Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

(Autonomous College Affiliated to University of Mumbai)

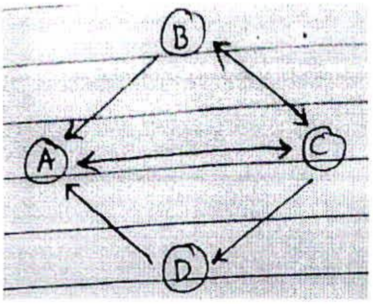
End Semester Examination December 2023

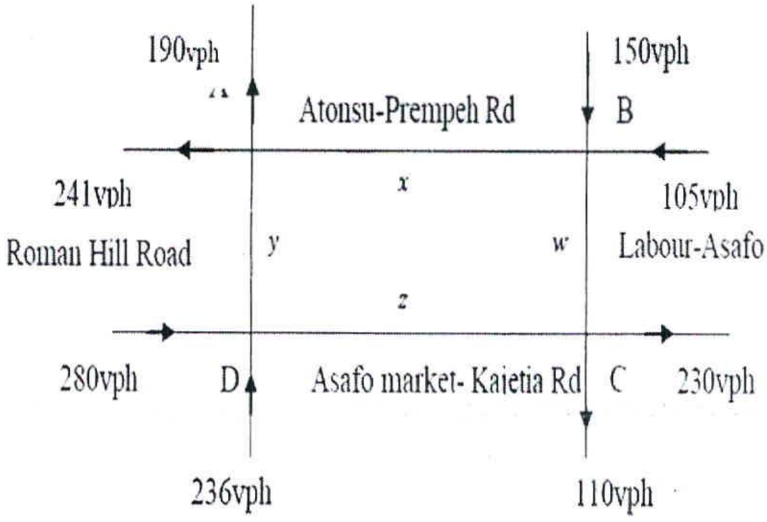
Max. Marks: - 100	**Duration: 3 Hours**
Class: S.E	**Semester: III**
Course Code: - MA201	**Branch: EXTC (A & B Division)**
Name of the Course: Linear Algebra	

Instructions:

1) **All Questions are Compulsory.**
2) **Assume suitable data if necessary.**

Q No.		Max. Mks	CO	BL
Q.1	a) Construct an orthonormal basis of R^3 by applying Gram Schmidt process where $u_1 = (1, 0, 0)$, $u_2 = (3, 7, -2)$, $u_3 = (0, 4, 1)$.	8	4	3
	b) State only one axiom that fails to hold for each of the following sets W to be subspaces of the respective real vector spaces V with Standard operations. 1) $W = \{(x, y) \mid x^2 = y^2\}$, $V = R^2$ 2) $W = \{(x, y) \mid x\, y \geq 0\}$, $V = R^2$ 3) $W = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$, $V = R^3$ 4) $W = \{f \mid f(x) \leq 0 \text{ for all } x\}$, $V = F(-\infty, \infty)$	8	4	3
	c) Show that the vectors (1, 2, 1) , (2, 1, 0) , (1, -1, 2) form a basis of R^3	7	4	2
	OR			
	c) Determine the linear dependence or independence of vectors (2,-1,3,2), (1, 3, 4, 2) and (3, -5, 2, 2). Find the relation between them if dependent.	7	4	2
	d) Find the basis for row space and column space of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ -2 & 1 & 7 & 2 \\ -1 & -4 & -1 & 3 \\ 3 & 2 & -7 & -1 \end{bmatrix}$	6	4	1
	e) Find the least square solution of AX= B where $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$	5	4	3

Q.2	a) Find the highest Page Rank from the given directed graph. Do till 3 iterations. 	8	6	3
	OR			
	a) Solve the following system of differential equation $y' = \begin{bmatrix} 4 & -5 \\ -2 & 1 \end{bmatrix}$ y Using diagonalisation with initial conditions $y_1(0) = -4$ and $y_2(0) = 3$.	8	6	3
	b) Define monic polynomial and show that A = $\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is a derogatory matrix.	7	6	3
	c) Find the singular value decomposition of the matrix A = $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$	8	6	2
	d) i) If λ *is an eigen value of a non singular matrix A then prove that* $\frac{\lvert A\rvert}{\lambda}$ is an eigen value of adj.A.	3	6	1
Q.3	a) Apply Crout's Method to solve the following equations 3x + 2y + 7z = 4 2x + 3y + z = 5 3x + 4y + z = 7	8	2	3
	b) Show that the matrix A = $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the diagonal matrix and the transforming matrix.	8	5	3
Q.4	a) Given the Hill 2-cipher key A = $\begin{bmatrix} 1 & 1 \\ 2 & 6 \end{bmatrix}$ Compute A^{-1} *modulo* 27 and hence decode the following message X , N , U , F , Y , V , C , R , S , Q , E , J	8	3	3

Q.5	a) Show that the system of equations $ax + by + cz = 0$ $bx + cy + az = 0$ $cx + ay + bz = 0$ has a non- trivial solution if $a + b + c = 0$ or if $a = b = c$ Find the non-trivial solution when the condition is satisfied.	8	1	3
	OR			
	a) Investigate for what values of λ and μ, the equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have i) no solution ii) a unique solution iii) infinite number of solutions.	8	1	3
	b) The diagram in the Figure below describes the four one-way streets in Kumasi (vph is number of vehicles per hour). Determine the amount of traffic between each of four intersections. 	8	1	3

*********** All the Best **********