3. DSP Algorithms

ES Weightage 20 Marks - NUM + Theory

***→ Linear FIR Filtering Using Overlap Save & Overlap Add Method

- Real Time Signals are long
- We can not wait till entire signal is available
- Convolution is required to be performed on-line
- Hence Overlapping Approach is used
- Overlap Save ⇒ Overlap Discard
- Overlap Add
- Linear Convolution is performed using Circular Convolution

How to decide Block Size

For Linear Convolution

$$L = L_1 + L_2 - 1$$

$$\underbrace{N}_{\text{Block Size}} = \underbrace{L}_{\text{Length of } x(n)} + \underbrace{M}_{\text{Length of } h(n)} - 1$$

Example Perform Linear Convolution using.

① Overlap Save Method
② Overlap Add Method

$x(n) = \{3, 4, 1, 2\}$

$h(n) = \{1, 2, 1\}$

Solution :- We select Block Size : $N = 4$

Here length of $h(n) = M = 3$

choose N = Block Length : $N = L + M - 1$

calculate L = Actual data of $x(n)$: $4 = L + 3 - 1$

given M = Length of $h(n)$: $\therefore L = 2$

① Overlap - Save Method

We divide $x(n)$ into multiple Overlapping blocks.

Since there is No-overlapping for first Block

we perform Zero-Padding :-

$\therefore x_1(n) =$

0	0	3	4

$x_2(n) =$

3	4	1	2

$x_3(n) =$

1	2	0	0

Now, we perform Linear Convolution using Circular Convolution

$\therefore h(n) = \{1, 2, 1, 0\}$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 4 \\ 3 \\ 10 \end{bmatrix}$$

$$\therefore y_1(n) = \{11, 4, 3, 10\}$$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 12 \\ 8 \end{bmatrix}$$

$$\therefore y_2(n) = \{8, 12, 12, 8\}$$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}$$

$$\therefore y_3(n) = \{1, 4, 5, 2\}$$

Final Answer is obtained by performing Overlap-Save (Overlap-Disc

11	4	3	10

8	12	12	8

Discard

1	4	5	2

$\therefore y(n)$ ~~{11, 4, 11, 22, 13, 12, 5, 2}~~

$$\therefore y(n) = \{3, 10, 12, 8, 5, 2\}$$

Date :

Cross Check

	3	4	1	2
1	3	4	1	2
2	6	8	2	4
1	3	4	1	2

$\therefore y(n) = \{3, 10, 12, 8, 5, 2\}$

② Overlap-Add Method

We divide $x(n)$ into Multiple Overlapping blocks.

$\therefore x_1(n) = \boxed{3 \mid 4 \mid 0 \mid 0}$

$\therefore x_2(n) = \boxed{1 \mid 2 \mid 0 \mid 0}$

Now we perform Linear Convolution using Circular Convolution

$\therefore h(n) = \{1, 2, 1, 0\}$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 11 \\ 4 \end{bmatrix}$$

$\therefore y_1(n) = \{3, 10, 11, 4\}$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}$$

$$\therefore y_2(n) = \{1, 4, 5, 2\}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

Overlap - Add Operation

3	10	11	4		
		+			
		1	4	5	2

$$y(n) = \{3, 10, 12, 8, 5, 2\}$$

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$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$h(n) = \{1, 1, 1\}$

1. Overlap Save Method
2. Overlap Add Method

Solution:- We select block size $= N = 4$
Here length of $h(n) = M = 3$

$$N = L + M - 1$$
$$4 = L + 3 - 1$$
$$\boxed{\therefore L = 2}$$

(1) Overlap-Add Method

$x_1(n) =$

1	2	0	0

$x_2(n) =$

3	4	0	0

$x_3(n) =$

5	6	0	0

$x_4(n) =$

7	8	0	0

Example:- Let $x(n) = \{2, 1, -1, 2, 1, 1\}$ and $h(n) = \{1, 2, 3\}$

Find Response of the system by Overlap-Add & Overlap-Save Method

Solution:- We know, $L = L_1 + L_2 - 1$

Consider $N = L + M - 1$

Here the Length of $h(n) = M = 3$

Hence $M - 1 = 3 - 1 = 2$.

