

3. DSI Project

EG Weightage 20 Marks - NUM+Thes

★ ★ ★ → Linear FIR Filtering Using Overlap Save & Overlap Add Method

- Real Time Signals are long
- We can not wait till entire signal is available
- Convolution is required to be performed on-line
- Hence Overlapping Approach is used
- Overlap Save \Leftrightarrow Overlap Discard
- Overlap Add
- Linear Convolution is performed using Circular Convolution

How to decide Block Size

For Linear Convolution

$$L = L_1 + L_2 - 1$$

$$N = L + M - 1$$

Block Size Length of $x(n)$ Length of $h(n)$

Example Perform Linear Convolution using.

① Overlap Save Method

② Overlap Add Method

$$x(n) = \{3, 4, 1, 2\}$$

$$h(n) = \{1, 2, 1\}$$

Solution - We select Block Size $\hat{=} N = 4$
 Here length of $h(n) = M = 3$

Given
 Calculate
 Find

$$N = \text{Block Length} \hat{=} N = L + M - 1$$

$$\text{Actual data of } x(n) \quad 4 = L + 3 - 1$$

$$\text{Length of } h(n) \quad \therefore L = 2$$

1	2	3	4
3	1	0	1
2	1	1	0
1	0	1	0

① Overlap-Save Method

We divide $x(n)$ into multiple Overlapping blocks:

1	2	3	4
3	1	0	1
2	1	1	0
1	0	1	0

Since there is No-overlapping for first block

We perform Zero-Padding

$$\therefore x_1(n) = [0 \ 0 \ 3 \ 4]$$

$$x_2(n) = [3 \ 4 \ 1 \ 2]$$

$$x_3(n) = [1 \ 2 \ 0 \ 0]$$

Now, we perform Linear Convolution using Circular Convolution

$$\therefore h(n) = \{1, 2, 1, 0\}$$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 4 \\ 3 \\ 10 \end{bmatrix}$$

$$\therefore y_1(n) = \{11, 4, 3, 10\}$$

$$\begin{bmatrix} 1 & 0 & 11 & 2 \\ 2 & 1 & 0 & 8 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 12 \\ 8 \end{bmatrix}$$

$$\therefore y_2(n) = \{8, 12, 12, 8\}$$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}$$

$$\therefore y_3(n) = \{1, 4, 5, 2\}$$

Final Answer is obtained by performing Overlap-Save (Overlap-Delay)

$$\boxed{11 \quad 4 \quad 3 \quad 10}$$

$$\boxed{8 \quad 12 \quad 12 \quad 8}$$

$$\boxed{1 \quad 4 \quad 5 \quad 2}$$

$$\therefore y(n) = \{11, 4, 11, 22, 13, 12, 5, 2\}$$

$$\therefore y(n) = \{-3, 10, 12, 8, 5, 20\}$$

Gross check

		3	4	1	1	2	
2							
8	1	3	4	1	8	2	
2		6	8	2	4		
1	3	4	1	2	..		

$$\therefore y(n) = \{3, 10, 12, 8, 5, 2\}$$

② Overlap-Add Method

We divide $x_1(n)$ into multiple overlapping blocks.

$$\therefore x_1(n) = [3 \quad 4 \quad 0 \quad 0]$$

$$\therefore x_2(n) = [1 \quad 2 \quad 0 \quad 0]$$

Now we perform Linear Convolution using Circular Convolution

$$h(n) = \{1, 2, 1, 0\}$$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 11 \\ 4 \end{bmatrix}$$

$$\therefore y_1(n) = \{3, 10, 11, 4\}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 1 \\ 2 & 1 & 0 & 1 & 2 \\ 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{array} \right] \left[\begin{array}{c} 1 \\ 2 \\ 0 \\ 0 \end{array} \right] = \left[\begin{array}{c} 1 \\ 4 \\ 5 \\ 2 \end{array} \right]$$

$$\therefore y_2(n) = \{1, 4, 5, 2\}$$

~~$$\left[\begin{array}{cccc|c} 2 & 0 & 3 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{c} 2 \\ 2 \\ 1 \\ 0 \end{array} \right]$$~~

obtains the $(n) \times M$ matrix \mathbf{w}

Overlap - Add Operation

$$\left[\begin{array}{cccc|c} 3 & 10 & 11 & 8 & 4 \end{array} \right] = (n) \times 5$$

