and is called linear con

$$y(n) = h(n) * x(n)$$

Convolution of Finite Length Sequences:

Example 1.10:

The impulse response of a LTI system

$$h(n) = \left\{1, \frac{1}{2}\right\}$$

Find the response of the system when input

$$x(n) = \{1, 2, 3\}$$

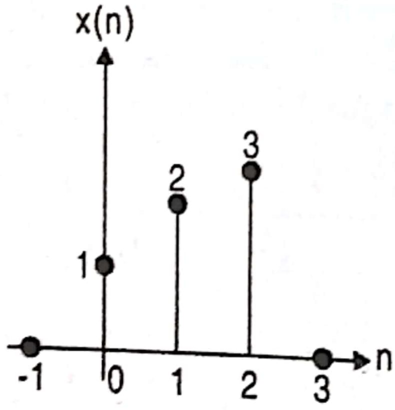
by (1) Fold, shift, multiply and sum concept

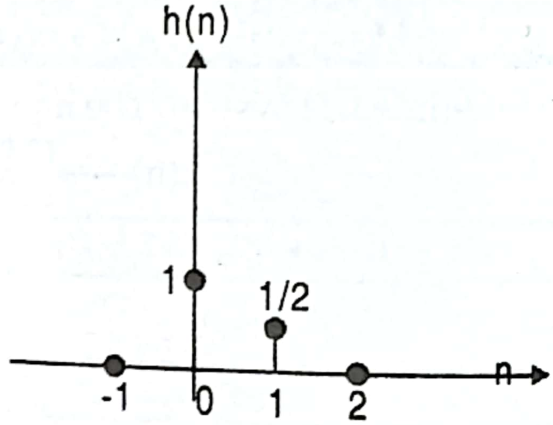
(2) Tabulation technique.

Solution:

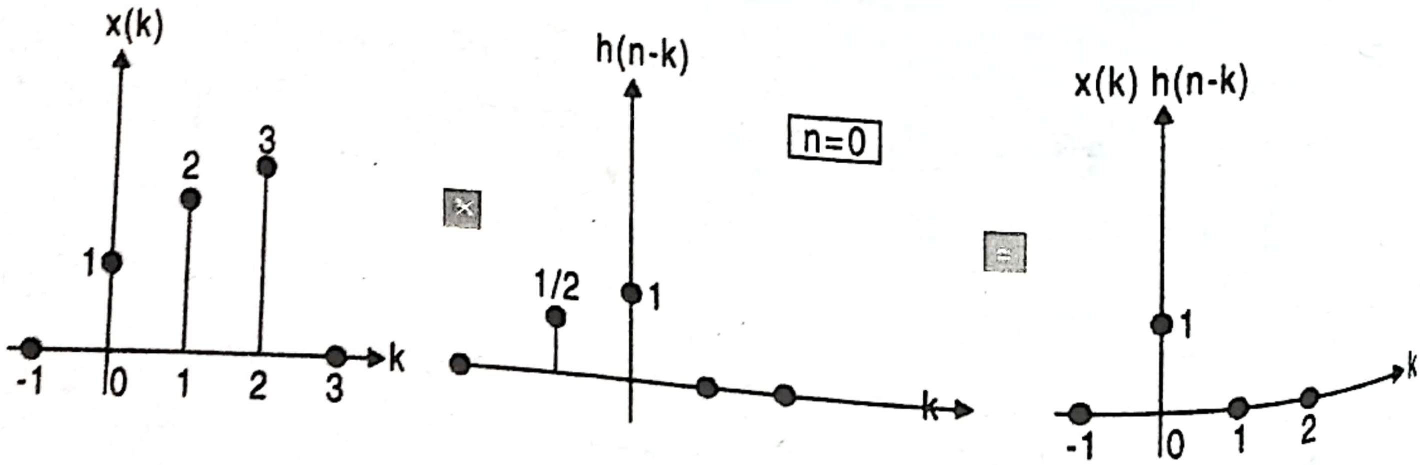
$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\ h(n-k)$$

