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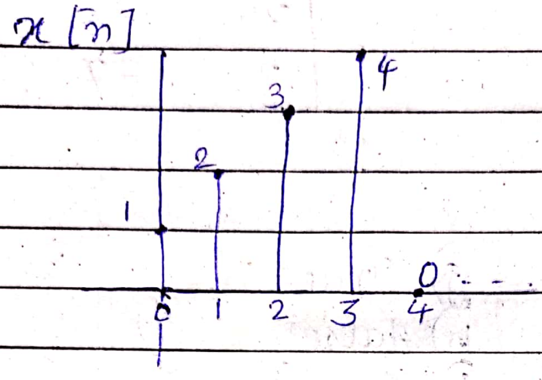
FOSIP Assignment: 2

Q1 a) $x[n] = \delta[n] + 2U[n-1] + U[n-2] + \delta[n-3] - 3U[n-4]$



$x[n] = \{1, 2, 3, 4\}$

To find $x[k]$, by D.F.T

$x(k) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nk}$ where $N = 4$, $W_N = e^{-j(\frac{2\pi}{N})}$

$x(k) = \sum_{0}^{3} x[n] \cdot W_N^{nk}$

$x(k) = x[0] \cdot W_N^{0} + x[1] \cdot W_N^{k} + x[2] \cdot W_N^{2k} + x[3] \cdot W_N^{3k}$

$x(k) = 1 + 2 \cdot W_N^{k} + 3W_N^{2k} + 4W_N^{3k}$

$k = 0$, $x(0) = 1 + 2 + 3 + 4 = 10$

$k = 1$, $x(1) = 1 + 2W_N + 3W_N^{2} + 4W_N^{3}$

where $W_N = e^{-j(\frac{2\pi}{4})} = e^{-j\pi/2} = -j$

$W_N^{2} = (-j)^{2} = -1$

$W_N^{3} = j$

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$\therefore x(1) = 1 + 2(-j) + 3(-1) + 4(j) = -2 + 2j$

$k = 2,\ x(2) = 1 + 2W_N^2 + 3W_N^4 + 4W_N^6$
$= 1 + 2(-1) + 3(-1)^2 + 4(-1)^3 = -2$

$k = 3,\ x(3) = 1 + 2W_N^3 + 3W_N^6 + 4W_N^9$
$= 1 + 2(j) + 3(j)^2 + 4(j)^3 = -2 - 2j$

$k = 4,\ x(4) = 1 + 2W_N^4 + 3W_N^8 + 4W_N^{12}$
$= 1 + 2 + 3 + 4$
$= 10$

$$\therefore x[k] = \begin{cases} 10 & k = 0 \\ -2 + 2j & k = 1 \\ -2 & k = 2 \\ -2 - 2j & k = 3 \end{cases}$$

(b) $x[n] = 3\cos(0.5\pi n)$

$\omega = 0.5\pi \Rightarrow 2\pi F_1 = 0.5\pi \Rightarrow F_1 = \frac{0.5}{2} = \frac{5}{20} = \frac{1}{4}$

$N = 4$ $\therefore$ Period of signal $= 4$

$\therefore$ We will calculate DFT for 4 samples

$n = 0,\ x[0] = 3\cos(0) = 3$
$n = 1,\ x[1] = 3\cos(0.5\pi) = 0$
$n = 2,\ x[2] = 3\cos(\pi) = -3$
$n = 3,\ x[3] = 3\cos(1.5\pi) = 0$
$n = 4,\ x[4] = 3\cos(2\pi) = 3$

values repeat

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$x[n] = \{3, 0, -3, 0\}$

By DFT:

$$x[k] = \sum_{0}^{N-1} x[n] \cdot W_N^{nk} \quad \text{where } N = 4$$

$$W_N = e^{-j(2\pi/N)}$$

$$x[k] = \sum_{0}^{3} x[k] \cdot W_N^{nk}$$

$$\therefore x[k] = x[0] \cdot W_N^{0} + x[1] W_N^{k} + x[2] \cdot W_N^{2k} + x[3] W_N^{3k}$$

In matrix form:

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$$x[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & k=0 \\ 6 & k=1 \\ 0 & k=2 \\ 6 & k=3 \end{bmatrix}$$

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(c) $x[n] = \{1,2,3,4,0,0,0,0\}$

By DFT, $x[k] = \sum_{0}^{N-1} x[n] \cdot W_N^{nk}$ where $N=8$ & $W_N = e^{-j(\frac{2\pi}{N})}$

$\therefore X[k] = x[0] \cdot W_N^0 + x[1] \cdot W_N^k + x[2] \cdot W_N^{2k} + x[3] \cdot W_N^{3k}$
$+ x[4] \cdot W_N^{4k} + x[5] \cdot W_N^{5k} + x[6] \cdot W_N^{6k} +$
$x[7] \cdot W_N^{7k}$

