→ Convolution / Response of a System by using DIT-FFT

Eg - 10 Marks - Numerical

$$\begin{tikzcd}
x(n) \arrow[d, "FFT"'] & * & h(n) \arrow[d, "FFT"'] & = & y(n) \\
X[k] & & H[k] & & Y[k] \arrow[u, "IFFT"']
\end{tikzcd}$$

Linear Convolution

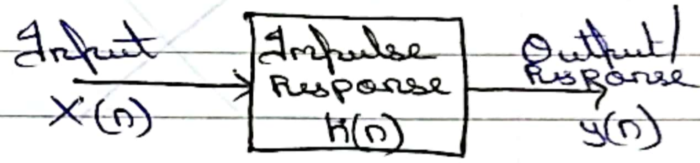
$$L = L_1 + L_2 - 1$$

Circular Convolution

$$L = L_1 = L_2$$

Selecting N-Point FFT

$$N \geq L$$



EG
10 Marks

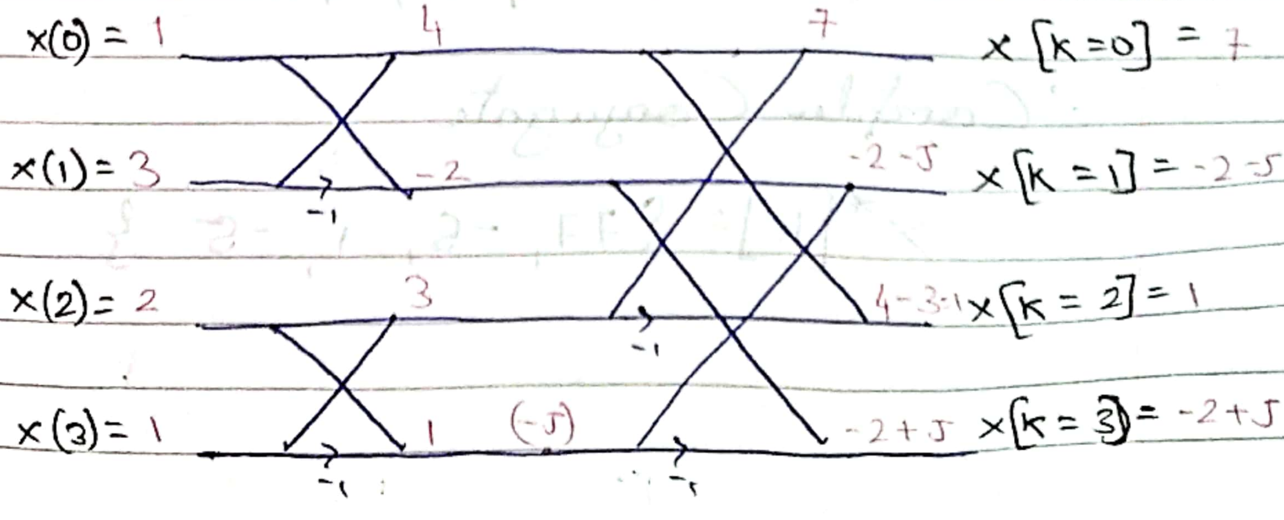
Determine Circular Convolution of Two sequence using $\boxed{\text{DIT-FFT}}$

$$x_1(n) = \{1, 2, 3, 1\}$$
$$x_2(n) = \{4, 3, 2, 2\}$$

Solution :- Hence $L = L_1 = L_2 = 4$
Hence we use 4-point FFT

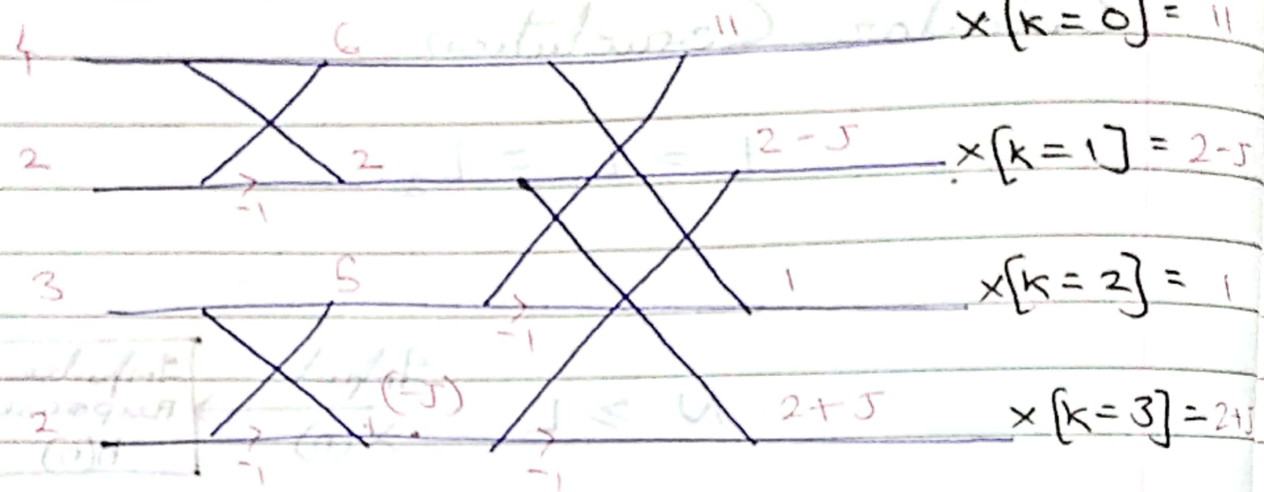
$\boxed{\text{STEP - I}}$ To obtain $X_1[k]$

$$X_1(n) = \{\underset{0}{1}, \underset{1}{2}, \underset{2}{3}, \underset{3}{1}\}$$



STEP - II To Obtain $X_2[k]$

$\therefore x_2(n) = \{ \underset{0}{4}, \underset{1}{3}, \underset{2}{2}, \underset{3}{2} \}$



STEP-III Perform Convolution

$$Y[k] = X_1[k] \cdot X_2[k]$$

$$= \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix} \cdot \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix} = \begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

