

END Semester Examination

April / May - 2018

Max. Marks: 100

Class: SE (Comp and IT)

Course Code: BS41

Name of the Course: Applied Mathematics-II

Duration: 3 Hrs

Semester: IV

Branch: Comp and IT

Instructions:

- (1) All questions are compulsory
(2) Assume suitable data if necessary

| Q No. | | Max Marks | CO | | | | | | | | | | | | | | |
|-------|--|-----------|-----|----|----|----|----|----|-----|----|----|----|----|----|----|----|-----|
| Q.1 | a) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ | 06 | CO1 | | | | | | | | | | | | | | |
| | b) i) If λ be an Eigen value of a non-singular square matrix A, then show that $\frac{ A }{\lambda}$ is an Eigen value of the adj A. | 03 | CO1 | | | | | | | | | | | | | | |
| | ii) prove that the characteristic roots of a Hermitian matrix are all real. | 03 | CO1 | | | | | | | | | | | | | | |
| | c) Show that matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable. | 08 | CO1 | | | | | | | | | | | | | | |
| | Find the transforming matrix and the diagonal matrix. | | | | | | | | | | | | | | | | |
| | OR | | | | | | | | | | | | | | | | |
| | c) Find the singular value decomposition of $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$ | 08 | CO1 | | | | | | | | | | | | | | |
| Q.2 | a) Compute Spearman's rank correlation coefficient from the following data: <table><tr><td>X :</td><td>10</td><td>12</td><td>18</td><td>18</td><td>15</td><td>40</td></tr><tr><td>Y :</td><td>12</td><td>18</td><td>25</td><td>25</td><td>50</td><td>25</td></tr></table> | X : | 10 | 12 | 18 | 18 | 15 | 40 | Y : | 12 | 18 | 25 | 25 | 50 | 25 | 06 | CO2 |
| X : | 10 | 12 | 18 | 18 | 15 | 40 | | | | | | | | | | | |
| Y : | 12 | 18 | 25 | 25 | 50 | 25 | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | |
|----------|--|----------|------------|----|----|----------|-----|-----|-----|----------------|-------------------|----|----|----|----|----|----|----|-----|
| | <p>b) If θ be the acute angle between the 2 regression lines in the case of two variables x and y, show that</p> $\tan\theta = \frac{1-r^2}{r} \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}, \text{ where } r, \sigma_x, \sigma_y \text{ have their usual meanings. Explain the significance when } r = 0 \text{ and } r = \pm 1.$ | 06 | CO2 | | | | | | | | | | | | | | | | |
| | <p>c) i) The coefficient of rank correlation of the marks obtained by 10 students in Physics and Chemistry was found to be 0.5. It was later discovered that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 3 instead of 7. Find the correct coefficient of rank correlation.</p> <p>ii) Write any four properties of coefficients of Regressions with proof.</p> | 04 04 | CO2 CO2 | | | | | | | | | | | | | | | | |
| | OR | | | | | | | | | | | | | | | | | | |
| | <p>c) Find the equation of lines of Regression for the following data</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr> <tr> <td>y</td><td>11</td><td>14</td><td>14</td><td>15</td><td>12</td><td>17</td><td>16</td></tr> </table> | x | 5 | 6 | 7 | 8 | 9 | 10 | 11 | y | 11 | 14 | 14 | 15 | 12 | 17 | 16 | 08 | CO2 |
| x | 5 | 6 | 7 | 8 | 9 | 10 | 11 | | | | | | | | | | | | |
| y | 11 | 14 | 14 | 15 | 12 | 17 | 16 | | | | | | | | | | | | |
| Q.3 | <p>a) A random variable X has the following probability distribution given below:-</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>$X=x$</td><td>-2</td><td>3</td><td>1</td></tr> <tr> <td>$P(X=x)$</td><td>1/3</td><td>1/2</td><td>1/6</td></tr> </table> <p>Find</p> <ul style="list-style-type: none"> [i] The Moment Generating Function [ii] The First four moments about origin [iii] The First four moments about mean. <p>b) The daily consumption of electric power (in million kwh) is a R.V. X with probability density function given by</p> $f(x) = \begin{cases} kxe^{-\frac{x}{3}} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$ <p>Find the value of k, the expectation of x and the probability that on a given day, the electric consumption is more than expected value.</p> <p>c) The probability density of two Random variables X & Y is given by $f(x,y) = 1/8 (6 - x - y)$, $0 < x < 2, 2 < y < 4$ 0, otherwise</p> <p>Find i) $P(x < 1, y < 3)$ ii) $P(x < 1 / y < 3)$ iii) Marginal density function of x iv) Check for independence of x & y.</p> | $X=x$ | -2 | 3 | 1 | $P(X=x)$ | 1/3 | 1/2 | 1/6 | 06 06 08 | CO3 CO3 CO3 | | | | | | | | |
| $X=x$ | -2 | 3 | 1 | | | | | | | | | | | | | | | | |
| $P(X=x)$ | 1/3 | 1/2 | 1/6 | | | | | | | | | | | | | | | | |