$\Rightarrow X[k] = 1 + 2W_N^k + 3W_N^{2k} + W_N^{3k}$

$X[0] = 1+2+3+4 = 10$

$X[1] = 1 + 2W_N + 3W_N^2 + 4W_N^3$
$= 1 + 2(0.707 - j0.707) + 3(-j) + 4(-0.707 - j0.707)$
$= -0.414 - j\,7.242$

$X[2] = 1 + 2W_N^2 + 3W_N^4 + 4W_N^6$
$= 1 + 2(-j) + 3(-1) + 4(j)$
$= -2 + 2j$

$X[3] = 1 + 2W_N^3 + 3W_N^6 + 4W_N^9$
$= 1 + 2(-0.707 - j0.707) + 3j + 4(0.707 - j0.707)$
$= 2.414 - j1.242$

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$X[4] = 1 + 2W_N^4 + 3W_N^8 + 4W_N^{12}$
$= 1 + 2(-1) + 3(-1)^2 + 4(-1)^3 = -2$

$X[5] = 1 + 2W_N^5 + 3W_N^{10} + 4W_N^{15}$
$= 1 + 2(-0.707 + j0.707) + 3(-j) + 4(0.707 + j0.707)$

$= 2.414 + 1.242j$

$X[6] = 1 + 2W_N^6 + 3W_N^{12} + 4W_N^{18}$
$= 1 + 2(j) + 3(j)^2 + 4(j)^3 = -2-2j$

$X[7] = 1 + 2W_N^7 + 3W_N^{14} + 4W_N^{21}$
$= 1 + 2(0.707 + j0.707) + 3(j) + 4(-0.707 + j0.707)$
$= -0.414 + \cancel{j0.7}\ j7.242$

$$X[k] = \begin{cases} 10 & k=0 \\ -0.414 - j7.242 & k=1 \\ -2+2j & k=2 \\ 2.414 - j1.242 & k=3 \\ -2 & k=4 \\ 2.414 + j1.242 & k=5 \\ -2-2j & k=6 \\ -0.414 + 7.242j & k=7 \end{cases}$$

Q2 $x[n] = \{1,2,3,4\}$
(a) $P[k] = 8\,x[k]$

To find, By DFT $= \sum_{0}^{N-1} x[n] \cdot W_N^{nk}$

$x[k] = x[0] \cdot W_N^0 + x[1] \cdot W_N^k + x[2] \cdot W_N^{2k} +$

$x[3] \cdot W_N^{3k}$

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$= 1 + 2W_N^{k} + 3W_N^{2k} + 4W_N^{3k}$

$$X[k] = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{cases} 10 & k=0 \\ -2+2j & k=1 \\ -2 & k=2 \\ -2-2j & k=3 \end{cases}$$

To find $P[k]$ when $P[k] = 8 \cdot x[k]$

By scaling & linearity property

If $x_1[n] \longrightarrow X_1[k]$, then $DFT\{a\, x_1[n]\} = a X_1[k]$

$\Rightarrow IDFT\{P[k]\} = IDFT\{8\, x[k]\}$

$P[n] = 8\, x[n] = \{8, 16, 24, 32\}$

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(b) $g[k] = 8 + x[k]$

$g[k] = 8 \cdot \delta[k] + x[k]$

By Scaling & Linearity property

$IDFT\{g[k]\} = IDFT\{8 \cdot \delta[k] + x[k]\}$

$g[n] = 2U[n] + x[n]$

$= \{3, 4, 5, 6, 2, 2, \ldots\}$

Q.3 $X[k] = \{1, 2, 3, 4\}$

To find $x[n]$; by IDFT,

$$x[n] = \frac{1}{N} \sum_{n=0}^{N-1} x(k) \cdot W_N^{-nk} \quad \text{where } N = 4$$

$$x(n) = \frac{1}{N} \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} & W_N^{-3} \\ W_N^0 & W_N^{-2} & W_N^{-4} & W_N^{-6} \\ W_N^0 & W_N^{-3} & W_N^{-6} & W_N^{-9} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \frac{1}{4} \begin{array}{c} \begin{bmatrix} 10 \\ -2-2j \\ -2 \\ -2+2j \end{bmatrix} \end{array} \begin{array}{l} n=0 \\ n=1 \\ n=2 \\ n=3 \end{array}$$

