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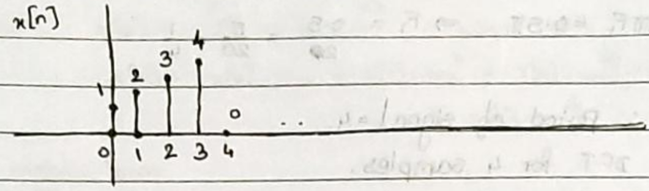
FOSIP : Assignment 2

Q1) a) $x[n] = \delta[n] + 2u[n-1] + u[n-2] + \delta[n-3] - 3u[n-4]$



$x[n] = \{1, 2, 3, 4\}$

To find $x[k]$, by DFT,

$$x(k) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nk} \qquad \text{where } N = 4,\ W_N = e^{-j\left(\frac{2\pi}{N}\right)}$$

$$x(k) = \sum_{0}^{3} x[n] \cdot W_N^{nk}$$

$$x(k) = x[0] \cdot W_N^{0} + x[1] \cdot W_N^{k} + x[2] \cdot W_N^{2k} + x[3] \cdot W_N^{3k}$$

$$\Rightarrow x(k) = 1 + 2 \cdot W_N^{k} + 3W_N^{2k} + 4W_N^{3k}$$

$k=0$, $x(0) = 1 + 2 + 3 + 4 = 10$

$k=1$, $x(1) = 1 + 2W_N + 3W_N^2 + 4W_N^3$

where $W_N = e^{-j\left(\frac{2\pi}{4}\right)} = e^{-j\pi/2} = -j$

$W_N^2 = (-j)^2 = -1$

$W_N^3 = j$

$\therefore x(1) = 1 + 2(-j) + 3(-1) + 4(j) = -2 + 2j$

$k=2$, $x(2) = 1 + 2 \cdot W_N^2 + 3W_N^4 + 4 \cdot W_N^6$

$= 1 + 2(-1) + 3(-1)^2 + 4 \cdot (-1)^3 = \underline{\underline{-2}}$

$k=3$, $x(3) = 1 + 2 \cdot W_N^3 + 3 \cdot W_N^6 + 4 \cdot W_N^9$

$= 1 + 2(j) + 3(j)^2 + 4(j)^3 = \underline{\underline{-2-2j}}$

$k=4$, $x(4) = 1 + 2 \cdot W_N^4 + 3 \cdot W_N^8 + 4 \cdot W_N^{12}$

$= 1 + 2 + 3 + 4 = 10$

$$\therefore x[k] = \begin{cases} 10 & k=0 \\ -2+2j & k=1 \\ -2 & k=2 \\ -2-2j & k=3 \end{cases}$$

b) $x[n] = 3\cos(0.5\pi n)$

$\omega = 0.5\pi \Rightarrow 2\pi F_1 = 0.5\pi \Rightarrow F_1 = \frac{0.5}{2} = \frac{5}{20} = \frac{1}{4}$

$\Rightarrow N = 4 \quad \therefore$ Period of signal $= 4$.

$\therefore$ we will calculate DFT for 4 samples.

$n=0, \quad x[0] = 3\cos(0) = 3$

$n=1, \quad x[1] = 3\cos(0.5\pi) = 0$

$n=2, \quad x[2] = 3\cos(\pi) = -3$

$n=3, \quad x[3] = 3\cos(1.5\pi) = 0$

$n=4, \quad x[4] = 3\cos(2\pi) = 3$

$\vdots$

values repeat.

$\Rightarrow x[n] = \{3, 0, -3, 0\}$

By DFT, $X[k] = \sum_{0}^{N-1} x[n].W_N^{nk}$ where $N = 4$, $W_N = e^{-j(\frac{2\pi}{N})}$

$$\Rightarrow X[k] = \sum_{0}^{3} x[n].W_N^{nk}$$

$$\therefore X[k] = x[0].W_N^{0} + x[1].W_N^{k} + x[2].W_N^{2k} + x[3].W_N^{3k}$$

In matrix form:

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$$\Rightarrow X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & k=0 \\ 6 & k=1 \\ 0 & k=2 \\ 6 & k=3 \end{bmatrix}$$

c) $x(n) = \{1, 2, 3, 4, 0, 0, 0, 0\}$

By DFT, $X(k) = \sum_{0}^{N-1} x(n) . w_N^{nk}$ where $N = 8$, & $w_N = e^{-j\left(\frac{2\pi}{N}\right)}$

