Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

(Autonomous College Affiliated to University of Mumbai)

End Semester Examination-December 2022

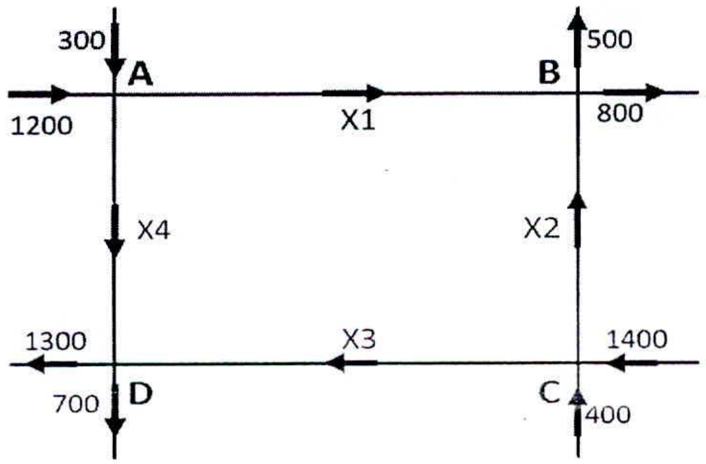
Max. Marks: - 100
Class: S.E. (EXTC A & B Division)
Course Code: - MA201
Name of the Course: Linear Algebra

Duration: 3 Hours
Semester: III
Branch :- EXTC

Instructions:
1) All Questions are Compulsory.
2) Assume suitable data if necessary.

Q No.		Max. Marks	CO
Q.1	a) Construct an orthonormal basis of R^3 by applying Gram Schmidt process where $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$, $u_3 = (0, 0, 1)$.	8	4
	b) Show that the set of real numbers (x, y) with cperation i) $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and $k(x,y) = (2kx, 2ky)$ ii) $(x_1, y_1\ z_1) + (x_2, y_2\ z_2) = (z_1 + z_2, y_1 + y_2, x_1 + x_2)$ and $k(x, y, z) = (kx, ky, kz)$ are not vector spaces in R^2 and R^3	7	4
	c) For what value of λ, the following vectors are linearly dependent? $(\lambda, -\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, \lambda, -\frac{1}{2})$, $(-\frac{1}{2}, -\frac{1}{2}, \lambda)$.	6	4
	d) Determine whether the following vectors span P_2. (P_2 means vector space consisting of all polynomials of second order.) $P_1 = 1 - x + 2x^2$, $P_2 = 5 - x + 4x^2$, $P_3 = -2 - 2x + 2x^2$ **OR** d) Verify whether (1, 2), (3, 6) are linearly independent when placed with initial points at the origin by analytical and geometrical method.	5	4
	e) Find the basis for the column space of $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ -2 & 1 & 7 & 2 \\ -1 & -4 & -1 & 3 \\ 3 & 2 & -7 & -1 \end{bmatrix}$	4	4

Q.2	a) Solve the following system of differential equation $y' = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$ y using diagonalisation with initial conditions $y_1(0) = 1$ and $y_2(0) = 6$.	8	6
	b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$. Hence find A^{50}	8	6
	OR		
	b) Find the Eigen values and Eigen vectors of A^3 where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ Is A derogatory matrix?		
	c) Find the Singular Value Decomposition of the matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$	8	6
	d) Prove that Eigen values of an orthogonal matrix are 1 or -1.	4	6
Q.3	a) Apply Gauss Jacobi Method to solve the following equations $5x + y + z = 10$ $2x + 4y = 12$ $x - y + 5z = -1$ upto 5 iterations.	8	2
	OR		
	a) Solve the following system of linear equations using Gauss Elimination Method: $x + 2y + 3z + 4w = 30$ $2x + 3y + 6z + 5w = 46$ $3x + 4y + 8z - 6w = 9$ $4x - y - z + 2w = 7$		
	b) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the diagonal matrix and the transforming matrix.	8	5
	c) Fit a curve of the form $y = a + bx + cx^2$ for the following data using Least square Method. X : -2 -1 0 1 2 Y : 15 1 1 3 19	5	4

Q.4	a) Using a suitable 2 x 2 matrix , Encode and decode the message VERY * GOOD	7	3
Q.5	a) Reduce the following matrix to Row Echelon form and find its rank $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	6	1
	b) If x_1, x_2, x_3, x_4 are the number of vehicles travelling through each road per hour. Find x_1, x_2, x_3, x_4 from the traffic diagram given below: 	8	1

*********** All the Best **********