



Dsip imp sums imp notes

Computer Engineering (University of Mumbai)



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1

Monday

SEPTEMBER DSP NOTES

Day 244 - Left 121

PLTU

Week-3

2014

Labor Day (US)

08

Important Formulae

09

• Energy

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

E is finite
P is 0

11

• Power

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

P is finite
E is ∞

12

01

$$\omega = \frac{K}{N}$$

02

• Linear Convolution

03

$$y(n) = \sum_{m=-\infty}^{\infty} x(m) h(n-m)$$

04

$$m_L = x_L$$

$$m_L = x_L + h_L$$

05

$$m_H = x_H$$

$$m_H = x_H + h_H$$

06

• Correlation

$$\gamma_{xy}(n) = \sum_{m=-\infty}^{\infty} x(m) h(m-n)$$

$$m_L = x_L - (y_L + N_2 - 1) \quad m_L = x_L$$

$$m_H = m_L + (N_1 + N_2 - 2) \quad m_H = x_H$$

SEPTEMBER

2014



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wk	M	T	W	T	F	S	S
36	1	2	3	4	5	6	7
37	8	9	10	11	12	13	14
38	15	16	17	18	19	20	21
39	22	23	24	25	26	27	28
40	29	30	*	*	*	*	*

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DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi n k}{N}} \quad \text{where } 0 \leq k \leq N-1$$

IDFT

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi n k}{N}}$$

Karl Pearson's Coefficient

$$\gamma = \frac{\sum XY}{\sqrt{\sum X^2} \sqrt{\sum Y^2}}$$

Values to Remember .

- $e^0 = 1$

- $e^{-j\frac{\pi}{2}n} = (-j)^n$
- $e^{j\frac{\pi}{2}k} = (j)^k$

- $e^{-j\pi n} = (-1)^n$
- $e^{j\pi k} = (-1)^k$

- $e^{-j\frac{3\pi}{2}n} = (j)^n$
- $e^{j\frac{3\pi}{2}k} = (-j)^k$

$$\omega_2^0 = 1 \quad \omega_8^0 = 1 @ \quad \omega_8^3 = -(1+j)$$

OCTOBER 2014						
Wk	M	T	W	T	F	S
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2	6	7	8	9	10	11
3	13	14	15	16	17	18
4	20	21	22	23	24	25
5	27	28	29	30	31	1

$$\omega_4^0 = 1 \quad \omega_8^1 = \frac{1-j}{\sqrt{2}}$$

$$\omega_4^1 = -j \quad \omega_8^2 = (-j)$$

The number one thing about trouble is...don't get into more - Dave Stockton

3 Wednesday
SEPTEMBER

Day 246 - Left 119

08 **BIBO Stability Condition**

09

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

10

Thursday

SEPTEMBER

4

Day 247 - Left 118

014 Energy & Power Walk

State whether Energy or Power signal?

$$x(n) = \cos(3\pi n/4)$$

$$\rightarrow \text{Energy}(E) = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |\cos(\frac{3\pi n}{4})|^2$$

$$= \sum_{n=-\infty}^{\infty} \cos^2(\frac{3\pi n}{4})$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2} (\cos 2(\frac{3\pi n}{4}) + 1) \quad \text{using}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(1 + \cos \frac{3\pi n}{2} \right)$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} 1 + \frac{1}{2} * \sum_{n=-\infty}^{\infty} \cos \frac{3\pi n}{2}$$

$$= \infty + 0 \longrightarrow \sum_{-\infty}^{\infty} \frac{\cos \frac{3\pi n}{2}}{2} = 0$$

$$E = \infty$$

$$\rightarrow \text{Power}(P) = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N \left| \cos \left(\frac{3\pi n}{4} \right) \right|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N \cos^2 \left(\frac{3\pi n}{4} \right)$$

OCTOBER 2014						
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12	13	14	15	16	17	18
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14	27	28	29	30	31	*

Happiness is a long walk with a putter. - Greg Norman

5 Friday
SEPTEMBER

Day 248 - Left 117

$\cos 2\theta = 2 \cos^2 \theta - 1$
using

2014

$$\begin{aligned}
 & \text{06} \quad \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1}{2} \left(1 + \cos \frac{3\pi n}{2} \right) \\
 & \text{09} \quad \cancel{\lim_{N \rightarrow \infty}} \frac{1}{2} \cdot \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(1 + \cos \frac{3\pi n}{2} \right) \\
 & \text{10} \quad = \frac{1}{2} \lim_{N \rightarrow \infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{N} 1 + \sum_{n=-N}^{N} \cos \frac{3\pi n}{2} \right) \\
 & \text{11} \quad = \frac{1}{2} \lim_{N \rightarrow \infty} \frac{1}{2N+1} \left(2N+1 + \sum_{n=-N}^{N} \cos \frac{3\pi n}{2} \right) \\
 & \text{01} \quad = \frac{1}{2} \lim_{N \rightarrow \infty} \left(1 + \frac{1}{2N+1} \sum_{n=-N}^{N} \cos \frac{3\pi n}{2} \right)
 \end{aligned}$$

Putting $N = \infty$.

$$\sum_{n=-\infty}^{\infty} \cos \frac{3\pi n}{2} = 0$$

$$\text{04} \quad \frac{1}{2} \cancel{\lim_{N \rightarrow \infty}} \left(1 + \frac{1}{2N+1} \cdot \sum_{n=-\infty}^{\infty} \cos \frac{3\pi n}{2} \right)$$

$$\text{05} \quad \frac{1}{2} \cancel{\lim_{N \rightarrow \infty}} (1 + 0 \cdot 0)$$

$$\text{06} \quad P = \boxed{\frac{1}{2}}$$

SEPTEMBER 2014

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02	8	9	10	11	12	13	
03	15	16	17	18	19	20	21
04	22	23	24	25	26	27	28
05	29	30					

Energy is infinite

& Power is finite

The above signal is a Power Signal.

