Synoptic :→

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Max. Marks: 100	**Duration:** 3 Hours
Class: SE (Comp and IT)	**Semester:** IV
Course Code: BS41	**Branch:** Comp and IT
Name of the Course: Applied Mathematics-II	

Instructions:
(1) All questions are compulsory
(2) Assume suitable data if necessary

Q1 :→

Ans a) The characteristic eqn is $|A-\lambda I|=0$

ie $\begin{vmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$

we get $\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

This eqn is satisfied by A. — (2) marks

Dividing $\lambda^8 - 5\lambda^7 + 7\lambda^6 - 3\lambda^5 + \lambda^4 - 5\lambda^3 + 8\lambda^2 - 2\lambda + 1$

by $\lambda^3 - 5\lambda^2 + 7\lambda - 3$

we get quotient is $\lambda^5 + \lambda$

Remainder $\lambda^2 + \lambda + 1$ — (1) mark

In terms of the matrix A This means

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$= (A^3 - 5A^2 + 7A - 3I)(A^5 + A) + (A^2 + A + I)$ — (1)

But $A^3 - 5A^2 + 7A - 3I = 0$

$\therefore$ (1) $\Rightarrow$ LHS $= A^2 + A + I$ — (2)

$\therefore A^2 = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$ — (2) marks

$\therefore$ (2) $\Rightarrow A^2+A+I = \begin{pmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{pmatrix} + \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{pmatrix}$ is the given Expression. — (1) mark

Q1 :→

ns b) i) By defn $A \cdot \text{adj}A = |A| \cdot I$

premultiply by A^{-1}

$(A^{-1}A)\ \text{adj}A = A^{-1}|A|$

$\Rightarrow \text{adj}A = |A|\,A^{-1}$ — (2) marks

$\therefore \text{adj}A X = |A|\,A^{-1}X = |A| \cdot \frac{1}{\lambda} X$

$= \frac{|A|}{\lambda} X$

$\therefore \left(\text{adj}A - \frac{|A|}{\lambda}\right) X = 0$

Since X is not a zero vector

$\therefore \text{adj}A - \frac{|A|}{\lambda} = 0 \Rightarrow \text{adj}A = \frac{|A|}{\lambda}$ — (1) mark

Hence $\frac{|A|}{\lambda}$ is an eigen value of adj$A$

Ans

ii) $AX = \lambda X$

$\therefore (AX)^\theta = (\lambda X)^\theta \Rightarrow X^\theta A^\theta = \bar{\lambda} X^\theta$

$\therefore X^\theta A = \bar{\lambda} X^\theta$ ($\because A$ is a Hermitian)

$\Rightarrow X^\theta A X = \bar{\lambda} X^\theta X \Rightarrow X^\theta (AX) = \bar{\lambda}(X^\theta X)$ — (2) marks

$\Rightarrow X^\theta \lambda X = \bar{\lambda}(X^\theta X)$

$\therefore \lambda x^{\theta}x - \bar{\lambda} x^{\theta}x = 0$

$\Rightarrow (\lambda - \bar{\lambda}) x^{\theta}x = 0$

$\Rightarrow \lambda = \bar{\lambda}$

we conclude that $\lambda = \bar{\lambda}$ — 1 mark

showing that λ is real.

Q1 :→

Ans c) $\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$

$\therefore \lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\Rightarrow \lambda = 0, 3, 15$

Here all Eigen values are distinct, the matrix A is diagonalizable

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $[A - \lambda I]X = 0$ — 3 marks

$\therefore \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

put $\lambda = 0$ $\therefore$ Eigen vectors are $[1, 2, 2]$

$\lambda = 3$,, ,, $[2, 1, -2]$

$\lambda = 15$,, ,, $[2, -2, 1]$ — 3 marks

Since $M^{-1}AM = D$, the given matrix A will be diagonalised to the diagonal matrix

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ by $M = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ — 2 marks or

Q1 :→

Ans c) $A'A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

∴ The charac. eqn of $A'A$ is $\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$

