

Assignment 5

Q1

INDIA

Size = 5 characters = 5 bytes = 40 bits

Symbol	Probability
I	0.4
N	0.2
D	0.2
A	0.2
*	

① Interval for 'I'

lower range = 0 upper range = 1

No. of sub-intervals = 5

Step value = $\frac{1-0}{5} = 0.2$

Interval for char 'I' is [0 to 0.4]

② Interval for 'N'

lower range = 0 upper range = 0.4

No. of sub-intervals = 5

Step value = $0.4 - 0 = 0.08$

Interval of for char 'N' is [0.16 to 0.24]

(3) Interval for 'D'

Lower range = 0.16 upper range = 0.24
No. of sub-intervals = 5

$$\text{Step-value} = \frac{0.24 - 0.16}{5} = \frac{0.08}{5} = 0.016$$

Interval for char 'D' is [0.208 to 0.224]

(4) 'T'

Range : [0.208 to 0.224]

$$\begin{aligned}\text{Step value} &= \frac{0.224 - 0.208}{5} \\ &= 0.0032\end{aligned}$$

Interval : [0.208 to 0.2144]

(5) 'A'

Range [0.208 to 0.2144]

$$\text{Step value} = 0.00128$$

Interval : [0.21312 to 0.2144]

0.21312 < Arithmetic Codeword < 0.2144

Arithmetic Codeword : 0.21313

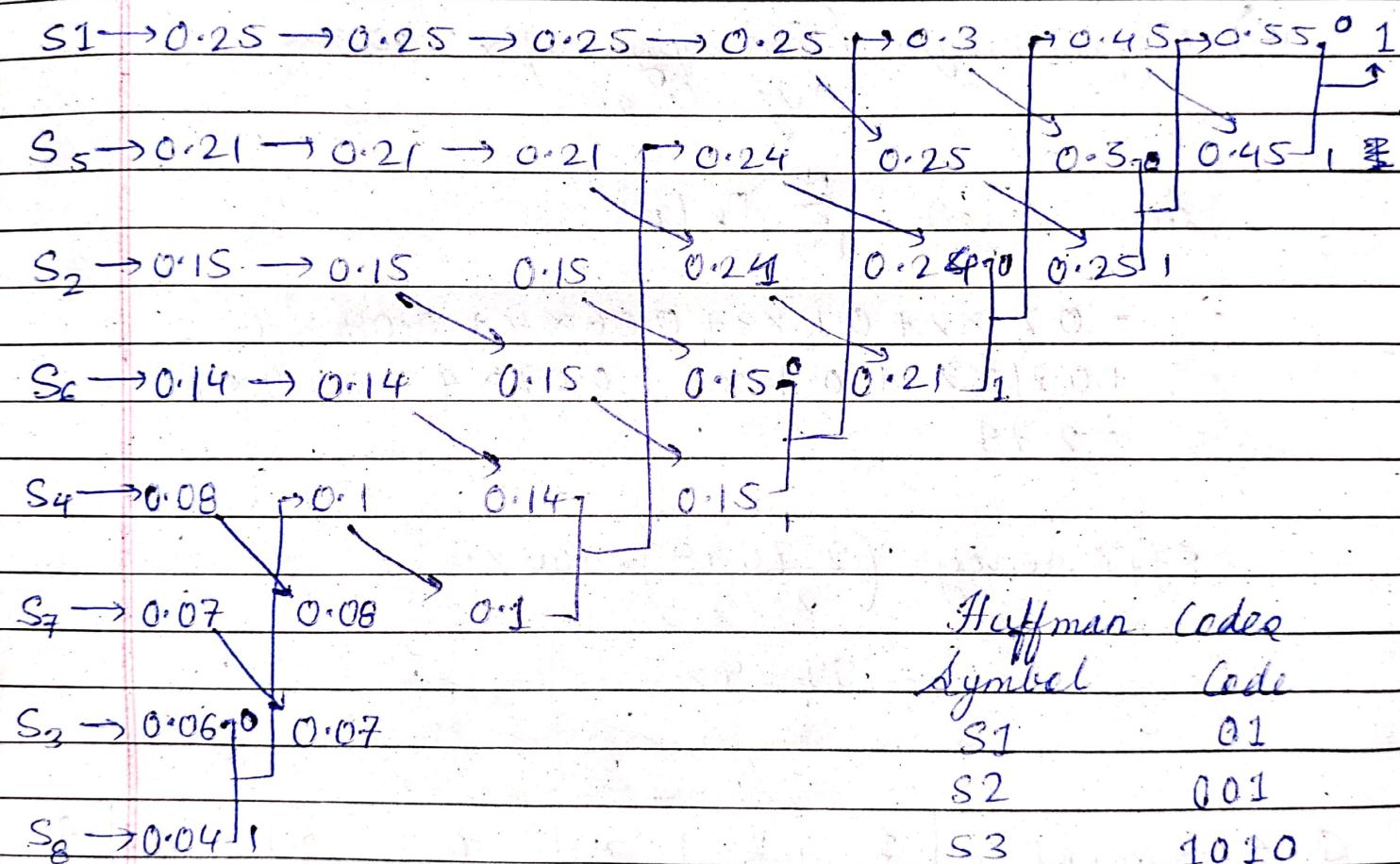
BPP = Size of compressed message in bits
Total no. of characters

