\delta(n-2)$.

→ $y(n) =$

	0	0	0	3
−	2	0	0	0
+	0	1	0	0
+	0	0	4	0
	−2,	1,	4,	3

$\therefore y(n) = \{-2, 1, 4, 3\}$.

Correlation formula:

$$r_{xy}(n) = \sum_{m=-\infty}^{\infty} x(m)\, y(m-n) \qquad -\infty \le n \le \infty$$

$m_L = x_L = 0$

$m_H = x_H = 3$

$n_L = x_1 - (y_1 + N_2 - 1)$

$= 0 - (0 + 4 - 1) = -3$

$n_H = n_L + (N_1 + N_2 - 2)$

$= -3 + (4 + 4 - 2) = 3$

$\therefore r_{xy}(n)$ when $-3 \le n \le 3$

for $n = -3$

$$r_{xy}(-3) = \sum_{m=-\infty}^{\infty} x(m)\, y(m-n)$$

$$= \sum_{m=0}^{3} x(m)\, y(m-n),$$

$r_{xy}(-3) = x(0)\,y(0+3) + x(1)\,y(1+3) + x(2)\,y(2+3) + x(3)\,y(3+3)$

$r_{xy}(-3) = 2 * 3 + 3 * 3 + 1 \times 0 + 4 \times 0$

$\boxed{r_{xy}(-3) = 6}$

Similarly for

II $n=-2$

$$\gamma_{xy}(-2) = \sum_{m=0}^{3} x(m)\,y(m-n)$$

$$= \sum x(0)y(0-(-2)) + x(1)y(1-(-2)) + x(2)y(2-(-2)) + x(3)y(3-(-2))$$

$$= 2*4 + 3*3 + 1\times 0 + 4\times 0$$

$$\therefore \gamma_{xy}(-2) = 17$$

$\therefore$ So on for $\gamma_{xy}(-1, 0, 1, 2, 3)$

Shortcut for Cross correlation

→ $x(n) = \{\underset{\uparrow}{2}, 3, 1, 4\}$ (indices 0, 1, 2, 3)

$y(n) = \{\underset{\uparrow}{-2}, 1, 4, 3\}$

Take $y(-n) = \{3, 4, 1, \underset{\uparrow}{-2}\}$ (indices −3, −2, −1, 0)

$x(n)$ ↓ \ $y(-n)$ →	3	4	1	−2
2	6	8	2	−4
3	9	12	3	−6
1	3	4	1	−2
4	12	16	4	−8

min limit is −3

Max limit is +3 $\{6, 17, 17, \underset{\uparrow}{15}, 11, 2, -8\}$

BIBO Stability of A LTI System

Q. Test stability of LTI system whose impulse response is

$$h(n) = 0.2^n u(+n) + 3^n u(-n).$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad \text{(BIBO Stability condition)}.$$

$$\therefore \sum_{n=-\infty}^{\infty} |0.2^n u(+n) + 3^n u(-n)|$$

$$= \sum_{n=-\infty}^{\infty} |0.2^n u(n)| + \sum_{n=-\infty}^{\infty} |3^n u(-n)|$$

$$= \sum_{n=0}^{\infty} 0.2^n + \sum_{n=-\infty}^{0} 3^n \quad \text{Let } P = \frac{1}{n}. \qquad P = \frac{1}{n}$$

$$= \sum_{n=0}^{\infty} 0.2^n + \sum_{P=0}^{\infty} \left(\frac{1}{3}\right)^P$$

$$= \frac{1}{1-0.2} + \frac{1}{1-\frac{1}{3}} = \frac{1}{0.8} + \frac{3}{2} = 2.75$$

$$\left(\because \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}\right)$$

$$\therefore \sum_{n=-\infty}^{\infty} |h(n)| = 2.75 < \infty$$

$\therefore$ It is Stable

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Week 37

2014

Sampling of Discrete Time Signal

Q Consider Analog Signal

$x(t) = 2\sin(80\pi t)$

If Sampling Frequency $F_s = 60\,Hz$.

Find Sampled version of discrete time signal $x(n)$ & also find an alias frequency corresponding to $F_s = 60\,Hz$.

