Detailed Answers

MATH1200 **Practice Exam 1** **Winter Term 2025**

NAIT

1. Find the derivatives of the following functions.

a. $y = \dfrac{-4x^2+3}{\sin(3x)}$ $y' =$ ______

Use Q.R $\Rightarrow$ $y'(x) = \dfrac{-8x\cdot\sin(3x) - (-4x^2+3)\cdot 3\cdot\cos(3x)}{\sin^2(3x)} \Rightarrow \boxed{y'(x) = \dfrac{-8x\cdot\sin(3x) - 3(-4x^2+3)\cdot\cos(3x)}{\sin^2(3x)}}$

b. $y = \ln\left(\sqrt[4]{3x-2}\right)$ $y' =$ ______

Use logarithmic properties $\Rightarrow$ $y = \frac{1}{4}\cdot\ln(3x-2)$

chain Rule $\Rightarrow$ $y'(x) = \frac{1}{4}\cdot\frac{3}{3x-2} \Rightarrow \boxed{y'(x) = \dfrac{3}{4\cdot(3x-2)}}$

c. $y = 2\cos^3\left(4\pi t^2\right)$ $y' =$ ______

$y = 2\cdot\left[\cos(4\pi t^2)\right]^3$

chain Rule $\Rightarrow$ $y'(t) = 2\cdot(3)\cdot\cos^2(4\pi t^2)\cdot\left(-\sin(4\pi t^2)\cdot 8\pi t\right) \Rightarrow \boxed{y'(t) = -48\pi t\cdot\cos^2(4\pi t^2)\cdot\sin(4\pi t^2)}$

d. $f(x) = \sqrt{x}\cdot e^{-\tan(x^2)} = x^{\frac{1}{2}}\cdot e^{-\tan(x^2)}$

P.R $\Rightarrow$ $f'(x) = \frac{1}{2}\cdot x^{-\frac{1}{2}}\cdot e^{-\tan(x^2)} + x^{\frac{1}{2}}\cdot e^{-\tan(x^2)}\cdot\left(-\sec^2(x^2)\cdot 2x\right)$

$\Rightarrow \boxed{f'(x) = e^{-\tan(x^2)}\cdot\left(\frac{1}{2\sqrt{x}} - 2x^{3/2}\cdot\sec^2(x^2)\right)}$ $y' =$ ______

2. The velocity of a flying object "v" in m/s, is given by $v = t\ln(t)$, (where t is the time in seconds). For what value of "t" is the acceleration of the object equal to **0.25 m/s²**? **Round your answer to 3 significant digits.**

$a(t) = v'(t) = \frac{d}{dt}\left[t\cdot\ln(t)\right] \overset{P.R}{=} (1)\cdot\ln(t) + \cancel{t}\cdot\frac{1}{\cancel{t}}$

$\Rightarrow a(t) = \ln(t) + 1$

Find $t = ?$ s.t $a(t) = 0.25 \Rightarrow \ln(t) + 1 = 0.25 \Rightarrow \ln(t) = -0.75$

$\Rightarrow t = e^{-0.75} = \boxed{0.472 \text{ sec}}$

it takes the flying object 0.472 sec to reach an accel. of 0.25 m/s²

3. Given $f(x,y) = 4y^2\tan(2x) - xe^{-4y} + \ln(4x)$, find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

Treat y-constant

$\frac{\partial f}{\partial x} = 4y^2 \cdot \sec^2(2x)\cdot(2) - (1)\cdot e^{-4y} + \frac{4}{4x} \Rightarrow \boxed{\frac{\partial f}{\partial x} = 8y^2\sec^2(2x) - e^{-4y} + \frac{1}{x}}$

$\frac{\partial f}{\partial y} = 8y\cdot\tan(2x) - x\cdot e^{-4y}(-4) \Rightarrow \boxed{\frac{\partial f}{\partial y} = 8y\cdot\tan(2x) + 4xe^{-4y}}$

Treat (keep) x-constant

4. Find the following integrals. Show your work! **Simplify** and **express your answers with positive exponents only**.

(a) $\int\left(\frac{4}{\sqrt[3]{x}} - \frac{5}{3x^{10}} + 3x^4 + \pi\right)dx = \int\left(4\cdot x^{-1/3} - \frac{5}{3}\cdot x^{-10} + 3x^4 + \pi\right)\cdot dx$

$= 4\cdot\frac{x^{2/3}}{2/3} - \frac{5}{3}\cdot\frac{x^{-9}}{-9} + \frac{3x^5}{5} + \pi\cdot x + C$