(a) $p[n] = x[n-1]$

By time shift property, If $x[n] \rightarrow x[k]$

then $DFT\{x[n-m]\}$

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Similarly, $DFT\{p[n]\} = DFT\{x[n-1]\}$

$$p[k] = W_N^{k}\, x[k]$$

$$= \{-1, -2j, -3, +4j\}$$

(b) $q[n] = x[n+1]$

By time Shift Property,

$$DFT\{q[n]\} = DFT\{x[n+1]\}$$

$$q[k] = W_N^{-k} \cdot x[k]$$

$$= \{1, 2j, 3, -4j\}$$

Q.4 $a[n] = \{1, 2, 3, 4\}$

a) By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{nk}$

$$X[k] = x[0] \cdot W_N^{0} + x[1] W_N^{k} + x[2] \cdot W_N^{2k} + x[3] \cdot W_N^{3k}$$

$$= 1 + 2W_N^{k} + 3W_N^{2k} + 4W_N^{3k}$$

$$= \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{matrix} k=0 \\ k=1 \\ k=2 \\ k=3 \end{matrix}$$

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b) $b[n] = \{3, 4, 1, 2\}$

$= x[n-2]$

By Time Shifting Property

$DFT\{b[n]\} = DFT\{x[n-2]\}$

$\Rightarrow B[k] = W_N^{2k} \cdot X[k]$

$= (-1)^k \cdot X[k]$

c) $c[n] = \{4, 6, 4, 6\}$

$= x[n-2] + x[n]$

$DFT\{c[n]\} = DFT\{x[n-2] + x[n]\}$

By Time Shifting Property

$\Rightarrow C[k] = W_N^{2k} \cdot X[k] + X[k]$

$= [1 + (-1)^k] \cdot X[k]$

d) $d[n] = \{-2, -2, 2, 2\}$

$= x[n] - x[n-2]$

By Time Shifting property

$DFT\{d[n]\} = DFT\{x[n] - x[n-2]\}$

$D[k] = X[k] - W_N^{2k} X[k]$

$= (1 - (-1)^k) X[k]$

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(e) $e[n] = \{5, 3, 5, 7\}$

$e[n] = x[n] + x[n-1]$

$DFT\{e[n]\} = DFT\{x[n] + x[n-1]\}$

$E[k] = X[k] + W_N^k . X[k]$

Q.5 $x[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}$

a) $a[n] = \{1, 1, 1, 1, 1, 1, 1, 1\} = x[n] + x[n-4]$

By Time Shifting Property

$A[n] = X[k] + W_N^{4k} . X[k]$

b) $b[n] = \{1, 1, 1, 1, -1, -1, -1, -1\}$
$= x[n] - x[n-4]$

By Time Shifting property

$DFT\{b[n]\} = DFT\{x[n] - x[n-4]\}$

$B[k] = X[k] - W_N^{4k} X[k]$

c) $c[n] = \{1, 0, 0, 0, -1, 0, 0, 0\}$

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d) $d[n] = \{2, 0, 0, 0, 0, 2, 2, 2\}$
$= 2x[n-5]$

$DFT\{d[n]\} = DFT\{2 \cdot x[n-5]\}$

$D[n] = 2 \cdot W_N^{5k} X[k]$

Q.6 $X[k] = \{1, 2, 3, 4\}$

a) $p[n] = (-1)^n x[n]$
$= W_N^{-2n} \cdot x[n]$

$DFT\{p[n]\} = DFT\{W_N^{-2n} \cdot x[n]\}$

$P[k] = X[k-2]$
$= \{3, 4, 1, 2\}$

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Q.7 $x[n] = \{1,1,1,1,0,0,0,0\}$

