Adwait Purao 202130010

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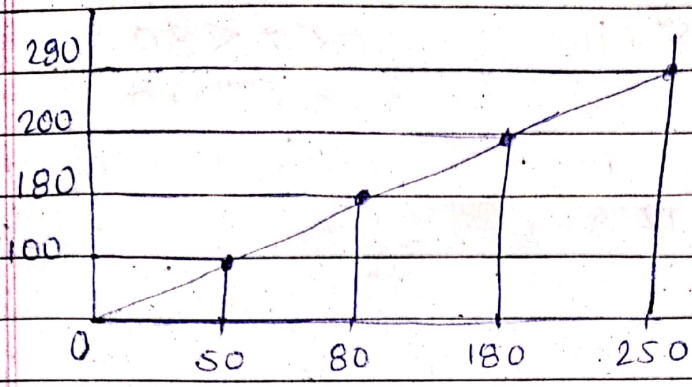
Assignment 3

Q.1



Case 1: $0 \leq r \leq 50$ $\quad [0,50] \rightarrow [0,100]$

$$\alpha = \frac{100}{50} = 2$$

Case 2: $50 \leq r \leq 80$ $\quad [50-80] \rightarrow [100-150]$

$$\text{slope} = \frac{150-100}{80-50} = \frac{5}{3}$$

$$S = (r-50)\cdot\frac{5}{3} + 100$$

Case 3: $80 \leq r \leq 1$ $\quad [80-130] \rightarrow [150-200]$ ~~$[100-150]$~~

$$\text{Slope} = \frac{200-150}{130-80} = 1$$

Case 4: $150 < r \leq 200$ $\quad [130-250] - [200-250]$

$$\text{Slope} = \frac{250-200}{250-130} = \frac{50}{120} = \frac{5}{12}$$

$$S = (r-130)\cdot\frac{5}{12} + 200$$

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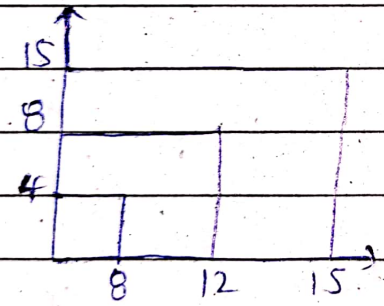
$$S = \begin{cases} 2r & 0 \le r \le 50 \\ (r-50)\cdot \frac{5}{3} + 100 & 50 < r \le 80 \\ (r+70) & 80 \le r < 130 \\ (r-130)\cdot \frac{5}{12} + 200 & 130 \le r \le 250 \end{cases}$$

(Q.2) $F = \begin{bmatrix} 7 & 12 & 2 & 3 & 4 \\ 10 & 15 & 1 & 6 & 7 \\ 12 & 4 & 6 & 19 & 12 \\ 8 & 2 & 7 & 15 & 2 \\ 11 & 13 & 3 & 3 & 5 \end{bmatrix}$



$a = 8,\ b = 12,\ (L-1) = 15$

$S_1 = 4,\ S_2 = 8$

$$\alpha = \frac{S_1}{a} = \frac{4}{8} = \frac{1}{2}$$

$$\beta = \frac{S_2 - S_1}{b - a} = \frac{8-4}{12-8} = 4/4 = 1$$

$$\gamma = \frac{(L-1) - S_2}{(L-1) - b} = \frac{15-8}{15-12} = \frac{7}{3}$$

$$S = \begin{cases} 1/2\, r & 0 \le r \le 8 \\ 4 + 1(r-8) & 8 \le r \le 12 \\ 8 + 7/3(r-12) & 12 \le r \le 15 \end{cases}$$

$$S = \begin{cases} 1/2\, r \\ r - 4 \\ 8 + \frac{7}{3} r - \frac{84}{3} \end{cases}$$

$$S = \begin{cases} 1/2\, r & 0 \le r \le 8 \\ r - 4 & 8 \le r \le 12 \\ 24 + 7r - 84 & 12 \le r \le 15 \end{cases}$$

$$S = \begin{cases} 1/2\, r & 0 \le r \le 8 \\ r - 4 & 8 \le r \le 12 \\ 7r - 60 & 12 \le r \le 15 \end{cases}$$

