An Information Model of Image Segmentation Algorithm Based on Redundancy Minimization

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***Abstract*—This paper is devoted to the study of the information-theoretical approach to the problem of image segmentation quality. We consider a system that includes a segmentation algorithm with a parameter on which the number of segments depends, and a procedure for selecting parameter value that provides the segmentation quality measure with minimum. As a quality measure, we use an information redundancy index. To study the properties of the system, a new simplified mathematical model is proposed. It is shown that for the proposed model, the redundancy measure has a minimum. The validity of the model is confirmed by a computational experiment. An experiment conducted on images from the Berkeley Segmentation Dataset showed that a segmented image corresponding to a minimum of the redundancy measure gives the highest information similarity to the ground truth images available in the BSDS500 database.**

Keywords—image segmentation, segmentation quality, information redundancy measure, variation of information

I. INTRODUCTION

When segmenting images, the problem of setting the parameters of the applied algorithms arises. For different tasks of image analysis, various quality criteria should be selected. The criterion for the quality of the partition can be a visual assessment of an expert or some quantitative measure. In segmentation studies, the result is usually compared to an image segmented by an expert and accepted as a ground truth [1]. There may be several ground truth images obtained by different experts.

The quality can be represented by measures describing boundary detection error, region consistency, and segment overlap (Szymkiewicz–Simpson coefficient). In papers [2] and [1], the authors used the precision-recall framework for comparing segment boundaries. In [3], Martin et al. proposed global and local consistency errors as the measures for comparing segments in the output and ground-truth segmentations.

If we consider the segmentation operation as a process of clustering pixels, then set-theoretic, statistical, and information-theoretic measures are used to compare the results of data clustering [4]. The most commonly used are chi-square measure, Rand Index and its variants [5], Fowlkes-Mallows measure [6] mutual information, and normalized mutual information [7], the variation of information [8]. These measures allow comparing different versions of partitioning image into non-overlapping regions. Based on measures of similarity or difference, some automated segmentation methods have been proposed.

In [9], the authors presented a two-step algorithm for segmentation of MR and CT images, based on the representation of the segmentation process by models of direct and inverse information channels. In the first step, quasi-homogeneous regions are distinguished in the image based on the condition of maximum mutual information between the input image and the resulting partition. In the second step, the brightness levels of the histogram of the input image are clustered, providing a minimum of loss of mutual information between the clustering result and the partition obtained in the first step. Since the dependencies of the mutual information at the input and output of the direct and inverse channels do not have extrema, the authors proposed to use additional conditions (such as the number of segments, the probability of error, the ratio of the mutual information of the inverse and direct channels) for obtaining the best segmentation result.

In paper [10], Zhang et al. proposed a theoretical information method for quantitative evaluating segmentation results. The combined entropy-based quality measure takes into account the heterogeneity of the characteristics of the pixels within the segments and the complexity of partitioning an image into segments. The best segmentation results, according to the authors, corresponding to local minima of the combined entropy measure. The proposed measure can be used for comparing the quality of different segmentations generated by the same algorithm at variant parameter values, or segmentations obtained by different algorithms.

A heuristic algorithm based on an iterative application of the mean shift procedure [11] is described in [12]. As a criterion for choosing the best segmentation, the authors use the relative rate of decreasing entropy of the segmented image between iterations. The threshold value is selected empirically. In [18], Cai et al. use the total entropy of the image as a criterion to stop the improved iterative particle swarm optimization algorithm.

Some publications are devoted to the approach to automatic image segmentation using classifiers, which, trained on a certain set of features, predict the quality of the result. In [13], Kohlberger proposed a method for estimating segmentation error without ground truth images, based on a regression algorithm. The method includes the steps of computing features characterizing the result of segmentation and training the regression algorithm on a variety of images with known ground truth segmentations to predict errors. The method can be used to choose parameters of the segmentation algorithm or the best result when analyzing the input image by several segmentation algorithms working in parallel. In [14], when choosing the parameters of the algorithm, the similarity of the segmentation result with the original image was evaluated. As a measure of similarity, the authors proposed to use the weighted uncertainty index computed using the values of the normalized mutual information [7] between the color channels of the original and segmented images. At the training phase, using a