(a) $a[n] = \{1,0,0,0,0,1,1,1\}$
$= x[-n]$

$DFT\{a[n]\} = DFT\{x[-n]\}$

$A[n] = X[-k]$

(b) $b[n] = \{2,1,1,1,0,1,1,1\}$
$= x[n] + x[-n]$

$DFT\{b[n]\} = DFT\{x[n] + x[-n]\}$

$B[n] = X[k] + X[-k]$

Q.8 $X[k] = \{1,2,3,4\}$
(a) $p[n] = x[-n]$

$DFT\{p[n]\} = DFT\{x[-n]\}$

$P[k] = X[-k]$
$= \{1,4,3,2\}$

(b) $q[n] = x[-n+1]$

$DFT\{q[n]\} = DFT\{x[-n+1]\}$

$Q[k] = W_N^{-k} X[-k]$
$= \{1, -4j, -3, 2j\}$

c) $r[n] = x[-n-1]$

$DFT\{r[n]\} = DFT\{x[-n-1]\}$

$R[k] = W_N^k X[-k]$
$= \{1, 4j, -3, -2j\}$

Q.9 $x[n] = \{1,1,1,1,0,0,0,0\}$

a) $p[n] = \{1,0,0,0,0,1,1,1\}$
$= x[-n]$

$DFT\{p[n]\} = DFT\{x[n]\}$

$\Rightarrow P[k] = X[-k]$

b) $q[n] = \{0,0,1,1,1,1,0,0\}$
$= x[n-2]$

$DFT\{q[n]\} = DFT\{x[n-2]\}$

$Q[n] = W_N^{2k} \cdot X[k]$

Q10) a) $P[k] = \{0, 2+j, -1, j\}$

By Symmetry Property, $P[k] = P^*[-k]$

$P^*[-k] = \{0, -j, \quad , -1, 2-j, \quad \}$

Comparing, $P[k]$ & $P^*[k]$, we get

$P[1] = -j$ & $P[4] = 2-j$

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$P[k] = \{0, -j, 2+j, -1, 2-j, j\}$

b) $g[k] = \{1, 2, \_, \_, 0, 1-j, -2, \_\}$

By Symmetry Property, $g[k] = g^*[-k]$

$g[-k] = \{1, \_, -2, 1-j, 0, \_, \_, 2\}$

$g^*[-k] = \{1, \_, -2, 1+j, 0, \_, \_, 2\}$

Comparing $g[k]$ & $g^*[-k]$, we get

$g[2] = -2$, $g[3] = 1+j$, $g[7] = 2$

Q12) $N = 8$

$X[k] = \{1, 4+2j, 6+4j, 2j, 6, -2j, 6-4j, 4-2j\}$

$P[n] = \frac{1}{2}\{x[n] + x[-n]\} \Rightarrow P[n]$ = even signal of $x[n]$

$X[-k] = \{1, 4-2j, 6-4j, -2j, 6, 2j, 6+4j, 4+2j\}$

By Even signal property

$X[k] = X[-k]$

$DFT\{p[n]\} = DFT\left\{\frac{1}{2}\{x[n] + x[-n]\}\right\}$

$P[k] = \frac{1}{2}\{X[k] + X[-k]\}$

$$P[k] = \frac{1}{2}\{2, 8, 12, 0, 12, 0, 12, 8\}$$

$$= \{1, 4, 6, 0, 6, 0, 6, 4\}$$

Q 13) $x[n] = \{1+j, 2+2j, 3+3j, 4+2j\}$

a) By DFT, $X[k] = \sum_{0}^{N-1} x[n] \cdot W_N^{nk}$

$$X[k] = x[0] \cdot W_N^0 + x[1] \cdot W_N^k + x[2] \cdot W_N^{2k} + x[3] \cdot W_N^{3k}$$

$$= (1+j) + (2+2j) W_N^k + \cancel{(3+3j) \cdot W}$$

$$(3+3j) W_N^{2k} + (4+2j) W_N^{3k}$$

$$X[k] = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix}$$

$$= \begin{bmatrix} 10+8j \\ -2 \\ -2 \\ -2-4j \end{bmatrix} \begin{matrix} k=0 \\ \cancel{k=1} \\ \cancel{k=2} \\ \cancel{k=3} \end{matrix}$$

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(b) By complex conjugate sequence property

$$DFT\{x^*[n]\} = X^*[-k]$$
$$= \begin{cases} 10-8j & k=0 \\ -2+4j \\ -2 \\ -2 \end{cases}$$

(c) $x[n] = \begin{cases} 1+1j & n=0 \\ 2+2j \\ 3+3j \\ 4+2j \end{cases} = p[n] + j\,q[n]$ — (1)

$x^*[n] = p[n] - j\cdot q[n]$ — (2)

Add eq(1) & eq(2)

$x[n] + x^*[n] = 2p[n]$

DFT on both sides

$X[k] + X^*[-k] = 2P[k]$

Subtract (2) from (1)

$x[n] - x^*[n] = 2j\cdot q[n]$ — (3)

$$P[k] = \frac{1}{2}\{X[k] + X^*[-k]\}$$
$$= \begin{cases} 10 \\ -2+2j \\ -2 \\ -2-2j \end{cases}$$

Consider eq ③

$$q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$$

$$Q[k] = \frac{1}{2j}\{X[k] - X^*[-k]\}$$

$$= \frac{1}{2j}\left\{\begin{bmatrix}10+8j\\-2\\-2\\-2+4j\end{bmatrix} - \begin{bmatrix}10-8j\\-2+4j\\-2\\-2\end{bmatrix}\right\} = \begin{bmatrix}8j\\-2j\\0\\2j\end{bmatrix} \quad k=0$$