O/p image =

A = ~~4 8 12~~

$$A = \begin{bmatrix} 4 & 8 & 1 & 2 & 2 \\ 6 & 15 & 1 & 3 & 4 \\ 8 & 2 & 3 & 15 & 8 \\ 4 & 1 & 4 & 15 & 1 \\ 7 & 10 & 2 & 2 & 3 \end{bmatrix}$$

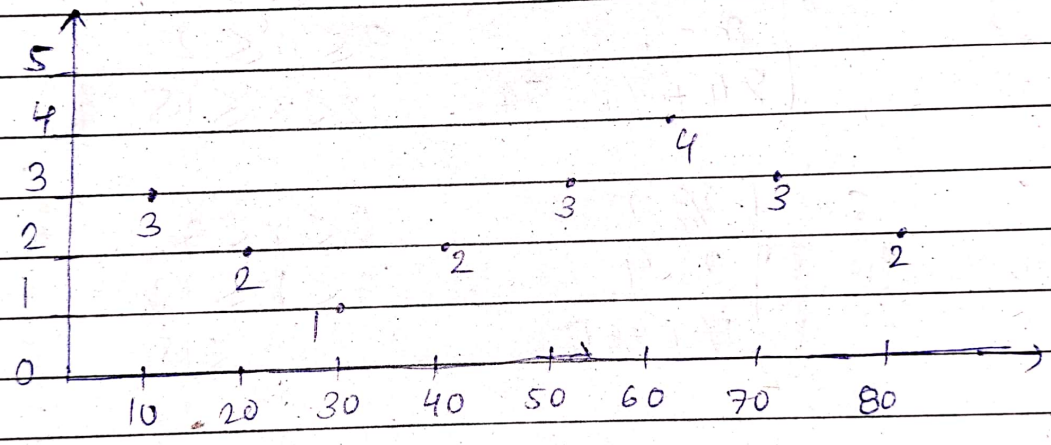
Q.3

$F =$

0	30	0	10	0
10	70	40	50	20
20	60	60	80	80
70	40	0	10	0
50	60	70	60	50

Frequency:

Pq.	0	10	20	30	40	50	60	70	80
Freq.	5	3	2	1	2	3	4	3	2



Threshold = 40

To find o/p by thresholding

$$S = \begin{cases} 0, & r < 40 \\ 80, & r \geq 40 \end{cases}$$

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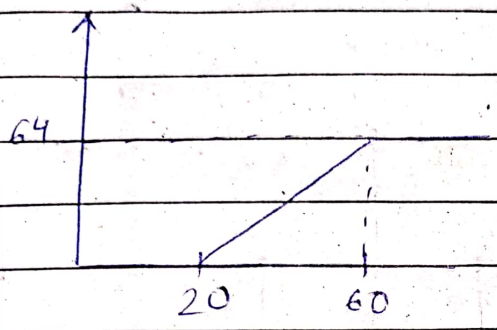
o/p Image = $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 80 & 80 & 80 & 0 \\ 0 & 80 & 80 & 80 & 80 \\ 80 & 80 & 0 & 0 & 0 \\ 80 & 80 & 80 & 80 & 80 \end{bmatrix}$

4 a)



$$S = \begin{cases} 0 & 0 < r \leq 20 \\ 1.6r - 32 & 20 < r \leq 60 \\ 64 & 60 < r \end{cases}$$

(i) $20 < r \leq 60$

[20,60] (0,64)

$$\text{Slope} = \frac{64-0}{60-20} = \frac{64}{40} = 1.6$$

$$S = 1.6r - 32$$

(b) $f = \begin{bmatrix} 20 & 50 & 30 & 50 \\ 30 & 60 & 50 & 30 \\ 30 & 90 & 60 & 40 \\ 20 & 50 & 40 & 50 \end{bmatrix}$

