→ Classification of Signals

- Continuous Time / Discrete Time
- Continuous Value / Discrete Value

→ Classification of D.T. Signals

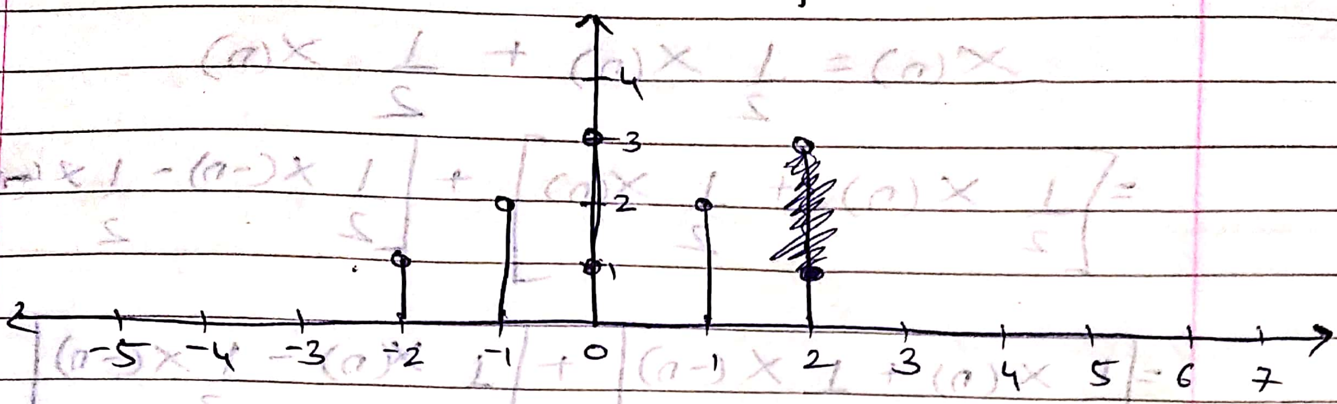
- Even / Odd Signals
- Periodic / Non-Periodic Signals 10 Marks
- Energy / Power Signals

→ Even / Odd Signals

- A D.T. Signal $x(n)$ is said to be Even or Symmetric if

$$x(n) = x(-n)$$

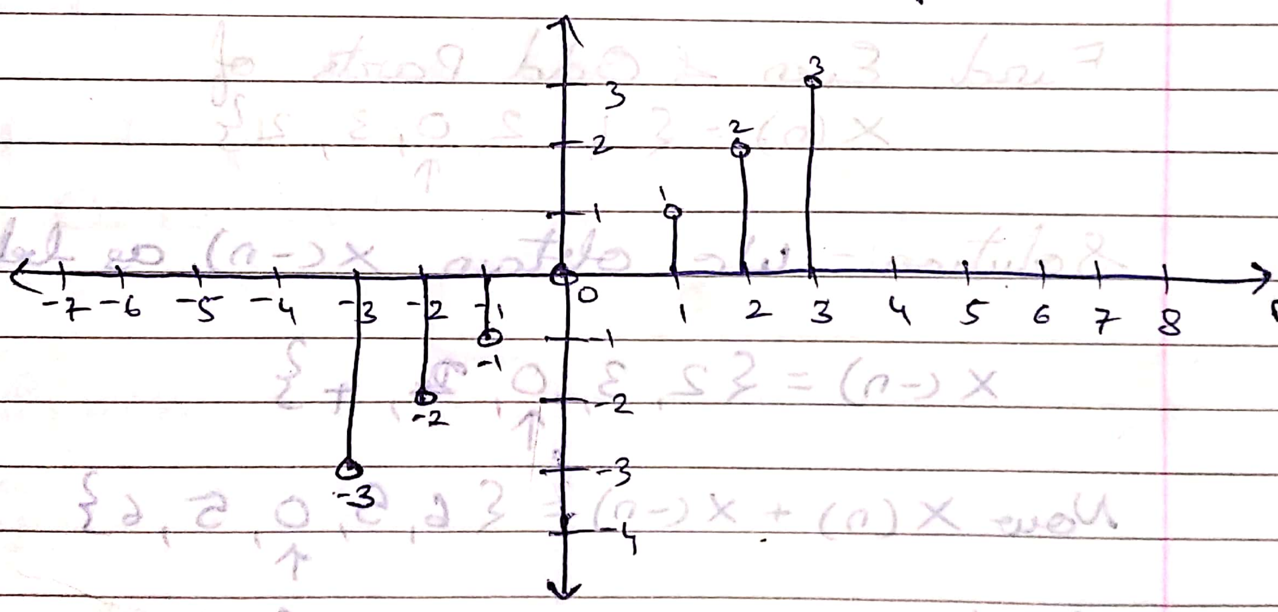
Example: $x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$