Q14) $x[n] = \{1,2,3,4\}$ ~~$X[k] = \{8,-2,$~~

$X[k] = \{8,-2,0,-2\}$

b) By circular convolution property of DFT

$$Q[k] = x[n] \circledast x[n] = \{64,4,0,4\}$$

a) $P[k] = X[k] \cdot X[k]$

By circular convolution property of IDFT

$p[n] = x[n] \circledast x[n]$

$x[n] = \{1,2,3,4\} \Rightarrow x[-n] = \{1,4,3,2\}$

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$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 26 \\ 28 \\ 26 \\ 20 \end{bmatrix}$$

Q15) $x[n] = \{1,2,3,2\}$ $\quad h[n] = \{1,2,3,4\}$

(a) Using DFT

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{nk}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & 1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & 1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$Y[k] = X[k] \cdot H[-k]$$

$$= \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 80 \\ 4-4j \\ 0 \\ 4+4j \end{bmatrix}$$

By IDFT,

$$y[n] = \frac{1}{N} \sum_{k=0}^{3} W_N^{-nk} Y[k]$$

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$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & 1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 20 \\ 4-4j \\ 0 \\ 4+4j \end{bmatrix} = \begin{bmatrix} 22 \\ 20-2j \\ 18 \\ 20+2j \end{bmatrix}$$

Q 16) $x[n] = \{1, 2, 3, 2\}$

By DFT, $X[k] = \sum_{0}^{N-1} x[n] \cdot W_N^{nk}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} \begin{matrix} k=0 \\ k=1 \\ k=2 \\ k=3 \end{matrix}$$

a) $E = \frac{1}{4} [8^2 + 2^2 + 0^2 + 2^2] = 18$

b) $E = [1^2 + 2^2 + 3^2 + 2^2] = 18$

Q 17) $x[n] = \{1, -2, 3, -4, 5, -6\}$

a) $X[0] = -3$

b) $X[3] =$

c) $E = \sum_{0}^{5} |x[n]|^2 = [1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2] = 91$

Energy in time domain = Energy in frequency domain.

$\therefore E = \sum_{0}^{5} |X[k]|^2 = 91$

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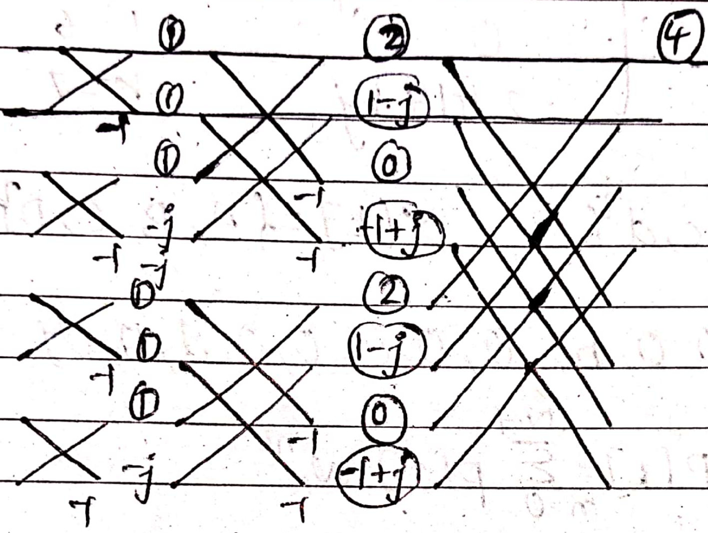
Q 13) $x[n] = \{ \underset{0}{1}, \underset{1}{1}, \underset{2}{1}, \underset{3}{1}, \underset{4}{0}, \underset{5}{0}, \underset{6}{0}, \underset{7}{0} \}$

$X[k] = G[k] + W_N^k H[k]$ (By DITFFT eqn.)

~~$x[0] = 1$~~

$x[0] = 1$
$x[4] = 0$
$x[2] = 1$
$x[6] = 0$
$x[1] = 1$
$x[5] = 0$
$x[3] = 1$
$x[7] = 0$



$X(0) = 4$
$X(1) = (1-j) + W_N^1 (1-j)$
$X(2) = 0$
$X(3) = (-1+j) + W_N^3 (-1+j)$
$X(4) = 0$
$X(5) = (1-j) - W_N^1 (1-j)$
$X(6) = 0$
$X(7) = (-1+j) - W_N^3 (-1+j)$

$$X[1] = (1-j) + (0.707 - 0.707j)(1-j)$$
$$= 1 - 2.414j$$

$$X[5] = (1-j) - (0.707 - 0.707j)(1-j)$$
$$= 1 + 0.414j$$

$$X[3] = (-1+j) + (0.707 - 0.707j)(-1+j)$$
$$= 0.414 + i$$

$$X[7] = (-1+j) + (0.707 + 0.707j)(-1+j)$$
$$= -2.414 + j$$

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$$X[k] = \begin{cases} 4 & k=0 \\ 1-2.414j & k=1 \\ 0 & k=2 \\ 0.414+j & k=3 \\ 0 & k=4 \\ 1+0.414j & k=5 \\ 0 & k=6 \\ -2.414+j & k=7 \end{cases}$$

