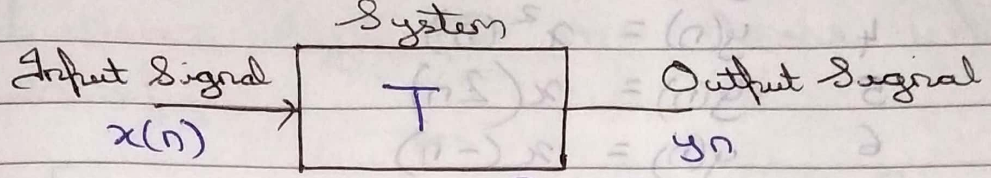
2. Discrete Time System (20 - Marks)

System :- Any means (Hardware / Software) by which signal is generated or processed.



$$y(n) = T[x(n)]$$

Classification of System (Eg - 10M Numerical)

1. Static / Dynamic System
2. Causal / Non - Causal System
3. Linear / Non - Linear System [$y = mx$]
4. Time Variants / Invariant Systems
5. Stable / Unstable System

1. Static / Dynamic System

→ Static Systems → Their output depends only on current input and NOT on Past / Future input

→ Dynamic Systems → Their output may depend on Past / Future input

Static System ⇒ Memory Less

Current → n

Past → $(n-1), (n-2), (n-3) \ldots$

Future → $(n+1), (n+2), (n+3) \ldots$

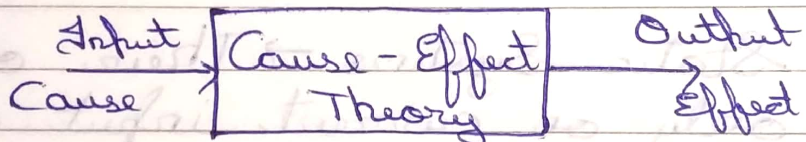
Examples

1. $y(n) = 2x(n)$ — Static
2. $y(n) = 2x(n) + 5x(n-1)$ — Dynamic
3. $y(n) = 2x(n) + 5x(n+1)$ — Dynamic
4. $y(n) = x^2(n)$ — Static
5. $y(n) = x(2n)$ — Dynamic
6. $y(n) = x(-n)$ — Dynamic

2. Causal / Non-Causal System

→ Causal System → Their output may depends on current & past input. But NOT on future Input.

→ Non-Causal System → Their output may depend on Future Input.



Non-Causal System – Predictive Anticipatory

Examples :-

1. $y(n) = 2x(n)$ — Causal
2. $y(n) = 2x(n) + 5x(n-1)$ — Causal
3. $y(n) = 2x(n) + 5x(n+1)$ — Non-Causal
4. $y(n) = x^2(n)$ — Causal
5. $y(n) = x(2n)$ — Non-Causal
6. $y(n) = x(-n)$ — Causal for $n \geq 0$

Note :- All Static Systems → Causal

3. Linear / Non-Linear Systems

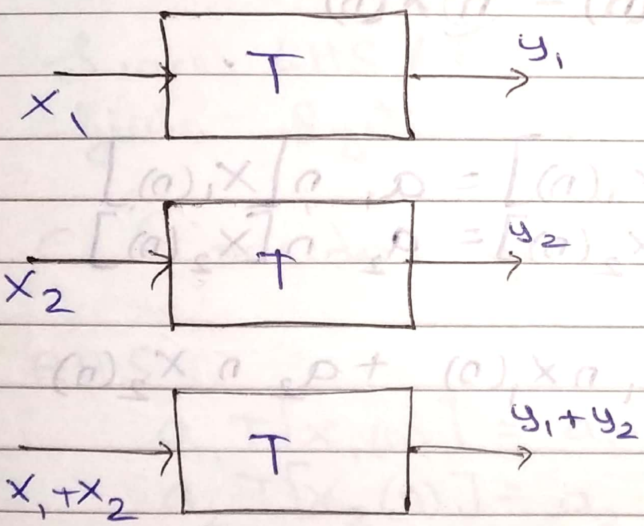
EQ - Seperate Numerical
(10 Marks)