$\therefore X(k) = x[0].w_N^0 + x[1].w_N^k + x[2].w_N^{2k} + x[3]w_N^{3k} + x[4].w_N^{4k} + x[5].w_N^{5k}$
$+ x[6].w_N^{6k} + x[7].w_N^{7k}$

$\cancel{\Rightarrow X[0] = x[0].w_N^0 + x[1]}$

$\Rightarrow X[k] = 1 + 2.w_N^k + 3w_N^{2k} + 4.w_N^{3k}$

$X[0] = 1+2+3+4 = \underline{10}$

$X[1] = 1 + 2w_N + 3w_N^2 + 4w_N^3$
$= 1 + 2(0.707 - j0.707) + 3(-j) + 4(-0.707 - j0.707)$
$= \underline{-0.414 - j7.242}$

$X[2] = 1 + 2w_N^2 + 3w_N^4 + 4w_N^6$
$= 1 + 2(-j) + 3(-1) + 4(j) = \underline{-2 + 2j}$

$X[3] = 1 + 2w_N^3 + 3w_N^6 + 4w_N^9$
$= 1 + 2(-0.707 - j0.707) + 3(j) + 4(0.707 - j0.707) = \underline{2.414 - j1.242}$

$X[4] = 1 + 2w_N^4 + 3w_N^8 + 4w_N^{12}$
$= 1 + 2(-1) + 3(-1)^2 + 4(-1)^3 = \underline{\underline{-2}}$

$X[5] = 1 + 2.w_N^5 + 3w_N^{10} + 4w_N^{15}$
$= 1 + 2(-0.707 + j0.707) + 3(-j) + 4(0.707 + j0.707) = \underline{2.414 + j1.242}$

$X[6] = 1 + 2w_N^6 + 3w_N^{12} + 4.w_N^{18}$
$= 1 + 2(j) + 3(j)^2 + 4(j)^3 = \underline{\underline{-2-2j}}$

$X[7] = 1 + 2w_N^7 + 3w_N^{14} + 4w_N^{21}$
$= 1 + 2(0.707 + j0.707) + 3(j) + 4(-0.707 + j0.707)$
$= -0.414 + j7.242$

$$X(k) = \begin{cases} 10 & k=0 \\ -0.414 - j7.242 & k=1 \\ -2+2j & k=2 \\ 2.414 - j1.242 & k=3 \\ -2 & k=4 \\ 2.414 + j1.242 & k=5 \\ -2-2j & k=6 \\ -0.414 + j7.242 & k=7. \end{cases}$$

Q2) $x[n] = \{1,2,3,4\}$

a) $P[k] = 8\, x(k)$

To find, By DFT = $\sum_{n=0}^{N-1} x[n]\cdot W_N^{nk}$

$$\Rightarrow x(k) = x[0]\cdot W_N^{0} + x[1]\cdot W_N^{k} + x[2]\cdot W_N^{2k} + x[3]\cdot W_N^{3k}$$

$$= 1 + 2W_N^{k} + 3W_N^{2k} + 4W_N^{3k}$$

$$X[k] = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix}\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = \begin{bmatrix} 10 & k=0 \\ -2+2j & k=1 \\ -2 & k=2 \\ -2-2j & k=3 \end{bmatrix}$$

To find $P[k]$ when $P[k] = 8\cdot x[k]$

By Scaling and Linearity Property,

If $x_1[n] \rightarrow X_1[k]$, then $DFT\{a\, x_1[n]\} = a X_1[k]$

$$\Rightarrow IDFT\{P[k]\} = IDFT\{8\, x[k]\}$$

$$\Rightarrow p[n] = 8\, x[n] = \{8, 16, 24, 32\}$$

b) $Q[k] = 8 + x[k]$

$$\Rightarrow Q[k] = 8\cdot\delta[k] + x[k]$$

By Scaling and Linearity Property,

$$IDFT\{Q[k]\} = IDFT\{8\cdot\delta[k] + x[k]\}$$

$$Q[n] = 2\cdot u[n] + x[n]$$

$$= \{3, 4, 5, 6, 2, 2, \dots\}$$

Q3) $X[k] = \{1, 2, 3, 4\}$

To find $x[n]$, by IDFT, $x[n] = \frac{1}{N}\sum_{n=0}^{N-1} X[k]\cdot W_N^{-nk}$ where $N=4$ & $W_N = e^{-j\left(\frac{2\pi}{N}\right)}$

$$x(n) = \frac{1}{N}\begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} & W_N^{-3} \\ W_N^0 & W_N^{-2} & W_N^{-4} & W_N^{-6} \\ W_N^0 & W_N^{-3} & W_N^{-6} & W_N^{-9} \end{bmatrix}\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 10 \\ -2-2j \\ -2 \\ -2+2j \end{bmatrix} \begin{array}{l} n=0 \\ n=1 \\ n=2 \\ n=3 \end{array}$$

a) $p[n] = x[n-1]$

By Time Shift Property, If $x[n] \rightarrow x[k]$ then $DFT\{x[n-m]\} = W_N^{mk}\, x[k]$

