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Assignment 1

Q.2.1

$T(x[n]) = g[n]x[n]$; $g[n]$ given

Stable

Causal: depends on present values only
∴ causal linear

$x_1[n] \rightarrow \boxed{} \rightarrow T(x_1[n]) = g[n]x_1[n]$

$x_2[n] \rightarrow \boxed{} \rightarrow T(x_2[n]) = g[n]$

$T(x_1[n] + x_2[n]) = g[n](x_1[n] + x_2[n])$

$= g[n] \cdot x_1[n] + g[n] \cdot x_2[n]$

$= T(x_1[n]) + T(x_2[n])$

∴ linear

Time Invariant

$x[n] \rightarrow \boxed{} \rightarrow T(x[n]) = g[n] \cdot x[n]$

delayed $x[n-m] \rightarrow \boxed{} \rightarrow T'(x[m]) =$

$g[n]x[n-m]$

$\neq y[n-m] = g[n-m]x[n-m]$

∴ Not time invariant

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Memoryless - depends on n^{th} value of x, so memoryless

(b) $T(x[n]) = \sum_{k=n_0}^{n} x[k]$

Stable:

Causal:
depends on future value $\therefore$ Not causal

Linear:
$$x_1[n] \rightarrow \boxed{} \rightarrow T(x_1[n]) = \sum_{k=n_0}^{n} x_1[k]$$

$$x_2[n] \rightarrow \boxed{} \rightarrow T(x_2[n]) = \sum_{k=n_0}^{n} x_2[k]$$

$$T(x_1[n] + x_2[n]) = \sum_{k=n_0}^{n} x_1[k] + \sum_{k=n_0}^{n} x_2[k]$$

$\therefore$ Linear

Time Invariant

$$x[n] \rightarrow \boxed{} \rightarrow T(x[n]) = \sum_{k=n_0}^{n} [k]$$

delay by m,

$$x[n-m] \rightarrow \boxed{} \rightarrow T'(x[n]) = \sum_{k=n_0}^{n} x[k-m] = \sum_{k=0}^{n-n_0} x[k]$$

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$\therefore$ Not time invariant $\neq y[n-n_0] = \sum_{k=n_0}^{n-n_0} x[k]$

Memoryless $\because$ depends on past $\therefore$ Not memoryless

(c) $T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]$

Stable:

Causal: depends on future value.
$\therefore$ Not causal

Linear: $x_1[k] \rightarrow \boxed{} \rightarrow T'(x_1[k]) \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$x_2[k] \rightarrow \boxed{} \rightarrow T(x_2[k]) = \sum_{k=n-n_0}^{n+n_0} x_2[k]$

$(x_1[k] + x_2[k]) \rightarrow \boxed{} \rightarrow T(x_1[k] + x_2[k]) =$

$\sum_{k=n-n_0}^{n+n_0} x_1[k] + \sum_{k=n-n_0}^{n+n_0} x_2[k]$

$\therefore$ & Linear

Time Invariant
$x[n] \rightarrow \boxed{} \rightarrow T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]$

delay $x[n-n_0] \rightarrow \boxed{} \rightarrow T'(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k-n_0]$

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$$= \sum_{k=n-n_0}^{n} x[k] = y[n-n_0]$$

$\therefore$ Time Invariant

Memory less $\therefore$ depend on future $\therefore$ Not memoryless

(d) $T(x[n]) = x[n-n_0]$

Stable:

Causal: if ~~ny~~ $n_0 \geq 0$ causal.
else not causal

Linear: $x_1[n] \rightarrow \boxed{} \rightarrow T(x_1[n]) = x_1[n-n_0]$

$x_2[n] \rightarrow \boxed{} \rightarrow T(x_2[n]) = x_2[n-n_0]$

$x_1[n] + x_2[n] \rightarrow \boxed{} \rightarrow T(x_1[n] + x_2[n])$

$= x_1[n-n_0] + x_2[n-n_0]$

$\therefore$ Linear

Time Invariant:

