Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

(Autonomous College Affiliated to University of Mumbai)

End Semester Examination

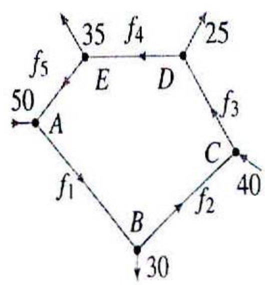
11 December 2023

Max. Marks: 100	**Duration: 3Hrs**
Class: FYMCA	**Semester: I**
Name of the Course: Linear Algebra	**Course Code: MA501**

Instruction:

1. **All questions are compulsory.**
2. **Assume suitable data if necessary.**
3. **Use of scientific calculator is allowed.**

Q No.		Max. Marks	CO	BL
Q.1	(a) Find the value of λ and μ such that the system $$\begin{aligned} x+2y+\lambda z&=1\\ x+2\lambda y+z&=\mu\\ \lambda x+2y+z&=1\end{aligned}$$ have (i) no solution (ii) only one solution, (iii) many solutions.	8	CO1	2
	(b) For what values of k, the following system has non-trivial solution: $$\begin{aligned} 2x+3ky+(3k+4)z&=0\\ x+(k+4)y+(4k+2)z&=0\\ x+2(k+1)y+(3k+4)z&=0.\end{aligned}$$	8	CO1	2
Q.2	(a) Solve the following system of equations using Gauss Jordan method: $$\begin{aligned} 2x+y+2z+w&=6\\ 6x-6y+6z+12&=36\\ 4x+3y+3z-3w&=-1\\ 2x+2y-z+w&=10.\end{aligned}$$	8	CO2	2
	(b) Solve the following system of equations using the Gauss-Seidel method up to four iterations: $$\begin{aligned} 10x+y+z&=12\\ x+10y+2&=12\\ x+y+10z&=12.\end{aligned}$$	8	CO2	2

	(c) Solve the following system of equations using LU Decomposition method: $$x + 2y + 4z = 3$$ $$3x + 8y + 14z = 13$$ $$2x + 6y + 13z = 4$$ **Or** (c) A traffic circle has five one-way streets, and vehicles enter and leave as shown in the following diagram.  Then (*i*) compute the possible flows, and (*ii*) conclude which road has the heaviest flow.	8	CO3	3
Q.3	Consider the key matrix $$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$ (i) Encode the message NOW_STUDY using the above key matrix modulo 27.	5	CO3	3
	(ii) Decode the message BOW_LTYDY using the above key matrix modulo 27.	5	CO3	3
Q.4	(a) Let A be an $m \times n$ matrix. For which column b in $\mathbf{R}^m$ is $U = \{x : x \in \mathbf{R}^m, Ax = b\}$ a subspace of $\mathbf{R}^n$. Justify your answer.	5	CO4	2
	(b) Which of the following subsets U of V are linearly independent? Justify. (i) $V = P_3$, $U = \{x^2 - x + 3, 2x^2 + x + 5, x^2 + 5x + 1\}$	5	CO4	2
	(ii) $V = M_{22}$, $U = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.	5	CO4	2

	(c) Find the least square approximating line $y = mx + c$ for the following sets of data point: $$(1,1),\ (3,2),\ (4,3),\ (6,4)$$ **Or** (c) Using Gram-Schmidt algorithm find orthogonal basis for $U = \{(1,1,1),(0,1,1)$. Find the vector in U closest to $x = (-1,2,1)$.	7	CO4	2
Q.5	(a) Find eigen values and eigen vectors of $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.	7	CO5	2
	(b) Using Cayley Hamilton theorem, find A^{-1} for $$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$.	7	CO5	3
	(c) Find e^A for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. **Or** (c) Solve the following system of differential equations: $$f_1' = -f_1 + 5f_2,$$ $$f_2' = f_1 + 3f_2$$ under the conditions $f_1(0) = 1$ and $f_2(0) = -1$.	7	CO6	3
Q.6	(a) Find the normal form of a matrix $$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$ Hence conclude its rank.	7	CO6	3