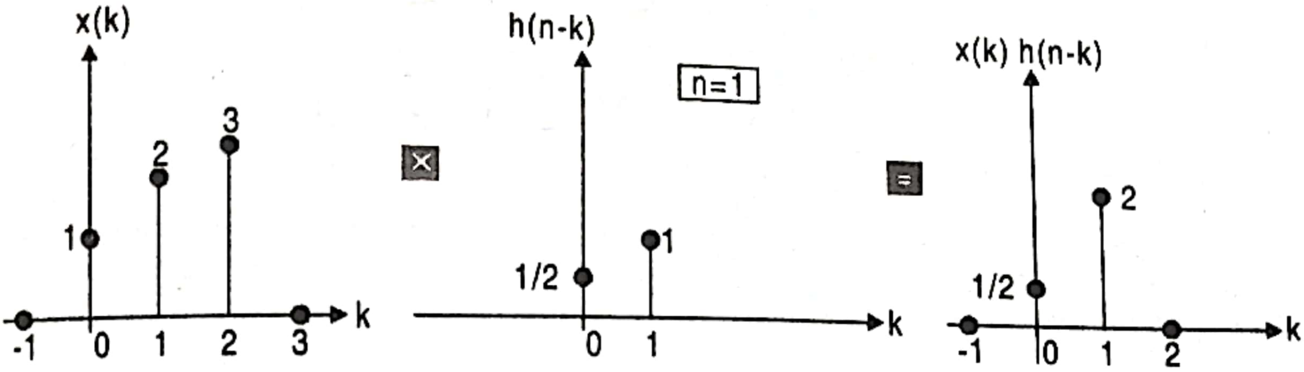




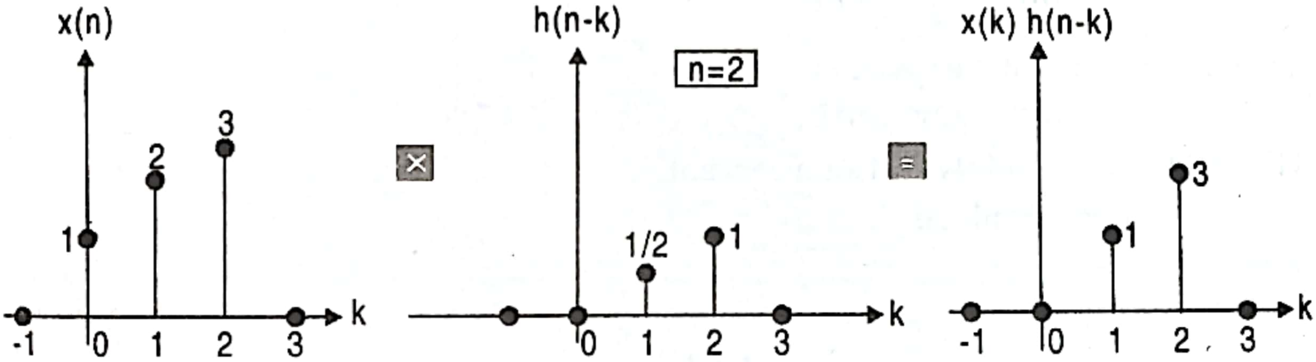
$$y(0) = \Sigma\, x(k)\, h(0-k)$$
$$= 1 \times 1$$
$$= 1$$