Similarly, $DFT\{p[n]\} = DFT\{x[n-1]\}$

$$p[k] = W_N^{k}\, x[k]$$

$$= \{1, -2j, -3, +4j\}$$

b) $q[n] = x[n+1]$

By Time Shift Property,

$$DFT\{q[n]\} = DFT\{x[n+1]\}$$

$$q[k] = W_N^{-k} \cdot x[k]$$

$$= \{1, 2j, 3, -4j\}$$

Q4) $a[n] = \{1, 2, 3, 4\}$

a) By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{nk}$

$$X[k] = x[0] \cdot W_N^{0} + X[1] \cdot W_N^{k} + x[2] \cdot W_N^{2k} + x[3] \cdot W_N^{3k}$$

$$= 1 + 2W_N^{k} + 3W_N^{2k} + 4W_N^{3k}$$

$$= \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \begin{array}{l} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array}$$

b) $b[n] = \{3, 4, 1, 2\}$
$= x[n-2]$

By Time Shifting property,
$DFT\{b[n]\} = DFT\{x[n-2]\}$
$\Rightarrow B[k] = W_N^{2k} \cdot X[k]$
$= (-1)^k \cdot X[k]$

c) $c[n] = \{4, 6, 4, 6\}$
$= x[n-2] + x[n]$
$DFT\{c[n]\} = DFT\{x[n-2] + x[n]\}$
By Time Shifting property,
$\Rightarrow C[k] = W_N^{2k} \cdot X[k] + X[k]$
$= [1 + (-1)^k] \cdot X[k]$

d) $d[n] = \{-2, -2, 2, 2\}$
$= x[n] - x[n-2]$
By Time Shifting property.
$DFT\{d[n]\} = DFT\{x[n] - x[n-2]\}$
$\Rightarrow D[k] = X[k] - W_N^{2k} X[k]$
$= [1 - (-1)^k] X[k]$

e) $e[n] = \{5, 3, 5, 7\}$
$e[n] = x[n] + x[n-1]$
$DFT\{e[n]\} = DFT\{x[n] + x[n-1]\}$
$E[k] = X[k] + \cancel{W_N^k\, E[k]}\; W_N^k \cdot X[k]$

Q5) $x[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}$
a) $a[n] = \{1, 1, 1, 1, 1, 1, 1, 1\}$
$= x[n] + x[n-4]$
By Time ~~Sharing~~ shifting property
$A[n] = X[k] + W_N^{4k} \cdot X[k] \quad \cancel{= 2X[k]}$

b) $b[n] = \{1, 1, 1, 1, -1, -1, -1, -1\}$

$= x[n] - x[n-4]$

By Time Shifting prop.