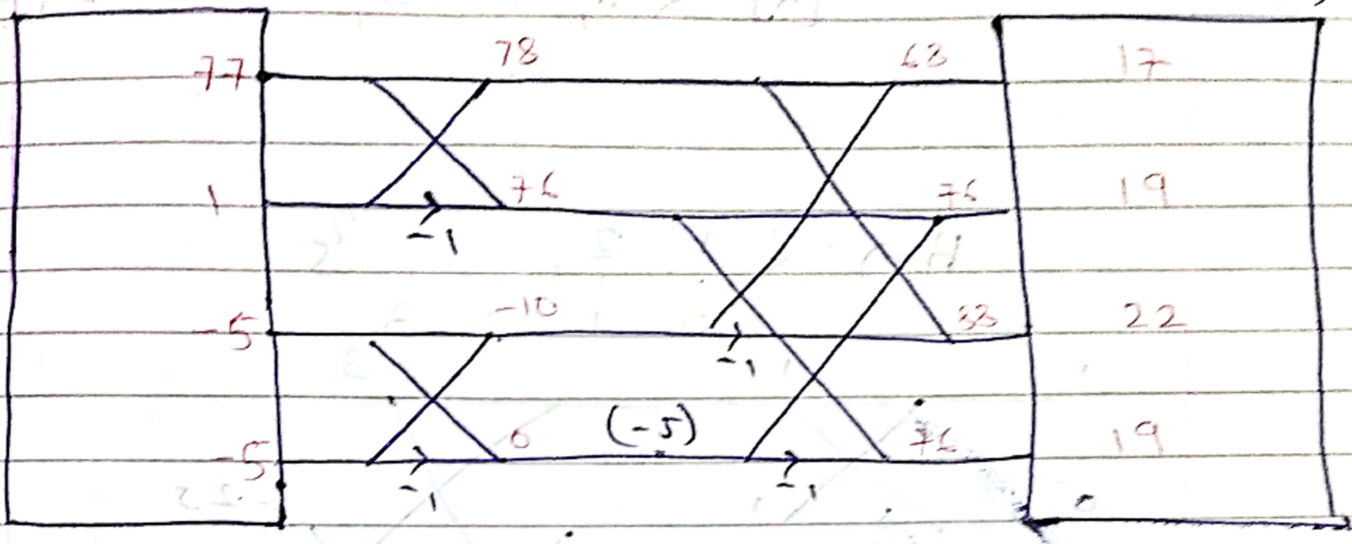
STEP - IV Obtain $y(n)$ using IFFT

$\therefore Y[k] = \{ 77, -5, 1, -5 \}$

$\therefore$ Complex Conjugate

$\therefore Y^*[k] = \{ 77, -5, 1, -5 \}$

Complex Conjugate   Multiply $\left(\frac{1}{N}\right)$



$\therefore y(n) = \{17, 19, 22, 19\}$

EQ
10 Marks

Given $h(n) = \{1, 2\}$. Find the response of the system to the input $x(n) = \{1, 2, 3\}$ using FFT & IFFT

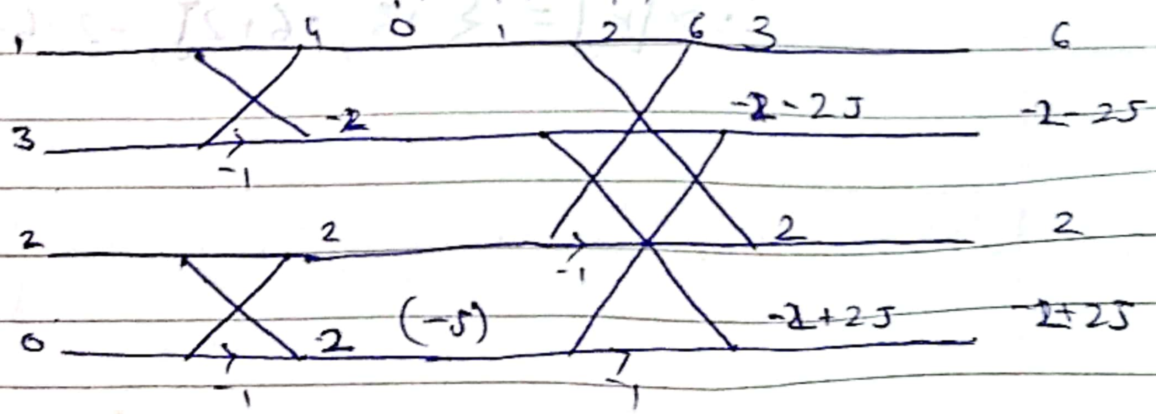
Solution:-

Here $L = L_1 + L_2 - 1$
$= 3 + 2 - 1$
$= 4$

Hence we use 4-point FFT

STEP-I To obtain $X[k]$

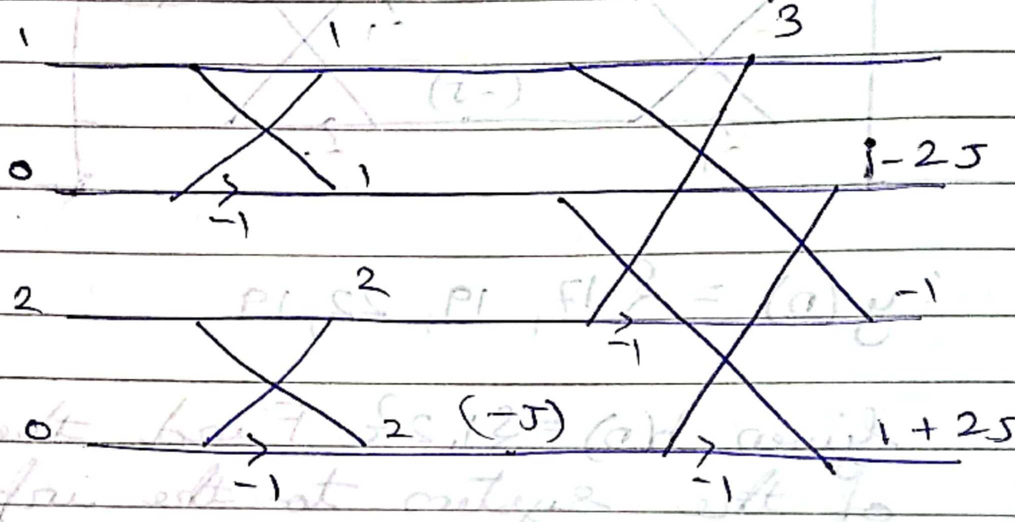
$x(n) = \{1, 2, 3, 0\}$



$\therefore X[k] = \{6, -2-2J, 2, -2+2J\}$

STEP-II To obtain H[k]

$$H(n) = \{\underset{0}{1}, \underset{1}{2}, \underset{2}{0}, \underset{3}{0}\}$$