We select $L = 2$ = Length of Current Samples

$\therefore$ Block size $= N = L + M - 1$

$= 2 + 3 - 1$

$= 4$

For overlap-Save Method:

Prev-Overlap / Zero	Actual / Current Samples
$(M-1)$	(L)

For Overlap-Add Method

Actual / Current Samples (L)	Zeros (M-1)

$\longleftarrow N \longrightarrow$

I Overlap-Save or Overlap-Discard

- We first obtain input Blocks

$x_1(n) = \boxed{0}\boxed{0}\boxed{2}\boxed{1}$

$x_2(n) = \boxed{2}\boxed{1}\boxed{-1}\boxed{2}$

$x_3(n) = \boxed{-1}\boxed{2}\boxed{1}\boxed{1}$

$x_4(n) = \boxed{1}\boxed{1}\boxed{0}\boxed{0}$

- Now we perform Block-Wise Convolution
  Here $h(n) = \{1, 2, 3, 0\}$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 2 \\ 5 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ 7 \\ 3 \end{bmatrix}$$

$$y_3(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 9 \end{bmatrix}$$

$$y_4(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

- Finally, we perform overlap-save to obtain $y(n)$

$y_1(n) = \boxed{\cancel{8}}\boxed{\cancel{3}}\boxed{2}\boxed{5}$

$y_2(n) = \boxed{\cancel{3}}\boxed{\cancel{11}}\boxed{7}\boxed{3}$

$y_3(n) = \boxed{\cancel{4}}\boxed{\cancel{3}}\boxed{2}\boxed{9}$

$y_4(n) = \boxed{\cancel{1}}\boxed{\cancel{3}}\boxed{5}\boxed{3}$

$y(n) = \{2, 5, 7, 3, 2, 9, 5, 3\}$

Cross Check at exam

	2	1	-1	2	1	1
1	2	1	-1	2	1	1
2	4	2	-2	4	2	2
3	6	3	-3	6	3	3

$y(n) = \{2, 5, 7, 3, 2, 9, 5, 3\}$

(II) Overlap-Add Method

- We first obtain Input Blocks.

$x_1(n) = $ | 2 | 1 | 0 | 0 |

$x_2(n) = $ | -1 | 2 | 0 | 0 |

$x_3(n) = $ | 1 | 1 | 0 | 0 |

- Now, we perform Block-wise Convolution

Here $h(n) = \{$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \\ 3 \end{bmatrix}$$

$$\therefore y_2(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 6 \end{bmatrix}$$

$$\therefore y_3(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

- Finally, we perform Overlap-Add to obtain $y(n)$

$y_1(n) = \boxed{2}\boxed{5}\boxed{8}\boxed{3}$

+

$y_2(n) = \boxed{-1}\boxed{0}\boxed{1}\boxed{6}$

+

$y_3(n) = \boxed{1}\boxed{3}\boxed{5}\boxed{3}$

$y(n) = \{2, 5, 7, 3, 2, 9, 5, 3\}$

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$x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$h(n) = \{1, 1, 1\}$

Perform Overlap Save & Overlap-Add

Solution

$L = L_1 + L_2 + 1$

$N = L + M - 1$

Given $M = 3$; $M - 1 = 3 - 1 = 2$

Choose $L = M - 1 = 2$

$N = 2 + 3 - 1$

$= 4$

$\longleftarrow N \longrightarrow$

Overlap $= M - 1$

Actual $= L$

Date :

$\boxed{\text{I}}$ Overlap-Save or Overlap-Add

- We first Obtain Input Blocks.

$x_1(n) = \boxed{0 \mid 0 \mid 1 \mid 2}$

$x_2(n) = \boxed{1 \mid 2 \mid 3 \mid 4}$

$x_3(n) = \boxed{3 \mid 4 \mid 5 \mid 6}$

$x_4(n) = \boxed{5 \mid 6 \mid 7 \mid 8}$

$x_5(n) = \boxed{7 \mid 8 \mid 0 \mid 0}$

- Now we perform Block-wise convolution.

$\therefore$ Here $h(n) = \{1, 1, 1, 0\}$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\therefore y_2(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 6 \\ 9 \end{bmatrix}$$

$$\therefore y_3(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 12 \\ 15 \end{bmatrix}$$

$$\therefore y_4(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 20 \\ 19 \\ 18 \\ 21 \end{bmatrix}$$

$$\therefore y_5(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \\ 15 \\ 8 \end{bmatrix}$$

- Finally we perform Overlap - Save to obtain $y(n)$

$\therefore y_1(n) = \boxed{\cancel{3}}\ \boxed{2}\ \boxed{1}\ \boxed{3}$

$\therefore y_2(n) = \boxed{\cancel{8}}\ \boxed{7}\ \boxed{6}\ \boxed{9}$

$\therefore y_3(n) = \boxed{\cancel{14}}\ \boxed{13}\ \boxed{12}\ \boxed{15}$

$\therefore y_4(n)$ $\boxed{\cancel{20}}\ \boxed{19}\ \boxed{18}\ \boxed{21}$

$\therefore y_5(n)$ $\boxed{\cancel{7}}\ \boxed{15}\ \boxed{15}\ \boxed{8}$

$\therefore y(n) = \{1, 3, 6, 9, 12, 15, 18, 21, 15, 8\}$

II Overlap - Add Method

- We first obtain Input Blocks.

