SY - SE-EXTC - III - LA

SE EXTC SEM-III ESE Synoptic Linear Algebra (1)

Q1 :→

Ans a) Step I: $v_1 = u_1 = (1, 1, 1)$

Step II: $v_2 = u_2 - \text{proj}\, u_2 = u_2 - \frac{(u_2, v_1)}{\|v_1\|^2} \cdot v_1$

Now, $(u_2, v_1) = 2$, $\boxed{\|v_1\| = 3}$

$\therefore v_2 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(2m)

Step III $v_3 = u_3 - \text{proj}\, u_3$

$v_3 = u_3 - \frac{(u_3, v_1)}{\|v_1\|^2} \cdot v_1 - \frac{(u_3, v_2)}{\|v_2\|^2} \cdot v_2$

Now $(u_3, v_1) = 1$, $\boxed{(u_3, v_2) = \frac{1}{3}}$, $\boxed{\|v_2\|^2 = \frac{2}{3}}$

$\therefore v_3 = (0, 0, 1) - \frac{1}{3}(1, 1, 1) - \frac{1}{2}\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$\Rightarrow v_3 = \left(0, -\frac{1}{2}, \frac{1}{2}\right)$

(03 m)

Hence v_1, v_2, v_3 form the orthogonal basis for R^3

Now, The norms of these vectors are

$\|v_1\| = \sqrt{3}$, $\|v_2\| = \frac{\sqrt{6}}{3}$, $\|v_3\| = \frac{1}{\sqrt{2}}$

Hence orthonormal basis for R^3 is

$q_1 = \frac{v_1}{\|v_1\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$, $q_2 = \frac{v_2}{\|v_2\|} = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$$q_3 = \frac{v_3}{\|v_3\|} = \left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$ — 3m

Q1:→

4ns b) i) If $u=(2,3)$, $v=(6,9)$ Then

$u+v=(2,3)+(6,9)=(2+6, 3+9)=(8,12)$

and $5u=5(2,3)=(20,30)$

we shall consider the axiom M_3

$m(u)=m(x,y)=(2mx, 2my)$

$k(mu)=k(2mx, 2my)=(4kmx, 4kmy)$

But $(km)(u)=km(x,y)=(2kmx, 2kmy)$

Thus $k(mu)\neq km.u$

Further we see that $1(x,y)=(2x,2y)\neq (x,y)$

∴ There is no multiplicative identity

Hence this is not a vector space.

ii). For second space, consider the axiom A_1

Here we have if $u=(2,3,5)$, $v=(6,9,12)$

Then $u+v=(2,3,5)+(6,9,12)$

$= 5+12, 3+9, 2+6 = (17,12,8)$

Now

$u+v=(x_1,y_1,z_1)+(x_2,y_2,z_2)$

$=(z_1+z_2, y_1+y_2, x_1+x_2)$

$(u+v)+w = (z_1+z_2, y_1+y_2, x_1+x_2)+(x_3,y_3,z_3)$

$=(x_1+x_2+x_3, y_1+y_2+y_3, z_1+z_2+z_3)$ — 4m

$\therefore v+w = (x_2, y_2, z_2) + (x_3, y_3, z_3)$ (2)

$= (z_2+z_3, y_2+y_3, x_2+x_3)$

$u+(v+w) = (x_1, y_1, z_1) + (z_2+z_3, y_2+y_3, x_2+x_3)$

$= (z_1+x_2+x_3, y_1+y_2+y_3, x_1+z_2+z_3)$

Thus $u+(v+w) \neq (u+v)+w$

Hence this set is not a vector space.

for Ex: If $u(2,3,5)$, $v=(3,-1,2)$

& $w=(-2,1,0)$ Then

$u+v = (2,3,5) + (3,-1,2)$

$= (5+2, 3-1, 2+3) = (7,2,5)$

$(u+v)+w = (7,2,5) + (-2,1,0)$

$= (5,3,5)$

Consider $v+w = (3,-1,2)+(-2,1,0)$

$= (2,0,1)$

And $u+(v+w) = (2,3,5)+(2,0,1)$

$= (6,3,4)$

Thus $(u+v)+w \neq u+(v+w)$

not vector space — (3M)

Q1 :→

Ans c) $k_1v_1+k_2v_2+k_3v_3=0$

$\Rightarrow k_1(\lambda, -\frac{1}{2}, -\frac{1}{3}) + k_2(-1, \lambda, -\frac{1}{2}) + k_3(-\frac{1}{2}, -\frac{1}{2}, \lambda) =$