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classifier applied to expert estimations of segmentations performed for various parameter values, the areas of under-segmentation, over-segmentation, and optimal segmentation were outlined in the space “parameter-uncertainty index”. At the processing phase, an iterative procedure using the graph-cut algorithm [15] was applied to obtain parameter value providing the optimal image partition. In recent years, methods for predicting the segmentation quality based on neural networks have been developed [Robinson R. 2018]. In this work, the Dice similarity coefficient was used as a quality measure. The drawback of the classification-based approach is related to the necessity of a training procedure. The segmentation algorithms show acceptable results only for those types of images that were involved in the training procedure.

In paper [16], based on the results of [17], it was proposed to consider the minimum of information redundancy measure as a criterion of image segmentation quality. This work deals with the study of an image segmentation system based on the information redundancy criterion. To study the properties of the system, we propose a simplified mathematical model of the segmentation process. It is shown that the redundancy measure for the proposed model has a minimum. The adequacy of the model is confirmed by a computational experiment on images from the Berkeley Segmentation Dataset [1]. It has been found that segmented images corresponding to a minimum of the redundancy measure, provide a minimum to dissimilarity measure when compared with ground truth images.

II. 2. Model of a Segmentation System

To study the properties of a segmentation system, it is necessary to form its mathematical model. This section proposes and explores a simplified information model that is not related to a particular segmentation algorithm.

A. Image Segmentation Problem

The segmentation operation can be described by the following model [16]:

$$V = F(U,t), \tag{1}$$

where $U:\mathbb{Z}^2 \to \mathbb{Z}$ is an input image, $V:\mathbb{Z}^2 \to \mathbb{Z}$ is a segmented image, $F:\mathbb{Z}^2 \times \mathbb{R} \to \mathbb{Z}$ is an operator describing a segmentation algorithm, $t \in \mathbb{R}$ is a parameter. The segmentation problem is formulated as follows. Let a set of Q images $\mathfrak{V} = \{V_1, V_2, ..., V_q, ..., V_Q\}$ be obtained from a given input image U using the segmentation algorithm (1) for various values of the parameter t. It is necessary to choose an image $V_{q\min}$ providing measure $M(U,V_q)$ with minimum:

$$q_{\min} = \arg\min_{V_q} \left[M\left(U, V_q\right) \right],\ q = 1,2,...,Q. \tag{2}$$

The next subsection will present a criterion of segmentation quality proposed earlier in [16].

B. Information Criterion of Segmentation Quality

The criterion of the minimum redundancy of information was the basis of the theoretical information model of the human visual system in [17]. In [16], it was proposed to apply the criterion of minimum information redundancy to an image segmentation algorithm. To apply the information-theoretic approach, a probabilistic model of the relationship between the source and segmented images is necessary.

For simplicity, we will consider the segmentation process of grayscale images. Let the source and segmented images be the input and output of the stochastic information system. Discrete random variables U and V with values u and v describe the grayscale values of the system input and output, respectively. The segmentation operation will be represented by an information channel model:

$$V_q = F(U + \eta, t_q), \tag{3}$$

where U is the signal at the channel input, V_q is the channel output when $t = t_q$, F is the transformation operator, η is the channel noise. The variables V and η are assumed to be independent. As a quality criterion of image segmentation, we propose to consider the minimum of information channel redundancy, defined as [17]:

$$R = 1 - \frac{I(U,V)}{C(V)} \tag{4}$$

where $I(U,V)$ is the mutual information between system input and output, $C(V)$ is the channel capacity. Let $C(V) = H(V)$, where $H(V)$ is the entropy of the system output. Then, given that $I(U;V) = H(V) - H(V \mid U)$, expression (4) will take the form:

$$R = \frac{H(V \mid U)}{H(V)} \tag{5}$$

where $H(V \mid U)$ is the conditional entropy of the channel output, provided that the input is U. In the next section, we will propose a simplified qualitative model of grayscale image segmentation operation and show that such a model is characterized by a minimum of the redundancy measure.

