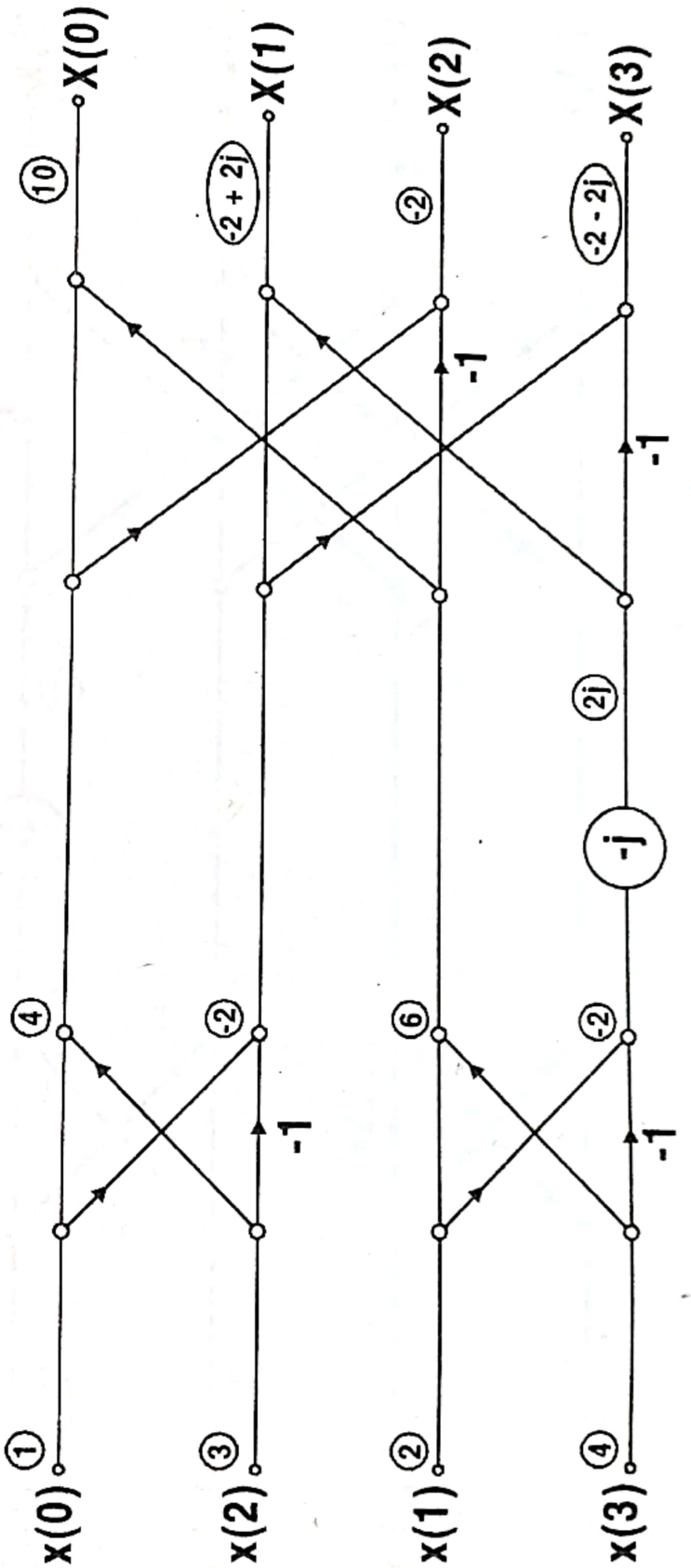
Example 4.18:

Find the DFT of the following sequences using DIT-FFT

$x(n) = \{1, 2, 3, 4\}$

Solution:

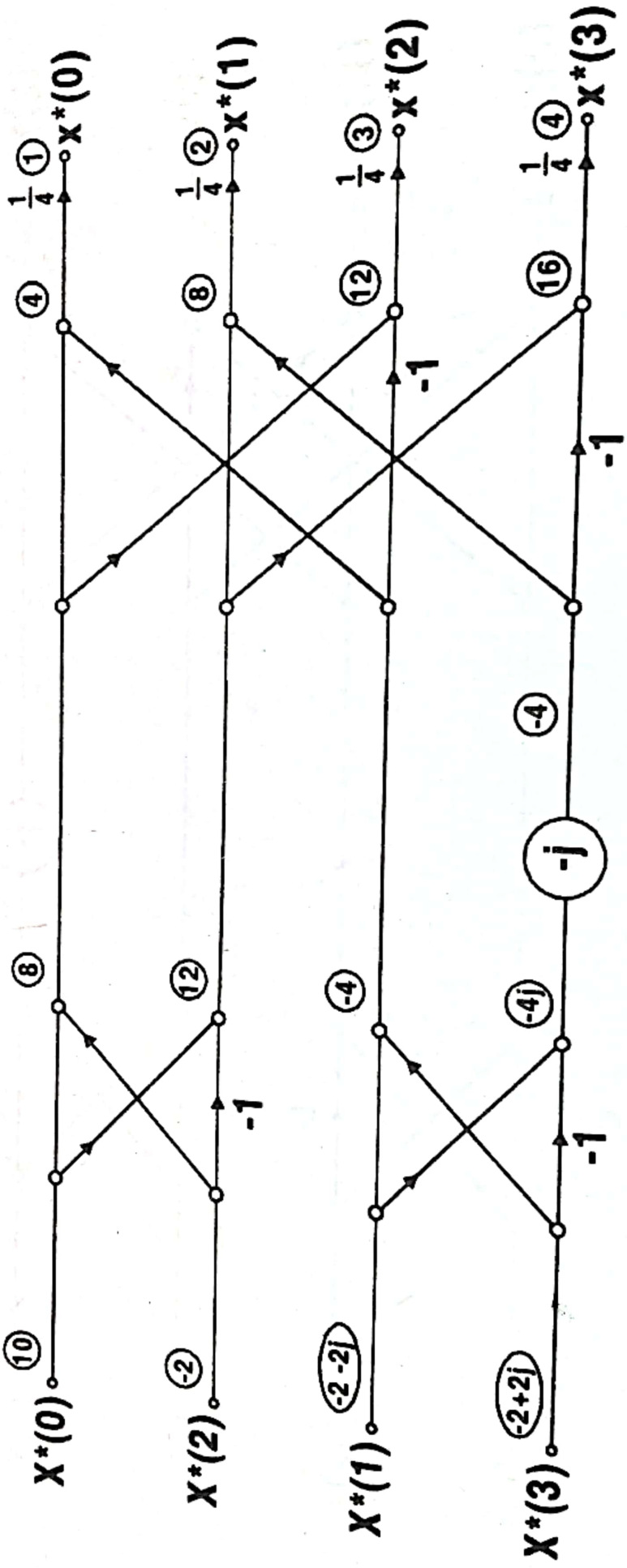


Example 4.19:

Find the IDFT of

$$X(k) = \{10, -2 + 2j, -2, -2 - 2j\}$$ using IDIT-FFT.

