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**Batch:** B2

LU Decomposition

**Code:**

```
printf("Adwait Purao\n")
printf("UID:2021300101 Batch:B2\n")

A = [1 5 1;2 1 3;3 1 4]
B = [14;13;17]

u11 = A(1,1)
u12 = A(1,2)
u13 = A(1,3)
l21 = A(2,1)/u11
l31 = A(3,1)/u11
u22 = A(2,2)-l21*u12
u23 = A(2,3)-l21*u13
l32 = (A(3,2)-l31*u12)/u22
u33 = A(3,3)-l31*u13-l32*u23

L = [1 0 0;l21 1 0;l31 l32 1]
U = [u11 u12 u13;0 u22 u23;0 0 u33]

printf("\nMatrix L")
disp(L)
printf("\nMatrix U")
disp(U)

y11 = B(1,1)
y21 = B(2,1) - l21*y11
y31 = B(3,1) - l31*y11 - l32*y21
Y = [y11;y21;y31]
printf("\nMatrix Y")
disp(Y)

x31 = Y(3,1)/U(3,3)
x21 = (Y(2,1)-U(2,3)*x31)/U(2,2)
x11 = (Y(1,1)-U(1,2)*x21 - U(1,3)*x31)/U(1,1)

X = [x11 x21 x31]
printf("\nMatrix X")
disp(X)
```

Screenshot of code:

```
printf("Adwait-Purao\n")
printf("UID:2021300101-Batch:B2\n")

A = [1 5 1; 2 1 3; 3 1 4]
B = [14; 13; 17]

u11 = A(1,1)
u12 = A(1,2)
u13 = A(1,3)
l21 = A(2,1)/u11
l31 = A(3,1)/u11
u22 = A(2,2) - l21*u12
u23 = A(2,3) - l21*u13
l32 = (A(3,2) - l31*u12)/u22
u33 = A(3,3) - l31*u13 - l32*u23

L = [1 0 0; l21 1 0; l31 l32 1]
U = [u11 u12 u13; 0 u22 u23; 0 0 u33]
printf("\nMatrix-L")
disp(L)
printf("\nMatrix-U")
disp(U)

y11 = B(1,1)
y21 = B(2,1) - l21*y11
y31 = B(3,1) - l31*y11 - l32*y21
Y = [y11; y21; y31]
printf("\nMatrix-Y")
disp(Y)

x31 = Y(3,1)/U(3,3)
x21 = (Y(2,1) - U(2,3)*x31)/U(2,2)
x11 = (Y(1,1) - U(1,2)*x21 - U(1,3)*x31)/U(1,1)

X = [x11 x21 x31]
printf("\nMatrix-X")
disp(X)
```

**Output:**

Adwait Purao  
UID:2021300101 Batch:B2

Matrix L

1.	0.	0.
2.	1.	0.
3.	1.5555556	1.

Matrix U

1.	5.	1.
0.	-9.	1.
0.	0.	-0.5555556

Matrix Y

14.
-15.
-1.6666667

Matrix X

1.	2.	3.
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**Problem solved on paper:**

Adwait Purao 101

$$x + 5y + z = 14$$

$$2x + y + 3z = 13$$

$$3x + y + 4z = 17$$

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ -l_{21}u_{11} & l_{21}u_{12} + u_{22} & \\ -l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$$u_{11} = 1 \quad u_{12} = 5 \quad u_{13} = 1$$

$$l_{21} = \frac{2}{u_{11}} = \frac{2}{1} = 2 \quad \boxed{l_{21} = 2}$$

$$1 = l_{21}u_{12} + u_{22}$$

$$u_{22} = 1 - 2 \cdot 5 = -9 \quad \boxed{u_{22} = -9}$$

$$3 = l_{21}u_{13} + u_{23}$$

$$u_{23} = 3 - 2 \cdot 1 = 1 \quad \boxed{u_{23} = 1}$$

Adwairt Pwarp 101

$$\bar{z} = d_{31} \cdot u_{11} \quad d_{31} = \frac{3}{u_{11}} = 3/1$$

$$d_{31} = 3$$

$$d_{31} \cdot u_{12} + d_{32} \cdot u_{22} = 3 \cdot 5 + d_{32}(-9) = 1$$

$$d_{32} = \frac{14}{9}$$

$$u_{33} = 4 - d_{31} \cdot u_{13} - d_{32} \cdot u_{23} = 4 - 3 \cdot 1 - \frac{14}{9} \cdot 1 = -\frac{5}{9}$$

$$u_{33} = -\frac{5}{9}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

$$AX = B \Rightarrow LUX = B$$

$$UX = Y$$

$$LY = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$$y_1 = 14, \quad y_2 + 2y_1 = 13, \quad y_3 = -15$$

$$y_3 = 17 - \frac{14}{9} \cdot (-15) - 3 \cdot 14 = 17 + \frac{70}{3} - 42$$

$$y_3 = -5/3$$

Adwairt Pwarp 101

$$UX = Y \Rightarrow \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -15 \\ -5/3 \end{bmatrix}$$

$$-\frac{5}{9}z = -\frac{5}{3} \Rightarrow z = 3$$

$$-9y + z = -15 \Rightarrow -9y = -18, \quad y = 2$$

$$x + 5y + z = 14, \quad x = 14 - 5 \times 2 - 3 = 7$$

$$x = 7, \quad y = 2, \quad z = 3$$