Applying above transformation

$S = \begin{bmatrix} 0 & 48 & 16 & 48 \\ 16 & 64 & 48 & 16 \\ 16 & 48 & 64 & 32 \\ 0 & 48 & 32 & 48 \end{bmatrix}$

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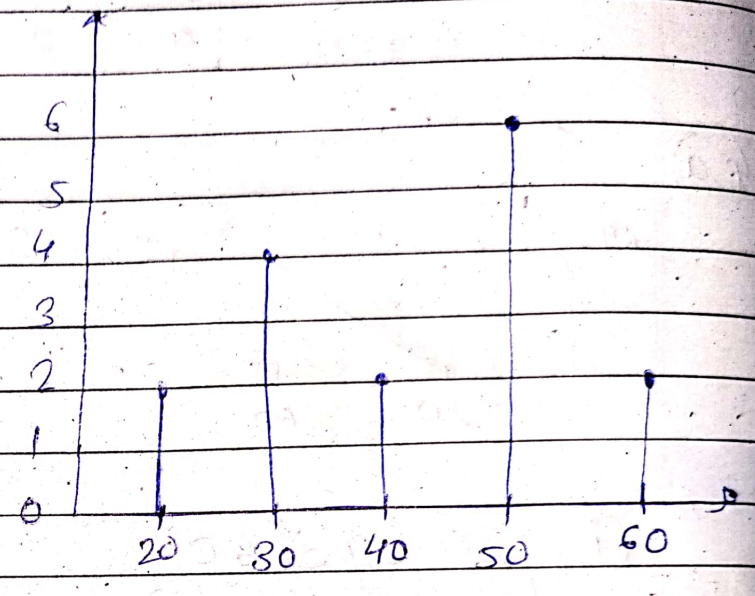
(c) (i) i/p image

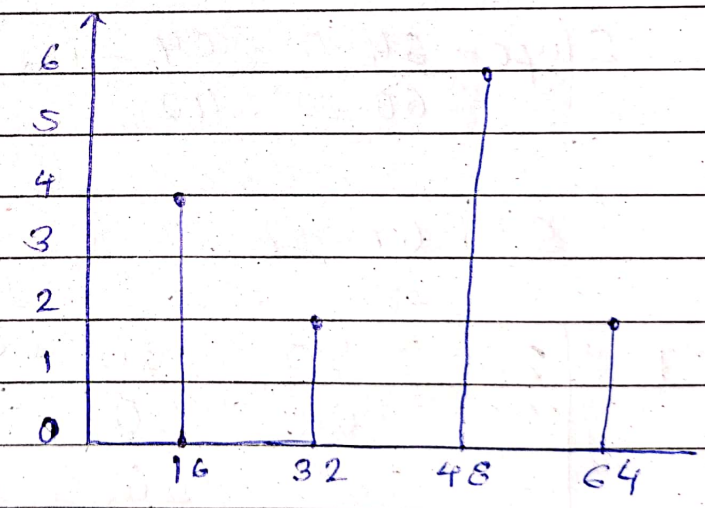
value	freq.
20	2
30	4
40	2
50	6
60	2

Histogram



(ii) o/p image

value	freq.
0	2
18	4
32	2
48	6
64	2



1) o/p image is stretched from [20-60] to [16-64]

2) i/p image has limited range

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9.5) $F(x,y) = \begin{bmatrix} 20 & 90 & 30 & 50 \\ 30 & 60 & 50 & 30 \\ 30 & 50 & 60 & 40 \\ 20 & 50 & 40 & 90 \end{bmatrix}$

Transformation function:

$$A(x,y) = 120 \log_{10}(1 + F(x,y))$$

$A(x,y) = \begin{bmatrix} 159 & 205 & 179 & 205 \\ 179 & 214 & 205 & 179 \\ 179 & 209 & 214 & 194 \\ 159 & 205 & 194 & 205 \end{bmatrix}$