CROSS
Correlation

Saturday
SEPTEMBER
Day 249 - Left 116

6

Onam (IN)

$$x(n) = \{2, 3, 1, 4\}$$

$$y(n) = 3\delta(n-3) - 2\delta(n) + \delta(n-1) + 4\delta(n-2).$$

$$\begin{array}{r} \rightarrow y(n) = \\ \quad \begin{array}{r} 0 \ 0 \ 0 \ 3 \\ - 2 \ 0 \ 0 \ 0 \\ \hline 2 \ 0 \ 0 \ 0 \\ + 0 \ 1 \ 0 \ 0 \\ + 0 \ 0 \ 4 \ 0 \\ \hline - 2, 1, 4, 3 \end{array} \end{array} \therefore y(n) = \{-2, 1, 4, 3\}.$$

Correlation formula:

$$\gamma_{xy}(n) = \sum_{m=-\infty}^{\infty} x(m)y(m-n) \quad -\infty \leq n \leq \infty$$

$$\begin{aligned} m_L &= x_L = 0 & n_L &= x_1 - (y_1 + N_2 - 1) \\ m_H &= x_H = 3 & & = 0 - (0 + 4 - 1) = -3 \end{aligned}$$

$$\begin{aligned} n_H &= n_L + (N_1 + N_2 - 2) \\ &= -3 + (4 + 4 - 2) = 3 \end{aligned}$$

$\therefore \gamma_{xy}(n)$ when $-3 \leq n \leq 3$

$$\gamma_{xy}(-3) = \sum_{m=-\infty}^{\infty} x(m)y(m-n)$$

$$= \sum_{m=0}^3 @ x(m)y(m-n),$$

$$\begin{aligned} \gamma_{xy}(-3) &= x(0)y(0+3) + x(1)y(1+3) + x(2)y(2+3) \\ &\quad + x(3)y(3+3) \end{aligned}$$

$$\gamma_{xy}(-3) = 2 \times 3 + 3 \times 3 + 1 \times 0 + 4 \times 0$$

$$\gamma_{xy}(-3) = 6$$

OCTOBER 2014						
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42	13	14	15	16	17	18
43	20	21	22	23	24	25
44	27	28	29	30	31	*

Have you ever noticed what golf sounds backwards? - Al Boliska

7

Sunday
SEPTEMBER

Day 250 - Left 115

08 Similarly for

II

$$n = -2$$

$$09 \quad \gamma_{xy}(-2) = \sum_{m=0}^3 x(m)y(m-n)$$

$$10 \quad = x(0)y(0 - (-2)) + x(1)y(1 - (-2)) \\ + x(2)y(2 - (-2)) + x(3)y(3 - (-2))$$

11

$$= 2 * 4 + 3 * 3 + 1 * 0 + 4 * 0$$

12

$$\therefore \gamma_{xy}(-2) = 17$$

01

So in for $\gamma_{xy}(-1, 0, 1, 2, 3)$

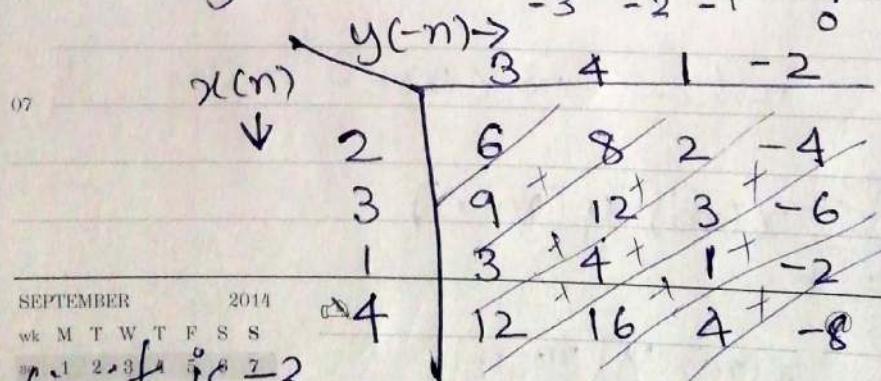
02

Shortcut for Cross Correlation

$$\rightarrow x(n) = \{2, 3, 1, 4\}$$

$$y(n) = \{-2, 1, 4, 3\}$$

$$06 \quad \text{Take } y(-n) = \{3, 4, 1, -2\}$$



07
SEPTEMBER 2014

wk M T W T F S S

1 2 3 4 5 6 7

8 9 10 11 12 13 14

15 16 17 18 19 20 21

22 23 24 25 26 27 28

29 30 31

min limit is -3

Max limit is 3 $\{6, 17, 17, 15, 11, 2, -8\}$

I've had a good day when I don't fall out of the cart. - Buddy Hackett

BIBO Stability of A LTI System

Monday
SEPTEMBER
Day 251 - Left 114

8

Q Test Stability of LTI system whose impulse response is

$$h(n) = 0.2^n u(n) + 3^n u(-n)$$

$$\rightarrow \sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad (\text{BIBO stability condition}).$$