Linear systems are those which satisfy

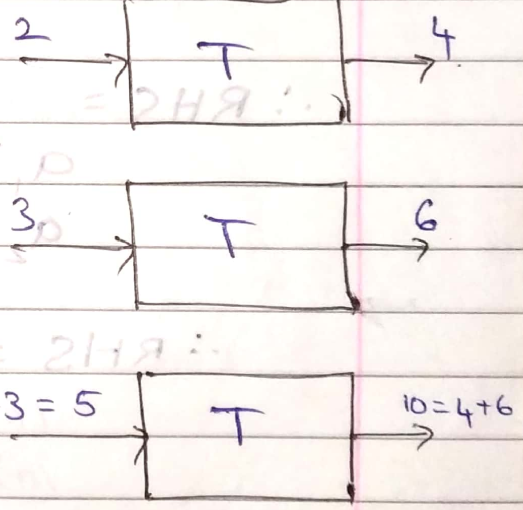
$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
(Superposition Theorem)

Otherwise it is called as Non-Linear Systems

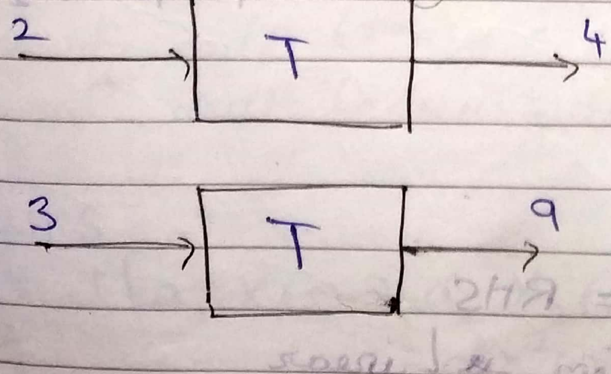
Explaination

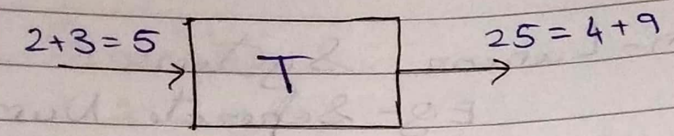


Example - Linear
$y(n) = 2x(n)$



Example - Non Linear
$y(n) = x^2(n)$





EG (10 M) Check if linear / non-linear

a. $y(n) = n.x(n)$

b. $y(n) = e^{x(n)}$

c $y(n) = x^2(n)$

Solution:-

Condition for Linearity

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

a. Consider $y(n) = n\,x(n)$

$\therefore$ RHS =

$$a_1 T[x_1(n)] = a_1\ n[x_1(n)]$$
$$a_2 T[x_2(n)] = a_2\ n[x_2(n)]$$

$\therefore$ RHS $= a_1 n x_1(n) + a_2 n x_2(n)$

LHS

$$T[a_1 x_1(n) + a_2 x_2(n)] = n[a_1 x_1(n) + a_2 x_2(n)]$$
$$= a_1 n x_1(n) + a_2 n x_2(n)$$

Since LHS = RHS

Given System is Linear

e^{2+3} $e^2 \cdot e^3$

b. Consider $y(n) = e^{x(n)}$

∴ RHS

$$a_1 T[x_1(n)] = a_1 e^{x_1(n)}$$
$$a_2 T[x_2(n)] = a_2 \cdot e^{x_2(n)}$$

$$\therefore RHS = a_1 e^{x_1(n)} + a_2 e^{x_2(n)}$$

LHS

$$T[a_1 x_1(n) + a_2 x_2(n)] = e^{a_1 x_1(n) + a_2 x_2(n)}$$
$$= e^{a_1 x_1(n)} \cdot e^{a_2 x_2(n)}$$

Since LHS ≠ RHS
given System is Non Linear

c. Consider $y(n) = x^2(n)$

RHS

$$a_1 T[x_1(n)] = a_1 \cdot x_1^2(n)$$
$$a_2 T[x_2(n)] = a_2 \cdot x_2^2(n)$$

$$\therefore RHS = a_1 x_1^2(n) + a_2 x_2^2(n)$$

LHS

$$T[a_1 x_1(n) + a_2 x_2(n)] = [a \cdot x_1(n) + a_2 x_2(n)]^2$$
$$= a^2 x_1^2(n) + 2 a \cdot x_1(n) \cdot a x_2(n) + a_2^2 x_2^2(n)$$