∴ $(3-\lambda)^2 - 1 = 0 \Rightarrow \lambda = 4, 2$

If $\lambda_1 = 4$ ∴ $\boxed{\sigma_1 = 2}$

$\lambda_2 = 2$ ∴ $\boxed{\sigma_2 = \sqrt{2}}$ ∴ $D = \begin{bmatrix} 2 & 0 \\ 0 & \sqrt{2} \end{bmatrix}$

② mark

for $\lambda = 4$ ∴ $\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

∴ $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

By $R_2 + R_1$ ∴ $\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow \boxed{x_1 = x_2}$

Let $x_1 = t$ ∴ $x_2 = t$ ∴ $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Also $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

① mark

It is clear that $(v_1 \cdot v_2) = (1, 1)(1, -1) = 0$

& hence v_1, v_2 are orthogonal

Now $\|v_1\| = \sqrt{2}$ And $v_2 = \sqrt{2}$

∴ $v_1 = \frac{v_1}{\|v_1\|} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$, $v_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

∴ $V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

① mark

$$u_1 = \frac{1}{\sigma_1} A v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

1 mark

Since A is a 3×2 matrix, U must be a 3×3 matrix

Let the third column of U be $u_3 = [a\ b\ c]$

But $(u_1 \cdot u_3) = 0$ ($\because u_1, u_2, u_3$ are orthogonal)

$\therefore \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \cdot (a, b, c) = 0$

$\Rightarrow a + b + 0c = 0$ —— (1)

And $u_2 \cdot u_3 = 0$

$(0, 0, 1) \cdot (a, b, c) = 0a + 0b + c = 0$ —— (2)

2 marks

Solving (1) & (2) by Cramer's Rule

$\boxed{a_1 = 1}$, $\boxed{b_1 = -1}$, $\boxed{c = 0}$

$\therefore \|u_3\| = \sqrt{2}$

$\therefore$ The normalised vector u_3 is

$$u_3 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

To make matrix multiplication possible, we take a zero row vector in D

$$\therefore A = UDV' = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

1 mark

Q2 :→

Ans a) prepare the table
$N=6, \quad \Sigma D^2 = 13.50$ — ② marks

There are two items in X series having equal values at the rank 4 ∴ each is given the rank $= \frac{4+5}{2} = \frac{9}{2} = 4.5$

||ly There are 3 items in Y series at the rank 3 ∴ $\frac{3+4+5}{3} = 4$ — ② marks

$$\therefore R = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right]}{N^3 - N}$$

Here $m_1 = 2, \; m_2 = 3$ — ① mark

$\therefore R = 0.5429$ — ① mark

Q2 :→

Ans b) Line of Regression of y on x

is $y - \bar{y} = r \frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$\Rightarrow m_1 = r \frac{\sigma_y}{\sigma_x}$ — ①

And $x - \bar{x} = r \frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$\therefore m_2 = \frac{1}{r} \frac{\sigma_y}{\sigma_x}$ — ②

we know $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

— ② marks

$$\therefore \tan\theta = \frac{\frac{1}{r}\frac{\sigma_y}{\sigma_x} - r\frac{\sigma_y}{\sigma_x}}{1 + \cancel{r}\frac{\sigma_y}{\sigma_x}\cdot\frac{1}{\cancel{r}}\frac{\sigma_y}{\sigma_x}} = \frac{\left(\frac{1}{r}-r\right)\frac{\sigma_y}{\sigma_x}}{1+\frac{\sigma_y^2}{\sigma_x^2}}$$

$$= \frac{\frac{1-r^2}{r}\cdot\frac{\sigma_y}{\sigma_x}\cdot\sigma_x^2}{\sigma_x^2+\sigma_y^2} = \frac{1-r^2}{r}\cdot\frac{\sigma_x\cdot\sigma_y}{\sigma_x^2+\sigma_y^2}$$ — ③

— ① mark

If $r=0$ Then there is no relationship between the two variables and they are independent.

put $r=0$ in ③ $\therefore \tan\theta=\infty$

$\therefore \boxed{\theta=\frac{\pi}{2}}$ $\therefore$ lines are $\perp^{ar}$.

