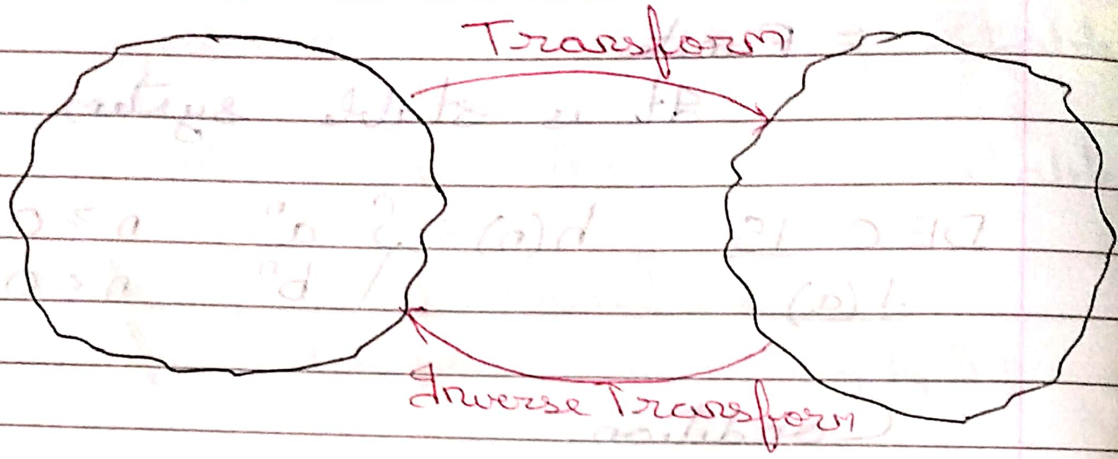
3. Discrete Fourier Transform [DFT]

→ What is Transform



→ Why Transform? :

- It is difficult to perform certain operations in its original domain.
- Here we transform it into Convenient & Domain
- Perform Operations in Transformed Domain
- Fetch Result back by Inverse Transform.

Time Domain

$y(n) = x(n) * h(n)$

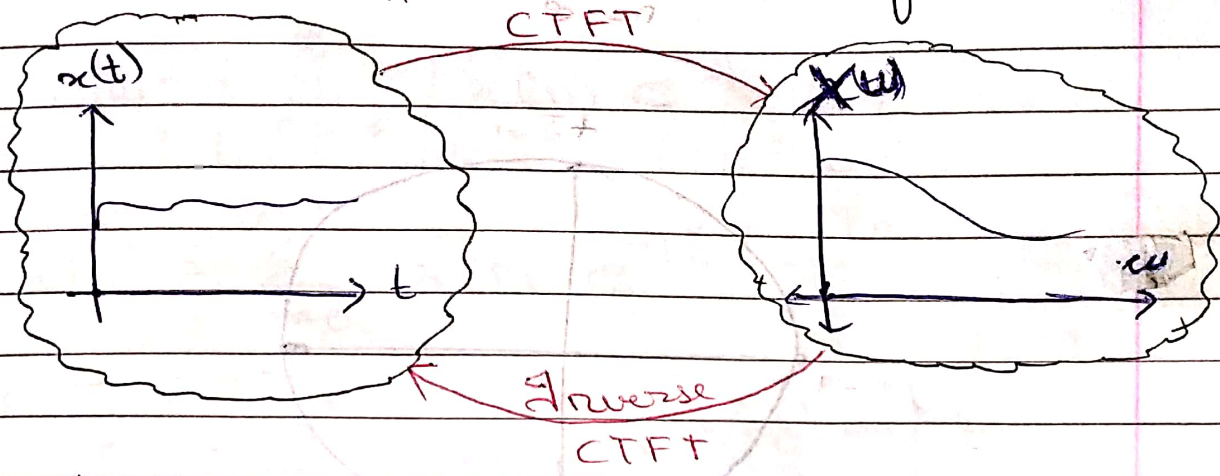
Shift, Multiply, Add

Frequency Domain

$Y[k] = X[k] \cdot H[k]$

Multiply

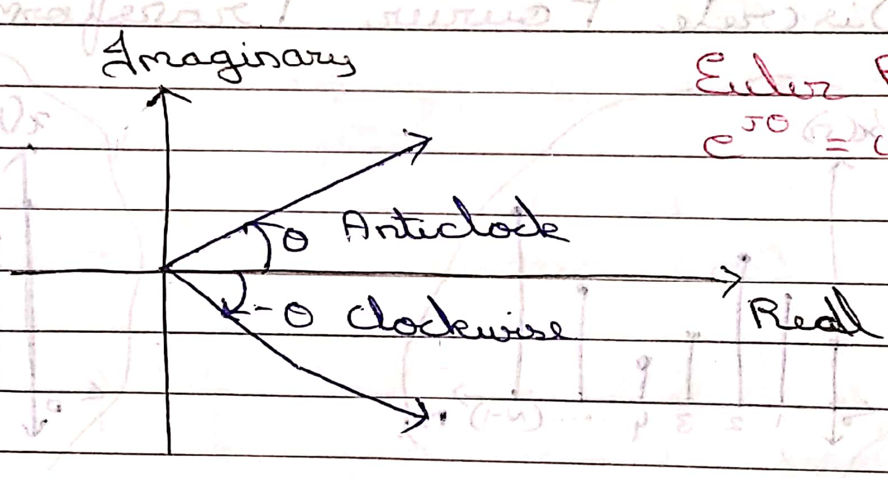
→ Continuous Time Fourier Transform [CTFT]



