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Vector space - Let V be a non-empty set of elements called vectors and k, m are scalars. If the set V satisfies the following axioms then it is called a vector space.

1. Closure axiom -

a. C_1 - if u and v are any 2 elements in V then u+v is also in V. Therefore, V is closed under addition.

b. C_2 - if k is any scalar then k.u is in V. Therefore, V is closed under scalar multiplication.

2. Addition axiom -

a. A_1 - $u + v = v + u$ (Commutativity)

b. A_2 - $u + (v+w) = (u+v) + w$ (Associativity)

c. A_3 - There is an element called zero in V such that
$u + 0 = u$ for every u in V.
(Existence of additive identity)

d. A_4 - There is an element -u in V corresponding to every u in V such that
$u + (-u) = 0$
(Existence of additive inverse)

3. Scalar multiplication axiom

a. M_1 - $k(u+v) = ku + kv$ (Distributivity)

b. M_2 - $(k+l)u = ku + lu$

c. M_3 - $(kl)u = k(lu)$ (Associativity)

d. M_4 - There is an element 1 in V such that
$u \cdot 1 = u$ for all u in V
(Existence of multiplicative identity)

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eg. if V is a vector space then show that
(i) Additive identity 0 is unique
(ii) Additive inverse of a vector u is unique.

→ if possible let there be 2 distinct additive identities 0 and 0'
then $u + 0 = u$ — ①

$u + 0' = u$ — ②

Substituting $u = 0'$, in ①

$0' + 0 = 0'$ — ③

Substituting $u = 0$ in ②

$0 + 0' = 0$ — ④

from ③ and ④,

$0 = 0'$

This shows that the two identities are unique.

if possible let u_1 and u_2 be two additive inverses of u.
then by the definition of inverse

$u + u_1 = 0$ — ①

$u + u_2 = 0$ — ②

Adding u_2 in ①,

$u + u_2 + u_1 = u_2$

$u_1 = u_2$

This shows that the additive inverse of a vector u is unique

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4. Check whether the set of all pairs of real numbers of the form $(1, x)$ with the operations $(1, y) + (1, y') = (1, y + y')$ and $k(1, y) = (1, ky)$ is a vector space.

→ given operations:

$$(1, y) + (1, y') = (1, y + y')$$
$$k(1, y) = (1, ky)$$

A. closure axiom

c_1 - if $u = (1, x)$ and $v = (1, y)$

$u + v = (1, x + y)$

$\therefore$ V is closed under addition.

B. c_2 - if k is a any scalar and $u = (1, x)$

$ku = k(1, x)$

$= (1, kx)$

$\therefore$ V is closed under scalar multiplication.

B. addition axiom

a_1 - $u + v = (1, x) + (1, y)$

$= (1, x + y)$

$= (1, y + x)$

$= v + u$

b_2 - $(u + v) + w = [(1, x) + (1, y)] + (1, z)$

$= (1, x + y) + (1, z)$

$= (1, x + y + z)$

$= (1, x) + (1, y + z)$

$= u + [v + w)$

a_3 - $u + 0 = (1, x) + (1, 0)$

$= (1, x) = u$ (Existence of addition identity)

a_4 - $u + -u = (1, x) + (1, -x) = (1, 0) = 0$

c. Scalar multiplication

M_1 - $k(u+v)$

$= k[(1,x) + (1,y)]$

$= k(1, x+y)$

$= (1, kx+ky)$

$= (1,kx) + (1,ky)$

$= ku + kv$ ... (ii)

M_2 - $(k+l)u = (k+l)(1,x)$

$= (1, (k+l)x)$

$= (1, kx + lx)$

$= (1,kx) + (1,lx)$

$= ku + lu$

Hence, all axioms are satisfied.

$\therefore$ V is a vector space.

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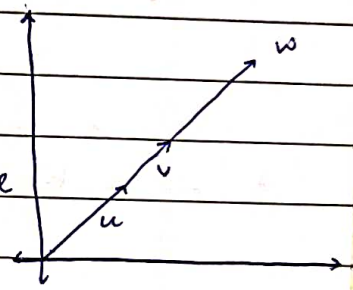
Subspace

A subset W of a vector space V is called a subspace of V if W is itself a vector space under addition and scalar multiplication defined on V.