$= \boxed{6x^{2/3} + \frac{5}{27\cdot x^9} + \frac{3x^5}{5} + \pi x + C}$

(b) $\int \frac{-x^2+10}{\left(x^3-30x+1\right)^3}\,dx$

divide by (-3) & multiply inside by -3

$= \frac{1}{-3}\int (x^3-30x+1)^{-3}(-3)(-x^2+10)\,dx = -\frac{1}{3}\int u^{-3}\cdot du$

$\boxed{\begin{aligned} u &= x^3-30x+1 \\ du &= (3x^2-30)\cdot dx \\ &= 3(x^2-10)\,dx \end{aligned}}$

$= -\frac{1}{3}\cdot\frac{u^{-2}}{-2} + C = \boxed{\frac{1}{6\cdot(x^3-30x+1)^2} + C}$

(c) Evaluate the definite integral $\int_0^1 x^2\left(\sqrt{2x^3+2}\right)dx$. **Round your answer to 3 significant digits.**

$I = \frac{1}{6}\int_0^1 (2x^3+2)^{\frac{1}{2}}(6)x^2\cdot dx = \frac{1}{6}\cdot\int_2^4 u^{1/2}\cdot du$

$\boxed{\begin{aligned} u &= 2x^3+2 \\ du &= 6x^2\cdot dx \\ x=0 &\Rightarrow u=2 \\ x=1 &\Rightarrow u=4 \end{aligned}}$

$= \frac{1}{6}\cdot\frac{u^{3/2}}{3/2}\Big|_2^4 = \frac{1}{9}\cdot\left(4^{3/2} - 2^{3/2}\right)$

$= \boxed{0.575}$

5. Find the equation of the curve whose slope is $\frac{1}{\sqrt{2x-1}}$ and that passes through the point $(1,-2)$.

Given $\frac{dy}{dx} = \frac{1}{\sqrt{2x-1}} \Rightarrow y = \int \frac{1}{\sqrt{2x-1}} \cdot dx$

Subst $\boxed{\begin{array}{l} u = 2x-1 \\ du = 2dx \end{array}}$

$y = \frac{1}{2}\int (2x-1)^{-\frac{1}{2}} dx \cdot (2)$

$\Rightarrow y = \frac{1}{2} \cdot \int u^{-\frac{1}{2}} \cdot du$

$y = \frac{1}{2} \cdot \frac{u^{1/2}}{1/2} + C \Rightarrow \boxed{y = \sqrt{2x-1} + C}$

Find C

Subst $x=1$, $y=-2$

$\Rightarrow -2 = 1 + C \Rightarrow \boxed{C=-3}$

$\Rightarrow \boxed{y = \sqrt{2x-1} - 3}$

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6. An object is accelerating according to $a(t) = 1 - 2(\sqrt{t})$ (m/s^2). **Determine the distance that the object traveled after 16 seconds,** given that the object has an initial displacement is 0 m and the object has an initial velocity 10 m/s. **Round answer to 3 significant digits.**

$\boxed{\begin{array}{l} t=0 \\ v=10 \\ s=0 \end{array}}$

$a(t) = 1 - 2\sqrt{t} \Rightarrow v(t) = \int (1 - 2 \cdot t^{1/2}) dt \Rightarrow v(t) = t - 2 \cdot \frac{t^{3/2}}{3/2} + C$

Use $v(t) = \int a(t)\, dt$

$\Rightarrow v(t) = t - \frac{4}{3} \cdot t^{3/2} + C$

Find C

$t=0$, $v=10 \Rightarrow \boxed{C=10}$

velocity of object $\Rightarrow \boxed{v(t) = t - \frac{4}{3} t^{3/2} + 10}$ $\left(\frac{m}{s}\right)$

Displacement of object

$s(t) = \int v(t)\, dt = \int \left(t - \frac{4}{3} t^{3/2} + 10\right) dt = \frac{t^2}{2} - \frac{4}{3} \cdot \frac{t^{5/2}}{5/2} + 10t + D$

Find D

Use $t=0$, $s=0 \Rightarrow \boxed{D=0}$

$\Rightarrow s(t) = \frac{t^2}{2} - \frac{8}{15} t^{5/2} + 10t + D \Rightarrow \boxed{s(t) = \frac{t^2}{2} - \frac{8}{15} t^{5/2} + 10t}$