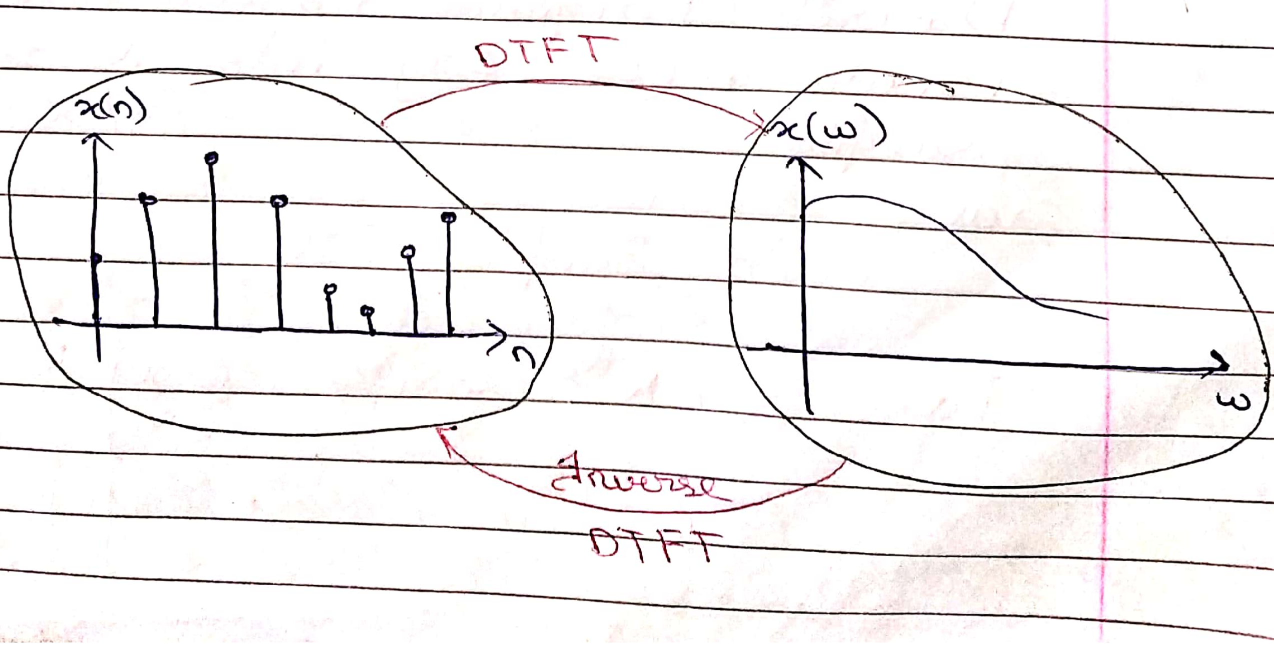
$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

Euler Rule

$e^{j\theta} = \cos\theta + j\sin\theta$

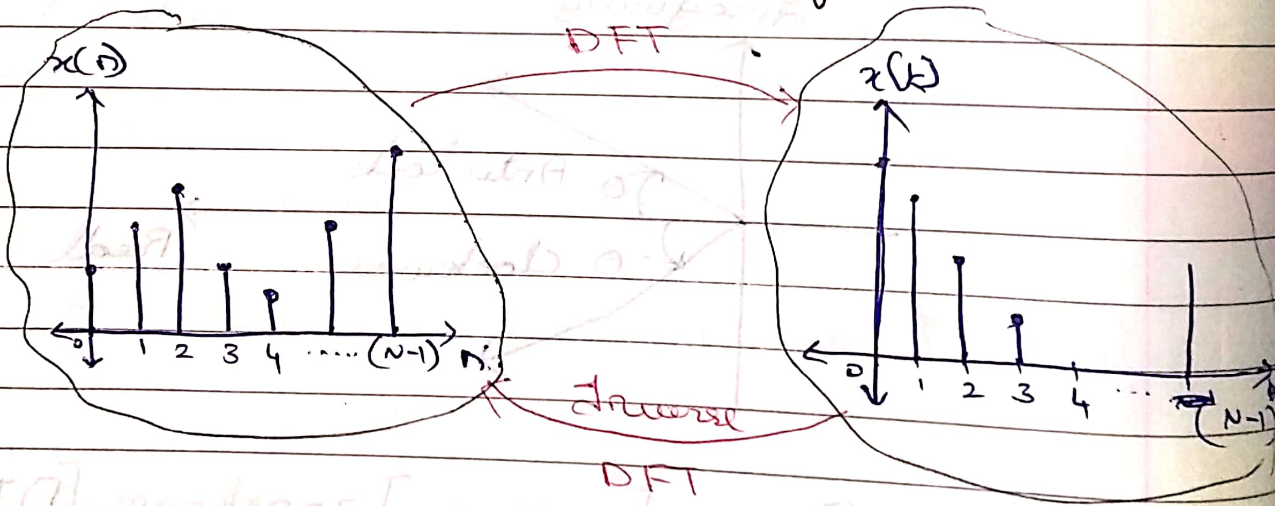


→ Discrete Time Fourier Transform [DTFT]