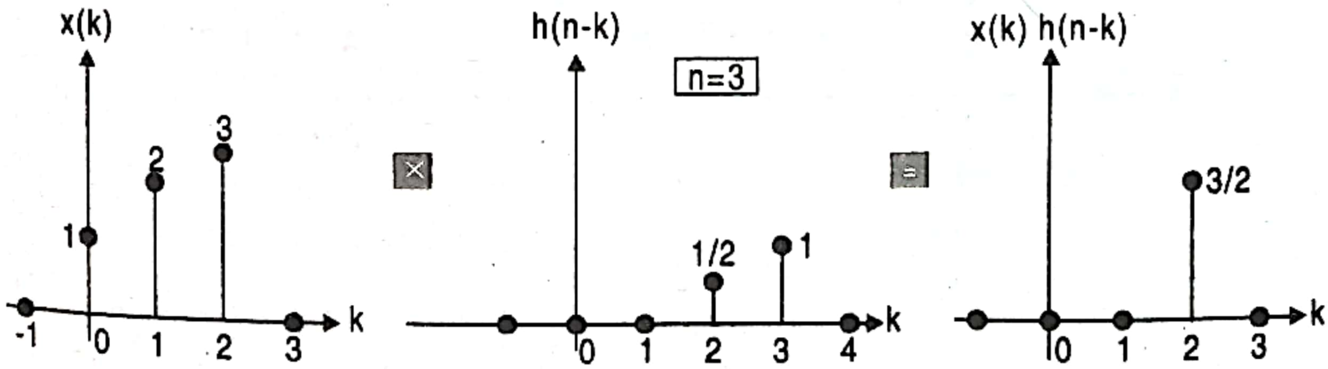
$$y(1) = \Sigma\, x(k)\, h(1-k)$$
$$= 1 \times \frac{1}{2} + 2 \times 1$$
$$= \frac{5}{2}$$



$$y(2) = \Sigma\, x(k)\, h(2-k)$$
$$= 2 \times \frac{1}{2} + 3 \times 1$$
$$= 4$$

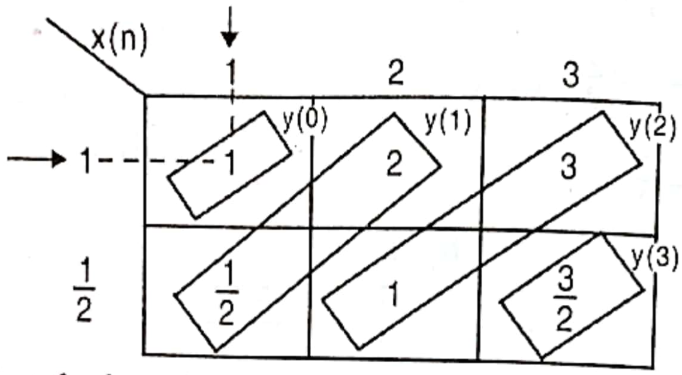


$$y(3) = \Sigma\, x(k)\, h(3-k)$$
$$= 3 \times \frac{1}{2}$$
$$= \frac{3}{2}$$



$$y(n) = \left\{ 1, \frac{5}{2}, 4, \frac{3}{2} \right\}$$

Tabulation Method:



Students should note that the length of the input array $x(n) = \{1, 2, 3\}$ $N_x = 3$ and the length of the impulse response array $h(n) = \left\{1, \frac{1}{2}\right\}$ $N_h = 2$ and the length of the output array $y(n) = \left\{1, \frac{5}{2}, 4, \frac{3}{2}\right\}$ is $N_x + N_h - 1 = 4$.

Convolution of Infinite Length Sequences:

Example 1.11:

The impulse response of a LTI system

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

Find the response of the system when input

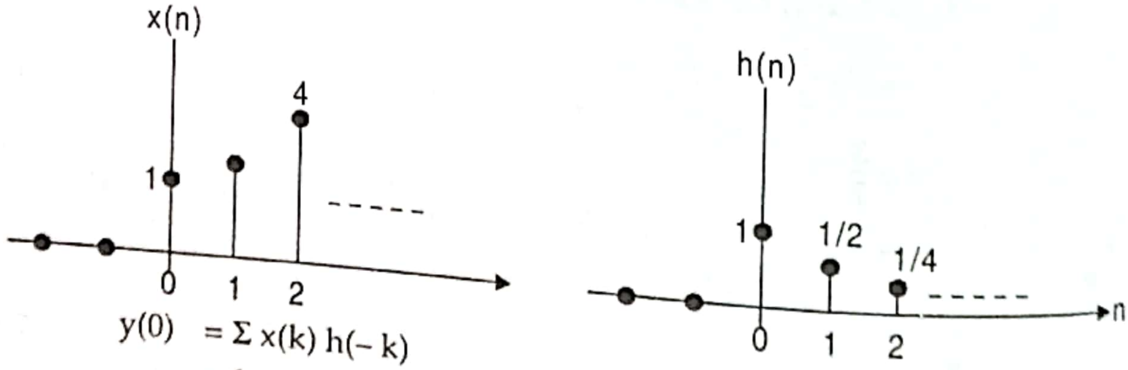
$$x(n) = (2)^n u(n) \text{ by}$$

(1) Fold, shift, multiply and sum concept

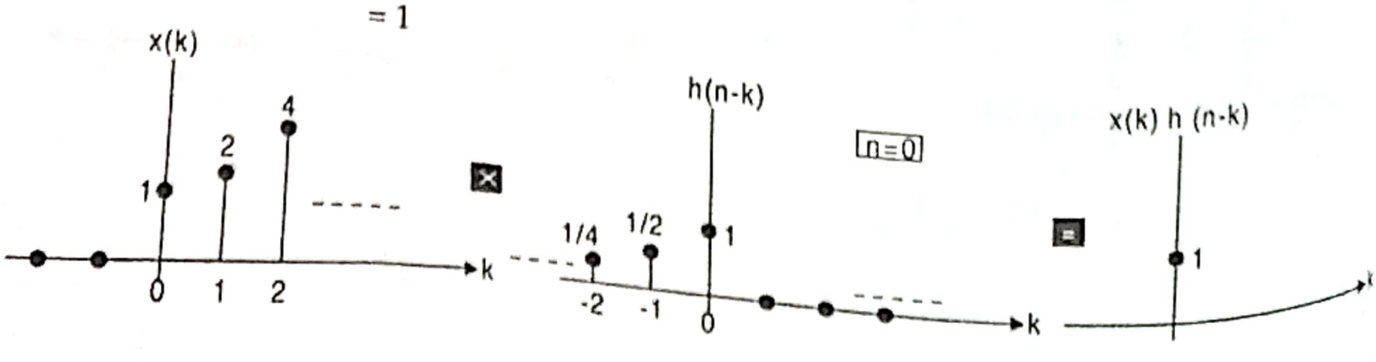
(2) Tabulation technique.

Solution:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\ h(n-k)$$



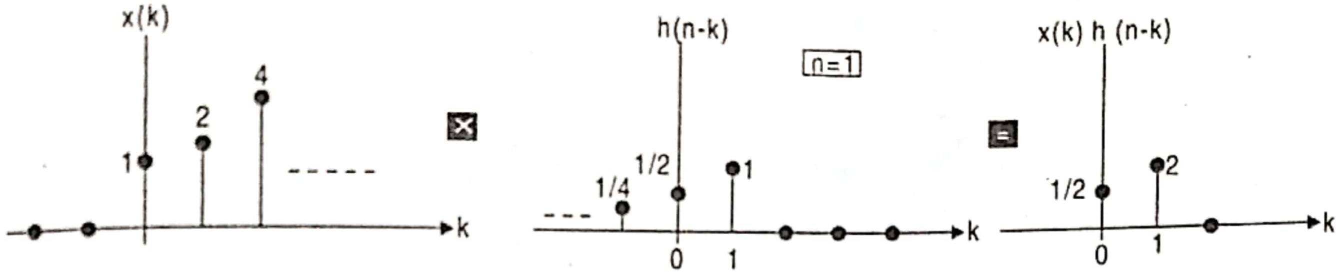
$$\begin{aligned} y(0) &= \Sigma\, x(k)\, h(-k) \\ &= 1 \times 1 \\ &= 1 \end{aligned}$$



