

2)

1:  $A \wedge B$  premise  
2:  $B$   $\wedge$  elim 1  
3:  $A$   $\wedge$  elim 1  
4:  $B \wedge A$   $\wedge$  intro 2,3

3)

1:	$A \rightarrow (B \rightarrow C)$	assumption
2:	$A \rightarrow B$	assumption
3:	$A$	assumption
4:	$B \rightarrow C$	$\rightarrow$ elim 1,3
5:	$B$	$\rightarrow$ elim 2,3
6:	$C$	$\rightarrow$ elim 4,5
7:	$A \rightarrow C$	$\rightarrow$ intro 3-6
8:	$(A \rightarrow B) \rightarrow (A \rightarrow C)$	$\rightarrow$ intro 2-7
9:	$(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$	$\rightarrow$ intro 1-8

4)

1:  $\neg(A \vee B)$  premise  
 2:  $A$  assumption  
 3:  $A \vee B$   $\vee$  intro 2  
 4:  $\perp$   $\neg$  elim 3,1  
 5:  $\neg A$   $\neg$  intro 2-4  
 6:  $B$  assumption  
 7:  $A \vee B$   $\vee$  intro 6  
 8:  $\perp$   $\neg$  elim 7,1  
 9:  $\neg B$   $\neg$  intro 6-8  
 10:  $\neg A \wedge \neg B$   $\wedge$  intro 5,9

5)

1:  $\neg(A \rightarrow B)$  premise  
 2:  $\neg A$  assumption  
 3:  $A$  assumption  
 4:  $\neg B$  assumption  
 5:  $\perp$   $\neg$  elim 3,2  
 6:  $B$  contra (classical) 4-5  
 7:  $A \rightarrow B$   $\rightarrow$  intro 3-6  
 8:  $\perp$   $\neg$  elim 7,1  
 9:  $A$  contra (classical) 2-8

6)

1:	$(A \rightarrow B) \rightarrow A$	premise
2:	$\neg A$	assumption
3:	$A$	assumption
4:	$\neg B$	assumption
5:	$\perp$	$\neg$ elim 3,2
6:	$B$	contra (classical) 4-5
7:	$A \rightarrow B$	$\rightarrow$ intro 3-6
8:	$A$	$\rightarrow$ elim 1,7
9:	$\perp$	$\neg$ elim 8,2
10:	$A$	contra (classical) 2-9

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7)

1:	$A \wedge (B \vee C)$	assumption
2:	$B \vee C$	$\wedge$ elim 1
3:	$B$	assumption
4:	$A$	$\wedge$ elim 1
5:	$A \wedge B$	$\wedge$ intro 4,3
6:	$(A \wedge B) \vee (A \wedge C)$	$\vee$ intro 5
7:	$C$	assumption
8:	$A$	$\wedge$ elim 1
9:	$A \wedge C$	$\wedge$ intro 8,7
10:	$(A \wedge B) \vee (A \wedge C)$	$\vee$ intro 9
11:	$(A \wedge B) \vee (A \wedge C)$	$\vee$ elim 2,3-6,7-10
12:	$(A \wedge (B \vee C)) \rightarrow ((A \wedge B) \vee (A \wedge C))$	$\rightarrow$ intro 1-11

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8)

1:	$(A \rightarrow B) \wedge (B \rightarrow C)$	assumption
2:	$\neg((S \rightarrow T) \vee (T \rightarrow S))$	assumption
3:	$S$	assumption
4:	$\neg T$	assumption
5:	$T$	assumption
6:	$S$	hyp 3
7:	$T \rightarrow S$	$\rightarrow$ intro 5-6
8:	$(S \rightarrow T) \vee (T \rightarrow S)$	$\vee$ intro 7
9:	$\perp$	$\neg$ elim 8,2
10:	$T$	contra (classical) 4-9
11:	$S \rightarrow T$	$\rightarrow$ intro 3-10
12:	$(S \rightarrow T) \vee (T \rightarrow S)$	$\vee$ intro 11
13:	$\perp$	$\neg$ elim 12,2
14:	$(S \rightarrow T) \vee (T \rightarrow S)$	contra (classical) 2-13
15:	$((A \rightarrow B) \wedge (B \rightarrow C)) \rightarrow ((S \rightarrow T) \vee (T \rightarrow S))$	$\rightarrow$ intro 1-14

9)