+

$$\left[\begin{array}{cccc|c} 1 & 4 & 5 & 2 \end{array} \right]$$

$$\text{matrix } y(n) = \{3, 10, 12, 8, 5, 2\}$$

DEC 15 ~~matrix \mathbf{w} contains overlap~~ ~~overlap~~ over

EG $\mathbf{w} \times (n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 10M $b(n) = \{1, 1, 1\}$

- ① Overlap Save Method
- ② Overlap Add Method

Solution: We select block size $= N = 4$
 Here length of $b(n) = M = 3$

$$N = L + M - 1$$

$$4 = L + 3 - 1$$

$$\therefore L = 2$$

① Overlap-Add Method

$$x_1(n) = [1 \ 2 \ 0 \ 0]$$

$$x_2(n) = [3 \ 4 \ 0 \ 0]$$

$$x_3(n) = [5 \ 6 \ 0 \ 0]$$

$$x_4(n) = [7 \ 8 \ 0 \ 0]$$

Example:- Let $x(n) = \{2, 1, -1, 2, 1, 13\}$ and $h(n) = \{1, 2, 3\}$

Find Response of the system by Overlap-Add & Overlap-Save Method

Solution:- We know, $L = L_1 + L_2 - 1$

$$\text{Consider } N = L + M - 1$$

Here the length of $h(n) = M = 3$

$$\text{Hence } N = 3 + 1 = 2 \cdot M$$

We select $L = 2 = \text{Length of Current Sample}$

$$\text{Block size} = N = L + M - 1$$

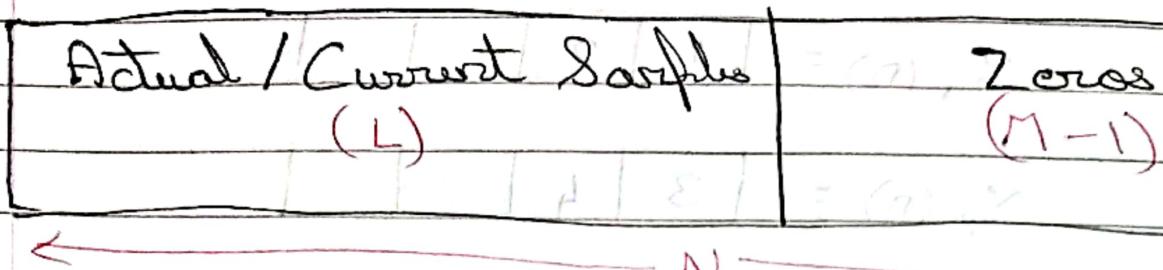
$$= 2 + 3 - 1$$

$$= 4$$

For overlap-save Method..

Prev-Overlap / Zero $(M-1)$	Actual / Current Sample (L)
--------------------------------	--------------------------------

For Overlap-Add Method



I Overlap-Save or Overlap-Discard

- We first obtain input Blocks

$$x_1(n) = \boxed{0 \ 0 \ 2 \ 1} = (n)d$$

$$x_2(n) = \boxed{0 \ 1 \ 1 \ 2} = (n)d$$

$$x_3(n) = \boxed{-1 \ 2 \ 1 \ 1}$$

$$x_4(n) = \boxed{1 \ 1 \ 0 \ 0}$$

- Now we perform Block-Wise Convolution
Here $b(n) = \{1, 2, 3, 0\}$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 2 \\ 5 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ 7 \\ 3 \end{bmatrix}$$

$$y_3(n) = \begin{bmatrix} 1 & 0 & 3 & 6 & 9 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 9 \end{bmatrix}$$

$$y_4(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

- Finally, we perform the overlap-save to obtain $y(n)$

$$y_1(n) = \boxed{8 \ 3} \boxed{2 \ 5}$$

$$y_2(n) = \boxed{3 \ 11 \ 7 \ 3}$$

$$y_3(n) = \boxed{4 \ 3} \boxed{2 \ 9}$$

$$y_4(n) = \boxed{1 \ 3 \ 5 \ 3}$$

$$y(n) = \{2, 5, 7, 3, 2, 9, 5, 3\}$$

(Cross check at exam)