$DFT\{b[n]\} = DFT\{x[n] - x[n-4]\}$

$\Rightarrow B[k] = X[k] - W_N^{4k}\, X[k]$

c) $c[n] = \{1, 0, 0, 0, -1, 0, 0, 0\}$

d) $d(n) = \{2, 0, 0, 0, 0, 2, 2, 2\}$

$= 2 \cdot x[n-5]$

$DFT\{d(n)\} = DFT\{2 \cdot x[n-5]\}$

$\Rightarrow D[n] = \cancel{2\, x[n-5]}\; 2 \cdot W_N^{5k} \cdot X[k]$

Q6) $X[k] = \{1, 2, 3, 4\}$

a) $p[n] = (-1)^n x[n]$

$= W_N^{-2n} \cdot x[n]$

$DFT\{p[n]\} = DFT\{W_N^{-2n} \cdot x[n]\}$

$P[k] = X[k-2]$

$= \{3, 4, 1, 2\}$ //

b) $q[n] = x[n] \cdot \cos\left(\frac{n\pi}{2}\right)$ //

Q7) $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

a) $a(n) = \{1, 0, 0, 0, 0, 1, 1, 1\}$

$= \cancel{x[n-5]}\; x[-n]$

$DFT\{a[n]\} = DFT\{x[-n]\}$

$\Rightarrow A[n] = X[-k]$ //

b) $b[n] = \{2, 1, 1, 1, 0, 1, 1, 1\}$

$= x[n] + x[-n]$

$DFT\{b[n]\} = DFT\{x[n] + x[-n]\}$

$B[n] = X[k] + X[-k]$ //

Q8) $x[k] = \{1, 2, 3, 4\}$

a) $p[n] = x[-n]$

$DFT\{p[n]\} = DFT\{x[-n]\}$

$P[k] = X[-k]$

$= \{1, 4, 3, 2\}$ //

b) $q[n] = x[-n+1]$

$DFT\{q[n]\} = DFT\{x[-n+1]\}$

$Q[k] = W_N^{-k}\ X[-k]$

$= \{1, -4j, -3, 2j\}$ //

c) $r[n] = x[-n-1]$

$DFT\{r[n]\} = DFT\{x[-n-1]\}$

$R[k] = W_N^{k}\ X[-k]$

$= \{1, 4j, -3, -2j\}$

Q9) $x[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}$

a) $p[n] = \{1, 0, 0, 0, 0, 1, 1, 1\}$

$= x[-n]$

$DFT\{p[n]\} = DFT\{x[n]\}$

$\Rightarrow$ ~~DFT~~ $P[k] = X[-k]$ //

b) $q[n] = \{0, 0, 1, 1, 1, 1, 0, 0\}$

$= x[+n-2]$

$DFT\{q[n]\} = DFT\{x[n-2]\}$

$Q[n] = W_N^{2k} \cdot X[k]$ //

Q10) a) $P[k] = \{0, \ \ , 2+j, -1, \ \ , j\}$

By Symmetry property, $P[k] = P^*[-k]$

~~$P^*[-k] = \{0, -j, \ , -1, 2-j, \ \}$~~ $P^*[-k] = \{0, -j, \ \ , -1, 2-j, \ \}$

Comparing, $P[k]$ & $P^*[k]$, we get

$P[1] = -j$ & $P[4] = 2-j$

$\Rightarrow P[k] = \{0, -j, 2+j, -1, 2-j, j\}$

b) $Q[k] = \{1, 2, \_, \_, 0, 1-j, -2, \_\}$

By Symmetry Property, $Q[k] = Q^*[-k]$

$Q[-k] = \{1, \;, 2, 1-j, 0, -, -, 2\}$

$Q^*[-k] = \{1, -, 3, 1+j, 0, -, -, 2\}$

Comparing $Q[k]$ & $Q^*[-k]$, we get

$\cancel{Q[1] = 2}$, $Q[2] = -2$, $Q[3] = 1+j$, $Q[7] = 2$

Q12) $N=8$,

$x[k] = \{1, 4+2j, 6+4j, 2j, 6, -2j, 6-4j, 4-2j\}$

$p[n] = \frac{1}{2}\{x[n] + x[-n]\}$ $\Rightarrow$ $p[n] =$ even signal of $x[n]$.

$x[-k] = \{1, 4-2j, 6-4j, -2j, 6, 2j, 6+4j, 4+2j\}$

By Even Signal Property,

$x[k] = X[-k]$

$\Rightarrow$ $DFT\{p[n]\} = DFT\{\frac{1}{2}(x[n] + x[-n])\}$

$P[k] = \frac{1}{2}\{x[k] + X[-k]\}$

$= 1$

$DFT\{p[n]\} = DFT\{\frac{1}{2}\{x[n] + x[-n]\}\}$

$P[k] = \frac{1}{2}\{x[k] + X[-k]\}$

$P[k] = \frac{1}{2}\{2, 8, 12, 0, 12, 0, 12, 8\}$

$= \{1, 4, 6, 0, 6, 0, 6, 4\}$ //

Q13) $x[n] = \{1+j,\ 2+2j,\ 3+3j,\ 4+2j\}$

a) By DFT, $X[k] = \sum_{0}^{N-1} x[n].W_N^{nk}$

$$X[k] = x[0].W_N^0 + x[1].W_N^k + x[2].W_N^{2k} + x[3].W_N^{3k}$$

$$= (1+j) + (2+2j)W_N^k + (3+3j).W_N^{2k} + (4+2j).W_N^{3k}$$

$$X[k] = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1+j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix}$$

$$= \begin{bmatrix} 10+8j \\ -2 \\ -2 \\ -2-4j \end{bmatrix} \quad k=0$$

b) By Complex Conjugate Sequence Property

$$DFT\{x^*[n]\} = X^*[-k]$$

$$= \begin{bmatrix} 10-8j \\ -2+4j \\ -2 \\ -2 \end{bmatrix} \quad k=0$$

c) $$x[n] = \begin{bmatrix} 1+1j \\ 2+2j \\ 3+3j \\ 4+2j \end{bmatrix} \quad n=0 \quad = p[n] + j.q[n] \quad —①$$

$$x^*[n] = p[n] - j.q[n] \quad —②$$

Add eq ① & ②

$$x[n] + x^*[n] = 2.p[n]$$

DFT on both sides.