$$\therefore \sum_{n=-\infty}^{\infty} |0.2^n u(n) + 3^n u(-n)|$$

$$= \sum_{n=-\infty}^{\infty} |0.2^n u(n)| + \sum_{n=-\infty}^{\infty} |3^n u(-n)|$$

$$= \sum_{n=0}^{\infty} 0.2^n + \sum_{n=-\infty}^0 3^n \quad P = \frac{1}{n} \quad \text{Let } P = \frac{1}{n}$$

$$= \sum_{n=0}^{\infty} 0.2^n + \sum_{P=0}^{\infty} \left(\frac{1}{3}\right)^P$$

$$\frac{1}{1-0.2} + \frac{1}{1-\frac{1}{3}} = \frac{1}{0.8} + \frac{3}{2} = 2.75$$

$$\left(\because \sum_{n=0}^{\infty} \gamma^n = \frac{1}{1-\gamma} \right)$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = 2.75 < \infty$$

∴ It is Stable

OCTOBER 2014							
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41	6	7	8	9	10	11	12
42	13	14	15	16	17	18	19
43	20	21	22	23	24	25	26
44	27	28	29	30	31	*	*

9

Tuesday
SEPTEMBER

D-252 Left 113

Sampling of Discrete Time Signal

@ Week 9
2014

Consider Analog Signal

$$x(t) = 2 \sin(80\pi t)$$

Sampling frequency $F_s = 60 \text{ Hz}$

- find Sampled Version of discrete time signal $x(n)$ & also find an alias frequency corresponding to $F_s = 60 \text{ Hz}$

→ $x(t) = 2 \sin(80\pi t)$

01

Replace t

02

$$\text{Sampling Period } (T) = \frac{1}{F_s} = \frac{1}{60}$$

03

$$\therefore x(n) = x(t) \rightarrow 2 \sin(80\pi \cdot t)$$

04

Replace t by nT

05

$$x(n) = 2 \sin(80\pi \cdot n \cdot T)$$

06

$$= 2 \sin\left(80\pi \cdot n \cdot \frac{1}{60}\right)$$

07

$$\therefore \boxed{x(n) = 2 \sin\left(\frac{4\pi n}{3}\right)}$$

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38	15	16	17	18	19	20
39	22	23	24	25	26	27
40	29	30				

@

Sampled Version

Wednesday

SEPTEMBER

Day 253 - Left 112

10

Now

$$\frac{\omega}{2\pi} = F$$

~~∴ ω~~

$$x(t) = 2 \sin 80\pi t$$

Comparing with standard eq

$$x(t) = A \sin \omega t$$

$$\therefore \underline{\underline{\omega = 80\pi}} .$$

$$\frac{80\pi}{2\pi} = F$$

$$F = 40 \text{ Hz}$$

actual signal frequency.

Aliasing Frequency: (F_a)

Aliased freq. is the absolute difference betn the actual signal frequency & the nearest integer multiple of the sampling frequency.

$$\text{ie } F_a = |F - N \times F_s|$$

$$\therefore F_a = |40 - N \times 60|$$

for nearest integer multiple $N=1$

$$\therefore F_a = |40 - 1 \times 60|$$

$$\underline{\underline{F_a = 20 \text{ Hz}}}$$

$$\therefore \underline{\underline{\text{aliasing frequency} = 20 \text{ Hz}}}$$

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4	20	21	22	23	24	25	26	
5	27	28	29	30	31			

11

**Compute DFT &
Sketch Magnitude &
Phase Spectrum**

Thursday

SEPTEMBER

Day 254 - Left 111

Week 33

2014

Q.

08 Compute DFT of seq. $x(n) = \{0, 2, 3, -1\}$

09 Sketch the magnitude & phase spectrum.

10 $\rightarrow x(n) = \{0, 2, 3, -1\} .$

11 Using Matrix Multiplication Method for
12 Calculating DFT.

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3-3j \\ 2 \\ -3+3j \end{bmatrix}$$

04 $\rightarrow X(k) = \{4, -3-3j, 2, -3+3j\} .$

DFT.

05 Now Magnitude Spectrum formula .

06 $|X(k)| = \sqrt{X_r^2(k) + X_i^2(k)}$

X_r is real part & X_i is imaginary part .

