MATH 1200 – Calculus   Sample EXAM 1

<u>Part A. Multiple Choice</u> : 6 marks per question.   Clearly indicate your answer.

1. $\frac{d}{dx}\left(-\cos^3 x\right)=$

a) $-3\sin^2 x\cos x$ **b)** $3\cos^2 x\sin x$ **c)** $3\cos^2 x$ **d)** $-3\cos^2 x\sin x$

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2. $\frac{d}{dx}\sin(-x)=$

a) $\cos(-x)$ **b)** $\cos(-1)$ **c)** $\cos x$ **d)** $-\cos(-x)$

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3. $\frac{d}{dx}\left(\ln\frac{x^2-5}{x^2}\right)=$

a) $\frac{1}{1-5x^{-2}}$ **b)** $\frac{-50x^3}{5x^2-1}$ **c)** $\frac{10}{x\left(x^2-5\right)}$ **d)** $\frac{x^2}{x^2-5}$

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4. $\frac{d}{dx}\left(e^{3x^{-1}}\right)=$

a) $-3e^{3x^{-1}}$ **b)** $-3e$ **c)** $-3x^{-2}e^{3x^{-1}}$ **d)** $-3xe^{3x^{-1}}$

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5. If $f(x,y)=xy^2-\cos y$, then $\frac{\partial f}{\partial y}$ is

a) $y^2+2xy+\sin y$ **b)** $\sin y+2xy$ **c)** $\sin y-2y$ **d)** $y^2-2y\sin x$

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6. $\int\frac{x}{\sqrt{x^2+1}}dx=$

a) $\frac{1}{2}\sqrt{x^2+1}+K$ **b)** $\frac{-1}{2}\sqrt{x^2+1}+K$ **c)** $\sqrt{x^2+1}+K$ **d)** $-\sqrt{x^2+1}+K$

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7. If you are 2 km from home, in the positive x-direction, and you begin walking according to a velocity function: $v(t)=-\frac{4}{(t+1)^2}$ km/h , the expression describing your displacement from home at any time, t, in hours, after you start walking is:

a) $\frac{8}{(t+1)^3}$ **b)** $-\frac{4}{(t+1)}-6$ **c)** $\frac{4}{(t+1)}-2$ **d)** $-\frac{8}{t^3}$

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8. If the current flowing through a capacitance, $C=0.2F$, is : $i(t)=t-\frac{t^2}{2}\ A$, then the voltage, $v_c(t)$, across the capacitance, assuming $v_c(0)=5\text{V}$, is:

a) $5\left(\frac{t^2}{2}-\frac{t^3}{6}\right)$ **b)** $5\left(\frac{t^2}{2}-\frac{t^3}{6}\right)+5$ **c)** $5\left(\frac{t^2}{2}-\frac{t^3}{6}+5\right)$ **d)** $\frac{t^2}{2}-\frac{t^3}{6}+5$

Part B. Long Answer Questions

The marks are shown with each question. To obtain marks for a question, the candidate must **show appropriate work; no work = no marks!**

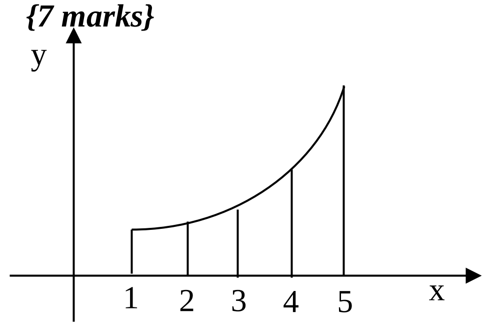
9. Find the equation of the line normal to $y = \dfrac{3}{\sqrt{x^3+1}}$ at $(2,1)$. ***{7 marks}***

10. If a current, $i(t)$, flows through an inductance, L = 0.5 H, and induces a voltage, $v_L(t) = 2\sqrt{t}$ V

    a) find the current as a function of time, if $i(0) = 0.5$ A ***{7 marks}***

    b) What is the current at t = 0.25s ? ***{4 marks}***

11. Find the approximate area under the curve $y = \dfrac{1}{4}x^2 + 1$ using rectangles. Assume the domain of y is $1 \le x \le 5$, and $\Delta x = 1$. Use the "left" height of each rectangle. ***{7 marks}***



12. Find the equation of the curve which passes through $(4,6)$ if its slope is given by $(2x+1)^{-1/2}$. ***{7 marks}***

13. Given Kirchhoff's voltage law for a series RLC circuit:

    Total voltage: $v(t) = v_R(t) + v_L(t) + v_C(t)$, express $v(t)$ in terms of $i(t)$, $\dfrac{di(t)}{dt}$, and $\int i(t)dt$ ***{6 marks}***

14. Evaluate the following definite integrals. **You must correctly show the integration and variable substitution processes to obtain marks.** Calculator answers are not acceptable.

    a) $\int_1^2 4x\left(x^2+2\right)^3 dx$ ***{7 marks}***

    b) $\int_{-4}^{-3} \left(x - x^2\right)dx$ ***{7 marks}***

Sample Exam 1

MATH 1200 – **Calculus**

Name: ______________________________

Class: __________

Pages: 4 Questions: 14 Total marks: ___/ 100

Part A. Multiple Choice : 6 marks per question. Clearly indicate your answer.