$\therefore x_1(n) = \boxed{1}\ \boxed{2}\ \boxed{0}\ \boxed{0}$

$\therefore x_2(n) = \boxed{3}\ \boxed{4}\ \boxed{0}\ \boxed{0}$

$\therefore X_3(n) = \boxed{5}\boxed{6}\boxed{0}\boxed{0}$

$\therefore X_4(n) = \boxed{7}\boxed{8}\boxed{0}\boxed{0}$

- Now we perform Block-wise Convolution

1, 1, 1, 0

Here $h(n) = \{1, \cancel{2}, \cancel{3}, 0\}$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$$

$$\therefore y_2(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 7 \\ 4 \end{bmatrix}$$

$$\therefore y_3(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ 11 \\ 6 \end{bmatrix}$$

$$\therefore y_4(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \\ 15 \\ 8 \end{bmatrix}$$

→ **Finally** we perform Overlap-Add to obtain $y(n)$

$\therefore y_1(n) =$ | 1 | 3 | 3 | 2 |

+

$\therefore y_2(n) =$ | 3 | 7 | 7 | 4 |

+

$\therefore y_3(n) =$ | 5 | 11 | 11 | 6 |

+

$\therefore y_4(n) = \{$ | 7 | 15 | 15 | 8 |

$\therefore y(n) = \{1, 3, 6, 9, 12, 15, 18, 21, 15, 8\}$

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$x(n) = \{7, 6, 4, 5, 2, 4, 5, 2, 3\}$
$h(n) = \{1, 2, 3, 1\}$
Fast-Overlap-Save Method

Solution

$\cancel{M = 3}$ | $M = 4$
$\therefore L = 2$ | $L = 3$
$\therefore N = L + M - 1$ | $N = L + M - 1$
$= 2 + 3 - 1$ | $= 4 + 3 - 1$
$=$ | $= 6$

- First we obtain ~~Overlap-Discard~~ input blocks

$x_1(n) = \boxed{0 \mid 0 \mid 0 \mid 7 \mid 6 \mid 4}$

$x_2(n) = \boxed{7 \mid 6 \mid 4 \mid 5 \mid 2 \mid 4}$

$x_3(n) = \boxed{5 \mid 2 \mid 4 \mid 5 \mid 2 \mid 3}$

$x_4(n) = \boxed{5 \mid 2 \mid 3 \mid 0 \mid 0 \mid 0}$

We need to make h(n) Length = 6

$\therefore h(n) = \{1, 2, 3, 1, 0, 0\}$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 7 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 33 \\ 18 \\ 4 \\ 7 \\ 20 \\ 37 \end{bmatrix}$$

$$\therefore y_2(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 6 \\ 4 \\ 5 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 26 \\ 39 \\ 41 \\ 38 \\ 30 \\ 27 \end{bmatrix}$$

$$\therefore y_3(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 4 \\ 5 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 22 \\ 23 \\ 26 \\ 24 \\ 26 \\ 26 \end{bmatrix}$$

$$\therefore y_4(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 22 \\ 17 \\ 11 \\ 3 \end{bmatrix}$$

→ Finally we obtain $y(n)$ by Overlap-Save Method

$\therefore y_1(n) = \boxed{33 \mid 18 \mid 4 \mid 7 \mid 20 \mid 37}$

$\therefore y_2(n) = \boxed{26 \mid 39 \mid 41 \mid 38 \mid 30 \mid 27}$

$\therefore y_3(n) = \boxed{22 \mid 23 \mid 26 \mid 24 \mid 26 \mid 26}$

$\therefore y_4(n) = \boxed{5 \mid 12 \mid 22 \mid 17 \mid 11 \mid 3}$

$\therefore y(n) = \{7, 20, 37, 38, 30, 27, 24, 26, 26, 17, 11, 3\}$

Cross Check at Exam.

	7	6	4	5	2	4	5	2	3
1	7	6	4	5	2	4	5	2	3
2	14	12	8	10	4	8	10	4	6
3	21	18	12	15	6	12	15	6	9
1	7	6	4	5	2	4	5	2	3

$\therefore y(n) = \{7, 20, 37, 38, 30, 27, 24, 26, 26, 17, 11, 3\}$