- A D.T. signal is said to be odd or antisymmetric if

$$x(n) = -x(-n)$$

Example :- $x(n) = \{-3, -2, -1, \underset{\uparrow}{0}, 1, 2, 3\}$



EG - Show that any D.T. Signal x(n) can be expressed as combination of Even & odd

i.e $x(n) = x_e(n) + x_o(n)$

where $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$ 5 Marks

Proof :- We express $x(n)$ as follows

$$x(n) = \frac{1}{2} x(n) + \frac{1}{2} x(n)$$

$$= \left[\frac{1}{2} x(n) + \frac{1}{2} x(n)\right] + \left[\frac{1}{2} x(-n) - \frac{1}{2} x(-n)\right]$$

$$= \left[\frac{1}{2} x(n) + \frac{1}{2} x(-n)\right] + \left[\frac{1}{2} x(n) - \frac{1}{2} x(-n)\right]$$

$$= \frac{1}{2}[x(n) + x(-n)] + \frac{1}{2}[x(n) - x(-n)]$$

$$x(n) = x_e(n) + x_o(n)$$

Hence Proved.

Eg :- Numerical ~~10~~ 5 Marks

Find Even & Odd Parts of

$$x(n) = \{4, 2, \underset{\uparrow}{0}, 3, 2\}$$

Solution :- We obtain $x(-n)$ as below

$$x(-n) = \{2, 3, \underset{\uparrow}{0}, 2, 4\}$$

Now $x(n) + x(-n) = \{6, 5, \underset{\uparrow}{0}, 5, 6\}$

Similarly $x(n) - x(-n) = \{2, -1, 0, 1, -2\}$

$$\therefore x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \{3, 2.5, \underset{\uparrow}{0}, 2.5, 3\}$$

$\therefore x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$= \{1, -0.5, 0, 0.5, -1\}$