Solution:

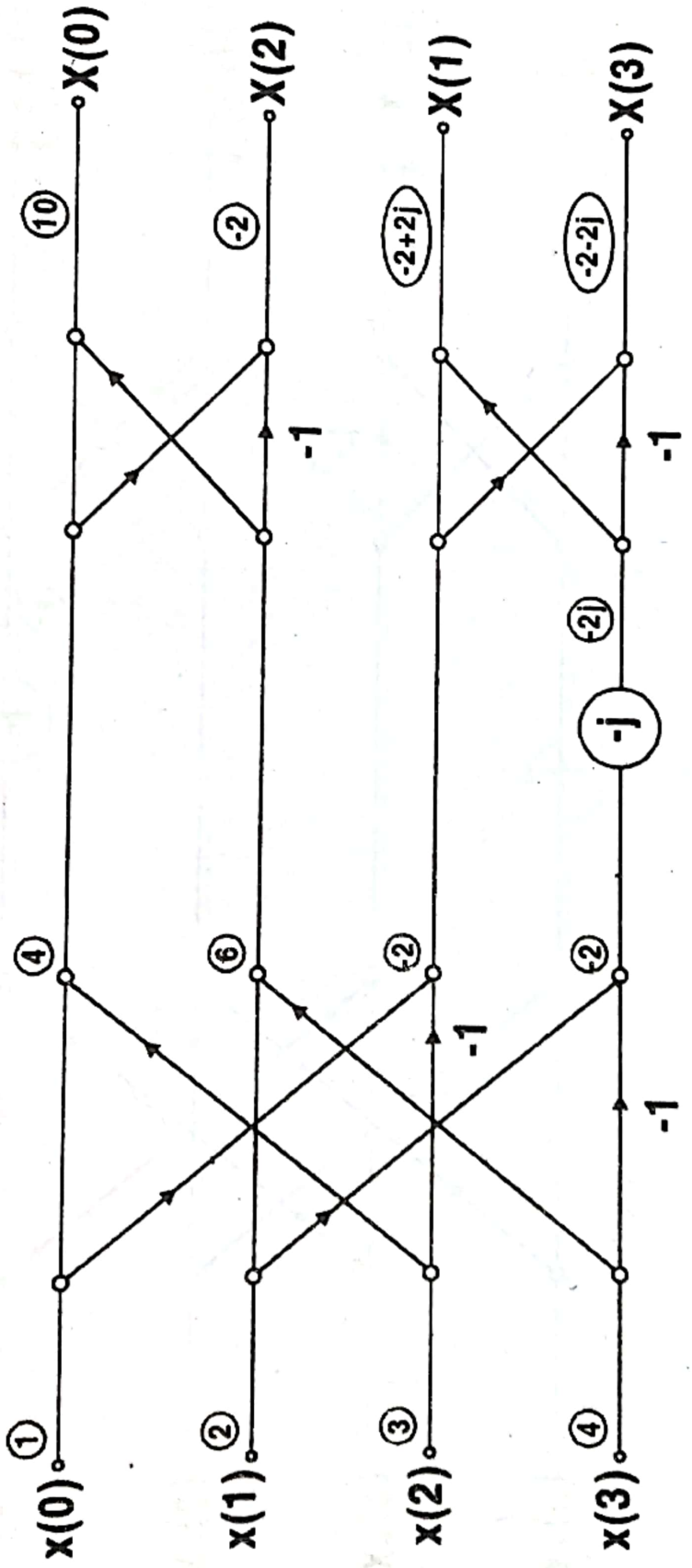


Example 4.20:

Find the DFT of

$x(n) = \{1, 2, 3, 4\}$ using DIF-FFT.

Solution:

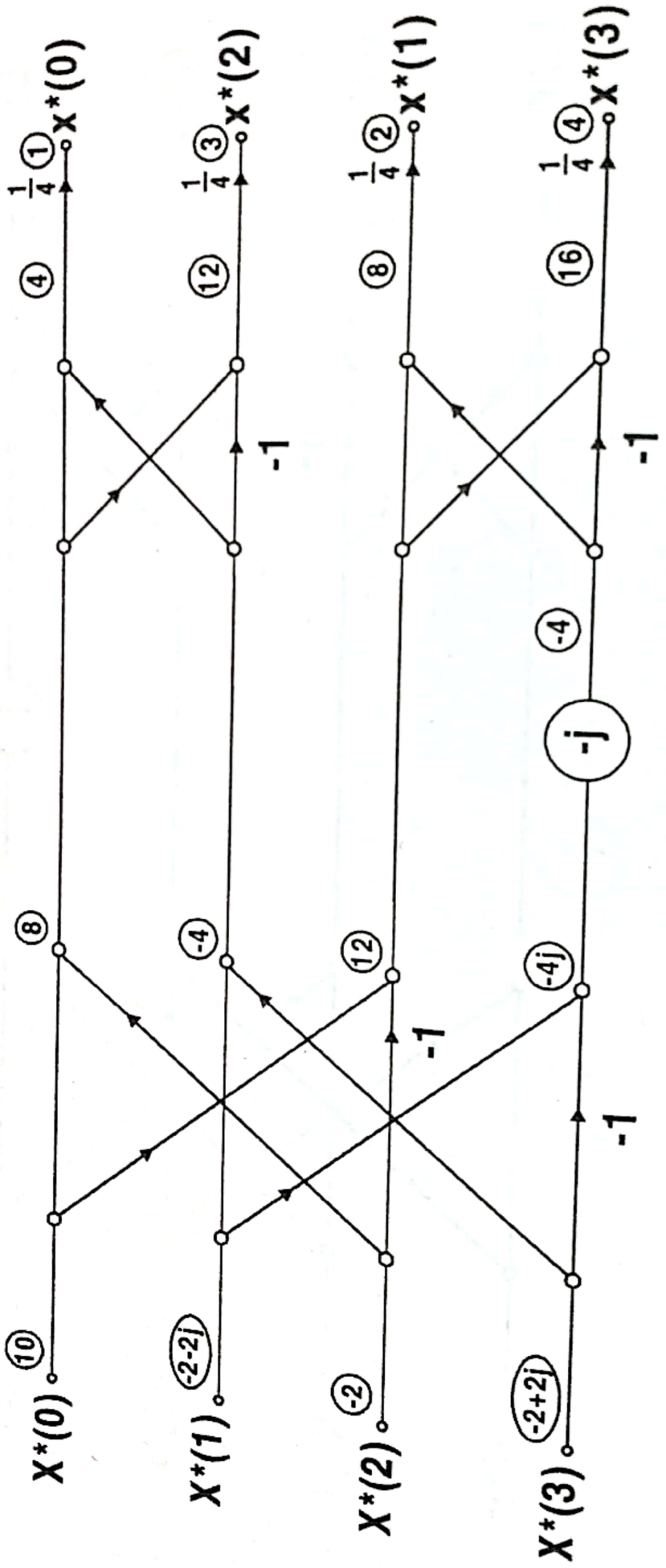


Example 4.21:

Find the IDFT of

$$X(k) = \{10, -2 + 2j, -2, -2 - 2j\}$$ using IDIF-FFT

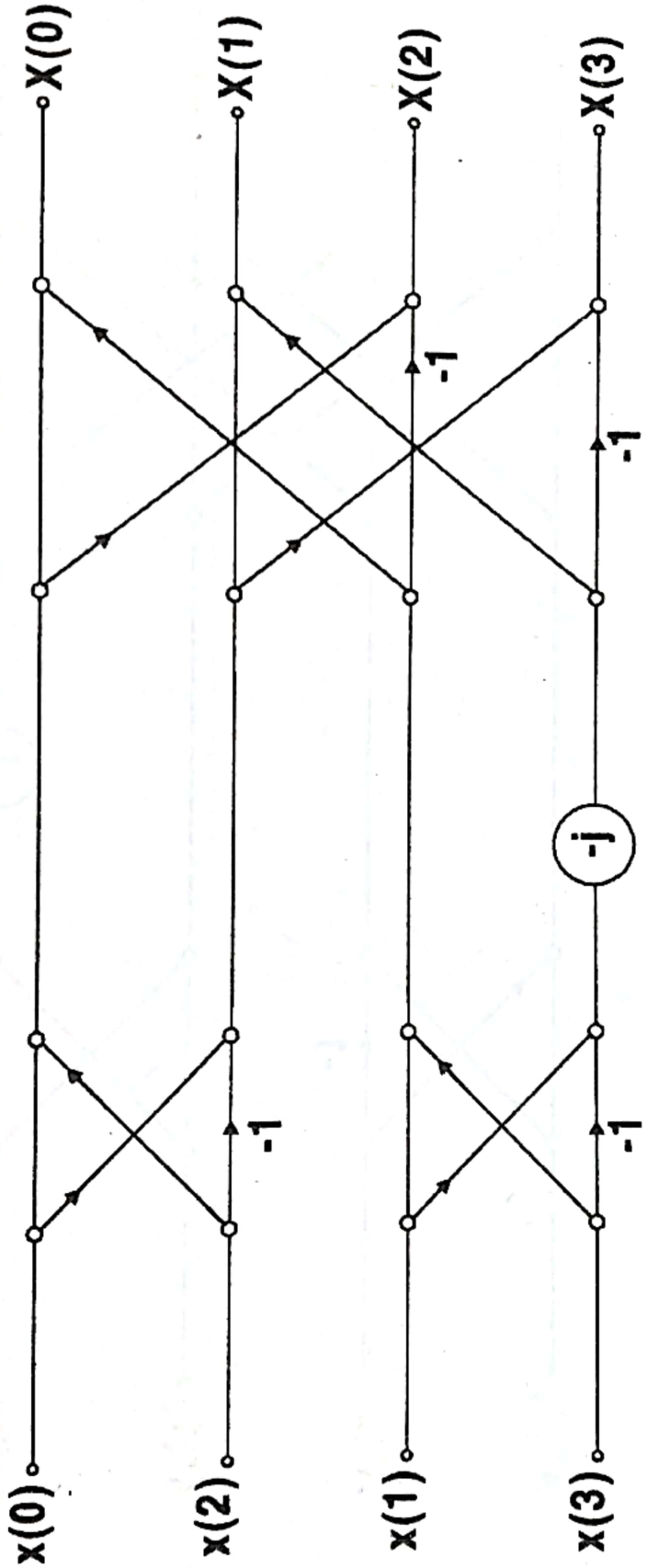
Solution:

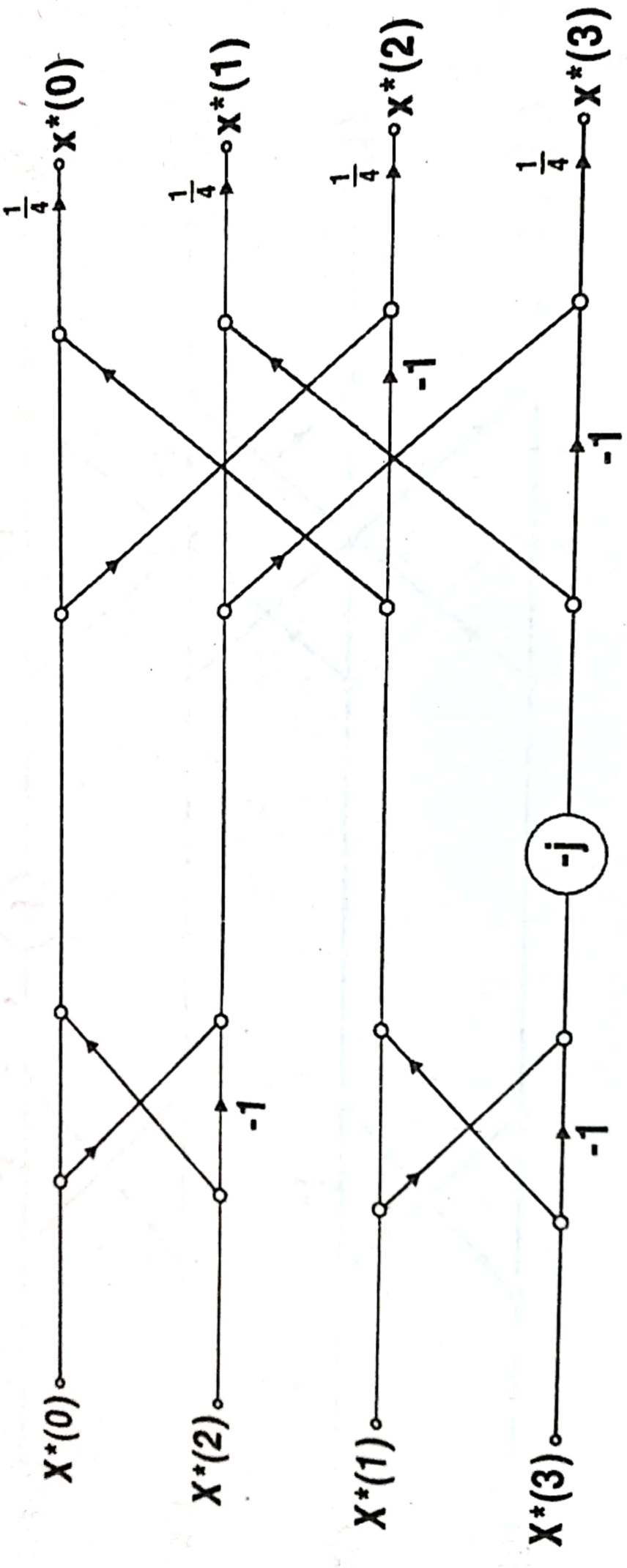


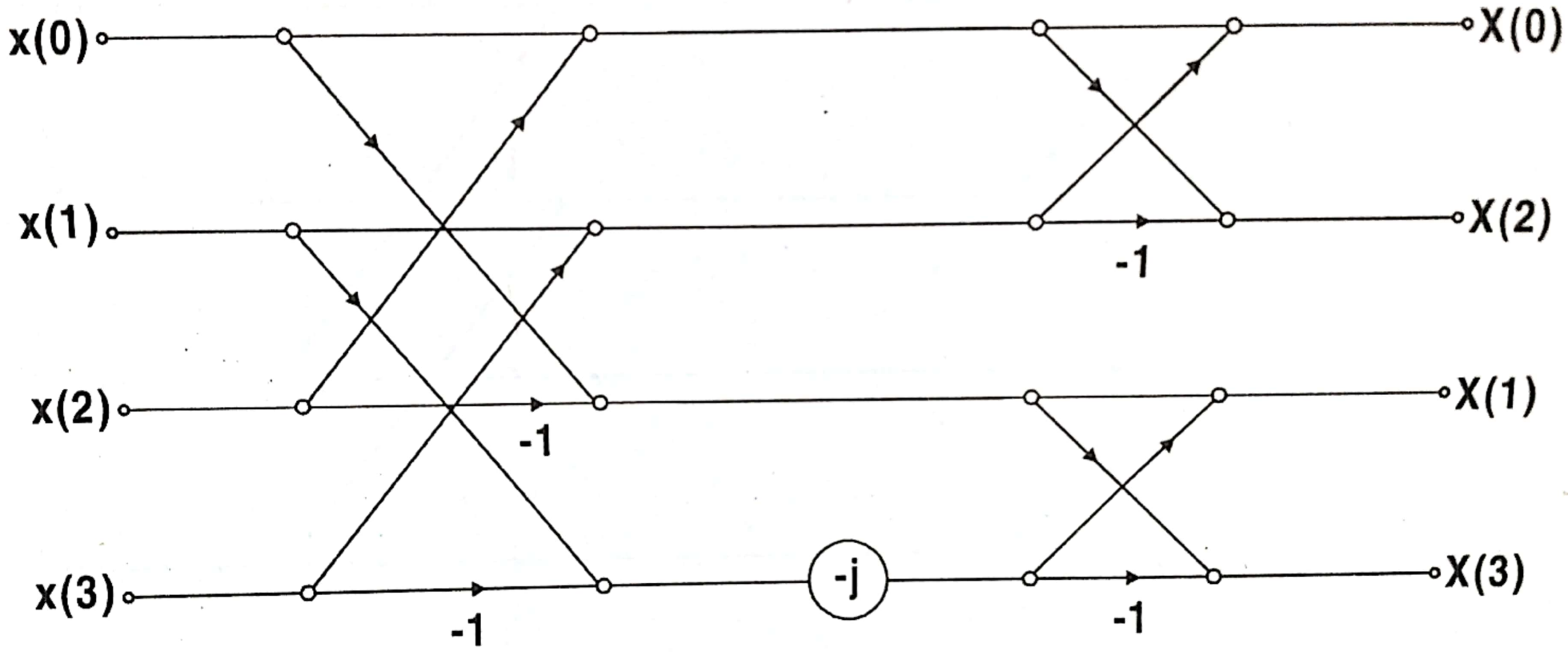
Students are advised to find the DFT and IDFT of

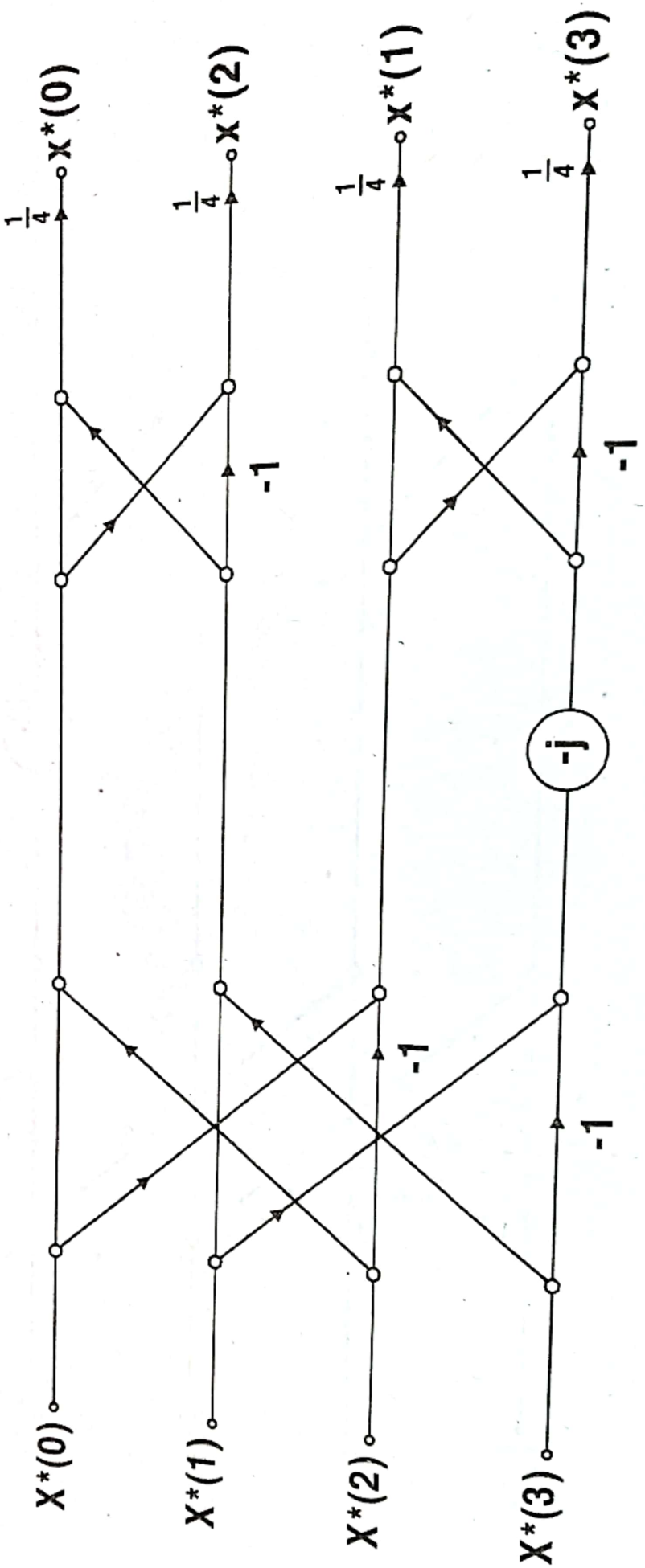
$x(n) = \{5, 6, 7, 8\}$

by using DIT and DIF algorithms.