$$y(1) = \Sigma\, x(k)\, h(1-k)$$

$$= 1 \times \frac{1}{2} + 2 \times 1$$

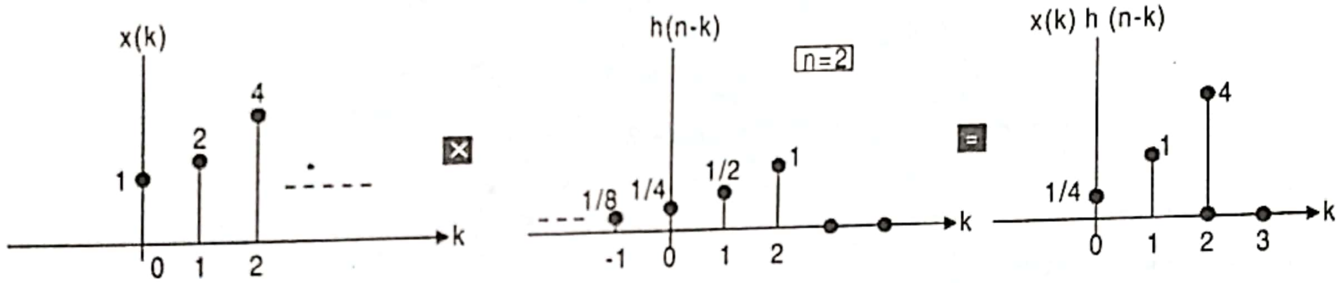
$$= \frac{5}{2}$$



$$y(2) = \Sigma\, x(k)\, h(2-k)$$

$$= 1 \times \frac{1}{4} + 2 \times \frac{1}{2} + 4 \times 1$$

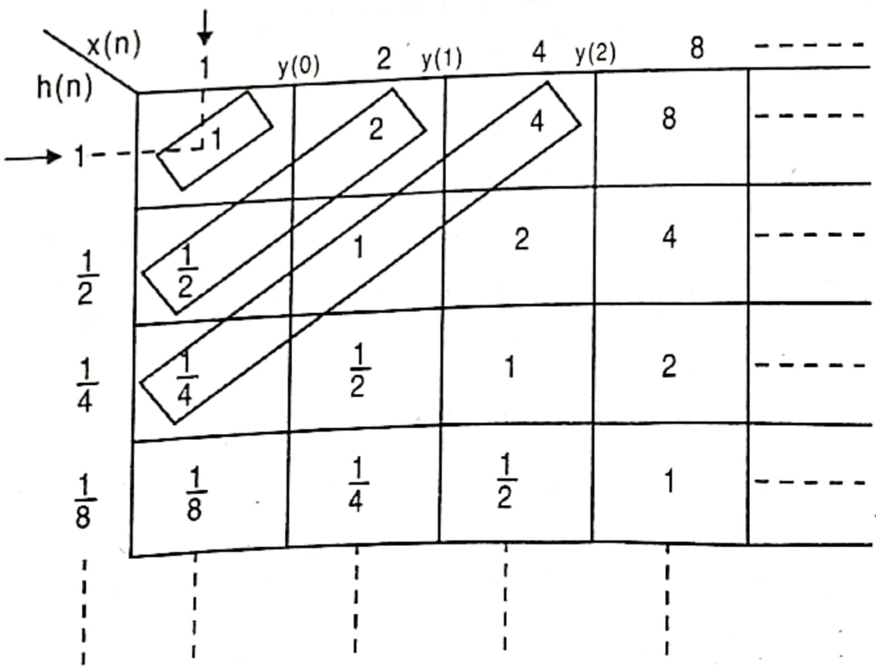
$$= \frac{21}{4}$$



This will be a never ending process

$$\therefore \quad y(n) = \left\{ 1, \frac{5}{2}, \frac{21}{4}, \ldots\ldots \right\}$$

Tabulation Method:



$$y(n) = \left\{1, \frac{5}{2}, \frac{21}{4}, \ldots\ldots\right\}$$