$\therefore \lambda k_1 - \frac{1}{2}k_2 - \frac{1}{2}k_3 = 0$, $-\frac{1}{2}k_1 + \lambda k_2 - \frac{1}{2}k_3 = 0$,
$-\frac{1}{2}k_1 - \frac{1}{2}k_2 + \lambda k_3 = 0$

$$\therefore \begin{bmatrix} \lambda & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \lambda & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \lambda \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_2 - R_1$
$R_3 - R_2$

$$\begin{bmatrix} \lambda & -1/2 & -1/2 \\ -\lambda-(\frac{1}{2}) & \lambda+(1/2) & 0 \\ 0 & -(\frac{1}{2})-\lambda & \lambda+(1/2) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{3M}$$

$\therefore \lambda k_1 - \frac{1}{2}k_2 - \frac{1}{2}k_3 = 0$ —— (1)

$-(\lambda+\frac{1}{2})k_1 + (\lambda+\frac{1}{2})k_2 = 0$ —— (2)

$\Rightarrow -(\lambda+\frac{1}{2})(k_1 - k_2) = 0$

And $-(\lambda+\frac{1}{2})k_2 + (\lambda+\frac{1}{2})k_3 = 0$

$\Rightarrow -(\lambda+\frac{1}{2})(k_2 - k_3) = 0$ —— (3)

If $\lambda \neq -\frac{1}{2}$ Then from (2) & (3) we get $k_1 = k_2$, $k_2 = k_3$ — 3M

Then from (1) $\lambda k_1 - \frac{1}{2}k_1 - \frac{1}{2}k_1 = 0$

$\Rightarrow (\lambda - 1)k_1 = 0$

If $\lambda \neq 1$, $k_1 = 0$ This means If $\lambda = \frac{1}{2}$ or 1

k_1, k_2, k_3 are not zero & vectors are dependent

Q1:→

Ans d) If P_1, P_2, P_3 span the entire vector space P_2 Then any vector in P_2 must be expressible in terms of P_1, P_2, P_3

If $P = b_1 + b_2 x + b_3 x^2$ is any vector in P_2 we must be able to find k_1, k_2, k_3 such that $P = k_1 P_1 + k_2 P_2 + k_3 P_3$

ie $(b_1, b_2, b_3) = k_1(1, -1, 2) + k_2(5, -1, 4) + k_3(-2, -2, 2)$

$= (k_1 + 5k_2 - 2k_3,\ -k_1 - k_2 - 2k_3,\ 2k_1 + 4k_2 + 2k_3)$

$\therefore b_1 = k_1 + 5k_2 - 2k_3$, $b_2 = -k_1 - k_2 - 2k_3$

$b_3 = 2k_1 + 4k_2 + 2k_3$ —— 3m

The system is consistent if $\begin{vmatrix} 1 & 5 & -2 \\ -1 & -1 & -2 \\ 2 & 4 & 2 \end{vmatrix} \neq 0$

But $\Delta = 1(-2+8) - 5(-2+4) - 2(-4+2)$

$\boxed{\Delta = 0}$

Hence the eqns are not consistent

This means P_1, P_2, P_3 do not span P_2.

2m

or

Ans d) Analytical: let $v_1 = (1, 2)$ $v_2 = (3, 6)$,

$\therefore k_1 v_1 + k_2 v_2 = 0 \Rightarrow k_1(1, 2) + k_2(3, 6) = 0$

$$\therefore \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$$

By $R_2 - 2R_1$ $\begin{pmatrix} 1 & 3 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\therefore k_1 + 3k_2 = 0 \Rightarrow k_1 = -3k_2$

vectors are dependent

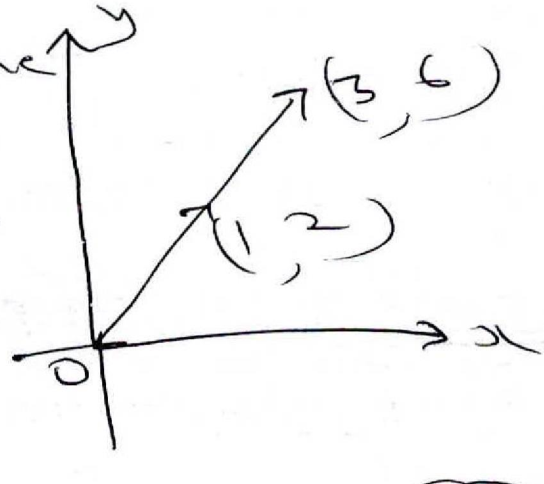
put $k_2 = t$ Then $k_1 = -3t$

from (1), we get $-3tv_1 + tv_2 = 0$

$\Rightarrow 3v_1 - v_2 = 0 \Rightarrow v_2 = 3v_1$ — (3m)

