Linear Algebra & Differential Calculus.

Q.1

$$V = r^n(3\cos^2\theta - 1)$$

$$\frac{\partial V}{\partial r} = nr^{n-1}(3\cos^2\theta - 1)$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = \frac{\partial}{\partial r}\left(nr^{n+1}(3\cos^2\theta - 1)\right) = n(n+1)V \quad -(1)$$

└ (02M)

$$\frac{\partial V}{\partial \theta} = r^n(-6\cos\theta\sin\theta)$$

$$\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) = \frac{\partial}{\partial \theta}\left(-6r^n\sin^2\theta\cos\theta\right)$$

$$= -6r^n(2\sin\theta\cos^2\theta - \sin^3\theta)$$

$$\therefore \frac{1}{\sin\theta}\left(\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right)\right) = -6V \quad -(2)$$

└ (02M)

$$\therefore n(n+1)V - 6V = 0 \Rightarrow n = 2 \text{ or } n = -3.$$

└ (01M)

Q.2

Rank A = 3 → (05M)

[Till Row echelon → 03M]

Q2

OR

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 5 & 7 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ b \\ b^2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & a-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ b-1 \\ b^2-2b-3 \end{bmatrix}$$ (02M)

(i) If $a \neq 1$,

Then Rank A = 3 = Rank [A : B] = No of unknowns

$\Rightarrow$ unique solution — (01M)

(ii) If $a = 1$; $b \neq -1$, & $b \neq 3$

Then Rank A = 2 and Rank [A : B] = 3

$\Rightarrow$ No solution — (01M)

(iii) If $a = 1$; $b = -1$ or $b = 3$

Then Rank A = 2 and Rank [A : B] = 2

= No of unknowns

$\Rightarrow$ Infinite solution — (01M)

Q.3 A is skew symmetric $\Rightarrow A^t = -A$

To show $(I-A)(I+A)^{-1}$ is orthogonal

ie $\left[(I-A)(I+A)^{-1}\right]\left[(I-A)(I+A)^{-1}\right]^t = I$ } (01M)

Now $\left[(I-A)(I+A)^{-1}\right]^t = (I+A)^{-1})^t (I-A)^t$

$= \left((I+A)^t\right)^{-1}(I+A)$

$= \left(I+(A^t)\right)^{-1}(I+A)$ $\left(\because (A^{-1})^t = (A^t)^{-1}\right)$

$= (I-A)^{-1}(I+A)$ ——— (1)

(02M)

$(I-A)(I+A)^{-1}(I-A)^{-1}(I+A)$

$= (I-A)\left[(I-A)(I+A)\right]^{-1}(I+A)$

$= (I-A)\left[(I+A)(I-A)\right]^{-1}(I+A)$ $\left[\begin{array}{l} I-A^2 = \\ (I-A)(I+A) \\ = (I+A)(I-A) \end{array}\right]$

$= (I-A)(I-A)^{-1}(I+A)^{-1}(I+A) = I$ (02M)

Q.4 {WORKING TO BE SHOWN FOR Atleast 2 Iterations}

$x_1 = 0.85$

$y_1 = -1.0275$

$z_1 = 1.0109$ (01M)

$x_2 = 1.0025$

$y_2 = -0.9998$

$z_2 = 0.9998$ (02M)

$x_3 = 1.000$

$y_3 = -1.000$

$z_3 = 1.000$ (02M)