$$\begin{array}{r|rrrrrr} & 1 & 2 & 1 & -1 & 2 & 1 & 1 \\ \hline 1 & 1 & 2 & 1 & -1 & 2 & 1 & 1 \\ 2 & 2 & 4 & 2 & -2 & 4 & 2 & 2 \\ 3 & 6 & 3 & -3 & 6 & 3 & 3 \end{array}$$

$$y(n) = \{2, 5, 7, 3, 2, 9, 5, 3\}$$

(II) Overlap-Add Method

- We first obtain Input Blocks

$$\mathbf{x}_1(n) = \boxed{2 \ 1 \ 0 \ 0}$$

$$\mathbf{x}_2(n) = \boxed{8 \ 0 \ -1 \ 2 \ 0 \ 0}$$

$$\mathbf{x}_3(n) = \boxed{1 \ 3 \ 8 \ 1 \ 1 \ 0 \ 0}$$

- Now we perform Block-wise Convolution

$$\text{Here } b(n) = \{$$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \\ 3 \end{bmatrix}$$

$$\therefore y_2(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 6 \end{bmatrix}$$

$$\therefore y_3(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

$$\{8, 2, 5, 8, 1, F, 2, 5, 2\} = (a)$$

- Finally, we perform Overlap-Add to obtain $y(n)$

$$y_1(n) = \boxed{2 \ 5 \ 8 \ 3}$$

+

$$y_2(n) = \boxed{-1 \ 0 \ 1 \ 6}$$

+

$$y_3(n) = \boxed{1 \ 3 \ 5 \ 3}$$

$$y(n) = \{2, 5, 7, 3, 2, 9, 5, 3\}$$

DEC 15 $X(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(b) 10M $h(n) = \{1, 1, 1\}$

Perform Overlap-Save & Overlap-Add

Solution:-

$$L = L_1 + L_2 + 1 \text{ (with)} \dots$$

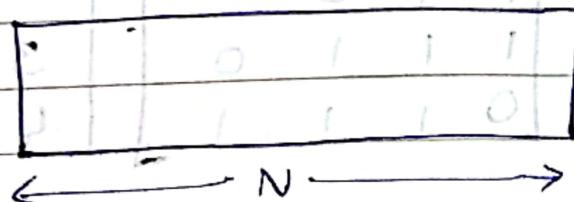
$$N = L + M - 1$$

$$\text{Given: } M = 3 \quad | \quad M - 1 = 3 - 1 = 2$$

$$\text{Choose } L = M - 1 = 2$$

$$N = 2 + 3 - 1$$

$$\begin{array}{cccc|cc} 2 & 1 & 1 & 1 & 4 & 0 & 1 \\ 5 & 0 & 1 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 1 & 0 & 0 & 0 \end{array}$$



$$\text{Overlap} = M - 1 = 2$$

$$\text{Actual} = L$$

$$\begin{array}{c|ccc|cc} n & 1 & 2 & 3 & 4 & 5 \\ \hline s_1 & 2 & 0 & 1 & 1 & 1 \\ s_2 & 1 & 1 & 0 & 1 & 0 \end{array}$$

I Overlap - Save or Overlap - Add

~~1. If all input blocks overlap by less than T~~
 - we first obtain Input Blocks.

$$x_1(n) = [0 \ 0 \ 1 \ 2 \ 3 \ 4] = (a)_g$$

$$x_2(n) = [3 \ 1 \ 0 \ 2 \ 3 \ 4] = (a)_g$$

$$x_3(n) = [2 \ 0 \ 1 \ 1 \ 3 \ 4 \ 5 \ 6] = (a)_g$$

$$x_4(n) = [2 \ P \ S \ E \ F \ 2 \ 5 \ 6 \ 7 \ 8] = (a)_g$$

$$x_5(n) = [2 \ A \ E \ S \ 1 \ 3 \ 7 \ 8 \ 0 \ 0] = (a)_g$$

~~2. Now we perform Block-wise convolution.~~

$$\therefore \text{Here } h(n) = \{1, 1, 1, 0\}$$

$$1 - M + 1 = 4$$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$$1 - E + S = 4$$

$$\therefore y_2(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 6 \\ 9 \end{bmatrix}$$

$$\therefore y_3(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 12 \\ 15 \end{bmatrix}$$

$$\therefore y_4(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 20 \\ 19 \\ 18 \\ 21 \end{bmatrix}$$

$$\therefore y_5(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \\ 15 \\ 8 \end{bmatrix}$$

