set of all $m \times n$ matrices also satisfy the above axioms. This space is denoted by M_{mn}.

EXERCISE - I

Examine in each case whether the set of real numbers with oporations of addition and scalar multiplication defined as follows is a vector space.

1. $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$; $k(x_1, y_1) = (3kx_1, 3ky_1)$ [**Ans.** : No. M_3 fails.]
2. $(x_1, y_1) + (x_2, y_2) = (x_1, y_1 + y_2)$; $k(x_1, y_1) = (kx_1, ky_1)$ [**Ans.** : No. M_2 fails]
3. $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$; $k(x_1, y_1) = (2kx_1, -ky_1)$

[**Ans.** : No. $m(x_1, y_1) = (2mx_1, -my_1)$]

and $k[m(x_1, y_1)] = k(2mx_1, -my_1) = (4km\, x_1, -kmy_1)$

But $(km)(x_1, y_1) = (2kmx_1, -kmy_1)$]

4. $(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0)$ and $k(x_1, 0) = (kx_1, 0)$. [**Ans.** : Yes]
5. $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$; $k(x_1, y_1) = (kx_1, y_1)$ [**Ans.** : No. M_2 fails.]
6. $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$; $k(x_1, y_1) = (k^2 x_1, k^2 y_1)$ [**Ans.** : No. M_2 fails.]
7. $(x_1, y_1) + (x_2, y_2) = (|x_1 + x_2|, |y_1 + y_2|)$; $k(x_1, y_1) = (kx_1, ky_1)$

[**Ans.** : No. A_2, A_3, A_4, M_1, M_2 fail.]

8. $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, 0)$; $k(x_1, y_1) = (kx_1, ky_1)$

[**Ans.** : No. There is no additive identity.]

9. $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$; $k(x_1, y_1) = (|k| x_1, |k| y_1)$

[**Ans.** : No. M_2 fails]

10. $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$k(x_1, y_1, z_1) = (kx_1, 0, 0)$ [**Ans.** : No. M_2 fails]

11. $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$k(x_1, y_1, z_1) = (2kx_1, 2ky_1, 2kz_1)$ [**Ans.** : No. M_4 fails]

12. $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2 - 1, y_1 + y_2 - 1, z_1 + z_2 - 1)$

$k(x_1, y_1, z_1) = (kx_1, ky_1, kz_1)$ [**Ans.** : No. M_1, M_2 fail]

13. $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$

$k(x_1, y_1, z_1) = (k^2 x_1, k^2 y_1, k^2 z_1)$ [**Ans.** : No. M_2 fails]

14. $(x_1, x_2, ..., x_n) + (y_1, y_2, ..., y_n) = (x_1 + y_1, x_2 + y_2, ..., x_n + y_n)$

$k(x_1, x_2, ..., x_n) = (3kx_1, 3kx_2, ..., 3kx_n)$ [**Ans.** : No, M_4 fails]

15. Examine whether the set of matrices of order 2 x 2 as defined below is a vector space.

(i) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with usual addition of matrices and scalar multiplication. [**Ans.** : Yes]

(ii) $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with usual addition of matrices and scalar multiplication. [**Ans.** : No. C_1 fails.]