$\boxed{s(16) = -258 \text{ meters}}$

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7. Given the current passing through a 12 mH inductor is given by $i(t) = 50e^{-25t} (A)$, find the voltage across the inductor $V_L(t)$ at $t = 2.35$ ms. **Round your answer to 3 significant digits.**

convert to seconds

$t = 2.35$ ms $= 0.00235$ sec

$\boxed{\begin{array}{l} \boxed{i(t) = 50 \cdot e^{-25t}} \\ L = 12 \text{ mH} \end{array}}$

Find $V_L(0.00235) = ?$

$V_L = L \cdot i'(t) = 12 \cdot (10^{-3}) \cdot 50 \cdot e^{-25t} \cdot (-25)$

$V_L(t) = -15 \cdot e^{-25t}$

$V_L(0.00235) = -15 \cdot e^{-25(0.00235)} = \boxed{-14.1 \text{ V}}$

6. An object has a velocity given by $v(t) = 2\left(\sqrt{4t+1}\right)$ $\left(m/s^2\right)$.

(a) Determine the distance that the object traveled after 4 sec, given that the object has an initial displacement of 5 m. $\boxed{s(0) = 5\ m}$

(b) Determine the acceleration of the object after 0.04 sec.

Round answers to 3 significant digits.

(a) $v(t) = 2\sqrt{4t+1} \rightarrow s(t) = \int v(t)\cdot dt = \frac{2}{4}\cdot\int (4t+1)^{1/2}\,dt\,(4)$

$\boxed{\begin{aligned} u &= 4t+1 \\ du &= 4\cdot dt \end{aligned}}$

$s(t) = \frac{1}{2}\cdot\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$

$\Rightarrow s(t) = \frac{1}{3}\cdot(4t+1)^{3/2} + C$

Find C : $\boxed{\begin{aligned} t&=0 \\ s&=5 \end{aligned}}$ $\Rightarrow$ $\rightarrow$ $5 = \frac{1}{3}\cdot\underbrace{(4\cdot 0+1)}_{1}^{3/2} + C \Rightarrow C = 5 - \frac{1}{3} = \boxed{\frac{14}{3}}$

Ans (a) $\boxed{s(t) = \frac{1}{3}(4t+1)^{3/2} + \frac{14}{3}\quad (m)}$ $\Rightarrow s(4) = \frac{1}{3}\cdot(4\cdot 4+1)^{3/2} + \frac{14}{3} = \boxed{28.0\ m}$

(b) Acceleration: $a(0.04) = ?$

$a(t) = v'(t) = \frac{d}{dt}\left[2\cdot(4t+1)^{1/2}\right] = \cancel{2}\left(\frac{1}{\cancel{2}}\right)(4t+1)^{-1/2}\cdot 4$

$\boxed{a(t) = \frac{4}{\sqrt{4t+1}}}$ $\frac{m}{s^2}$ $\Rightarrow a(0.04) = \frac{4}{\sqrt{4(0.04)+1}} = \boxed{3.71\ \frac{m}{s^2}}$

8. Find the area between the curves: $\begin{cases} y = x^2 - 2x - 1 \\ y = -x + 5 \end{cases}$. ***Show work for full marks!!***

y intercept $(0,-1)$

(a) Find the points of intersection between the two curves.
(b) Sketch the two curves and **highlight the area** between the two curves.
(c) Use a definite integral to determine the exact area of the shaded region between the 2 curves above (i.e. using integration). Express answer in exact form and rounded to 3 significant digits.

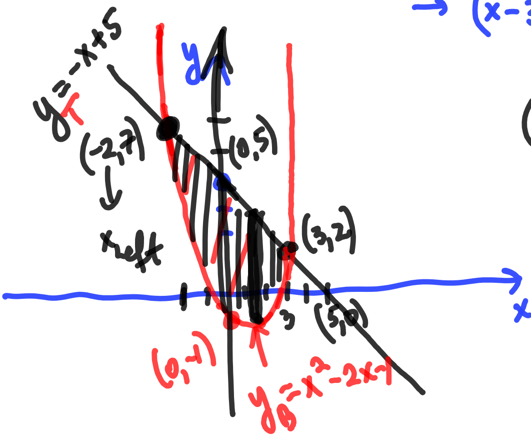
(a) $\begin{cases} y = x^2 - 2x - 1 \\ y = -x + 5 \end{cases} \rightarrow x^2 - 2x - 1 = -x + 5$

$\rightarrow x^2 - x - 6 = 0$

$\rightarrow (x-3)\cdot(x+2) = 0$

$\boxed{\begin{matrix} x = -2 \\ y = 2 + 5 \end{matrix}}$ & $\boxed{\begin{matrix} x = 3 \\ y = -3 + 5 \end{matrix}}$