Theorem - if W is a non-empty subset of vector space V then W is a subspace of V if

1. for all u and v in W, $u + v$ is also in W.
2. for any scalar k and any vector u in W, the ku is also in W.

eg. Show that a line through the origin in R^3 is a subspace of R^3.



→ Consider a line through the origin. if u and v are two vectors with origin as the initial point. Then $u + v$ also lies on this line.

Thus $u + v$ is closed under addition.

If k is any scalar, the $k \times v$ is also a vector on this line.

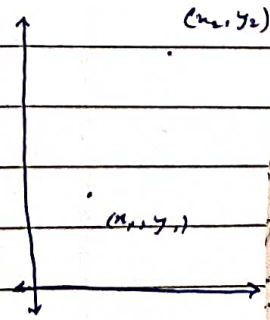
Thus W is closed under scalar multiplication.

This means that a line through the origin in R^3 is a subspace of R^3.

eg If W is a set of all points (x, y) in R^2 such that $x \geq 0$, $y \geq 0$. then W is not a subspace of R^2.

→ if (x_1, y_1), (x_2, y_2) are any two points on W. in the first quadrant then $(x_1, y_1) + (x_2, y_2)$ $= (x_1 + x_2, y_1 + y_2)$ is also a point on W in the first quadrant but

$$k(x, y) \notin (kx, ky)$$



for instance, $k = -2 \rightarrow k(x, y)$ is not in the first quadrant.

Hence, W is not closed under scalar multiplication.

eg. Let W be the set of all functions (i.e V) of the form $P(x) = a_0 + a_1 x \ldots a_n$ where n is a non-negative integer and $a_0, a_1 \ldots a_n$ are real numbers then W is a subspace of all real valued functions.

→ Similarly.

$$q(x) = b_0 + b_1 x + b_2 x + \ldots\ldots b_n x^n.$$

$$p(x) + q(x) = (a_0 + b_0) + (a_1 + b_1)x + \ldots\ldots (a_n + b_n)x^n$$

is also of the given form.

∴ $p(x) + q(x)$ is in W.

for any scalar k,

$$kp(x) = ka_0 + ka_1 x + \ldots\ldots ka_n x^n$$

is also of the given form.

∴ $kp(x)$ is in W

∴ W is a subspace of all real valued functions V.

eg. Show that any plane through the origin is a subspace of R^3.

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Span Spanned by a Vector

Let W be a ~~key~~ Subspace of V consisting of all linear combinations $v_1, v_2 \ldots v_n$ of S then W is called Span Spanned by S. We also say that the vectors $v_1, v_2 \ldots v_n$ Spans W.

To span means to furnish with something that extends / spreads over. "loosely speaking", to span means to generate or to create.

$$W = \text{Span}(S)$$
$$= \text{span}(v_1, v_2 \ldots v_n)$$
$$= L(S)$$

Q. Show that the vectors $v_1 = (1, 0, 1)$, $v_2 = (2, 1, 4)$ and $v_3 = (1, 1, 3)$ do not span the vector space in R^3.

→ if v_1, v_2, v_3 Span the vector space in R^3 then any vector in R^3 should be expressible in terms of v_1, v_2, v_3.

Let w_1, w_2 and w_3 be vectors in R^3, then we try to find Scalars k_1, k_2, k_3 such that

$$w = k_1 v_1 + k_2 v_2 + k_3 v_3$$
$$w = k_1(1, 0, 1) + k_2(2, 1, 4) + k_3(1, 1, 3)$$
$$= [(k_1 + 2k_2 + k_3), (k_2 + k_3), (k_1 + 4k_2 + 3k_3)]$$

$$\Rightarrow w_1 = k_1 + 2k_2 + k_3$$
$$w_2 = 0k_1 + k_2 + k_3$$
$$w_3 = k_1 + 4k_2 + 3k_3$$

NOTE - the system is consistent if

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 4 & 3 \end{vmatrix} \neq 0$$

$$= 0$$

Here, the given system is non-consistent.

This means there are no scalars k_1, k_2, k_3 satisfying the equation.

$\Rightarrow$ No vector can be expressed as a linear combination of v_1, v_2, v_3

Hence, v_1, v_2, v_3 do not span the vector space in R_3.