If $r=1$ or -1 in ③ $\therefore \tan\theta=0$

$\therefore \boxed{\theta=0}$ $\therefore$ lines are Coincide.

— ③ Marks

Q2 :→

Ans c) i) we know $R = 1 - \frac{6\Sigma D^2}{N^3-N}$

$\therefore \Sigma D^2 = \frac{495}{6}$

— ② marks

$\therefore$ correct ΣD^2 = Incorrect ΣD^2 − (Incorrect rank difference)^2 + (correct Rank Diff)^2

$= \frac{495}{6} - 3^2 + 7^2 = \frac{735}{6}$

— ① mark

$\therefore$ correct $R = 1 - 6\frac{(735/6)}{990} = 0.2576$ — ① mark

Q2: c.ii) Geometric mean of the coeff.
Ans

of regression is the coeff. of correlation

$$G.M = \sqrt{b_{yx}\, b_{xy}} = \sqrt{r\frac{\sigma_y}{\sigma_x}\, r\frac{\sigma_x}{\sigma_y}} \quad \text{— ① mark}$$

$$= \sqrt{r^2} = r$$

- If one coefficient of regression is greater than one, the other must be less than one.

Since $-1 \le r \le 1$, $r^2 \le 1$ — ① mark

but $r^2 = b_{yx} \cdot b_{xy}$ $\therefore b_{yx}\, b_{xy} \le 1$

$\therefore b_{yx} \le \frac{1}{b_{xy}}$ If $b_{yx} < 1$, $b_{xy} > 1$

- Coeff. of regression are indep. of change of origin but not of change of scale.
- Arithmetic mean of the coeff. of regression is greater than or equal to the coeff. of correlation

$$\frac{b_{yx} + b_{xy}}{2} \geq r \quad \therefore (\sigma_x - \sigma_y)^2 \geq 0$$ — ② mark

or

Q2:

Ans c) prepare the table

$\Sigma x = 56$, $\Sigma y = 99$, $\Sigma x^2 = 476$, — ② marks

$\Sigma y^2 = 1427$, $\Sigma xy = 811$

$\bar{x} = \frac{\Sigma x}{n}$ $\therefore \boxed{\bar{x} = 8}$, $\bar{y} = \frac{\Sigma y}{n} = 14.14$ — ① mark

$$\sigma_x^2 = \frac{\Sigma x^2}{n} - \bar{x}^2 = 4 \Rightarrow \boxed{\sigma_x = 2}$$

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - \bar{y}^2 \Rightarrow \boxed{\sigma_y = 1.98}$$

$$Cov(x,y) = \frac{\Sigma xy}{n} - \bar{x}\bar{y} = 2.74$$

(3) marks

$$r = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y} = 0.69$$

$\therefore y = 0.68x + 8.7$

And $x = 0.69y - 1.76$

(2) marks

Q3: →

Ans a) i) $M.GF = E(e^{tx}) = \Sigma e^{tx} \cdot P(x)$

$$= \frac{e^{-2t}}{3} + \frac{e^{3t}}{2} + \frac{e^{t}}{6}$$ — (1) mark

ii) Prepare the Table

$\mu_r' = \Sigma x^r P(x)$ — (1)

— (2) marks

put $r=1$ in (1) $\quad \therefore \mu_1' = \Sigma x P(x) = 1$

$r=2 \quad \mu_2' = 6$

$r=3 \quad \mu_3' = 11$

$r=4 \quad \mu_4' = 46$

— (1) mark

iii) The first four moments about mean are $\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 5$