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Q.6

$$F = \begin{bmatrix} 20 & 50 & 30 \\ 30 & 0 & 50 \\ 50 & 50 & 64 \end{bmatrix}$$

Normalized pixel value $= \frac{\text{I/P pixel value}}{\text{Max. value}}$

$$B = \begin{bmatrix} 0.313 & 0.781 & 0.469 \\ 0.469 & 0 & 0.781 \\ 0.469 & 0.781 & 1 \end{bmatrix}$$

$S = r^2$

$$C = \begin{bmatrix} 0.098 & 0.610 & 0.220 \\ 0.220 & 0 & 0.610 \\ 0.220 & 0.610 & 1 \end{bmatrix}$$

$$64 \times C = \begin{bmatrix} 6 & 39 & 14 \\ 14 & 0 & 39 \\ 14 & 39 & 64 \end{bmatrix}$$

Q.7

$$F = \begin{bmatrix} 0 & 1 & 0 & 2 & 1 & 6 \\ 2 & 3 & 5 & 5 & 1 & 6 \\ 2 & 4 & 4 & 3 & 2 & 1 \\ 1 & 3 & 5 & 3 & 0 & 1 \\ 0 & 1 & 2 & 6 & 6 & 2 \end{bmatrix}$$

$$S = \begin{cases} 7 & ; 3 \le r \le 5 \\ r & ; \text{else} \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 & 1 & 7 \\ 2 & 7 & 7 & 7 & 1 & 7 \\ 2 & 7 & 7 & 7 & 2 & 1 \\ 1 & 7 & 7 & 7 & 0 & 1 \\ 2 & 1 & 2 & 6 & 7 & 2 \end{bmatrix}$$

Q.8

$$F = \begin{bmatrix} 4 & 2 & 3 & 0 \\ 1 & 3 & 5 & 7 \\ 5 & 3 & 2 & 1 \\ 2 & 4 & 6 & 7 \end{bmatrix}$$

(a) $L = 2^3 = 8$

$L - 1 = 7$

$$S = \begin{cases} 7 & ; r > 4 \\ 0 & ; \end{cases}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 7 \\ 7 & 0 & 0 & 0 \\ 0 & 7 & 7 & 7 \end{bmatrix}$$

(b) $$S = \begin{cases} 7 & ; 2 < r < 5 \\ r & ; \text{else} \end{cases}$$

$$A = \begin{bmatrix} 7 & 2 & 7 & 0 \\ 1 & 7 & 5 & 7 \\ 5 & 7 & 2 & 1 \\ 2 & 7 & 6 & 7 \end{bmatrix}$$

(c) $r = [b2, b1, b0]$

If $b2 = 1$ $S = 7$

else $S = 0$

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$$F = \begin{bmatrix} 100 & 010 & 011 & 0 \\ 001 & 011 & 101 & 111 \\ 101 & 011 & 010 & 001 \\ 010 & 100 & 110 & 111 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 0 & 7 & 7 \\ 7 & 0 & 0 & 0 \\ 0 & 7 & 7 & 7 \end{bmatrix}$$

(a) $S = (L-1) - r$

$= 7 - r$

$$A = \begin{bmatrix} 3 & 5 & 4 & 7 \\ 6 & 4 & 2 & 0 \\ 2 & 4 & 5 & 6 \\ 5 & 3 & 1 & 0 \end{bmatrix}$$