$$x(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

→ Discrete Fourier Transform



Discrete Frequency Domain Also Divide ω (i.e 2π) into N- equal samples

∴ Each Sample = $\frac{2\pi}{N}$

Take its k^{th} sample $= \left(\frac{2\pi}{N}\right) k$

Relationship between DTFT & DFT

$$x(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$x[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j\left[\left(\frac{2\pi}{N}\right)k\right]n}$$

$$\boxed{x[k] = \sum_{n=0}^{N-1} x(n)\, e^{-j\left(\frac{2\pi}{N}\right)kn}}$$

$$\boxed{X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}}$$

Here $W_N^{k \cdot n} = \left[e^{-j\frac{2\pi}{N}}\right]^{kn}$

$W = e^{-j\left(\frac{2\pi}{N}\right)}$ = Twiddle Factor

If $N=2$ Then 2-point DFT
If $N=\cancel{3}\,4$ Then 4-point DFT
If $N=8$ Then 8-point DFT

Consider $N=2$ Point DFT

$$X[k] = \sum_{n=0}^{1} x(n)\, W_2^{kn}$$

$$X[k=0] = X(0)\,W^0 + X(1)\,W^0$$
$$X[k=1] = X(0)\,W^0 + X(1)\,W^1$$

$$X[k] = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} W^0 & W^0 \\ W^0 & W^1 \end{bmatrix} \end{array} \begin{bmatrix} x(0) \\ x(1) \end{bmatrix}$$

Example :- Find DFT of $x(n) = \{3, 4\}$

$$X[k] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$N = 4$ **4-Point DFT**

$$X[k] = \sum_{n=0}^{3} x(n)\, W_4^{kn}$$

$$\therefore X[k=0] = x(0)W^0 + x(1)W^0 + x(2)W^0 + x(3)W^0$$

$$X[k=1] = x(0)W^0 + x(1)W^1 + x(2)W^2 + x(3)W^3$$

$$X[k=2] = x(0)W^0 + x(1)W^2 + x(2)W^4 + x(3)W^6$$

$$X[k=3] = x(0)W^0 + x(1)W^3 + x(2)W^6 + x(3)W^9$$

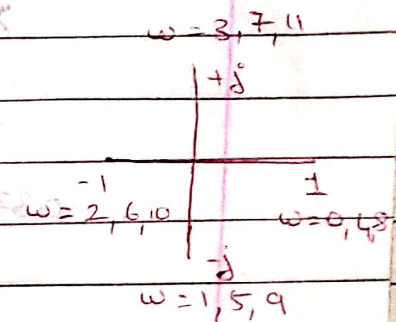
$$X[k] = \begin{array}{c|c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 \\ W^0 & W^2 & W^4 & W^6 \\ W^0 & W^3 & W^6 & W^9 \end{bmatrix} \end{array} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & +1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

Example :- Find DFT of $x(n) = \{1, 2, 3, 4\}$

Solution :- We know for 4-point DFT

$$X[k] = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 \\ W^0 & W^2 & W^4 & W^6 \\ W^0 & W^3 & W^6 & W^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$



$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$\therefore x[k] = \{10, -2+2j, -2, -2-2j\}$$

DEC 15
4(a) 10M

$h(n) = 0.3\,\delta(n) - \delta(n-1) + 0.38\,\delta(n-3)$

Sketch the magnitude spectrum of the filter using DFT.

Solution :- Given $h(n) = \{0.3, -1, 0, 0.38\}$

We first ~~of~~ obtain $H[k]$ using DFT

$$\cancel{X}[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0.3 \\ -1 \\ 0 \\ 0.38 \end{bmatrix} = \begin{bmatrix} -0.32 \\ \cancel{0.68 - j}\; 0.3 + 1.38j \\ \cancel{1.08}\; 0.92 \\ 0.3 - 1.38j \end{bmatrix}$$

