



BHARATIYA VIDYA BHAVAN'S
SARDAR PATEL INSTITUTE OF TECHNOLOGY

MUNSHI NAGAR, ANDHERI (WEST), MUMBAI – 400 058, India
(Autonomous College Affiliated to University of Mumbai)

End Semester Examination Mar 2023

Max. Marks: 100

Duration: 3hrs

Class: FYMCA

Semester: I

Course Code: MA1M1

Date: 06/3/2023

Subject: Linear Algebra

Time: 2–5 pm

Instructions: (1) All questions are compulsory.
(2) Use of scientific calculator is allowed.
(3) Assume any necessary data but justify the same.

Q.N		Marks	CO
1.	(a) Reduce the following matrix to row echelon form and find its rank. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$	[5]	1
	(b) Show that if $\lambda \neq -5$ the following system of equations has a unique solution. Find the solution when $\lambda = 0$. $3x - y + 4z = 3, x + 2y - 3z = -2, 6x + 5y + \lambda z = -3$	[15]	1
2.	Attempt any TWO . (a) Solve the following system of linear equations by LU decomposition. $2x_1 + 4x_2 + x_3 = 3, 3x_1 + 2x_2 - 2x_3 = -2, x_1 - x_2 + x_3 = 6$	[10]	2
	(b) Solve the following system of linear equations using Jacobi's method. $2x_1 + 3x_2 + x_3 = 9, x_1 + 2x_2 + 3x_3 = 6, 3x_1 + x_2 + 2x_3 = 8$ Use initial guess as (0, 0, 0). (Only 3 iterations)	[10]	2
	(c) Solve the following system of linear equations using Gauss Seidel method. $2x_1 + 3x_2 + x_3 = 9, x_1 + 2x_2 + 3x_3 = 6, 3x_1 + x_2 + 2x_3 = 8$ Use initial guess as (0, 0, 0). (Only 3 iterations).	[10]	2



BHARATIYA VIDYA BHAVAN'S
SARDAR PATEL INSTITUTE OF TECHNOLOGY

MUNSHI NAGAR, ANDHERI (WEST), MUMBAI – 400 058, India
(Autonomous College Affiliated to University of Mumbai)

3.	<p>(a) Assign each letter of the alphabet (A-Z) to a number (1-26) and space to 0. Encode the message KHARGHAR STATION using the encoding matrix $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$. Find the corresponding decoding matrix. Verify that this decoding matrix correctly decodes the encoded message.</p> <p>(b) Show that matrix $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ is diagonalizable. Find its diagonal form and transforming matrix.</p>	[8]	3
4.	<p>(a) Determine whether the vectors (2, 3, -1), (3, 4, 1) and (7, 10, -1) are linearly independent. If not, express one of the vectors as linear combination of others.</p> <p>(b) Show that the function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(a,b) = (a+b, a-b, b)$ is a linear transformation. Also find the rank and nullity of T.</p>	[8] [12]	4 4
5.	<p>Attempt any TWO.</p> <p>(a) Find the page having maximum rank in the following graph.</p> <div data-bbox="416 1203 772 1448" data-label="Diagram"> </div> <p>(b) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$, find A^{100}.</p> <p>(c) Solve the following system of differential equations using diagonalization.</p> $\frac{dy_1}{dx} = -2y_1 + y_2$ $\frac{dy_2}{dx} = y_1 - 2y_2$	[10] [10] [10]	6 6 6

---X---X---