1. $\frac{d}{dx}\left(-\cos^3 x\right) =$ $-3\cos^2 x\,(-\sin x) = 3\cos^2 x \sin x$

a) $-3\sin^2 x\cos x$ **b)** $3\cos^2 x\sin x$ **c)** $3\cos^2 x$ **d)** $-3\cos^2 x\sin x$

2. $\frac{d}{dx}\sin(-x) =$ $\cos(-x)\cdot(-1) = -\cos(-x)$

a) $\cos(-x)$ **b)** $\cos(-1)$ **c)** $\cos x$ **d)** $-\cos(-x)$

3. $\frac{d}{dx}\left(\ln\frac{x^2-5}{x^2}\right) =$ $\frac{1}{\frac{x^2-5}{x^2}}\left[\frac{x^2(2x)-(x^2-5)2x}{x^4}\right] = \frac{10}{x(x^2-5)}$

a) $\frac{1}{1-5x^{-2}}$ **b)** $\frac{-50x^3}{5x^2-1}$ **c)** $\frac{10}{x(x^2-5)}$ **d)** $\frac{x^2}{x^2-5}$

4. $\frac{d}{dx}\left(e^{3x^{-1}}\right) =$ $e^{3x^{-1}} = (3(-1)x^{-2})$

a) $-3e^{3x^{-1}}$ **b)** $-3e$ **c)** $-3x^{-2}e^{3x^{-1}}$ **d)** $-3xe^{3x^{-1}}$

5. If $f(x,y) = xy^2 - \cos y$, then $\frac{\partial f}{\partial y}$ is $= x(2y)-(-\sin y) = 2xy + \sin y$

a) $y^2+2xy+\sin y$ **b)** $\sin y + 2xy$ **c)** $\sin y - 2y$ **d)** $y^2 - 2y\sin x$

6. $\int \frac{x}{\sqrt{x^2+1}}\,dx = \int x(x^2+1)^{-\frac{1}{2}}dx = \int x\,u^{-\frac{1}{2}}\frac{du}{2x} = \frac{1}{2}\int u^{-\frac{1}{2}}du =$

a) $\frac{1}{2}\sqrt{x^2+1}+K$ b) $\frac{-1}{2}\sqrt{x^2+1}+K$ c) $\sqrt{x^2+1}+K$ d) $-\sqrt{x^2+1}+K$

$= \frac{1}{2}\cdot\frac{u^{1/2}}{\frac{1}{2}} + K$

$= \sqrt{x^2+1} + K$

7. If you are 2 km from home, in the positive x-direction, and you begin walking according to a velocity function: $v(t) = -\frac{4}{(t+1)^2}$ km/h , the expression describing your displacement from home at any time, t, in hours, after you start walking is:

a) $\frac{8}{(t+1)^3}$ b) $-\frac{4}{(t+1)}-6$ c) $\frac{4}{(t+1)}-2$ d) $-\frac{8}{t^3}$

$s(t) = -4\int (t+1)^{-2}dt = \frac{-4(t+1)^{-1}}{-1} + C = \frac{4}{t+1} + C$

@ $t=0$

$C = -2$

8. If the current flowing though a capacitance, $C = 0.2F$, is : $i(t) = t - \frac{t^2}{2}\ A$, then the voltage, $v_c(t)$, across the capacitance, assuming $v_c(0) = 5\,\text{V}$, is:

a) $5\left(\frac{t^2}{2}-\frac{t^3}{6}\right)$ b) $5\left(\frac{t^2}{2}-\frac{t^3}{6}\right)+5$ c) $5\left(\frac{t^2}{2}-\frac{t^3}{6}+5\right)$ d) $\frac{t^2}{2}-\frac{t^3}{6}+5$

$V_c = \frac{1}{C}\int i_c(dt) = \frac{1}{0.2}\int\left(t-\frac{t^2}{2}\right)dt = 5\left(\frac{t^2}{2}-\frac{t^3}{6}\right)+K$

@ $K=0$ $V_c = 5$

$V_c(t) = 5\left(\frac{t^2}{2}-\frac{t^3}{6}\right)+5$

Part B. Long Answer Questions

The marks are shown with each question. To obtain marks for a question, the candidate must **show appropriate work; no work = no marks!**