Vectors are not linearly independent

Geometrical: If we place the vectors with the initial points at the origin we see that the two vectors lie on the same line



$\therefore$ They are not independent. — (2m)

8) $\rightarrow$

Ans e) $R_2 + 2R_1$
$R_3 + R_1$
$R_4 - 3R_1$
$\begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 5 & 5 & 10 \\ 0 & -2 & -2 & 7 \\ 0 & -4 & -4 & -13 \end{pmatrix}$

$R_4 \to R_4 + R_2$
$R_2 \to R_4$
$\begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & -2 & -2 & 7 \\ 0 & 5 & 5 & 10 \end{pmatrix}$ — (2m)

R_3+2R_2
R_4-5R_2

$$\begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 25 \end{pmatrix}$$

4

$R_4 \to R_4 - 25R_3$

$$\begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

1st, 2nd, 4th columns are pivot columns
Thus " " " of the original matrix
form basis for the column space

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 3 \\ -1 \end{bmatrix} \right\}$$ 2m

Q2:→

Ans a) $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$\therefore \begin{vmatrix} 1-\lambda & 1 \\ 4 & -2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2+\lambda-6=0 \Rightarrow \lambda=2,-3$

For $\lambda_1=2$ $\therefore v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2=-3$ $\therefore v_2=\begin{bmatrix} 1 \\ -4 \end{bmatrix}$ 2m

$P=[v_1\ v_2]=\begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix}$ & $D=\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$

Since $A = PDP^{-1}$ we write $y' = PDP^{-1}y$

multiply by P^{-1} : $(P^{-1}y)' = D(P^{-1}y)$

let $z = P^{-1}y$

$\therefore z' = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} z$

$\Rightarrow \begin{bmatrix} z_1' \\ z_2' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$\Rightarrow z_1' = 2z_1$ and $z_2' = -3z_2$

on solving $z_1 = c_1 e^{2t}$, $z_2 = c_2 e^{-3t}$

Since $z = P^{-1}y$ premultiply by P

$\therefore Pz = y$

$\Rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = P \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} c_1 e^{2t} \\ c_2 e^{-3t} \end{bmatrix}$

$= \begin{bmatrix} c_1 e^{2t} + c_2 e^{-3t} \\ c_1 e^{2t} - 4c_2 e^{-3t} \end{bmatrix}$ — 4m

At $t = 0$, $y_1(0) = 1$, $y_2(0) = 6$

$\therefore 1 = c_1 + c_2$ $\Rightarrow \boxed{c_2 = -1}$

$6 = c_1 - 4c_2$ And $\boxed{c_1 = 2}$

$- \quad - \quad +$

$-5 = 5c_2$

$\therefore \vec{y} = \begin{bmatrix} 2e^{2t} - e^{-3t} \\ 2e^{2t} + 4e^{-3t} \end{bmatrix}$ — 2m

Ans b) $|A - \lambda I| = 0 \Rightarrow \lambda^3 - \lambda^2 - \lambda + 1 = 0$

By CH Thm, this eqn is satisfied by A.

$\therefore A^3 - A^2 - A + I = 0$ —— (1)

$\Rightarrow A^3 = A + A^2 - I$ —— (2)

we prove by mathematical induction

let the result be true for $n = k$

ie suppose $A^k = A^{k-2} + A^2 - I$ be true

multiply eqn by A $\therefore A^{k+1} = A^{k-1} + A^3 - A$

but by (1) $A^3 - A = A^2 - I$

$\therefore A^{k+1} = A^{k-1} + A^2 - I = A^{(k+1)-2} + A^2 - I$

Hence result is true for $n = k+1$

but by (2) " " for $n = 3$

By mathem. Induc. it is true for $n = 4, 5$

- - - - for all $n \geq 3$

Hence $A^n = A^{n-2} + A^2 - I$ —— (3)

To find A^{50} we put $n = 2, 4 - - 46, 48, 50$ in (3)

(1) $= A^2 = I + A^2 - I$

(2) $= A^4 = A^2 + A^2 - I$ 5m

(3) $= A^6 = A^4 + A^2 - I$

— — — ———

— — — — —

(23) $A^{46} = A^{44} + A^2 - I$

— — — —

(25) $A^{50} = A^{48} + A^2 - I$

Adding these results columnwise, Since

there are 25 eqnalities $A^{50} = 25A^2 - 24I$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \therefore A^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$$