$\therefore H[k] = \{3, 1-2J, -1, 1+2J\}$

STEP-III Perform Convolution

$$X[k] \cdot H[k] = Y[k]$$

$$\begin{bmatrix} 6 \\ -2-2J \\ 2 \\ -2+2J \end{bmatrix} \begin{bmatrix} 3 \\ 1-2J \\ -1 \\ 1+2J \end{bmatrix} = \begin{bmatrix} 18 \\ -6+2J \\ -2 \\ -6-2J \end{bmatrix}$$

$\therefore Y[k] = \{18, -6+2J, -2, -6-2J\}$

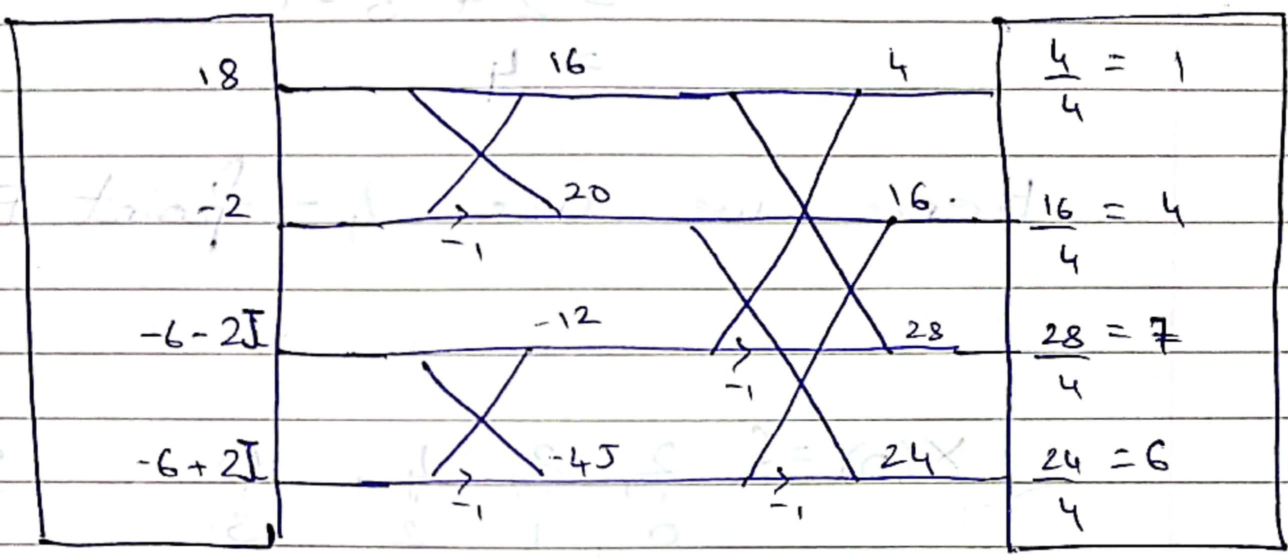
STEP-IV Obtain y(n) using IFFT

$\therefore X[K] = \{ 18, -6+2J, -2, -6-2J \}$

$\therefore$ Complex Conjugate

$\therefore X^*[K] = \{ 18, -6-2J, -2, -6+2J \}$

Complex Conjugate $\qquad$ Multiply $\left(\frac{1}{N}\right)$



$\therefore y(n) = \{ 1, 4, 7, 6 \}$

Cross Check

	1	2	3
1	1	2	3
2	2	4	6

$\therefore y(n) = \{ 1, 4, 7, 6 \}$

EQ 10.M Impulse Response of an FIR filter is given by $h(n) = \{1, 1\}$. Find the response of the system to the input $x(n) = \{2, 2, 4\}$ using FFT/IFFT.

Solution:-

Here $L = L_1 + L_2 - 1$

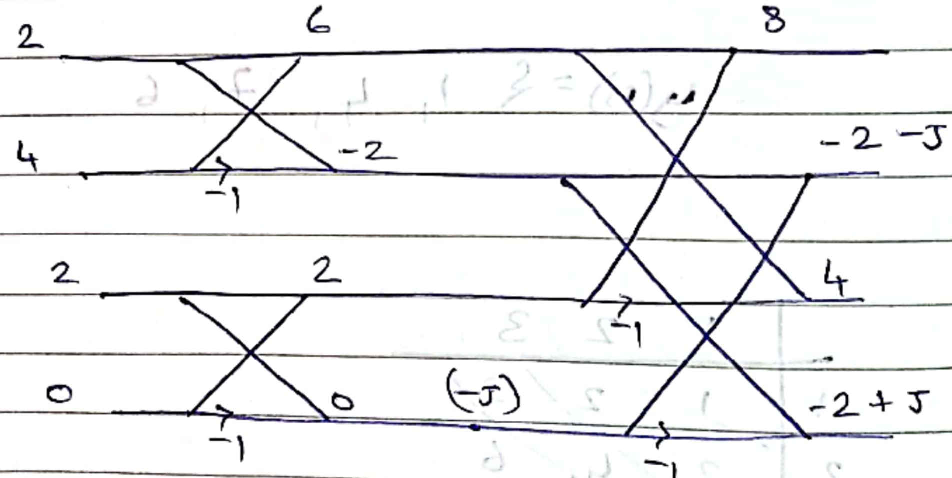
$= 2 + 3 - 1$

$= 4$

Hence we use 4-point FFT

$\boxed{\text{STEP-I}}$ To obtain $X[k]$

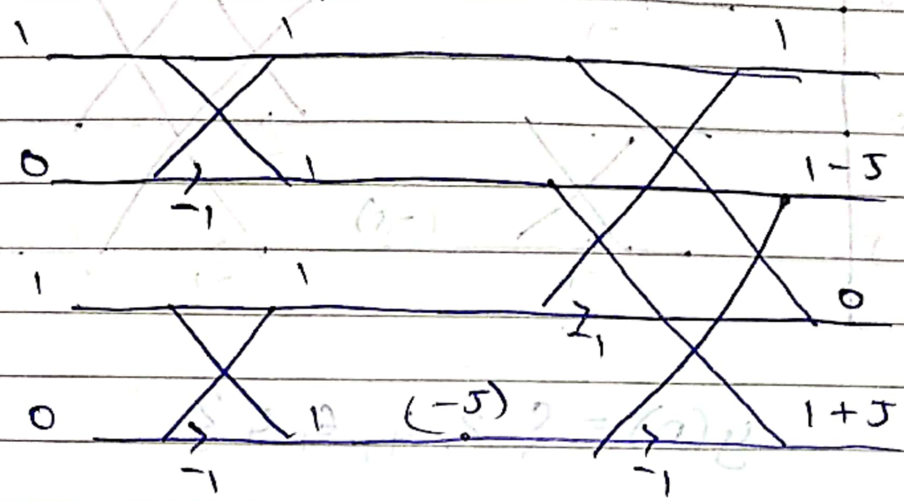
$$x(n) = \{\underset{0}{2}, \underset{1}{2}, \underset{2}{4}, \underset{3}{0}\}$$