$x(n) = x_e(n) + x_o(n)$

$= \{3, 2.5, 0, 2.5, 3\} + \{1, -0.5, 0, 0.5, -1\}$

$= \{4, 2, \underset{\uparrow}{0}, 3, 2\}$

★★ → Periodic / Non-Periodic Signal 10 Marks

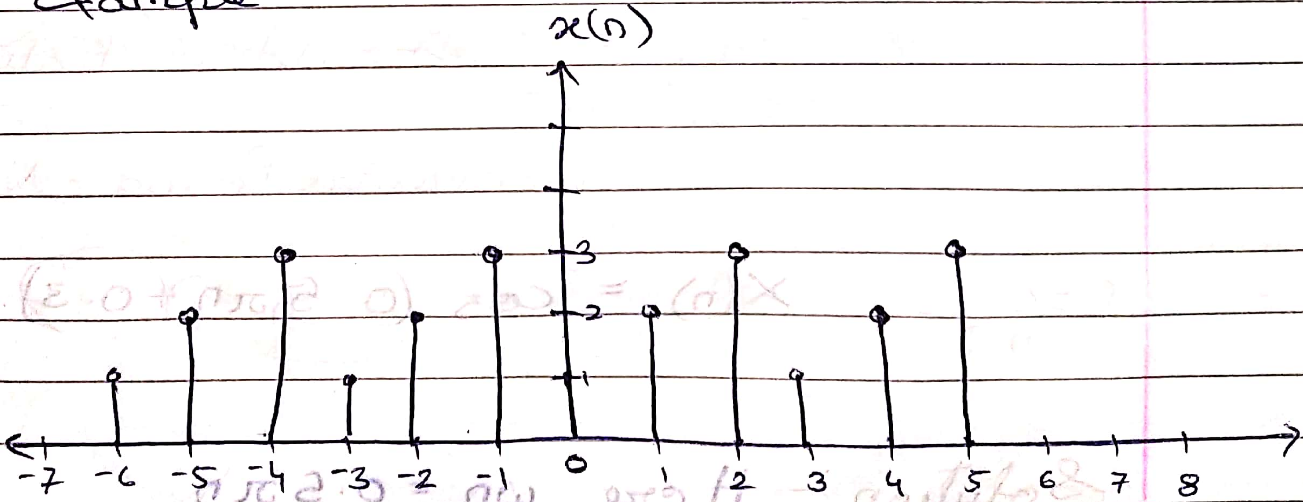
- A D.T. Signal $x(n)$ is said to be periodic with period $N > 0$ if

$$x(n) = x(n+N)$$

- Otherwise it is called as Non-periodic or aperiodic signal.

Shortcut:- π dikha to periodic warna non-periodic

Example



How to check?

Frequency should be Rational =

Ratio of two integration

Irrational number e.g. π

Explaination

Consider $x(n) = A \sin(\omega n + \phi)$
$= A \sin(2\pi f n + \phi)$

$\therefore x(n+N) = A \sin(2\pi f[n+N] + \phi)$

If it is periodic, then
$x(n) = x(n+N)$

$$A \sin(2\pi f n + \phi) = A \sin(2\pi f[n+N] + \phi)$$
$$\sin(2\pi f n + \phi) = \sin(2\pi f n + 2\pi f N + \phi)$$

$$\therefore 2\pi f N = 0, 2\pi, 4\pi, 6\pi, 8\pi, \ldots$$
$$= k(2\pi) \quad \text{where } k = 0, 1, 2, \ldots$$
$$\therefore fN = k$$
$$\therefore f = \frac{k}{N}$$

i.e. Frequency should be Rational Number

Fundamental Period $= N$

DEC 15
2(a) I

$x(n) = \cos(0.5\pi n + 0.3)$

Solution :- Here $\omega n = 0.5\pi \cancel{n}$
$$2\pi f = \frac{1}{2}\cancel{\pi}$$

$\therefore f = \frac{1}{4} = \frac{K}{N}$ = Rational No

Since its frequency is Rational Number given signal is periodic

$\therefore$ Its period $= N = 4$

Ex. $x(n) = \cos(0.5n + 0.3)$

Here $\omega n = 0.5n$

$\therefore 2\pi f = \frac{1}{2}$

$\therefore f = \frac{1}{4\pi}$ ⇐ Irrational Number

Since the frequency is Irrational Number given signal is non-periodic.

DEC 15 2(a) II $\quad x(n) = \cos(0.3\pi n) + 10\sin(0.25\pi n)$

Solution :- We first consider $\cos(0.3\pi n)$

Here $\omega n = 0.3\pi n$

$2\pi f = \frac{3\pi}{10}$

$f = \frac{3}{20} = \frac{K}{N}$

Since its frequency is rational given signal is periodic

$\therefore$ Its period $= N_1 = 20$

Now consider $10 \sin(0.25\pi n)$

Here $\omega n = 0.25\pi n$

$2\pi f = \frac{1}{4}\pi$

$\therefore f = \frac{1}{8} = \frac{k}{n}$

Since its frequency is Rational Number, given signal is periodic

$\therefore$ Its period $= N_2 = 8$

$\therefore$ Since all its signal are individually periodic overall signal is also periodic

Its period $= LCM(N_1, N_2) = LCM(20, 8) = 40$

MAY 16 3(B) 1 $x(n) = \sin(0.25\pi n + 0.4)$

Solution :- Here $\omega n = 0.25\pi n$

$\therefore 2\pi f = \frac{1}{4}\pi$

$\therefore f = \frac{1}{8} = \frac{k}{N}$ Rational No.