$x[n-m] \rightarrow \boxed{} \rightarrow T'(x[n]) = x[n-n_0-m]$

$= y[n-m]$ or $T(x[n-m])$

$\therefore$ Time Invariant

Memoryless $\therefore$ depend on past value $\therefore$ not memoryless

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(e) $T(x[n]) = e^{x[n]}$

Stable:

Causal: depend on present ∴ Causal

Linear: $x_1[n] \rightarrow \boxed{} \rightarrow T(x_1[n]) = e^{x_1[n]}$

$x_2[n] \rightarrow \boxed{} \rightarrow T(x_2[n]) = e^{x_2[n]}$

$T(x_1[n] + x_2[n]) = e^{x_1[n] + x_2[n]}$
$\neq T(x_1[n]) + T(x_2[n])$

∴ Not linear

Time Invariant:

$x[n-m] \rightarrow \boxed{} \rightarrow T'(x[n]) = e^{x[n-m]}$
$= T(x[n-m])$

∴ Time Invariant

Memoryless ∴ depends only on nth value of x ∴ memoryless

(f) $T(x[n]) = ax[n] + b$
Stable:

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Causal: doesn't depend on ~~futt~~ future value
∴ Causal

Linear: $x_1[n] \rightarrow \boxed{} \rightarrow T(x_1[n]) = a x_1[n] + b$

$x_2[n] \rightarrow \boxed{} \rightarrow T(x_2[n]) = a x_2[n] + b$

$T(x_1[n] + x_2[n]) = a(x_1[n] + x_2[n]) + b$

$\neq T(x_1[n]) + T(x_2[n])$

∴ Not linear

Time Invariant:

$x[n-m] \rightarrow \boxed{} \rightarrow T'(x[n]) = a x[n-m] + b$

$= T(x[n-m])$

∴ Time Invariant

Memoryless: depends on nth value of x

∴ Memoryless

(g) $T(x[n]) = x[-n]$

Stable:

Causal: $n<0$, depends on future value ∴ Not causal

Linear: $T(x_1[n]) = x_1[-n]$

$T(x_2[n]) = x_2[-n]$

$T(x_1[n] + x_2[n]) = x_1[-n] + x_2[-n]$

$= T(x_1[n]) + T(x_2[n])$

$\therefore$ Linear

Time Invariant:

$$x[n-m] \rightarrow \boxed{} \rightarrow T'(x[n]) = x[-n-m]$$

$$T(x[n-m]) = x[-(n-m)]$$
$$= x[-n+m]$$

$\therefore$ Not time invariant

Memoryless :- depends on other values than that of nth

$\therefore$ Not memoryless

(h) $T(x[n]) = x[n] + U[n+1]$

Stable:

Causal: doesn't depend on future of $n+1$

$\therefore$ Causal

Linear:

$$x_1[n] \rightarrow \boxed{} \rightarrow T(x_1[n]) = x_1[n] + U[n+1]$$
$$x_2[n] \rightarrow \boxed{} \rightarrow T(x_2[n]) = x_2[n] + U[n+1]$$

$$T(x_1[n] + x_2[n]) = x_1[n] + x_2[n] + U[n+1]$$
$$\neq T(x_1[n]) + T(x_2[n])$$

$\therefore$ Not Linear

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Time Invariant:

$x[n-m] \rightarrow \boxed{} \rightarrow T(x[n]) = x[n-m] + U[n+1]$

$y[n-m] = x[n-m] + U[n-m+1]$

$\therefore$ Not time Invariant

Memoryless : depends on n^{th} value of x only
memoryless

Q.2.2
(a) $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$

$h[k] \neq 0$ for $N_0 \leq k \leq N_1$

$y[n] = \sum_{k=N_0}^{N_1} h[k]\, x[n-k]$

$x[n] \neq 0$ for $N_2 \leq n \leq N_3$

$\therefore x[n-k] \neq 0$ for $N_2 \leq (n-k) \leq N_3$

min. value of $(n-k)$ is N_2

$\therefore$ lower bound on n, occurs for $k = N_0$ is
$N_4 = N_0 + N_2$

Similarly, $N_5 = N_1 + N_3$

o/p is non-zero for, $(N_0 + N_2) \leq n \leq (N_1 + N_3)$

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(b) $x[n]$ is non-zero
for $n_0 \leq n \leq (n_0+N-1)$

& $h[n]$ for $n_1 \leq n \leq (n_1+M-1)$

Similar to part A.

$$(n_0+n_1) \leq n \leq (n_0+n_1+M+N-2)$$

Q.2.3 $h[n] = a^{-n} U[-n]$ $\quad 0 < a < 1$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Step response result,

$$x[n] = U[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{\infty} a^{-k}\, U[-k]\, U[n-k]$$

For $n \leq 0$

$$y[n] = \sum_{k=-\infty}^{\infty} a^{-k} = \sum_{k=-n}^{\infty} a^{k}$$

$$= \frac{a^{-n}}{1-a}$$

For $n > 0$,

$$y[n] = \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$$

Q2.14 (a) $x[n] = (1/3)^n$, $y[n] = 2(1/3)^n$

System can be LTI but cannot be uniquely determined from info in the i/p - o/p ~~cont~~ constraint since it satisfies eigenfunction property

(b) $x[n] = (1/2)^n$, $y[n] = (1/4)^n$
System can't be LTI
For LTI, output in form $A(1/2)^n$.

(c) $x[n] = (2/3)^n U[n]$, $y[n] = 4(2/3)^n - 3(1/2)^n U[n]$

System can be LTI & there is only one LTI system that satisfies the i/p - o/p constraint

System can be found using DTFT

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$\frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

$$h[n] = \left(\frac{1}{2}\right)^n U[n]$$

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2.27

a $\left(\frac{1}{2}\right)^n \longrightarrow \left(\frac{1}{4}\right)^n$

$\left(\frac{1}{2}\right)^n$ is an eigen function of LTI.

Response would be of the form $A\left(\frac{1}{2}\right)^n$

O/p is $\left(\frac{1}{4}\right)^n$ ∴ Not LTI

b $e^{j\pi/8} U[n] \longrightarrow 2e^{j\pi/8} U[n]$

FT of $x[n]$ is ∞

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} e^{j\pi/8} U[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} e^{-j(\omega - \pi/8)n}$$

$$= \frac{1}{1 - e^{-j(\omega - \pi/8)n}}$$

∴ $y[n] = 2x[n]$

$$Y(e^{j\omega}) = \frac{2}{1 - e^{-j(\omega - \pi/8)}}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

∴ System is LTI & unique

(c) $e^{j\pi/8} \longrightarrow 2e^{j\pi/8}$

$x[n] = e^{j\pi/8}$

expected o/p, $y[n] = \gamma e^{j\pi/8}$, for system to be LTI

given o/p, $y[n] = 2e^{j\pi/8}$

$\therefore$ LTI

Not unique, since only constraint is

$$H(e^{j\omega})\Big|_{\omega=1/8} = 2$$

Q.2.30

(a) $T(x[n]) = (\cos \pi n)\, x[n]$

$\cos(\pi n)$ takes value $+1$ or -1

$\therefore T(x[n]) = (-1)^n x[n]$

Stable: Since magnitude doesn't change
$\therefore$ Stable

Linear: $y_1[n] = T(x_1[n]) = (-1)^n x_1[n]$
$y_2[n] = T(x_2[n]) = (-1)^n x_2[n]$

