(B) Find the ranks of the following matrices.

1. $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 6 \end{bmatrix}$ **(M.U. 1994)**

2. $\begin{bmatrix} 6 & 1 & 3 & 6 \\ 4 & 2 & 6 & 1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 13 \end{bmatrix}$ **(M.U. 2005)**

3. $\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$ **(M.U. 2005)**

4. $\begin{bmatrix} 2 & 3 & 1 & 4 \\ 5 & 2 & 3 & 0 \\ 9 & 8 & 0 & 8 \end{bmatrix}$ **(M.U. 2003)**

5. $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

6. $\begin{bmatrix} 3 & 4 & -2 & 1 \\ 5 & 8 & 4 & 2 \\ 8 & 12 & 2 & 3 \\ 13 & 20 & 6 & 5 \end{bmatrix}$

7. $\begin{bmatrix} 2 & 6 & -2 & 6 & 10 \\ -3 & 3 & -3 & -3 & -3 \\ 1 & -2 & 4 & 3 & 5 \\ 2 & 0 & 4 & 6 & 10 \\ 1 & 0 & 2 & 3 & 5 \end{bmatrix}$

8. $\begin{bmatrix} 3 & 2 & -4 & 3 & 6 \\ 1 & -2 & 3 & 4 & -3 \\ 2 & -4 & 6 & 8 & -6 \\ 3 & -6 & 9 & 12 & -9 \\ 5 & -2 & 2 & 11 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{bmatrix}$ **(M.U. 2003)**

[**Ans.** : **(1)** $r = 3$, **(2)** $r = 2$, **(3)** $r = 2$, **(4)** $r = 3$, **(5)** $r = 3$, **(6)** $r = 2$, **(7)** $r = 3$, **(8)** $r = 2$, **(9)** $r = 3$.]

(A) Solve the following equations : Class (b) : 6 Marks

1. $x_1 - 2x_2 + 3x_3 = 0,\ 2x_1 + 5x_2 + 6x_3 = 0.$ [**Ans.** : $x_1 = -3t,\ x_2 = 0,\ x_3 = t$]
2. $2x_1 - x_2 + 3x_3 = 0,\ 3x_1 + 2x_2 + x_3 = 0,\ x_1 - 4x_2 + 5x_3 = 0.$ [**Ans.** : $x_1 = -t,\ x_2 = t,\ x_3 = t$]
3. $x_1 - x_2 + x_3 = 0,\ x_1 + 2x_2 + x_3 = 0,\ 2x_1 + x_2 + 3x_3 = 0$ [**Ans.** : $x_1 = x_2 = x_3 = 0$]

4. $x_1 + x_2 + x_3 + x_4 = 0$, $2x_1 + x_2 - x_4 = 0$, $x_1 + 3x_2 + 2x_3 + 4x_4 = 0$.

[**Ans.** : $x_1 = t$, $x_2 = -t$, $x_3 = -t$, $x_4 = t$]

5. $7x_1 + x_2 - 2x_3 = 0$, $x_1 + 5x_2 - 4x_3 = 0$, $3x_1 - 2x_2 + x_3 = 0$, $2x_1 - 7x_2 + 5x_3 = 0$.

[**Ans.** : $x_1 = \frac{3}{17}t$, $x_2 = \frac{13}{17}t$, $x_3 = t$]

6. $4x - y + 2z + t = 0$, $2x + 3y - z - 2t = 0$, $7y - 4z - 5t = 0$, $2x - 11y + 7z + 8t = 0$.

[**Ans.** : $x = -\frac{5r + s}{14}$, $y = \frac{4r + 5s}{7}$, $z = r$, $t = s$]

7. $3x_1 + 4x_2 - x_3 - 9x_4 = 0$, $2x_1 + 3x_2 + 2x_3 - 3x_4 = 0$,
$2x_1 + x_2 - 14x_3 - 12x_4 = 0$, $x_1 + 3x_2 + 13x_3 + 3x_4 = 0$.

(M.U. 2004, 16) [**Ans.** : $x_1 = 11t$, $x_2 = -8t$, $x_3 = t$, $x_4 = 0$]

(B) Class (b) : 6 Marks

1. Find the value of λ for which the following equations have non-zero solutions. Also solve the equations.

$$x + 2y + 3z = \lambda x,\ 3x + y + 2z = \lambda y,\ 2x + 3y + z = \lambda z.$$

(M.U. 2003, 05, 06)

[**Ans.** : $\lambda = 6$, $x = y = z = t$]

Solve the following equations by Gauss-Jacobi's (Jacobi's) method : Class (c) : 8 Mar

1. $10x + y + 2z = 13$, $2x + 10y + 3z = 15$, $x + 3y + 10z = 14$.
2. $15x + y - z = 14$, $x + 20y + z = 23$, $2x - 3y + 18z = 35$.
3. $12x + 2y + z = 27$, $2x + 15y - 3z = 16$, $2x - 3y + 25z = 26$.
4. $8x - y + 2z = 13$, $x - 10y + 3z = 17$, $3x + 2y + 12z = 25$.
5. $14x - y + 3z = 18$, $2x - 14y + 3z = 19$, $x - 3y + 16z = 20$.
6. $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$.
7. $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$.

