



Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

(Autonomous College Affiliated to University of Mumbai)

End Semester Examination-December 2022

Max. Marks: - 100

Class: S.E. (EXTC A & B Division)

Course Code: - MA201

Name of the Course: Linear Algebra

Duration: 3 Hours

Semester: III

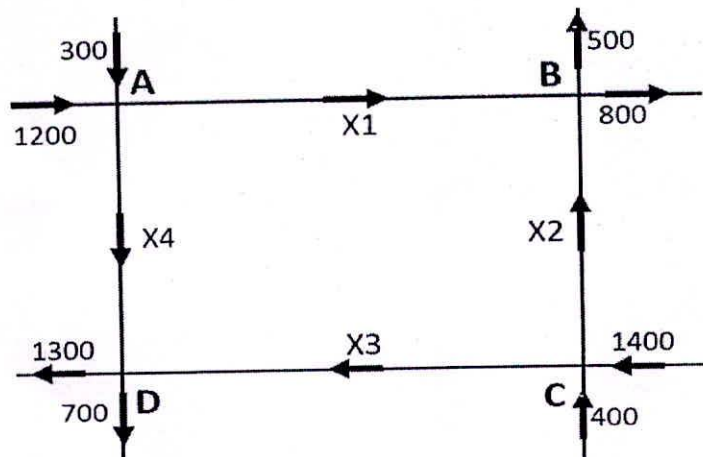
Branch: - EXTC

Instructions:

- 1) All Questions are Compulsory.
- 2) Assume suitable data if necessary.

Q No.		Max. Marks	CO
Q.1	a) Construct an orthonormal basis of \mathbb{R}^3 by applying Gram Schmidt process where $u_1 = (1, 0, 0)$, $u_2 = (3, 7, -2)$, $u_3 = (0, 4, 1)$.	8	4
	b) State only one axiom that fails to hold for each of the following sets W to be subspaces of the respective real vector spaces V with Standard operations. 1) $W = \{(x, y) \mid x^2 = y^2\}$, $V = \mathbb{R}^2$ 2) $W = \{(x, y) \mid xy \geq 0\}$, $V = \mathbb{R}^2$ 3) $W = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$, $V = \mathbb{R}^3$ 4) $W = \{f \mid f(x) \leq 0 \text{ for all } x\}$, $V = F(-\infty, \infty)$	7	4
	c) Show that $S = \{1 - t - t^2, -2 + 3t + 2t^3, 1 + t + 5t^3\}$ is linearly independent in P_3 . (P_3 means the set of all polynomials of degree 3 i.e. of the form $a_0 + a_1x + a_2x^2 + a_3x^3$)	6	4
	OR c) Determine the linear dependence or independence of vectors $(2, -1, 3, 2)$, $(1, 3, 4, 2)$ and $(3, -5, 2, 2)$. Find the relation between them if dependent.		
	d) Find the basis for the row and column space of $A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 3 & 1 & 5 \end{bmatrix}$	4	4
	e) Check whether the following set of vectors is a basis for P_2 . $S = \{1 - 3x + 2x^2, 1 + x + 4x^2, 1 - 8x + x^2\}$ Find the coordinate vector of $p = 1 - 2x + x^2$ with respect to the above basis.	5	4

Q.2	<p>a) Solve the following system of differential equation $y' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} y$</p> <p>b) Show that the matrix $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfies Cayley- Hamilton Theorem and hence find A^{-1} if it exists.</p> <p style="text-align: center;">OR</p> <p>b) If $A = \begin{bmatrix} \alpha & \alpha \\ \alpha & a \end{bmatrix}$ then prove that $e^A = e^\alpha \begin{bmatrix} \cosh \alpha & \sinh \alpha \\ \sinh \alpha & \cosh \alpha \end{bmatrix}$</p> <p>c) Find the Singular Value Decomposition of the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$</p> <p>d) Prove that the characteristic roots of a Hermitian matrix are all real.</p>	8	6														
Q.3	<p>a) Apply Gauss Jacobi Method to solve the following equations</p> $4x + y + 3z = 17$ $x + 5y + z = 14$ $2x - y + 8z = 12 \text{ upto 5 iterations.}$ <p style="text-align: center;">OR</p> <p>a) Apply Gauss Seidel Method to solve the following equations</p> $28x + 4y - z = 32$ $2x + 17y + 4z = 35$ $x + 3y + 10z = 24$ <p>Note :- upto 5 iterations</p> <p>b) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Find the diagonal matrix and the transforming matrix.</p> <p>c) Fit a curve of the form $y = a + b x + c x^2$ for the following data using Least square Method.</p> <table><tr><td>X</td><td>: 1.0</td><td>1.2</td><td>1.4</td><td>1.6</td><td>1.8</td><td>2</td></tr><tr><td>Y</td><td>: 2.345</td><td>2.419</td><td>2.592</td><td>2.863</td><td>3.233</td><td>3.702</td></tr></table>	X	: 1.0	1.2	1.4	1.6	1.8	2	Y	: 2.345	2.419	2.592	2.863	3.233	3.702	8	5
X	: 1.0	1.2	1.4	1.6	1.8	2											
Y	: 2.345	2.419	2.592	2.863	3.233	3.702											
		5	4														

Q.4	a) Using a suitable 2×2 matrix, Encode and decode the message NOW * STUDY	7	3
Q.5	<p>a) Reduce the following matrix to Row Echelon form and find its rank</p> $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 4 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & -8 & 0 \end{bmatrix}$ <p>b) If x_1, x_2, x_3, x_4 are the number of vehicles travelling through each road per hour. Find x_1, x_2, x_3, x_4 from the traffic diagram given below:</p>  <p>The traffic diagram shows a network of roads with inflows, outflows, and unknown flows x_1, x_2, x_3, x_4. The diagram consists of two horizontal roads and two vertical roads. The top horizontal road has an inflow of 1200 from the left and an outflow of 300 at junction A. The bottom horizontal road has an inflow of 1400 from the right and an outflow of 1300 to the left at junction D. The left vertical road has an inflow of 700 from the bottom and an outflow of x_4 at junction A. The right vertical road has an inflow of 400 from the bottom at junction C and an outflow of 500 from the top at junction B. The flow x_1 is the horizontal flow from junction A to junction B. The flow x_2 is the vertical flow from junction C to junction B. The flow x_3 is the horizontal flow from junction B to junction D.</p>	6	1
		8	1

***** All the Best *****