Since LHS ≠ RHS
Given System is Non-Linear

4. Time Variant / Invariant Systems

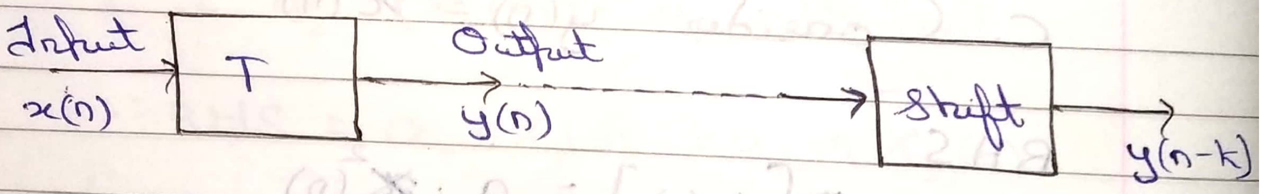
Time (Shift) Variant System
Input - output relationship varies (changes) with time.

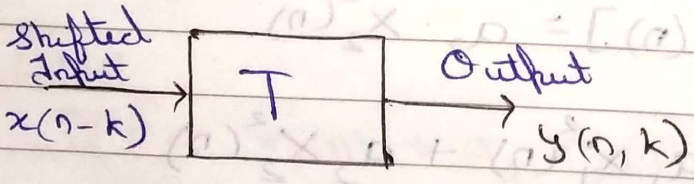
Time (Shift) Invariant System

Input - Output relationship does NOT vary with time.

Explaination - Time Invariant

Shifted Input - gives - Shifted Output





If $y(n-k) = y(n,k)$ Then Invariant
If $y(n-k) \neq y(n,k)$ Then Variant

Example :- $y(n) = x(2n)$

Solution

1. Shift Output

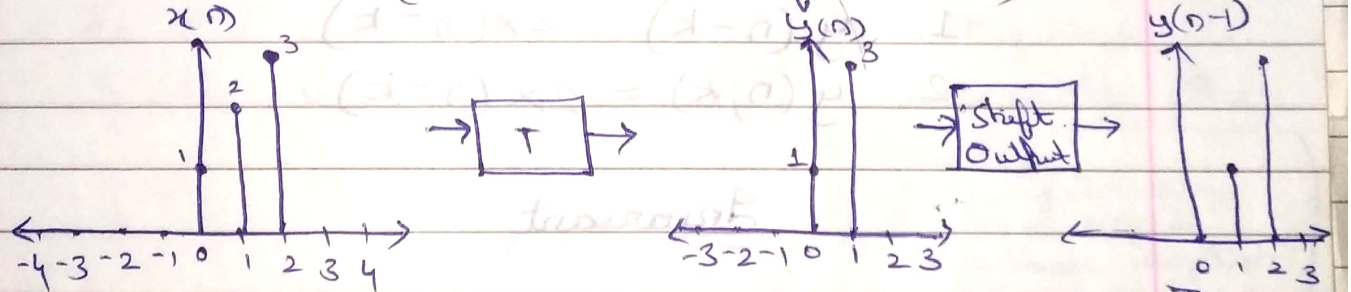
$$y(n-k) = x(2n-k)$$

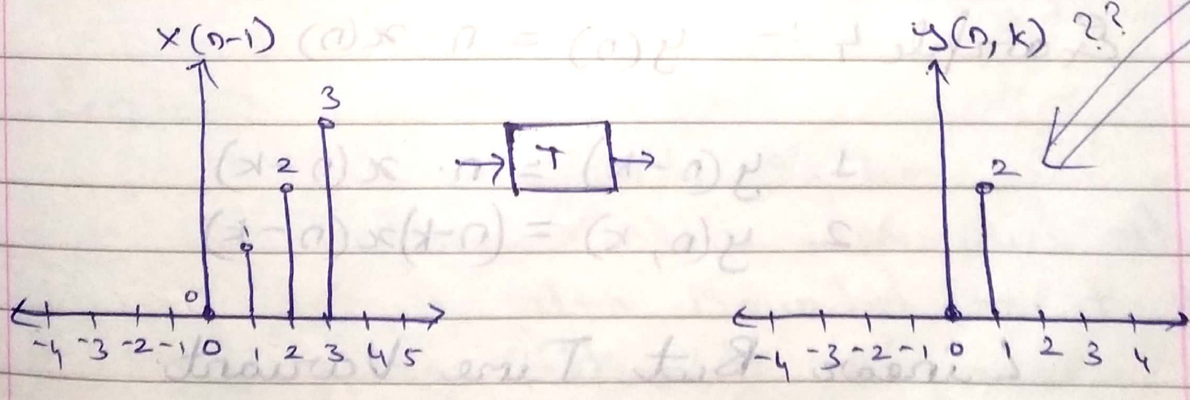
2. Shift Input (Replace n by n-k everywhere