$T(x_1[n] + x_2[n]) = (-1)^n (x_1[n] + x_2[n])$
$= y_1[n] + y_2[n]$

$\therefore$ Linear

Causal ∴ depends on current value ∴ Causal

Time Invariant:

$$y_1'[n] = (-1)^n x[n-m]$$

$$y[n-m] = (-1)^{n-m} x[n-m]$$

$$y[n-m] = (-1)^{n-m} x[n-m]$$

∴ Not time invariant

(b) $T(x[n]) = x[n^2]$

Stable: $x[n]$ is bounded, $x[n^2]$ also bounded
∴ Stable

Causal: depend on future value ∴ Causal

Linear: $y_1[n] = T(x_1[n]) = x_1[n^2]$
$y_2[n] = T(x_2[n]) = x_2[n^2]$

$$T(x_1[n] + x_2[n]) = x_1[n^2] + x_2[n^2]$$
$$= y_1[n] + y_2[n]$$

∴ Linear

Time Invariant:

$$y'[n] = T'(x[n]) = x[n^2 - m]$$
$$y[n-m] = x[(n-m)^2]$$

$\therefore$ Not time invariant

(c) $T(x[n]) = x[n] \sum_{k=0}^{\infty} \delta[n-k]$

$$\sum_{k=0}^{\infty} \delta[n-k] = U[n]$$

$$T(x[n]) = x[n]\, U[n]$$

Stable:

Causal: depend on present $\therefore$ Causal

Linear: $y_1[n] = x_1[n]\, U[n]$
$y_2[n] = x_2[n]\, U[n]$

$$T(x_1[n] + x_2[n]) = (x_1[n] + x_2[n])\, U[n]$$
$$= y_1[n] + y_2[n]$$

$\therefore$ Linear

Time invariant:

delay by m, $y'[n] = x[n-m]\, U[n]$

$$y[n-m] = x[n-m]\, U[n-m]$$

$\therefore$ Not time invariant

(d) $T(x[n]) = \sum_{k=n-1}^{\infty} x[k]$

Stable: $T(x[n]) = \infty$
∴ Not stable

Causal: depends on future
∴ Not causal

Linear:
$$T(x_1[n] + x_2[n]) = \sum_{k=n-1}^{\infty} (x_1[k] + x_2[k])$$
$$= T(x_1[n]) + T(x_2[n])$$

∴ Linear

Time Invariant:
$$T'(x[n]) = \sum_{k=n-1}^{\infty} x[k]$$

$$T(x[n-m]) = \sum_{k=n-m-1}^{\infty} x[k]$$

∴ Time Invariant

Q.2.35
(a) $x_1[n] = x_2[n] + x_3[n+4]$

$$T(x_1[n]) = T(x_2[n]) + T(x_3[n+4])$$

$$\neq y_2[n] + y_3[n+4]$$
∴ Not Linear

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(b) Response,

$$\cancel{\delta[n]} \quad \delta[n] = x_3[n+4]$$

$$T(\delta[n]) = y_3[n+4]$$
$$= 3\delta[n+6] + 2\delta[n+5]$$

(c) System is time invariant & not linear.
we can't use choice such as:

$$\delta[n] = x_1[n] - x_2[n]$$

&

$$\delta[n] = \frac{1}{2} x_2[n+1]$$

∴ to determine impulse response

Q. 2.59 $x[n] = y[n] - ay[n-1]$ & $y[0] = 1$

(a) Time Invariant
For, $x_1[n] = \delta[n]$, $y_1[0] = 1$

$$y_1[1] = ay[0] = a$$

For, $x_2[n] = \delta[n-1]$, delay by 1
$y_2[0] = 1$

$$y_2[1] = a \quad y_2[0] + x_2[1]$$

$$y_2[1] = a + 1$$

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$x_2[n] = x_1[n-1]$, $y_2[n] \neq y_2[n-1]$

$\therefore$ Not time invariant

(b) Linear

If i/p is ~~doubled~~ doubled o/p should also be doubled at each n.

but $y[0] = 1$ always $\therefore$ Not Linear

Let $x_3 = \alpha x_1[n] + \beta x_2[n]$

(c) $y[0] = 0$

$n \geq 0$

$y_3[n] = x_3[n] + a y_3[n-1]$

$= \alpha x_1[n] + \beta x_2[n] + a(x_3[n-1] + y_3[n-2])$

$= \alpha \sum_{k=0}^{n-1} a^k x_1[n-k] + \beta \sum_{k=0}^{n-1} a^k x_2[n-k]$

$= \alpha(h[n] - x_1[n]) + \beta(h[n] + x_2[n])$.