Q19) $x[n] = \{a, b, c, d\}$, $X[k] = \{A, B, C, D\}$

$p[n] = \{a, 0, 0, b, 0, 0, c, 0, 0, d, 0, 0\}$

By DFT, $P[k] = \sum_{n=0}^{N-1} p[n] \cdot W_N^{nk}$

$$P[k] = \sum_{r=0}^{N/3} p(3r) W_N^{3rk} + \sum_{r=0}^{N/3} p(3r+1) W_N^{(3r+1)k} +$$

$$\sum_{r=0}^{N/3} p(3r+2) W_N^{(3r+2)k}$$

$$= \sum_{r=0}^{N/3} p(3r) W_N^{3rk} + W_N^{k} \sum_{r=1}^{N/3} p(3r+1) \cdot W_N^{3rk} +$$

$$W_N^{2k} \sum_{r=0}^{N/3} p(3r+2) W_N^{3rk}$$

$$P[k] = DFT[p(3r)] + W_N^{k} DFT[p(3r+1)] + \cancel{W_N^{2k} DFT}$$

$$W_N^{2k} DFT[p(3r+2)]$$

$$= G[k] + W_N^{k} H[k] + W_N^{2k} I[k]$$

$$= DFT\{a,b,c,d\} + DFT\{W_N^{k}\{0,0,0,0\}\} +$$

$$DFT\{W_N^{2k}\{0,0,0,0\}\}$$

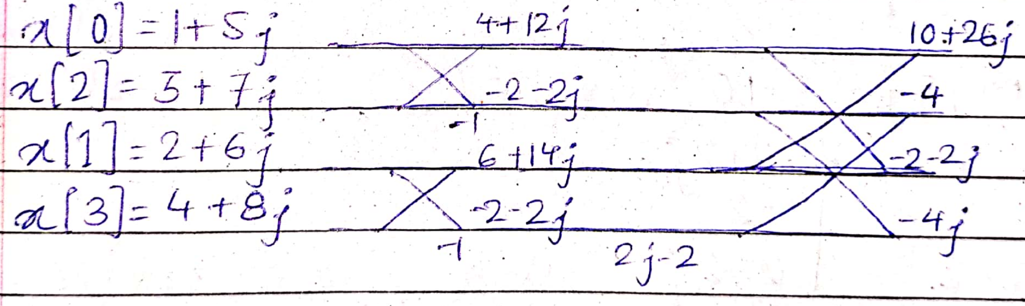
$$= \{A,B,C,D\} + 0 + 0$$

$$P[k] = \{A,B,C,D,A,B,C,D,A,B,C,D\}$$

Q.20 $p[n] = \{1,2,3,4\}$ $q[n] = \{5,6,7,8\}$

Let ~~p[n]~~ $x[n] = p[n] + j\,q[n]$

$\therefore x = \{1+5j,\ 2+6j,\ 3+7j,\ 4+8j\}$



$$X[k] = \begin{cases} 10+26j & k=0 \\ -4 & \\ -2-2j & \\ -4j & \end{cases}$$

(i) To find P[k]

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$$x[n] = p[n] + j \cdot q[n] \quad -(1)$$
$$x^*[n] = p[n] - j \cdot q[n] \quad -(2)$$

$$x[n] + x^*[n] = 2p[n]$$

$$p[n] = \frac{1}{2}\{x[n] + x^*[n]\}$$

$$P[K] = \frac{1}{2}\{X[K] + X^*[K]\}$$

$$P[k] = \frac{1}{2}\left\{\begin{bmatrix} 10+26j \\ -4 \\ -2-2j \\ -4j \end{bmatrix} + \begin{bmatrix} 10-26j \\ +4j \\ -2+2j \\ -2 \end{bmatrix}\right\}$$

$$P[k] = \begin{cases} 10 & k=0 \\ -2+2j \\ -2 \\ -2-2j \end{cases}$$

(1) − (2)

$$q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$$

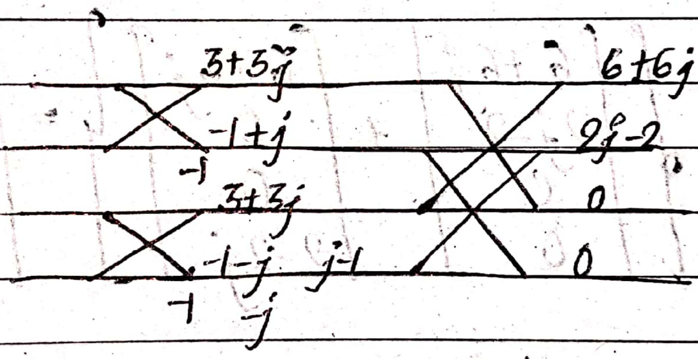
$$Q[k] = \frac{1}{2j}\{X[k] - X^*[k]\}$$