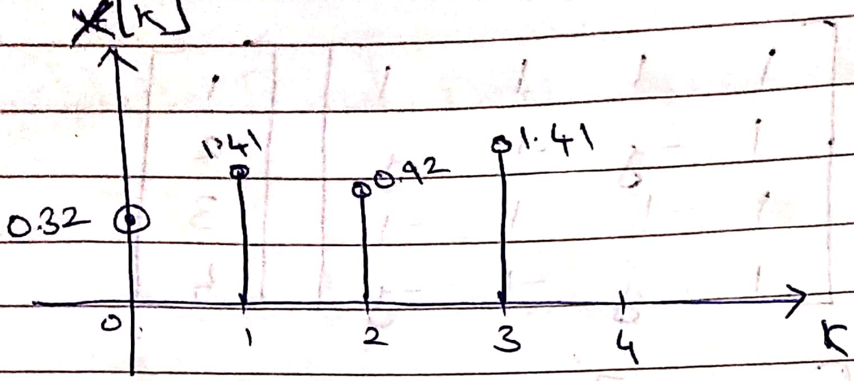
$$\therefore x[k] = \{-0.32, 0.3+1.38j, 0.92, 0.3-1.38j\}$$

For $k=0$ Magnitude $= 0.32$

$k=1$ Magnitude $= \sqrt{0.3^2 + 1.38^2} = 1.41$

$k=2$ Magnitude $= 0.92$

$k=3$ Magnitude $= \sqrt{0.3^2 + 1.38^2} = 1.41$



May 16 1(D) 5M

For causal signal $x(n) = \{2, 2, 4, 4\}$ compute four point DFT using DIT-FFT ← Simplicity

Solution :- For a 4-point DFT, we know

$$X[k] = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 \\ W^0 & W^2 & W^4 & W^6 \\ W^0 & W^3 & W^6 & W^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 12 \\ -2+2j \\ 0 \\ -2-2j \end{bmatrix}$$

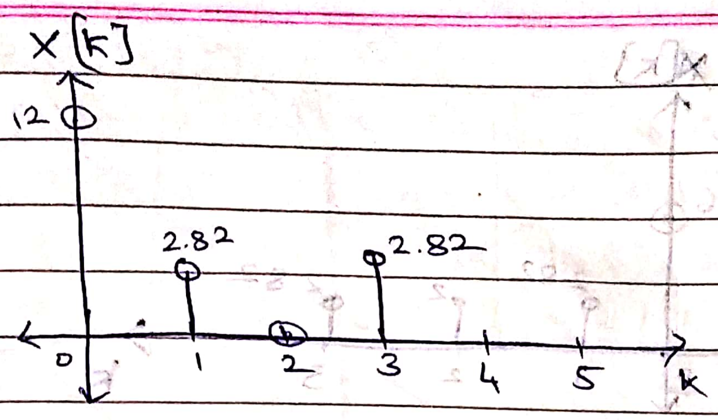
$\therefore X[k] = \{12, -2+2j, 0, -2-2j\}$

$\therefore$ For $k=0$ Magnitude $= 12$

$k=1$ Magnitude $= \sqrt{(-2)^2 + 2^2} = 2.82$

$k=2$ Magnitude $= 0$

$k=3$ Magnitude $= \sqrt{(-2)^2 + (-2)^2} = 2.82$



Example :- Find DFT of $x(n) = \{0, 1, 2, 3\}$
Also find Magnitude & ~~Space~~ Phase Spectra

<u>Solution</u> :- We first find DFT $X[k]$

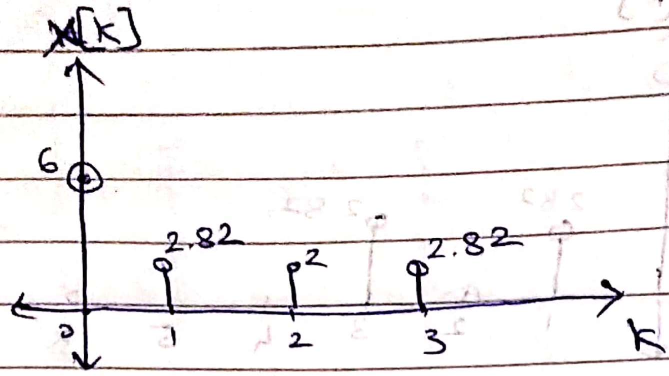
For 4-point DFT we know

$$X[k] = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 \\ W^0 & W^2 & W^4 & W^6 \\ W^0 & W^3 & W^6 & W^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2i \\ -2 \\ -2-2i \end{bmatrix}$$

$\therefore X[k] = \{6, -2+2i, -2, -2-2i\}$

For $k=0$ Magnitude $= 6$
$k=1$ Magnitude $= \sqrt{(-2)^2 + 2^2} = 2.82$
$k=2$ Magnitude $= 2$
$k=3$ Magnitude $= \sqrt{(-2)^2 + (-2)^2} = 2.82$