SEPTEMBER 2014						
S	M	T	W	T	F	S
1	2	3	4	5	6	7
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15	16	17	18	19	20	21
22	23	24	25	26	27	28
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07 i.e $X(0) = \sqrt{4^2 + 0^2} = 4 //$

$X(1) = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2} //$

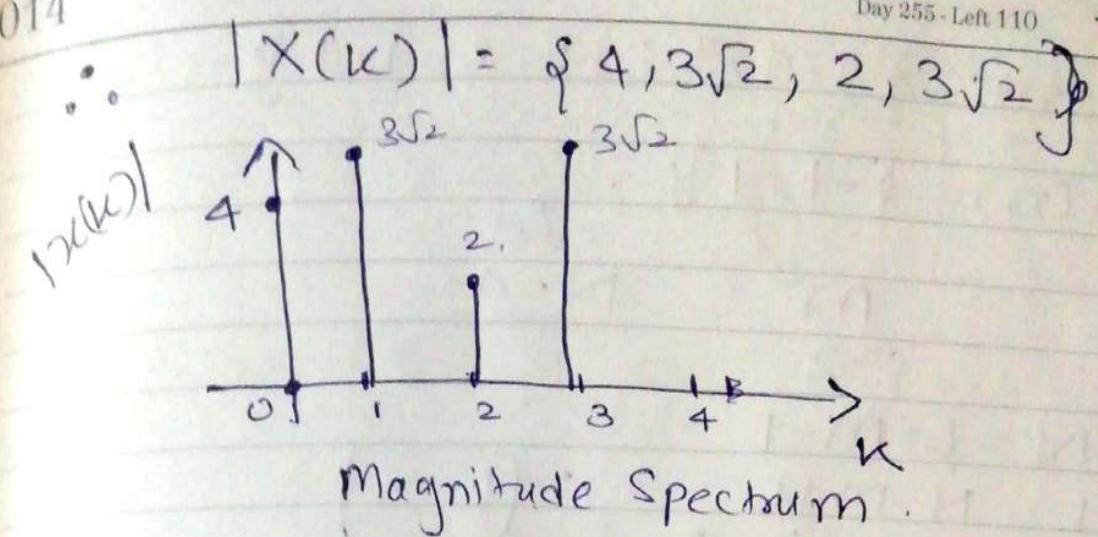
$X(2) = \sqrt{2^2 + 0^2} = 2 //$

$X(3) = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2} //$

Week 8
2014

Friday
SEPTEMBER
Day 255 - Left 110

12



Now Phase Spectrum formula.

$$\angle X(k) = \tan^{-1} \frac{X_i(k)}{X_R(k)}$$

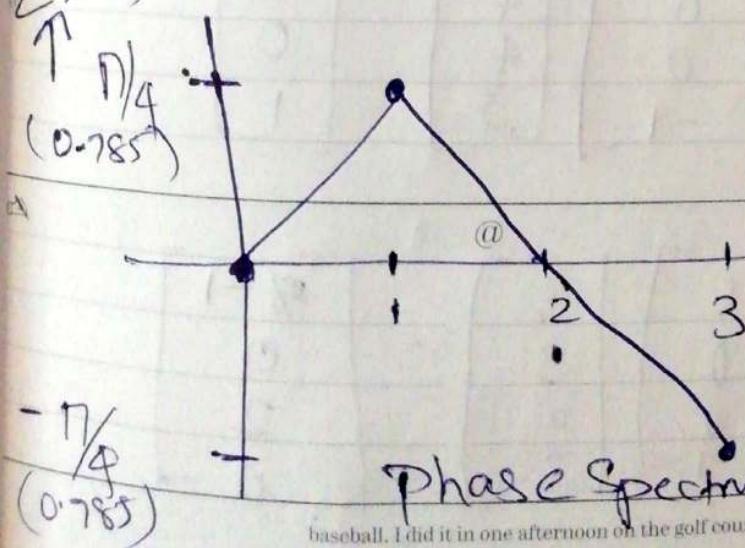
$$\angle X(0) = \tan^{-1} \left(\frac{0}{4} \right) = 0$$

$$\angle X(1) = \tan^{-1} \left(\frac{-3}{3} \right) = \tan^{-1}(1) = \frac{\pi}{4}$$

$$\angle X(2) = \tan^{-1} \left(\frac{0}{2} \right) = 0$$

$$\angle X(3) = \tan^{-1} \left(\frac{-3}{3} \right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\angle X(k) = (0, \frac{\pi}{4}, 0, -\frac{\pi}{4})$$



baseball. I did it in one afternoon on the golf course... - Hank Aaron

Overlap Save Method

13

Saturday
SEPTEMBER

Day 256 - Left 109

Week 2014

$$x(n) = \{4, 4, 3, 3, 2, 2, 1, 1\}$$

$$h(n) = \{-1, 1\}$$

$L = 3$ $M = 2$ $N = 4$

Let

$$\begin{aligned} N &= L + M - 1 \\ L &= N - M + 1 \\ &= 4 - 2 + 1 = 3 \end{aligned} \quad \therefore \boxed{L = 3}$$

$$x_1(n) = \{4, 4, 3\}$$

$$x_2(n) = \{3, 2, 2\}$$

$$x_3(n) = \{2, 1, 0\}$$

$$h(n) = \{-1, 1, 0\}$$

Matrix Multiplication Method to find Convolution.

$$y_1(n) = x_1(n) \otimes h(n)$$

$$-y_1(n) = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 0 \\ 1 \end{bmatrix}$$