$\therefore X[k] = \{8, -2-J, 4, -2+J\}$

STEP - II To obtain H[k]

$$h(n) = \{\underset{0}{1}, \underset{1}{1}, \underset{2}{0}, \underset{3}{0}\}$$



$$\therefore H[k] = \{1, 1-J, 0, 1+J\}$$

STEP - III Perform Convolution

$$Y[k] = X[k] \cdot H[k]$$

$$\begin{bmatrix} 1 \\ 1-J \\ 0 \\ 1+J \end{bmatrix} \begin{bmatrix} 8 \\ -2-J \\ 4 \\ -2+J \end{bmatrix} = \begin{bmatrix} 8 \\ -3+J \\ 0 \\ -3-J \end{bmatrix}$$

$$\therefore Y[k] = \{8, -3+J, 0, -3-J\}$$

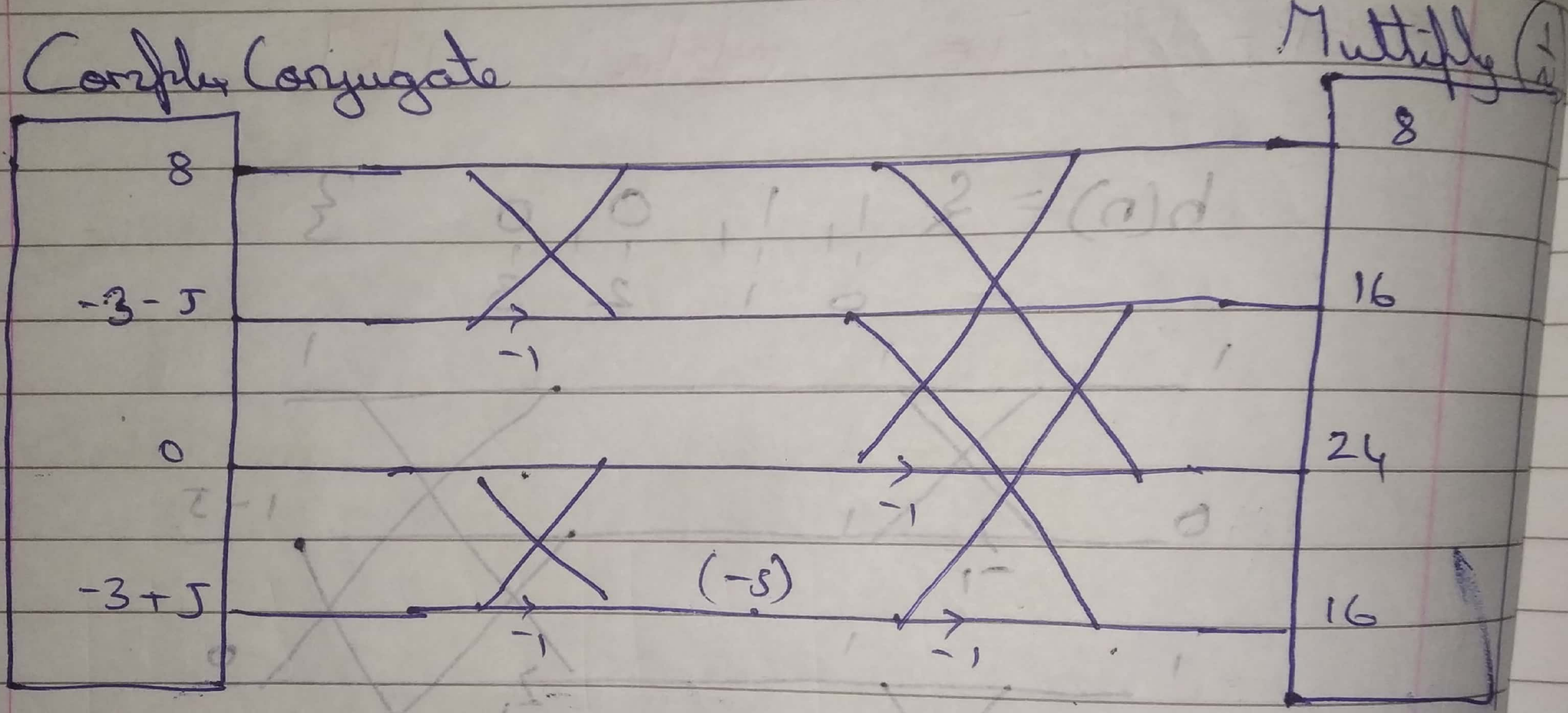
STEP - IV Obtain y(n) using IFFT

$$\therefore Y[k] = \{8, -3+J, 0, -3-J\}$$

Complex Conjugate

$$Y^*[k] = \{8, -3-J, 0, -3+J\}$$

Complex Conjugate



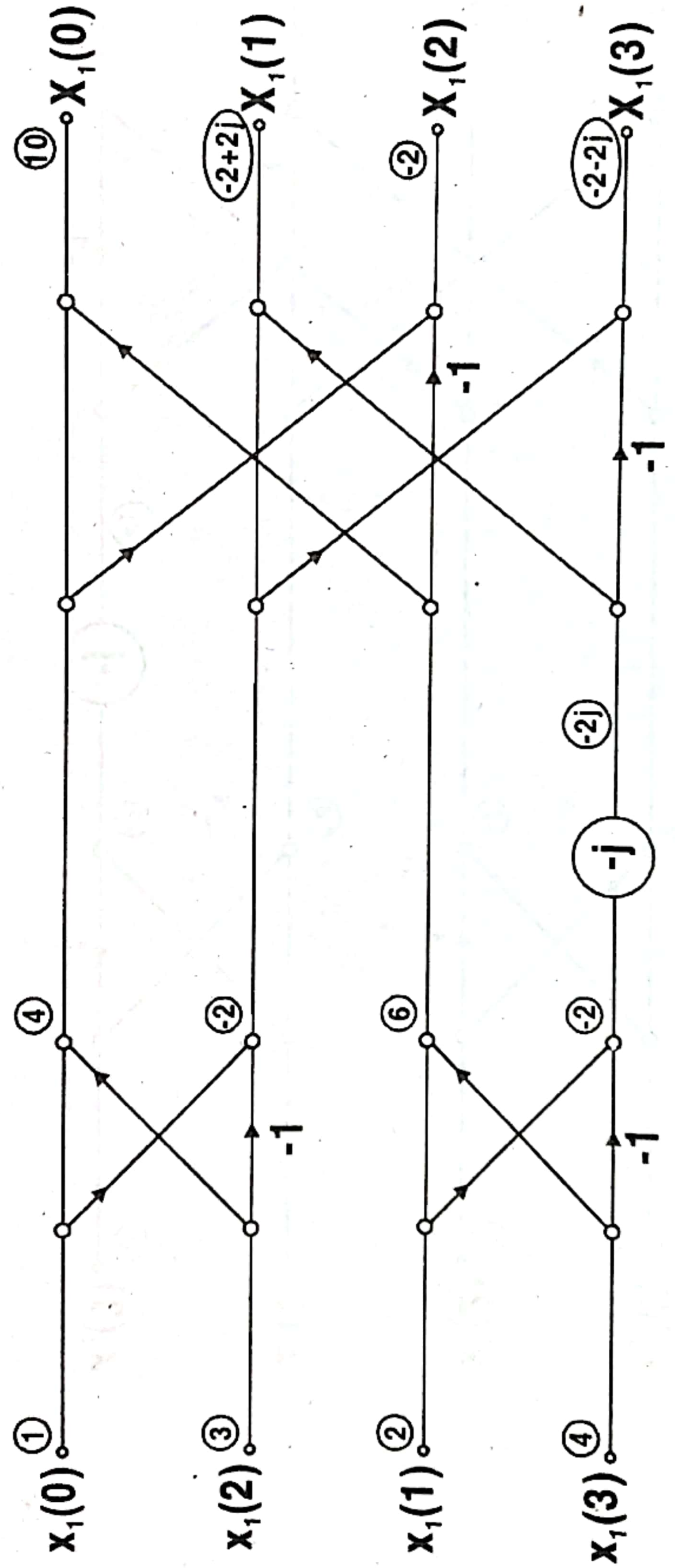
$y(n) = \{2, 4, 6, 4\}$

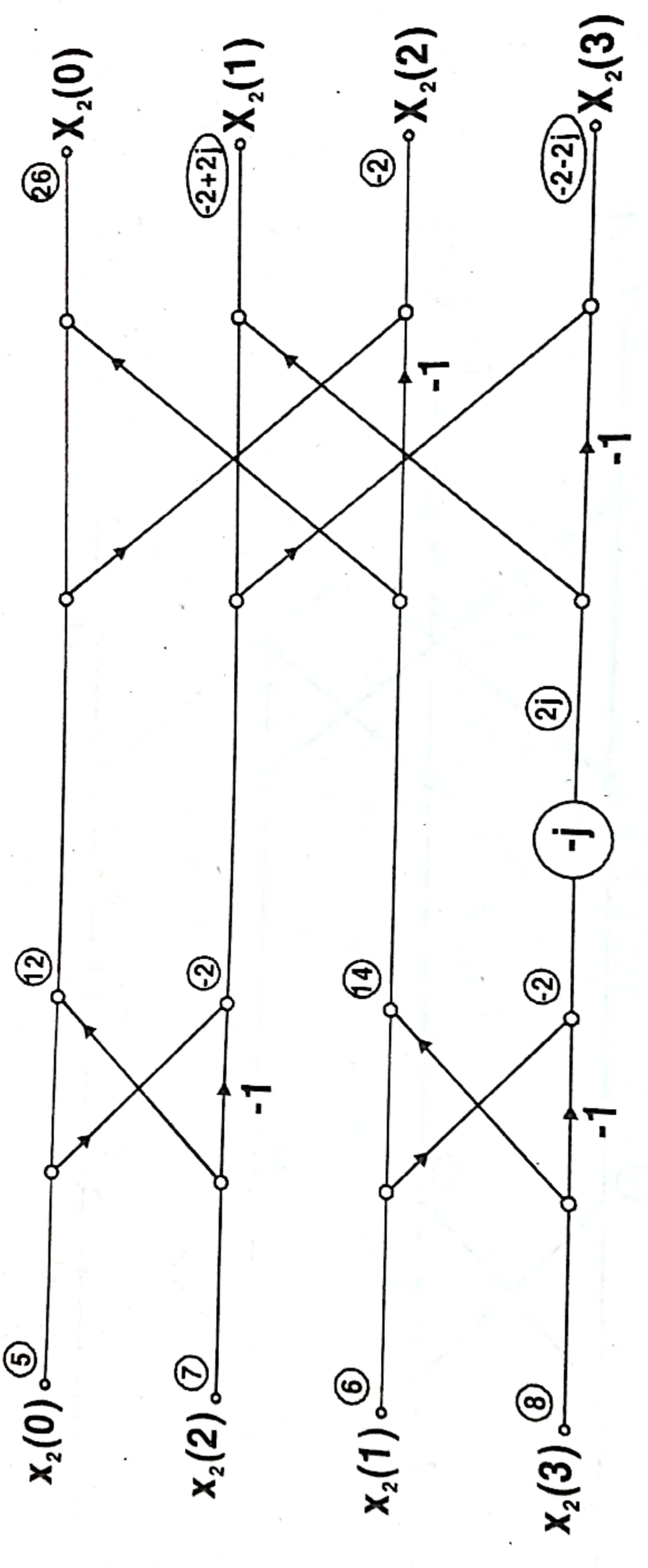
Example 4.23:

By means of DFT–IDFT or FFT–IFFT technique compute the circular convolution of the sequences.