Phase $k=0$ $\tan^{-1}\left(\frac{0}{6}\right)$ = Mag 6 $\angle 0$

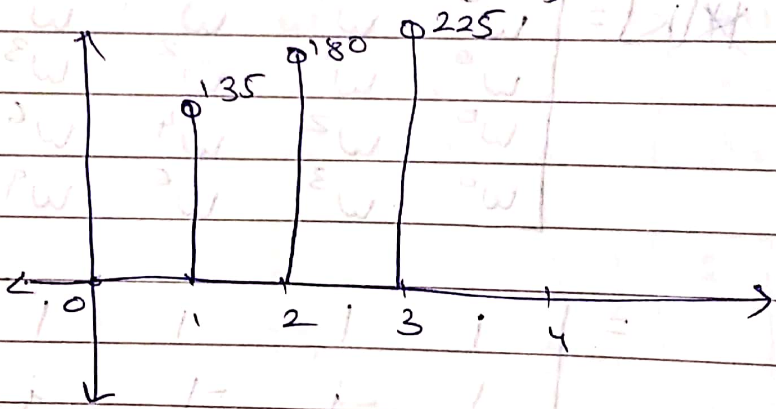
~~k = 1~~

$k=1$ $\tan^{-1}\left(\frac{2}{-2}\right)$ = Mag $2\sqrt{2}$ $\angle 135$

$k=2$ $\tan^{-1}\left(\frac{0}{-2}\right)$ = Mag 2 $\angle 180$

$k=3$ $\tan^{-1}\left(\frac{-2}{-2}\right)$ = Mag $2\sqrt{2}$ $\angle 225$

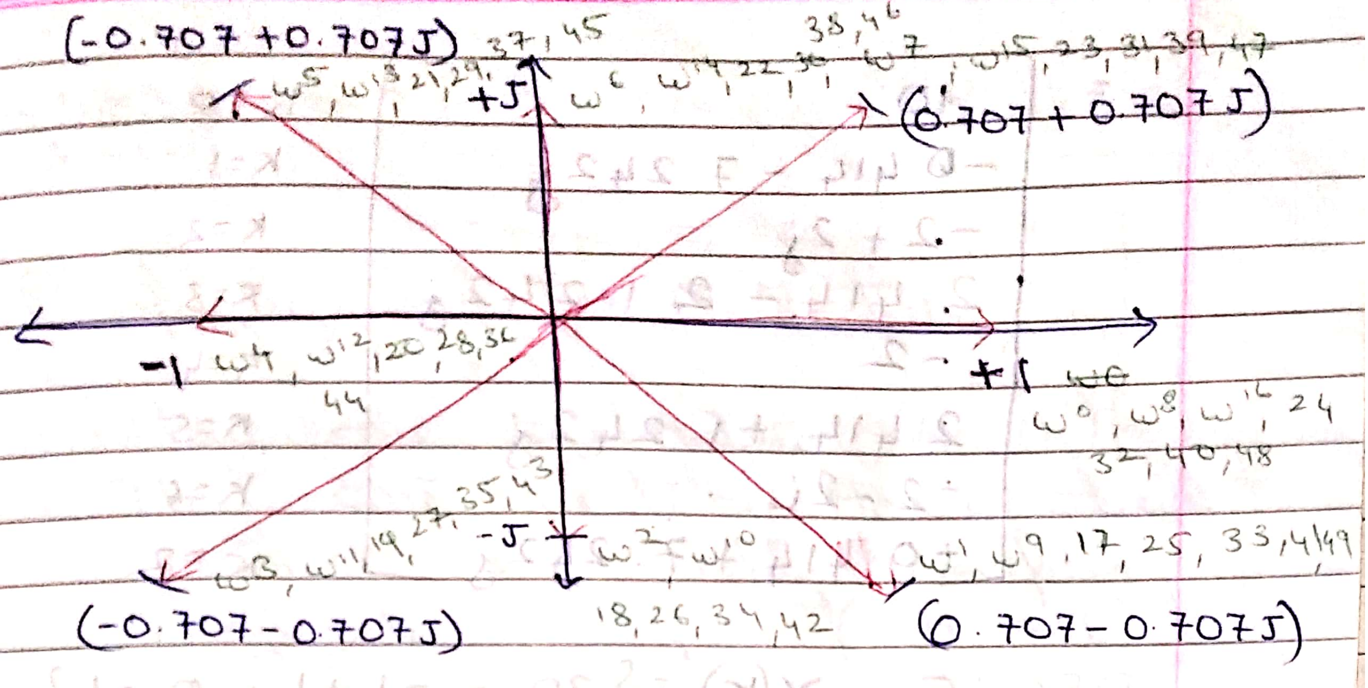
or $\angle -135$



→ N = 8 8 - Point DFT

Eg - Mostly Property Based (DIT - FFT)