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15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} @ \begin{bmatrix} 3 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

next shot is eighty percent of winning golf. - Ben Hogan

Sunday
SEPTEMBER
14

Day 257 - Left 108

Week 37
2014

$$y_3(n) = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$y_1(n) = \underbrace{\begin{matrix} 3 & -4 & 0 & 1 \\ \uparrow & \curvearrowleft & & \end{matrix}}_{\text{disorderd}} \quad (\because L=3)$$

$$\begin{matrix} -1 & 0 & 1 & 0 & 1 \end{matrix}$$

$$\begin{matrix} \uparrow \\ \text{discord} \end{matrix}$$

$$\begin{matrix} -2 & 1 & 0 & 1 \end{matrix}$$

$$\begin{matrix} \uparrow \\ \text{discord} \end{matrix}$$

$$y(n) = \{ -4, 0, 1, 0, 1, 0, 1, 0, 1 \}$$

@

OCTOBER 2014						
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41	6	7	8	9	10	11

Determine Response of LTI sys using RAD₂X:2 DITFFT

15 Monday
SEPTEMBER

Day 258 - Left 107

Week 10
2014

08 $x(n) = \{1, 2, 1\}$
 $h(n) = \{1, 3\}$

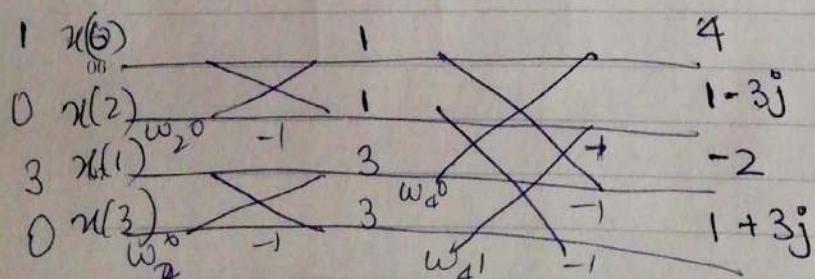
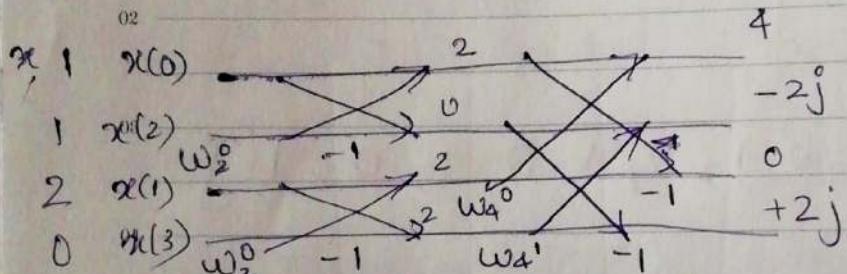
09 $\therefore N_1 = 3$

10 $N_2 = 2$

11 $N = \text{length Output Signal} = N_1 + N_2 - 1$
 $= 3 + 2 - 1 = \underline{\underline{4}}$

12 $\therefore x(n) = \{1, 2, 1, 0\}$
 $h(n) = \{1, 3, 0, 0\}$

01 \therefore Using 4 Point DITFFT



07 SEPTEMBER 2014
 wk M T W T F S S
 00 1 2 3 4 5 6 7
 01 8 9 10 11 12 13 14
 02 15 16 17 18 19 20 21
 03 22 23 24 25 26 27 28
 04 29 30 $H(k) = \{4, 1-3j, -2, 1+3j\}$