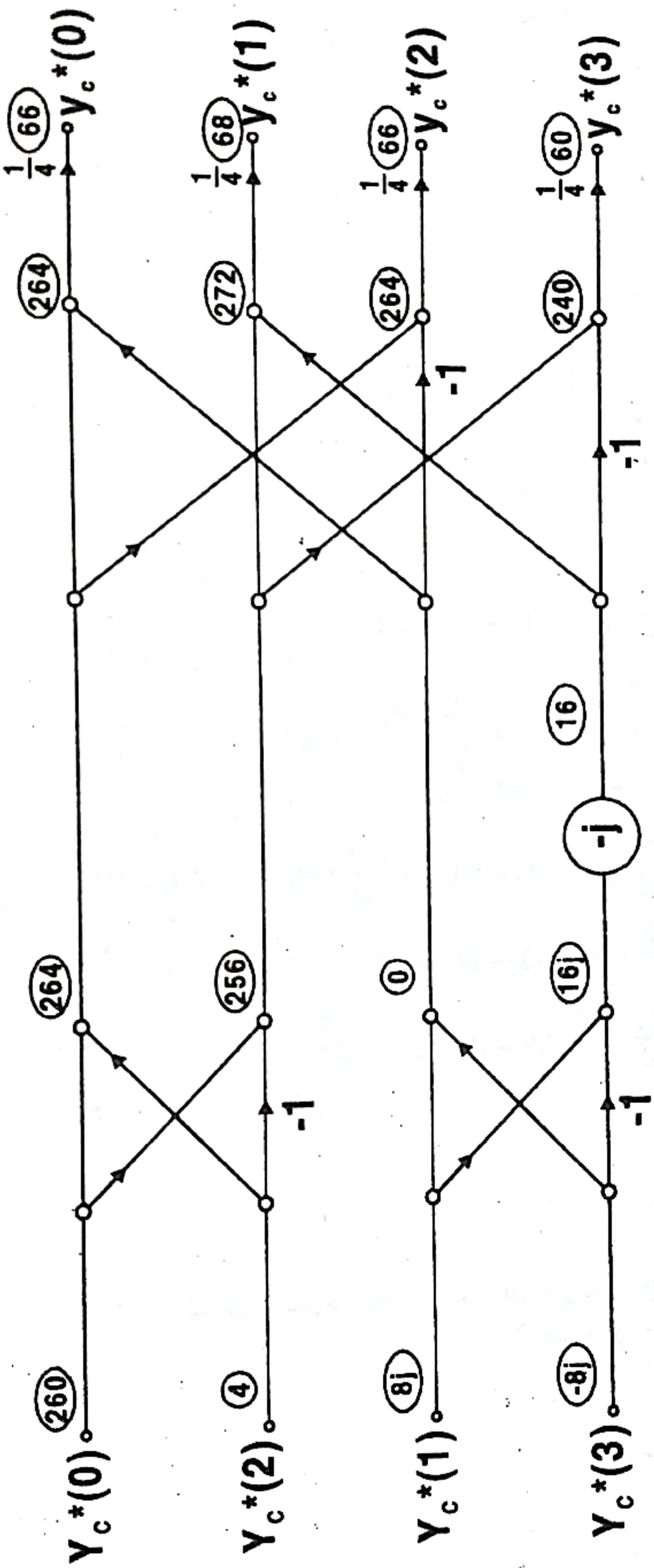
$$x_1(n) = \{1, 2, 3, 4\} \quad x_2(n) = \{5, 6, 7, 8\}$$

Solution:





$x_2(0)$ 5
$x_2(2)$ 7
$x_2(1)$ 6
$x_2(3)$ 8
12
-2
14
-2
-1
-1
-j
2j
-1
-1
26 $X_2(0)$
-2+2j $X_2(1)$
-2 $X_2(2)$
-2-2j $X_2(3)$


Y_c*(0)
Y_c*(2)
Y_c*(1)
Y_c*(3)
260
4
8j
-8j
264
256
0
16j
-1
-1
-j
16
-1
-1
264
272
264
240
1/4
66
68
66
60
y_c*(0)
y_c*(1)
y_c*(2)
y_c*(3)

Solution: Cross – Correlation

$$x_1(n) \circledcirc x_2(n) \longleftrightarrow X_1(k) \cdot X_2(-k)$$

i.e. $x_1(n) \circledast x_2(n)$

(1) $x_1(n) = \{1, 2, 3, 4\} \longleftrightarrow X_1(k) = \{10, -2 + 2j, -2, -2 -2j\}$

$x_2(n) = \{5, 6, 7, 8\} \longleftrightarrow X_2(k) = \{26, -2 + 2j, -2, -2 -2j\}$

$X_2(-k) = \{26, -2 -2j, -2, -2 +2j\}$

Let

$$R_{x_1x_2}(k) = X_1(k) X_2(-k)$$

$$= \{260, 8, 4, 8\}$$

Performing IDFT

$$r_{x_1x_2}(l) = \{70, 64, 62, 64\}$$

Solution: Auto – Correlation

$$x(n) \circledcirc x(n) \longleftrightarrow X(k) \cdot X(-k)$$

i.e. $x(n) \circledast x(-n)$

(1) $x_1(n) = \{1, 2, 3, 4\} \longleftrightarrow X_1(k) = \{10, -2+2j, -2, -2-2j\}$

$X_2(-k) = \{10, -2-2j, -2, -2+2j\}$

Let $Rx_1x_1(k) = X_1(k) \cdot X_1(-k)$

$= \{100, 8, 4, 8\}$

Performing IDFT

$r_{x_1x_1}(l) = \{30, 24, 22, 24\}$

(2) $x_2(n) = \{5, 6, 7, 8\} \longleftrightarrow X_2(k) = \{26, -2+2j, -2, -2-2j\}$

$X_2(-k) = \{26, -2-2j, -2, -2+2j\}$

$Rx_2x_2(k) = X_2(k) \cdot X_2(-k)$

$= \{676, 8, 4, 8\}$

Performing IDFT

$r_{x_2x_2}(l) = \{174, 168, 166, 168\}$

4.8 LINEAR CONVOLUTION/FILTERING USING DFT-IDFT OR FFT-IFFT TECHNIQUE

Zero pad the sequences so that their length becomes atleast $N_x + N_h - 1$ where N_x is the length of the first sequence and N_h is the length of the second sequence. $\hat{x}(n)$ and $\hat{h}(n)$ are the zero pad sequences.