$$y(n,k) = x(2(n-k))$$
$$= x(2n-2k)$$

Since Shifted Input does not give shifted output - given system is Time (Shift) Variant.

Explaination (Not needed for exam)





Example 2: $y(n) = x(n-1)$

Solution :

1. Shift Output
$y(n-k) = x(n-1-k)$

2. Shift Input
$y(n,k) = x((n-k)-1)$
$= x(n-k-1)$

Since Shifted Input gives Shifted Output
∴ given signal is Time (Shift) Invariant

EX3 :- $y(n) = 2x(n)$

1. $y(n-k) = 2x(n-k)$
2. $y(n,k) = 2x(n-k)$

Invariant

Example 4 :- $y(n) = n \cdot x(n)$

1. $y(n-k) = n \cdot x(n-k)$
2. $y(n,k) = (n-k)x(n-k)$

Linear But Time Variant.

Example 5 :- $y(n) = x(-n)$

1. $y(n-k) = x(-n-k)$
2. $y(n,k) = x(-(n-k))$
$= x(-n+k)$

Time Variant

5. Stable / Unstable System

Stable System
If Bounded Input gives Bounded Output then it is stable

Unstable System
If Bounded Input gives unbounded output then it is unstable.

Condition of ~~Un~~Stability (EQ)

BIBO
1. Bounded Input → Bounded Output (BIBO)
2. For LTI system if its Impulse Response is absolutely summable then it is Stable.

$$\boxed{\sum_{n=-\infty}^{\infty} h(n) < \infty}$$

Note:-
① FIR system are always stable
② Recursive System may be unstable

Example : $y(n) = x(2n)$
It is Stable System
If input $x(n)$ is bounded then $x(2n)$ is also Bounded as it is dropping alternate sequence.

Example : $y(n) = x(n-1)$
It is Stable System
If input $x(n)$ is bounded then $x(n-1)$ is also bounded as it is ~~shifted~~ in shifting with one unit.

Ex :- $y(n) = x^2(n)$ Stable

Ex :- $y(n) = e^{x(n)}$ Stable

Ex :- $y(n) = 2x(n) + y(n-1)$ Unstable

Ex :- Check if stable / unstable
Impulse Response $h(n) = \{2, 4, 5\}$

Solution :-
Condition for stability

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = 2+4+5$$
$$= 11$$
$$= \text{Finite}$$

Hence Stable

Eg
May 16
1(b)

5 M

$$h(n) = (0.3)^n u(n) + 5\delta(n)$$

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty} (0.3)^n u(n) + \sum_{n=-\infty}^{\infty} 5\delta(n)$$

$$= \sum_{n=0}^{\infty} (0.3)^n (1) + 5$$

$$= \frac{1}{1-0.3} + 5$$

$= \frac{1}{0.7} + 5$

$=$ Finite

$\therefore$ It is stable system.

<u>DEC 15</u>
<u>1(a)</u>

$$h(n) = \begin{cases} a^n & n \geq 0 \\ b^n & n < 0 \end{cases}$$

Condition

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{-1} b^n + \sum_{n=0}^{\infty} a^n$$

$$= \sum_{n=1}^{\infty} b^{-m} + \sum_{n=0}^{\infty} a^n$$

$$= \underbrace{\sum_{m=1}^{\infty} \left(\frac{1}{b}\right)^m}_{\text{Convergence if } \left(\frac{1}{b}\right) < 1} + \underbrace{\sum_{n=0}^{\infty} a^n}_{\text{Convergence if } a<1}$$

$$= \frac{(1/b)}{1-\left(\frac{1}{b}\right)} + \frac{1}{1-a}$$

Condition $a < 1$

$\frac{1}{a} < 1$ i.e. $b > 1$.