- Finally we perform Overlap-Save to obtain $y(n)$

$$\therefore y_1(n) = \boxed{3 \quad 2 \quad 1 \quad 3}$$

$$\therefore y_2(n) = \boxed{8 \quad 7 \quad 6 \quad 9}$$

$$\therefore y_3(n) = \boxed{14 \quad 13 \quad 12 \quad 15}$$

$$\therefore y_4(n) = \boxed{20 \quad 19 \quad 18 \quad 21}$$

$$\therefore y_5(n) = \boxed{7 \quad 15 \quad 15 \quad 8}$$

$$\therefore y(n) = \{1, 3, 6, 9, 12, 15, 18, 21, 15, 8\}$$

IV. Overlap - Add Method

- We first obtain Input Blocks

$$\therefore x_1(n) = \boxed{1 \quad 2 \quad 0 \quad 0}$$

$$\therefore x_2(n) = \boxed{3 \quad 4 \quad 0 \quad 0}$$

$$\therefore X_3(n) = \boxed{5 \ 6 \ 10 \ 00 \ 1} = (a)_M$$

$$\therefore X_4(n) = \boxed{0 \ 17 \ 8 \ 00 \ 1} = (a)_M$$

- Now we perform Block-wise:

Convolution

Here $h(n) = \{1, 2, 1, 0\}$

$$\therefore y_1(n) = \boxed{\begin{array}{cccc} 0 & 3 & 2 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \\ 0 & 3 & 2 \end{array}} \text{ on match -}$$

$$\therefore y_2(n) = \boxed{\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}} \boxed{1} = \boxed{1}$$

$$\therefore y_3(n) = \boxed{\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}} \boxed{3} = \boxed{3}$$

$$\therefore y_4(n) = \boxed{\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}} \boxed{5} = \boxed{5}$$

$$1 \ 1 \ 1 \ 5 \ 1 = (a)_M$$

$$1 \ 1 \ 1 \ 5 \ 1$$

$$= (a)_M$$

$$\therefore y_4(n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \\ 15 \\ 8 \end{bmatrix}$$

→ Finally, we perform Overlap-Add to obtain $y(n)$

$$\therefore y_1(n) = [1 \ 3 \ 3 \ 2]$$

$$\therefore y_2(n) = [3 \ 7 \ 7 \ 4]$$

$$\therefore y_3(n) = [8 \ 1 \ 3 \ 5 \ 11 \ 11 \ 6]$$

$$\therefore y_4(n) = [5 \ 0 \ 0 \ 1 \ 8 \ 7 \ 15 \ 15 \ 8]$$

$$\therefore y(n) = \{1, 3, 6, 8, 9, 12, 15, 18, 21, 15, 8, 3\}$$

~~$\text{May } 16 \quad x(n) = \{7, 6, 1, 15, 2, 4, 5, 2, 3\}$~~

~~$s(B) = 10M8 \quad h(n) = \{1, 2, 3, 1\}$~~

Fast-Overlap-Save Method

~~Solutions: $M=3 \quad L=2 \quad M=4$~~

~~$\therefore N=L+M-1 \quad L=3$~~

~~$\therefore N=L+M-1 \quad N=L+M-1$~~

~~$= 2+3-1 \quad = 4+3-1$~~

~~$= 6$~~

- First we obtain Overlap-Discard input blocks

$$x_1(n) = [0 \ 0 \ 0 \ 7 \ 6 \ 4]$$

$$x_2(n) = [7 \ 6 \ 4 \ 5 \ 2 \ 4]$$

$$x_3(n) = [5 \ 2 \ 4 \ 5 \ 2 \ 8]$$

$$x_4(n) = [5 \ 2 \ 3 \ 0 \ 0 \ 0]$$

$$[8 \ 8 \ 8 \ 1] = (a)_M$$

We need to make $h(n)$ Length = 6

$$\therefore h(n) = \{1, 2, 3, 1, 0, 0\} = (a)_M$$

$$\therefore y_1(n) = [1 \ 2 \ 0 \ 0 \ 1 \ 3 \ 2] \begin{array}{|c|} \hline 0 \\ \hline \end{array} = [33]$$