$$X[k] + X^*[-k] = 2.P[k]$$

Subtract ② from ①

$$x[n] - x^*[n] = 2j.q[n] \quad —③$$

~~DFT on both sides~~

$$P[k] = \frac{1}{2}\{X[k] + X^*[-k]\}$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Consider eq ③

$$q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$$

$$Q[k] = \frac{1}{2j}\{X[k] - X^*[-k]\}$$

$$= \frac{1}{2j}\left\{\begin{bmatrix} 10+8j \\ -2 \\ -2 \\ -2+4j \end{bmatrix} - \begin{bmatrix} 10-8j \\ -2+4j \\ -2 \\ -2 \end{bmatrix}\right\} = \begin{bmatrix} 8j \\ -2j \\ 0 \\ 2j \end{bmatrix} \quad k=0$$

Q14) $x[n] = \{1,2,3,4\}$ $\quad X[k] = \{8,-2,0,-2\}$

b) By circular convolution property of DFT

$$Q[k] = \underset{x[n] \circledast x[n]}{\cancel{X[k]\ X[k]}} = \{64, 4, 0, 4\}$$

a) $p[k] = \cancel{\ } X[k] \cdot X[k]$

By circular convolution property of IDFT

$p[n] = x[n] \circledast x[n]$

$x[n] = \{1,2,3,4\} \quad \rightarrow \quad x[-n] = \{1,4,3,2\}$

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 26 \\ 28 \\ 26 \\ 20 \end{bmatrix}$$

~~Q14) $x[n] = \{1,2,3,4\}$ $X[k] = \{8,-2,0,-2\}$.~~

Q15) $x[n] = \{1,2,3,2\}$ $h[n] = \{1,2,3,4\}$

a) $y(n) = \sum_{-\infty}^{\infty} x(m) \cdot h(m-n)$

$x(m)$	1	2	3	2	$y(n)$
$h(m)$	1	2	3	4	$y(0) = 22$
$h(m-1)$	4	1	2	3	$y(1) = 18$
$h(m-2)$	3	4	1	2	$y(2) = 18$
$h(m-3)$	2	3	4	1	$y(3) = 22$
$h(m-4)$	1	2	3	4	$y(4) = 22$
					$\vdots$

a) Using DFT

By DFT, $X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{nk}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & 1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$

$$H[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & 1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$Y[k] = X[k] \cdot H[k]$$

$$= \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 80 \\ 4-4j \\ 0 \\ 4+4j \end{bmatrix}$$

By IDFT,

$$y[n] = \frac{1}{N} \sum_{k=0}^{3} W_N^{-nk} \, Y[k]$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & +1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 20 \\ 4-4j \\ 0 \\ 4+4j \end{bmatrix} = \begin{bmatrix} 22 \\ 20-2j \\ 18 \\ 20+2j \end{bmatrix}$$

Q16) $x[n] = \{1, 2, 3, 2\}$

By DFT, $X[k] = \sum_{0}^{N-1} x[n] . W_N^{nk}$

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \begin{array}{ll} \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} & \begin{matrix} k=0 \\ k=1 \\ k=2 \\ k=3 \end{matrix} \end{array}$$