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eg. determine whether the following vectors span P_2 (P_2 vector space consisting of all polynomials of second order.

$$v_1 = 1 - x + 2x^2$$

$$v_2 = 5 - x + 4x^2$$

$$v_3 = -2 - 2x + 2x^2$$

Construct an orthonormal basis by applying Gram Schmidt to the set $S = \{(3,1), (4,2)\}$

Given $u_1 = (3,1)$ — (A) $\quad u_2 = (4,2)$ — (B)

Step 1. Let $v_1 = u_1 = (3,1)$ — (C)

Step 2. $v_2 = u_2 - \text{proj}_{u_2}$

$$= u_2 - \frac{(u_2 \cdot v_1)}{\|v_1\|^2} \cdot v_1$$

first find $(u_2 \cdot v_1)$ and $\|v_1\|^2$

$$(u_2 \cdot v_1) = (4,2) \cdot (3,1)$$

$$= 14$$

$$\|v_1\|^2 = (\sqrt{3^2 + 1^2})^2 = 10$$

$$v_2 = (4,2) - \frac{14}{10}(3,1)$$

$$= \left(\frac{-1}{5}, \frac{3}{5}\right)$$

Hence, $v_1 = (3,1)$

$v_2 = \left(\frac{-1}{5}, \frac{3}{5}\right)$ form an orthogonal basis for R^2.

Now the norms of this vectors are

$$\|v_2\| = \sqrt{\left(\frac{-1}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{2}{5}}$$

$$q_1 = \frac{v_1}{\|v_1\|} = \frac{(3,1)}{\sqrt{10}} = \left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$$

$$q_2 = \frac{v_2}{\|v_2\|} = \frac{(-1/5, 3/5)}{\sqrt{2/5}} = \left(\frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$$

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eg. Construct an orthonormal basis for the subspace of R^3 by applying Gram Schimat where $S = \{(1,2,0), (0,3,1)\}$

→ $u_1 = (1,2,0)$, $u_2 = (0,3,1)$

Step 1.

Let $v_1 = u_1 = (1,2,0)$

Step 2.

Let $v_2 = u_2 - \text{proj}\, u_2$

$= (0,3,1) - \frac{(u_2 \cdot v_1)}{\|v_1\|^2} \cdot v_1$

$u_2 \cdot v_1 = (0,3,1) \cdot (1,2,0) = 6$

$\|v_1\|^2 = 5$

$v_2 = (0,3,1) - \frac{6}{5} \cdot (1,2,0)$

$= \left(-\frac{6}{5}, \frac{3}{5}, 1\right)$

Norms -

$\|v_1\| = \sqrt{5}$

$\|v_2\| = \sqrt{\frac{14}{5}}$

$q_1 = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right)$

$q_2 = \left(\frac{-6}{\sqrt{70}}, \frac{3}{\sqrt{70}}, \sqrt{\frac{5}{14}}\right)$

Euclidean Inner product:

if $u = u_1, u_2 \ldots\ldots u_n$ and $v = v_1, v_2 \ldots\ldots v_n$ are any two vectors of R^n then the Euclidean Inner product of u, v is denoted by $u.v$

$$u.v = u_1v_1 + u_2v_2 \ldots\ldots u_nv_n$$

Let R^3 have the Euclidean Inner product. Use the Gram Schidt's process to transform the basis (u_1, u_2, u_3) into the orthonormal basis where

$$u_1 = (1, 0, 0)$$

$$u_2 = (3, 7, -2)$$

$$u_3 = (0, 4, 1)$$

Let $v_1 = u_1 = (1, 0, 0)$

$v_2 = u_2 - \text{proj}\, u_2$

$u_2 . v_1 = 3$

$||v_1|| = 1$

$v_2 = (3, 7, -2) - 3(1, 0, 0)$

$= (0, 7, -2)$

$$v_3 = u_3 - \frac{(u_3 . v_1)}{||v_1||^2} . v_1 - \frac{(u_3 . v_2)}{||v_2||^2} . v_2$$

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eg. If the vector space P_2 have the inner product defined by

$$(P, q) = \int_{-1}^{1} P(x)\, q(x)\, dx$$

Use Gram Schmidt's process to transfer the standard

→ Let $u_1 = 1$, $u_2 = x$, $u_3 = x^2$

Step 1 $v_1 = u_1 = 1$

Step 2 $v_2 = u_2 - \frac{(u_2 \cdot v_1)}{\|v_1\|^2} \cdot v_1$

$= u_2 - 0$

$= u_2$

$\|v_1\|^2 = \int_{-1}^{1} [v_1(x)]^2\, dx$

$= \int_{-1}^{1} 1^2\, dx = 2.$

Step 3 $v_3 = u_3 - \frac{(u_3 \cdot v_1)}{\|v_1\|^2} \cdot v_1 - \frac{(u_3 \cdot v_2)}{\|v_2\|^2} \cdot v_2$