→ $x(t) = 2\sin(80\pi t)$

~~Replace t~~ =

Sampling Period $(T) = \frac{1}{F_s} = \frac{1}{60}$

$\therefore x(n) = x(t) \rightarrow 2\sin(80\pi t)$

replace t by nT

$x(n) = 2\sin(80\pi \times n.T)$

$= 2\sin\left(\cancel{80}^{4}\pi \times n \times \frac{1}{\cancel{60}_{3}}\right)$

$\therefore \boxed{x(n) = 2\sin\left(\frac{4\pi n}{3}\right)}$

→ Sampled Version.

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My goal this year is basically to find the fairways - Lauri Peterson

Now $x(t) = 2\sin 80\pi t$

$\frac{\omega}{2\pi} = F$

Comparing with standard eq

$x(t) = A\sin\omega t$

$\therefore$ $\omega = 80\pi$

$\frac{80\pi}{2\pi} = F$

$\boxed{F = 40Hz}$ ← actual signal frequency.

Alias frequency: (F_a)

Aliased freq. is the absolute difference betn the actual signal frequency & the nearest integer multiple of the sampling frequency.

ie $F_a = |F - N * F_s|$

$\therefore F_a = |40 - N * 60|$

for nearest integer multiple $N = 1$

$\therefore F_a = |40 - 1 * 60|$

$F_a = 20Hz$

$\therefore$ Aliasing frequency = $\underline{20Hz}$

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Compute DFT & Sketch Magnitude & Phase Spectrum

Q. Compute DFT of seq. $x(n) = \{0, 2, 3, -1\}$ Sketch the magnitud & phase spectrum.

→ $x(n) = \{0, 2, 3, -1\}$.

Using Matrix Multiplicatⁿ Method for Calculating DFT.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3-3j \\ 2 \\ -3+3j \end{bmatrix}$$

→ $X(k) = \{4, -3-3j, 2, -3+3j\}$.

DFT.

Now Magnitude Spectrum Formula.

$$|X(k)| = \sqrt{X_r^2(k) + X_i^2(k)}$$

X_r is real part & X_i is imaginary part.

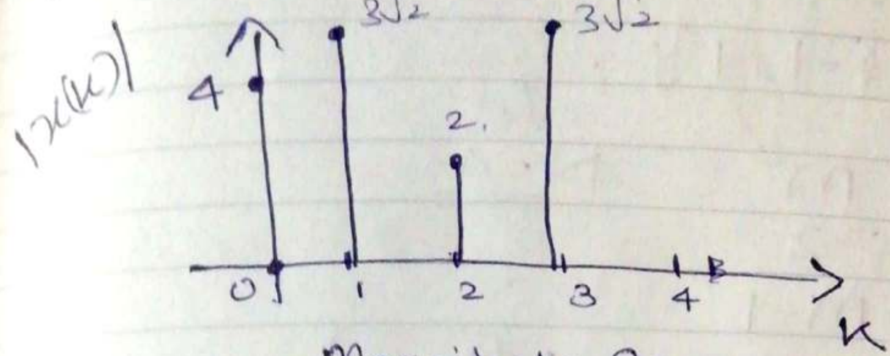
ie $X(0) = \sqrt{4^2 + 0^2} = 4$ //

$X(1) = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}$ //

$x(2) = \sqrt{2^2 + 0} = 2$ //

$x(3) = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$ //

$\therefore \; |X(k)| = \{4, 3\sqrt{2}, 2, 3\sqrt{2}\}$.



Magnitude Spectrum.

Now. Phase Spectrum formula.