x*(0)
x*(1)
x*(2)
x*(3)
1/4
1/4
1/4
1/4
-1
-1
-1
-1
-j
X*(0)
X*(2)
X*(1)
X*(3)


x(0)
x(1)
x(2)
x(3)
X(0)
X(2)
X(1)
X(3)
-1
-1
-1
-j
-1

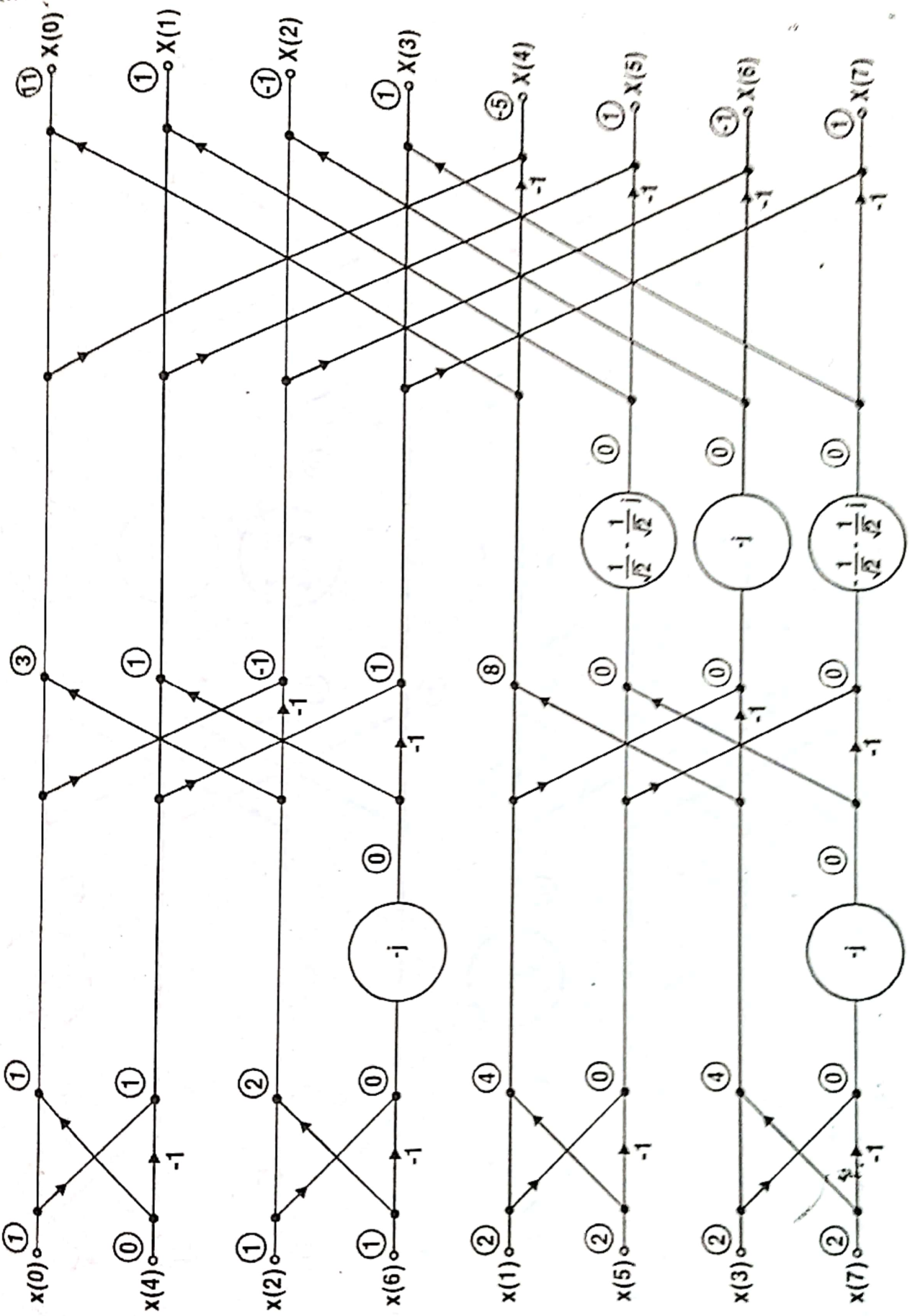