Since its frequency is Rational Number given signal is periodic

$\therefore$ Its period $= 8$

Solution:- We first consider $\cos(0.5 n \pi)$

Here $\omega n = 0.5 n \pi$

$\therefore 2\pi f = \frac{1}{2}\pi$

$\therefore f = \frac{1}{4} = \frac{k}{N}$

Since its frequency is Rational given signal is periodic

Its period $= N_1 = 4$

$\therefore$ Now Consider $\sin(0.25 \pi n)$

Here $\omega n = 0.25 \pi n$

$\therefore 2\pi f = \frac{1}{4}\pi$

$\therefore f = \frac{1}{8} = \frac{k}{n}$

Since its frequency is Rational Number, given signal is periodic

Its period $= N_2 = 8$

$\therefore$ Since all its signal are individually periodic overall signal is also periodic

$\therefore$ Its period $= LCM(N_1, N_2) = LCM(4, 8) = 8$

EG :- 15. 1 $x_1(n) = e^{j\left(\frac{\pi}{4}\right)n} = e^{j\omega n}$

$\therefore \omega n = \frac{\pi}{4} n$

$\therefore 2\pi f \cancel{n} = \frac{\pi}{4} \cancel{n}$

$\therefore f = \frac{1}{8} = \frac{k}{N}$

Since its frequency is Rational given signal is periodic.

→ # Energy & Power Signals

EG - Always Asked in Compulsory Question (5-Marks)

$$E = \sum_{n=-\infty}^{\infty} x(n)^2 \rightarrow \text{Magnitude Square}$$

If energy of a D.T. signal is finite then it is called as energy signal

$0 < E < \infty \Rightarrow$ Energy Signal

- Average Power of D.T. Signal

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)^2$$

If ~~period~~ power of a D.T. Signal is finite then it is called as Power Signal

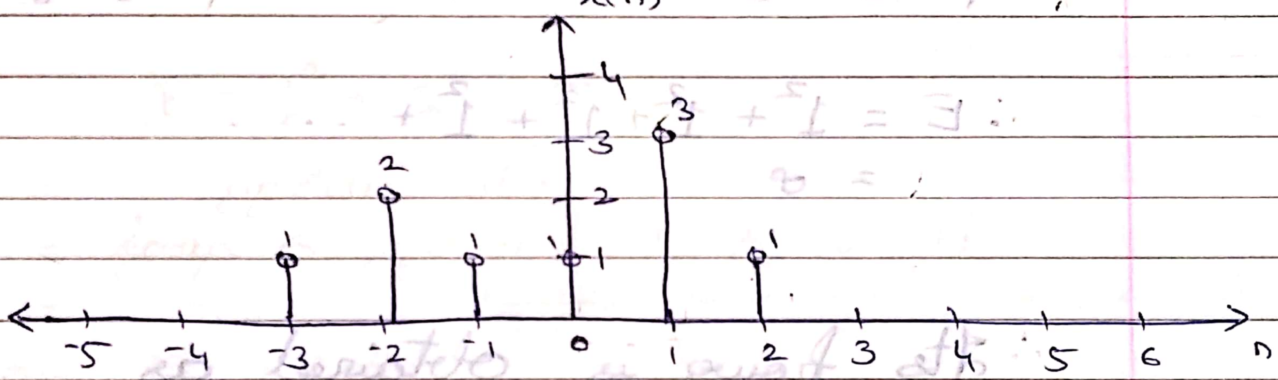
$0 < P < \infty \Rightarrow$ Power Signal

Eg - 5 Marks

- Justify - Unit Step is Power Signal
- Justify - Unit Ramp is neither energy nor power.
- If Energy is finite, power is zero
- Numerical Problem.