$$\angle X(k) = \tan^{-1} \frac{X_i(k)}{X_r(k)}.$$

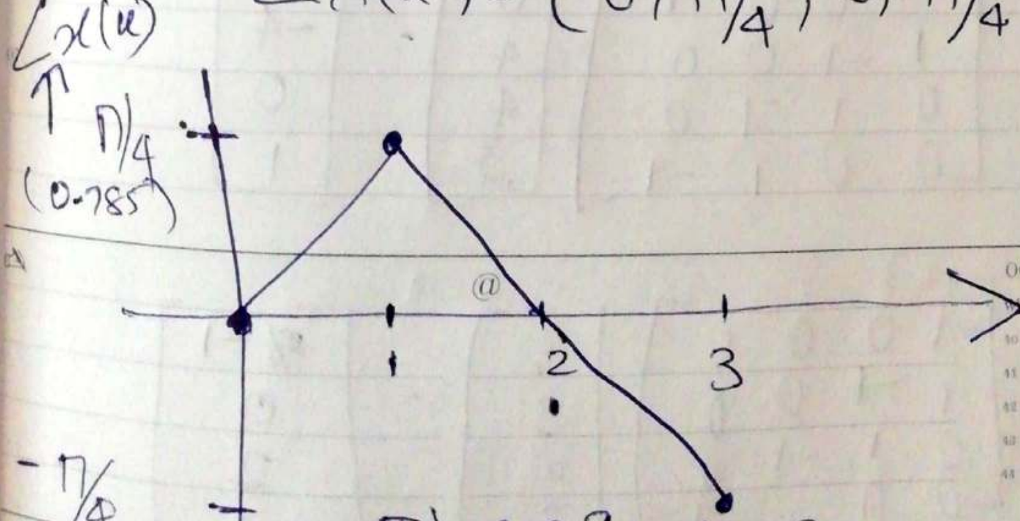
$$\angle X(0) = \tan^{-1}\left(\frac{0}{4}\right) = 0$$

$$\angle X(1) = \tan^{-1}\left(\frac{-3}{-3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\angle X(2) = \tan^{-1}\left(\frac{0}{2}\right) = 0$$

$$\angle X(3) = \tan^{-1}\left(\frac{-3}{3}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\angle X(k) = (0, \pi/4, 0, \pi/4).$$



Phase Spectrum.

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Overlap Save Method.

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$x(n) = \{4, 4, 3, 3, 2, 2, 1, 1\}$.

$h(n) = \{-1, 1\}$.

L $\quad$ $M = 2$ $\quad$ Let $\underline{N=4}$

$N = L + M - 1$

$L = N - M + 1$ $\quad \therefore \boxed{L = 3}$

$= 4 - 2 + 1 = 3$

$x_1(n) = \{0, 4, 4, 3\}$

$x_2(n) = \{3, 3, 2, 2\}$

$x_3(n) = \{2, 1, 1, 0\}$

$h(n) = \{-1, 1, 0, 0\}$.

Matrix Multiplicat^n Method to find Convolut^n.

$y_1(n) = x_1(n) \circledast h(n)$

$$y_1(n) = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 0 \\ 1 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

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M	T	W	T	F	S	S
1	2	3	4	5	6	7
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15	16	17	18	19	20	21
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29	30					

next shot is eighty percent of winning golf.. - Ben Hogan

$$y_3(n) = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$y_1(n) = 3 \; \underbrace{-4 \;\; 0 \;\; 1}$ ($\because L=3$).

$\uparrow$ discarded

$-1 \;\; 0 \;\; 1 \;\; 0$

$\uparrow$ discard

$-2 \;\; 1 \;\; 0 \;\; 1$

$\uparrow$ discard.

$$y(n) = \{-4, 0, 1, 0, 1, 0, 1, 0, 1\}$$

Determine Response of LTI sys using RADIX : 2 DITFFT

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Week 38
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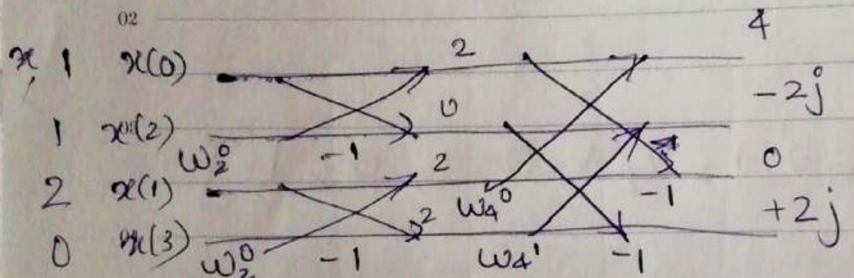
Q. $x(n) = \{1, 2, 1\}$
$h(n) = \{1, 3\}$