C. Information Model of a Segmentation System

To study the qualitative properties of the redundancy measure, it is necessary to build a model in the form of a joint two-dimensional discrete distribution of the segmentation system input and output. Such a model will allow us to study the dynamics of information measures (entropies) characterizing the process of image segmentation. To build a qualitative model, it is necessary to consider how the joint two-dimensional discrete distribution of gray levels of the input and output of the system changes depending on the number of outlined segments K in image V. Let the grayscale image U shown in Fig. 1 (a) be applied to the input of the segmentation system. Fig. 1 (b) - (d) show two-dimensional discrete distributions of grayscale values for three values of K. Here, the number of gray levels L in the image U is equal to 16. The figures show the specificity of gray levels distribution in the segments. Each segment has a significant number of pixels of the dominant gray tone level, forming distribution peaks, and a certain number of pixels with other levels. Each segment has a significant number of pixels of the dominant gray tone value, forming distribution peaks, and a certain number of pixels of other levels. When the sizes of the segments increase and their number decreases, the peaks are smoothed (see Fig. 1).

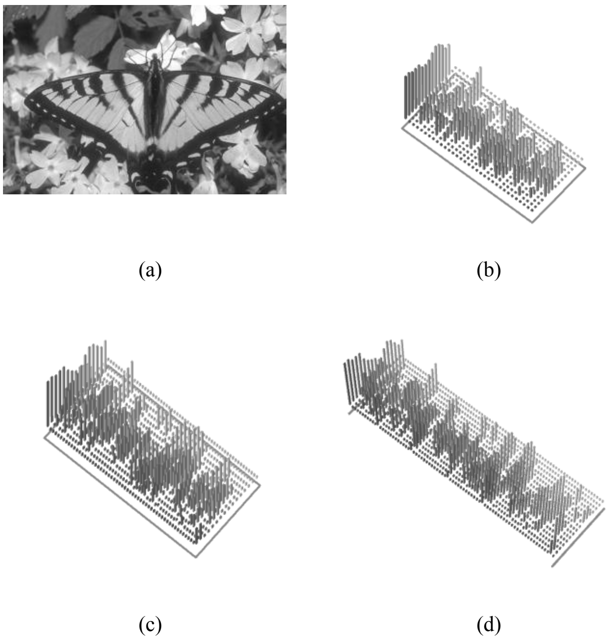

Fig. 1. Joint discrete distribution of gray levels of input and output of the segmentation system obtained for different numbers of outlined segments: (a) input image; (b) joint discrete distribution of gray levels of U and V for $K = 32$, (c) $K = 57$, and (d) for $K = 75$.

We assume that the image U has L, and V can have $1 \le l \le L$ gray levels. Suppose that the joint grayscale distribution of the images U and V can be represented by K components corresponding to the segments of image V. For simplicity, we assume that all components have the same size and consist of L components that correspond to the frequency of occurrence of pixel's gray level l in the k th segment. Each component has a peak corresponding to a dominant gray level. Let $P(u_l, v_k) = P$ if $l = k$. The relationship between the probabilities of the gray levels in the components is determined by the coefficient α, $0 < \alpha \le 1$. For example, in an image segment encoded with level v_1, $P(u_2, v_1) = P(u_3, v_1) = ... = P(u_L, v_1) = \alpha P(u_1, v_1) = \alpha P$, where $\alpha = \alpha(K)$ depends on the number of segments in the image V. The model of the described two-dimensional discrete distribution is shown in Fig. 2.

For such a model, the following relations hold $P(v_k) = (L-1)\alpha P + P$, $K\left[\alpha\left(L-1\right)+1\right]P = 1$, from where it follows that $P = 1/K\left[\alpha\left(L-1\right)+1\right]$.

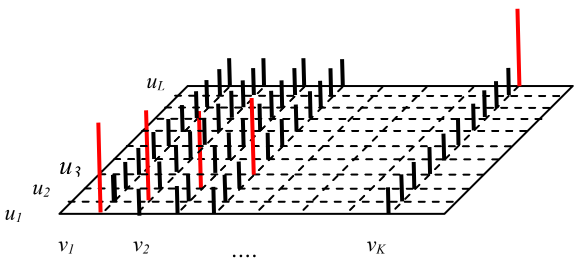

Fig. 2. Model of the discrete joint distribution of gray levels of images U and V.