Q.9

i/p	Norm	Freq.	PDF	CDF	o/p
0	0	70	0.7	0.7	2
1	1/3	20	0.2	0.9	3
2	2/3	7	0.07	0.97	3
3	1	3	0.03	1	3

o/p	freq.
0	0
1	0
2	70
3	30

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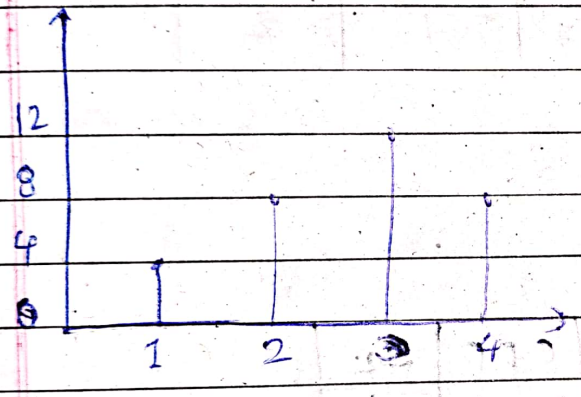
Q10.

i/p	Norm.	Freq.	PDF	CDF	Equalized
0	0	4	0.125	0.125	1
1	1/4	8	0.25	0.375	2
2	1/2	8	0.25	0.625	3
3	3/4	4	0.125	0.75	3
4	1	8	0.25	1	4

Equalized Histogram:



r_k	N_{sk}
0	0
1	4
2	8
3	12
4	8

Q.11

Input (A)

Grayscale	0	1	2	3	4	5	6	7
Freq.	2	3	5	6	7	12	14	13

Specified Image (B)

Grayscale	0	1	2	3	4	5	6	7
Freq.	13	12	14	14	11	0	0	0

(i) Equalize A:

Input	Norm	Freq.	PDF	CDF	S_k
0	0	2	0.031	0.031	0
1	1/7	3	0.046	0.077	1
2	2/7	5	0.078	0.155	1
3	3/7	6	0.093	0.248	2
4	4/7	9	0.14	0.388	3
5	5/7	12	0.187	0.575	4
6	6/7	14	0.218	0.793	6
7	7/7	13	0.207	1	7

(ii) Equalized B:

i/p	Norm	Freq.	PDF	CDF	S_k
0	0	13	0.203	0.203	1
1	1/7	12	0.187	0.39	3
2	2/7	14	0.218	0.608	4
3	3/7	14	0.218	0.826	6
4	4/7	11	0.171	1	7
5	5/7	0	0	1	7
6	6/7	0	0	1	7
7	7/7	0	0	1	7

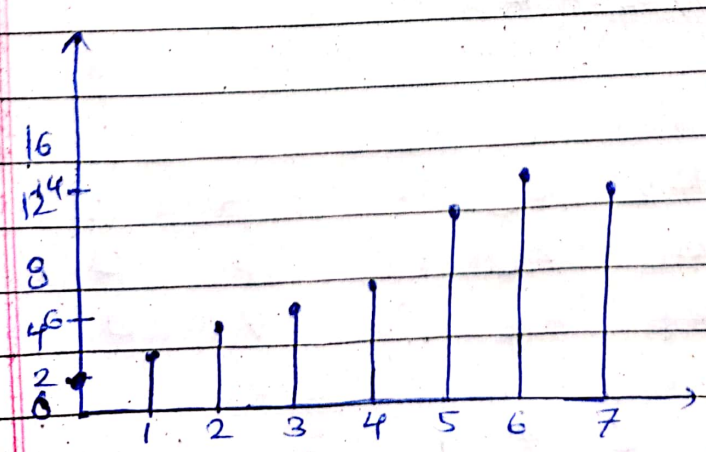
(iii) Equalize A as per B:

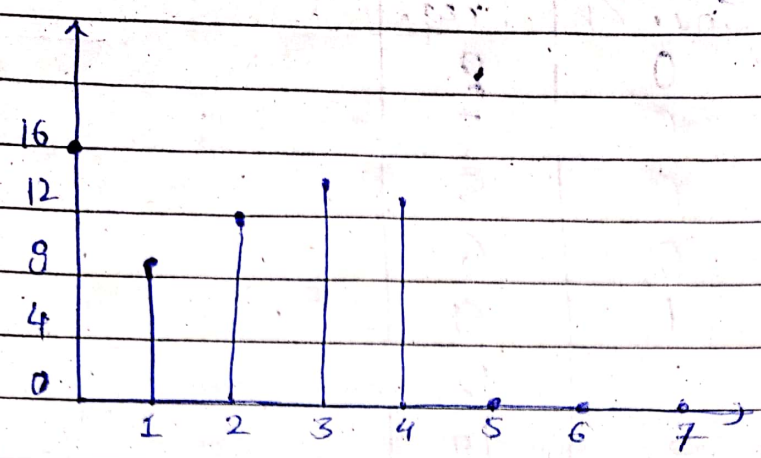
I/p	S_k	V_k	Inv. Z_k	Freq.
0	0	1	0	2
1	1	1	0	3
2	1	1	0	5
3	2	1	0	6
4	3	3	1	9
5	4	4	2	12
6	6	6	3	14
7	7	7	4	13

(iv) Freq. table of o/p image

0	16
1	9
2	12
3	14
4	13

Histogram (Input A)





Q12 $F(x,y) = \begin{bmatrix} 3 & 2 & 1 \\ 9 & 2 & 6 \\ 7 & 9 & 1 \end{bmatrix}$

a) Mask - 1 → z_1 z_2 z_3 z_4 . z_6 z_7 z_8 z_9 → 3 2 1 5 6 7 9 1

1 1 2 3 5 6 7 9

Median = (3 + 5) / 2 = 4

b) Mask-2 → z_1 z_2 z_3 z_4 z_5 z_6 z_7 z_8 z_9 ⇒ 3 2 1 5 2 6 7 9 1

1 1 2 2 3 5 6 7 9

Median = 3

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Q.13 $F = \begin{bmatrix} 0 & 1 & 0 & 6 & 7 \\ 2 & 0 & 1 & 6 & 5 \\ 1 & 1 & (3) & 5 & 6 \\ 1 & 0 & 6 & 6 & 5 \\ 2 & 5 & 6 & 7 & 6 \end{bmatrix}$

(a) Calculate median of : $\begin{bmatrix} 0 & 1 & 6 \\ 1 & (3) & 5 \\ 0 & 6 & 6 \end{bmatrix}$

0 0 1 1 (3) 5 6 6 6

Median = 3

replace F(2,2) with 3
A(2,2) = 3

(b) $\text{Min}_w = 0$
A(2,2) = 0

window = $\begin{bmatrix} 0 & 1 & 6 \\ 1 & 3 & 5 \\ 0 & 6 & 6 \end{bmatrix}$

(c) Max = 6
A(2,2) = 6

Median is the non-linear order statistic filter where o/p at (x,y) is median of neighbourhood of i/p pixel.

Max is non-linear ordered statistic filter where o/p pixel at (x,y) is obtained by selecting max of neighbourhood of i/p pixel.

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Med is non linear ordered statistic filter where o/p pixel at (x,y) is obtained by selecting med of neighbourhood of i/p pixel.

Q14 Prewit filter

$$F = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 6 & 7 \\ 7 & 1 & 3 \end{bmatrix}$$

$$G(x) = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad G(y) = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Repeated border & pixel values

$$F = \begin{bmatrix} 3 & 3 & 2 & 5 & 5 \\ 3 & 3 & 2 & 5 & 5 \\ 4 & 4 & 6 & 7 & 7 \\ 7 & 7 & 1 & 3 & 3 \\ 7 & 7 & 1 & 3 & 3 \end{bmatrix}$$

Applying G(x) & G(y) to each pixel

$$A(x) = \begin{bmatrix} 6 & 7 & 8 \\ 7 & 1 & -5 \\ 1 & -6 & -13 \end{bmatrix} \qquad A(y) = \begin{bmatrix} 0 & 7 & 7 \\ -5 & 1 & 6 \\ -10 & -5 & 5 \end{bmatrix}$$

$$A = |Ax| + |Ay|$$

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$$= \begin{bmatrix} 6 & 14 & 15 \\ 12 & 2 & 11 \\ 11 & 11 & 18 \end{bmatrix}$$