$$\boxed{BPP = 6.2}$$

$$\text{Compression} = \left(\frac{\text{Original size} - \text{Compressed size}}{\text{Original size}} \right) \times 100\%$$

$$\boxed{\text{Compression} = 20\%}$$

Q2. @ Huffman tree



Huffman Codes

Symbol	Code
S_1	01
S_2	001
S_3	1010
S_4	0000
S_5	11
S_6	100
S_7	0001
S_8	1011

(b) Min. no. of average bits of seq. per symbol = Entropy

$$= \sum_{k=1}^N P_k \log\left(\frac{1}{P_k}\right)$$

$$= 2.7698$$

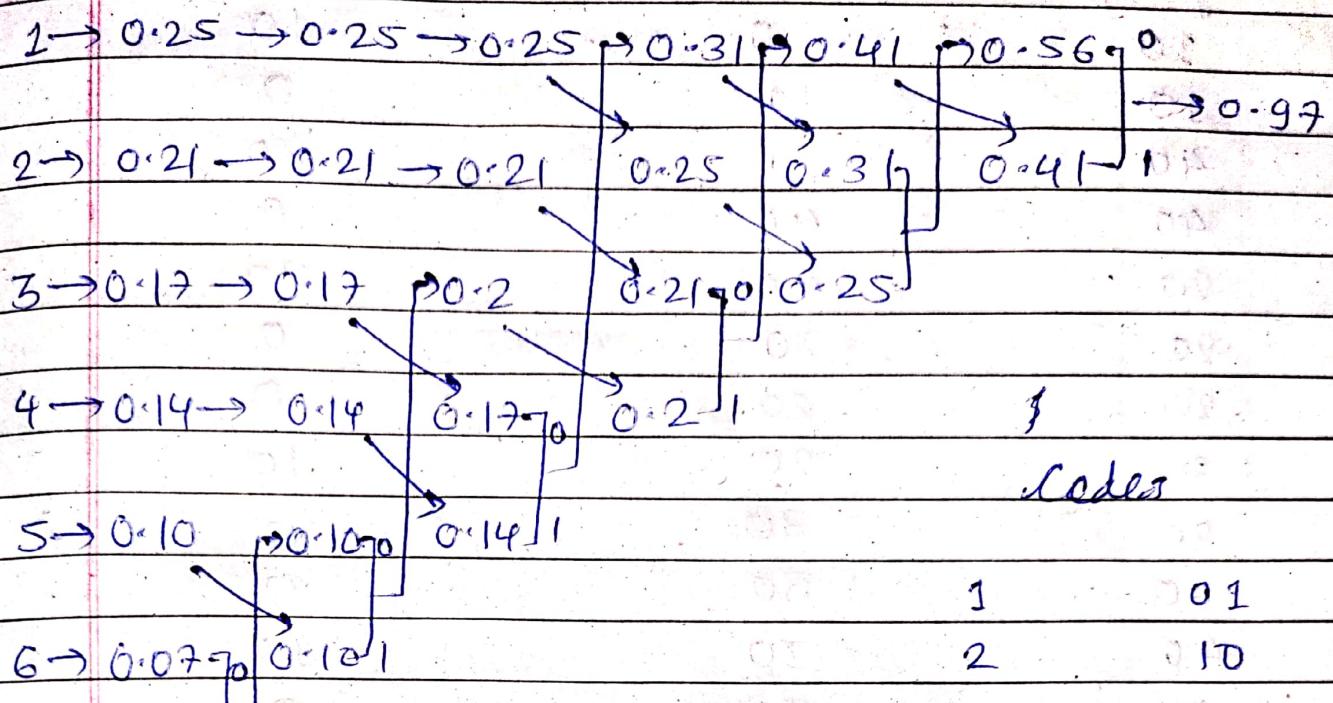
(c) Efficiency = $\left(\frac{\text{Entropy}}{\text{Avg. length}} \right) \times 100\%$

