

10/7/22

fe-exam



Sardar Patel Institute of Technology
 Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India
 (Autonomous College Affiliated to University of Mumbai)

End Semester Examination-JULY-2023

Max. Marks: - 100
 Class: S.Y.B.TECH
 Course Code: - MA201
 Name of the Course: Linear Algebra

Duration: 3 Hours

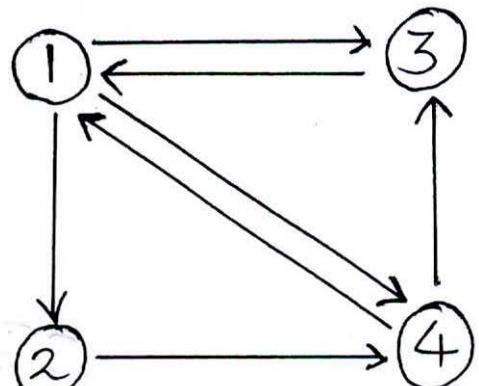
Semester: ~~II~~ III

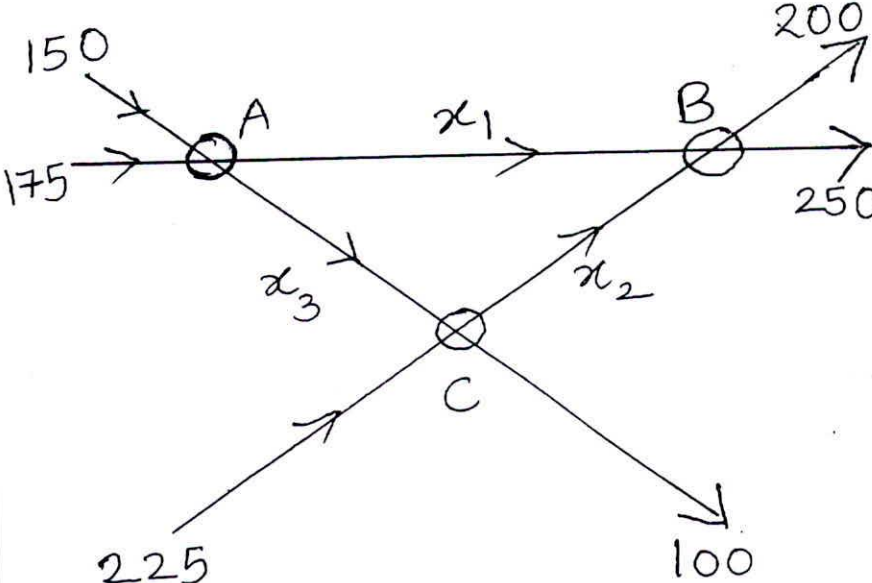
Branch: ~~ALL~~ ETC

Instructions:

- 1) All Questions are Compulsory.
- 2) Assume suitable data if necessary.

Q No.		Max. Mks	C O	BL
Q.1	a) Apply Gram-Schmidt orthogonalization process to transform the basis $S = \{(1, -1, 1), (1, 0, 1), (1, 1, 2)\}$ into an orthonormal basis.	8	4	3
	b) Check whether the set of all pairs of real numbers of the form $(1, x)$ with operations: $(1, y) + (1, y') = (1, y + y')$ and $k(1, y) = (1, ky)$ is a vector space.	8	4	2
	c) Verify whether the following vectors are linearly independent or linearly dependent. $v_1 = (4, 5, 1), v_2 = (0, -1, -1), v_3 = (3, 9, 4), v_4 = (-4, 4, 4)$	6	4	2
	OR	6	4	2
	c) If $v_1 = (4, 6, 8), v_2 = (2, 3, 4), v_3 = (-2, -3, -4)$ are three vectors with initial points at the origin, find whether they are in the same line.			
	d) Find the basis of row space and column space for the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & 10 & 25 \\ 20 & 10 & 0 \end{bmatrix}$	7	4	1
	e) Find the least square solution of $AX = B$ where $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$	5	4	2

Q2	<p>a) Find the highest Page Rank from the given directed graph.</p>  <p style="text-align: center;">OR</p> <p>a) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$</p>	8	6	3
	b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3 \tan A = A \tan 3$	5	6	2
	c) Solve the following system of differential equation $y' = \begin{bmatrix} 4 & -5 \\ -2 & 1 \end{bmatrix} y$ using diagonalization of matrices.	8	6	3
	d) Verify whether $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory or not.	5	6	1
Q.3	<p>a) Apply Gauss-Jacobi Method to solve the following equations</p> $27x + 6y - z = 85$ $6x + 15y + 2z = 72$ $x + y + 54z = 110$ <p style="text-align: right;">Note: Perform 5 Iterations only</p>	8	2	3
	<p>b) Show that the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ is diagonalisable. Find the Diagonal matrix and the transforming matrix.</p>	8	5	3
Q.4	<p>a) Given the Hill 2-cipher key $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$</p> <p>Compute $A^{-1} \text{ modulo } 27$ and hence decode the following message V,P,I,R,U,W,R,U,A,B</p>	8	3	3

Q.5	<p>a) Convert the matrix into reduced row echelon form and hence find its rank</p> $\begin{bmatrix} 2 & -2 & 4 & 2 \\ 2 & 1 & 10 & 7 \\ -4 & 4 & -8 & 4 \\ 4 & -1 & 14 & 6 \end{bmatrix}$ <p style="text-align: center;">OR</p> <p>a) Determine the solution of the following system of equations</p> $\begin{aligned} x + y - z + w &= 0 \\ x - y + 2z - w &= 0 \\ 3x + y + z &= 0 \end{aligned}$	8	1	3
	<p>b) Consider the network of three-one-way streets shown below. x_1, x_2, x_3 indicate the traffic flow (in vehicles per hour) along the stretches of road AB, AC and CB. The other number indicates the traffic flow rates into and out of the intersections A, B and C. Determine the amount of traffic between each of three intersections.</p> 	8	1	3