$= \alpha y_1[n] + \beta y_2[n]$

$n < 0$

$y_3[n] = \alpha^{-1}(y_3[n+1] - x_3[n])$

$= -\alpha \sum_{k=1}^{n} a^k x_1[n-k] - \beta \sum_{k=1}^{n} a^k x_2[n-k]$

$= a y_1[n] + \beta y_2[n]$

$n = 0$

$$y_3[n] = y_1[n] = y_2[n] = 0$$

$$\therefore y_3[n] = \alpha y_1[n] + \beta y_2[n] \text{ for all } n$$

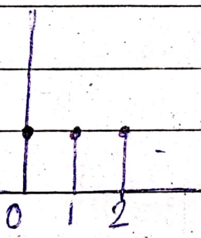
$\therefore$ Linear but not time invariant

Q2.30 $u[n] = u[n] - u[n-6]$

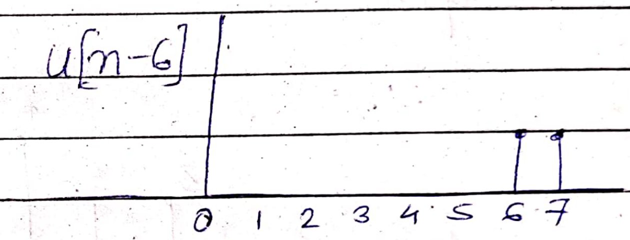
$w[n] = \delta[n] + 2\delta[n-2] + \delta[n-4]$

$q[n] = u[n] * w[n]$

$u[n]$

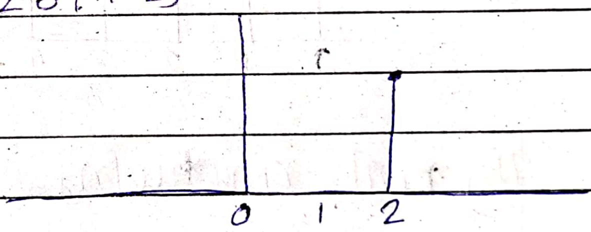


$u[n-6]$



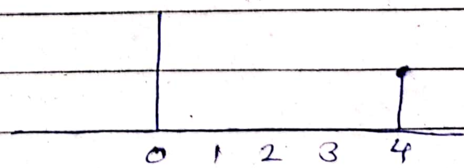
$\delta[n]$

$2\delta[n-2]$



$$u[n] = \{\underset{0}{1}, \underset{1}{1}, \underset{2}{1}, \underset{3}{1}, \underset{4}{1}, \underset{5}{1}\}$$

$\delta[n-4]$

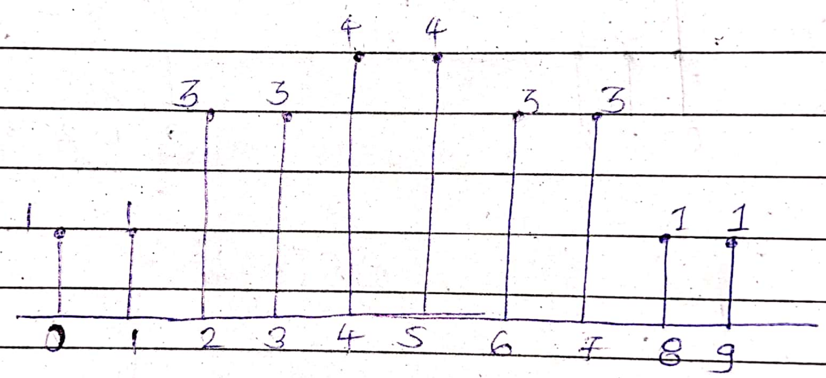


$$w[n] = \{\underset{0}{1}, \underset{1}{0}, \underset{2}{2}, \underset{3}{0}, \underset{4}{1}\}$$