a) $E = \frac{1}{4}\left[8^2 + 2^2 + 0^2 + 2^2\right] = \underline{18}$

b) $E = \left[1^2 + 2^2 + 3^2 + 2^2\right] = \underline{\underline{18}}$

Q17) $x[n] = \{1, -2, 3, -4, 5, -6\}$

a) $X[0] = \underline{\underline{-3}}$

b) $X[3] =$

c) $E = \sum_{0}^{5} |x[n]|^2$

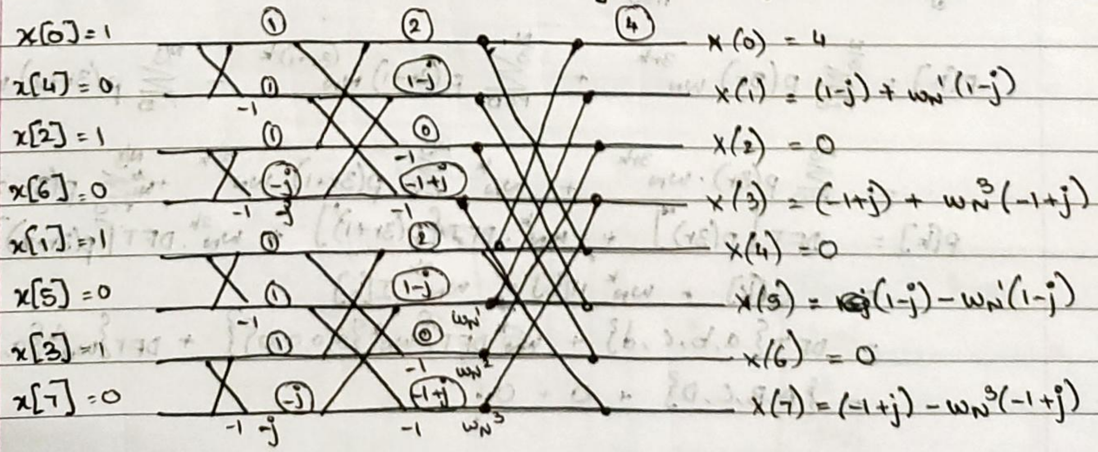
$= \left[1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2\right] = 91$

Energy in time domain = Energy in frequency domain.

$\therefore\ E = \sum_{0}^{5} |X[k]|^2 = \underline{91}$

Q18) $x[n] = \{\overset{0}{1}, \overset{1}{1}, \overset{2}{1}, \overset{3}{1}, \overset{4}{0}, \overset{5}{0}, \overset{6}{0}, \overset{7}{0}\}$

$X[k] = G[k] + W_N^k H[k]$ (by DITFFT eqn).



$$x[1] = (1-j) + (0.707 - j0.707)(1-j)$$
$$= 1 - 2.414j$$

$$x[5] = (1-j) - (0.707 - 0.707j)(1-j)$$
$$= 1 + 0.414j$$

$$x[3] = (-1+j) + (-0.707 - 0.707j)(-1+j)$$
$$= 0.414 + i$$

$$x[7] = (-1+j) + (0.707 + 0.707j)(-1+j)$$
$$= -2.414 + j$$

$$x[k] = \begin{cases} 4 & k=0 \\ 1 - 2.414j & k=1 \\ 0 & k=2 \\ 0.414 + j & k=3 \\ 0 & k=4 \\ 1 + 0.414j & k=5 \\ 0 \ \cancel{2.414+j} & k=6 \\ -2.414 + j & k=7 \end{cases}$$

Q 19) $x[n] = \{a, b, c, d\}$ $\quad X[k] = \{A, B, C, D\}$

$p[n] = \{a, 0, 0, b, 0, 0, c, 0, 0, d, 0, 0\}$

By DFT, $P[k] = \sum_{n=0}^{N-1} p[n] \cdot W_N^{nk}$

$$P[k] = \sum_{r=0}^{\frac{N}{3}} p(3r)\, W_N^{3rk} + \sum_{r=0}^{\frac{N}{3}} p(3r+1)\, W_N^{(3r+1)k} + \sum_{r=0}^{N/3} p(3r+2)\, W_N^{(3r+2)k}$$

$$= \sum_{r=0}^{N/3} p(3r) \cdot W_N^{3rk} + W_N^{k} \sum_{r=1}^{N/3} p(3r+1) \cdot W_N^{3rk} + W_N^{2k} \sum_{r=0}^{N/3} p(3r+2)\, W_N^{3rk}$$

$$P[k] = \text{DFT}[p(3r)] + W_N^{k} \cdot \text{DFT}[p(3r+1)] + W_N^{2k} \cdot \text{DFT}[p(3r+2)]$$

$$= G[k] + W_N^{k} H[k] + W_N^{2k} I[k]$$

$$= \text{DFT}\{a, b, c, d\} + \text{DFT}\{W_N^{k}\{0,0,0,0\}\} + \text{DFT}\{W_N^{2k}\{0,0,0,0\}\}$$

