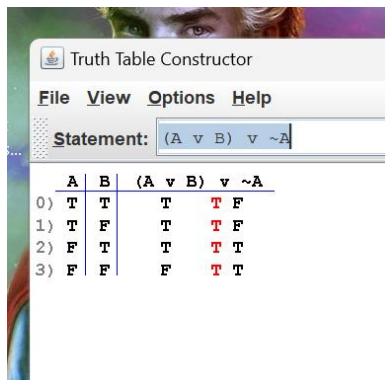


1)

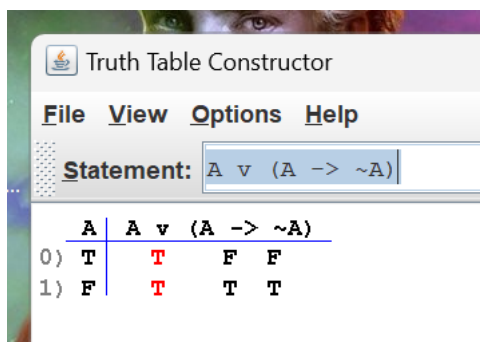


Truth Table Constructor

File View Options Help

Statement: $(A \vee B) \vee \sim A$

	A	B	$(A \vee B)$	\vee	$\sim A$
0)	T	T	T	T	F
1)	T	F	T	T	F
2)	F	T	T	T	T
3)	F	F	F	T	T

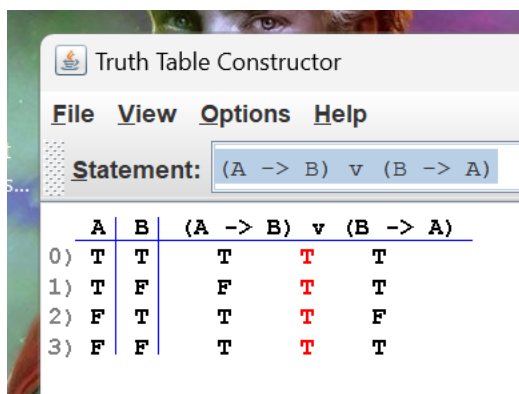


Truth Table Constructor

File View Options Help

Statement: $A \vee (A \rightarrow \sim A)$

	A	$A \vee (A \rightarrow \sim A)$
0)	T	T
1)	F	T



Truth Table Constructor

File View Options Help

Statement: $(A \rightarrow B) \vee (B \rightarrow A)$

	A	B	$(A \rightarrow B)$	\vee	$(B \rightarrow A)$
0)	T	T	T	T	T
1)	T	F	F	T	T
2)	F	T	T	T	F
3)	F	F	T	T	T

Ac
Re

Truth Table Constructor

File View Options Help

Statement: $(A \vee B) \vee (C \vee D)$

	A	B	C	D	$(A \vee B) \vee (C \vee D)$
0)	T	T	T	T	T
1)	T	T	T	F	T
2)	T	T	F	T	T
3)	T	T	F	F	T
4)	T	F	T	T	T
5)	T	F	T	F	T
6)	T	F	F	T	T
7)	T	F	F	F	T
8)	F	T	T	T	T
9)	F	T	T	F	T
10)	F	T	F	T	T
11)	F	T	F	F	T
12)	F	F	T	T	T
13)	F	F	T	F	T
14)	F	F	F	T	T
15)	F	F	F	F	F

Truth Table Constructor

File View Options Help

Statement: $(D \vee A) \vee (C \vee B)$

	A	B	C	D	$(D \vee A) \vee (C \vee B)$
0)	T	T	T	T	T
1)	T	T	T	F	T
2)	T	T	F	T	T
3)	T	T	F	F	T
4)	T	F	T	T	T
5)	T	F	T	F	T
6)	T	F	F	T	T
7)	T	F	F	F	T
8)	F	T	T	T	T
9)	F	T	T	F	T
10)	F	T	F	T	T
11)	F	T	F	F	T
12)	F	F	T	T	T
13)	F	F	T	F	T
14)	F	F	F	T	T
15)	F	F	F	F	F

Truth Table Constructor

File View Options Help

Statement: A

	A	A
0)	T	T
1)	F	F

Truth Table Constructor

File View Options Help

Statement: $A \vee \sim A$

	A	$A \vee \sim A$
0)	T	T
1)	F	T

Truth Table Constructor

File View Options Help

Statement: $(A \rightarrow B) \rightarrow A$

	A	B	$(A \rightarrow B)$	$(A \rightarrow B) \rightarrow A$
0)	T	T	T	T
1)	T	F	F	T
2)	F	T	T	F
3)	F	F	T	F

2)

$\{1,3,5,7,9,11,\dots\} \cup \{2,4,6,8,10,\dots\} = \mathbb{N}$ as it forms a set of all natural numbers.