$$+ [2 \ 1 \ 0 \ 0 \ 1 \ 3 \ 0] \begin{array}{|c|} \hline 0 \\ \hline \end{array} = [18]$$

$$[8 \ 21 \ 21 \ 21 \ 21 \ 21 \ 21] = [0 \ 2 \ 1 \ 0 \ 2 \ 1 \ 0] \begin{array}{|c|} \hline 0 \\ \hline \end{array} = [4]$$

$$+ [1 \ 3 \ 2 \ 1 \ 0 \ 0 \ 7] \begin{array}{|c|} \hline 7 \\ \hline \end{array} = [7]$$

$$[8 \ 21 \ 18 \ 21 \ 18 \ 21 \ 21] = [0 \ 1 \ 2 \ 1 \ 3 \ 0 \ 6] \begin{array}{|c|} \hline 6 \\ \hline \end{array} = [20]$$

$$+ [0 \ 0 \ 1 \ 3 \ 2 \ 1 \ 4] \begin{array}{|c|} \hline 4 \\ \hline \end{array} = [37]$$

$$\therefore y_2(n) = [1 \ 0 \ 2 \ 0 \ 3 \ 1] \begin{array}{|c|} \hline 3 \times 2 \\ \hline \end{array} = [26]$$

$$[2 \ 1 \ 8 \ 0 \ 3 \ 0] \begin{array}{|c|} \hline 3 \\ \hline \end{array} = [39]$$

$$[3 \ 2 \ 1 \ 0 \ 0 \ 1] \begin{array}{|c|} \hline 4 \\ \hline \end{array} = [41]$$

$$[1 \ 3 \ 2 \ 1 \ 0 \ 0] \begin{array}{|c|} \hline 5 \\ \hline \end{array} = [38]$$

$$[0 \ 1 \ 3 \ 2 \ 1 \ 0] \begin{array}{|c|} \hline 2 \\ \hline \end{array} = [30]$$

$$[0 \ 0 \ 2 \ 1 \ 3 \ 2 \ 1] \begin{array}{|c|} \hline 4 \\ \hline \end{array} = [27]$$

$$1 - M + S = n \quad 1 - M + S = 19$$

$$1 - M + S = [1 \ 0 \ 2 \ 0 \ 3 \ 2] \begin{array}{|c|} \hline 5 \\ \hline \end{array} = [22]$$

$$[2 \ 1 \ 0 \ 0 \ 1 \ 3] \begin{array}{|c|} \hline 2 \\ \hline \end{array} = [23]$$

$$[3 \ 2 \ 1 \ 0 \ 0 \ 1] \begin{array}{|c|} \hline 4 \\ \hline \end{array} = [26]$$

$$[1 \ 3 \ 2 \ 1 \ 0 \ 0] \begin{array}{|c|} \hline 5 \\ \hline \end{array} = [24]$$

$$[0 \ 1 \ 3 \ 2 \ 1 \ 0] \begin{array}{|c|} \hline 2 \\ \hline \end{array} = [26]$$

$$[0 \ 0 \ 1 \ 3 \ 2 \ 1] \begin{array}{|c|} \hline 3 \\ \hline \end{array} = [26]$$

$$\therefore y_4(n) = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 22 \\ 17 \\ 11 \\ 3 \end{bmatrix}$$

→ Finally we obtain $y(n)$ by Overlap-Save Method

$$\therefore y_1(n) = \boxed{33 \ 18 \ 4 \ 7 \ 20 \ 37}$$

$$\therefore y_2(n) = \boxed{26 \ 39 \ 41 \ 38 \ 30 \ 27}$$

$$\therefore y_3(n) = \boxed{22 \ 23 \ 26 \ 24 \ 26 \ 26}$$

$$\therefore y_4(n) = \boxed{5 \ 12 \ 22 \ 17 \ 11 \ 3}$$

$$\therefore y(n) = \{7, 20, 37, 38, 30, 27, 24, 26, 26, 17, 11, 3\}$$

Cross Check at Exam.

	7	6	4	5	2	4	5	2	3
1	7	6	4	5	2	4	5	2	3
2	14	12	8	10	4	8	10	4	6
3	21	18	12	15	6	12	15	6	9
1	7	6	4	5	2	4	5	2	3

$$\therefore y(n) = \{7, 20, 37, 38, 30, 27, 24, 26, 17, 11, 3\}$$