For such a model, the following relations hold $P(v_k) = (L-1)\alpha P + P$, $K\left[\alpha\left(L-1\right)+1\right]P = 1$, from where it follows that $P = 1/K\left[\alpha\left(L-1\right)+1\right]$. Taking into account the constructed model of the joint discrete distribution of gray levels of the input and output of the segmentation system, we find the expressions for the entropies in formula (5):

$$H(U,V) = -\sum_{l=1}^{L}\sum_{k=1}^{K} P(u_l, v_k)\log P(u_l, v_k) = \\ = \log K + \log\left[\alpha\left(L-1\right)+1\right] - \frac{\left(L-1\right)\alpha\log\alpha}{\alpha\left(L-1\right)+1}; \tag{6}$$

$$H(U) = -\sum_{l=1}^{L}\left\{\left[\sum_{k=1}^{K} P(u_l, v_k)\right]\log\left[\sum_{k=1}^{K} P(u_l, v_k)\right]\right\} = \\ = -LP_0\left[\left(K_0-1\right)\alpha_0+1\right]\log\left[\left(K_0-1\right)\alpha_0 P_0 + P_0\right]; \tag{7}$$

$$H(V) = -\sum_{k=1}^{K}\left\{\left[\sum_{l=1}^{L} P(u_l, v_k)\right]\log\left[\sum_{l=1}^{L} P(u_l, v_k)\right]\right\} = \log K; \tag{8}$$

$$H(V\,|\,U) = H(U,V) - H(U) = \\ \log K + \log\left[\alpha\left(L-1\right)+1\right] - \frac{\left(L-1\right)\alpha\log\alpha}{\alpha\left(L-1\right)+1} - H(U), \tag{9}$$

where P_0, K_0, and α_0 are the quantities corresponding to the image U. Substituting expressions (6) - (9) in (5), we obtain:

$$R = 1 + \frac{\log\left[\alpha\left(L-1\right)+1\right]}{\log K} - \frac{\left(L-1\right)\alpha\log\alpha}{\left[\alpha\left(L-1\right)+1\right]\log K} - \frac{H(U)}{\log K}. \tag{10}$$

It is noticed that when the number of segments K of the image U decrease, the shape of the distribution changes (see Fig. 1). In the model shown in Fig. 2, such a transformation of the distribution shape corresponds to an increase in the coefficient α. Such dependence can be represented by a monotonic function, for example:

$$\alpha\left(K\right) = \frac{a}{1+e^{c(K-b)}} + d, \tag{11}$$

Where $a, b, c,$ and d are the parameters, $d = 1 - a$. The dependence $\alpha(K)$ for various values of the parameter c is shown in Fig. 3.

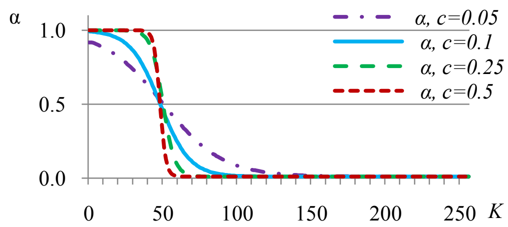

Fig. 3. Function $\alpha(K)$ for various values of the parameter c.

Next, we show that the criterion described by formulas (8-11) have a minimum. To analyze the function described by expression (10), we replace the integer variable K by the real variable z. For simplicity, we assume that "log" in expressions (6-10) is the natural logarithm. Then the derivative dR/dz takes the form:

$$R'(z) = -\frac{\log[\alpha(L-1)+1]}{z\log^2 z} + \frac{\alpha(L-1)\log\alpha}{z[\alpha(L-1)+1]\log^2 z} - \\ -\frac{ace^{c(z-b)}(L-1)^2\alpha\log\alpha}{[\alpha(L-1)+1]^2\left[e^{c(z-b)}+1\right]^2\log z} + \\ +\frac{ace^{c(z-b)}(L-1)\log\alpha}{[\alpha(L-1)+1]\left[e^{c(z-b)}+1\right]^2\log z} + \frac{H(U)}{z\log^2 z}. \qquad (12)$$