$U = \{1,2,3, \dots, 6, 7, 8\}$, so $\{1,3,5,7,9\}^c = \{2,4,6,8\}$ as complement of a set U means elements that belongs to the set U but are not in the complement set $\{1,3,5,7,9\}$.

$A = \{1,3,5,7,9\}$, $B = \{2,4,6,8\}$ and $C = \{1,2,3,4\}$ so $(A \cup B) = \{1,2,3,4,5,6,7,8,9\}$ not $\{1,2,3,4\}$.

$\{a,b,c,d,e\} \cap \{a,e,i,o,u\} = \{a,e\}$ as the intersection checks for elements that are common in both sets which are a and e.

$(X \cap Y) \cup Z = (X \cup Y) \cap Z$ is not valid as intersection and union are not the same and they have very different set results.

$\{1,2,3,4,5\} \cap \{1,4,8,12\} = \{1,4\}$ as 1 and 4 are common in both sets. So the statements $\{1,2,3,4,5\} \cap \{1,4,8,12\}$ is not equal to $\{1,2,3,4,5,8,12\}$ as it does union instead.

3)

$\{1,2,3,1,2,3,1,2,3\} = \{1,2,3\}$ set does not count duplicates

$\{n \in \mathbb{N} : n^2 = 25\} = \{5\}$ as the only natural number when squared to give 25 is 5. The set elements have to be represented in a set not by itself.

$\{2n : n \in \{1, 2, 3, \dots\}\} = \{2, 4, 6, \dots\}$ as 2 times the set elements give $2*1, 2*2, 2*3 = \{2, 4, 6\}$. The set elements have to be represented in a set not by itself.

4)

$A \cap (B \cup C) = (A \cap B) \cup C$ due to associative law.

$(A \setminus B) \cup (B \setminus A) = A \oplus B$ is not valid as the symmetric difference of sets A and B, denoted by $A \oplus B$.

$(A \cap A^c) \cup (B \cap B^c) = \emptyset$ as $A \cap A^c$ will be an empty set same as the other side resulting in a union of empty sets which is an empty set.

$A \cap B = (A^c)^c \cap B$ is not valid as the complement of A Complement is A so $A \cap B$ is not $= A \cup B$.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ as it represent distribution law.

$A \cap (B \cup C) \cap D = (A \cap B) \cup (C \cap D)$ is not valid as it does not follow any law and D would always be separate.

$(A \setminus B) \cap (B \setminus A) = \{\}$ as for example if $A = \{1, 2\}$ and $B = \{2, 3\}$ the $(A \setminus B)$ is $\{1\}$ and $(B \setminus A) = \{3\}$ resulting in no common elements so it is an empty set.

$A \cup B \subseteq A \cap B$ is not valid as it the union cannot be a subset of an intersection.

5)

$|\{\{1, 2, 3\}, \{1, 3, 2\}, \{2, 1, 3\}, \{3, 2, 1\}\}| = 1$ as there is only one distinct set although in different order

$|\{1, 2, 3, 4, 4, 5, 5\}| = 5$ as there are 5 distinct elements, the rest are duplicates.

$|\{6, 6, 6, 6, 6\}| = 1$ as there is only 1 distinct element so cardinality is 1.

$|\{a, b, c\}| = 3$ as there are 3 distinct elements

$\{1, 2, 3\} = 3$ is not valid as this is just a normal set that does not ask for the cardinality.

6)

The relation N is symmetric, reflexive and transitive as it is reflexive as a person (a) has the same name as themselves. It is symmetric as if a has the same name as b then b obviously has the same name as a. It is transitive as if a has the same name as b and b has the same c then a obviously has the same name as c.

The relation S is transitive as if a is shorter than b and b is shorter than c then a is obviously shorter than c. It is not reflexive a cannot be shorter than a. It is not symmetric as a is shorter than b so b is definitely not shorter than a.

The relation M is symmetric as if a is married to b then b is married to a. M is not reflexive as a person cannot be married to themselves. M is not transitive as if a is married to b and b is married to c then it does not mean a is married to c.

7)

The relation S is transitive as if a is shorter than b and b is smaller than c then a is obviously shorter than c. It is not reflexive a cannot be smaller than a. It is not symmetric as a is smaller than b so b is definitely not smaller than a.

The relation E is symmetric, reflexive and transitive as if x has the same number of letters as y then y has the same number of letters as x, it is reflexive as x will have the same number of letters as x and it is transitive as if x has the same number of letters as y and y has the same number of letters as z then x has the same number of letters as z.

