

5) Solve the following

$$(nz - ny)p + (nx - lz)q = ly - mx$$

Ans:

A.I.E

$$\frac{dx}{nz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

Choose lagrangian multipliers as

$$p' = l, \quad q' = m, \quad r' = n$$

$$p p' + q q' + r r' = 0$$

$$l x + m \underline{dy} + n dz = 0$$

ii) choose lagrangean multipliers as

$$P' = x, \quad Q' = y, \quad R' = z$$

$$6) (x^2 - y^2 - z^2) P + 2xy Q = 2xz \quad |H.W|$$

clue

Equate

$$\frac{x dx + y dy + z dz}{\quad} = \frac{dz}{2xz}$$

$$\Rightarrow \frac{dx}{2xy} = \frac{dz}{2xz}$$

$$7) (z^2 - 2yz - y^2)P + (xy + zx)Q = xy - zx \quad \text{H.W}$$

Clue

choose multiplier

i. x, y, z

$$ii) \text{Equate } \frac{dy}{xy + zx} = \frac{dz}{xy - zx}$$

$$\frac{dy}{x(y+z)} = \frac{dz}{x(y-z)}$$

$$\Rightarrow ydy - zdy = ydz + zdz$$

$$y dy - z dz - z dy - y dz = 0$$

$$y^2 - z^2 - \int d(yz) = 0$$

$$y^2 - z^2 - yz = b$$

8) Solve $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = xyz$

Ans:

Auxiliary equation

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{du}{xyz}$$

$\frac{dx}{x} = \frac{dy}{y}$ integrating $\log x - \log y = \log a$

$$\frac{x}{y} = a$$

$$\frac{dy}{y} = \frac{dz}{z}$$

integrating

$$\log y - \log z = \log b$$

$$y/z = b$$

$$\frac{dx+dy}{x+y} = \frac{dy+dz}{y+z}$$

Integ

$$\log x+y = \log y+z + \log C$$

$$\frac{x+y}{y+z} = C$$

$$\begin{aligned} u &= x \\ v &= y \\ w &= z \end{aligned}$$

$$\frac{yz dx + zxdy + xydz}{3xyz} = \frac{du}{xyz}$$

$$yz dx + zxdy + xydz = 3du$$

$$d(xyz) = 3du$$

Integrating

$$xyz = 3u + C$$

$$(xyz - 3u) = c$$

Solution

$$\phi(u, v, w) = 0$$

$$\phi\left(\frac{x}{y}, \frac{y}{z}, xyz - 3u\right) = 0$$