We introduce the following notation:

$$R'_1(z) = -\frac{\log[\alpha(L-1)+1]}{z\log^2 z};\quad R'_2(z) = \frac{\alpha(L-1)\log\alpha}{z[\alpha(L-1)+1]\log^2 z};$$

$$R'_3(z) = -\frac{ace^{c(z-b)}(L-1)^2\alpha\log\alpha}{[\alpha(L-1)+1]^2\left[e^{c(z-b)}+1\right]^2\log z};$$

$$R'_4(z) = \frac{ace^{c(z-b)}(L-1)\log\alpha}{[\alpha(L-1)+1]\left[e^{c(z-b)}+1\right]^2\log z};\ R'_5(z) = \frac{H(U)}{z\log^2 z}.$$

The function $\alpha(z)$ described by expression (11) decreases when z increases(see Fig. 3). It is shown that for small z and $\alpha = 1$ we have $R'(z) = 0$. Let $z = b$ and $\alpha = 0.5$. Using the notations introduced above, we get

$$R'_1(z) + R'_2(z) + R'_5(z) = \\ = \frac{(\alpha L - \alpha + 1)\log\dfrac{\alpha L}{\alpha L - \alpha + 1} - \log(\alpha)}{(\alpha L - \alpha + 1)z\log^2 z} = \\ = \frac{[0.5(L-1)+1]\log\dfrac{0.5L}{0.5(L-1)+1} - \log 0.5}{[0.5(L-1)+1]b\log^2 b} \approx 0.$$

At the same time,

$$R'_3(z) + R'_4(z) = \frac{ace^{c(z-b)}(L-1)\log\alpha}{[\alpha(L-1)+1]^2\left[e^{c(z-b)}+1\right]^2\log z} = \\ = \frac{ac(L-1)\log 0.5}{[0.5(L-1)+1]^2 4\log b} < 0.$$

At $z = K_0 = L$ and $\alpha = d$ we have the following:

$$R'_3(z) + R'_4(z) = \frac{ace^{c(L-b)}(L-1)\log d}{[d(L-1)+1]^2\left[e^{c(L-b)}+1\right]^2\log L} \approx 0$$

$$R'_1(z) + R'_2(z) + R'_5(z) = \\ = \frac{(Ld - d + 1)\log\dfrac{Ld}{Ld - d + 1} - \log(d)}{(Ld - d + 1)L\log^2 L} > 0.$$

Thus, $R'(z)$ is negative for $\alpha < 1$ and relatively small z, and it is positive for large z and $\alpha \approx d$. Therefore, there exists at least one point z at which $R'(z) = 0$ and the second derivative at this point is positive $R''(z) > 0$. This means that the function $R(z)$ has a minimum. The graphs of the function $R(K)$ defined by expression (10) for various values of the parameter c of the function $\alpha(K)$ given by expression (11) are shown in Fig. 4. The graphs of entropies (6)-(9), which determine the value of information redundancy (10) depending on the number of segments outlined in the input image, are depicted in Fig. 5.

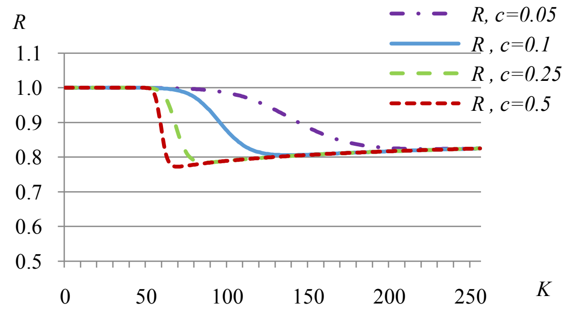

Fig. 4. Function $R(K)$ for various values of the parameter c.