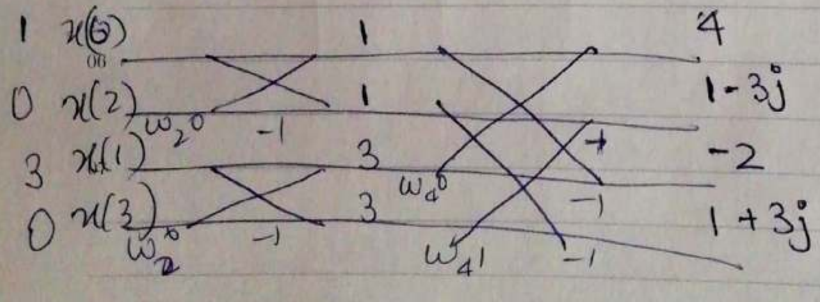
$\therefore N_1 = 3$
$N_2 = 2$
N = length Output Signal $= N_1 + N_2 - 1$
$= 3 + 2 - 1 = \underline{\underline{4}}$

$\therefore x(n) = \{1, 2, 1, 0\}$
$h(n) = \{1, 3, 0, 0\}$

$\therefore$ Using 4 Point DITFFT



$X(k) = \{4, -2j, 0, 2j\}$



$H(k) = \{4, 1-3j, -2, 1+3j\}$

Using Circular Convol^{n} ppty of DFT

$y(n) = x(n) \circledast h(n) \xrightarrow{DFT} X(k) \cdot H(k) = Y(k)$

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37	8	9	10	[illegible]	[illegible]	[illegible]	[illegible]
38	15	16	17	18	19	20	21
39	22	23	24	25	26	27	28
40	29	30					

Revolution is the festival of the oppressed - Germaine Greer

$16+4$ $\quad -6-2j-6+2j$ $\quad -6-2j$

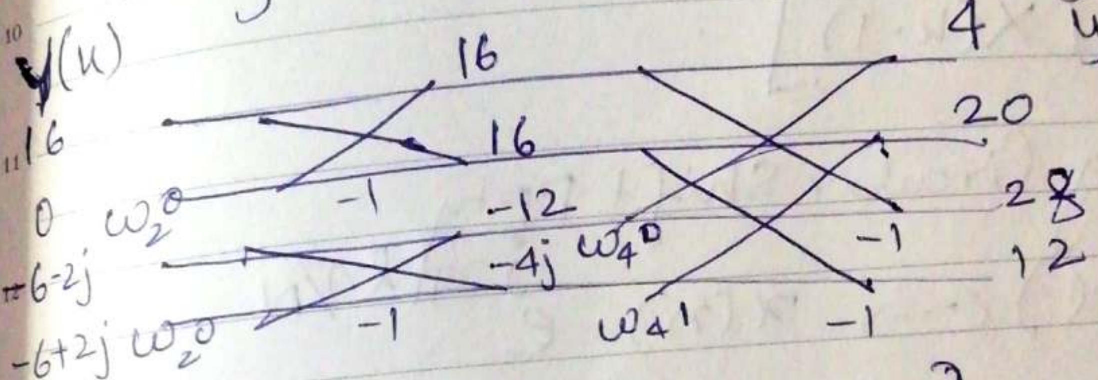
$16-4=12$

$Y(k) = X(k)\cdot H(k)$

$= \{4, -2j, 0, 2j\} \cdot \{4, 1-3j, -2, 1+3j\}$

$Y(k) = \{16, -6-2j, 0, -6+2j\}$

Taking Inverse DITFFT Bcoz we want y(n)



$\frac{1}{4}(4, 20, 28, 12)$

$y(n) = (1, 5, 7, 3)$

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41	6	7	8	9	10	11	12
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@

DFT Ppty SUMS

Q. If IDFT $[X(k)] = x(n) = \{2, 1, 2, 0\}$

$\rightarrow$ $x(n) = \{\underset{\uparrow}{2}, 1, 2, 0\}$

$\therefore N = 4$

(i) IDFT of $[X(k-1)]$

$\hookrightarrow$ Using Circular shift ppty

$$X(k-l) \Longleftrightarrow x(n) \cdot e^{j2\pi ln/N}$$

$$\Longleftrightarrow x(n) e^{j2\pi n/\cancel{4}}$$

$$\Longleftrightarrow x(n) e^{jn\cancel{\pi/2}}$$

$$\Longleftrightarrow x(n)\, j^n$$

$\therefore$ IDFT $[X(k-1)] = \{2, j, -2, 0\}$.

