# Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

(Autonomous College Affiliated to University of Mumbai)

**End Semester Examination-December 2022**

**Max. Marks: - 100**
**Class: S.E. (EXTC A & B Division)**
**Course Code: - MA201**
**Name of the Course: Linear Algebra**

**Duration: 3 Hours**
**Semester: III**
**Branch: - EXTC**

**Instructions:**
**1) All Questions are Compulsory.**
**2) Assume suitable data if necessary.**

| Q No. | | Max. Marks | CO |
|---|---|---|---|
| Q.1 | a) Construct an orthonormal basis of $R^3$ by applying Gram Schmidt process where $u_1 = (1, 0, 0)$, $u_2 = (3, 7, -2)$, $u_3 = (0, 4, 1)$. | 8 | 4 |
| | b) State only one axiom that fails to hold for each of the following sets W to be subspaces of the respective real vector spaces V with Standard operations. 1) $W = \{(x, y) \mid x^2 = y^2\}$, $V = R^2$<br>2) $W = \{(x, y) \mid x\,y \geq 0\}$, $V = R^2$<br>3) $W = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$, $V = R^3$<br>4) $W = \{f \mid f(x) \leq 0 \text{ for all } x\}$, $V = F(-\infty, \infty)$ | 7 | 4 |
| | c) Show that $S = \{1 - t - t^2, -2 + 3t + 2t^3, 1 + t + 5t^3\}$ is linearly independent in $P_3$. ($P_3$ means the set of all polynomials of degree 3 i.e. of the form $a_0 + a_1 x + a_2 x^2 + a_3 x^3$)<br>**OR**<br>c) Determine the linear dependence or independence of vectors (2,-1, 3,2), (1, 3, 4, 2) and (3, -5, 2, 2). Find the relation between them if dependent. | 6 | 4 |
| | d) Find the basis for the row and column space of<br>$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 3 & 1 & 5 \end{bmatrix}$ | 4 | 4 |
| | e) Check whether the following set of vectors is a basis for $P_2$.<br>$S = \{1 - 3x + 2x^2,\ 1 + x + 4x^2,\ 1 - 8x + x^2\}$<br>Find the coordinate vector of $p = 1 - 2x + x^2$ with respect to the above basis. | 5 | 4 |

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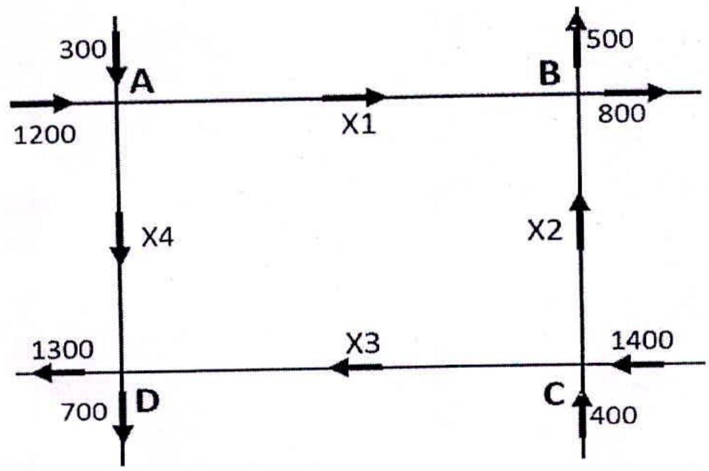
|---|---|---|---|
| **Q.2** | a) Solve the following system of differential equation $y' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} y$ | 8 | 6 |
| | b) Show that the matrix A = $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfies Cayley- Hamilton Theorem and hence find $A^{-1}$ if it exists.<br>**OR**<br>b) If A = $\begin{bmatrix} \alpha & \alpha \\ \alpha & a \end{bmatrix}$ then prove that $e^{A} = e^{\alpha} \begin{bmatrix} cosh\alpha & sinh\alpha \\ sinh\alpha & cosh\alpha \end{bmatrix}$ | 8 | 6 |
| | c) Find the Singular Value Decomposition of the matrix A = $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ | 8 | 6 |
| | d) Prove that the characteristic roots of a Hermitian matrix are all real. | 4 | 6 |
| **Q.3** | a) Apply Gauss Jacobi Method to solve the following equations<br>$4x + y + 3z = 17$<br>$x + 5y + z = 14$<br>$2x - y + 8z = 12$ upto 5 iterations.<br>**OR**<br>a) Apply Gauss Seidel Method to solve the following equations<br>$28x + 4y - z = 32$<br>$2x + 17y + 4z = 35$<br>$x + 3y + 10z = 24$<br>**Note :-** upto 5 iterations | 8 | 2 |
| | b) Show that the matrix A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Find the diagonal matrix and the transforming matrix. | 8 | 5 |
| | c) Fit a curve of the form $y = a + b\,x + c\,x^2$ for the following data using Least square Method.<br>X : 1.0   1.2   1.4   1.6   1.8   2<br>Y : 2.345   2.419   2.592   2.863   3.233   3.702 | 5 | 4 |

| | | | |
|---|---|---|---|
| **Q.4** | a) Using a suitable 2 x 2 matrix , Encode and decode the message NOW * STUDY | 7 | 3 |
| **Q.5** | a) Reduce the following matrix to Row Echelon form and find its rank $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 4 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & -8 & 0 \end{bmatrix}$ | 6 | 1 |
| | b) $If\ x_1, x_2, x_3, x_4$ are the number of vehicles travelling through each road per hour. Find $x_1, x_2, x_3, x_4$ from the traffic diagram given below: | 8 | 1 |




*********** All the Best **********