$$\text{Avg. length} = \sum_{k=1}^N P_k L_k$$

$$= 0.25 \times 2 + 0.15 \times 3 + 0.06 \times 4 + 0.08 \times 4 \\ + 0.21 \times 2 + 0.14 \times 3 + 0.09 \times 4 + 0.04 \times 4 \\ = 2.79$$

$$\text{Efficiency} = \left(\frac{2.7698}{2.79} \right) \times 100\% \\ = 99.24\%$$

Q. 3 Symbol	1	2	3	4	5	6	7
Probability	0.25	0.21	0.17	0.14	0.10	0.07	0.03

Codes

1	01
2	10
3	000
4	001
5	111
6	1100
7	1101

Q.4

$$F = \begin{bmatrix} 10 & 10 & 40 & 40 \\ 20 & 20 & 20 & 30 \\ 30 & 30 & 40 & 40 \\ 50 & 50 & 60 & 80 \end{bmatrix}$$

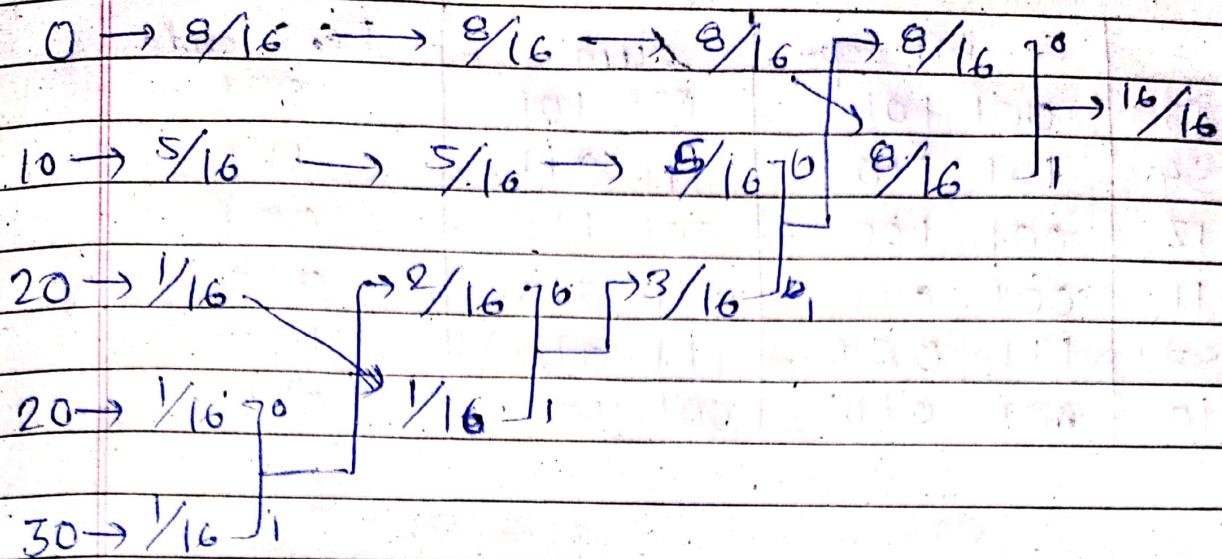
DPCM Encoding

$f(x,y)$	$f^{\wedge}(x,y)$	$c(x,y) = f(x,y) - f^{\wedge}(x,y)$
10	0	10
10	10	0
40	10	30
40	40	0
20	40	-20
20	20	0
20	20	0
30	20	10
30	30	0
30	30	0
40	30	10
40	40	0
50	40	10
50	50	0
60	50	10
80	60	20

Coded Image = $\begin{bmatrix} 10 & 0 & 30 & 0 \\ -20 & 0 & 0 & 10 \\ 0 & 0 & 10 & 0 \\ 10 & 0 & 10 & 20 \end{bmatrix}$

Q.5

$\begin{bmatrix} 10 & 0 & 30 & 0 \\ -20 & 0 & 0 & 10 \\ 0 & 0 & 10 & 0 \\ 10 & 0 & 10 & 20 \end{bmatrix}$	Sym.	Prob.
	0	5/16
	10	5/16
	20	1/16
	-20	1/16
	30	3/16



Symbol	Code	Encoded Image
0	1	00 1 0101 1
10	00	0100 1 1 00 1
20	011	1 1 00 1 00 011
-20	0100	00 1 00 011
30	0101	

$$\text{BPP} = \frac{\text{Size of compressed image}}{\text{No. of pixel}} = \frac{29}{16} = 1.81$$

Sample

$$\text{Compression ratio} = \frac{(96 - 29)}{96} \times 100\% = 69.79\%$$

Q.6