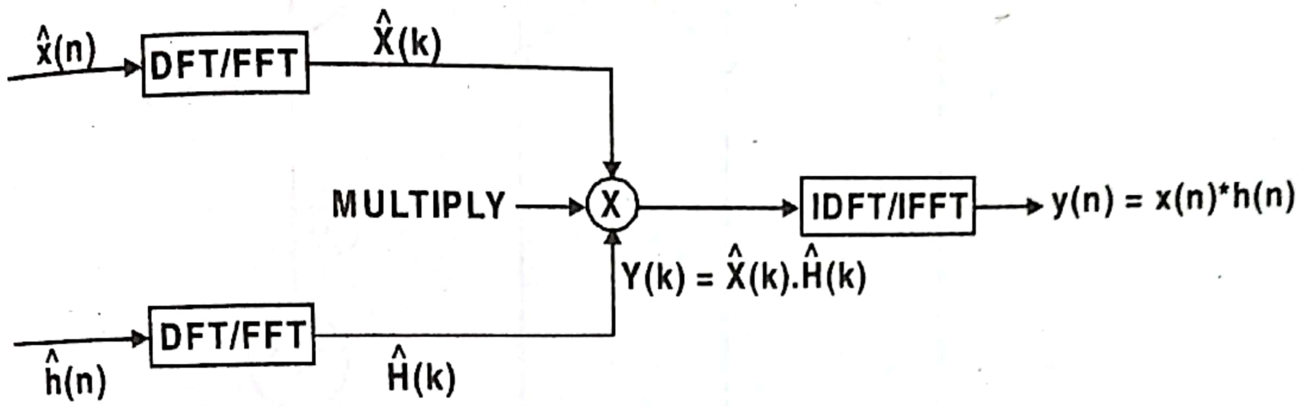


Fig. 4.49

Example 4.27:

By means of DFT–IDFT or FFT–IFFT technique compute linear convolution

$$x(n) = \{1, 2, 3\} \quad h(n) = \left\{1, \frac{1}{2}\right\}$$

Solution:

$$x(n) = \{1, 2, 3\}$$

$$N_x = 3$$

$$h(n) = \left\{1, \frac{1}{2}\right\}$$

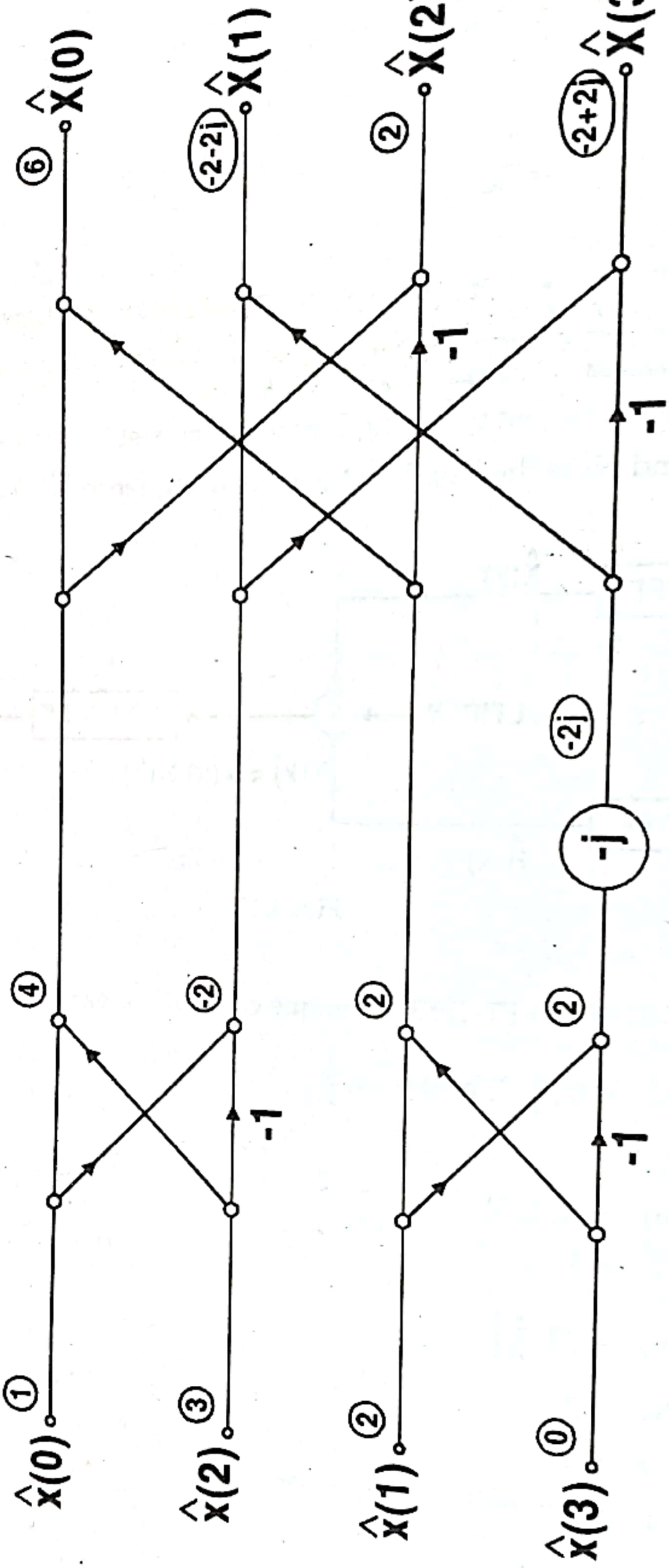
$$N_h = 2$$

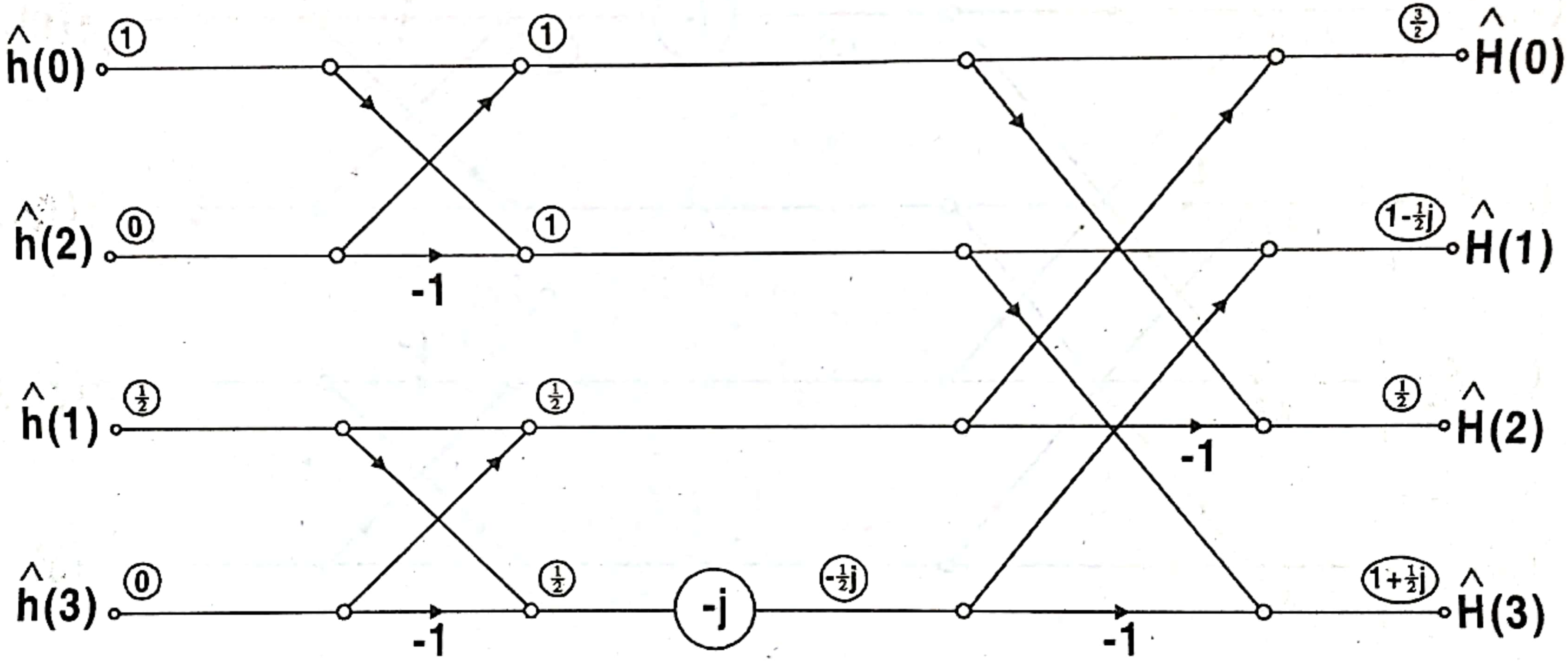
$$N_x + N_h - 1 = 4$$

$\therefore$ We pad trailing zeros to both the sequences so that their lengths become atleast 4.

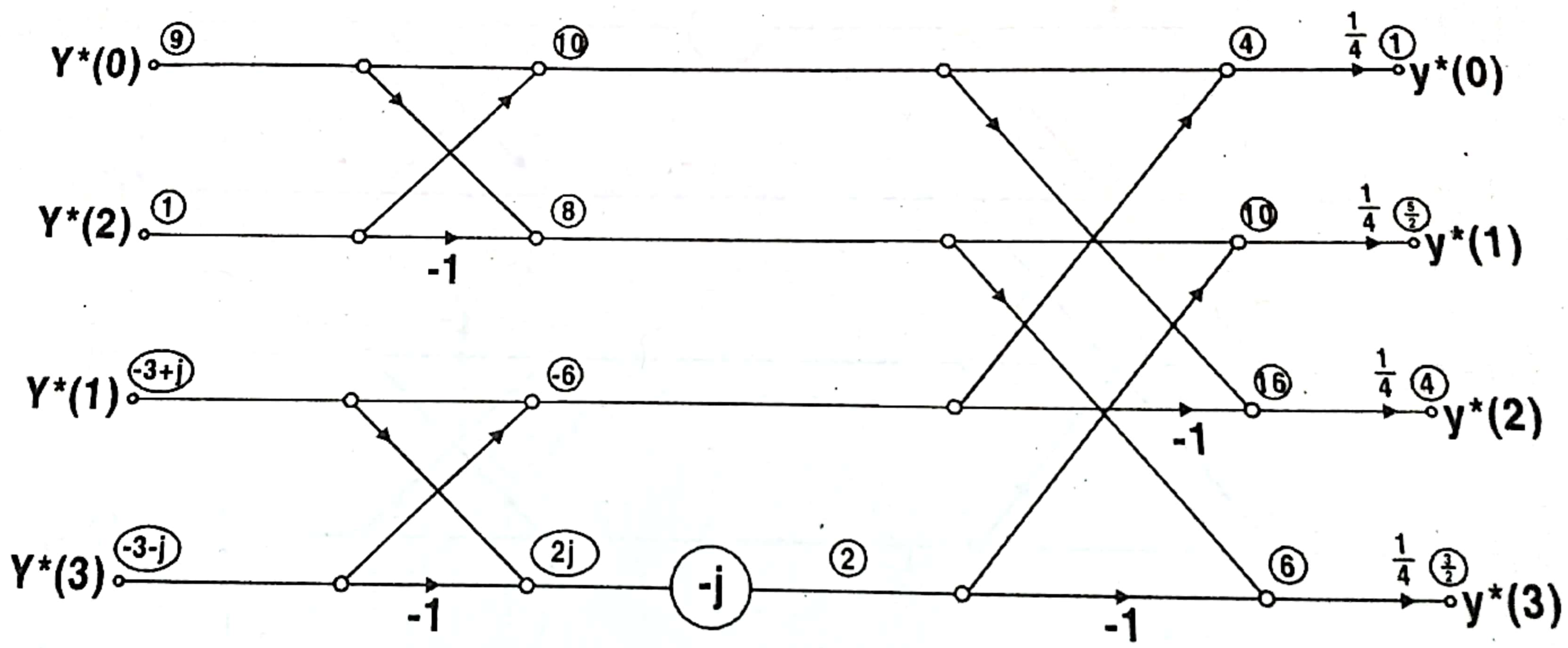
$$\hat{x}(n) = \{1, 2, 3, 0\}$$

$$\hat{h}(n) = \left\{1, \frac{1}{2}, 0, 0,\right\}$$


$\hat{x}(0)$
$\hat{x}(2)$
$\hat{x}(1)$
$\hat{x}(3)$
1
3
2
0
4
-2
2
2
-1
-1
-j
-2j
-1
-1
6
-2-2j
2
-2+2j
$\hat{X}(0)$
$\hat{X}(1)$
$\hat{X}(2)$
$\hat{X}(3)$



ĥ(0)
ĥ(2)
ĥ(1)
ĥ(3)
1
0
1/2
0
1
1
1/2
1/2
-1
-1
-j
-1/2 j
-1
-1
3/2
1-1/2 j
1/2
1+1/2 j
Ĥ(0)
Ĥ(1)
Ĥ(2)
Ĥ(3)


Y*(0)
Y*(2)
Y*(1)
Y*(3)
9
1
-3+j
-3-j
10
8
-6
2j
-1
-1
-j
2
4
10
16
6
-1
-1
1/4
1
5/2
4
3/2
y*(0)
y*(1)
y*(2)
y*(3)