	1	1	1	1	1	1
1	1	1	1	1	1	1
0	0	0	0	0	0	0
2	2	2	2	2	2	2
0	0	0	0	0	0	0
1	1	1	1	1	1	1

$$q[n] = \{\underset{0}{1}, \underset{1}{1}, \underset{2}{3}, \underset{3}{3}, \underset{4}{4}, \underset{5}{4}, \underset{6}{3}, \underset{7}{3}, \underset{8}{1}, \underset{9}{1}\}$$

$q[n]$



(b) $r[n] = r[n] * u[n] = \sum_{0}^{n-1} q[k]$

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(c) let $a[n] = v[-n]$
& $b[n] = w[-n]$

then:

$$a[n] * b[n] = \sum_{k=-\infty}^{\infty} a[k]\, b[n-k]$$

$$= \sum_{k=-\infty}^{\infty} v[-k]\, w[k-n]$$

$r = -k$

$$= \sum_{r=-\infty}^{\infty} \cancel{v[r} \; v[r]\, w[-n-r]$$

$$= q[-n]$$

$$q[-n] = v[-n] * w[-n]$$

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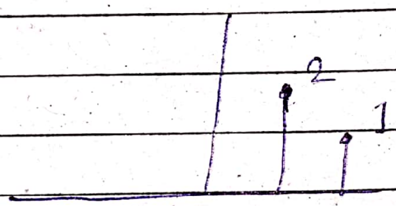
Q. 2.22

(a) $x[n] = \{0, 1\}$
$h[n] = \{2, 1\}$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

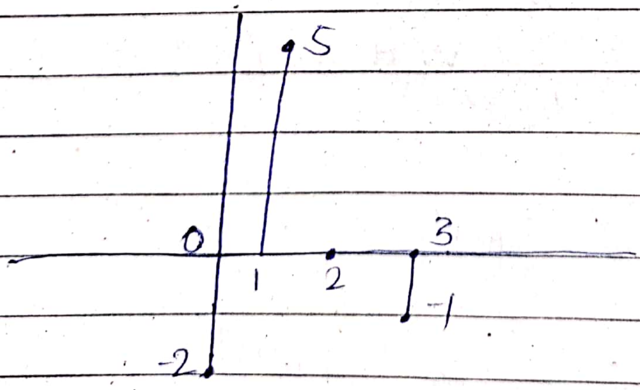
$$y[n] = \{0, 2, 1\}$$



(b) $x[n] = \{2, -1\}$
$h[n] = \{-1, 2, 1\}$

	2	-1
-1	-2	1
2	4	-2
1	2	-1

$y[n] = \{-2, 5, 0, -1\}$



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(c) $x[n] = \{1,1,1,1,1\}$

$h[n] = \{0,0,1,1,1,1,1,1,0,0,0,1,1,1,1,1,1\}$

	1	1	1	1
0	0	0	0	0
0	0	0	0	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$y[n] = \{0,0,1,2,3,4,4,4,3,2,1,1,2,3,4,4,4,3,2,1\}$

(d) $x[n] = \{1,2,1,1\}$
$\quad\quad -2 \; -1 \; 0 \; 1 \; 2$

$h[n] = \{0,0,1,-1,0,0,1,1\}$

(d)

	1	2	1	1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

$y[n] = \{0, 0, 1, 1, -1, 0, 0, 3, 3, 2, 1, 1\}$