$= u_3 - \frac{2/3}{2} \cdot 1 - 0$

$= x^2 - \frac{1}{3}$

$\|v_3\| =$

If A be a m×n matrix. The subspace of R^n spanned by A is called the rowspace of A. The subspace of R^m spanned by the column vector of A is called as a column space of A. Dimension of

$$\dim(\text{rowspace}(A)) = \dim(\text{columnspace}(A)) = \text{rank}(A)$$

Q. Find the basis for the rowspace of the following matrix

$$A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$$

→ Reduce the matrix to Row Echelon Form

$R_4 \rightarrow R_4 - R_1$

$R_5 \rightarrow R_5 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & -2 & -4 & 0 & 0 \\ 0 & -1 & -2 & -2 & -3 \end{bmatrix}$$

$R_4 \rightarrow R_4 + 2R_2$

$R_5 \rightarrow R_5 + R_2$

$$\begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$$

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Find the basis for the column space of the following matrix

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 3 & 1 & 5 \end{bmatrix}$$

→ Reduce to Row Echelon

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & 3 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - 3R_2$

$$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Check pivot columns

if any column has 1 in the diagonal positions.

(make steps and skip if any ~~no~~ column does not have 0)

$w_1 = [1, 0, \overset{1}{0}, \overset{2}{0}]$

$w_2 = [0, 1, 0^1, 0^3]$

It is very clear that first and second column are pivot columns. Thus, first and second columns of the original matrix form a basis for the column space.

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eg. find basis for the colum space for the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ -2 & 1 & 7 & 2 \\ -1 & -4 & -1 & 3 \\ 3 & 2 & -7 & -1 \end{bmatrix}$$

→ Reduce the matrix to Row Echelon form

We get

$$\begin{bmatrix} (1) & 2 & -1 & 4 \\ 0 & (1) & 1 & -3 \\ 0 & 0 & 0 & (1) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

column 1, 2 and 4 are the pivot columns.

Thus, 1st, 2nd and 3rd column of the original matrix form a basis for the column space of A.

$$w_1 = (1, -2, 1, 3]$$
$$w_2 = [2, 1, -4, 2]$$
$$w_3 = [4, 2, 3, -1]$$

eg. find a basis for row space of $A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 3 & 1 & 5 \end{bmatrix}$

LEAST

If A be a $m \times n$ matrix and b be a vector in R^n. To find the least square solution of $AX = b$

1. Compute the matrix A^TA and A^Tb
2. Form the augmented matrix $[A^TA \mid A^Tb]$
3. Reduce the matrix $[A^TA \mid A^Tb]$ to reduced row echelon form
4. This equation is always consistent and any solution $\hat{x}$ is a least square solution

eg. find the least square solution of $AX = B$ where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

→ Compute A^TA and A^TB

$$A^TA = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 1 & 6 \end{bmatrix}$$

$$A^TB = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

form the augmented matrix and row reduce

$$\left[\begin{array}{cc|c} 6 & 1 & 9 \\ 1 & 6 & 4 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cc|c} 1 & 6 & 4 \\ 6 & 1 & 4 \end{array}\right]$$

$R_2 \rightarrow R_2 - 6R_1$

$$\left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & -35 & -15 \end{array}\right]$$

$R_2 \rightarrow \frac{R_2}{-35}$

$$\left[\begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 1 & 3/7 \end{array}\right]$$

$R_1 \rightarrow R_1 - 6R_2$

$$\left[\begin{array}{cc|c} 1 & 0 & 10/7 \\ 0 & 1 & 3/7 \end{array}\right]$$

∴ The least Square Solution

$$\hat{x} = \begin{bmatrix} 10/7 \\ 3/7 \end{bmatrix}$$

that is a unique least Square Solution

eg. find the least square solution of $AX = b$ where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & -3 \end{bmatrix} \quad b = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

→ $A^TA = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 3 & -3 \\ 3 & 5 & -7 \\ -3 & -7 & 11 \end{bmatrix}$$

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$$A^T b = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}$$

We form an augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 3 & -3 & 6 \\ 3 & 5 & -7 & 0 \\ -3 & -7 & 11 & 6 \end{array}\right]$$