$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3 = -5$

$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4 = 35$

(2) marks

Q3 :→

Ans b) $\int_x f(x)dx = 1 = \int_0^\infty k x e^{-x/3} dx = 1$

$$\boxed{k = \frac{1}{9}}$$

Now p.d.f. is $f(x) = \frac{1}{9} x e^{-x/3}$, $x > 0$ — 2 marks

$E(x) = \int_x x f(x) dx = \frac{1}{9}\int_0^\infty x^2 e^{-x/3} dx$ — 2 marks

$\boxed{E(x) = 6}$

Now $P(x > 6) = \int_6^\infty f(x) dx = \int_6^\infty \frac{1}{9} x e^{-x/3} dx$

$= \frac{3}{e^2}$ — 2 marks

Q3 (c) :→

Ans i) $P(x < 1, y < 3) = P(0 < x < 1, 2 < y < 3)$

$= \int_{x=0}^{1}\int_{y=2}^{3} f(x,y) dx dy = \int_0^1 \left[\int_2^3 \frac{1}{8}(6 - x - y) dy\right] dx$ — 2 marks

$= \frac{3}{8}$

ii) $P(x < 1 / y < 3) = \frac{P(x < 1, y < 3)}{P(y < 3)}$

$\therefore P(y < 3) = P(2 < y < 3) = \int_{y=2}^{3} f(y) dy$ — 2 marks

Now $f(y) = \int_x f(x,y) dx = \int_0^2 \frac{1}{8}(6 - x - y) dx$

$\boxed{f(y) = \frac{1}{4}(5 - y)}$

$\therefore P(y<3) = \int_2^3 \frac{1}{4}(5-y)\,dy = \frac{5}{8}$

$\therefore P(x<1 / y<3) = \frac{3}{5}$ — 2 marks

iii) $f(x) = \int_y f(x,y)\,dy = \int_2^4 \frac{1}{8}(6-x-y)\,dy$

$= \frac{1}{4}(3-x)$ — 1 mark

iv) $\frac{1}{8}(6-x-y) \neq \frac{1}{4}(3-x)\ \frac{1}{4}(5-y)$

$\Rightarrow f(x,y) \neq f(x) \cdot f(y)$ $\therefore$ X & Y are Not independent. — 1 mark

or

Q3:→

Ans c) P (success in the first trial) $= \frac{1}{n}$

P (Failure " ") $= 1 - \frac{1}{n}$

If there is failure in the first trial, the key is eliminated. There are now $(n-1)$ keys — 2 mark

$\therefore$ P (success in the second trial) $= \frac{1}{n-1}$

P (Failure " 1st & success in the second trial) $= \left(1-\frac{1}{n}\right)\left(\frac{1}{n-1}\right)$

$= \frac{n-1}{n} \cdot \frac{1}{n-1} = \frac{1}{n}$ — 2 marks

P (Failure in the 1st trial, failure in the second trial & success in the third trial)

$= \left(1-\frac{1}{n}\right)\left(1-\frac{1}{n-1}\right)\left(\frac{1}{n-2}\right) = \frac{1}{n}$

$\therefore X = 1 \quad 2 \quad 3 \quad 4 \quad \cdots \quad n$

$P(X=x): \frac{1}{n} \quad \frac{1}{n} \quad \frac{1}{n} \quad \frac{1}{n} \quad \cdots \quad \frac{1}{n}$ — (1) mark

$E(X) = \Sigma x P(x) = \frac{1}{n}\cdot 1 + \frac{1}{n}\cdot 2 + \cdots \frac{1}{n}\cdot n$

$= \frac{1}{n}(1+2+\cdots n) = \frac{1}{\cancel{n}} \cdot \frac{\cancel{n}}{2}(n+1)$

$= \frac{n+1}{2}$ — (1) mark

$E(X^2) = \Sigma P(x) x^2 = \frac{1}{n}\cdot 1^2 + \frac{1}{n}\cdot 2^2 + \frac{1}{n}\cdot 3^2 + \cdots \frac{1}{n}\cdot n^2$

$= \frac{(n+1)(2n+1)}{6}$ — (1) mark

$Var(X) = E(X^2) - [E(X)]^2 = \frac{n^2-1}{12}$ — (1) mark

Q4:→

Ans a) i) If the car is not used then $(x) = 0$

$P(x) = \frac{e^{-m} m^x}{x!} \Rightarrow P(0) = \frac{e^{-1.5}(1.5)^0}{0!}$

$= e^{-1.5} = 0.2231$

Number of days in a year when the demand is zero $= 365 \times 0.2231 = 81.4315$ — (3) mark

ii) Some demand is refused if the no. of demands is more than two ie $x > 2$

$\therefore P(x>2) = P(3) + P(4) + \cdots$

$= 1 - [P(0) + P(1) + P(2)]$ — (2) mark

$= 0.1912625$

No. of days in a year when some demand of car is refused $= 365 \times 0.1912625 = 69.81 = 70$ days — (1) mark