Explanation

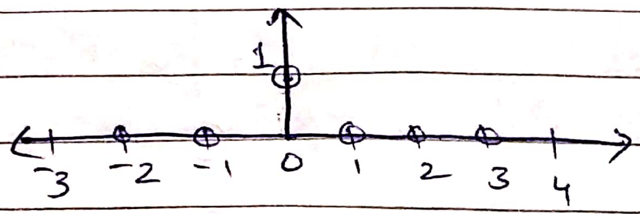
Consider following D.T. Signal



$$E = 1^2 + 2^2 + 1^2 + 1^2 + 3^2 + 1^2$$
$$= 1 + 4 + 1 + 1 + 9 + 1$$
$$= 17 \therefore \text{Finite Energy}$$

$\therefore$ Hence given signal is Finite Energy Signal.

Ex :- Justify - Unit Impulse is Energy Signal



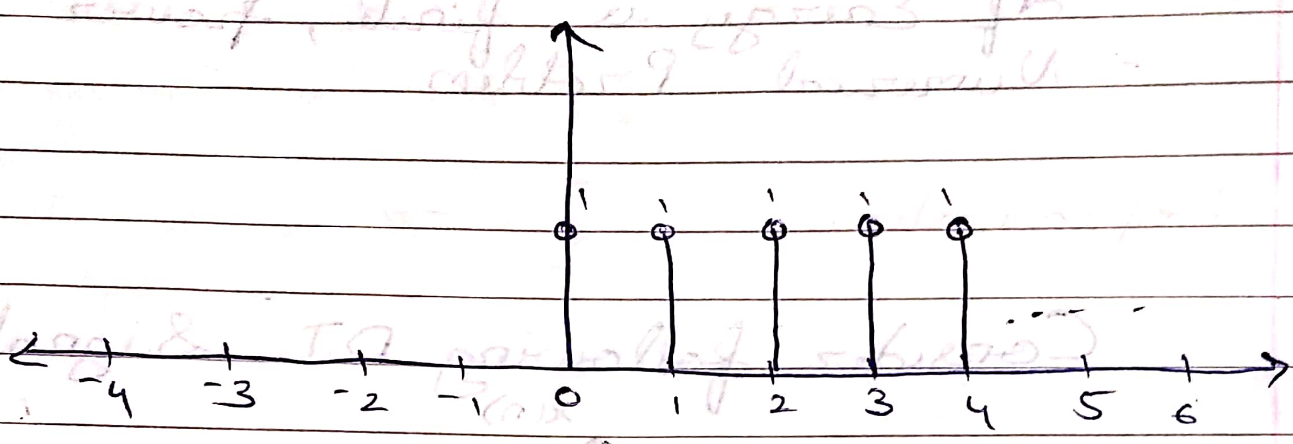
$\therefore E = 1^2 = 1 \therefore$ Finite Energy

Since Energy is finite, Unit Impulse is Energy Signal

*** Eg :- Justify Unit Step is Power Signal

- Consider Unit Step Signal

$$x(n) = u(n)$$



$$\therefore E = 1^2 + 1^2 + 1^2 + 1^2 + \dots$$
$$= \infty = \text{Infinity Energy}$$

$\therefore$ It is NOT Energy Signal

$\therefore$ Its power is obtained as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} (N+1)$$