Using Circular Convolution property of DFT

05 $y(n) = x(n) \otimes h(n) \xrightarrow{\text{DFT}} X(k) \cdot H(k) = Y(k)$

Revolution is the festival of the oppressed - Germaine Greer

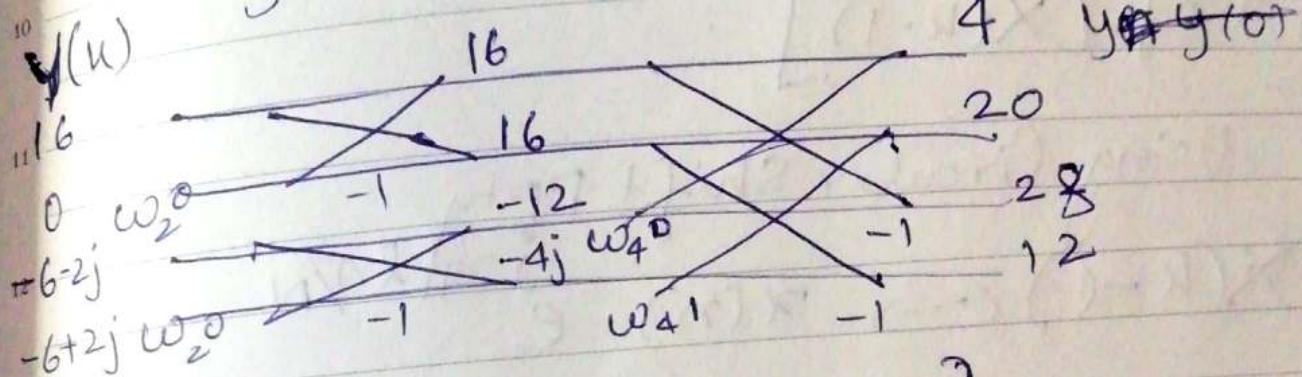
Tuesday
SEPTEMBER 16
Day 259 - Left 106

16, 4
~~-6 - 2j~~
~~-6 + 2j~~
Week 38
2014 16 - 4 = 12

$$Y(u) = X(u) \cdot H(u)$$

$$Y(u) = \{16, -6 - 2j, 0, -6 + 2j\} \cdot \{4, 1 - 3j, -2, 1 + 3j\}$$

Taking Inverse DIT FFT. Bcoz we want $y(n)$



$$\frac{1}{4} (4, 20, 28, 12)$$

$$y(n) = (1, 5, 7, 3)$$

@

OCTOBER 2014						
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40	.	.	1	2	3	4
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42	13	14	15	16	17	18
43	20	21	22	23	24	25
44	27	28	29	30	31	.

DFT PPTy SUMS

17

Wednesday
SEPTEMBER

Day 260 - Left 105

Week 2
2014

Q. If IDFT $[x(k)] = x(n) = \{2, 1, 2, 0\}$

$$\Rightarrow x(n) = \{2, 1, 2, 0\} \\ \therefore N=4$$

i) IDFT of $[x(k-1)]$

Using Circular Shift Ppty
 $\downarrow j^{2\pi ln/N}$

$$x(k-1) \Leftrightarrow x(n) \cdot e^{-j\frac{2\pi n}{N}}$$

$$\Leftrightarrow x(n) e^{-j\frac{2\pi n}{4}}$$

$$\Leftrightarrow x(n) e^{-j\frac{\pi n}{2}}$$

$$\Leftrightarrow x(n) j^n$$

$$\therefore \text{IDFT } [x(k-1)] = \{2, j, -2, 0\}$$

ii) IDFT of $[x(k) \text{ circularly convolved with } x(k)]$

$$\text{DFT } [x_1(n) \cdot x_2(n)] = \frac{1}{N} [x_1[n] \otimes x_2[n]]$$

$$\text{IDFT } [x_1[n] \otimes x_2[n]] = N \cdot [x_1(n) \cdot x_2(n)]$$

SEPTEMBER 2014						
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15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

$$\begin{aligned} & \therefore = 4 \cdot [x_1(n) \cdot x_2(n)] \\ & = 4 \cdot [4, 1, 4, 0] \\ & = \underline{16, 4, 16, 0} \end{aligned}$$

Reality is merely an illusion, albeit a very persistent one - Albert Einstein

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Thursday

8/3

8/3

8/3

8/3

8/3
Thursday

SEPTEMBER

18

Week - 38

2014

Day 261 - Left 104

(iii) IDFT

$$[x(u) \cdot x(u)]$$

$$\text{DFT } [x_1(n) * x_2(n)] \Leftrightarrow x_1(u) \cdot x_2(u).$$

$$\text{IDFT } x_1(u) \cdot x_2(u) = [x_1(n) * x_2(n)]$$

$$\begin{aligned} &= \begin{bmatrix} 2 & 0 & 2 & 1 \\ 1 & 2 & 0 & 2 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 9 \\ 4 \end{bmatrix} // \end{aligned}$$

$$\therefore y(n) = x(n) * x(n) = [8, 4, 9, 4].$$

iv

Signal Energy

using Parseval's Energy Theorem.

$$E = \sum_{n=0}^{N-1} |x(n)|^2$$

$$= \sum_{n=0}^3 |x(n)|^2 =$$

$$E = 2^2 + 1^2 + 2^2 + 0^2$$

$$\boxed{E = 9}$$

@

OCTOBER

2014

	M	T	W	T	F	S	S
40				1	2	3	4
41	6	7	8	9	10	11	12
42	13	14	15	16	17	18	19
43	20	21	22	23	24	25	26
44	27	28	29	30	31		

19

Friday

SEPTEMBER

Day 262 - Left 103

Karl Pearson's Coefficient

Week 2
2014

Q. Evaluate Karl's Coefficient for two given sequences.

09-

$$X(n) = \{1, 3, 4, 2\}$$

10

$$Y(n) = \{1, 2, 2, 1\}$$

11

→ Formula to be used:

12

~~$$\gamma = \frac{\sum xy}{\sqrt{\sum x^2} \sqrt{\sum y^2}}$$~~

01

02

$$\text{where } X = (x(n) - \bar{x}) \text{ & } Y = (y(n) - \bar{y})$$

03

	X(n)	y(n)	X	Y	XY	X ²	Y ²
1	1	1	-1.5	-0.5	0.75	2.25	0.25
2	3	2	0.5	0.5	0.25	0.25	0.25
3	4	2	1.5	0.5	0.75	2.25	0.25
4	2	1	-0.5	-0.5	0.25	0.25	0.25
Σ	10	6			2	5	1
Avg	2.5	1.5					

07

Putting values in formula

$$\gamma = \frac{\sum xy}{\sqrt{\sum x^2} \sqrt{\sum y^2}}$$

$$\gamma = \frac{2}{\sqrt{5} \sqrt{1}} = \underline{\underline{0.894}}$$

08

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$$\therefore \underline{\underline{\gamma = 0.894}}$$