$$F = \begin{bmatrix} 13 & 54 & 12 \\ 13 & 11 & 57 \\ 11 & 10 & 12 \end{bmatrix}$$

(a)	6-bit rep.	sum	IGS code
13	001 101	001 101	001
54	101 110	111 011	111
12	001 100	001 111	001
11	001 011	010 010	010
57	111 001	111 011	111
10	001 010	001 101	001

$$\text{Encoded Image} = \begin{bmatrix} 001 & 111 & 001 \\ 001 & 010 & 111 \\ 010 & 001 & 001 \end{bmatrix} \quad B = 27 - 3$$

$$\text{Compression factor} = \frac{(54 - 27)}{54} \times 100$$

(b) IGS (3-bit)	Decoded	% pixel value
001	001 000	16
111	111 000	56
001	001 000	8
010	010 000	16
111	111 000	56
001	001 000	8

$$\text{Decoded Image} = \begin{bmatrix} 8 & 56 & 8 \\ 8 & 16 & 56 \\ 16 & 8 & 8 \end{bmatrix}$$

$$MSE = \frac{1}{M \times N} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - f^*(x,y)]^2$$

$$MSE = \frac{1}{9} [25 + 4 + 16 + 25 + 25 + 1 + 25 + 4 + 16]$$

$$MSE = 15.67$$

$$PSNR = \frac{(57)^2}{15.67} = 207.3$$

Q. 7

- (i) All image compression techniques are invertible
- False
- (ii) Runlength coding is less than but may not give data compression always - True
- (iii) Runlength coding always gives data compression - False
- (iv) LSSY is not suitable for compressing executable files - True
- (v) Variable length coding procedure can be used to compress a histogram equalized image with 2^n gray levels - True
- (vi) Shrinking of an image is lossy compression - True
- (vii) Compression of an image is only possible if pixel values are occurring consecutively - False