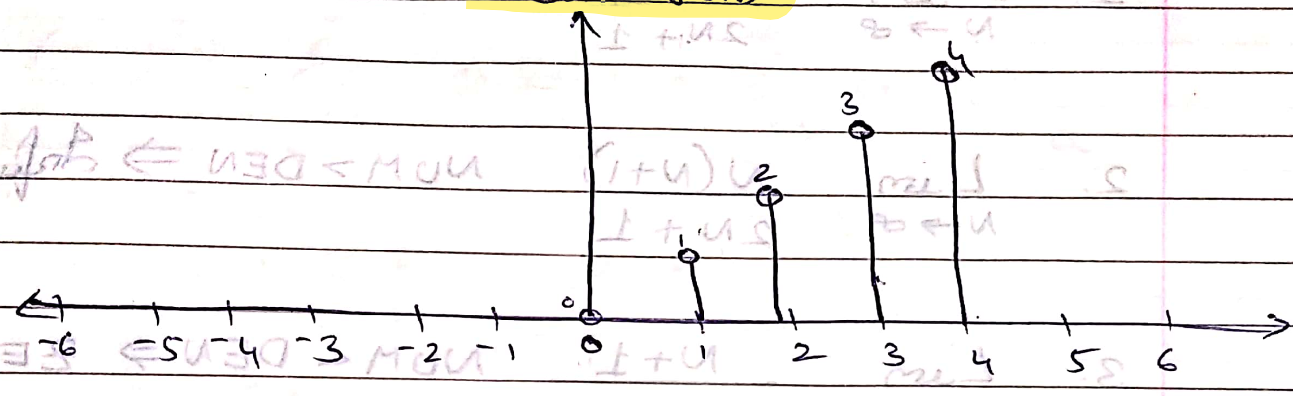
$$= \lim_{N \to \infty} \frac{N+1}{2N+1} = \lim_{N \to \infty} \frac{1+(1/N)}{1+(1/N)} = \frac{1}{2} = \text{Finite}$$

$\therefore$ Since its Power is Finite, Unit Step is Power Signal

EG :- Justify - Unit Ramp is Neither Energy Nor Power

- Consider Unit Ramp Signal

$x(n) = u_r(n)$



$\therefore E = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 \ldots$

$= \infty$ = Infinite Energy

$\therefore$ Since its Energy is Infinite it is not ~~E N~~ Energy Signal

Its Power is obtained as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} n^2$$

We know Ramp $x(n) = n$

Since order of NUM is greater than DEN Limit gives infinite Power.

Hence Unit Ramp is Neither Energy Nor Power Signal.

* Mathematics Review *

Limits

1. $\lim\limits_{N \to \infty} \dfrac{N+1}{2N+1}$ NUM = DEN ⇒ Finite

2. $\lim\limits_{N \to \infty} \dfrac{N(N+1)}{2N+1}$ NUM > DEN ⇒ Infinite

3. $\lim\limits_{N \to \infty} \dfrac{N+1}{N(2N+1)}$ NUM < DEN ⇒ ZERO

Geometric Series

1. $\sum\limits_{n=0}^{\infty} a^n = \dfrac{1}{1-a}$

2. $\sum\limits_{n=1}^{\infty} a^n = \dfrac{a}{1-a}$

→ Numerical Problems

DEC 15 5 Marks
1 (B)

Find Energy of Signal

$x(n) = 0.5^n u(n) + 8^n u(-n-1)$

→ We Consider $(0.5)^n \cdot u(n)$

$$\therefore E_1 = \sum_{n=-\infty}^{\infty} x(n)^2$$

$$= \sum_{n=0}^{\infty} \left[(0.5)^n\right]^2$$

$$= \sum_{n=0}^{\infty} \left[(0.5)^2\right]^n$$

$$\boxed{= \sum_{n=0}^{\infty} (0.25)^n = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}}$$

$$= \frac{1}{1-0.25} = \frac{1}{0.75} = \frac{4}{3}$$

$$\therefore E_1 = \frac{4}{3}$$

Now, we consider $8^n \, u(-n-1)$

$$E = \sum_{n=-\infty}^{\infty} x(n)^2 = \sum_{n=-\infty}^{-1} \left[(8)^n\right]^2$$

