Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

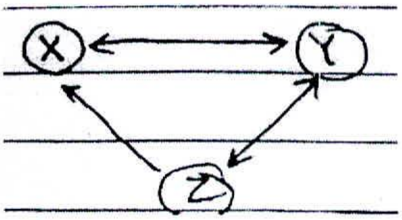
(Autonomous College Affiliated to University of Mumbai)

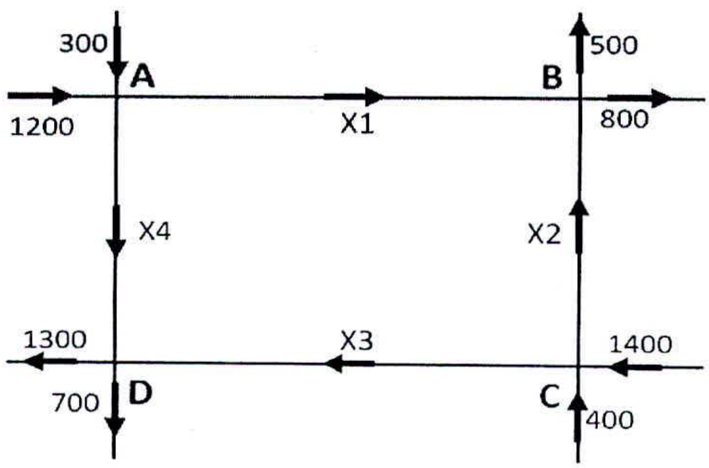
RE-EXAMINATION - July 2023

Max. Marks: - 100 | **Duration: 3 Hours**
Class: S.E | **Semester: IV**
Course Code: - MA201 | **Branch: - COMPS, AIML, DS**
Name of the Course: Linear Algebra

Instructions:
1) All Questions are Compulsory.
2) Assume suitable data if necessary.

Q No.		Max. Mks	CO	BL
Q.1	a) *If* $u = (1,2,2)$, $v = (3,4,6)$ Then prove that w = (5, 8, 10) is a linear combination of u and v but w = (6, 7, -4) is not a linear combination of u and v.	8	4	3
	b) Construct an orthonormal basis of R^3 by applying Gram Schmidt process where $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$, $u_3 = (1, 2, 1)$	8	4	3
	c) If W is the set of all points (x, y) in R^2 such that $x \geq 0$, $y \geq 0$ then show that W is not a subspace of R^2.	6	4	2
	OR			
	c) Show that the vectors $v_1 = (1, 0, 1)$, $v_2 = (2, 1, 4)$ and $v_3 = (1, 1, 3)$ do not span the vector space R^3.	6	4	2
	d) Find the null space of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$	6	4	2
	e) If V is a vector space, Then show that I) Additive identity 0 is unique. ii) Additive inverse of a vector u is unique.	6	4	1

Q.2	a) Find the highest Page Rank from the given directed graph. Do till 3 iterations.  	8	6	3
	OR			
	a) Solve the following system of differential equation $y' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} y$	8	6	3
	b) Verify Cayley- Hamilton's Theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Also find the inverse of A.	8	6	2
	c) Find the Eigen value and eigen vector of the matrix $A=\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$	8	6	2
	d) Prove that the matrices A and $P^{-1}AP$ have the same characteristic roots.	4	6	2
Q.3	a) Apply Gauss Jacobi Method to solve the following equations $15x + 2y + z = 18$ $2x + 20y - 3z = 19$ $3x - 6y + 25z = 22$ **Note:** - Take 6 iterations.	8	2	2
	b) Show that the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalisable. Find the diagonal matrix and the transforming matrix.	8	5	3
Q.4	Using a suitable 2 x 2 matrix , Encode and decode the message NOW * STUDY	8	3	3

Q.5	a) Reduce the following matrix to Row Echelon form and find its rank $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 4 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & -8 & 0 \end{bmatrix}$	6	1	2
	OR			
	b) Discuss the system of equations for all values of k, $2x + 3ky + (3k + 4)\,z = 0$ $x + (k + 4)\,y + (4k + 2)\,z = 0$ $x + 2(k + 1)\,y + (3k + 4)\,z = 0$	6	1	2
	b) *If* x_1, x_2, x_3, x_4 are the number of vehicles travelling through each road per hour. Find x_1, x_2, x_3, x_4 from the traffic diagram given below:  	8	1	3

*********** All the Best **********