(ii) IDFT of [X(k) circularly convolved with X(k)]

$$DFT[x_1(n) \cdot x_2(n)] = \frac{1}{N}[X_1[k] \circledast X_2[k]]$$

$$IDFT[X_1[k] * X_2[k]] = N \cdot [x_1(n) \cdot x_2(n)]$$

$$\therefore \quad = 4 \times [x(n) \cdot x(n)]$$

$$= 4\,[4, 1, 4, 0]$$

$$= \underline{16, 4, 16, 0}$$

(iii) IDFT $[X(k) \cdot X(k)]$

$$DFT\,[x_1(n) \circledast x_2(n)] \Leftrightarrow X_1(k) \cdot X_2(k).$$

$$IDFT\ X_1(k) \cdot X_2(k) = [x_1(n) \circledast x_2(n)]$$

$$= \begin{bmatrix} 2 & 0 & 2 & 1 \\ 1 & 2 & 0 & 2 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 9 \\ 4 \end{bmatrix}$$

$$\therefore y(n) = x(n) \circledast x(n) = \{8, 4, 9, 4\}.$$

(iv) Signal Energy.

using Parsevals Energy Theorem.

$$E = \sum_{n=0}^{N-1} |x(n)|^2$$

$$= \sum_{n=0}^{3} |x(n)|^2 =$$

$$E = 2^2 + 1^2 + 2^2 + 0^2$$

$$\boxed{E = 9}$$

Karl Pearson's Coefficient

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Day 262 - Left 103

2014

Q. Evaluate Karl's Coefficient for two Causal Sequences.

$x(n) = \{1, 3, 4, 2\}$

$y(n) = \{1, 2, 2, 1\}$

→ Formula to be used:

$$\cancel{\frac{\sum xy}{\sqrt{}}} \quad r = \frac{\sum XY}{\sqrt{\sum X^2}\sqrt{\sum Y^2}}$$

where $X = (x(n) - \bar{x})$ & $Y = (y(n) - \bar{y})$

	x(n)	y(n)	X	Y	XY	X^2	Y^2
	1	1	-1.5	-0.5	0.75	2.25	0.25
	3	2	0.5	0.5	0.25	0.25	0.25
	4	2	1.5	0.5	0.75	2.25	0.25
	2	1	-0.5	-0.5	0.25	0.25	0.25
Σ	10	6			2	5	1
Avg	2.5	1.5					

Putting values in formula $r = \dfrac{\sum XY}{\sqrt{\sum x^2}\sqrt{\sum y^2}}$

$$r = \frac{2}{\sqrt{5}\sqrt{1}} = \underline{0.894}$$

$\therefore \underline{r = 0.894}$

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40	29	30					

Overlap Add Method

Q. $x(n) = \{3, 4, 2, 1, 2, 2, 1, 1\}$

$h(n) = \{1, -1\}$

L $\quad M = 2 \quad N = 4$

$L = N - M + 1 = 4 - 2 + 1$

$\underline{\underline{L = 3}}$.

$x_1(n) = \{3, 4, 2, 0\}$

$x_2(n) = \{1, 2, 2, 0\}$

$x_3(n) = \{1, 1, 0, 0\}$

$h(n) = \{1, -1, 0, 0\}$.

Using matrix multiplicat^n to find Circular Convol^n

$y_1(n) = x_1(n) \circledast h(n)$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \\ -2 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

$$y_3(n) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

21 Sunday SEPTEMBER

Day 264 - Left 101

$y_1(n)$ 3 1 -2 -2 $y_2(n)$=1 1 0 -2 $y_3(n)$=1, 0, -1, 0

	3	1	-2	-2						
+				1	1	0	-2			
+							1	0	-1	0
	3	1	-2	-1	1	0	-1	0	-1	0

$\therefore$ $y(n) = \{3, 1, -2, -1, 1, 0, -1, 0, -1, 0\}$.