We put $n = -m$

$\therefore$ when $n = -1$ $m = 1$

when $n = -\infty$ $m = \infty$

$$\therefore E = \sum_{m=1}^{\infty} \left[(8)^{-m}\right]^2$$

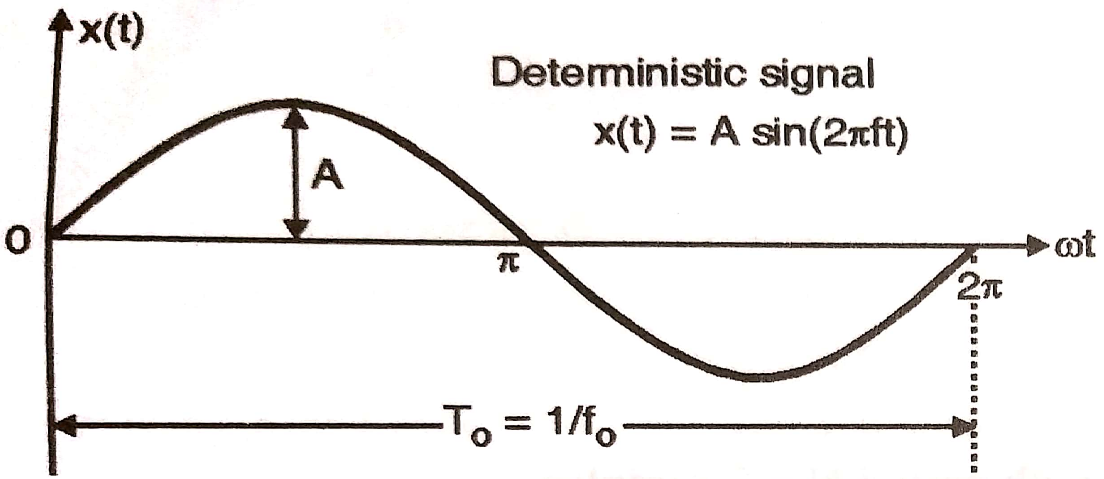
Magnitude Square

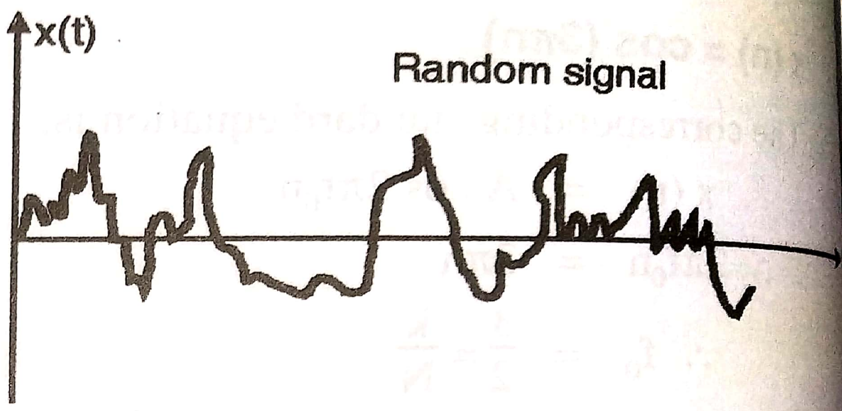
$$= \sum_{n=1}^{\infty} \left(\frac{1}{64}\right)^m = \boxed{\sum_{n=1}^{\infty} a^m = \frac{a}{1-a}}$$

$$\therefore E_2 = \frac{1/64}{1-1/64} = \frac{1/64}{63/64} = \frac{1}{63}$$

$$\therefore \text{Total Energy} = E_1 + E_2 = \frac{4}{3} + \frac{1}{63} = \frac{85}{63} = 1.3492$$

Sr. No.	Power signals	Energy signals
1.	The signal having finite non-zero power are called as power signals.	The signals having a finite non-zero energy are called as energy signals.
2.	Almost all the periodic signals in practice are power signals.	Almost all the non-periodic signals are energy signals.
3.	Power signals can exist over an infinite time. They are not time limited.	Energy signals exist over a short period of time. They are time limited.
4.	Energy of a power signal is infinite.	Power of an energy signal is zero.


x(t)
Deterministic signal
x(t) = A sin(2πft)
A
0
π
2π
ωt
T0 = 1/f0


x(t)
Random signal