$\boxed{(-2, 7)}$ $\boxed{(3, 2)}$



(c) $A = \int_{x_{Left}}^{x_{Right}} (y_T - y_B)\cdot dx = \int_{-2}^{3} \left[\underbrace{-x+5-(x^2-2x-1)}_{-x+5-x^2+2x+1}\right]\cdot dx$

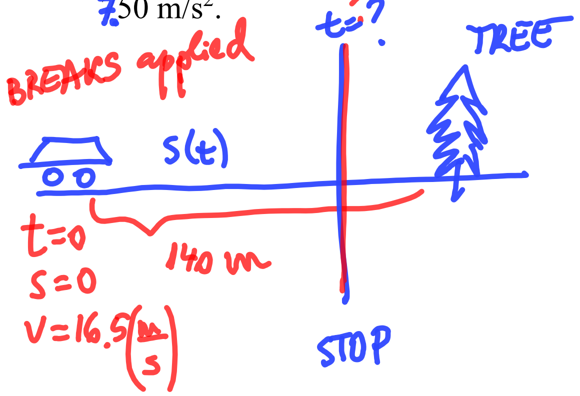
$y_T \rightarrow -x+5$, $y_B \rightarrow x^2-2x-1$

$= \int_{-2}^{3} (-x^2 + x + 6)\cdot dx$

$\left(-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right)\Big|_{-2}^{3} = -\frac{3^3}{3} + \frac{3^2}{2} + 6(3) - \left(-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 6(-2)\right) = \frac{27}{2} - \left(-\frac{22}{3}\right)$

$= \boxed{\frac{125}{6}}$ OR 20.8 unit^2

9. A car is headed directly towards a tree. If the car is 140 m away from the tree and travelling at 16.5 m/s when breaks are applied, will the car hit the tree? Assume the car was decelerating at 7.50 m/s^2.



$a = -7.50 \left(\frac{m}{s^2}\right)$

The car stops when $\boxed{v(t) = 0}$ ← Find t s.t. $\boxed{v(t) = 0}$

$v(t) = \int a \cdot dt = \int -7.50 \cdot dt = -7.50 \cdot t + C \Rightarrow v(t) = -7.50t + C$

$t = 0$
$v = 16.5$

$16.5 = -7.50(0) + C$

$\boxed{C = 16.5}$

velocity: $\boxed{v(t) = -7.50t + 16.5 \quad \left(\frac{m}{s}\right)}$

Find $t = ?$ s.t $\boxed{v(t) = 0}$ $\Rightarrow$ $-7.50 \cdot t + 16.5 = 0 \Rightarrow t = \frac{16.5}{7.50}$

$\boxed{t = 2.20 \text{ sec}}$

Displacement:

$s(t) = \int v(t) \cdot dt = \int (-7.50t + 16.5) \cdot dt$

$s(t) = -\cancel{7.50} \cdot \frac{t^2}{\cancel{2}} + 16.5t + D \Rightarrow$ $\boxed{s(t) = -3.75t^2 + 16.5t \text{ (m)}}$

$t=0$

$s=0$

$0 = 0 + 0 + D$

$\boxed{D=0}$

$s(2.20) = -3.75(2.2)^2 + 16(2.2)$

$= 18.5 \text{ m} < 140 \text{ m}$

Since displacement is less than 140m

$\Rightarrow$ the car will not hit the tree

10. Find the equation of the **tangent line** to the curve $y = \ln(2x^2 - 1)$ at the point where $x = -1$. Write the equation of the line in the slope-intercept form $y = mx + b$.

$y = \ln(2x^2 - 1)$

Point $x = -1$

$y = \ln(2 \cdot (-1)^2 - 1) = \ln(1) = 0 \Rightarrow \boxed{(-1, 0)}$

Deriv: $\boxed{y'(x) = \frac{4x}{2x^2 - 1}}$

Slope of line tangent to $y = \ln(2x^2 - 1)$ is $m = y'(-1)$

$m = \frac{4(-1)}{2(-1)^2 - 1} = \underset{m}{\boxed{-4}}$

$\Rightarrow$ Equation of tangent line at $(-1, 0)$ is

T: $y = m \cdot x + b$

To find b: substitute $x = -1$, $y = 0$ into (*)

(*) $y = -4 \cdot x + b$

$0 = -4(-1) + b \Rightarrow \boxed{b = -4}$

$\rightarrow$ Eq. of tang. line: $\boxed{y = -4x - 4}$