[**Ans.** : Actual values are : **(1)** $x = 1, y = 1, z = 1$; **(2)** $x = 1, y = 1, z = 2$; **(3)** $x = 2, y = 1, z = 1$; **(4)** $x = 1, y = -1, z = 2$; **(5)** $x = 1, y = -1, z = 1$; **(6)** $x = 1, y = 2, z = 3$; **(7)** $x = 1, y = -1, z = 1$.

(B) Solve the following systems of linear equations by Gauss-Seidel Method by taking three iterations only : Class (c) : 8 Marks

1. $4x + y + z = 5,\quad x + 6y + 2z = 19,\quad x + 2y + 5z = -10.$ **(M.U. 1997, 98, 2000)**
2. $x_1 + 10x_2 + 4x_3 = 6,\quad 2x_1 - 4x_2 + 10x_3 = -15,\quad 9x_1 + 2x_2 + 4x_3 = 20.$ **(M.U. 1997, 99)**
3. $10x_1 - 5x_2 - 2x_3 = 3,\quad 4x_1 - 10x_2 + 3x_3 = -3,\quad x_1 + 6x_2 - 10x_3 = -3.$ **(M.U. 1999, 2013)**

[**Ans. : (1)** $x = 1{\cdot}2060,\quad y = 4{\cdot}2060,\quad z = -3{\cdot}9484$

(2) $x_1 = 2{\cdot}7403,\quad x_2 = 0{\cdot}9970,\quad x_3 = -1{\cdot}6493$

(3) $x_1 = 0{\cdot}9147,\quad x_2 = 0{\cdot}9368,\quad x_3 = -0{\cdot}9536$]

27. Find the characteristic equation, eigen values and eigen vectors of the following matrices.

(i) $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

(M.U. 1993)

(iv) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ **(v)** $\begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

(vii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ **(viii)** $\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$ **(ix)** $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

(M.U. 1996, 2004, 05, 06)

(x) $\begin{bmatrix} -3 & -9 & -12 \\ 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ **(xi)** $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ **(xii)** $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$

(xiii) $\begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$ **(xiv)** $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ **(xv)** $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

(M.U. 1999, 2002)

(xvi) $\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$

[**Ans. : (i)** $\lambda = 2, 2, 3$; $(5, 2, -5)'$, $(1, 1, -2)'$

(ii) $\lambda = 1, 2, 4$; $(0, 0, 0)'$, $(0, -1, 1)'$.

(iii) $\lambda = 2, 3, 6$; $(1, 0, -1)'$, $(1, 1, 1)'$, $(1, -2, 1)'$.

(iv) $\lambda = 1, 2, 2$; $(1, 1, -1)'$, $(2, 1, 0)'$.

(v) $\lambda = 2, 0, 1$; $(2, -1, 0)'$, $(4, -1, 0)'$, $(4, 0, -1)'$.

(vi) $\lambda = 1, 2, 3$; $(1, 0, -1)'$, $(0, 1, 0)'$, $(1, 0, 1)'$.

(vii) $\lambda = 5, -3, -3$; $(1, 2, -1)'$, $(2, -1, 0)'$, $(3, 0, 1)'$.

(viii) $\lambda = 1, -1, 2$; $(1, 2, 1)'$, $(0, -1, 1)'$, $(1, 1, 0)'$.

(ix) $\lambda = 1, 1, 1$; $(1, 1, 1)'$.

(x) $\lambda = 0, 1, 1$; $(3, -1, 0)'$, $(12, -4, -1)'$.

(xi) $\lambda = 1, 2, 3$; $(1, -1, 0)'$, $(2, -1, -2)'$, $(1, -1 -2)'$.

(xii) $\lambda = 0, 1, -2$; $(-10, -3, 11)'$, $(1, 0, 0)'$, $(-4, -3, 7)'$.

(xiii) $\lambda = 0, -1, 2$; $(-1, 0, 1)'$, $(-1, -1, 1)'$, $(4, 1, -3)'$.

(xiv) $\lambda = 1, -1, 4$; $(-6, -2, 7)'$, $(0, 1, -1)'$, $(-3, -1, 1)'$.

(xv) $\lambda = -2, 1, 3$; $(11, 1, -14)'$, $(1, -1, -1)'$, $(1, 1, 1)'$.]

(xvi) $\lambda = -1, -1, +1$.]