8 - point DFT ⇒ Divide ($\omega = 2\pi$) into P equal samples



Example :- Find DFT of

$x(n) = \{1, 2, 3, 4, 0, 0, 0, 0\}$

Solution

$$
X[k] = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 \\
W^0 & W^1 & W^2 & W^3 & W^4 & W^5 & W^6 & W^7 \\
W^0 & W^2 & W^4 & W^6 & W^8 & W^{10} & W^{12} & W^{14} \\
W^0 & W^3 & W^6 & W^9 & W^{12} & W^{15} & W^{18} & W^{21} \\
W^0 & W^4 & W^8 & W^{12} & W^{16} & W^{20} & W^{24} & W^{28} \\
W^0 & W^5 & W^{10} & W^{15} & W^{20} & W^{25} & W^{30} & W^{35} \\
W^0 & W^6 & W^{12} & W^{18} & W^{24} & W^{30} & W^{36} & W^{42} \\
W^0 & W^7 & W^{14} & W^{21} & W^{28} & W^{35} & W^{42} & W^{49}
\end{bmatrix}
\end{array}
\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x7 \end{bmatrix}
$$

$$
= \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & (0.707-0.707j) & -J & (-0.707-0.707J) & -1 & (-0.707+0.707J) & J & (+0.707+0.707J) \\
1 & -J & -1 & +J & +1 & -J & -1 & J \\
1 & (-0.707-0.707J) & J & (+0.707-0.707J) & -1 & (0.707+0.707J) & -J & (-0.707+0.707J) \\
1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\
1 & (-0.707+0.707J) & -J & (0.707+0.707J) & -1 & (0.707-0.707J) & J & (-0.707-0.707J) \\
1 & J & -1 & -J & +1 & +J & -1 & -J \\
1 & (0.707+0.707J) & +J & (-0.707+0.707J) & -1 & (-0.707-0.707J) & -J & (0.707-0.707J)
\end{bmatrix}
\end{array}
\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$= \begin{bmatrix} 10 \\ -0.414 - 7.242j \\ -2 + 2j \\ 2.414 - 1.242j \\ -2 \\ 2.414 + 1.242j \\ -2 - 2i \\ -0.414 + 7.242j \end{bmatrix} \begin{matrix} k=0 \\ k=1 \\ k=2 \\ k=3 \\ k=4 \\ k=5 \\ k=6 \\ k=7 \end{matrix}$$

DEC 15
4(b) → (i)

$X(K) = \{20, 0, -4+4j, 0, -4\}$

Find $x[k]$ for $k = 5, 6, 7$

Using Complex Conjugate Symmetry Property of DFT, we get

$x[k]$ for $k = 5, 6, 7$ as below

$$x[k] = \begin{bmatrix} 20 & k=0 \\ 0 & k=1 \\ -4+4j & k=2 \\ 0 & k=3 \\ -4 & k=4 \\ 0 & k=5 \\ -4-4j & k=6 \\ 0 & k=7 \end{bmatrix}$$

$\therefore x[k=5] = 0$

$\therefore x[k=6] = -4-4j$

$\therefore x[k=7] = 0$

MAY '18
5 (a)

Properties of DFT Matrix

1. They are symmetric $(A = A^T)$
2. They are unitary Hence their inverse is simply complex conjugate Transpose. $(AA^\circ = I)$
3. $\frac{N}{2}$ (Row & Column) is alternate
   $+1, -1$
4. There is complex conjugate symmetry about $\frac{N}{2}$ (Row & Column)

→ Inverse DFT

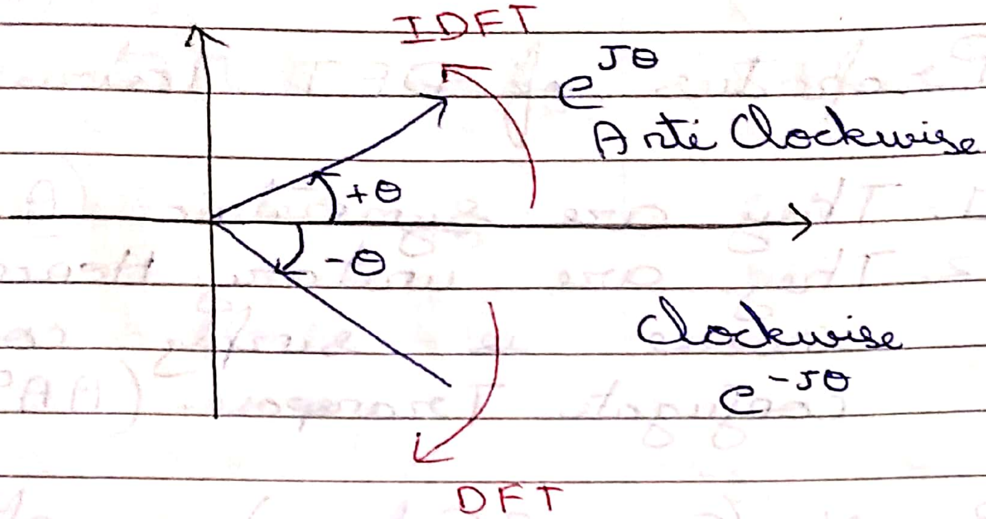
$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x[k]\, e^{j\left(\frac{2\pi}{N}\right)kn}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} x[k]\, W^{*kn}$$

Observations

1. Polarity Change
   ⇒ Complex Conjugate

2. Additional Factor $\left(\frac{1}{N}\right)$ in Inverse DFT



Example :- Find IDFT $x(n)$ of
$x[k] = \{10, -2+2j, -2, -2-2j\}$

Solution

$$x(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & +j & -1 & -j \\ 1 & -1 & +1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix}\begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ 12 \\ 16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$\therefore x(n) = \{1, 2, 3, 4\}$