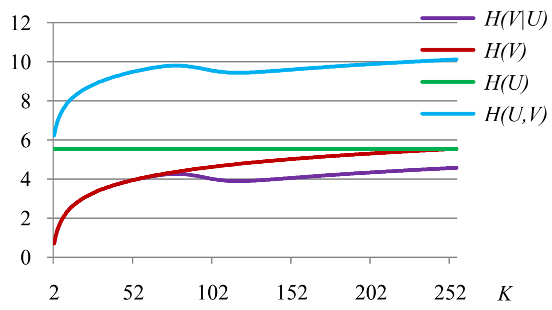

Fig. 5. Graphs of the entropies that determine the value of information redundancy $R(K)$ at the parameter value $c = 0.1$ depending on the number of segments K.

Fig. 6 and Fig.7 show graphs of the information redundancy and entropy magnitudes depending on the number of segments allocated in the image shown in Fig. 1 (a). Fig. 4 and Fig. 6, Fig. 5 and Fig. 7 demonstrate a qualitative similarity between the dynamics of the information redundancy and entropy magnitudes depending on the number of segments in the partitions of the hypothetical and real images.

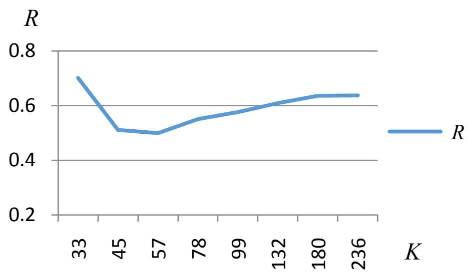

Fig. 6. Graph of the function $R(K)$ depending on the number of segments K in the partitions of the image shown in Fig. 1(a).

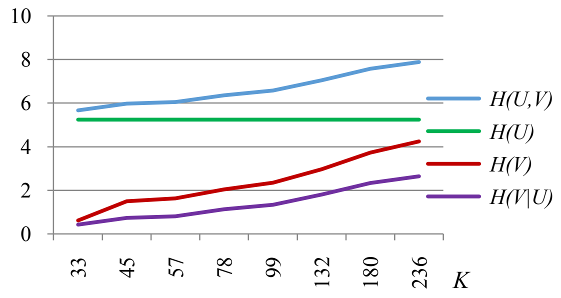

Fig. 7. Graphs of entropies characterizing $R(K)$ for the image shown in Fig. 1(a).

Next, we show that the segmented image $V(K_{\min})$ corresponding to the minimum of the redundancy measure $R(K_{\min})$ provides a sufficiently small information difference from the input image U. To measure the difference between the original and the segmented image, we will use the normalized variation of information

$$VI_n(U,V) = \frac{VI(U,V)}{H(U,V)}, \tag{13}$$

where the variation of information defined as follows [8]:

$$\begin{aligned} VI(U,V) &= H(U) + H(V) - 2I(U,V) = \\ &= H(U) + H(V)[2R-1]. \end{aligned} \tag{14}$$

Here, $H(U)$ and $H(V)$ are the entropies of images U and V, $I(U;V)$ is the mutual information. Let us estimate the value of $VI_n(U,V)$ for the images U and V. For this purpose, using formulas (4), (5), and (14) we represent $VI_n(U,V)$ using information redundancy $R(K)$:

$$VI_n(U,V) = 1 - \frac{[1-R(K)]}{H(U)/H(V) + R(K)} \tag{15}$$

As before, we replace the integer variable K with the real variable z. Differentiating $VI_n(U,V)$ with respect to z, we get:

$$\begin{aligned} VI_n'(U,V) &= \frac{[R(z)-1]H'(V)H(U)}{[H(U)+R(z)H(V)]^2} + \\ &+ \frac{R'(z)H(V)[H(U)+H(V)]}{[H(U)+R(z)H(V)]^2}, \end{aligned} \tag{16}$$

where $R'(z)$ and $H'(V)$ are derivatives of the redundancy measure and entropy of the system output with respect to z. For small K, $R(z) \approx 1$, and $H(V) \ll H(U)$. In this case, it follows from (15) that $VI_n(U,V) \approx 1$. Further, for $z < z_{\min}$, both terms in right hand part of the expression (16) are negative. Therefore, the value of the normalized variation of information decreases. Also, $|1-R(z)| < |1-R(z_{\min})|$, $\frac{H(U)}{H(V(z))} + R(z) > \frac{H(U)}{H(V(z_{\min}))} + R(z_{\min})$, and