Evaluating Nyquist Rate:

Consider Analog Signal

Q: $x(t) = 5\cos 2\pi(1000t) + 10\cos 2\pi(5000t)$ to be sampled.

(i) Evaluate Nyquist Rate.

(ii) If the signal is sampled at 4kHz, will the signal be recovered from its samples?

(i) $x(t) = 5\cos 2\pi(\underset{F_1}{1000}t) + 10\cos 2\pi(\underset{F_2}{5000}t)$.

Cmpr eq^n with stndrd eq^n $x(t) = A\cos\omega t$.

$\omega = 2\pi f$

ie $A\cos 2\pi ft$.

$\therefore F_1 = 1000\text{ Hz}$

& $F_2 = 5000\text{ Hz}$

$\therefore f_{max} = max(F_1, F_2) = 5000\text{ Hz}$

For Proper reconstruction of signal,

$F_s \geq 2 * F_{max}$

$F_s \geq 2 * 5000\text{ Hz}$

$F_s \geq 10{,}000\text{ Hz}$.

The minimum value of Sampling frequency is called as Nyquist Rate.

$\therefore$ Nyquist Rate = ~~10kHz~~ 10,000Hz = 10kHZ.

(ii) Since, 4000Hz < Nyquist Rate

$\therefore$ Reconstruction of signal NOT Possible.

OCTOBER							2014
wk	M	T	W	T	F	S	S
40			1	2	3	4	5
41	6	7	8	9	10	11	12
42	13	14	15	16	17	18	19
43	20	21	22	23	24	25	26
44	27	28	29	30	31		

Albert Einstein

23 Tuesday SEPTEMBER
Day 266 - Left 99
2014

Evaluate DFT (5 marks wala.)

Q. Evaluate DFT of $x(n) = \cos(0.25\pi n)$

Taking $N = 4$.

$\therefore x(0) = \cos(0.25\pi \times 0) = 1$

$x(1) = \cos(0.25\pi \times 1) = 0.707$

$x(2) = \cos(0.25\pi \times 2) = 0$

$x(3) = \cos(0.25\pi \times 3) = -0.707$

$\therefore x(n) = \{1, 0.707, 0, -0.707\}$

$$x(k) = W_0 \times x(n)$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0.707 \\ 0 \\ -0.707 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - 1.414j \\ 1 \\ 1 + 1.414j \end{bmatrix}$$

$X(k) = \{1, 1 - j1.414, 1, 1 + 1.414j\}$

$-0.707j$
$-0.707j$

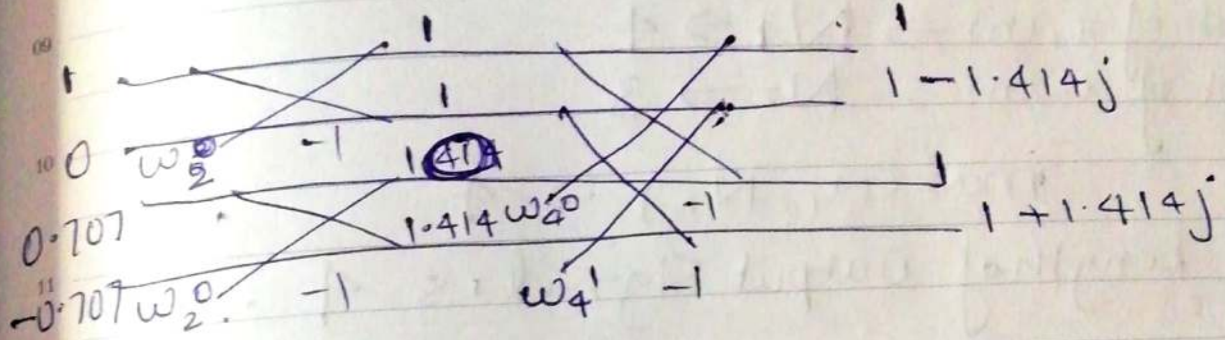
Alternate Method for Finding DFT is using DIT-FFT (Butterfly Diagram)

SEPTEMBER 2014

wk	M	T	W	T	F	S	S
36	1	2	3	4	5	6	7
37	8	9	10	11	12	13	14
38	15	16	17	18	19	20	21
39	22	23	24	25	26	27	28
40	29	30					

-0.707
$+0$
$0\,j + 0.707$
0

$x(n) = \{1, 0.707, 0, -0.707\}$.