@

SEPTEMBER 2014						
wk	M	T	W	T	F	S
06	1	2	3	4	5	6
07	7	8	9	10	11	12
08	13	14	15	16	17	18
09	19	20	21	22	23	24
10	25	26	27	28	29	30

Overlap Add Method

week 38
2014

Saturday
SEPTEMBER
Day 263 - Left 102

20

$$Q. \quad x(n) = \{3, 4, 2, 1, 2, 2, 1, 1\}$$

$$h(n) = \{1, -1\}$$

$$L \quad M=2 \quad N=4$$

$$L = N - m + 1 = 4 - 2 + 1$$

$$\underline{L=3}$$

$$x_1(n) = \{3, 4, 2, 0\}$$

$$x_2(n) = \{1, 2, 2, 0\}$$

$$x_3(n) = \{1, 1, 0, 0\}$$

$$h(n) = \{1, -1, 0, 0\}$$

Using matrix multiplication to find Circular Convolution

$$y_1(n) = x_1(n) \oplus h(n)$$

$$\therefore y_1(n) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \\ -2 \end{bmatrix}$$

$$\therefore y_2(n) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$y_3(n) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

OCTOBER 2014						
wk	M	T	W	T	F	S
1					1	2
2					3	4
3					5	
4					6	7
5					8	9
6					10	11
7					12	
8					13	14
9					15	16
10					17	18
11					19	
12					20	21
13					22	23
14					24	25
15					26	
16					27	28
17					29	30
18					31	

21

Sunday
SEPTEMBER

Day 264 - Left 101

$$y_1(n) \quad 3 \quad 1 \quad -2 \quad -2 \quad y_2(n)=1 \quad 1 \quad 0 \quad -2 \quad y_3(n)=1, 0, -1, 0$$

$$\begin{array}{r}
 09 \quad 3 \ 1 \ -2 \ -2 \\
 + \quad \quad \quad 1 \ 1 \ 0 \ -2 \\
 \hline
 10 \quad 3 \ 1 \ -2 \ -1 \ 1 \ 0 \ -1 \ 0 \ -1 \ 0
 \end{array}$$

$$\therefore y(n) = \{3, 1, -2, -1, 1, 0, -1, 0, -1, 0\}.$$

Evaluating Nyquist Rate

Monday
SEPTEMBER
Day 265 - Left 100

22

Week - 39
2014 Consider Analog Signal

$$x(t) = 5\cos 2\pi(1000t) + 10\cos 2\pi(5000t)$$

D: to be sampled

i) Evaluate Nyquist Rate.

ii) If the signal is sampled at 4kHz, will the signal be recovered from its samples?

$$\rightarrow x(t) = 5\cos 2\pi(1000t) + 10\cos 2\pi(5000t)$$

$\uparrow \quad \uparrow$
 $F_1 \quad F_2$

Compare with standard eqn $x(t) = A\cos \omega t$.

$$\omega = 2\pi f$$

i.e. $A\cos 2\pi ft$.

$$F_1 = 1000 \text{ Hz}$$

$$\& F_2 = 5000 \text{ Hz}$$

$$\therefore F_{\max} = \max(F_1, F_2) = 5000 \text{ Hz}$$

For proper reconstruction of signal,

$$f_s \geq 2 * F_{\max}$$

$$f_s \geq 2 * 5000 \text{ Hz}$$

$$f_s \geq 10000 \text{ Hz}$$

The minimum value of Sampling frequency is called as Nyquist Rate.

$$\therefore \text{Nyquist Rate} = \underline{\underline{10,000 \text{ Hz}}} = 10 \text{ kHz}$$

ii) Since, $4000 \text{ Hz} < \text{Nyquist Rate}$

∴ Reconstruction of signal
NOT Possible.

OCTOBER 2014						
wk	M	T	W	T	F	S
40				1	2	3
41	4	5	6	7	8	9
42	10	11	12	13	14	15
43	16	17	18	19	20	21
44	22	23	24	25	26	27
	28	29	30	31		

23

Tuesday SEPTEMBER

Day 266 - Left 99

Evaluate DFT (5 marks)

wala.

Week 8
2014

Q8. Evaluate DFT of $x(n) = \cos(0.25\pi n)$

Taking $N=4$

$$\therefore x(0) = \cos(0.25\pi \times 0) \\ = 1$$

$$x(1) = \cos(0.25\pi \times 1) \\ = 0.707$$

$$x(2) = \cos(0.25\pi \times 2) \\ = 0$$

$$x(3) = \cos(0.25\pi \times 3) \\ = -0.707$$

$$\therefore x(n) = \{1, 0.707, 0, -0.707\}$$

$$x(k) = W_0 \times x(n)$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & \frac{1}{j} & 1 & \frac{1}{j} \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0.707 \\ 0 \\ -0.707 \end{bmatrix} = \begin{bmatrix} 1 \\ 1-j1.414j \\ 1 \\ 1+j1.414j \end{bmatrix}$$

$$x(k) = \{1, 1-j1.414, 1, 1+j1.414\}$$

Alternate Method for Finding DFT

is using [@]
DIT-FFT
(Butterfly Diagram)