Example - Obtain Circular Convolution of
$x_1(n) = \{1, 2, 3, 4\}$ and
$x_2(n) = \{1, 2\}$ using DFT
$y(n) = \{-7, 1, 0, 0\}$

Solution :-

Circular Convolution $L = L_1 = L_2$

$x_2(n) = \{1, 2, 0, 0\}$

Using DFT

$$\begin{array}{ccccc} x_1(n) & \circledast & x_2(n) & = & y(n) \\ \big\downarrow \text{DFT} & & \big\downarrow \text{DFT} & & \big\uparrow \text{IDFT} \\ X_1[k] & \cdot & X_2[k] & = & Y[k] \end{array}$$

Step I - Find $X_1[k]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & +1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$\therefore X_1[k] = \{10, -2+2j, -2, -2-2j\}$

Step II - Find $X_2[k]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1-2j \\ -1 \\ 1+2j \end{bmatrix}$$

$X_2[k] = \{3, 1-2j, -1, 1+2j\}$

Step 3 :- Find $Y[k]$ / Perform Convolution

But $Y[k] = X_1[k] \cdot X_2[k]$

$\therefore x_1[k] = \{10, -2+2j, -2, -2-2j\}$
$x_2[k] = \{3, 1-2j, -1, 1+2j\}$

$\therefore y[k] = \{30, 2+6j, 2, 2-6j\}$

Step IV:- Inverse DFT of $y[k]$

$$\therefore y(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 30 \\ 2+6j \\ 2 \\ 2-6j \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 40-4i \\ 20-4i \\ 24+4i \\ 36+4i \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 36 \\ 16 \\ 28 \\ 40 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 7 \\ 10 \end{bmatrix}$$

$\therefore y(n) = \{9, 4, 7, 10\}$

May 16
5 (A)

$x_1(n) = \{5, 6, 2, 1\}$
$x_2(n) = \{3, 2, 1, 4\}$

Solution

Circular Convolution $L = L_1 = L_2 = 4$ point

Using DFT

~~$Y[k]$~~ $x_1(n) \circledast x_2(n) = y(n)$

$X_1[k] \cdot X_2[k] = Y[k]$

Step 1 :- Find $x_1[k]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ \cancel{4-6i}\ 3-5j \\ 0 \\ 3+5j \end{bmatrix}$$

$$\therefore x_1[k] = \{14, 3-5j, 0, 3+5j\}$$

Step 2 :- Find $x_2[k]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 2+2j \\ -2 \\ 2-2j \end{bmatrix}$$

Step 3 :- Perform Convolution

$$\therefore x_1[k] \cdot x_2[k] = y[k]$$

$$\therefore y[k] = \begin{bmatrix} 14 \\ 3-5j \\ 0 \\ 3+5j \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 2+2j \\ -2 \\ 2-2j \end{bmatrix} = \begin{bmatrix} 140 \\ 16-4i \\ 0 \\ 16+4i \end{bmatrix}$$

$$\therefore y[k] = \{140, 16-4i, 0, 16+4i\}$$

Step 4 :- Find Inverse DFT

$$\therefore y(n) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 140 \\ 16-4i \\ 0 \\ 16+4i \end{bmatrix} \leftarrow y(k)$$

$$= \frac{1}{4}\begin{bmatrix} 172 \\ 148 \\ 108 \\ 132 \end{bmatrix} = \begin{bmatrix} 43 \\ 37 \\ 27 \\ 33 \end{bmatrix}$$

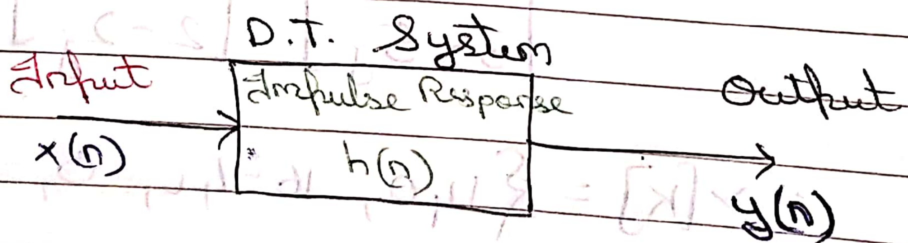
$$\therefore y(n) = \{43, 37, 27, 33$$

Cross Check Using Circular Matrix

$$y(n) = \begin{bmatrix} 5 & 1 & 2 & 6 \\ 6 & 5 & 1 & 2 \\ 2 & 6 & 5 & 1 \\ 1 & 2 & 6 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 43 \\ 37 \\ 27 \\ 33 \end{bmatrix}$$

$$\therefore y(n) = \{43, 37, 27, 33\}$$

→ Response of FIR System



* Response of D.T. System can be obtained by Convolution

$$x(n) * h(n) = y(n)$$

In Frequency Domain we get

$$X[k] \cdot H[k] = Y[k]$$

Eg (OLD) Impulse Response of FIR Filter is given by $h(n) = \{1, 1\}$

Find the response of the system to input $x(n) = \{2, 2, 4\}$ using DFT

Solution:- Response of the system can be obtained by solving convolution in Frequency Domain

$$\begin{array}{ccccc} x(n) & * & h(n) & = & y(n) \\ \downarrow \text{DFT} & & \downarrow \text{DFT} & & \uparrow \text{IDFT} \\ X[k] & \cdot & H[k] & = & Y[k] \end{array}$$