$\therefore \rightarrow \quad P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

(7)

Ans b) $p = 0.01, \quad q = 0.99, \quad n = 10$ — (2) marks

$P(X=x) = {}^{10}C_{x} (0.01)^{x} (0.99)^{10-x}$

Now $P(X>1) = 1 - [P(X \leq 1)]$ — (3) marks

$= 0.00425$

$M.G.F = M_0(t) = q + pe^{t}$

$= 0.99 + 0.01e^{t}$ — (1) mark

Q4: $\rightarrow$

Ans c) we have S.N.V, $z = \frac{X-m}{\sigma} = \frac{X-42}{24}$

i) when $X = 50$ $\quad \therefore z = \frac{50-42}{24} = \frac{1}{3} = 0.33$ — (2) mark

$P(z \geq 50)$ = area to the right of 0.33

$= 0.5 -$ (area between $z=0$ and $z=0.33$)

$= 0.5 - 0.1293 = 0.3707$ — (2) marks

ii) when $X = 30$ and $X = 54$

$z = \frac{30-42}{24} = -0.5$ And $z = \frac{54-42}{24} = 0.5$

$\therefore P(30 \leq z \leq 54)$ = (area betn $z=-0.5$ to $z=0.5$)

$= 2$ (area betn $z=0$ & $z=0.5$)

$= 2(0.1915) = 0.3830$ — (2) marks

No. of students getting more than 50 marks

$= NP = 1000 \times 0.3707 = 371$

$\therefore$ No. of students getting marks betn 30 & 54 $= NP = 1000 \times 0.3830$

$= 383$ — (2) marks

or

Q4:

Ans c) i) mean $= \dfrac{\Sigma fx}{\Sigma f}$ and ——— (1)

$(q+p)^n = {}^nC_0 q^n p^0 + {}^nC_1 q^{n-1} p^1 + {}^nC_2 q^{n-2} p^2 + {}^nC_3 q^{n-3} p^3 + - - - + p^n$ — (1) mark

Then prepare the table ——— (1) mark

$\Sigma fx = nq^{n-1}p + n(n-1)q^{n-2}p^2 + \dfrac{n(n-1)(n-2)}{2} q^{n-3}p^3 + - - - np^n$

$= np\left[q^{n-1} + (n-1)q^{n-2}p + \dfrac{(n-1)(n-2)}{2} q^{n-3}p^2 + - - p^{n-1}\right]$

$= np\,(q+p)^{n-1} = np.1 = np$ ——— (2)

And $\Sigma f = q^n + nq^{n-1}p + \dfrac{n(n-1)}{2} q^{n-2}p^2 + - - p^n$

$= (q+p)^n = 1.$ ——— (3)

$\therefore$ mean $= np$ — (2) marks

c ii) $P(x) = \dfrac{e^{-m} m^x}{x!} \Rightarrow P(x+1) = \dfrac{e^{-m} m^{x+1}}{(x+1)!}$

$\therefore \dfrac{P(x+1)}{P(x)} = \dfrac{e^{-m} m^{x+1}}{(x+1)!} \times \dfrac{x!}{e^{-m} m^x}$

$\boxed{P(x+1) = \dfrac{m}{x+1} \cdot P(x)}$ → Recurrence relation — (1) — (2) marks

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$\therefore\ P(x) = \frac{e^{-2}\, 2^{x}}{x!}$

$x=0 \quad \therefore P(0) = e^{-2}$

$x=0$ in ① $\quad \therefore P(1) = \frac{2}{1} P(0) = 2e^{-2}$

$x=1$ in ① $\quad \therefore \quad P(2) = 2e^{-2}$

$P(3) = \frac{4}{3} e^{-2}$ ― ② marks

$P(4) = \frac{2e^{-2}}{3}$

Q5 (a):