SEPTEMBER 2014

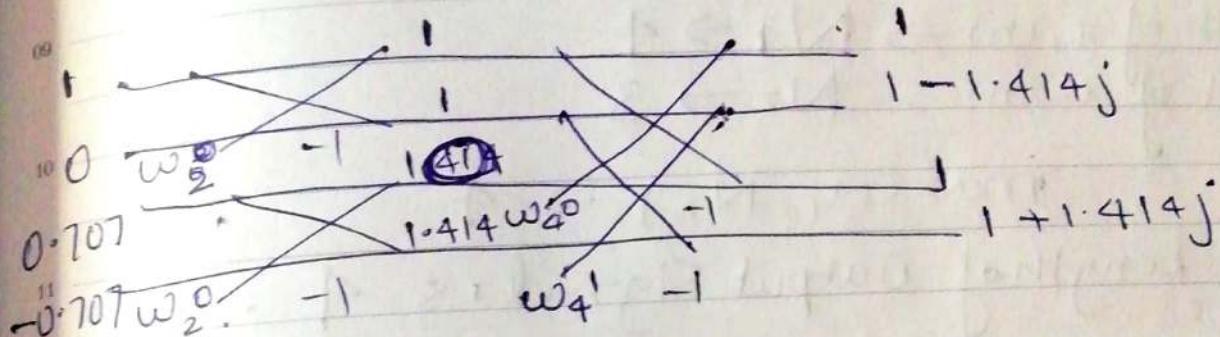
Wk	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21

Week - 39
2014

Wednesday
SEPTEMBER 24
Day 267 - Left 98

Heritage Day (ZA)

$$x(n) = \{ 1, 0.707, 0, -0.707 \}$$



$$x(k) = \{ 1, 1 - 1.414j, 1, 1 + 1.414j \}$$

Same Arg ↑

OCTOBER

25

Thursday
SEPTEMBER

Day 268 - Left 97

Circular Convolution
(5 marks)

2014

08 $x_1(n) = \{3, 2, 4, 1\}$

$x_2(n) = \{2, 1, 3\}$

09 Length of $x_1(n) \rightarrow N_1 \rightarrow 4$

10 Length of $x_2(n) \rightarrow N_2 \rightarrow 3$

11 $\max(N_1, N_2) \rightarrow 4$

12 ∴ Length of Output Signal is 4.

01 $x_1(n) = \{3, 2, 4, 1\}$

01 $x_2(n) = \{2, 1, 3, 0\}$

02
$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 2 & 0 & 3 & 1 \\ 1 & 2 & 0 & 3 \\ 3 & 1 & 2 & 0 \\ 0 & 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 10 \\ 19 \\ 12 \end{bmatrix}$$

↑
'0' Padded

05 ∴ $y(n) = x_1(n) \odot x_2(n) = \{19, 10, 19, 12\}$

06

07

Check whether the system is
nonlinear

Week-39

2014

$$y(n) = x(n) + 2x(n-2)$$

Friday SEPTEMBER Day 269 - Left 96

26

I) Static or Dynamic.

$$y(n) = x(n) + 2x(n-2)$$

Let's take $n = 1$

$$= x(1) + 2x(1-2) = x(1) + 2x(-1)$$

↑
Post input value

∴ Since calculation of output signal requires past input values, the signal is Dynamic.

II) Linear or Nonlinear

$$y(n) = x(n) + 2x(n-2) \rightarrow ①$$

Step I)

$$y_1(n) = x_1(n) + 2x_1(n-2)$$

$$y_2(n) = x_2(n) + 2x_2(n-2)$$

$$y'(n) = y_1(n) + y_2(n)$$

$$= x_1(n) + x_2(n) + 2x_1(n-2) + 2x_2(n-2)$$

Step II) Replace $x(n) = x_1(n) + x_2(n)$ in eqn ①.

$$\therefore y'(n) = x_1(n) + x_2(n) + 2x_1(n-2) + 2x_2(n-2)$$

$$\therefore y'(n) = y(n)$$

System is linear

OCTOBER 2014						
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42	13	14	15	16	17	18
43	20	21	22	23	24	25
	27	28	29	30	31	

27

Saturday
SEPTEMBER
Day 270 - Left 95.

201

III Causal or Non Causal

$$y(n) = x(n) + 2x(n-2)$$

Let's Putting n=1

$$\begin{aligned} y(1) &= x(1) + 2x(1-2) \\ &= x(1) + 2x(-1) \end{aligned}$$

↑ ↑
current input past input

∴ No future input value is involved
System is Causal.

IV Shift Variant or Shift Invariant

$$y(n) = x(n) + 2x(n-2)$$

Step I) Delaying Output Signal by k units
ie n = n-k (Replace n = n-k)

$$y(n-k) = x(n-k) + 2x(n-2-k)$$

Step II) Delaying Input Signal (Replace)

$$y(n) = x(n-k) + 2x(n-2-k)$$

SEPTEMBER 2014						
wk	M	T	W	T	F	S
26	1	2	3	4	5	6
27	8	9	10	11	12	13
28	15	16	17	18	19	20
29	22	23	24	25	26	27
30	29	30				

$y(n-k) = y(n)$ ∵ the System is
Shift Invariant

Illusion and wisdom combined are the charm of life and art. - Joseph Joubert