9. Find the equation of the line normal to $y = \frac{3}{\sqrt{x^3+1}}$ at $(2,1)$. *{7 marks}*

$y = 3(x^3+1)^{-1/2}$

$\frac{dy}{dx} = 3\left(-\frac{1}{2}\right)(x^3+1)^{-3/2}(3x^2)$

$y' = -\frac{9}{2}x^2(x^3+1)^{-3/2}$

$y'\Big|_{x=2} = -0.6667 \Rightarrow m_1$

$m_2 = -\frac{1}{m_1} = -\frac{1}{-0.667} = 1.5$

$y - y_1 = m(x-x_1)$

$y - 1 = \frac{3}{2}(x-2)$

$\boxed{y = \frac{3}{2}x - 2}$

10. If a current, $i(t)$, flows through an inductance, L = 0.5 H, and induces a voltage, $v_L(t) = 2\sqrt{t}$ V

a) find the current as a function of time, if $i(0) = 0.5$ A *{7 marks}*

$$V_L = L \cdot \frac{di}{dt} \quad \therefore \quad i_L = \frac{1}{L}\int v_L\,dt = \frac{1}{0.5}\int 2t^{1/2}\,dt = 4\int t^{1/2}\,dt$$

$$i_L = 4 \cdot \frac{t^{3/2}}{\frac{3}{2}} + C = \frac{8}{3}t^{3/2} + C$$

Evaluate c

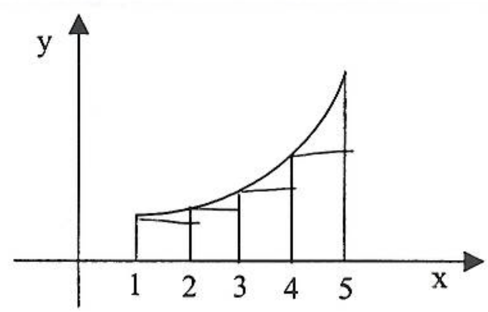
$$0.5 = \frac{8}{3}(0)^{3/2} + c \Rightarrow c = 0.5 \Rightarrow \boxed{i_L = \frac{8}{3}t^{3/2} + 0.5}$$

b) What is the current at t = 0.25s ? *{4 marks}*

$$i(0.25) = \frac{8}{3}(0.25)^{3/2} + 0.5 = 0.833 \text{ Amps}$$

11. Find the approximate area under the curve $y = \frac{1}{4}x^2 + 1$ using rectangles. Assume the domain of y is $1 \le x \le 5$, and $\Delta x = 1$. Use the "left" height of each rectangle. *{7 marks}*

n	x(n)	y(x_n)
1	1	1.25
2	2	2
3	3	3.25
4	4	5



$$\therefore A \approx (1.25 + 2 + 3.25 + 5) = \underline{11.5 \text{ units}^2}$$

12. Find the equation of the curve which passes through $(4,6)$ if its slope is given by $(2x+1)^{-1/2}$ *{7 marks}*

$$y' = (2x+1)^{-\frac{1}{2}}$$

$$y = \int (2x+1)^{-\frac{1}{2}}\,dx$$

$$y = (2x+1)^{\frac{1}{2}} + C$$

use the point (4, 6)

$$6 = (2 \cdot 4 + 1)^{1/2} + C$$

$$\boxed{C = 3}$$

$$\therefore \boxed{y = \sqrt{2x+1} + 3}$$

13. Given Kirchhoff's voltage law for a series RLC circuit:

Total voltage: $v(t) = v_R(t) + v_L(t) + v_C(t)$

Express $v(t)$ in terms of $i(t)$, $\frac{di(t)}{dt}$, and $\int i(t)dt$ **{6 marks}**

$KVL \Rightarrow v(t) = V_R(t) + V_L(t) + V_C(t)$

$$\therefore \boxed{v(t) = iR + L \cdot i' + \frac{1}{C}\int i\, dt}$$

14. Evaluate the following definite integrals. **You must correctly show the integration and variable substitution processes to obtain marks.** Calculator answers are not acceptable.

a) $\int_1^2 4x(x^2+2)^3 dx$ = **{7 marks}**

Aside
$u = x^2 + 2$
$du = 2x\,dx$
$\frac{du}{2x} = dx$
$x = 1 \Rightarrow u = 3$
$x = 2 \Rightarrow u = 6$

$$= \int_3^6 4\cancel{x} \cdot u^3 \cdot \frac{du}{2\cancel{x}} = 2\int_3^6 u^3\, du$$

$$= 2 \cdot \frac{u^4}{4}\Big|_3^6 = \frac{1}{2}\left[6^4 - 3^4\right]$$

$$= \frac{1}{2}\left[1296 - 81\right] = \frac{1215}{2} = \underline{608}$$

b) $\int_{-4}^{-3} (x - x^2)dx$ **{7 marks}**

$$= \int_{-4}^{-3} x\,dx - \int_{-4}^{-3} x^2 dx = \frac{x^2}{2}\Big|_{-4}^{-3} - \frac{x^3}{3}\Big|_{-4}^{-3}$$

$$= \frac{1}{2}\left[(-3)^2 - (-4)^2\right] - \frac{1}{3}\left[(-3)^3 - (-4)^3\right] = \frac{-7}{2} - \frac{1}{3}\left[-27 + 64\right] = \underline{\underline{15..}}$$