$\therefore X(k) = \{1, 1-1.414j, 1, 1+1.414j\}$.

Same Ans ↑

25 Thursday SEPTEMBER

Day 268 - Left 97

2014

Circular Convolution. (5 marks).

08 $x_1(n) = \{3, 2, 4, 1\}$.

$x_2(n) = \{2, 1, 3\}$.

09 Length of $x_1(n) \rightarrow N_1 \rightarrow 4$

10 length of $x_2(n) \rightarrow N_2 \rightarrow 3$

$\max(N_1, N_2) \rightarrow 4$

11 $\therefore$ Length of Output Signal is 4.

$6+12+1$

$3+4+3$

12 $\therefore x_1(n) = \{3, 2, 4, 1\}$.

$9+2+8$

01 $x_2(n) = \{2, 1, 3, 0\}$

$6+4+2$

'0' Padded

02 03 04

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 2 & 0 & 3 & 1 \\ 1 & 2 & 0 & 3 \\ 3 & 1 & 2 & 0 \\ 0 & 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 10 \\ 19 \\ 12 \end{bmatrix}$$

05 $\therefore y(n) = x_1(n) \circledast x_2(n) = \{19, 10, 19, 12\}$.

06

07

SEPTEMBER 2014

wk M T W T F S S

Check whether the system is
Wale sums

$y(n) = x(n) + 2x(n-2)$

I) Static or Dynamic.

$y(n) = x(n) + 2x(n-2)$.

Let's take $n = 1$

$= x(1) + 2x(1-2) = x(1) + 2x(-1)$

↑ Past input value.

∴ Since calculation of output signal requires past input values, the signal is **Dynamic**.

II) Linear or Nonlinear

$y(n) = x(n) + 2x(n-2) \rightarrow (1)$

step 1)

$y_1(n) = x_1(n) + 2x_1(n-2)$

$y_2(n) = x_2(n) + 2x_2(n-2)$

$y'(n) = y_1(n) + y_2(n)$

$= x_1(n) + x_2(n) + 2x_1(n-2) + 2x_2(n-2)$

Step II) Replace $x(n) = x_1(n) + x_2(n)$ in eqⁿ (1).

$\therefore y''(n) = x_1(n) + x_2(n) + 2x_1(n-2) + 2x_2(n-2)$

$\therefore y'(n) = y''(n)$

System is linear

OCTOBER							2014
wk	M	T	W	T	F	S	S
40			1	2	3	4	5
41	6	7	8	9	10	11	12
42	13	14	15	16	17	18	19
43	20	21	22	23	24	25	26
44	27	28	29	30	31		

Intolerance betrays want of faith in one's cause. - Mahatma Gandhi

III Causal or Non Causal

$$y(n) = x(n) + 2x(n-2)$$

Lets Putting n=1

$$y(1) = x(1) + 2x(1-2)$$
$$= \underbrace{x(1)}_{\text{current input}} + 2\underbrace{x(-1)}_{\text{Past input}}$$

∴ No future input value is involved
System is Causal.

IV) Shift Variant or Shift Invariant.

$$y(n) = x(n) + 2x(n-2)$$

Step I) Delaying Output Signal by k units
ie n = n-k (Replace n = n-k)

$$y(n-k) = x(n-k) + 2x(n-k-2)$$

Step II) Delaying Input Signal ~~(Replace~~

$$y'(n) = x(n-k) + 2x(n-2-k)$$

$$y(n-k) = y'(n)$$ ∴ the System is Shift Invariant

SEPTEMBER 2014

wk	M	T	W	T	F	S	S
36	1	2	3	4	5	6	7
37	8	9	10	11	12	13	14
38	15	16	17	18	19	20	21
39	22	23	24	25	26	27	28
40	29	30					

Illusion and wisdom combined are the charm of life and art. - Joseph Joubert