The relation D is symmetric as if $a + b$ is 10 then $b + a$ is also 10. D is not reflexive as $a + a$ cannot be 10 and it is not transitive as if $a + b$ is 10 and $b + c$ then it does not mean $a + c$ is 10.

10)

Line	Statement	Justification
1:	$A \vee B$	assumption
2:	A	assumption
3:	$\neg A$	assumption
4:	\perp	\neg elim 2,3
5:	$\neg\neg A$	\neg intro 3-4
6:	$\neg\neg A \vee \neg\neg B$	\vee intro 5
7:	B	assumption
8:	$\neg B$	assumption
9:	\perp	\neg elim 7,8
10:	$\neg\neg B$	\neg intro 8-9
11:	$\neg\neg A \vee \neg\neg B$	\vee intro 10
12:	$\neg\neg A \vee \neg\neg B$	\vee elim 1,2-6,7-11
13:	$(A \vee B) \rightarrow (\neg\neg A \vee \neg\neg B)$	\rightarrow intro 1-12

11)

Proof #1

File Edit Backward Forward Window Help

1: $\neg A \vee (B \wedge C)$, A premises
2: $\neg A$ assumption
3: \perp \neg elim 1.2,2
4: C contra (constructive) 3
5: $B \wedge C$ assumption
6: C \wedge elim 5
7: C \vee elim 1.1,2-4,5-6

12)

Proof #1

File Edit Backward Forward Window Help

1: A premise
2: B assumption
3: C assumption
4: D assumption
5: A hyp 1
6: $D \rightarrow A$ \rightarrow intro 4-5
7: $C \rightarrow (D \rightarrow A)$ \rightarrow intro 3-6
8: $B \rightarrow (C \rightarrow (D \rightarrow A))$ \rightarrow intro 2-7

13)

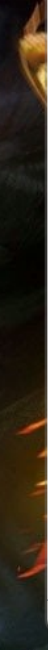
The screenshot shows a window titled "Proof #1" with a menu bar containing "File", "Edit", "Backward", "Forward", "Window", and "Help". The main area displays a formal proof with 12 lines. Lines 2 through 8 are enclosed in a box, and lines 3 through 4 are further enclosed in a smaller box. The proof steps are as follows:

Line	Formula	Justification
1:	$\neg((A \rightarrow B) \vee (B \rightarrow A))$	assumption
2:	A	assumption
3:	B	assumption
4:	A	hyp 2
5:	$B \rightarrow A$	\rightarrow intro 3-4
6:	$(A \rightarrow B) \vee (B \rightarrow A)$	\vee intro 5
7:	\perp	\neg elim 6,1
8:	B	contra (constructive) 7
9:	$A \rightarrow B$	\rightarrow intro 2-8
10:	$(A \rightarrow B) \vee (B \rightarrow A)$	\vee intro 9
11:	\perp	\neg elim 10,1
12:	$((A \rightarrow B) \vee (B \rightarrow A))$	contra (classical) 1-11

14)

1:	$(A \vee B) \vee (C \vee D)$	assumption
2:	$A \vee B$	assumption
3:	A	assumption
4:	$A \vee C$	\vee intro 3
5:	$(B \vee D) \vee (A \vee C)$	\vee intro 4
6:	B	assumption
7:	$B \vee D$	\vee intro 6
8:	$(B \vee D) \vee (A \vee C)$	\vee intro 7
9:	$(B \vee D) \vee (A \vee C)$	\vee elim 2,3-5,6-8
10:	$C \vee D$	assumption
11:	C	assumption
12:	$A \vee C$	\vee intro 11
13:	$(B \vee D) \vee (A \vee C)$	\vee intro 12
14:	D	assumption
15:	$B \vee D$	\vee intro 14
16:	$(B \vee D) \vee (A \vee C)$	\vee intro 15
17:	$(B \vee D) \vee (A \vee C)$	\vee elim 10,11-13,14-16
18:	$(B \vee D) \vee (A \vee C)$	\vee elim 1,2-9,10-17
19:	$((A \vee B) \vee (C \vee D)) \rightarrow ((B \vee D) \vee (A \vee C)) \rightarrow$ intro 1-18	

15)



1:	$\neg\neg(A \vee \neg A) \rightarrow (A \vee \neg A)$	premise
2:	$\neg(A \vee \neg A)$	assumption
3:	A	assumption
4:	$A \vee \neg A$	\vee intro 3
5:	\perp	\neg elim 4,2
6:	$\neg A$	\neg intro 3-5
7:	$A \vee \neg A$	\vee intro 6
8:	\perp	\neg elim 7,2
9:	$\neg\neg(A \vee \neg A)$	\neg intro 2-8
10:	$A \vee \neg A$	\rightarrow elim 1,9