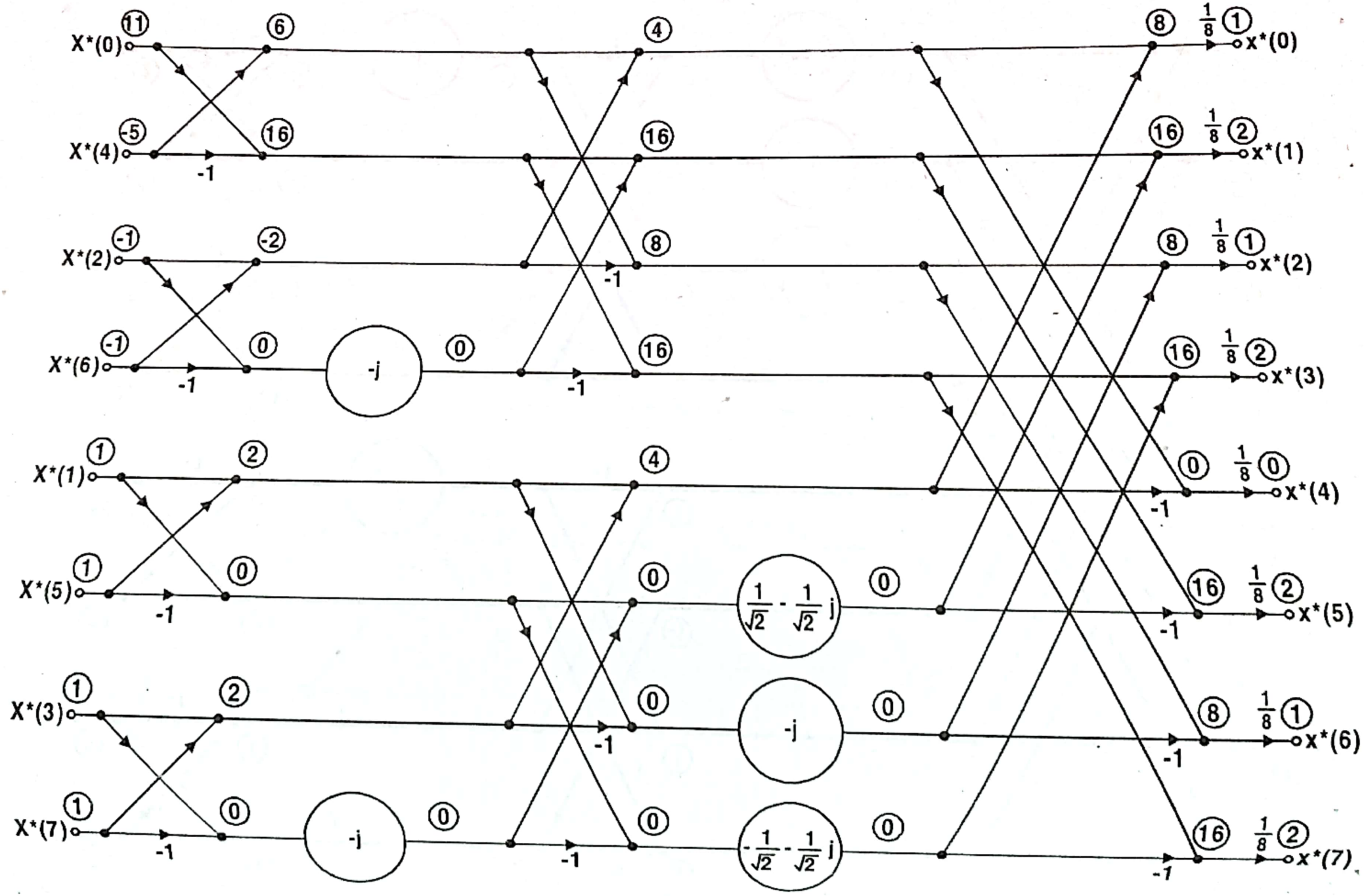
Example 4.22:

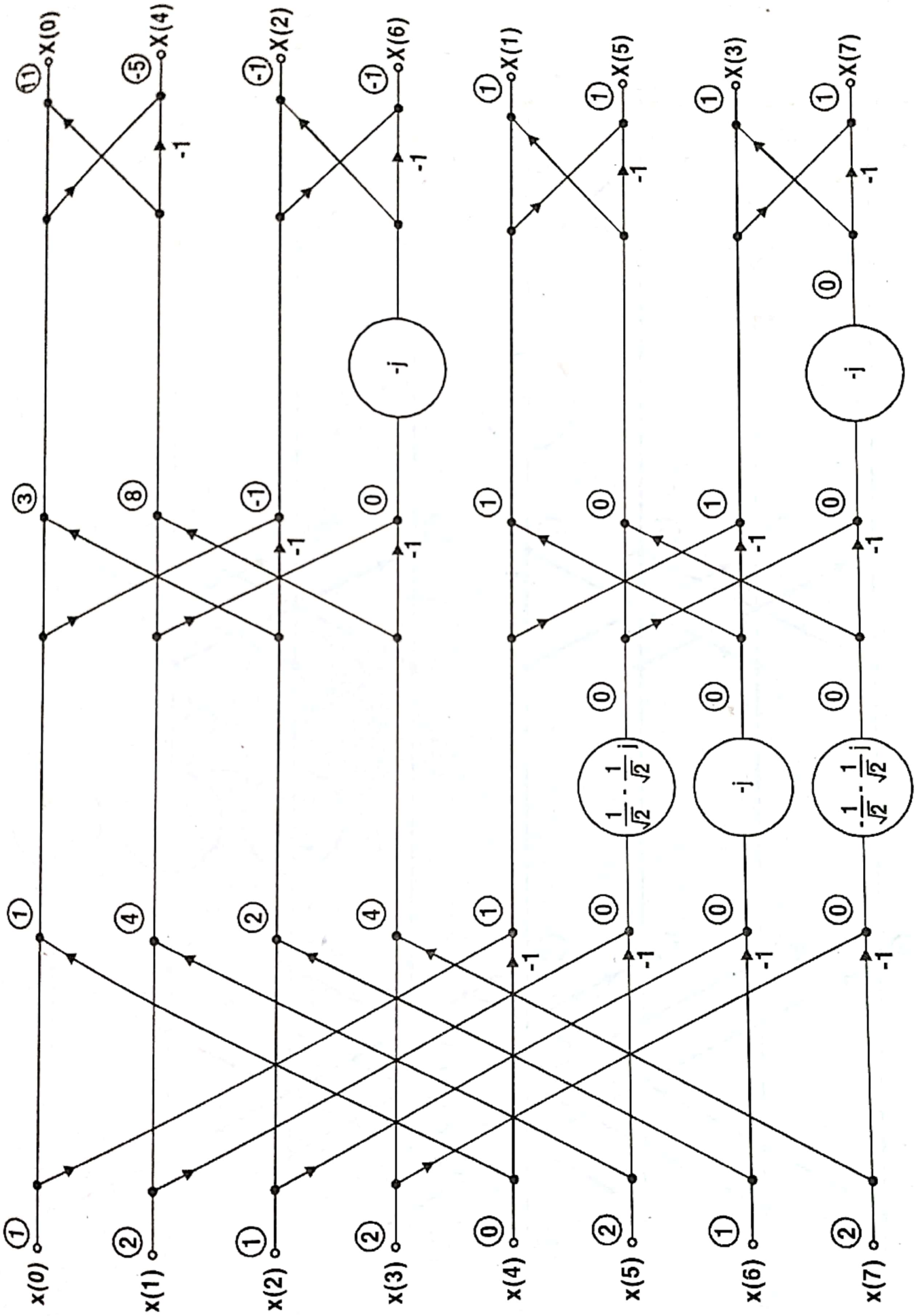
Find DFT of the following using DIT-FFT.