Converting to reduced row echelon form

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Solving the system $x_1 + x_3 = 5$

$x_2 - 2x_3 = -3$

Put random values and solve

$x_3 = 0$ $\quad$ $x_3 = 1$

$x_2 = -3$ $\quad$ $x_2 = -1$

$x_1 = 5$ $\quad$ $x_1 = 4$

$$\hat{x} = \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

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Q: find the least square solution of $AX = b$ for

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

for decoding - $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

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Application of Matrices to Coding and Decoding

Cryptography - A study of coding and decoding a message is called as cryptography

Plain text - message to be sent

Codes are ciphers

code messages are cipher codes

Encoding / Enciphering - process of converting plain text by coding

Decoding / Deciphering - process of getting plain text from cipher text

Method - consists of the following steps -

1. To convert the words / ~~sentences~~ letters by numbers
2. To encode the message
3. To decode the message
4. To replace numbers by words / letters

A → 1
B → 2
⋮
Z → 26
(*) Space → 27

NOTE - (i) The selected matrix is called as an encoding matrix and its inverse is called as a decoding matrix.

(ii) Encoding matrix is also known as key

eg Using a suitable 2×2 matrix encode and decode the message 'WE GO' (one word)

→ 1. Converting words into number then the transformation is

WE GO
23 5 7 15

Then we write this as a 2×1 matrix

$\begin{bmatrix} 23 \\ 15 \end{bmatrix}, \begin{bmatrix} 7 \\ 15 \end{bmatrix}$

2. We now premultiply each of the above column vectors by the encoding matrix.

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 23 & 7 \\ 5 & 15 \end{bmatrix} = \begin{bmatrix} 28 & 22 \\ 5 & 15 \end{bmatrix}$$

The columns of this matrix give the encoded message
The above message is then transmitted linearly

28 5 22 15

3. The received message is now written in a column matrix $\begin{bmatrix} 28 & 22 \\ 5 & 15 \end{bmatrix}$

This is then premultiplied by the inverse of the coding matrix $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 28 & 22 \\ 5 & 15 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 28 & -6 \\ 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 28 & 22 \\ 5 & 15 \end{bmatrix} = \begin{bmatrix} 23 & 7 \\ 5 & 15 \end{bmatrix}$$

4. The columns of the above matrix in linear form is

23 5 7 15

it is transformed

23 5 7 15

W E G O

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Use a suitable 2×2 matrix to encode and decode the message

NOW * STUDY

For 2×2 matrix multiplication, the last number has to have a pair ⇒ NOW * STUDY *

1. Replacing letters by numbers

N O W * S T U D Y *

14 15 23 27 19 20 21 4 25 27

We write the sequence in a 2×1 column matrix

$$\begin{bmatrix}14\\15\end{bmatrix}, \begin{bmatrix}23\\27\end{bmatrix}, \begin{bmatrix}19\\20\end{bmatrix}, \begin{bmatrix}21\\4\end{bmatrix}, \begin{bmatrix}25\\27\end{bmatrix}$$

2. We now premultiply each of the matrices by the encoding matrix

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}14 & 23 & 19 & 21 & 25\\15 & 27 & 20 & 4 & 27\end{bmatrix}$$

$$\begin{bmatrix}29 & 50 & 39 & 25 & 52\\15 & 27 & 20 & 4 & 27\end{bmatrix}$$

Columns give the encoded message. Message is transmitted as

29 15 50 27 39 20 25 4 52 27

3. The received message is then written in a sequence of 2×1 matrices

$$\begin{bmatrix}29\\15\end{bmatrix}\begin{bmatrix}50\\27\end{bmatrix}\begin{bmatrix}39\\20\end{bmatrix}\begin{bmatrix}25\\4\end{bmatrix}\begin{bmatrix}52\\27\end{bmatrix}$$

Premultiplying the above matrix by the inverse of the coding matrix

$$\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}\begin{bmatrix}29 & 50 & 39 & 25 & 52\\15 & 27 & 20 & 4 & 27\end{bmatrix}$$

$$\begin{bmatrix}14 & 23 & 19 & 21 & 25\\15 & 27 & 20 & 4 & 27\end{bmatrix}$$

Columns are written in linear form

14 15 23 27 19 20 21 4 25 27

N O W * S T U D Y *

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eg. To encode and decode the given message using suitable 2×2 matrix MOVE