1:	$\neg(A \vee \neg A)$	assumption
2:	$\neg A$	assumption
3:	$A \vee \neg A$	$\vee$ intro 2
4:	$\perp$	$\neg$ elim 3,1
5:	$A$	contra (classical) 2-4
6:	$A \vee \neg A$	$\vee$ intro 5
7:	$\perp$	$\neg$ elim 6,1
8:	$A \vee \neg A$	contra (classical) 1-7
9:	$\neg(B \vee \neg B)$	assumption
10:	$\neg B$	assumption
11:	$B \vee \neg B$	$\vee$ intro 10
12:	$\perp$	$\neg$ elim 11,9
13:	$B$	contra (classical) 10-12
14:	$B \vee \neg B$	$\vee$ intro 13
15:	$\perp$	$\neg$ elim 14,9
16:	$B \vee \neg B$	contra (classical) 9-15
17:	$(A \vee \neg A) \wedge (B \vee \neg B)$	$\wedge$ intro 8,16
18:	$((A \vee \neg A) \wedge (B \vee \neg B)) \vee ((R \wedge S) \rightarrow T)$	$\vee$ intro 17

10)

1:	$P \rightarrow Q$	assumption
2:	$\neg Q$	assumption
3:	$P$	assumption
4:	$Q$	$\rightarrow$ elim 1,3
5:	$\perp$	$\neg$ elim 4,2
6:	$\neg P$	$\neg$ intro 3-5
7:	$\neg Q \rightarrow \neg P$	$\rightarrow$ intro 2-6
8:	$(P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$	$\rightarrow$ intro 1-7
9:	$\neg Q \rightarrow \neg P$	assumption
10:	$P$	assumption
11:	$\neg Q$	assumption
12:	$\neg P$	$\rightarrow$ elim 9,11
13:	$\perp$	$\neg$ elim 10,12
14:	$Q$	contra (classical) 11-13
15:	$P \rightarrow Q$	$\rightarrow$ intro 10-14
16:	$(\neg Q \rightarrow \neg P) \rightarrow (P \rightarrow Q)$	$\rightarrow$ intro 9-15
17:	$((P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)) \wedge ((\neg Q \rightarrow \neg P) \rightarrow (P \rightarrow Q))$	$\wedge$ intro 8,16

11)

1:	$A \wedge B$	assumption
2:	$A$	$\wedge$ elim 1
3:	$A \rightarrow B$	assumption
4:	$A$	hyp 2
5:	$(A \rightarrow B) \rightarrow A$	$\rightarrow$ intro 3-4
6:	$(A \wedge B) \rightarrow ((A \rightarrow B) \rightarrow A)$	$\rightarrow$ intro 1-5

12)

1:	$\neg((A \rightarrow B) \vee (B \rightarrow A))$	assumption
2:	$A$	assumption
3:	$\neg B$	assumption
4:	$B$	assumption
5:	$A$	hyp 2
6:	$B \rightarrow A$	$\rightarrow$ intro 4-5
7:	$(A \rightarrow B) \vee (B \rightarrow A)$	$\vee$ intro 6
8:	$\perp$	$\neg$ elim 7,1
9:	$B$	contra (classical) 3-8
10:	$A \rightarrow B$	$\rightarrow$ intro 2-9
11:	$(A \rightarrow B) \vee (B \rightarrow A)$	$\vee$ intro 10
12:	$\perp$	$\neg$ elim 11,1
13:	$(A \rightarrow B) \vee (B \rightarrow A)$	contra (classical) 1-12

13)

1:	$(P \rightarrow Q) \wedge (Q \rightarrow R)$	premise
2:	$P$	assumption
3:	$P \rightarrow Q$	$\wedge$ elim 1
4:	$Q \rightarrow R$	$\wedge$ elim 1
5:	$Q$	$\rightarrow$ elim 3,2
6:	$R$	$\rightarrow$ elim 4,5
7:	$(P \rightarrow R)$	$\rightarrow$ intro 2-6

14)

1:	$(E \rightarrow (F \rightarrow G)), E \rightarrow F, E$	premises
2:	$F$	$\rightarrow$ elim 1.2,1.3
3:	$F \rightarrow G$	$\rightarrow$ elim 1.1,1.3
4:	$G$	$\rightarrow$ elim 3,2

15)

1:	$\neg(P \vee Q)$	assumption
2:	$\neg P$	assumption
	...	
3:	$P$	
4:	$P \vee Q$	$\vee$ intro 3
5:	$\perp$	$\neg$ elim 4,1
6:	$P$	contra (classical) 2-5
7:	$P \vee Q$	$\vee$ intro 6
8:	$\perp$	$\neg$ elim 7,1
9:	$P \vee Q$	contra (classical) 1-8
10:	$\neg(R \vee S)$	assumption
11:	$\neg S$	assumption
	...	
12:	$S$	
13:	$R \vee S$	$\vee$ intro 12
14:	$\perp$	$\neg$ elim 13,10
15:	$S$	contra (classical) 11-14
16:	$R \vee S$	$\vee$ intro 15
17:	$\perp$	$\neg$ elim 16,10
18:	$R \vee S$	contra (classical) 10-17
19:	$(P \vee Q) \wedge (R \vee S)$	$\wedge$ intro 9,18
20:	$((P \vee Q) \wedge (R \vee S)) \vee \neg(P \wedge R)$	$\vee$ intro 19