BHARATIYA VIDYA BHAVAN'S

SARDAR PATEL INSTITUTE OF TECHNOLOGY

MUNSHI NAGAR, ANDHERI (WEST), MUMBAI – 400 058, India

(Autonomous College Affiliated to University of Mumbai)

End Semester Examination May 2022

Max. Marks: 60	Duration: 2 hrs
Class: FYMCA	Semester: I
Course Code: MA1M1	Date: 10/5/2022
Subject: Linear Algebra	Time: –

Instructions: (1) All questions are compulsory.
(2) Use of scientific calculator is allowed.
(3) Assume any necessary data but justify the same.

Q.N		Marks	CO
1.	(a) Reduce the following matrix to row echelon form and find its rank. $A=\begin{bmatrix} 0 & -8 & -6 \\ -2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$	[5]	1
	(b) For what values of λ the equations $3x-2y+\lambda z=1$ $2x+y+z=2$ $x+2y-\lambda z=-1$ will have no unique solution? Will the equations have any solution for this value of λ?	[10]	1
2.	Solve the following system using Jacobi's method and Gauss Seidel method(3 iterations only). $2x_1+3x_2+x_3=9$ $x_1+2x_2+3x_3=6$ $3x_1+x_2+2x_3=8$ Use the initial guess $\overline{X}^{(0)}=0$.	[15]	2
3.	Solve the following system of differential equations using diagonalization. $\frac{dy_1}{dt}=-3y_1-2y_2+2y_3$ $\frac{dy_2}{dt}=2y_2$ $\frac{dy_3}{dt}=-4y_1-y_2+3y_3$	[15]	6

4.	Attempt ant THREE.	[5]	1
	(a) If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & -1\\ 3 & 1 & 1\end{bmatrix}$, find two matrices P and Q such that PAQ is in normal form.		
	(b) Assign each letter of the alphabet(**A-Z**) to a number (**1–26**) and **space** to **0**.Encode the message THIS IS SPIT using the encoding matrix $\begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}$. Find the corresponding decoding matrix. Verify that this decoding matrix correctly decodes the message.	[5]	3
	(c) Apply the Gram Schmidt orthogonalization process to find the orthogonal basis and then the orthogonormal basis for the subspace U of R^4 spanned by $v_1=(1, 1, 1, 1)$, $v_2=(1, 1, 2, 4)$, $v_3=(1, 2, -4, -3)$	[5]	4
	(d) Show that following vectors are linearly dependent and find relation between them. $X_1=(1\ \ 2\ \ 3),\ X_2=(3\ \ -2\ \ 1),\ X_3=(1\ \ -6\ \ -5)$	[5]	4
	(e) Given $A=\begin{bmatrix}2 & 3\\ -3 & -4\end{bmatrix}$. Find the matrix A^{100}.	[5]	6

---x—x---