$$= \frac{1}{2j}\left\{\begin{bmatrix}10+26j\\-4\\-2-2j\\-4j\end{bmatrix} - \begin{bmatrix}10-26j\\+4j\\-2+2j\\-2\end{bmatrix}\right\}$$

$$= \begin{cases}26 & k=0\\ -2+2j\\ -2\\ -2-2j\end{cases}$$

Q21 $x[n] = \{(1+2j), (1+j), (2+j), (2+2j)\}$

$x[0] = 1+2j$
$x[2] = 2+j$
$x(1) = 1+j$
$x[3] = 2+2j$



$$X[k] = \begin{cases}6+6j & K=0\\ 2j-2\\ 0\\ 0\end{cases} \qquad X^*[k] = \begin{cases}6+6j & K=0\\ 0\\ 0\\ 2+2j\end{cases}$$

$x[n] = p[n] + jq[n]$ —(1)

$x^*[n] = p[n] - jq[n]$ (2)

① + ②

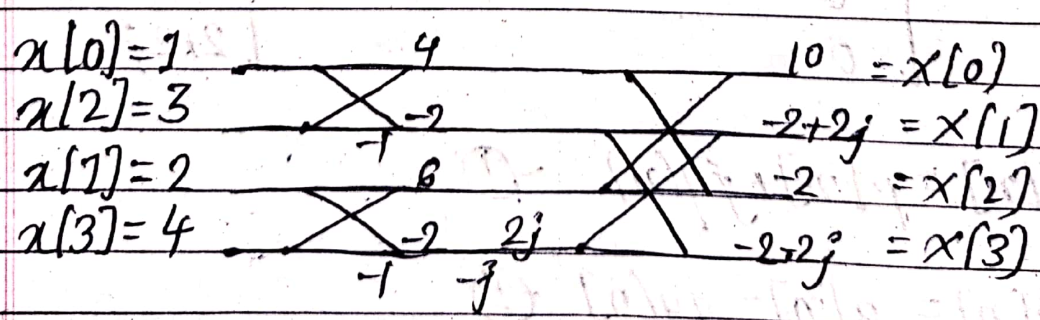
$$p[n] = \frac{1}{2}\{x[n] + x^*[n]\}$$

$$q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$$

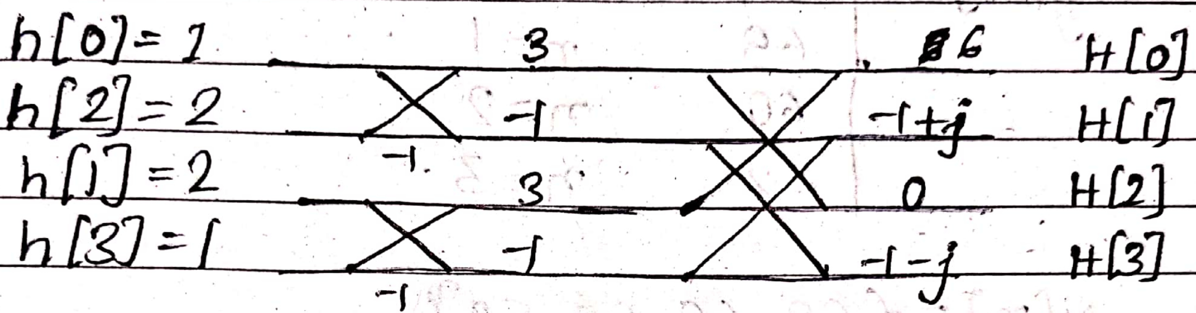
$$P[k] = \frac{1}{2}\left\{\begin{bmatrix} 6+6j \\ 2j-2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6+6j \\ 0 \\ 0 \\ -2-2j \end{bmatrix}\right\} = \begin{bmatrix} 6+6j \\ -1+j \\ 0 \\ -1-j \end{bmatrix}$$

$$Q[k] = \frac{1}{2j}\left\{\begin{bmatrix} 6+6j \\ 2j-2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6+6j \\ 0 \\ 0 \\ -2-2j \end{bmatrix}\right\} = \begin{bmatrix} 6 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$

