26/7/23

Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058, India

(Autonomous College Affiliated to University of Mumbai)

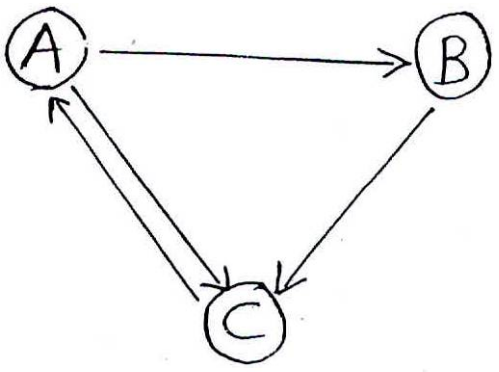
End Semester Re-Examination-JULY-2023

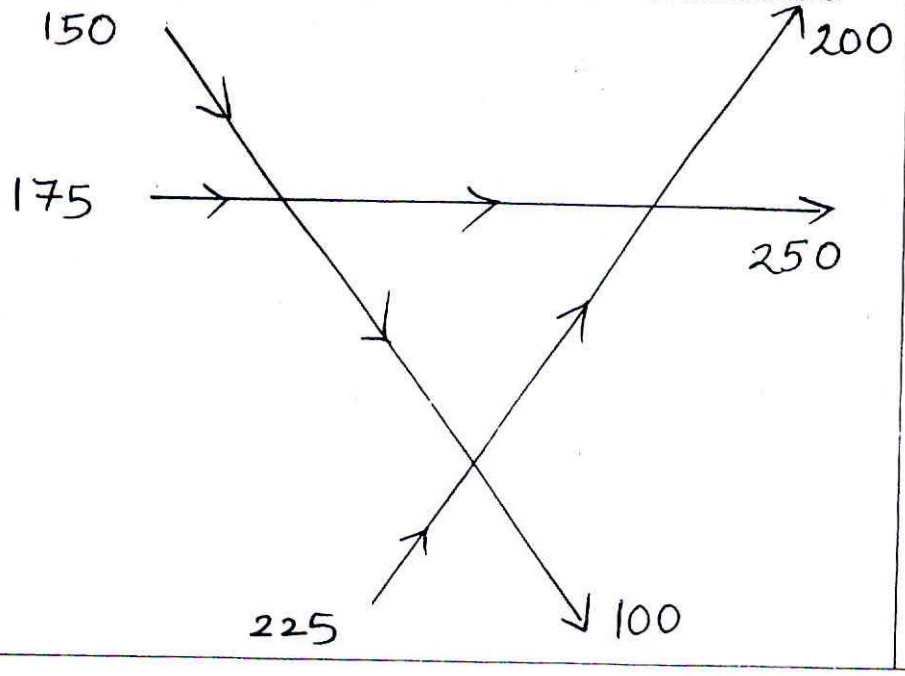
Max. Marks: - 100
Class: S.Y.B.TECH
Course Code: - MA201
Name of the Course: Linear Algebra

Duration: 3 Hours
Semester: IV
Branch: -ALL

Instructions:
1) All Questions are Compulsory.
2) Assume suitable data if necessary.

Q No.			Max. Mks	CO	BL
Q.1	a)	Apply Gram-Schmidt orthogonalization process to transform the Basis $S = \{(1,0,1), (1,0,-1), (0,3,4)\}$ into an orthonormal basis.	8	4	3
	b)	Check whether the vector $V = \{x, y, z \in R^3 : 2x + 3y^3 - 4z^3 = 0\}$ is vector space or not?	5	4	2
	c)	Verify whether the following vectors are linearly independent or linearly dependent. $v_1 = (1,2,3),\ v_2 = (3,-2,1),\ v_3 = (1,-6,5)$	8	4	2
		OR			
	c)	If $v_1 = (4,6,8),\ v_2 = (2,3,4),\ v_3 = (-2,-3,-4)$ are three vectors with initial points at the origin, find whether they are in the same line.	8	4	2
	d)	Find the basis of row space and column space for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$	8	4	1
	e)	Find the least square solution of $AX = B$ where $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$	5	4	2

Q2	a) Find the highest Page Rank from the given directed graph.  **OR** a) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$	8 8	6 6	3 3
	b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then Verify Cayley-Hamilton Theorem for A.	5	6	2
	c) Solve the following system of differential equation $y' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} y$ using diagonalization of matrices.	8	6	3
	d) Verify whether $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory or not.	5	6	1
Q.3	a) Apply Gauss-Jacobi Method to solve the following equations $6x + 2y - z = 4$ $x + 5y + z = 3$ $2x + y + 4z = 27$ Note: Perform 5 Iterations only	8	2	3
	b) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Find the Diagonal matrix and the transforming matrix.	8	5	3
Q.4	a) Given the Hill 2-cipher key $A = \begin{bmatrix} 1 & 1 \\ 2 & 6 \end{bmatrix}$ Compute A^{-1} *modulo* 27 and hence decode the following message X,N,U,F,Y,V,C,R,S,Q,E,J	8	3	3

Q.5	a) Convert the matrix into reduced row echelon form and hence find its rank $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$ **OR**	8	1	3
	a) Determine the solution of the following system of equations $5x + 2y - 3z = 0$ $3x + y + z = 0$ $2x + y + 6z = 0$	8	1	3
	b) Consider the network of three-one-way streets shown below. x_1, x_2, x_3 indicate the traffic flow (in vehicles per hour) along the stretches of road AB, AC and CB. The other number indicates the traffic flow rates into and out of the intersections A, B and C. Determine the amount of traffic between each of three intersections.  	8	1	3

☺ All the Best ☺