# Practice Exam #1

**Name:** ____________________________ **Total:** __/36

**Simplify all fractions. All numerical answers must be expressed to 3 sig. dig. unless otherwise specified.**

1. Integrate $y=\int\left(5x^{7/4}-9x-\pi\right)dx$ *[3 marks]*

2. Integrate $y=\int\frac{6x^2+2}{\left(4x^3+4x+7\right)^3}\,dx$ *[3 marks]*

3. Integrate $y=\int\frac{3x^5-9x^{-3}+\sqrt{x}}{4x^2}\,dx$ *[3 marks]*

4. Find the equation of the curve whose slope is $2\left(x^2+1\right)^2$ and passes through $(-1,-2)$

5. At the end of graduation ceremonies, a student throws his cap jubilantly into the air. The cap is thrown **UPWARDS** at 10 m/s from a height of 3 meters. What is the velocity of the cap as it hits the ground ? **Use** $a_g = -9.81\ m/s^2$ ***[5 marks]***

6. A current $i = \frac{t}{\sqrt{t^2+2}}$, in $\mu A$, is sent through an electric dryer circuit containing a previously uncharged $4\mu F$ capacitor. Find the capacitor voltage, $v_C(t)$. ***[4 marks]***

7. Approximate the integral $y = \int_{4.0}^{6.4} \frac{50}{5x+4}\, dx$ using the **Trapezoid Rule** with $n = 4$. Give your final answer to three significant digits. ***[3 marks]***

8. Evaluate $\int_2^5 6\sqrt[4]{x}\,dx$ Show intermediate calculations, as well as final answer. ***[3 marks]***

9. Find the area bounded by $y = 3 - x^2$ and $y = -1$ . Show the sketch with the appropriate area shaded, and significant points found. Then, using definite integrals, find the area. ***[5 marks]***

10. Given $z = 4\sin(2x) - 5x^2y^4 + 3e^{-2y}$ , find the partial derivative $\dfrac{\partial z}{\partial x}$ ***[3 marks]***

# Practice Exam #1 - Detailed Answers

**Name:** ____________________________ **Total:** __/36

**Simplify all fractions. All numerical answers must be expressed to 3 sig. dig. unless otherwise specified.**

1. Integrate $y = \int \left(5x^{7/4} - 9x - \pi\right) dx$ *[3 marks ]*

$$= 5x^{11/4} * \frac{4}{11} - 9x^2 * \frac{1}{2} - \pi x + K$$

$$= \frac{20}{11} x^{11/4} - \frac{9}{2} x^2 - \pi x + K$$

2. Integrate $y = \int \frac{6x^2 + 2}{\left(4x^3 + 4x + 7\right)^3} dx$ *[3 marks ]*

$u = 4x^3 + 4x + 7$

$du = (12x^2 + 4) dx$

$du = 2(6x^2 + 2) dx$

$$\frac{1}{2} = \int (4x^3 + 4x + 7)^{-3} \, 2(6x^2 + 2) dx$$

$$= \frac{1}{2} \int u^{-3} du = \frac{1}{2} u^{-2} * \frac{1}{-2} + K$$

$$= \frac{-1}{4} (4x^3 + 4x + 7)^{-2} + K$$

3. Integrate $y = \int \frac{3x^5 - 9x^{-3} + \sqrt{x}}{4x^2} dx$ *[3 marks ]*

$$= \int \frac{3x^5}{4x^2} - \frac{9x^{-3}}{4x^2} + \frac{1x^{1/2}}{4x^2} dx$$

$$= \frac{3}{4} \int x^3 - \frac{9}{4} \int x^{-5} + \frac{1}{4} \int x^{-3/2} dx$$

$$= \frac{3}{16} x^4 + \frac{9}{16} x^{-4} - \frac{1}{2} x^{-1/2} + K$$

9

4. Find the equation of the curve whose slope is $2\left(x^2+1\right)^2$ and passes through $(-1,-2)$

**[4 marks]**

$$m_{tan} = y' = 2(x^2+1)^2$$
$$= 2(x^4+2x^2+1)$$
$$= 2x^4+4x^2+2$$
$$y = \int(2x^4+4x^2+2)dx$$
$$y = \frac{2}{5}x^5+\frac{4}{3}x^3+2x+K$$
$$-2 = \frac{-2}{5}+\frac{-4}{3}+{}^{-}2+K$$
$$1.7\overline{3} = \frac{26}{15} = K$$
$$\boxed{\therefore y = \frac{2}{5}x^5+\frac{4}{3}x^3+2x+1.73}$$

5. At the end of graduation ceremonies, a student throws his cap jubilantly into the air. The cap is thrown **UPWARDS** at 10 m/s from a height of 3 meters. What is the velocity of the cap as it hits the ground? **Use** $a_g = -9.81\ m/s^2$