3m

or

Ans b) $|A-\lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1, 2, 3$$

eigen values of A^3 are $1^3, 2^3, 3^3$

for $\lambda = 1$ $[A - \lambda I]X = 0$ gives

2m

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_3 - 2R_2$ $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$\Rightarrow x_3 = 0 \quad \therefore x_1 + x_2 + x_3 = 0 \Rightarrow x_1 + x_2 = 0$

let $x_2 = -1 \quad \therefore \boxed{x_1 = 1}$

$\therefore X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ 2m

Hence eigen values of A^3 are 1, 8, 27

for $\lambda = 2$ $\therefore X_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

4m

for $\lambda = 3$, $\therefore X_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

If $AX = \lambda X$ Then $A^n X = \lambda^n X$

All eigenvalues of A are all distinct

Non Derogatory

Q2:→

6

Ans c) $A'A = \begin{bmatrix} 4 & 6 \\ 6 & 13 \end{bmatrix}$ $\begin{vmatrix} 4-\lambda & 6 \\ 6 & 13-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = 16, 1$ $\lambda^2 - 17\lambda + 16 = 0$

$D = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$, for $\lambda = 16$ $\therefore v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\lambda = 1$ $v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

since $(v_1 \cdot v_2) = 0$, v_1, v_2 are orthogonal

$\|v_1\| = \sqrt{5}$, $\|v_2\| = \sqrt{5}$

$v_1 = \frac{v_1}{\|v_1\|} = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$, $v_2 = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$V = \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$ —— 6m

The columns of U are $u_1 = \frac{1}{\sigma_1} A v_1 = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$u_2 = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$ $\therefore U = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$

$\therefore A = UDV' = \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$

2m

Ans d) :→ $AA' = A'A = I$, If λ is an eigen value of matrix A & X is eigen vector Then $AX = \lambda X$ —①

$(AX)' = (\lambda X)' = X'A' = \lambda X'$ —② from ① & ②

$\Rightarrow X'X(\lambda^2 - 1) = 0 \Rightarrow \lambda = \pm 1$

4m

Ans a) $x = \frac{1}{5}(10 - y - z)$, $y = \frac{1}{4}(12 - 2x)$

$z = \frac{1}{5}(-1 - x - y)$

Step I: Let $x_0 = y_0 = z_0 = 0$ $\therefore x_1 = \frac{1}{5}(10) = 2$

$y_1 = 3$, $z_1 = -0.5$

Step II: $x_2 = 1.5$, $y_2 = 2$, $z_2 = -1.2$

Step III: $x_3 = 1.84$, $y_3 = 2.25$, $z_3 = -0.9$

Step IV: $\rightarrow$ $x_4 = 1.73$, $y_4 = 2.08$, $z_4 = -1.018$

Step V: $\rightarrow$ $x_5 = 1.7876$, $y_5 = 2.135$, $z_5 = -0.962$

Then ~~Exact~~ values of fifth iteration ——— 8m

OR

Ans a)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 6 & 5 \\ 3 & 4 & 8 & -6 \\ 4 & -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 30 \\ 46 \\ 9 \\ 7 \end{bmatrix}$$

By $R_2 - 2R_1$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 0 & -3 \\ 0 & -2 & -1 & -18 \\ 0 & -9 & -13 & -14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 30 \\ -14 \\ -81 \\ -113 \end{bmatrix}$$
——— 4m

By $R_4 - 4R_3$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & -1 & -12 \\ 0 & 0 & 0 & 169 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 30 \\ -14 \\ -53 \\ 702 \end{bmatrix}$$

$\therefore w = \frac{702}{169} = 4.1538$, $z = 3.1544$

$x = 0.846$ ——— 4m

Q3:→

Ans b) $\begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$

(7)

$\Rightarrow \lambda = 1, 2, 3$ Since all eigen values are distinct $\therefore$ matrix A is diagonalizable

let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that $[A - \lambda I] X = 0$ — 3m

put $\lambda = 1$ $\therefore X = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$ And put $\lambda = 3$

$\lambda = 2$ $\therefore X = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ $\therefore X = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ — 3m

Since $M^{-1}AM = D$, the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

will be diagonalised to the diagonal matrix

$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ by the transforming matrix

$M = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ — 2m

Ans c) prepare the Table

$\Sigma x = 0$, $\Sigma y = 39$, $\Sigma xy = 10$, $\Sigma x^2 = 10$, $\Sigma x^2 y = 140$

$\Sigma x^3 = 0$, $\Sigma x^4 = 34$ — 2m

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$\boxed{a = -\frac{37}{35}}$ $\boxed{b = 1}$ $\boxed{c = \frac{31}{7}}$