Q.15)

Q.15) $F = \begin{bmatrix} 2 & 3 & 4 & 2 \\ 5 & 5 & 2 & 4 \\ 3 & 6 & 3 & 5 \\ 5 & 3 & 5 & 5 \end{bmatrix}$

LPF $\longrightarrow$

$$\begin{bmatrix} 1.67 & 2.33 & 2.22 & 1.33 \\ 2.67 & 3.67 & 3.78 & 2.22 \\ 3 & 4.1 & 4.22 & 2.67 \\ 1.89 & 2.78 & 3 & 2 \end{bmatrix}$$

$\cong$

$$\begin{bmatrix} 2 & 2 & 2 & 1 \\ 3 & 4 & 4 & 2 \\ 3 & 4 & 4 & 3 \\ 2 & 3 & 3 & 2 \end{bmatrix}$$

HPF

$$W = \frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

HPF $\longrightarrow$

$$\begin{bmatrix} -0.33 & 0.67 & 1.78 & 0.67 \\ 2.33 & 1.33 & -1.78 & 1.78 \\ 0 & 1.89 & -1.22 & 2.33 \\ 3.1 & 0.22 & 2 & 3 \end{bmatrix}$$

$\cong$

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 1 & -2 & 2 \\ 0 & 2 & -1 & 2 \\ 3 & 0 & 2 & 3 \end{bmatrix}$$

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LPF + HPF = $\begin{bmatrix} 2 & 3 & 4 & 2 \\ 5 & 5 & 2 & 4 \\ 3 & 6 & 3 & 5 \\ 5 & 3 & 5 & 5 \end{bmatrix}$

Q16) $F = \begin{bmatrix} 4 & 8 & 2 \\ 7 & 5 & 6 \\ 6 & 3 & 7 \end{bmatrix}$

Padded image

$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 8 & 2 & 0 \\ 0 & 7 & 5 & 6 & 0 \\ 0 & 6 & 3 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(i) $W_1 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ o/p $S_1 = \begin{bmatrix} 0 & 10 & 0 \\ 9 & 0 & 7 \\ 9 & 0 & 0 \end{bmatrix}$

(ii) $W_2 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ o/p $S_2 = \begin{bmatrix} 1 & 11 & 0 \\ 4 & 0 & 3 \\ 5 & 1 & 8 \end{bmatrix}$

(iii) $W_3 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ o/p $S_3 = \begin{bmatrix} 1 & 21 & 0 \\ 13 & 0 & 10 \\ 14 & 0 & 19 \end{bmatrix}$

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Q17 $F = \begin{bmatrix} 4 & 8 & 2 \\ 7 & 5 & 6 \\ 6 & 3 & 7 \end{bmatrix}$

Applying W_1, W_2, W_3 at (1,1)

$W_1 = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ $W_2 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$W_3 = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

$S_1(1,1) = 1$ $S_2(1,1) = -4$ $S_3(1,1) = 16$

Q18 $F = \begin{bmatrix} 4 & 8 & 2 \\ 7 & 5 & 6 \\ 6 & 3 & 7 \end{bmatrix}$

Padded Image

$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 8 & 2 & 0 \\ 0 & 7 & 5 & 6 & 0 \\ 0 & 6 & 3 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(i) Laplacian filtered image

$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ $S_1 = \begin{bmatrix} 1 & 21 & -6 \\ 13 & -4 & 10 \\ 14 & -16 & 19 \end{bmatrix}$

(ii) High Boost

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 12 & 40 & -3 \\ 30 & -3 & 23 \\ 33 & -7 & 42 \end{bmatrix}$$

Q 19

a) False
b) True
c) True
d) False
e) True
f) True
g) True
h) False
i) True
j) True
k) True
l) True