$$\left| \frac{[1-R(z)]}{\frac{H(U)}{H(V(z))} + R(z)} \right| < \left| \frac{[1-R(z_{\min})]}{\frac{H(U)}{H(V(z_{\min}))} + R(z_{\min})} \right|$$

whence it follows that if $z < z_{\min}$, then $VI_n(U,V(z_{\min})) < VI_n(U,V(z))$. At the point corresponding to $R(z_{\min})$, taking into account (8), we obtain

$$VI_n'(U,V) = \frac{[R(z_{\min})-1]H(U)}{z_{\min}[H(U)+R(z_{\min})\log z_{\min}]^2} < 0.$$

For $z > z_{\min}$, we get $R(z) > R(z_{\min})$ and the first term in expression (16) satisfies the condition $\left| \frac{[R(z)-1]H(U)}{z[H(U)+R(z)\log z]^2} \right| < \left| \frac{[R(z_{\min})-1]H(U)}{z[H(U)+R(z_{\min})\log z_{\min}]^2} \right|$.

Also, for $z > z_{\min}$, the second term on the right-hand part of expression (16) is positive and its value increases if z increases, while the absolute value of the first term decreases. Therefore, the rate of decrease of $VI_n(U,V)$ diminishes when $z \geq z_{\min}$. This indicates that a segmented image corresponding to a minimum of information redundancy has rather small information dissimilarity from the image received at the input of the segmentation system (2), (3). The graphs of the dependence of the dissimilarity measure $VI_n(U,V)$ from the number of segments in the output image V for various values of the parameter c in (11) are shown in Fig. 8.

The efficiency of the information redundancy criterion for image segmentation is confirmed by the computational experiment described in the next section.

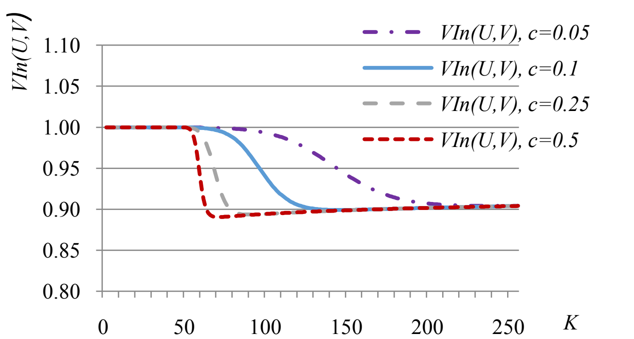

Fig. 8. The graphs of the dependence of the dissimilarity measure $VI_n(U,V)$ from the number of segments in the output image V for various values of the parameter c in (11).

III. Computational Experiment

In this paper, to illustrate the validity of the information model of the segmentation process, we used images from the BSDS500 database [1]. Each of the tested images was segmented using a modified SLIC algorithm with different values of the parameter of the post-processing procedure [16]. As a result of the segmentation of the image U, we obtained a set of Q images $\mathfrak{V} = \{V_1, V_2, ..., V_Q\}$. For input image U and each of the segmented images $V_q, q = 1,2,...,Q$ the redundancy measure R is computed. To involve all color components of images, we use here the weighted version of the redundancy measure:

$$R_w(U, V_q) = \frac{R_L H_L(U) + R_a H_a(U) + R_b H_b(U)}{H_L(U) + H_a(U) + H_b(U)}$$

where R_i is the redundancy measure computed for color component $i \in \{L, a, b\}$ of images U and V_q, H_i is the entropy of the color component i of the input image. We select image V_q giving minimum to R_W: $R_W(U, V_q) = R_{\min}$. Dependency of measure R_W on number of segments K for the test color version of image from Fig. 1(a) is depicted in Fig. 9. Minimum of R_W is reached at $K = 87$.