$x(n) = \{1, 2, 1, 2, 0, 2, 1, 2\}$

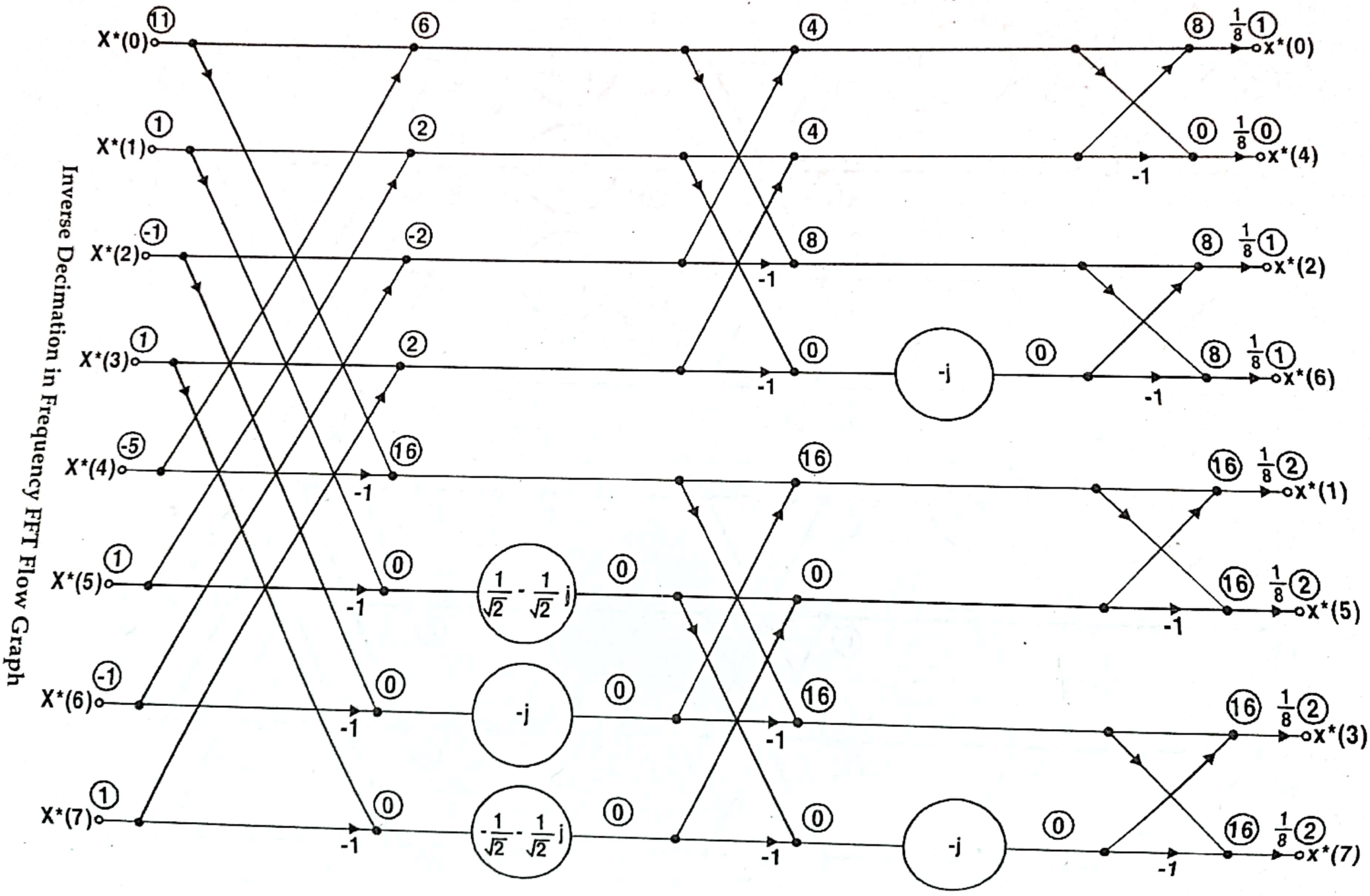
Solution:




X*(0)
X*(4)
X*(2)
X*(6)
X*(1)
X*(5)
X*(3)
X*(7)
-j
-j
-j
x*(0)
x*(1)
x*(2)
x*(3)
x*(4)
x*(5)
x*(6)
x*(7)



Decimation in Frequency FFT Flow Graph



Inverse Decimation in Frequency FFT Flow Graph

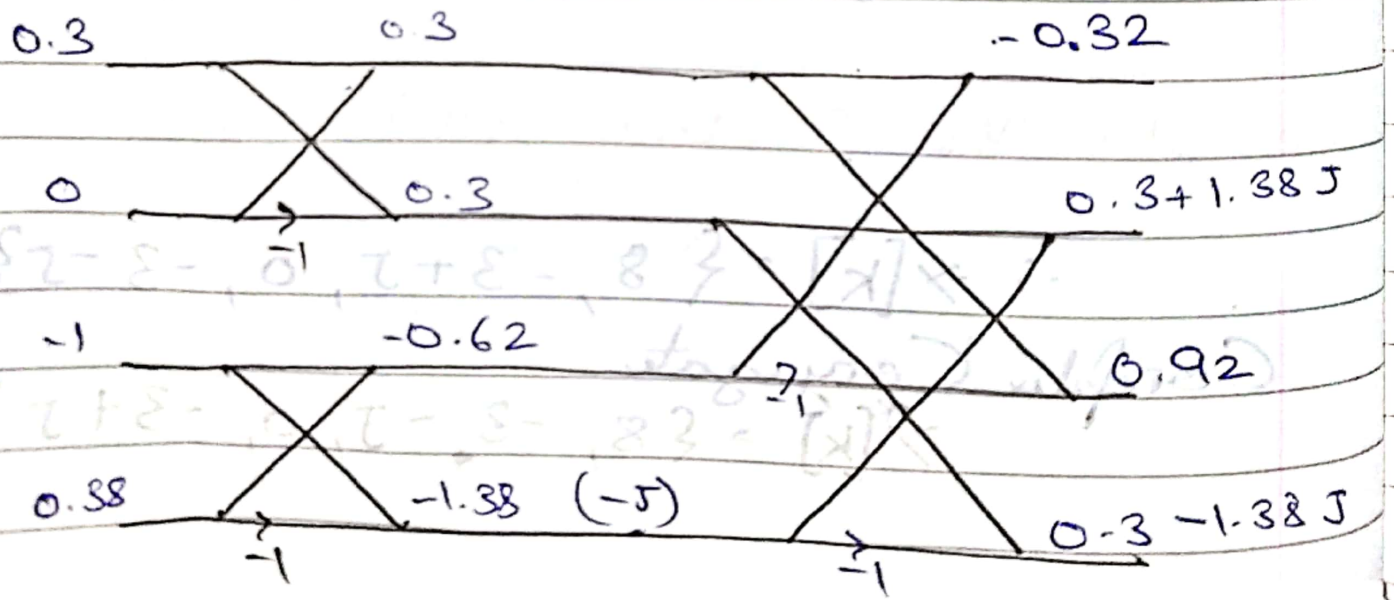
→ Spectral Analysis using FFT

EG 10 M

For the causal LTI digital filter with impulse response given by $h(n) = 0.3\delta(n) - \delta(n-1) + 0.38\delta(n-2)$ sketch the magnitude spectrum of the filter using DIT-FFT