For Linear Convolution $L = L_1 + L_2 - 1$

$$= 3 + 2 - 1$$

$$= 4$$

Hence we use 4-Point DFT

$\boxed{\text{STEP - I}}$ Obtain $X[k]$ using DFT

$$\therefore X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ -2-2j \\ 4 \\ -2+2j \end{bmatrix}$$

$$= \{8, -2-2j, 4, -2+2j\}$$

Step II : Obtain H[k] using DFT

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$= \{2,\ 1-j,\ 0,\ 1+j\}$$

Step III: Perform Convolution

$$Y[k] = X[k] \cdot H[k]$$

$$= \begin{bmatrix} 8 \\ -2-2j \\ 4 \\ -2+2j \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$= \begin{bmatrix} 16 \\ -4 \\ 0 \\ -4 \end{bmatrix}$$

Step IV: Obtain Response y(n) using IDFT

$$\therefore y(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 16 \\ -4 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 24 \\ 16 \end{bmatrix}$$

$$\frac{1}{4}\begin{bmatrix} 16-4-4 & = & 8 \\ 16-4j+4j & = & 16 \\ 16+4\cancel{j}\,4 & = & 24 \\ 16+4j-4j & = & 16 \end{bmatrix} = $$

$$y[k] = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 4 \end{bmatrix} = \{2, 4, 6, 4\}$$

Cross cheak by Linear.

	1	1
2	2	2
2	2	2
4	4	4

$y[k] = \{2, 4, 6, 4\}$

Q. Given $h(n) = \{1, 2\}$ find response of the system to i/p $x(n) = \{1, 2, 3\}$ using DFT

Response system can be obtain by taking convolution in the Frequency Do

By Linear Convolution

$L_1 = x(n) = \{1, 2, 3\} = 3$

$L_2 = h(n) = \{1, 2\} = 2$

$L = L_1 + L_2 - 1$

$4 = 4$

Hence we use 4 point DFT.

Step

Find $X[k]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 1-2j-3 \\ 1-2+3 \\ 1+2j-3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2-2j \\ 2 \\ -2+2j \end{bmatrix}$$

Step-2 Find $H[k]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1-2j \\ -1 \\ 1+2j \end{bmatrix}$$

Step-3 Find Convolution in Frequency

$Y[k] = X[k] \cdot H[k]$

$$\begin{bmatrix} 3 \\ 1-2j \\ -1 \\ 1+2j \end{bmatrix} \begin{bmatrix} 6 \\ -2-2j \\ 2 \\ -2+2j \end{bmatrix} = \begin{bmatrix} 18 \\ -6+2j \\ -2 \\ -6-2j \end{bmatrix}$$

Step-4

~~Find Convolution~~ in ~~Frequency~~

Obtain responce $y(n)$

$$\frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & +j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix} \begin{bmatrix} 18 \\ -6+2j \\ -2 \\ -6-2j \end{bmatrix} = \begin{bmatrix} 18-6+\cancel{2j}-2-6-\cancel{2j} \\ 18+j(-6+2j)+2-j(-6- \\ 18+6-\cancel{2j}-2+6+\cancel{2j} \\ 18-j(-6+2j)+2+j(-6- \end{bmatrix}$$

DATE

$$= \frac{1}{4}\begin{bmatrix}4\\24\\28\\16\end{bmatrix} = \begin{bmatrix}1\\6\\7\\4\end{bmatrix}$$

cross check

	1	2	3
1	1	2	3
2	2	4	6

Transfer Function of D.T system

D.T system

$$x(n) \rightarrow \boxed{\text{Impulse Responce } h(n)} \rightarrow \text{o/p responce } y(n)$$

Response of D.T signal can be obtained by Convolution

$$x(n) * h(n) = y(n)$$

In frequency Domain we get

$$X[k] \cdot H[k] = Y[k]$$

$$\text{Transfer Function} = \frac{Y[k]}{X[k]} = \frac{\text{o/p}}{\text{I/p}}$$

Hence Transfer Function = DFT of Impulse Responce

Q. Obtain Transfer function of the system in Frequency Domain if its.

Soln Transfer function = DFT of h(n)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+2+1+2 = 6 \\ 1-2j-1+2j = 0 \\ 1-2+1-2 = -2 \\ 1+2j-1-2j = 0 \end{bmatrix}$$

$XH[k] = \{6, 0, -2, 0\}$