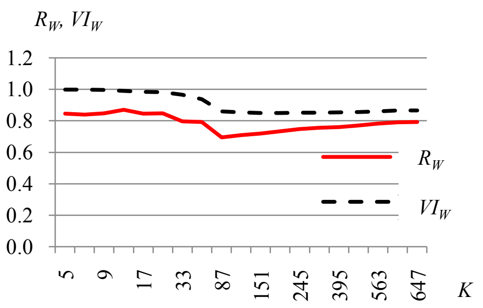

Fig. 9. Dependency of redundancy R_W and normalized variation of information $VI_W(U, V_q)$ on the number of segments K for the color version of image shown in Fig. 1(a).

Next, we compared the segmented images $V_q, q = 1,2,...,Q$ with the original image U and with the ground truth segmentations $V_s^{GT}, s = 1,2,...,S$ from the BSDS500 database. To compare the images, we used the normalized variation of information (13). Here, in order to involve all color components of images we use the weighted index based on this measure:

$$VI_W(U, V_q) = \frac{VI_{nL} H_L(U) + VI_{na} H_a(U) + VI_{nb} H_b(U)}{H_L(U) + H_a(U) + H_b(U)}, \quad (17)$$

$$VI_{ni}(U, V_q) = \frac{H_i(U) + H_i(V_q) - 2I_i(U, V_q)}{H_i(U, V_q)} \quad (18)$$

where $VI_W(U, V_q)$ is the weighted variation of information, VI_{ni} is the normalized distance between color component i of image U and component i of image V_q, $I_i(U, V_q)$ is the mutual information, and $H_i(U, V_q)$ is the joint entropy. In order to estimate the distance between the input and the segmented images, we compute weighted normalized variation of information (17, 18). The curve representing $VI_W(U, V_q)$ as the function of the number of segments is depicted in Fig. 9 by a dashed line. One can see that distance between the input and segmented images decreases when K grows and become nearly stable at $K_{\min}$ corresponding to minimal redundancy value.

To show the efficiency of the proposed quality model, we use the following relative difference [16]:

$$\Delta K_{rel} = \frac{K_{\min} - K_{\min}^{GT}}{K_{\max}} \quad (19)$$

where $K_{\min}$ is a number of segments corresponding to $R_{\min}$; $K_{\min}^{GT}$ is a number of segments in image V_q, which corresponds to the minimum of distance $VI_w(V_s^{GT}, V_q)$; $K_{\max}$ is the largest possible number of segments in images V_q obtained from input image U. Histogram of ΔK_{rel} values (19) computed for 54 test images and 270 ground truth segmentations (5 ground truth segmentations per each of the test images) is depicted in Fig 10. Fig. 10 shows a sufficiently large group of test images with magnitude of ΔK_{rel} close to zero. The ground truth segmentations of these images are close enough in the sense of measure (17, 18) to segmentations, which minimize information redundancy measure R_w.

IV. Conclusions

In this paper, a simplified mathematical model of digital image segmentation algorithms with a parameter was proposed. We used information redundancy as a measure of segmentation quality. It was shown that for the proposed information model, there exists a minimum of the redundancy measure, which allows getting the best segmentation of an image. The model validity was confirmed by a computational experiment on images from the Berkeley Segmentation Dataset database. A segmented image corresponding to a minimum of redundancy measure gave a minimum to information dissimilarity measure when compared with ground truth segmentations. Future research will be related to the study of more complicated segmentation models and properties of the quality measure based on information redundancy.

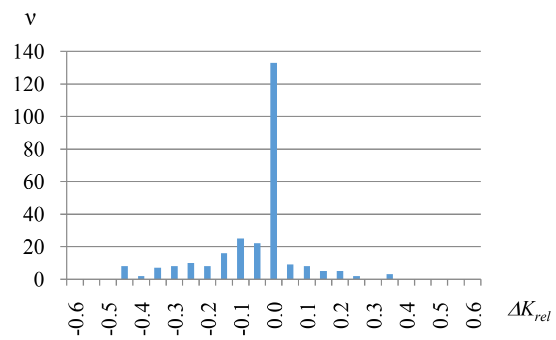

Fig. 10. Histogram of ΔK_{rel} values, ν is a frequency of occurrence of a particular ΔK_{rel} value.

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