**[5 marks]**

S↑ V↑ a↑

$$a = {}^{-}9.81$$
$$v = \int -9.81\,dt$$
$$v = -9.81t + K_1 \qquad \text{at } t=0\ \ v = {}^{+}10\text{m/s}$$
$$10 = 0 + K_1 \qquad K_1 = 10$$
$$\therefore v = -9.81t + 10$$

$$s = \int(-9.81t+10)dt$$
$$s = -4.905t^2+10t+K_2 \qquad \text{at } t=0,\ ht = s = 3$$
$$3 = 0+0+K_2 \qquad K_2 = 3$$
$$s = -4.905t^2+10t+3$$

At ground, $ht = s = 0$ ; find $t$

$$0 = -4.905t^2+10t+3 \quad \text{(Quadratic)} \quad t>0$$
$$t = 2.304 \text{ seconds}$$

$$v\Big|_{t=2.304s} = \boxed{{}^{-}12.6\text{ m/s}}$$

6. A current $i = \frac{t}{\sqrt{t^2+2}}$ , in $\mu A$, is sent through an electric dryer circuit containing a previously uncharged $4\mu F$ capacitor. Find the capacitor voltage, $v_C(t)$. ***[4 marks]***

$$V_c = \frac{1}{C}\int i\,dt = \frac{1\mu A}{4\mu F}\int (t^2+2)^{-1/2}\ t\,dt$$

$$u = t^2+2$$
$$du = 2t\,dt$$

$$= \frac{1}{2}\frac{1}{4}\int (t^2+2)^{-1/2}\ 2t\,dt$$

$$= \frac{1}{8}\int u^{-1/2}\,du = \frac{1}{8}u^{1/2} * \frac{2}{1} + K$$

$$V_C = \frac{1}{4}\sqrt{(t^2+2)} + K$$

$$\boxed{\therefore\ V_c = \frac{1}{4}\sqrt{t^2+2} - 0.354}$$

at $t=0, V_c = 0$

$$0 = \frac{1}{4}\sqrt{2} + K$$

$$-\frac{\sqrt{2}}{4} = K$$

$$-0.354 = K$$

7. Approximate the integral $y = \int_{4.0}^{6.4} \frac{50}{5x+4}\,dx$ using the **Trapezoid Rule** with $n = 4$. Give your final answer to three significant digits. ***[3 marks]***

$$\Delta x = \frac{6.4-4}{4} = 0.6$$

$f(4) = \frac{50}{24} = 2.083\ \ *1$
$f(4.6) = 1.852\ \ *2$
$f(5.2) = 1.667\ \ *2$
$f(5.8) = 1.515\ \ *2$
$f(6.4) = 1.389\ \ *1$

$\Sigma = 13.54$

$$y_{TRAP} = \frac{\Delta x}{2} * \Sigma\text{pattern} = \frac{0.6}{2} * 13.54$$

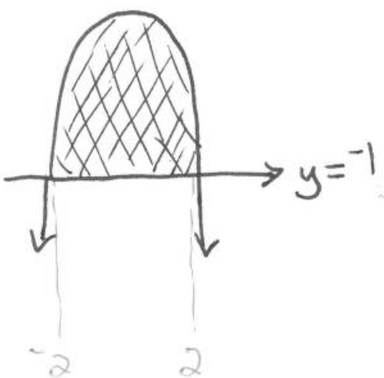
$$= 4.06$$

8. Evaluate $\int_2^5 6\sqrt[4]{x}\,dx$ Show intermediate calculations, as well as final answer. ***[3 marks]***

$$\int_2^5 6x^{1/4}\,dx = 6x^{5/4} * \frac{4}{5}\Big]_2^5 = \frac{24}{5}x^{5/4}\Big]_2^5$$

$$= [35.888] - [11.416] = \boxed{24.5}$$

9. Find the area bounded by $y = 3 - x^2$ and $y = -1$. Show the sketch with the appropriate area shaded, and significant points found. Then, using definite integrals, find the area. ***[5 marks]***




$$y = y$$

$$3 - x^2 = -1$$

$$0 = x^2 - 4$$

$$0 = (x+2)(x-2) \quad x = -2, 2$$

$$A = \int_{-2}^{2} (3-x^2) - (-1)\,dx$$

$$= \int_{-2}^{2} (4-x^2)\,dx = 4x - \frac{1}{3}x^3\Big]_{-2}^{2}$$

$$= \left[\frac{16}{3}\right] - \left[\frac{-16}{3}\right] = \frac{32}{3} = \boxed{10.\overline{6} \text{ sq. units}}$$

10. Given $z = 4\sin(2x) - 5x^2y^4 + 3e^{-2y}$, find the partial derivative $\frac{\partial z}{\partial x}$ ***[3 marks]***

$$z = 4\sin(2x) - \boxed{5y^4}x^2 + \boxed{3e^{-2y}}$$

$$\frac{\partial z}{\partial x} = 8\cos(2x) - 10y^4x + 0$$