Q22 $h[n] = \{1, 2, 2, 1\}$ $\quad x[n] = \{1, 2, 3, 4\}$



$$X[k] = \begin{cases} 10 & k=0 \\ -2+2j \\ -2 \\ -2-2j \end{cases}$$

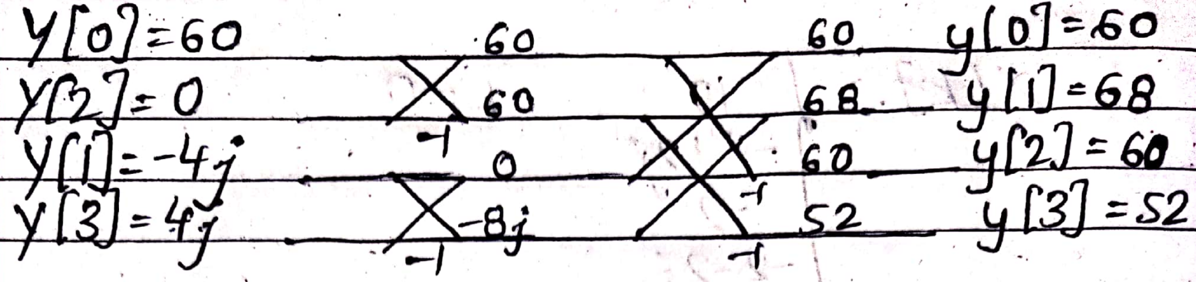


$$H[k] = \begin{cases} 6 & k=0 \\ -1+j \\ 0 \\ -1-j \end{cases}$$

$$Y[k] = X[k] \cdot H[k]$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -1+j \\ 0 \\ -1-j \end{bmatrix} = \begin{bmatrix} 60 \\ -4j \\ 0 \\ 4j \end{bmatrix}$$

Find $y[n]$ by IFFT



$$y[n] = \begin{cases} 60 & n=0 \\ 68 & n=1 \\ 60 & n=2 \\ 52 & n=3 \end{cases}$$

$$y[n] = \{60, 68, 60, 52\}$$

Q23) $h[n] = \{1, 0, 2\}$ $\quad x[n] = \{1, 2, 3, 4, 0, 0, 1, 2, 3, 4\}$

$L = 10$, $M = 3$

(I) Select N

$N = L + M - 1$

$8 = L + 3 - 1 \Rightarrow L = 6$

(II) Decompose $x[n]$

$x_1[n] = \{1, 2, 3, 4, 0, 0\}$

$x_2[n] = \{1, 2, 3, 4, 0, 0\}$

(III) Zero Padding $x[n]$

$x_1[n] = \{1, 2, 3, 4, 0, 0, 0, 0\}$

$x_2[n] = \{1, 2, 3, 4, 0, 0, 0, 0\}$

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$h[n] = \{1,0,2,0,0,0,0,0\}$

$\Rightarrow h[-n] = \{1,0,0,0,0,0,2,0\}$

(IV) $y_1[n] = y_2[n]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 8 \\ 6 \\ 8 \\ 0 \\ 0 \end{bmatrix}$$

$y_1[n] = y_2[n] = \{1,2,5,8,6,8,0,0\}$

$y_1[n-l] = \{0,0,0,0,0,1,2,5,8,6,8,0,0\}$

$y_1[n-L] = \{1,2,5,8,6,8,1,2,5,8,6,8,0,0\}$

$L = 6$

$h[n] = \{1,0,2\}$ $\quad x[n] = \{1,2,3,4,0,0,1,2,3,4\}$

(i) Select N (let $N=8$)

$N = L+M-1 \Rightarrow 8 = L+3-1 \Rightarrow L = 6$

(ii) Decompose $x[n]$

$x_1[n] = \{1,2,3,4,0,0\}$
$x_2[n] = \{1,2,3,4,0,0\}$
$x_3[n] = \{0,0,0,0,0,0\}$

(iii) Modify i/p sequence

$x_1[n] = \{0,0,1,2,3,4,0,0\}$
$x_2[n] = \{0,0,1,2,3,4,0,0\}$
$x_3[n] = \{0,0,0,0,0,0,0,0\}$

$h[n] = \{1,0,2,0,0,0,0,0\}$

$h[-n] = \{1,0,0,0,0,0,2,0\}$

(iv) Using $\sum_{n=0}^{M-1} x[m].h[n-m]$

$$y_1[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} =$$

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$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 5 \\ 8 \\ 6 \\ 8 \end{bmatrix}$$

~~By~~ Sum $y_2[n] = \{0, 0, 1, 2, 5, 8, 6, 8\}$

$y_3[n] = \{0, 0, 0, 0, 0, 0, 0, 0\}$

Merging $y_1[n], y_2[n], y_3[n]$

$y[n] = \{1, 2, 5, 8, 6, 8, 1, 2, 5, 8, 6, 8, 0, 0, 0, 0, 0\}$