Solution - given $h(n) = \{0.3, -1, 0, 0.38\}$

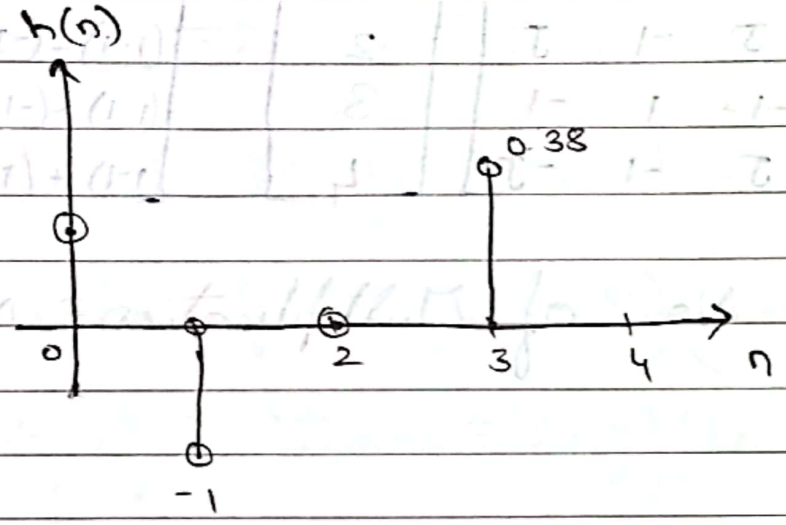
We first obtain $H[k]$ using DIT-FFT



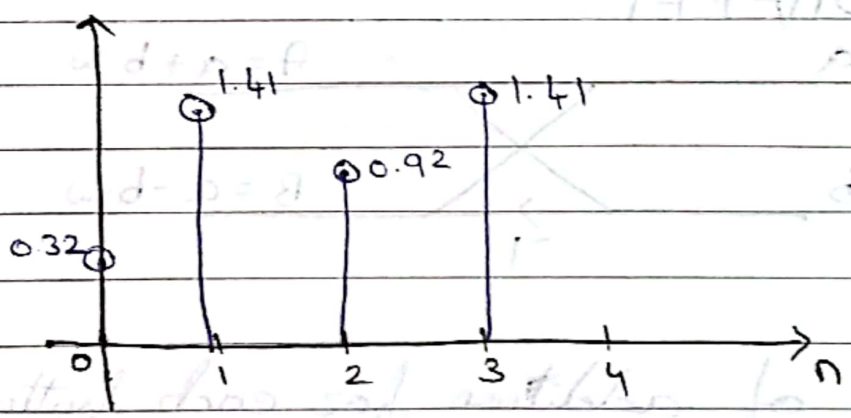
$\therefore H[k] = \{-0.32,\ 0.3+1.3j,\ 0.92,\ 0.3-1.3j\}$

Magnitude $|H[k]| = \begin{bmatrix} 0.32 \\ 1.41 \\ 0.92 \\ 1.41 \end{bmatrix}$ Phase $\angle H[k] = \begin{bmatrix} 180 \\ 77.74 \\ 0 \\ -77.74 \end{bmatrix}$

Time Domain:

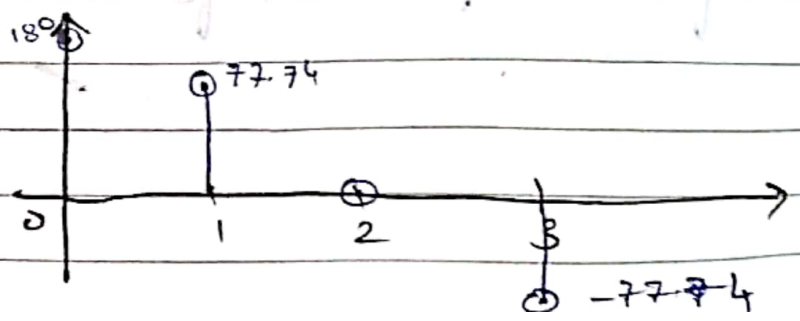


Magnitude Spectrum



Phase Spectrum

$\angle H[k]$



Comparision of Real & Complex Multiplication & Addition of DFT and FFT

★★★ (EC Expected This time - 5 Marks Theory)

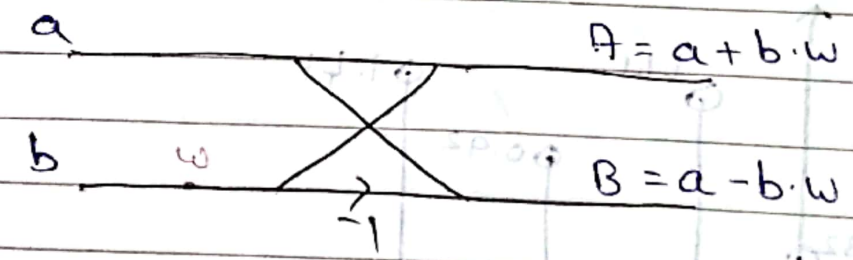
- Consider N=4 DFT

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} (1\cdot1)+(1\cdot2)+(1\cdot3)+(1\cdot4) \\ (1\cdot1)+(-j\cdot2)+(-1\cdot3)+(j\cdot4) \\ (1\cdot1)+(-1\cdot2)+(1\cdot3)+(-1\cdot4) \\ (1\cdot1)+(j\cdot2)+(-1\cdot3)+(-j\cdot4) \end{bmatrix}$$

Total No. of Multiplication = $N \times N = N^2$

Total No. of Addition = $(N-1) \times N = N^2 - N$

- For DIT-FFT



No. of addition for each butterfly = 2

No. of multiplication for each butterfly = 1

Radix-2 FFT

8-Point $2^{(3)} = 8$ 3-Stages

4-Point $2^{(2)} = 4$ 2-Stages

* 16-Point $2^{(4)} = 16$ 4-Stages

$\therefore N$-Point $\Rightarrow$ No. of Stages $= \log_2 N$

$\therefore$ No. of Butterfly for each stage $= \frac{N}{2} \Rightarrow$ Radix-2

Total number of Butterfly $= \left(\frac{N}{2}\right)\left(\log_2 N\right)$

Total No. of Additions $= (2)\left(\frac{N}{2}\right)\left(\log_2 N\right) = N \log_2 N$

Total No. of Multiplication $= (1)\left(\frac{N}{2}\right)\left(\log_2 N\right) = \frac{N}{2} \log_2 N$

	DFT	FFT
Multiplication	N^2	$\frac{N}{2}\log_2 N$
Addition	$N(N-1)$	$N\log_2 N$