Ans a) i) Let $x_1, x_2 - - - x_n$ be a random sample size n from a large population $X_1, X_2 - - - X_n$ of size N with mean μ

The sample mean $\bar{x}$ is

$\bar{x} = \frac{x_1 + x_2 + - - - x_n}{n}$ ― ② marks

$\Rightarrow E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(x_i)$ ― ①

Since x_i is a sample value from the population $X_1, X_2, - - -, X_n$ it can take any one of the values $X_1, X_2 - - - X_n$ each with equal prob. $\frac{1}{N}$

$\therefore E(x_i) = \frac{1}{N}(X_1 + X_2 + - - X_n)$ ― ② marks

$E(x_i) = \mu$ ― ②

Using ② in ①

$\therefore E(\bar{x}) = \frac{1}{\cancel{n}} \cdot \cancel{n}\mu = \mu$

― ① mark

a) ii) (Liapounoff's form)

If $X_1, X_2 --- X_n$ are independent random variates with $E(X_i) = \mu_i$ and $Var(X_i) = \sigma_i^2$, $i = 1, 2 --- n$ Then under certain general conditions $S_n = X_1 + X_2 + --- X_n$ is a Nml. variate with $\mu = \Sigma \mu_i$ and var $\sigma^2 = \Sigma \sigma_i^2$ as n tends to ∞.

— ① mark

Q5 :→

Ans b) step I) Null Hypo: H_0 : The die is unbiased

Alt. " H_a : " " not unbiased

step II) : Calculation of test statistic :

On the hypothesis that the die is unbiased we should expect the freq. of each no. to be $132/6 = 22$

prepare the table

$$\Sigma (O-E)^2 = 196$$

$$\chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{196}{22} = 8.91$$

— ③ marks

step III : Level of significance : $\alpha = 0.05$

$\nu = n-1 = 6-1 = 5$

— ① mark

step IV : For 5 d.f. at 5% LOS, the table value is 11.07

— ① mark

step V : Since the calculated value is less than table value the Null Hypo is accepted.

∴ The die is unbiased.

— ① mark

Ans c) prepare the table

$\bar{x} = a + \frac{\Sigma d_i}{n} = 48 + \frac{10}{9} = 49.11$

$\Sigma(x_i - \bar{x})^2 = \Sigma d_i^2 - \frac{(\Sigma d_i)^2}{n} = 54.89$

$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = 6.099$

(4) marks

Steps

i) : Null Hypo (H_0) : $\mu = 47.5$

Alt. (H_a) : $\mu \neq 47.5$

ii) $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{49.11 - 47.5}{\sqrt{6.099}/\sqrt{8}} = 1.84$

(2) marks

iii) LOS : $\alpha = 0.05$

iv) $\nu = n-1 = 8$ d.f. is 2.306

v) Decision : Since the calculated value of $|t| = 1.84$ is less than the table value $t_\alpha = 2.306$, the Null Hypo is accepted.

∴ The mean of Nine items does not differ significantly from assumed population mean 47.5.

(2) marks

or

Ans c i) Null Hypo (H_0) : $\mu = 42000$

Alt. " $\mu \neq 42000$

Test statistic : $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{42500 - 42000}{4000/\sqrt{81}}$

$\Rightarrow |z| = 1.125$

$\alpha = 0.05$

(2) marks

Critical value : 1.96

Decision : computed value is less than critical value ∴ The Null Hypo. is accepted. — 1 mark

Ans c ii) :→

Null Hypo. : $\mu_1 = \mu_2$

Alt. " $\mu_1 \neq \mu_2$

$$S_p = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} = 40.19$$

$$SE = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 20.8$$ — 3 marks

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE} = 5.288$$

$\alpha = 0.05$

Critical value is 2.16

Decision : Since the computed value is greater than the t_2 value ∴ The Hypo. is rejected. The diff. is significant. — 2 marks