$\therefore y = a + bx + cx^2 = -\frac{37}{35} + x + \frac{31}{7}x^2$ — 3m

Q4:→

Ans a) V corresponds to 22, E to 5, R to 18 ---- D to 4

$\therefore$ 22 5 18 25 27 7 15 15 4

write all four steps — 7m

Q5:→

Ans a) By $R_2 + 2R_4$, $R_3 - R_4$
$$\begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

By $R_2 - 3R_4$, $R_3 + 2R_4$
$$\begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$
By R_{24}
$$\begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

no. of Non zero rows is 2 $\therefore$ rank = 2 — 6m

Ans b) :→ At X, input = output

At node A, $x_4 + x_1 = 300 + 1200 = 1500$

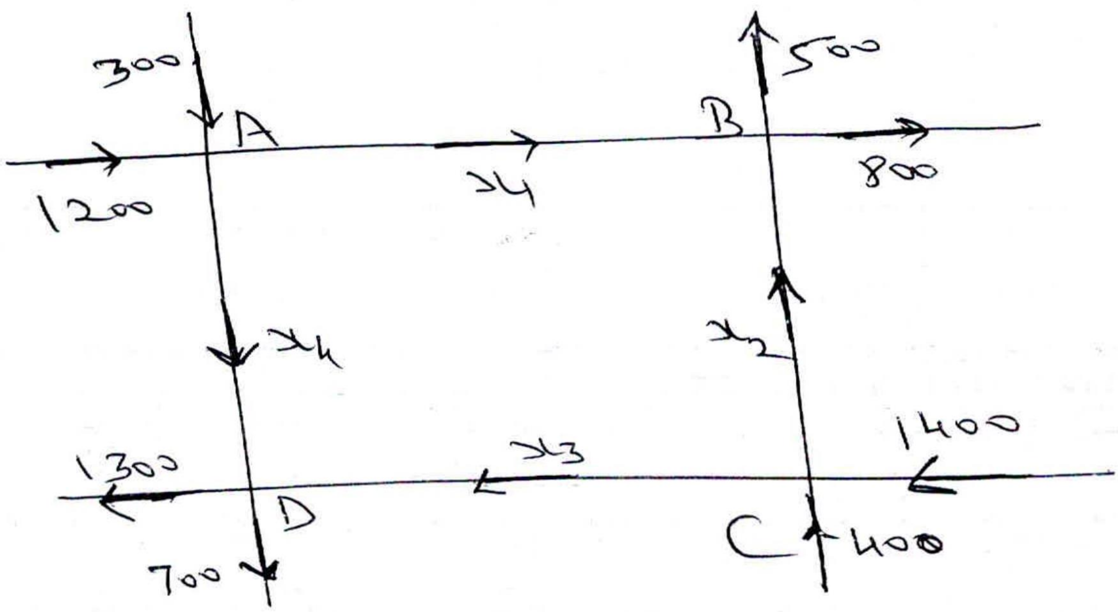
B $x_1 + x_2 = 800 + 500 = 1300$

C $x_2 + x_3 = 1400 + 400 = 1800$

D $x_3 + x_4 = 1300 + 700 = 2000$

Application of Solving system of Eqns in Traffic Control:

Ex:→ If x_1, x_2, x_3, x_4 are the number of vehicles travelling through each road per hour. Find x_1, x_2, x_3, x_4 from the traffic diagram given below:



Soln:→ At x^n, input = output

At Node A $\quad x_1 + x_4 = 300 + 1200 = 1500$

B, $\quad x_1 + x_2 = 800 + 500 = 1300$

C, $\quad x_2 + x_3 = 1400 + 400 = 1800$

D, $\quad x_3 + x_4 = 1300 + 700 = 2000$

$$\therefore \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1500 \\ 1 & 1 & 0 & 0 & 1300 \\ 0 & 1 & 1 & 0 & 1800 \\ 0 & 0 & 1 & 1 & 2000 \end{array}\right]$$

Reducing

By $R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_2$
$R_4 \to R_4 - R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1500 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 + x_4 = 1500$

$x_2 - x_4 = -200$

$x_3 + x_4 = 200$

Expressing each variable in terms of x_4

$\therefore x_1 = 1500 - x_4$ —— (1)

$x_2 = x_4 - 200$ —— (2)

$x_3 = 200 - x_4$ —— (3)

Now we can assume that $x_4 = 100$

$\therefore$ we get $x_4 = 100$

$x_3 = 100$

$x_2 = -100$

$x_1 = 1400$

Here value of x_2 as negative indicates that cars are going in opposite direction as that which is considered.