| | | | | | | | | | | | | | | | | | | | |
|--|--|-----------------|-----|----|----|----|-----|-------|-------|-----------|----|----|----|----|----|----|-----|----|-----|
| | <p style="text-align: center;">OR</p> <p>c) A woman with n keys with her, wants to open the door of her house by trying keys independently and randomly one by one. Find the mean and the variance of the number of trials required to open the door, if unsuccessful keys are kept aside.</p> | 08 | CO3 | | | | | | | | | | | | | | | | |
| Q.4 | a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the number of days in a year on which (i) neither car is on demand (ii) a car demand is refused. | 06 | CO4 | | | | | | | | | | | | | | | | |
| | b) A transmission channel has a per digit error probability 0.01. Calculate the probability of more than one error in 10 received digits. Using Binomial Distribution. Also find Moment generating function. | 06 | CO4 | | | | | | | | | | | | | | | | |
| | c) In an intelligence test administered to 1000 students, the average was 42 and standard deviation was 24. Find the number of students i) exceeding the score 50 and ii) between 30 and 54. | 08 | CO4 | | | | | | | | | | | | | | | | |
| | <p style="text-align: center;">OR</p> <p>c) i) Derive the mean of Binomial Distribution.</p> | 04 | CO4 | | | | | | | | | | | | | | | | |
| | ii) Derive Recurrence Relation, find the Probability of $x = 1, 2, 3, 4$ from it if variance of the Poisson distribution is 2. | 04 | CO4 | | | | | | | | | | | | | | | | |
| Q.5 | a) i) Prove that the sample mean is an unbiased estimator of the population mean. ii) State Central Limit Theorem | 05 | CO5 | | | | | | | | | | | | | | | | |
| | b) A die was thrown 132 times and the following frequencies were observed. | 01 | CO5 | | | | | | | | | | | | | | | | |
| | <table border="1"><tr><td>Number obtained</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>Total</td></tr><tr><td>Frequency</td><td>15</td><td>20</td><td>25</td><td>15</td><td>29</td><td>28</td><td>132</td></tr></table> | Number obtained | 1 | 2 | 3 | 4 | 5 | 6 | Total | Frequency | 15 | 20 | 25 | 15 | 29 | 28 | 132 | 06 | CO5 |
| | Number obtained | 1 | 2 | 3 | 4 | 5 | 6 | Total | | | | | | | | | | | |
| Frequency | 15 | 20 | 25 | 15 | 29 | 28 | 132 | | | | | | | | | | | | |
| Test the hypothesis that the die is unbiased.(using Chi- square test) | | | | | | | | | | | | | | | | | | | |

| <p>c) Nine items of a sample had the following values:- 45, 47, 50, 52, 48, 47, 49, 53, 51 Does the mean of nine items differ significantly from the assumed population mean 47.5?</p> | 08 | CO5 | | | | | | | | | | | | |
|--|-------|-------|-------|----------------|---|---|----------------------------|------|------|-------------------------------|----|----|----|-----|
| <p style="text-align: center;">OR</p> <p>i) A tyre company claims that the lives of tyres have mean 42,000 kms with S.D. of 4000 kms. A change in the production process is believed to result in better product. A test sample of 81 new tyres has mean life of 42,500 kms. Test at 5% level of significance that new product is significantly better than the old one.</p> <p>ii) Samples of two types of electric bulbs were tested for length of life and the following data were obtained.</p> | 03 | CO5 | | | | | | | | | | | | |
| <table border="1" data-bbox="446 750 1128 963"> <thead> <tr> <th></th><th>Type1</th><th>Type2</th></tr> </thead> <tbody> <tr> <td>No. of Samples</td><td>8</td><td>7</td></tr> <tr> <td>Mean of Samples (in hours)</td><td>1134</td><td>1024</td></tr> <tr> <td>Standard deviation (in hours)</td><td>35</td><td>40</td></tr> </tbody> </table> <p>Test at 5% level of significance whether the difference in sample means is significant. (Table value of t for 13 d.f. is 2.16)</p> | | Type1 | Type2 | No. of Samples | 8 | 7 | Mean of Samples (in hours) | 1134 | 1024 | Standard deviation (in hours) | 35 | 40 | 05 | CO5 |
| | Type1 | Type2 | | | | | | | | | | | | |
| No. of Samples | 8 | 7 | | | | | | | | | | | | |
| Mean of Samples (in hours) | 1134 | 1024 | | | | | | | | | | | | |
| Standard deviation (in hours) | 35 | 40 | | | | | | | | | | | | |

.....All the Best.....