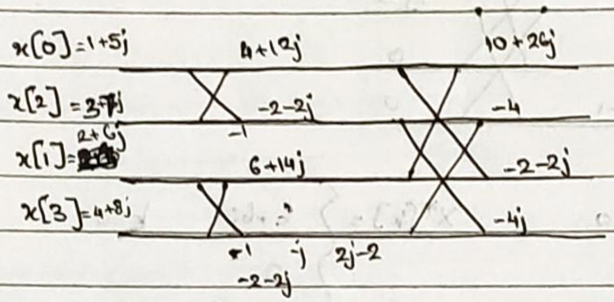
$$= \{A, B, C, D\} + 0 + 0$$

$$P[k] = \{A, B, C, D, A, B, C, D, A, B, C, D\}$$

Q20) $p[n] = \{1,2,3,4\}$ $\quad q[n] = \{5,6,7,8\}$

Let $x[n] = p[n] + j\,q[n]$

$\therefore\ x = \{1+5j,\ 2+6j,\ 3+7j,\ 4+8j\}$



$$X[k] = \begin{cases} 10+26j & k=0 \\ -4 \\ -2-2j \\ -4j \end{cases}$$

(i) To find $p[k]$

$$x[n] = p[n] + j.q[n] \quad —①$$

$$+\quad x^*[n] = p[n] - j\,q[n] \quad —②$$

$$x[n] + x^*[n] = 2\,p[n]$$

$$p[n] = \frac{1}{2}\{x[n] + x^*[n]\}$$

$$P[k] = \frac{1}{2}\{X[k] + X^*[k]\}$$

$$P[k] = \frac{1}{2}\left\{\begin{bmatrix} 10+26j \\ -4 \\ -2-2j \\ -4j \end{bmatrix} + \begin{bmatrix} 10-26j \\ +4j \\ -2+2j \\ -2 \end{bmatrix}\right\} \Rightarrow P[k] = \begin{cases} 10 & k=0 \\ -2+2j \\ -2 \\ -2-2j \end{cases}$$

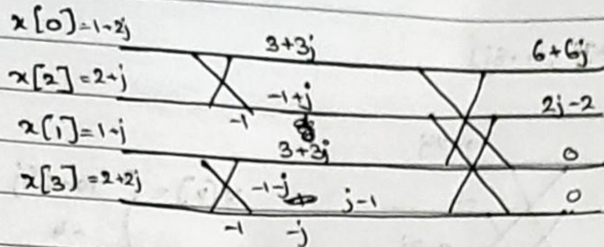
① - ②

$$q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$$

$$Q[k] = \frac{1}{2j}\{X[k] - X^*[k]\}$$

$$= \frac{1}{2j}\left\{\begin{bmatrix} 10+26j \\ -4 \\ -2-2j \\ -4j \end{bmatrix} - \begin{bmatrix} 10-26j \\ +4j \\ -2+2j \\ -2 \end{bmatrix}\right\} = \begin{bmatrix} 26 & k=0 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Q21) $x[n] = \{(1+2j), (1+j), (2+j), (2+2j)\}$



$$X[k] = \begin{cases} 6+6j & k=0 \\ 2j-2 \\ 0 \\ 0 \end{cases} \qquad X^*[k] = \begin{cases} 6+6j & k=0 \\ 0 \\ 0 \\ 2+2j \end{cases}$$

$x[n] = p[n] + j\,q[n]$ —①

$x^*[n] = p[n] - j\,q[n]$ —②

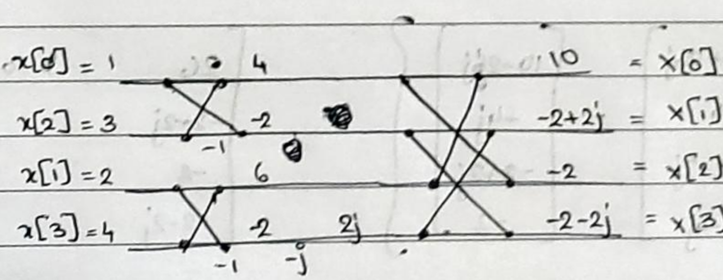
① + ②

$$p[n] = \frac{1}{2}\{x[n] + x^*[n]\} \qquad q[n] = \frac{1}{2j}\{x[n] - x^*[n]\}$$

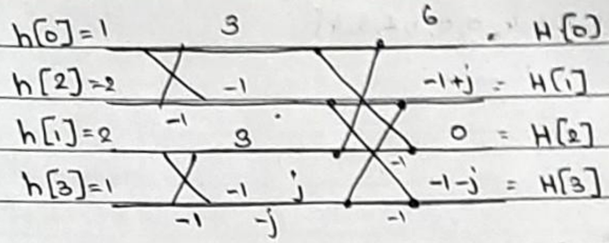
$$P[k] = \frac{1}{2}\left\{\begin{bmatrix} 6+6j \\ 2j-2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6+6j \\ 0 \\ 0 \\ -2-2j \end{bmatrix}\right\} = \begin{bmatrix} 6+6j \\ -1+j \\ 0 \\ -1-j \end{bmatrix}$$

$$Q[k] = \frac{1}{2j}\left\{\begin{bmatrix} 6+6j \\ 2j-2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6+6j \\ 0 \\ 0 \\ -2-2j \end{bmatrix}\right\} = \begin{bmatrix} 6 \\ 1+j \\ 0 \\ 1-j \end{bmatrix}$$

