



BHARATIYA VIDYA BHAVAN'S
SARDAR PATEL INSTITUTE OF TECHNOLOGY

MUNSHI NAGAR, ANDHERI (WEST), MUMBAI – 400 058, India
(Autonomous College Affiliated to University of Mumbai)

End Semester (Special) Examination Aug 2023

Max. Marks: 100

Class: FYMCA

Course Code: MA501

Subject: Linear Algebra

Duration: 3 hrs

Semester: I

Date: /8/2023

Time: –

Instructions: (1) All questions are compulsory.
(2) Use of scientific calculator is allowed.
(3) Assume any necessary data but justify the same.

| Q.N | | Marks | CO |
|-----|---|--------------------------|--------------------|
| 1. | (a) Reduce the following matrix to row echelon form and find its rank. $A = \begin{bmatrix} 0 & -8 & -6 \\ -2 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ (b) For what values of λ the system of equations $3x - y + 4z = 3, x + 2y - 3z = -2, 6x + 5y + \lambda z = -3$ has a unique solution? Find that unique solution. | [5] [15] | 1 1 |
| 2. | Solve the following system $3x_1 + x_2 + 2x_3 = 11$ $2x_1 + 3x_2 + x_3 = 11$ $x_1 + 2x_2 + 3x_3 = 14$ Use the initial guess $\bar{X}^{(0)} = 0$. (a) use Jacobi's method (3 iterations only). (b) Use Gauss Seidel method (3 iterations only). | [10] [10] | 2 2 |
| 3. | (a) Assign each letter of the alphabet (A-Z) to a number (1-26) and space to 0. Encode the message MY NAME IS VED using the encoding matrix $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$. Find the corresponding decoding matrix. Verify that this decoding matrix correctly decodes the message. $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ (b) Show that matrix A is diagonalizable. Find its diagonal form and transforming matrix. | [8] [12] | 3 5 |



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|------|--|------|---|--|--|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|------|---|
| 4. | (a) Show that following vectors are linearly dependent and find relation between them. $X_1 = (1 \ 2 \ 3)^T$, $X_2 = (3 \ -2 \ 1)^T$, $X_3 = (1 \ -6 \ -5)^T$ | [8] | 4 | | | | | | | | | | | | | | | | | | | | |
| | (b) Show that the function $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(a, b, c) = (a, b)$ is a linear transformation. Also find the rank and nullity of T . | [12] | 4 | | | | | | | | | | | | | | | | | | | | |
| 5. | Attempt any TWO. (a) Solve the following system of differential equations using diagonalization. $\frac{dy_1}{dt} = -3y_1 - 2y_2 + 2y_3$ $\frac{dy_2}{dt} = 2y_2$ $\frac{dy_3}{dt} = -4y_1 - y_2 + 3y_3$ | [10] | 6 | | | | | | | | | | | | | | | | | | | | |
| | (b) Given $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find the matrix A^{50} . | [10] | 6 | | | | | | | | | | | | | | | | | | | | |
| | (c) Given the following link Matrix of a digraph with vertices A, B and C. <table style="margin-left: auto; margin-right: auto;"> <tr><td style="text-align: right;">From</td><td></td><td></td><td></td></tr> <tr><td></td><td>A</td><td>B</td><td>C</td></tr> <tr><td style="text-align: right;">A</td><td>0</td><td>1</td><td>1</td></tr> <tr><td style="text-align: right;">B</td><td>1</td><td>0</td><td>0</td></tr> <tr><td style="text-align: right;">C</td><td>1</td><td>1</td><td>0</td></tr> </table> | From | | | | | A | B | C | A | 0 | 1 | 1 | B | 1 | 0 | 0 | C | 1 | 1 | 0 | [10] | 6 |
| From | | | | | | | | | | | | | | | | | | | | | | | |
| | A | B | C | | | | | | | | | | | | | | | | | | | | |
| A | 0 | 1 | 1 | | | | | | | | | | | | | | | | | | | | |
| B | 1 | 0 | 0 | | | | | | | | | | | | | | | | | | | | |
| C | 1 | 1 | 0 | | | | | | | | | | | | | | | | | | | | |
| | Which vertex has maximum page rank. | | | | | | | | | | | | | | | | | | | | | | |

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