Q22) $h[n] = \{1, 2, 2, 1\}$ $\qquad x[n] = \{1, 2, 3, 4\}$



$$X[k] = \begin{cases} 10 & k=0 \\ -2+2j \\ -2 \\ -2-2j \end{cases}$$

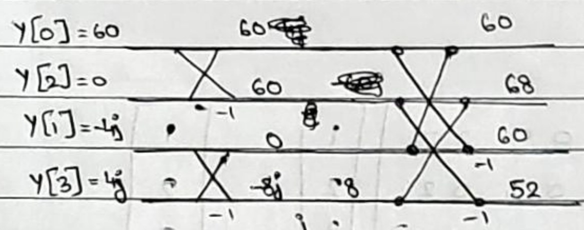


$$H[k] = \begin{cases} 6 & k=0 \\ -1+j \\ 0 \\ -1-j \end{cases}$$

$$Y[k] = X[k] \cdot H[k]$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -1+j \\ 0 \\ -1-j \end{bmatrix} = \begin{bmatrix} 60 \\ -4j \\ 0 \\ 4j \end{bmatrix}$$

Find y[n] by IFFT



$y[0] = 60$
$y[1] = 68$
$y[2] = 60$
$y[3] = 52$

$$y[n] = \begin{cases} 60 & n=0 \\ 68 & n=1 \\ 60 & n=2 \\ 52 & n=3 \end{cases}$$

$y[n] = \{60, 68, 60, 52\}$

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Q23) $h[n] = \{1, 0, 2\}$ $\quad x[n] = \{1, 2, 3, 4, 0, 0, 1, 2, 3, 4\}$

$L = 10, \quad M = 3$

(I) Select N

$N = L + M - 1$

$8 = L + 3 - 1 \Rightarrow \underline{\underline{L = 6}}$

(II) Decompose $x(n)$

$x_1[n] = \{1, 2, 3, 4, 0, 0\}$

$x_2[n] = \{1, 2, 3, 4, 0, 0\}$

(III) Zero padding.

$x_1[n] = \{1, 2, 3, 4, 0, 0, 0, 0\}$

$x_2[n] = \{1, 2, 3, 4, 0, 0, 0, 0\}$

$h(n) = \{1, 0, 2, 0, 0, 0, 0, 0\}$ $\quad \Rightarrow h[-n] = \{1, 0, 0, 0, 0, 0, 2, 0\}$

(IV) $y_1[n] = y_2[n]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 8 \\ 6 \\ 8 \\ 0 \\ 0 \end{bmatrix}$$

$y_1[n] = y_2[n] = \{1, 2, 5, 8, 6, 8, 0, 0\}$

$y_1[n-1] = \{0, 0, 0, 0, 0, 1, 2, 5, 8, 6, 8, 0, 0\}$

$y_1[n-L] = \{1, 2, 5, 8, 6, 8, 1, 2, 5, 8, 6, 8, 0, 0\}$

$L = 6$

$h[n] = \{1, 0, 2\}$ $\quad x[n] = \{1, 2, 3, 4, 0, 0, 1, 2, 3, 4\}$

(i) Select N $\quad$ (let $N = 8$)

$N = L + M - 1 \Rightarrow 8 = L + 3 - 1 \rightarrow L = 6.$

(ii) Decompose $x[n]$.

$x_1[n] = \{1,2,3,4,0,0\}$

$x_2[n] = \{1,2,3,4,0,0\}$

$x_3[n] = \{0,0,0,0,0,0\}$

(iii) Modify i/p sequence

$x_1[n] = \{0,0,1,2,3,4,0,0\}$

$x_2[n] = \{0,0,1,2,3,4,0,0\}$

$x_3[n] = \{0,0,0,0,0,0,0,0\}$

$h[n] = \{1,0,2,0,0,0,0,0\} \Rightarrow h[-n] = \{1,0,0,0,0,0,2,0\}$

(iv) Using $\sum_{n=0}^{M-1} x[m] \cdot h[n-m]$

$$y_1[n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 5 \\ 8 \\ 6 \\ 8 \end{bmatrix}$$

Sum $y_1[n] = \{0,0,1,\cancel{0},2,5,8,6,8\}$

$y_2[n] = \{0,0,0,0,0,0,0,0\}$

Merging $y_1[n]$, $y_2[n]$, $y_3[n]$

$y[n] = \{1,2,5,8,6,8,1,2,5,8,6,8,0,0,0,0,0\}$